Bound-State Effects on Kinematical Distributions of Top-Quarks at Hadron Colliders

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based on arXiv:1007.0075

in collaboration with
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HP².3rd

The 3rd International Workshop on
High Precision for Hard Processes at the LHC

Florence, Italy, 14-17 September 2010

Outline :

1. Introduction
2. Bound-state effects on ttbar production at Hadron colliders
3. Differential cross-section / Even Generation
4. Summary

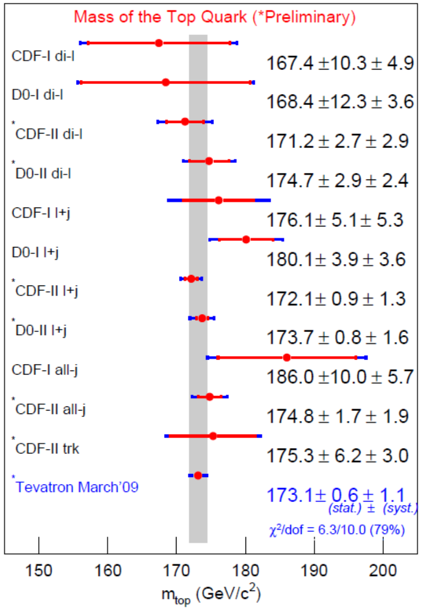
1. Introduction

- Properties of top-quark

  - Mass measurement (CDF and D0 combined)

$$m_t = 173.1 \pm 0.6(\text{stat.}) \pm 1.1(\text{syst.})\ [\text{GeV}]$$

arXiv:0903.2503



  - Decay-width (SM) : $$\Gamma_t \simeq \frac{G_F m_t^3}{8\sqrt{2}\pi}|V_{tb}|^2 \sim 1.5\,[\text{GeV}]$$

$$\Gamma_t \gg \Lambda_{\text{QCD}}$$

a unique property : top-quarks decay before hadronization,
spin information is preserved in decay products

  - Cross-Section at the LHC

$$\sigma_{tt}(\text{LHC14TeV}) \sim 800\,\text{pb}$$

$$\sim 8\text{M}/\text{year}\,(\text{L} = 10\text{fb}^{-1})$$

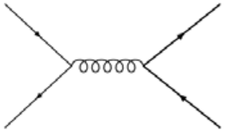
LHC = top factory,
detail study can be possible

1. Introduction

- Pair production at hadron colliders

Tevatron LHC

$$q\bar{q} \to t\bar{t}$$

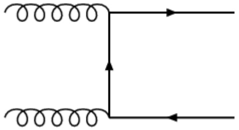


Color: Octet
|J|=1

85% 10%

.. ..

$$gg \to t\bar{t}$$



Color: Singlet, Octet
|J|=0,1,2,,,

15% 90%

- Hadronic cross-section

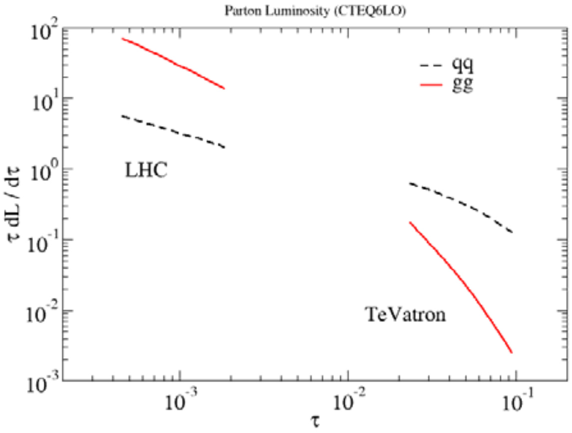
i=qq,gg,,,

$$\sigma_{t\bar{t}}(s) = \sum_i \int d\tau \frac{dL_i}{d\tau}(\tau)\hat{\sigma}_i(\hat{s} = \tau s)$$

partonic cross-section

partonic luminosity

$$\frac{dL_i}{d\tau}(\tau, \mu_F) = \int dx_1 dx_2 \delta(\tau - x_1 x_2) f_a(x_1, \mu_F) f_b(x_2, \mu_F)$$



1. Introduction

(apologize for the incomplete list)

- Theoretical efforts on the top-quark pair production at hadron colliders

  - NLO QCD corrections : Dawson,Ellis,Nason('88), Beenakker etal.('90)
    (analytic): Cazkon,Mitov('08)
  - Threshold resummation [(N)NLL] : Kidonakis Sterman('97),Bonciani etal.('98),,,
    Cacciari etal('08), Moch,Uwer('08), Kidonakis,Vogt('08)
  - Building blocks for NNLO correction : Korner etal('06),Dittmaier etal('07),Cazkon etal('07)
  - 1-loop electroweak correction : Bernreuther,Fuecker,Si('06),Kuhn,Scharf,Uwer('06),
    Moretti,Nolten,Ross('06)
  - NLO correction to the productions and decays : Bernreuther etal('10), Melnikov,Schulze('09)
  - Bound-state effects (Coulomb summation) : Catani,Mangano,Nason,Trentadue('96),
    Hagiwara,Sumino,HY('08), Kiyo etal('08),
    Sumino,HY('10)

1. Introduction

- High Precision top-quark physics at Hadron colliders
  - Precise mass (and width) determination :

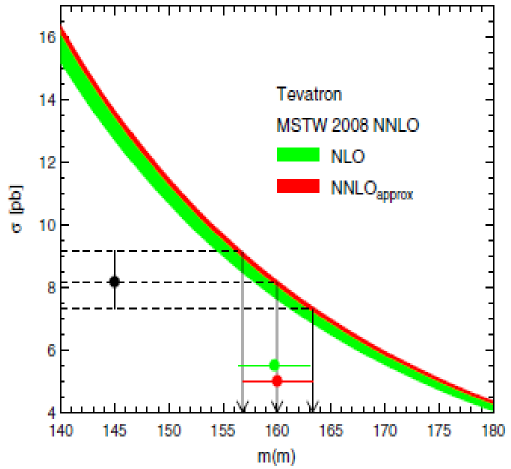
important : definition of the mass in an infra-safe manner

Langenfeld,Moch,Uwer('09) determination of the $\overline{\text{MS}}$ mass

$$\bar{m}(\bar{m}) = 160.0^{+3.3}_{-3.2}\,[\text{GeV}] \quad \text{(NNLO)}$$

what about the threshold mass ?

