

Transverse-momentum resummation for Drell-Yan lepton pair production at NNLL accuracy

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In collaboration with:

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Outline

- 1 Drell-Yan q_T distribution and fixed order results
- 2 Transverse-momentum resummation
- 3 Resummed results
- 4 Conclusions and Perspectives



Motivations

The study of Drell-Yan lepton pair production is well motivated:

- Large production rates and clean experimental signatures:
 - Important for detector calibration.
 - Possible use as luminosity monitor.
- Transverse momentum distributions needed for:
 - Precise prediction for M_W .
 - Beyond the Standard Model analysis.
- Test of perturbative QCD predictions.
- Constrain for fits of PDFs.



State of the art: fixed order calculations

Historically the Drell-Yan process [Drell,Yan('70)] was the first application of parton model ideas developed for deep inelastic scattering.

- QCD corrections:
 - Total cross section known up to NNLO ($\mathcal{O}(\alpha_S^2)$)
[Hamberg,Van Neerven,Matsuura('91)], [Harlander,Kilgore('02)]
 - Rapidity distribution known up to NNLO
[Anastasiou,Dixon,Melnikov,Petriello('03)]
 - Fully exclusive NNLO calculation completed
[Melnikov,Petriello('06)], [Catani,Cieri,de Florian,G.F., Grazzini('09)]
 - Vector boson transverse-momentum distribution known up to NLO ($\mathcal{O}(\alpha_S^2)$)
[Ellis et al.('83)], [Arnold,Reno('89)], [Gonsalves et al.('89)]
- Electroweak corrections are known at $\mathcal{O}(\alpha)$
[Dittmaier et al.('02)], [Baur et al.('02)]



The Drell-Yan q_T distribution

$$h_1(p_1) + h_2(p_2) \rightarrow V(M) + X \rightarrow l_1 + l_2 + X$$

where $V = \gamma^*, Z^0, W^\pm$ and $l_1 l_2 = l^+ l^-, \ell \nu_\ell$

According to the QCD factorization theorem:

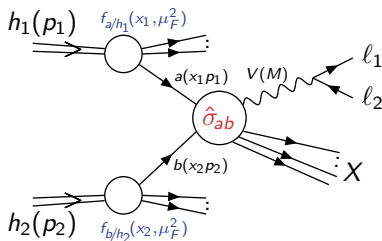
$$\frac{d\sigma}{dq_T^2}(q_T, M, s) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \frac{d\hat{\sigma}_{ab}}{dq_T^2}(q_T, M, \hat{s}; \alpha_S, \mu_R^2, \mu_F^2) + \mathcal{O}\left(\frac{\Lambda^2}{M^2}\right).$$

The standard fixed-order QCD perturbative expansions gives:

$$\int_{Q_T^2}^{\infty} dq_T \frac{d\hat{\sigma}_{q\bar{q}}}{dq_T^2} \sim \alpha_S \left[c_{12} \log^2(M^2/Q_T^2) + c_{11} \log(M^2/Q_T^2) + c_{10}(Q_T) \right] \\ + \alpha_S^2 \left[c_{24} \log^4(M^2/Q_T^2) + \dots + c_{21} \log(M^2/Q_T^2) + c_{20}(Q_T) \right] + \mathcal{O}(\alpha_S^3)$$

Fixed order calculation theoretically justified only in the region $q_T \sim M_V$

For $q_T \rightarrow 0$, $\alpha_S^n \log^m(M^2/q_T^2) \gg 1$: need for resummation of logarithmic corrections



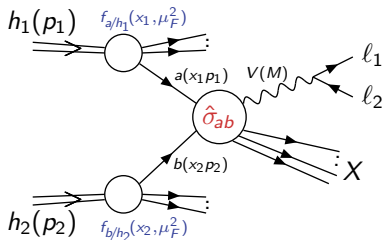
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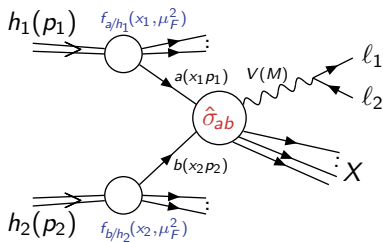
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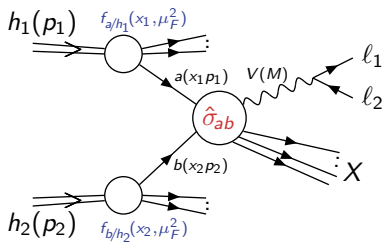
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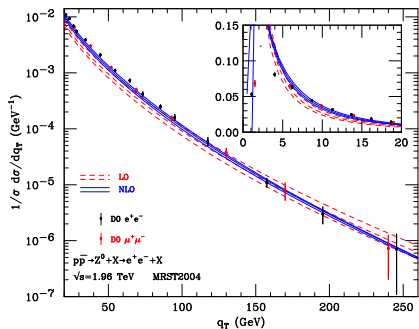
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Fixed order results: q_T spectrum of Drell-Yan l^+l^- pairs at $\sqrt{s} = 1.96$ TeV



