Gauge invariance in effective theories of QCD

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Outline

- SCET and its building blocks
- Gauge invariance for covariant gauges
- Gauge invariance for singular gauges
  (Light-cone gauge)
- A new Wilson line in SCET: T
- Conclusions

SCET, an effective theory of QCD

- SCET (soft collinear effective theory) is an effective theory of QCD
- SCET describes interactions between low energy ,"soft" partonic fields and collinear fields (very energetic in one light-cone direction)
- SCET and QCD have the same infrared structure: matching is possible
- SCET helps in the proof of factorization theorems and identification of relevant scales

SCET: Kinematics

Bauer, Fleming, Pirjol, Stewart, '00

	SCET $[\lambda \sim m/Q \ll 1]$	
	n-collinear $(\xi_n,\, A_n^\mu)$	$p_n^\mu \sim Q(\lambda^2, 1, \lambda)$
	$\bar{n}$-collinear $(\xi_{\bar{n}},\, A_{\bar{n}}^\mu)$	$p_{\bar{n}}^\mu \sim Q(1, \lambda^2, \lambda)$
Crosstalk:	soft $(q_s,\, A_s^\mu)$	$p_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$

Light-cone coordinates

$$p^\mu = (+, -, \perp)$$

$$n^\mu = (1, \vec{n}), \qquad \bar{n}^\mu = (1, -\vec{n})$$

$$(\vec{n}^2 = 1,\, n^2 = 0,\, \bar{n}^2 = 0)$$

$$\psi(x) = \sum_n \sum_p e^{-ipx} \psi_{n,p}(x)$$

$$\bar{n}p \sim Q$$

$$p_\perp \sim \lambda Q$$

$$np \sim \lambda^2 Q$$

$$\psi = \left(\frac{\not{n}\not{\bar{n}}}{4} + \frac{\not{\bar{n}}\not{n}}{4} \right) \psi = \xi + \phi$$

Integrated out with EOM

SCET

Bauer, Fleming, Pirjol, Stewart, '00

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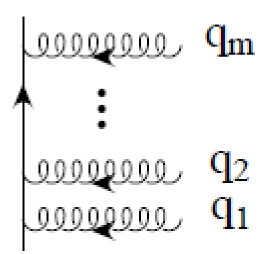
Leading order Lagrangian (n-collinear)

$$\mathcal{L}_{c,n} = \bar{\xi}_{n,p'} \left\{ i\, n\cdot D + g n\cdot A_{n,q} + \left(\mathcal{P}\!\!\!/_\perp + g A\!\!\!/^\perp_{n,q} \right) W \frac{1}{\bar{\mathcal{P}}} W^\dagger \left(\mathcal{P}\!\!\!/_\perp + g A\!\!\!/^\perp_{n,q'} \right) \right\} \frac{\bar{n}\!\!\!/}{2} \xi_{n,p}$$

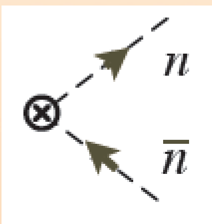
$$W_n(x) = \bar{P} \exp\left(-ig \int_0^\infty ds\ \ \bar{n}\cdot A\ \ (\bar{n}s + x) \right)$$

