# A Holographic Quantum Hall Fluid

Oren Bergman
Technion - Israel Institute of Technology

AdS/CFT and the Holographic States of Matter
GGI Florence, Sept. 2010

# Outline

1. Introduction
2. The quantum Hall effect
3. A holographic model
4. Conclusions

Based on arXiv:1003.4965, with Jokela, Lifschytz, Lippert

# 1. Introduction

Many interesting physical phenomena are associated with systems in which the fundamental degrees of freedom are strongly coupled, for example:

- Confinement in QCD
- High $T_c$ superconductivity
- Fractional Quantum Hall Effect

Weak coupling techniques (pert. theory, fermi liquid theory) fail to explain these phenomena.

Other approaches:

- Low energy effective theories: physical but not microscopic (QCD Chiral Lagrangian, Landau-Ginzburg model of superconductivity).

- Phenomenological models: microscopic but not physical (NJL model, Laughlin's wavefunction).

- Non-perturbative techniques like lattice gauge theory work for some things (finite temp.) but not for others (finite density).

A new tool from string theory: holographic duality.

| Gauge field theory in d spacetime dimensions | = | Quantum gravity in d+1 spacetime dimensions |
| --- | --- | --- |
| strong coupling | | classical, Einstein |

There is a precise dictionary, many examples known.

This is a microscopic description: the field theory Lagrangian (if known) gives the micro. dof's.

So far no real physical systems, only "phenomenological models".

Holographic models have been quite successful in exhibiting qualitative (and some quantitative) properties of QCD at strong coupling, like transport properties of the QGP and properties of the hadronic phase.

Can these successes be extended to other strongly coupled (fermionic) systems?

Can we exhibit strong-coupling phenomena from the realm of condensed matter physics, like high temp. superconductivity and the FQHE?

In some cases the phenomena are unique to strong coupling (confinement). In other cases (High Tc, FQHE, some aspects of QGP) similar phenomena occur also at weak coupling, but the strong coupling version is different in an interesting way.

In these cases we would like the holographic description to make these differences manifest.

Holographic superconductors: superconductivity in a strongly interacting system (of fermions?).

These probably have nothing to do with the real High Tc superconductors, but they do exhibit some properties distinct from BCS (weak-coupling) supercondcutors.

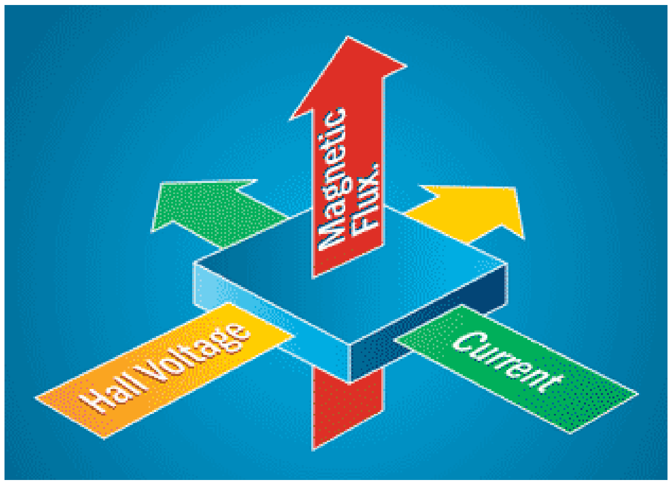
In this talk I will describe a holographic model of a quantum Hall fluid of strongly interacting charged fermions.

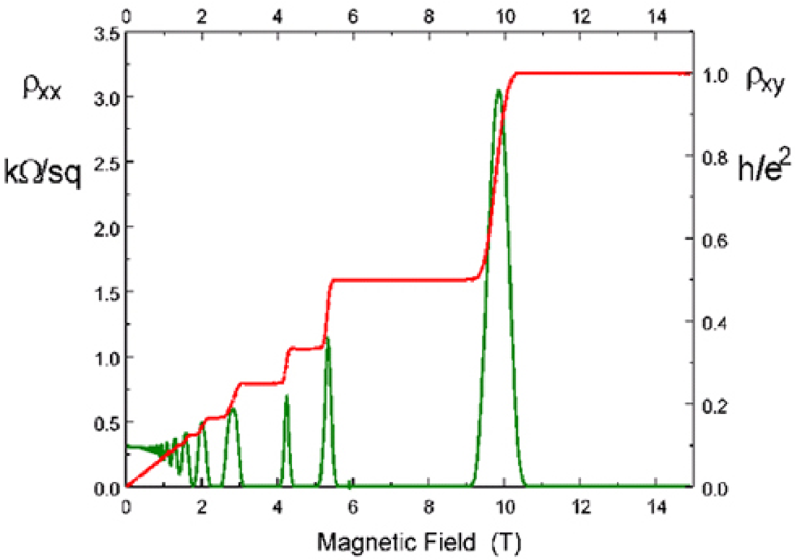
This will not describe the observed fractional quantum Hall states, but it will be different from the weakly coupled integer quantum Hall effect.