PHYSICAL REVIEW D 80, 054009 (2009)



(b)

- Spin correlations :

Bernreuther etal('04,'10), Mahlon,Parke('10)

large spin correlation in the events with low m_{tt}

⇒ Threshold events contains rich information on the top-quark precision physics

2. Bound-state effects at Hadron colliders

- NLO correction near partnic threshold : $\left(\beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}} \to 1\right)$

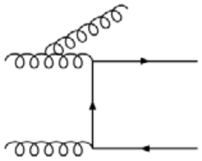
$$\hat{\sigma}_i^{c,(1)} \sim \hat{\sigma}_i^{c,(0)} \left[\underline{A_i \ln^2\left(8\beta^2\right) + B_i^{(c)} \ln\left(8\beta^2\right)} + \underline{C_i^{(c)} \frac{\pi^2}{\beta}} + \underline{D_i^{(c)}} + \mathcal{O}(\beta) \right]$$

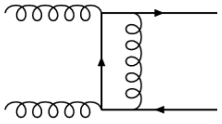
i=qq,gg

Threshold logs: emission of soft and/or collinear gluon in initial-state and final-state

Coulomb singularity: Coulomb gluon exchange between t and t-bar

Hard correction: process dependent





- Factorization of each contributions :

Beneke,Falgari,Schwinn ('09)

2. Bound-state effects at Hadron colliders

☐ Coulomb corrections to all-orders

color-factor

$$\begin{cases} \text{singlet} & C^{(1)} = -C_F \\ \text{octet} & C^{(8)} = C_A/2 - C_F \end{cases}$$

- Coulomb singularity $\propto C^{(c)}\dfrac{\alpha_s}{\beta}$ $\qquad \mathcal{O}(1)$ for $\beta \simeq \alpha_s$

- Summation of ladder diagrams = Sommerfeld factor

Sommerfeld,Sakharov (QED)

$$S(z) = \frac{z}{1-\exp[z]}, \quad z = C^{(c)}\pi\alpha_s/\beta$$

- Green's function formalism (NRQCD)

Fadin,Khoze('87)

$$\left[(E+i\Gamma_t) - \left\{-\frac{\nabla^2}{m_t} + V^{(c)}_{QCD}(r)\right\}\right] G^{(c)}(E,\vec{x}) = \delta^3(\vec{x})$$

Schrodinger's Eq.

finite width effects by complex energy

$$G(E,\vec{x}) = \sum_n \frac{\Psi_n(\vec{x})\Psi_n(0)^*}{E - E_n + i\Gamma_n/2} + \text{continuum}$$

Optical theorem

$$\sigma_{\text{tot}}(s) \propto \text{Im}[G(E,\vec{r}=\vec{0})]$$

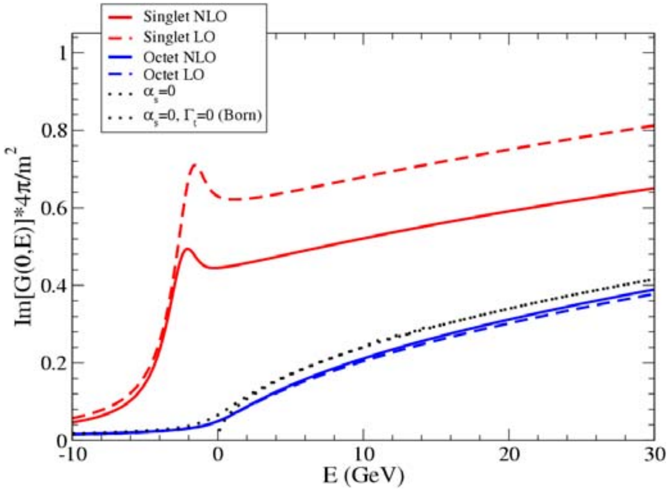
2. Bound-state effects at Hadron colliders

QCD potential

- Perturbative QCD potential (NLO), since an IR cut-off by $r \lesssim \frac{1}{\Gamma_t}$

$$V^{(c)}_{\text{QCD}}(r) = C^{(c)}\frac{\alpha_s(\mu_B)}{r} \times \left[1 + \frac{\alpha_s}{\pi}v^{(c)}_1(r) + \cdots\right]$$

$$\begin{cases} \text{singlet} & C^{(1)} = -4/3 \\ \text{octet} & C^{(8)} = 1/6 \end{cases}$$



- Scales : $m_t \gg \mu_B > E_B \simeq \Gamma_t \gg \Lambda_{\text{QCD}}$
  - Binding energy : $E_B \simeq m_t\alpha_s^2 \simeq 2\text{GeV}$ — If $\Gamma_t > E_B$, top-quark decays before bound-state formation
  - Bohr radius : $\mu_B \simeq m_t\alpha_s \simeq 20-30\text{GeV}$ — typical momentum of Coulomb gluon

2. Bound-state effects at Hadron colliders

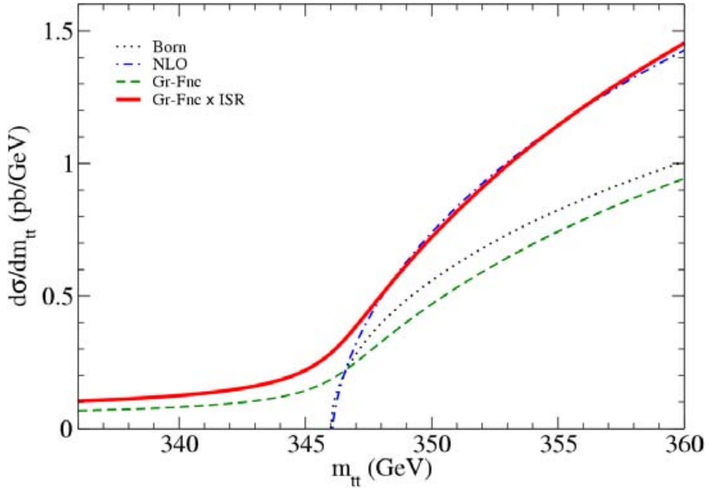
m_t=173 GeV, Γ_t=1.5 GeV, CTEQ6M

- ttbar invariant-mass distributions

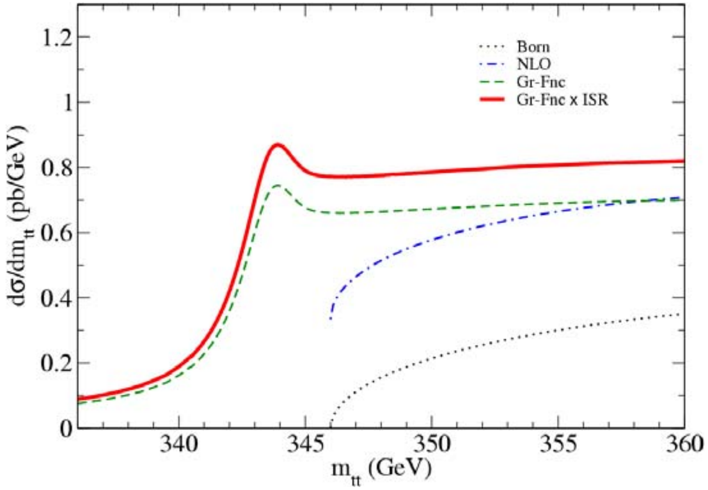
Black : Born
Blue : O(as) corr. (NLO)
Green : Gr-Fnc. without ISR
Red : Gr-Fnc. with ISR