$$iD^\mu = i\partial^\mu + g A^\mu_{us}$$

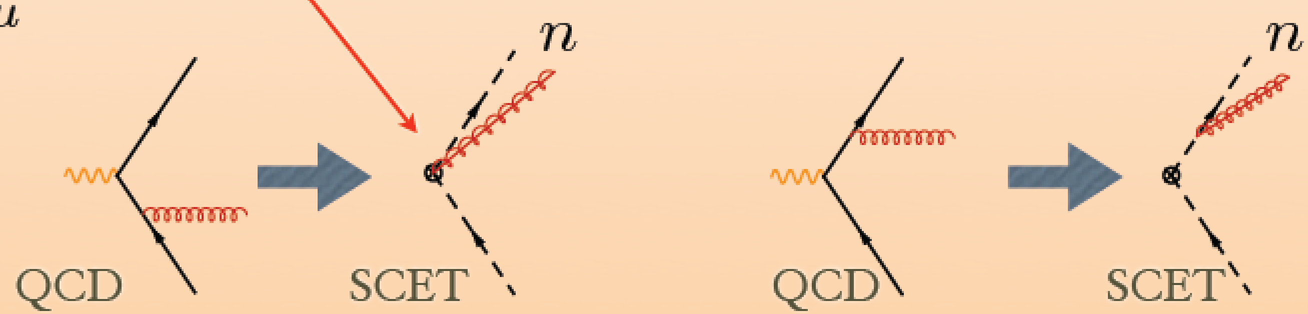


SCET

Production Current: $Q \gg m$



$$\underbrace{\bar\psi\, \Gamma^\mu \psi}_{\mathcal{J}_i^\mu} \to (\bar\xi_n W_n)_\omega\, \Gamma^\mu (W^\dagger_{\bar n} \xi_{\bar n})_{\bar\omega} = (\bar\xi_n W_n)_\omega\, Y_n^\dagger \Gamma^\mu Y_{\bar n} (W^\dagger_{\bar n} \xi_{\bar n})_{\bar\omega}$$



$$\mathcal{J}_i^\mu(0) = \int d\omega\, d\bar\omega\, C(\omega, \bar\omega, \mu)\, J_i^{(0)\mu}(\omega, \bar\omega, \mu)$$

SCET building blocks

The SCET Lagrangian is formed by gauge invariant building blocks.

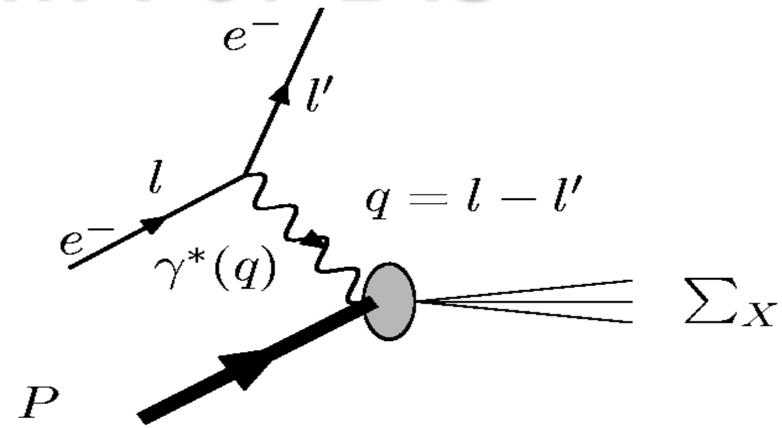
Gauge Transformations:

$$\xi \to U\xi$$
$$W_n^+ \to W_n^+ U^+$$

$\longrightarrow$ $W_n^+ \xi$ Is gauge invariant

Factorization Theorem For DIS

$$F_1(x,Q^2) = \sum_f \int_x^1 \frac{dy}{y} C_f\left(\frac{x}{y}, \frac{Q^2}{\mu^2}\right) q_f(y,\mu^2) \ ,$$



- PDF In Full QCD

$$q(x) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P|\overline{\psi}(\lambda n)\ \not{n} e^{-ig\int_0^\lambda d\lambda' n\cdot A(\lambda' n)} \psi(0)|P\rangle \ ,$$

- Factorization In SCET $\quad F_1^N(Q^2) = H(Q^2/\mu^2)\phi_N(Q^2/\mu^2)S_N(Q^2/\mu^2)J_N(Q^2/\mu^2) \ ,$

[Neubert et.al]

- PDF In SCET: $\phi(x,\mu_I^2) = \left\langle P \left| \bar{\xi}_{\bar{n}} W_{\bar{n}} \delta\left(x - \frac{n\cdot\mathcal{P}_+}{n\cdot p}\right) \frac{\not{n}}{\sqrt{2}} W_{\bar{n}}^\dagger \xi_{\bar{n}} \right| P \right\rangle$

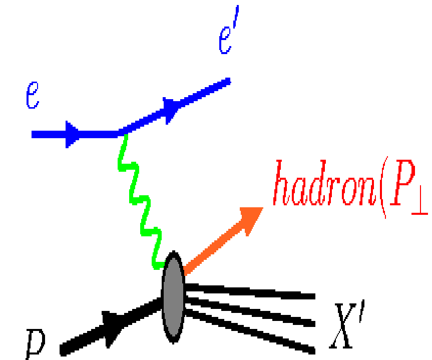
[Stewart et.al]

$\phi(x,\mu_f^2)$ is gauge invariant because each building block is gauge invariant