The model will also suggest a general strategy for finding other holographic models, which may come closer.

# 2. The Quantum Hall Effect

Electrons in 2+1 dimensions with a perpendicular B field:




Plateaus in the Hall conductivity at:

$$\sigma_{xy} = \nu \frac{e^2}{h} \qquad \nu \equiv \frac{n_e}{2\pi B} = \begin{cases} \text{integers} \\ \text{certain fractions} \end{cases}$$

At these values $\sigma_{xx} = 0$ , indicating a gapped state.

## Integer QHE: $\nu = 1, 2, 3, \ldots$

Free electrons, Landau problem (+ impurities)

## Fractional QHE: $\nu = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \ldots$

Interaction dominated

Laughlin state: $\psi_{\nu=\frac{1}{k}}(z_i) = \prod_{i<j}^{N} (z_i - z_j)^k e^{-eB\sum |z_i|^2}$ $\qquad k \in 2\mathbb{Z}+1$

Effective field theory: $J^\mu = \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda$

$$\mathcal{L} = \frac{k}{4\pi}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda - \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda$$

Fractionally charged excitations: $Q = \frac{e}{k}$ (observed experimentally!)

# 3. A holographic model

Basic ingredients for the boundary theory:

- charged fermions in 2+1 dimensions
- background magnetic field $B$
- finite charge density $J^0$

Goals:

- find states with $\sigma_{xx} = 0$ and quantized $\sigma_{xy}$ } modest
- fractionally charged excitations
- plateau transitions (varying $B$) less modest
- 1/3, 2/5, 3/7,... well...

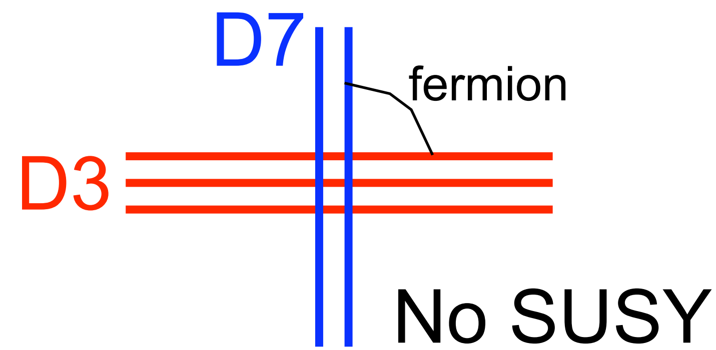
Top-down approach:

$$\begin{array}{ccc} \text{string theory} & \longrightarrow & \text{micro. dof's} \\ \downarrow & & \downarrow \\ \text{bulk gravity} & & \text{boundary QFT} \end{array}$$

# D3-D7' model:

Rey, Kraus et. al.

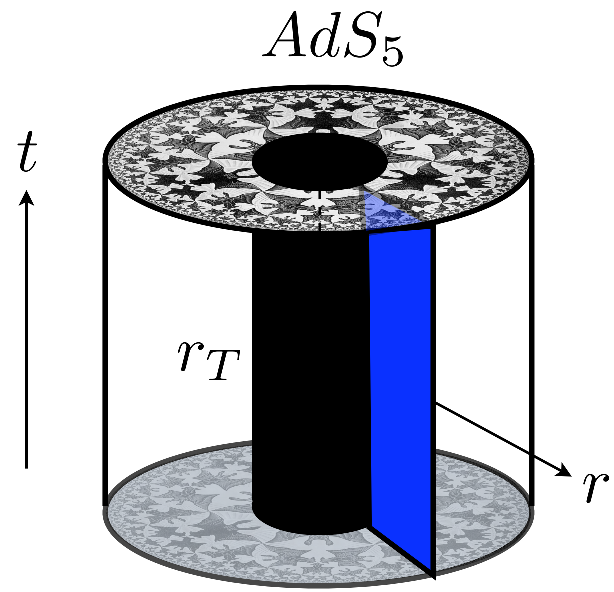
| | $0$ | $1$ | $2$ | $3$ | $r$ | $S^5$ |
|---|---|---|---|---|---|---|
| $N_c$ D3's | ● | ● | ● | ● | | |
| $N_f$ D7's | ● | ● | ● | | ● | $M_4$ |