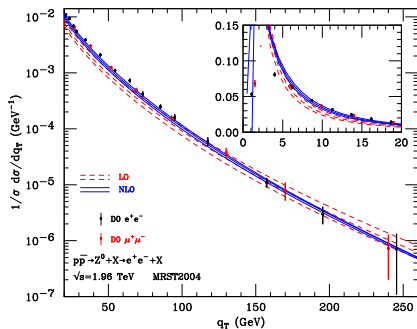
- D0 data normalized to 1: [D0 Coll. ('08, '10)]
- Factorization and renormalization scale variations:
 $\mu_F = \mu_R = m_Z, \quad m_Z/2 \leq \mu_F, \mu_R \leq 2m_Z,$
 $1/2 \leq \mu_F/\mu_R \leq 2.$
 LO and NLO scale variations bands overlap only for $q_T > 60$ GeV
- Good agreement between NLO results and data up to $q_T \sim 20$ GeV.
- In the small q_T region ($q_T \lesssim 20$ GeV) LO and NLO result diverges to $+\infty$ and $-\infty$ (accidental partial agreement at $q_T \sim 5 - 7$ GeV): need for resummation.

In the small q_T region ($q_T \lesssim 20$ GeV) effects of soft-gluon resummation are essential
 At Tevatron 90% of the W^\pm and Z^0 are produced with $q_T \lesssim 20$ GeV



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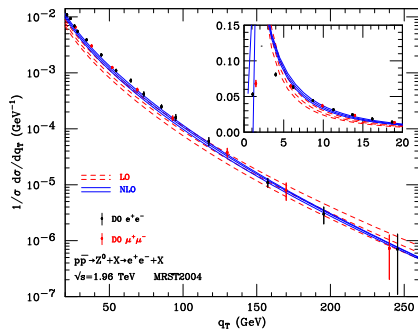
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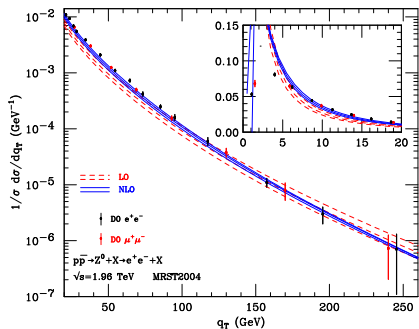
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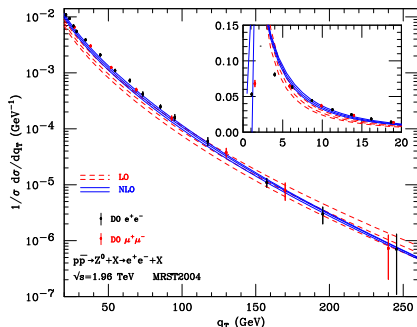
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State of the art: transverse-momentum resummation

- The method to perform the resummation of the large logarithms of q_T is known
[Parisi,Petronzio('79)], [Kodaira,Trentadue('82)], [Altarelli et al.('84)], [Collins,Soper,Sterman('85)], [Catani,de Florian,Grazzini('01)]
- Various phenomenological studies of the vector boson transverse momentum distribution exist
[Balasz,Qiu,Yuan('95)], [Balasz,Yuan('97)], [Ellis et al.('97)], [Kulesza et al.('02)]
- Recently various results for transverse momentum resummation in the framework of Effective Theories appeared [Gao,Li,Liu('05), Idilbi, Ji, Yuan('05), Mantry,Petriello('10), Becher,Neubert('10)].
- In this study we apply for Drell-Yan transverse-momentum distribution the resummation formalism developed by [Catani,de Florian, Grazzini('01)] already applied for the case of Higgs boson production [Bozzi,Catani,de Florian, Grazzini('03,'06,'08)].