Factorization Theorem For SIDIS in QCD: Covariant gauge

- In Full QCD And At Low Transverse Momentum:

$$F(x_B, z_h, P_{h\perp}, Q^2) = \sum_{q=u,d,s,\ldots} e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\ell}_\perp$$
$$\times q\left(x_B, k_\perp, \mu^2, x_B\zeta, \rho\right) \hat{q}_T\left(z_h, p_\perp, \mu^2, \hat{\zeta}/z_h, \rho\right) S(\vec{\ell}_\perp, \mu^2, \rho)$$
$$\times H\left(Q^2, \mu^2, \rho\right) \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\ell}_\perp - \vec{P}_{h\perp}) \,,$$



Ji, Ma,Yuan '04

- "Naïve" Transverse Momentum Dependent PDF (TMDPDF):

$$q \approx Q / S$$

$$Q(x, k_\perp, \mu, x\zeta) = \frac{1}{2}\int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \left\langle P \left| \overline{\psi}_q(\xi^-, 0, \vec{b}_\perp) \mathcal{L}_v^\dagger(\infty; \xi^-, 0, \vec{b}_\perp) \gamma^+ \mathcal{L}_v(\infty; 0) \psi_q(0) \right| P \right\rangle$$

$$\mathcal{L}_v(\infty; \xi) = \exp\left(-ig \int_0^\infty d\lambda v \cdot A(\lambda v + \xi)\right) .$$

Analogous to the W in SCET

This result is true only in "regular" gauges:
Here all fields vanish at infinity

Transverse Gauge Link in QCD

Ji, Ma, Yuan
Ji, Yuan
Belitsky, Ji, Yuan
Cherednikov, Stefanis

- For gauges not vanishing at infinity [Singular Gauges] like the Light-Cone gauge (LC) one needs to introduce an additional Gauge Link which connects $(\infty, \vec{0}_\perp)$ with $(\infty, \vec{b}_\perp)$ to make it Gauge Invariant $A^{\mu\perp}(r_\perp)$
- In LC Gauge This Gauge Link Is Built From The Transverse Component Of The Gluon Field:

$$\tilde{\mathcal{F}}_{i/h}(x, \boldsymbol{k}_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + i\boldsymbol{k}_\perp\boldsymbol{\xi}_\perp} \left\langle h|\bar{\psi}_i(\xi^-, \boldsymbol{\xi}_\perp)[\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger_{[n]}[\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger_{[\boldsymbol{l}]}\right.$$
$$\left.\times\gamma^+[\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \boldsymbol{0}_\perp]_{[\boldsymbol{l}]}[\infty^-, \boldsymbol{0}_\perp; 0^-, \boldsymbol{0}_\perp]_{[n]}\psi_i(0^-, \boldsymbol{0}_\perp)|h\right\rangle$$

$$[\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \boldsymbol{\xi}_\perp]_{[\boldsymbol{l}]} \equiv \mathcal{P}\exp\left[ig\int_0^\infty d\tau\ \boldsymbol{l}\cdot\boldsymbol{A}_a t^a(\boldsymbol{\xi}_\perp + \boldsymbol{l}\tau)\right]$$

Gauge Invariant TMDPDF In SCET?

Are TMDPDF fundamental matrix elements in SCET?

Are SCET matrix elements gauge invariant?

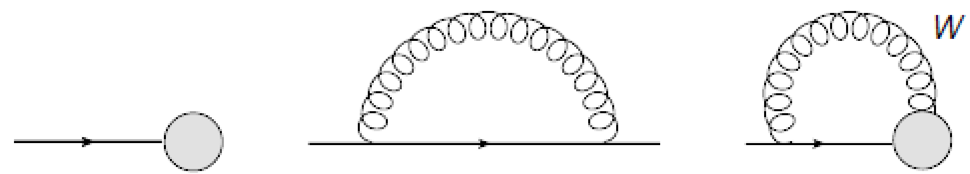
Where are transverse gauge link in SCET?

$$W^{\dagger}\xi \xrightarrow{LC\ gauge} \xi$$

Gauge invariance of SCET building blocks

We calculate $\left\langle 0 \middle| W_{\bar{n}}^{\dagger} \xi_{\bar{n}} \middle| q \right\rangle$ at one-loop in Feynman Gauge and in LC gauge