4d N=4 SU($N_c$) SYM + 3d $N_f$ fundamental fermions

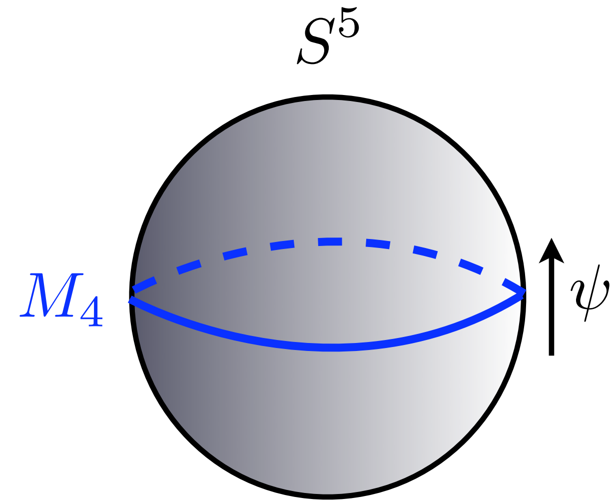
Probe approximation: $N_c \to \infty \, , \quad N_f = \text{finite}$

In this limit the dynamics of the fermions are described by the embedding of the D7's in the near horizon background of the D3's.




$r_T \propto T$

$\times$




$$\int_{S^5} F_5 = N_c$$

The D7-brane embedding is specified by $\psi(r), z(r)$.

But the embeddings are unstable:

$m_\psi^2 < -9/4$ (BF bound for AdS₄)

“slipping” mode

## Stability:

We can stabilize by turning on a (sufficient) worldvolume flux.

Myers, Wapler

For example:

$M_4 = S^4 \subset S^5$ with $\displaystyle\int_{S^4} \mathrm{Tr}(F \wedge F) \neq 0 \quad (N_f \geq 2)$

$M_4 = S^2 \times S^2 \subset S^5$ with $\displaystyle\int_{S^2_{(i)}} F = f_i \quad (N_f = 1)$

We'll take $M_4 = S^2_{(1)} \times S^2_{(2)}$ with $f_1 \neq 0, f_2 = 0$.

(We'll soon see why.)

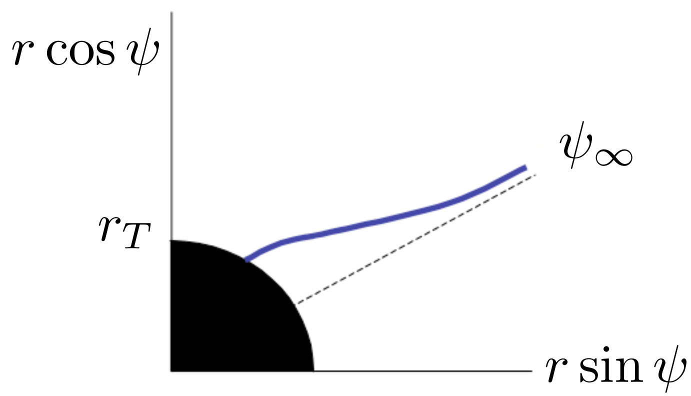
# Embeddings:

$h(r) = 1 - r_T^4/r^4 \quad (r_T \propto T)$

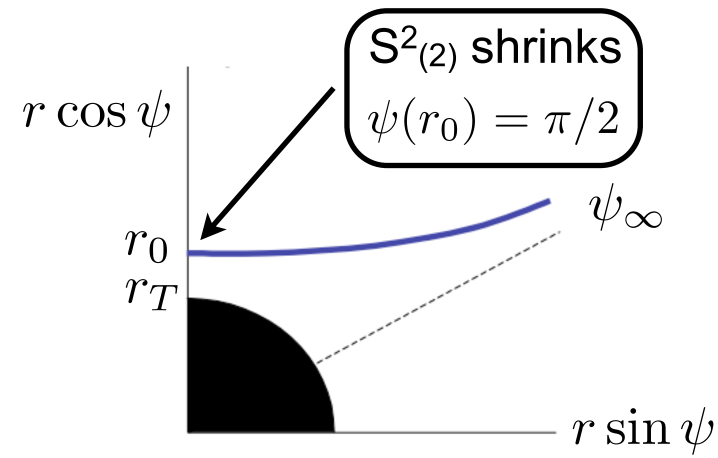
$$\begin{aligned} S_{DBI} &= -T_7 \int \sqrt{-\det(g+F)} \\ &= -\mathcal{N} \int dr\, r^2 \cos^2\psi \sqrt{\left(4\sin^4\psi + f_1^2\right)\left(1 + r^2 h(r) \psi'(r)^2\right)} \end{aligned}$$