ISR : up to O(a_s) (soft/collinear)

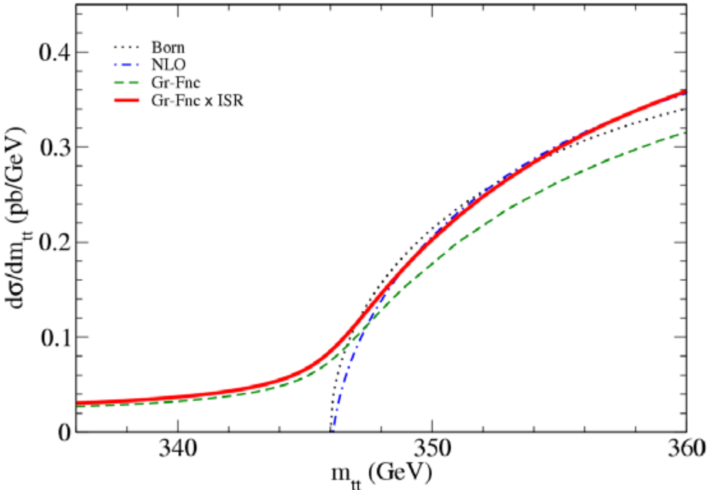
gg->tt, color-octet



gg->tt, color-singlet



qq->tt, color-octet



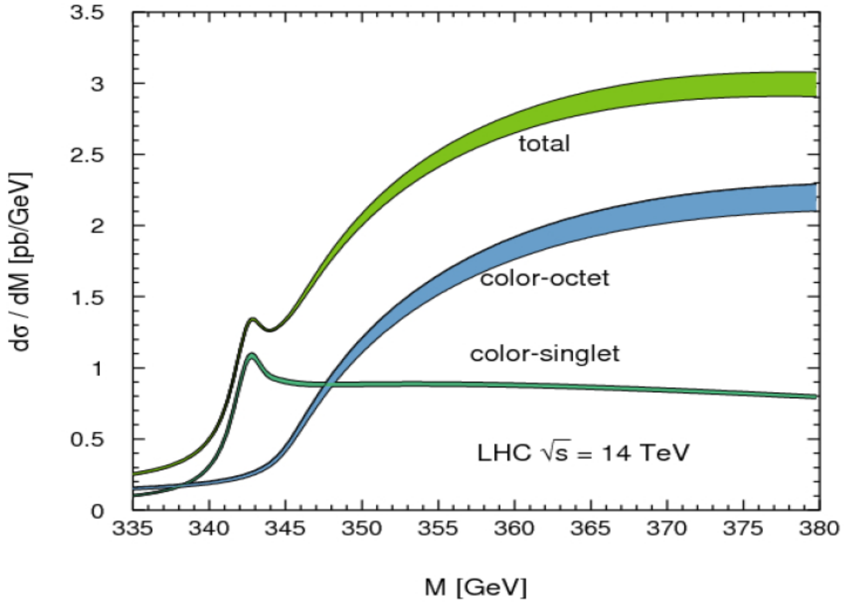
2. Bound-state effects at Hadron colliders

- Kiyo,Kuhn,Moch,Steinhauser,Uwer ('08)

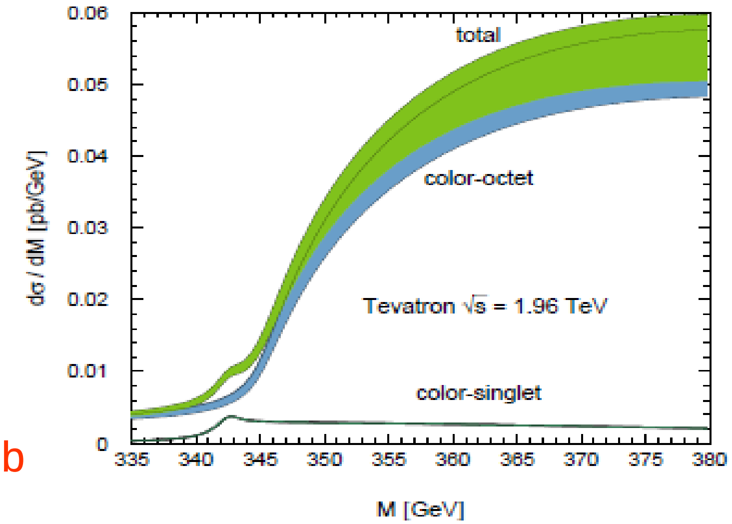
  + full O(a_s) ISR (non-singular term)

  + Resummed ISR (NLL)

but mostly on the normalization of the distribution



- In total at the LHC :
  - BS effects deform the invariant-mass distribution near threshold
  - form a broad resonance peak below threshold (observable in principal)
  - Enhance the total cross-section by O(1%), ~ 10 pb



3. Differential Cross-section

Sumino,HY arXiv:1007.0075

☐ Coulomb correction to the differential cross-sections

- Fully differential distributions are useful for simulation studies with kinematical cuts,,,
- Formalism well-developed in e^+e^- collider case Jezabek,Kuhn,Teubner('92)
  Sumino,Fujii,Hagiwara,Murayama,Ng('93)
- Coulomb correction affects the top-quark momentum distributions
  via the momentum-space Green functions

$$\frac{d\sigma}{d^3\vec{p}} \propto |\tilde{G}(E,\vec{p})|^2 \quad \Leftrightarrow \quad \sigma_{\mathrm{tot}} \propto \mathrm{Im}[G(E,\vec{r}=\vec{0})]$$ (optical theorem)