Transverse momentum resummation

$$\frac{d\hat{\sigma}_{ab}}{dq_T^2} = \frac{d\hat{\sigma}_{ab}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}_{ab}^{(fin)}}{dq_T^2};$$

The finite component $\left(\lim_{Q_T \rightarrow 0} \int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}_{ab}^{(fin)}}{dq_T^2} \right]_{f.o.} = 0 \right)$
 ensure to reproduce the fixed order calculation at large q_T

Resummation holds in impact parameter space:

$$\frac{d\hat{\sigma}_{ab}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int_0^\infty db \frac{b}{2} J_0(bq_T) \mathcal{W}_{ab}(b, M), \quad q_T \ll M \Leftrightarrow Mb \gg 1, \quad \log M^2/q_T^2 \gg 1 \Leftrightarrow \log Mb \gg 1$$

In the Mellin moments space we have the exponentiated form:

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$$\mathcal{G}_N(\alpha_S, L) = L g^{(1)}(\alpha_S L) + g_N^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g_N^{(3)}(\alpha_S L) + \dots; \quad \mathcal{H}_N(\alpha_S) = \sigma^{(0)}(\alpha_S, M) \left[1 + \frac{\alpha_S}{\pi} \mathcal{H}_N^{(1)} + \left(\frac{\alpha_S}{\pi} \right)^2 \mathcal{H}_N^{(2)} + \dots \right]$$

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$$\frac{d\hat{\sigma}_{ab}}{dq_T^2} = \frac{d\hat{\sigma}_{ab}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}_{ab}^{(fin)}}{dq_T^2}; \quad \text{The finite component } \left(\lim_{Q_T \rightarrow 0} \int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}_{ab}^{(fin)}}{dq_T^2} \right]_{f.o.} = 0 \right)$$

ensure to reproduce the fixed order calculation at large q_T

Resummation holds in impact parameter space:

$$\frac{d\hat{\sigma}_{ab}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int_0^\infty db \frac{b}{2} J_0(bq_T) \mathcal{W}_{ab}(b, M), \quad q_T \ll M \Leftrightarrow Mb \gg 1, \quad \log M^2/q_T^2 \gg 1 \Leftrightarrow \log Mb \gg 1$$

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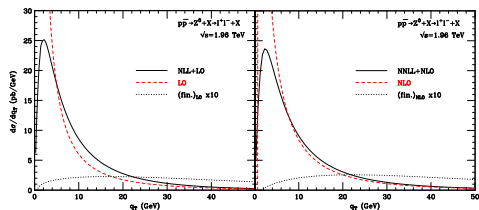
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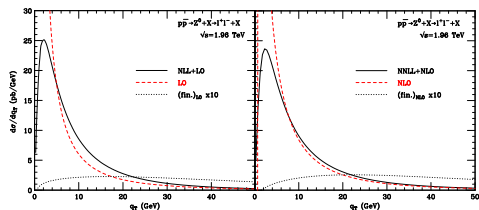
Resummed results: q_T spectrum of Drell-Yan l^+l^- pairs at $\sqrt{s} = 1.96$ TeV