In Feynamn Gauge



$$I_w = \frac{\alpha_s}{4\pi} C_F \, i\not{p} \left[\frac{1}{\epsilon_{\rm UV}} + 1 - \ln \frac{-p^2}{\mu^2} \right],$$

$$W_{n,y} = \mathrm{P} \exp \left(ig \int_{-\infty}^{y} \mathrm{d}s \ \bar{n} \cdot A_n(s\bar{n}) \right)$$

$$I_{\bar{n},Fey} = -2g^2 C_F \mu^{2\varepsilon} i \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + i0)(k^+ + i0)} \frac{p^+ + k^+}{(p+k)^2 + i0}$$

$$I_{\bar{n}} = -\frac{g^2}{8\pi^2} C_F \gamma^\mu \Big[-\frac{1}{\epsilon^2} - \frac{1}{\epsilon} + \frac{1}{\epsilon} \ln \frac{-p_1^2}{\mu^2} - \frac{1}{2} \ln^2 \frac{-p_1^2}{\mu^2} + \ln \frac{-p_1^2}{\mu^2} - 2 + \frac{\pi^2}{12} \Big].$$

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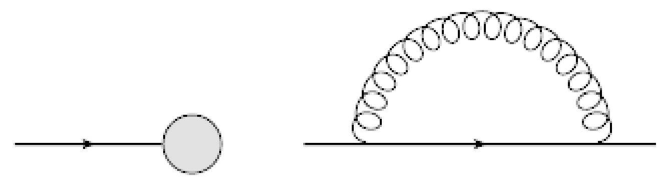
In LC Gauge



$$A^{+} = 0 \rightarrow W_{\bar{n}} = W_{\bar{n}}^{\dagger} = 1$$

$$\tilde{D}_{\mu\nu}(k) = \frac{-i}{k^2 + i0}\left(g_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{[k^+]} \right)$$

Prescription	$1/[k^+]$
$+i0$	$1/(k^+ + i0)$
$-i0$	$1/(k^+ - i0)$
PV	$1/2(1/(k^+ + i0) + 1/(k^+ - i0))$
ML	$1/(k^+ + i0\mathrm{Sgn}(k^-))$

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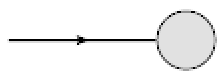
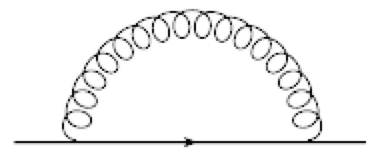
In LC Gauge





$$\tilde{D}_{\mu\nu}(k) = \frac{-i}{k^2 + i0}\left(g_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{\left[k^+\right]} \right)$$

$$\Sigma_{LC}(p) = \Sigma_{Fey}(p) + \Sigma_{Ax}^{(\mathrm{Pres})}(p) = \left(I_{w,Fey}(p) + I_{w,Ax}^{(\mathrm{Pres})}(p) \right) \frac{ip^2}{p^+} \frac{\not{n}}{\sqrt{2}}$$

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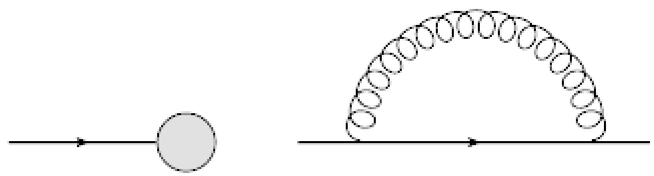
We calculate $\langle 0 | W_{\bar{n}}^{\dagger} \xi_{\bar{n}} | q \rangle$ at one-loop in Feynman Gauge and in LC gauge

In LC Gauge



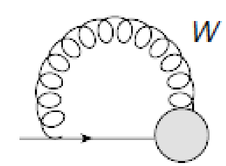
$$\Sigma_{LC}(p) = \Sigma_{Fey}(p) + \Sigma_{Ax}^{(\mathrm{Pres})}(p) = \left(I_{w,Fey}(p) + I_{w,Ax}^{(\mathrm{Pres})}(p) \right) \frac{ip^2}{p^+} \frac{\not{n}}{\sqrt{2}}$$

The gauge invariance is ensured when

$$I_{w,Ax}^{(\mathrm{Pres})} = 4ig^2 C_F \mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{p^+ + k^+}{\left(k^2 + i0\right)\left((p+k)^2 + i0\right)\left[k^+\right]}$$

$$-\frac{1}{2} I_{w,Ax}^{(\mathrm{Pres})} = I_{\bar{n},Fey}$$

$$I_{\bar{n},Fey} = -2ig^2 C_F \mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{p^+ + k^+}{\left(k^2 + i0\right)\left((p+k)^2 + i0\right)\left(k^+ + i0\right)}$$



Gauge invariance is realized only with one prescription!!