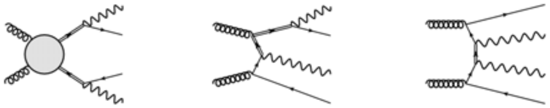
- In contrast to the e+e- case, at hadron colliders;
  - Color of the ttbar pair can be singlet or octet
  - (partonic) collision energy is not fixed
- Take into account the "Leading-order" contribution in both region :
  - Threshold region : $(\alpha_s/\beta)^n$ but not α_s^n, β^n
  - High-energy region : β^n but not α_s^n

- note, $\Gamma_t/m_t \sim \alpha_W \sim \alpha_s^2$

3. Differential Cross-section

- Description
  - Start from Matrix-elements for gg/qq to bWbW process to take into account the off-shellness of top-quarks. (Decay of W-bosons can be also incorporated at ME level)
  - Resonant diagrams and also non-resonant diagrams exist

$$\mathcal{M}^{(c)}(I \to bWbW) = \mathcal{M}^{(c)}_{t\bar{t}} + \mathcal{M}^{(c)}_{\mathrm{nr}}$$



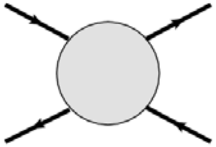
  - A correction factor to the resonant part by the momentum-space Green functions

$$\mathcal{M}^{(c)}_{t\bar{t}} \to \mathcal{M}^{(c)}_{t\bar{t}} \times \frac{\tilde{G}^{(c)}(E+i\Gamma_t,\vec{p})}{\tilde{G}_0(E+i\Gamma_t,\vec{p})}$$

- Two a bit complicated topics in the following pages :
  - Green's functions at high-energy
  - Phase-space suppression effects

3. Differential Cross-section

☐ Green's function which connects smoothly to the high-energy



$$G^{(c)}(E,\vec{p}) = \langle f | \frac{1}{m_{tt} - H + i\Gamma_t} | i \rangle$$

Total energy of the ttb system $m_{tt} = 2m_t + E$

Hamiltonian of the ttb system
$$\begin{aligned} H &= 2\sqrt{\vec{p}^2 + m_t^2} + V(\vec{r}) \\ &\simeq 2m_t + \frac{\vec{p}^2}{m_t} - \frac{\vec{p}^4}{8m_t^3} + \cdots + V(\vec{r}) \end{aligned}$$

higher-order terms are non-negligible at HE

- Our prescription :

Solving the on-shell relation with free Hamiltonian; $m_{tt} = H_0$ ➡ $E + \dfrac{E^2}{4m_t} = \dfrac{\vec{p}^2}{m_t}$

thus, define $E' = E + \dfrac{E^2}{4m_t}$ and $G'^{(c)}(E,\vec{p}) = \langle f | \dfrac{1}{E' - \frac{\vec{p}^2}{m_t} - V(\vec{r}) + i\Gamma_t} | i \rangle$

which has correct pole structure and well-known functional form

3. Differential Cross-section

☐ Phase-space suppression effects

- Reduction of the running decay-width
  due to the suppression of the available phase-space in top decays

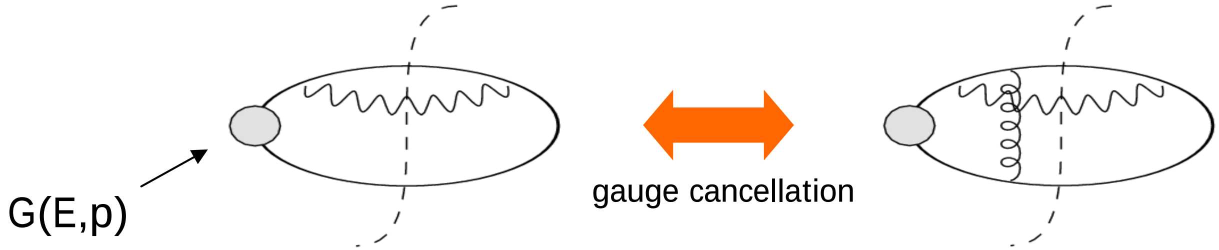
$$\left(\int d\Phi_{bW}\right)^2 |\mathcal{M}_{t\bar{t}}|^2 \propto \frac{\sqrt{s_t}\,\Gamma_t(s_t)}{m_t\Gamma_t}\,\frac{\sqrt{s_{\bar{t}}}\,\Gamma_t(s_{\bar{t}})}{m_t\Gamma_t} \qquad \Gamma_t(s) \propto s^{\frac{3}{2}}$$

near the threshold, $\langle s_t - m_t^2 \rangle \propto -\dfrac{\pi\alpha_s}{\vec{q}^2}$

- However, surprisingly, it is known to cancel with t-bbar (tbar-b) Coulomb int.

  and only the time dilatation effect remains (gauge invariance)

Jezabek,Kuhn,Teubner('92), Sumino etal('93)
Modritsch,Kummer('94)

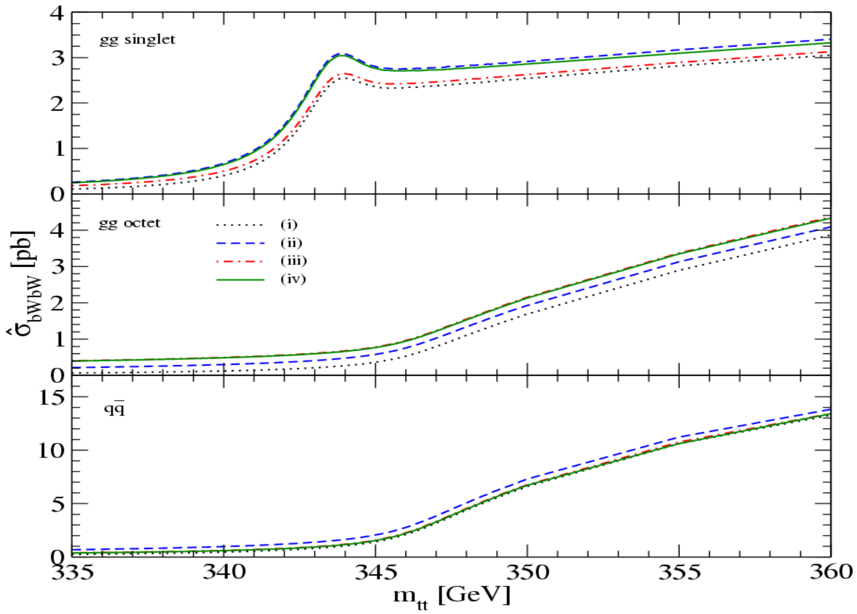


3. Differential Cross-section

- Our prescription = "multiply a factor which cancel this suppression, and add a counter-term in non-res amp"