- Left side: NLL+LO result compared with fixed LO result. Resummation cures the fixed order divergence at $q_T \rightarrow 0$.
- Right side: NNLL+NLO result compared with fixed NLO result.
- The q_T spectrum is slightly harder at NNLL+NLO accuracy than at NLL+LO accuracy.
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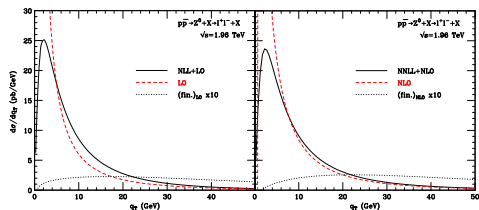
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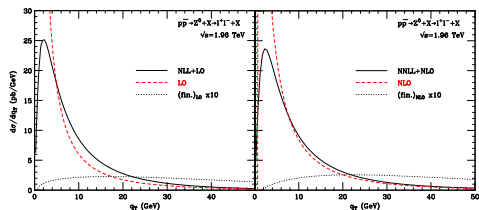
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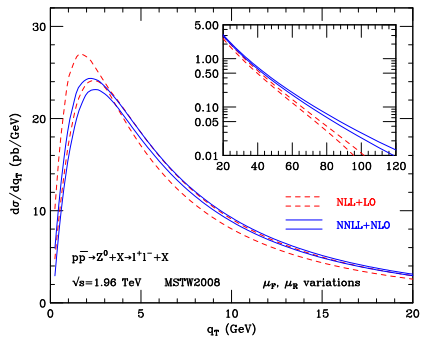
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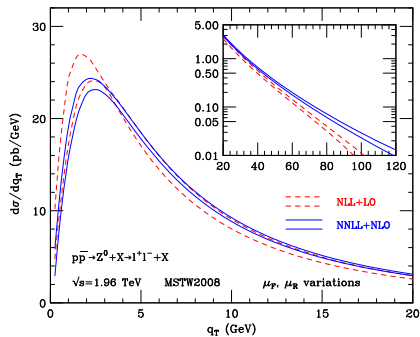
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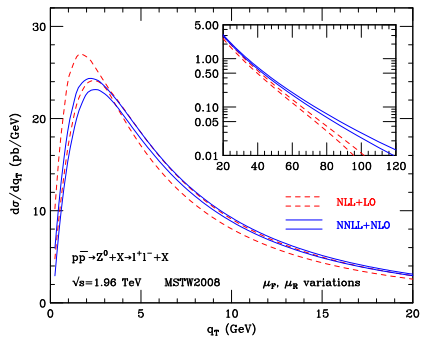
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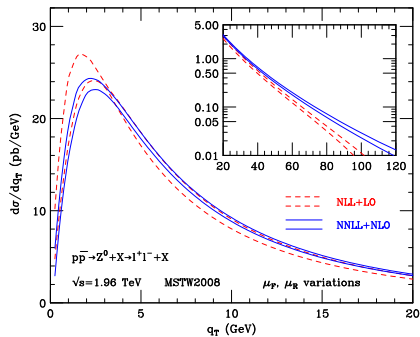
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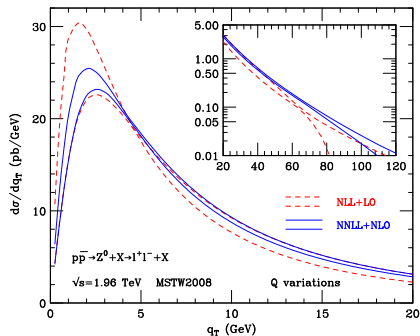
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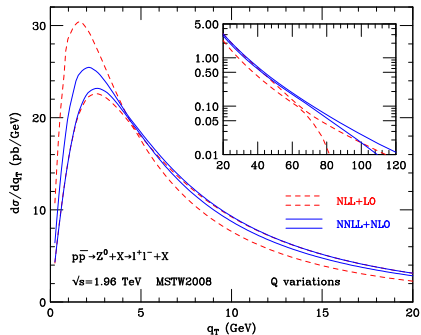
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- Uncertainty bands obtained by performing resummation scale variations (estimate of higher-order logarithmic contributions):
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- The resummation scale dependence at NNLL+NLO (NLL+LO) is about $\pm 5\%$ ($\pm 12\%$) around the peak and $\pm 5\%$ ($\pm 16\%$) in the $q_T \gtrsim 20$ GeV region and it is larger than the renormalization and factorization scale dependence.
- Going from the NLL+LO to the NNLL+NLO calculation the resummation scale dependence is reduced by roughly a factor 2 in the wide region $5 \text{ GeV} \lesssim q_T \lesssim 50 \text{ GeV}$.



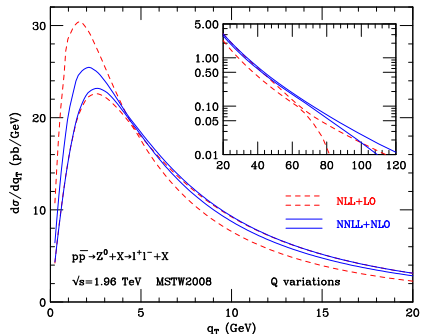
Resummed results: q_T spectrum of Drell-Yan l^+l^- pairs at $\sqrt{s} = 1.96$ TeV