Gauge invariance in SCET

The SCET matrix element $\langle 0|W_{\bar{n}}^{\dagger}\xi_{\bar{n}}|q\rangle$ is not gauge invariant . Using LC gauge the result of the one-loop correction depends on the used prescription.

$$I_{w,Ax}^{+i0}(p) = -I_{w,Ax}^{-i0}(p) \longrightarrow$$

Gauge invariance is violated with -i0 prescription. The same occurs with PV, ML

Gauge invariance in SCET

In order to restore gauge invariance we have to introduce a new Wilson line, T, in SCET matrix elements

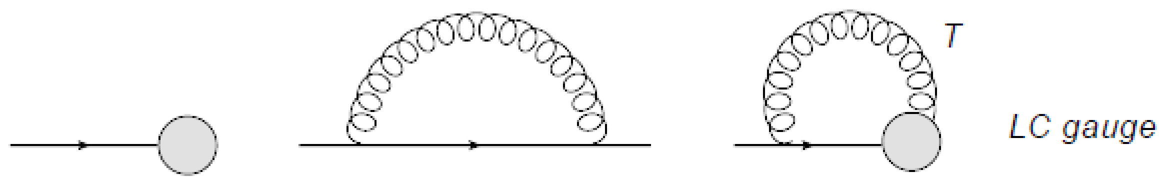
$$T_{\bar{n}}^{\dagger}(x^{+},x_{\perp})=P\exp\left[ig\int_{0}^{\infty}d\tau\,\mathbf{l}_{\perp}\cdot\mathbf{A}_{\perp}(\infty^{-},x^{+};\mathbf{l}_{\perp}\tau+\mathbf{x}_{\perp})\right]$$

And the new gauge invariant matrix element is $\boxed{\langle 0|\,T_{\bar{n}}^{\dagger}W_{\bar{n}}^{\dagger}\xi_{\bar{n}}\,|q\rangle}$

The T–Wilson Line

In covariant gauges $T = T^{\dagger} = 1$, so we recover the SCET results $\langle 0 | W_{\bar{n}}^{\dagger} \xi_{\bar{n}} | q \rangle$

In LC gauge

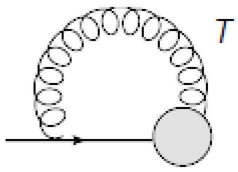


$$\langle 0 | T_{\bar{n}}^{\dagger} \xi_{\bar{n}} | q \rangle$$

The T–Wilson Line

$$I_{T,Ax}^{(\mathrm{Pres})} = 2C_F g^2 \mu^{2\varepsilon} i \int \frac{d^d k}{(2\pi)^d} \frac{p^+ + k^+}{(k^2 + i0)((p+k)^2 + i0)} \left[\frac{C_\infty^{(\mathrm{Pres})}}{k^+ - i0} - \frac{C_\infty^{(\mathrm{Pres})}}{k^+ + i0} \right]$$

Prescription	C∞
+i0	0
–i0	1
PV	1/2
ML	$\Theta(k^-)$



$$\boxed{I_{\bar{n},Fey} = \frac{-1}{2} I_{w,Ax}^{(\mathrm{Pres})} + I_{T,Ax}^{(\mathrm{Pres})}}$$

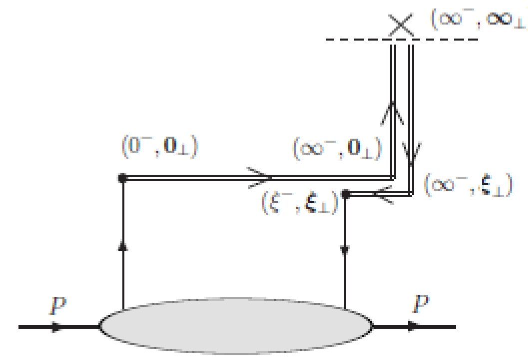
All prescription dependence cancels out and gauge invariance is restored no matter what prescription is used

$$\langle 0 | W_{\bar{n}}^\dagger \xi_{\bar{n}} | q \rangle \Longrightarrow \langle 0 | T_{\bar{n}}^\dagger W_{\bar{n}}^\dagger \xi_{\bar{n}} | q \rangle$$