$$\tilde{\mathcal{M}}_{t\bar{t}} = \mathcal{M}_{t\bar{t},\mathrm{BS}} \times \left[\frac{m_t\Gamma_t}{\sqrt{s_t}\Gamma_t(s_t)}\frac{m_t\Gamma_t}{\sqrt{\bar{s}_t}\Gamma_t(\bar{s}_t)}\right]^{\frac{1}{2}}$$

$$\tilde{\mathcal{M}}_{nr} = \mathcal{M}_{t\bar{t},\mathrm{tree}} \times \left(1 - \left[\frac{m_t\Gamma_t}{\sqrt{s_t}\Gamma_t(s_t)}\frac{m_t\Gamma_t}{\sqrt{\bar{s}_t}\Gamma_t(\bar{s}_t)}\right]^{\frac{1}{2}}\right) + \mathcal{M}_{nr}$$



(i) $|\mathcal{M}_{t\bar{t}}|^2$

(ii) $|\tilde{\mathcal{M}}_{t\bar{t}}|^2$

(iii) $|\mathcal{M}_{t\bar{t}} + \mathcal{M}_{nr}|^2$

(iv) $|\tilde{\mathcal{M}}_{t\bar{t}} + \tilde{\mathcal{M}}_{nr}|^2$

Singlet : (ii) ~ (iv)

Octet : (iii) ~ (iv)

3. Event Generation

Sumino,HY arXiv:1007.0075

☐ Event Generator (Born + Coulomb) :

Specialized to include the LO effects in both threshold and high-energy region

- Full gg/qq→bWbW amplitudes, plus W-decays at the Matrix-Element level
- Color decomposition in gg→bWbW process
- Bound-state correction to the double-resonant amplitudes
- Color-dependent K-factors to reproduce NLO m_{tt} dist. near threshold

- On the other hand : General-purpose Monte-Carlo's :

MadGraph/MadEvent, Sherpa,,, (PYTHIA, HERWIG,,,)
Tree-level, ○non-resonant effects, off-shell effect,,,

MCFM,MC@NLO,,,
NLO, ×non-resonant effects, Breit-Wigner,,,

3. Event Generation

Sumino,HY arXiv:1007.0075

☐ Event Generator (Born + Coulomb) :

- Matrix-Elements : based on **MadGraph/HELAS** code.

  generated by “`pp>(w+>mu+vm)(w->mu-vm~)bb`”

  add color decomposition for gg.

- Green’s Function : pre-tabulated by solving Schrodinger Eq.

  in coordinate-space, then taking Fourier trans.

  NLO QCD potential

- Phase-Space Integration/Event Generation :

  BASES/SPRING, or put them into **MadEvent**

- LHEF interface to parton-shower & hadronization simulators,

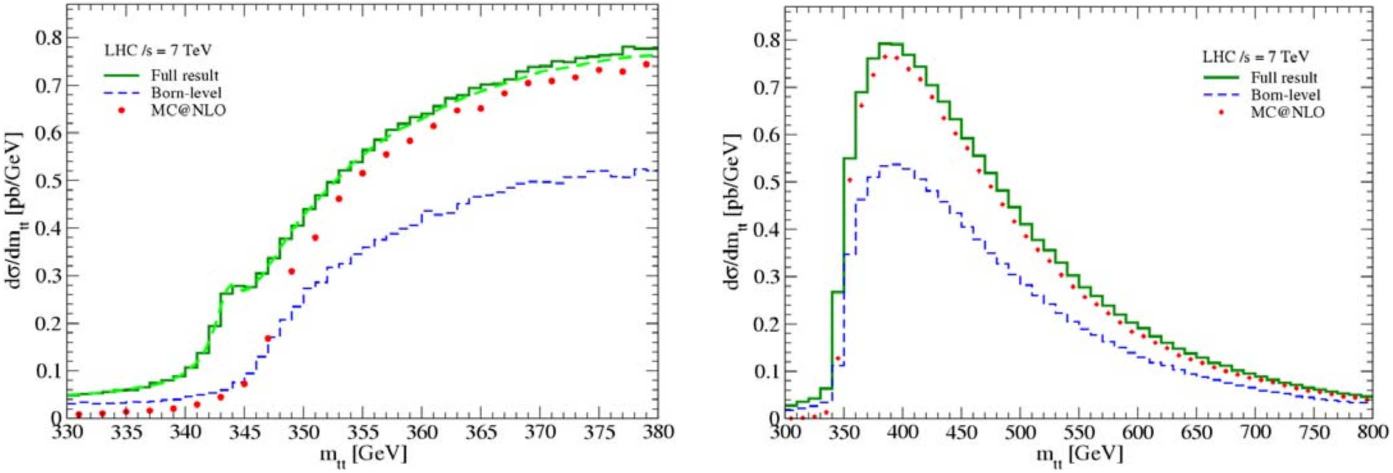
  PYTHIA,HERWIG,,,

available from http://madgraph.kek.jp/~yokoya/TopBS

3. Event Generation

☐ Some Examples (at partonic-level)

(1) ttbrar invariant-mass (m_{tt}) distribution :



- Check with previous results by Hagiwara,Sumino,HY('08) and Kiyo et.al.('08)

- Effectively, well reproduce **MC@NLO** results at large m_{tt} by taking the scales as

$$\mu = m_t \quad (\mu = \sqrt{m_t^2 + p_T^2} \quad \text{in MC@NLO})$$

- The only generator which describes the threshold enhancement and resonance

3. Event Generation

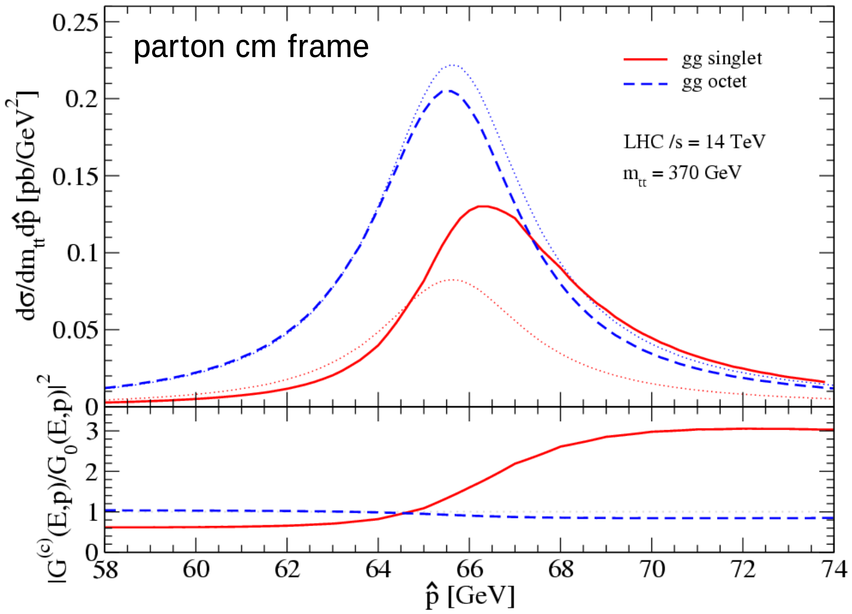
- Some Examples (at partonic-level)