- Uncertainty bands obtained by performing resummation scale variations (estimate of higher-order logarithmic contributions):
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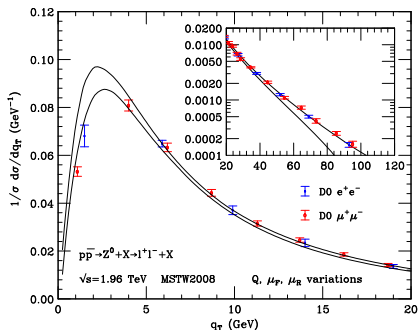
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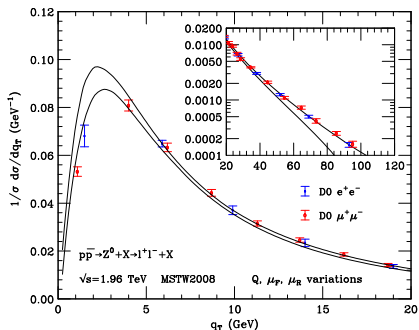
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- D0 data compared with our NNLL+NLO result.
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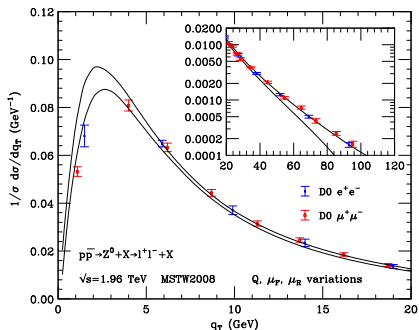
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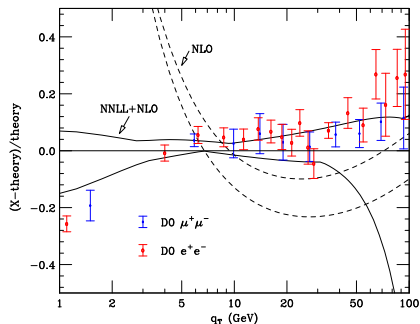
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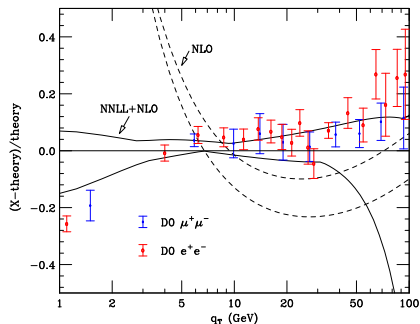
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- Fractional difference with respect to the reference result: NNLL+NLO, $\mu_R = \mu_F = 2Q = m_Z$.
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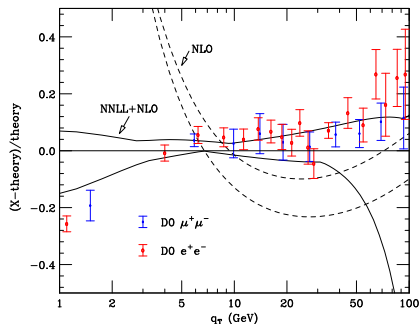
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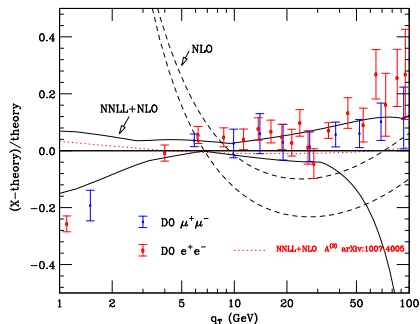
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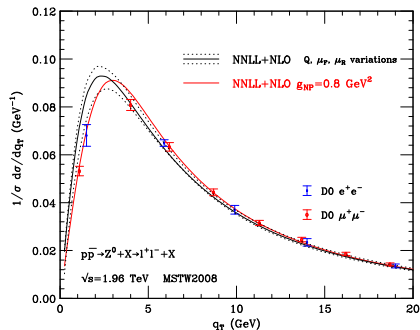
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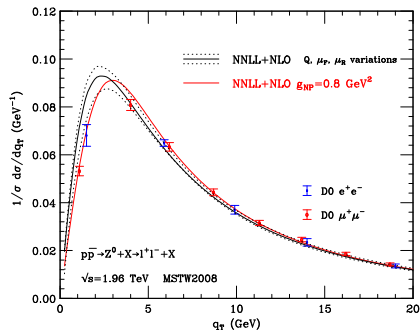
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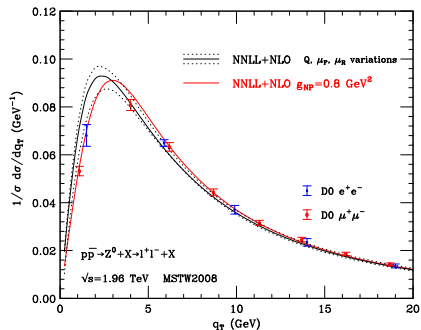
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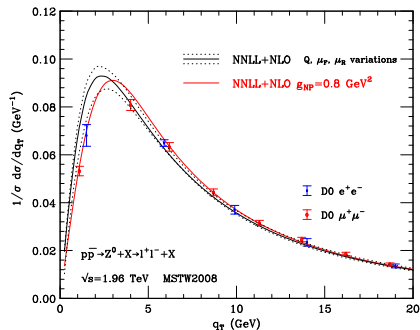
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- We have compared LO and NLO fixed order prediction to Tevatron data finding good agreement down to transverse momenta of the order $q_T \sim 20 \text{ GeV}$.
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