Covariant Gauges — In All Gauges

Applications

•TMDPDF

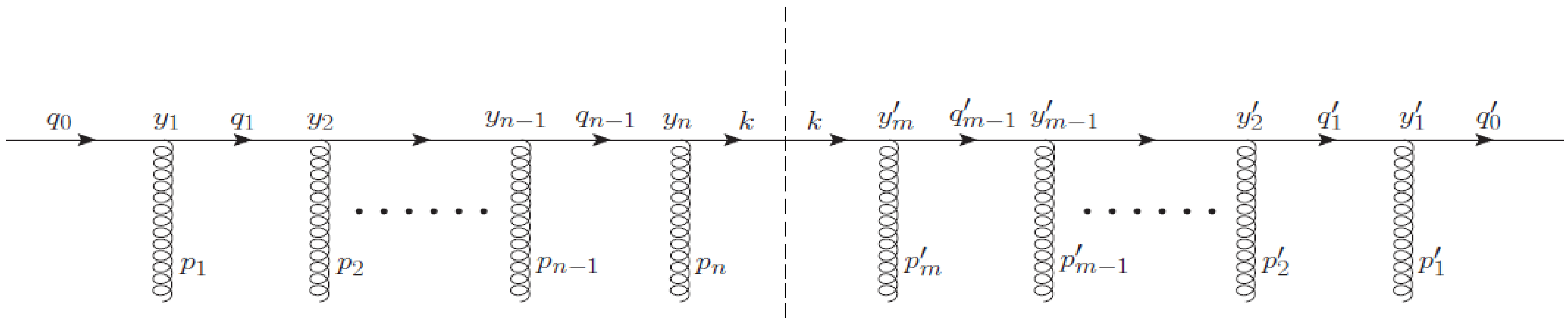


$$\underline{\chi}_{\bar{n}}(y) \equiv T_{\bar{n}}^{\dagger}(y^{+}, \mathbf{y}_{\perp}) W_{\bar{n}}^{\dagger}(y) \xi_{\bar{n}}(y)$$

$$\phi_{q/P} = \langle P_{\bar{n}} | \underline{\bar{\chi}}_{\bar{n}}(y) \delta\left(x - \frac{n\mathcal{P}}{np} \right) \delta^{(2)}(p_{\perp} - \mathcal{P}_{\perp}) \frac{\not{n}}{\sqrt{2}} \underline{\chi}_{\bar{n}}(0) | P_{\bar{n}} \rangle$$

We Can Define A Gauge Invariant TMDPDF In SCET (And Factorize SIDIS)

•Application To Heavy-Ion Physics



$$\sum_{m=1,n=1}^{\infty} \frac{d^2 \mathcal{A}_{nm}}{d^2 k_\perp} = \frac{\sqrt{2}}{L^3 N_c} \int dy^+ dy_\perp dy'_\perp \, e^{-ik_\perp \cdot (y_\perp - y'_\perp)} \left\langle \mathrm{Tr}\left[\left(W_F^\dagger [y^+, y'_\perp] - 1 \right) \left(W_F [y^+, y_\perp] - 1 \right) \right] \right\rangle$$

D´Eramo, Liu, Rajagopal

$$W_F\left[y^+, y_\perp\right] \equiv P\left\{ \exp\left[ig \int_0^{L^-} dy^- \, A^+(y^+, y^-, y_\perp) \right] \right\}$$

In LC Gauge The Above Quantity Is Meaningless. If We Add To It The T-Wilson line Then We Get A Gauge Invariant Physical Entity.

Conclusions

The usual SCET building blocks have to be modified introducing a New Gauge Link, the T-Wilson line.

Using the new formalism we get gauge invariant definitions of non-perturbative matrix elements. In particular the T is compulsory for matrix elements of fields separated in the transverse direction. These matrix elements are relevant in semi-inclusive cross sections or transverse momentum dependent ones.

It is possible that the use of LC gauge helps in the proofs of factorization. The inclusion of T is so fundamental.

Work in progress in this direction.