(2) Top-quark momentum distribution ; $\frac{d\sigma}{d|\vec{p}|} \propto |\tilde{G}(E,\vec{p})|^2$

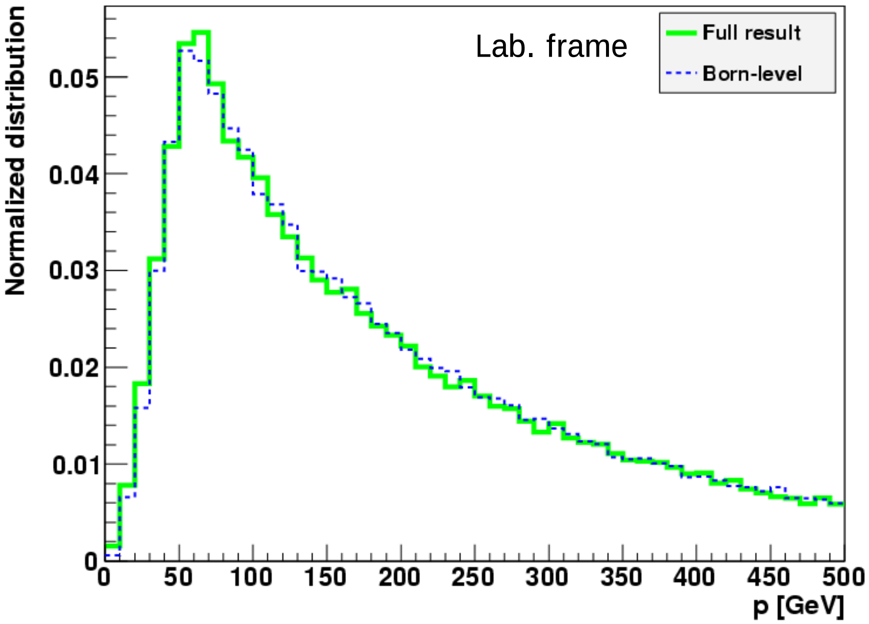
color-singlet : $\delta p > 0$

color-octet : $\delta p < 0$

Fixed m_{tt} (=370 GeV), color individual



m_{tt} integrated-out, color-sum



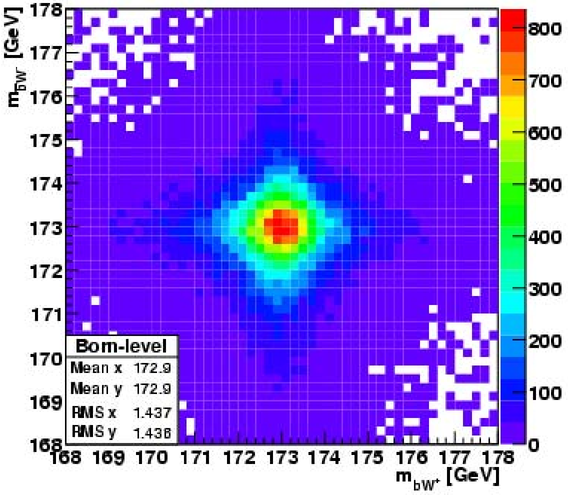
3. Event Generation

☐ Some Examples (at partonic-level)

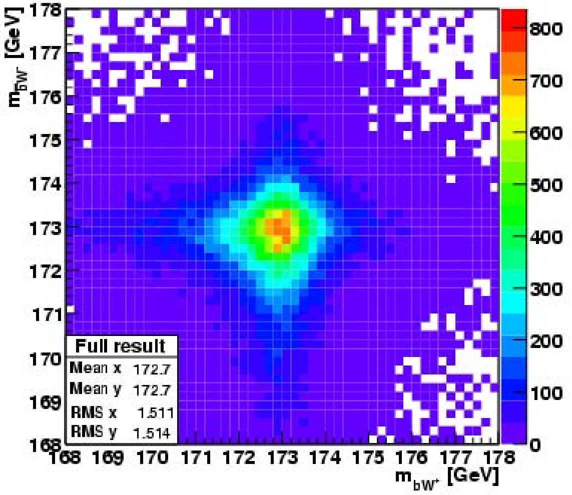
(3) (bW)-(bW) double invariant-mass distribution of top-quarks ; $m_{bW} = (p_b+p_W)^2$

limiting for the events with m_{tt}<370 GeV (10% of total events)

Born



Coulomb



$$\delta m \simeq -200\,[\mathrm{MeV}]$$

correlated deviation : one top-quark is still on-shell,
but the other invariant-mass is reduced

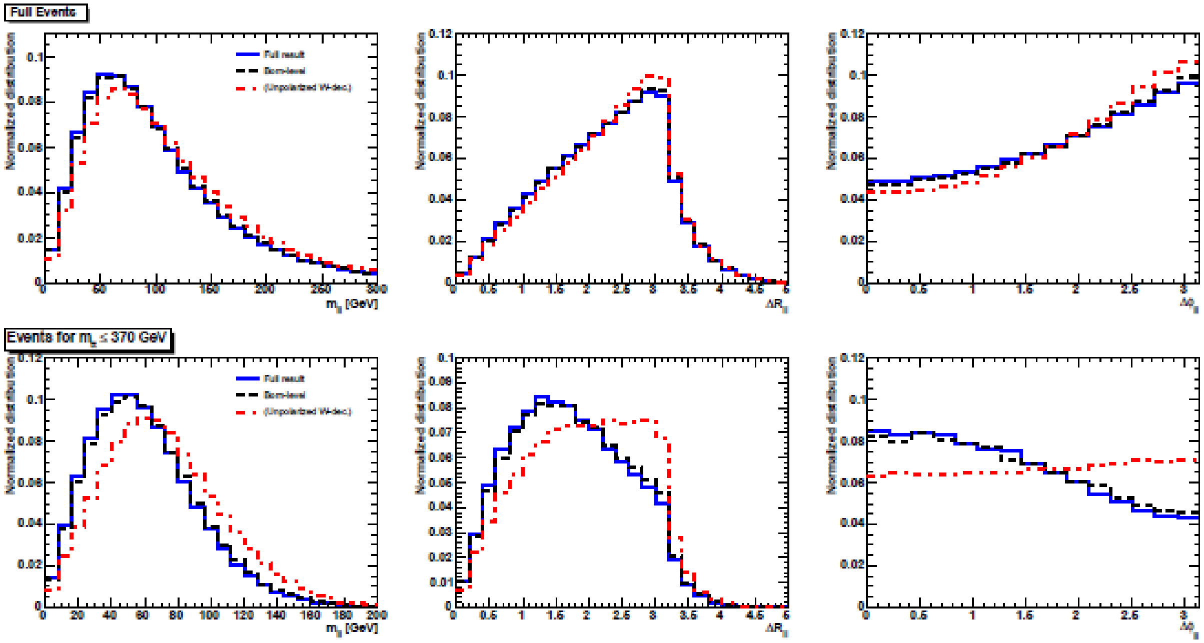
3. Event Generation

☐ Some Examples (at partonic-level)

(4) lepton angular distributions (di-lepton case)

- small bound-state effects in final lepton angular distributions
- bWbW MEs plus W-decay in Parton-Shower → Wrong (no W polarization)

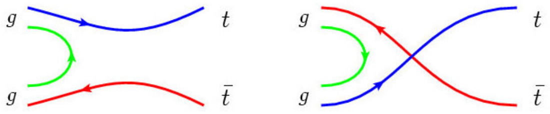
- must use 6-body MEs



3. Event Generation

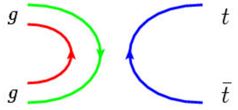
☐ Color decomposition & Color-flow assignment

in gluon-fusion process,



$$\begin{aligned}\mathcal{M}_{gg\to t\bar{t}} &= (T^aT^b)_{ij}\mathcal{M}_1 + (T^bT^a)_{ij}\mathcal{M}_2 \\ &= \frac{1}{2}\{T^a,T^b\}_{ij}\mathcal{M}_S + \frac{1}{2}[T^a,T^b]_{ij}\mathcal{M}_A\end{aligned}$$

symmetric part : $\frac{1}{2}\{T^a,T^b\} = \underbrace{\frac{1}{2N_c}\delta^{ab}\delta_{ij}}_{\text{color-singlet}} + \frac{1}{2}d^{abc}T^c_{ij}$



ratio in the amplitude squared : $\left|\frac{1}{2N_c}\delta_{ab}\delta_{ij}\right|^2 \Big/ \left|\frac{1}{2}d^{abc}T^c_{ij}\right|^2 = \frac{2}{N_c^2-4}$

this is zero in large-N limit, but not in QCD

our color-singlet events have correct color-flow assignment in the LHEF record

4. Summary

- At the LHC, gluon-fusion process dominates
  and the ttbar pair can be color-singlet

  The bound-state effects are calculated for the mtt distribution at Hadron Colliders up to NLO (Green's func., gluon radiation, hard-correction).

  Large correction in mtt dist. near threshold,
  and there appears a broad resonance below threshold.

- Differential cross-sections are also calculated including BS effects,
  non-resonant parts as well as decays of W's
  - incorporate momentum-space Green functions for color-singlet and octet
  - smooth interpolation to the high-energy region
  - non-resonant diagrams and phase-space suppression effect are considered

- Event Generator including Bound-state effects and NLO K-factors

Back-up slides

$\overline{\mathrm{MS}}$ mass

$$m_t^{\mathrm{pole}} = \bar{m}(\bar{m}) + \bar{m}(\bar{m})\left[\frac{\alpha_s}{\pi}d_1 + \left(\frac{\alpha_s}{\pi}\right)^2 d_2 + \cdots\right] \qquad d_1 = \frac{4}{3},,,$$

known up to 3-loop

difference between the two scheme is large, $\delta m \sim 10\,[\mathrm{GeV}]$

- Extracted from the total cross-section at the Tevatron

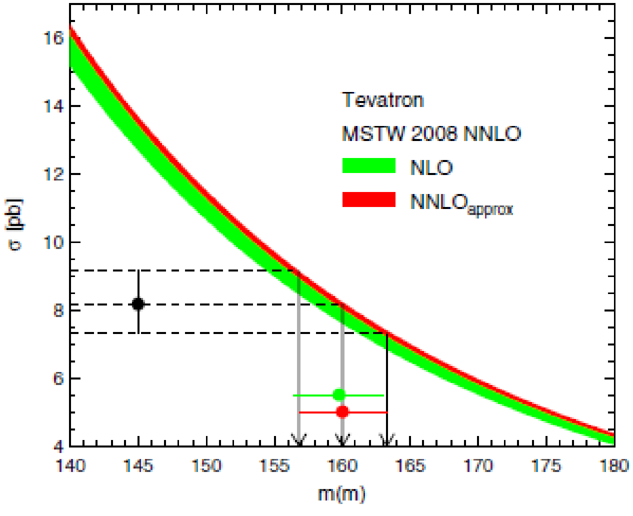
Langenfeld,Moch,Uwer ('09)

$$\bar{m}(\bar{m}) = 160.0^{+3.3}_{-3.2}\,[\mathrm{GeV}]$$
NNLO

$$159.8^{+3.3}_{-3.3}$$
NLO

$$159.2^{+3.5}_{-3.4}$$
LO

PHYSICAL REVIEW D 80, 054009 (2009)



(b)

- The corresponding value in pole mass scheme (NNLO)

$$m_t^{\mathrm{pole}} = 168.9^{+3.5}_{-3.3}\,[\mathrm{GeV}]$$

is consistent with current world average.

☐ Short-distance mass measurement

- Pole mass is known to be ambiguous due to its sensitivity to small momenta (IR-Renormalon), bad perturbative convergency.
- Use short-distance mass which don't have such ambiguity.

  Threshold mass (PS mass, 1S mass, Kinetic mass,,,), MSbar mass, Jet-mass,,,
- Example: 1S-mass ; *defined as a half of the 1S toponium mass* Hoang,Teubner ('99)

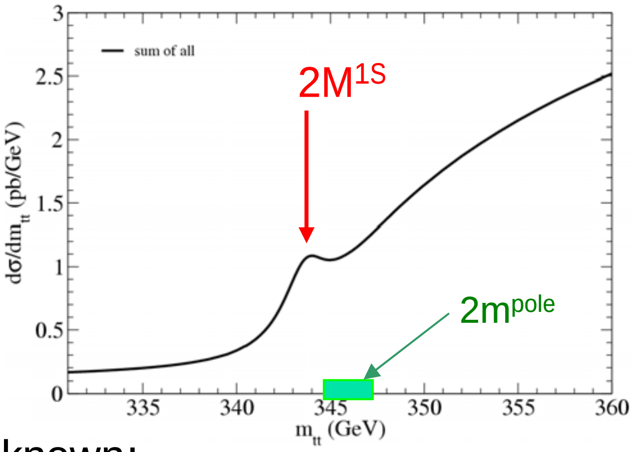
◆ Suppose, you measure the peak in m_{tt} dist. at the LHC

SD mass could be obtained very cleanly.

Peak consists of color-singlet resonance

→ Free from low momentum color-connection

Theoretically good perturbative convergence



Relation to the pole mass or the other SD mass is well known;

$$M^{1S}(\mu) = m_t^{\text{pole}} - \frac{m_t^{\text{pole}} C_F^2 \alpha_s^2}{8}\left[1+\frac{\alpha_s}{\pi}d_1+\left(\frac{\alpha_s}{\pi}\right)^2 d_2+\cdots\right]$$

☐ Threshold events are worthwhile!

- But not easy in real life ;

  Small fraction of the threshold events

  Errors and combinatorials disturb the m_{tt} measurement

  → Large-m_{tt} events contribute as significant BG

- Challenging task at the LHC ;

  How to pick-up the threshold events in good accuracy?

  How much are the errors in measuring m_{tt} for each decay channel?

  How to treat additional jets from initial- and final- state radiation?

- Statistics may not be a problem (not a physics for first few years.)

- ☐ Which decay-channel is better to see threshold events?

Key is the Jet Energy Scale, especially for B-jets.

- all-hadron : 6 jets
- lepton+jets : 4 jets (2-fold sol.)
- di-lepton : 2 jets (8-fold sol.)

- Reference : Systematics errors in top-quark mass measurement

for ATLAS Borjanovic etal('05)

- all-hadron : $\delta m_{t,(\mathrm{sys.})} \sim 3\,[\mathrm{GeV}]$
- lepton+jets : $\delta m_{t,(\mathrm{sys.})} \sim 1.5\,[\mathrm{GeV}]$
- di-lepton : $\delta m_{t,(\mathrm{sys.})} \sim 1.5\,[\mathrm{GeV}]$

$$\delta m_{tt,(\mathrm{sys.})} \sim \sqrt{2}\delta m_{t,(\mathrm{sys.})} ?\ \delta m_{t,(\mathrm{sys.})} ?$$

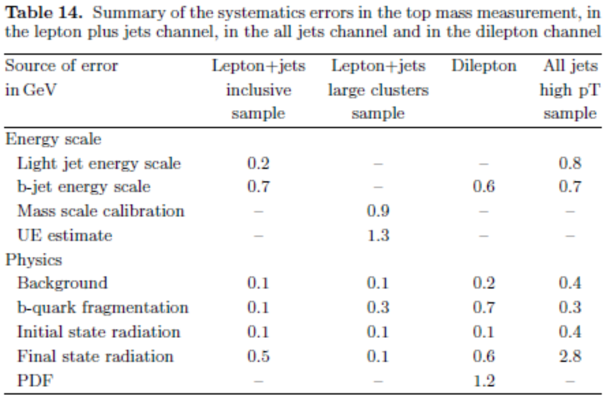


Table 14. Summary of the systematics errors in the top mass measurement, in the lepton plus jets channel, in the all jets channel and in the dilepton channel

Source of error in GeV	Lepton+jets inclusive sample	Lepton+jets large clusters sample	Dilepton	All jets high pT sample
Energy scale				
Light jet energy scale	0.2	–	–	0.8
b-jet energy scale	0.7	–	0.6	0.7
Mass scale calibration	–	0.9	–	–
UE estimate	–	1.3	–	–
Physics				
Background	0.1	0.1	0.2	0.4
b-quark fragmentation	0.1	0.3	0.7	0.3
Initial state radiation	0.1	0.1	0.1	0.4
Final state radiation	0.5	0.1	0.6	2.8
PDF	–	–	1.2	–

☐ Selecting threshold events; di-lepton example

- Mahlon,Parke('10) :

  (for the purpose of spin correlation study)

  Solve the system with the on-shell conditions for 4 particles (t,tbar,W^+,W^-).

  Take the average of the (at most) 8-fold solutions.

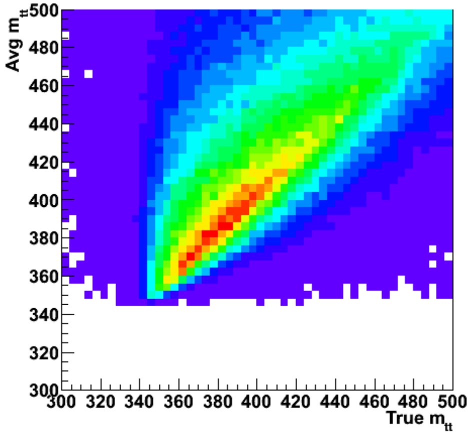
  Events with m_{tt}(Avg)<400GeV contain less events with m_{tt}(True)>400 GeV.

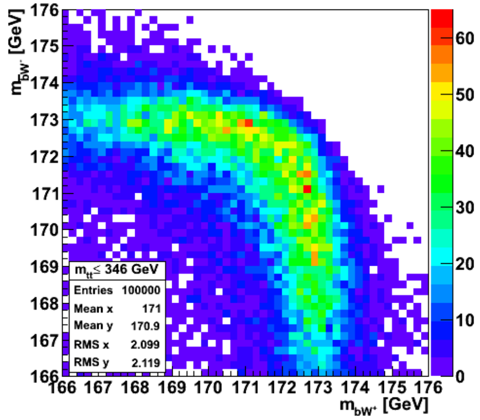
- However, for the threshold events, (both of) top-quarks on-shell condition cannot be used to reconstruct momenta.

  (It is still OK to use them for the selection of threshold events)

  → How to reconstruct m_{tt},in di-lepton channel?

using parton-level momenta





events for $m_{tt}<2m_t$