$$\psi(r) \sim \psi_\infty + \frac{m}{r^\alpha}$$

$\psi_\infty, \alpha$ fixed by $f_1$, and $m$ = "mass" parameter




gapless state



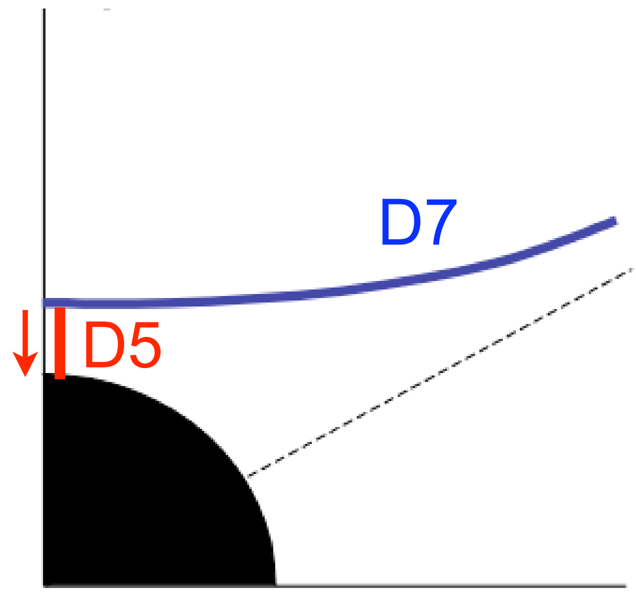

gapped state

# Why $S^2 \times S^2$ ? Why $f_2 = 0$ ?

The gapped state corresponds to $S^2_{(2)}$ shrinking at $r=r_0$.

A non-zero flux $f_2$ therefore requires magnetic sources at $r=r_0$, provided by D5-branes ending on the D7-brane.

These pull the D7 into the horizon, eliminating the gap:




$S^4$ is ruled out for a similar reason.

# Currents and background fields

We want to study this system at finite charge density and background magnetic field, and to compute current densities in a background electric field.

| <u>boundary</u> | | <u>bulk (background + D7)</u> |
|---|---|---|
| global symmetry | $\longleftrightarrow$ | gauge symmetry |
| conserved current $J^\mu$ | $\longleftrightarrow$ | gauge field $A_\mu$ |

Strictly speaking, the boundary theory doesn't have a dynamical gauge field, but we may consider background fields by allowing spacetime-dependent boundary values:

$$A_i(x_\mu, r) = \epsilon_{ij} x_j B + t E_i + \underbrace{a_i(r)}_{\text{currents}} \qquad A_0(x_\mu, r) = \underbrace{a_0(r)}_{\text{charge}}$$

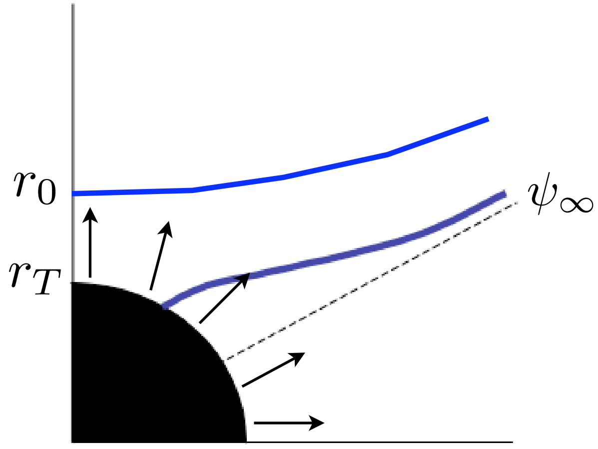
# CS term:

"axion"

$$S_{CS} = -\frac{T_7}{2}\int C_4 \wedge F \wedge F = \mathcal{N}\int dr\, c(r)\left(Ba_0'(r) + \vec{E}\times\vec{a}'(r)\right)$$

$$c(r) \equiv \int_{S^2\times S^2} C_4 = \psi(r) - \psi_\infty - \frac{1}{4}\left(\sin 4\psi(r) - \sin 4\psi_\infty\right)$$

$c(r_{min}) =$ amount of $F_5$ flux captured by the D7-brane




- $c(r_0)$ fixed and quantized by $f_1$
- $c(r_T)$ temp. dependent, unquantized

The currents are given by the constants of the motion:

$$J^\mu = \left.\frac{\partial \mathcal{L}}{\partial a'_\mu(r)}\right|_{EOM} = J^\mu_{DBI} + J^\mu_{CS}$$

$$r^2 \cos^2\psi \sqrt{4\sin^4\psi + f_1^2} \times [\text{mess depending on } a'_\mu(r)] \qquad\qquad 2c(r)\epsilon^{\mu\nu\lambda}F_{\nu\lambda}$$

In particular: $J^0_{CS} \propto B$ and $\vec{J}_{CS} \perp \vec{E}$ (Hall current).

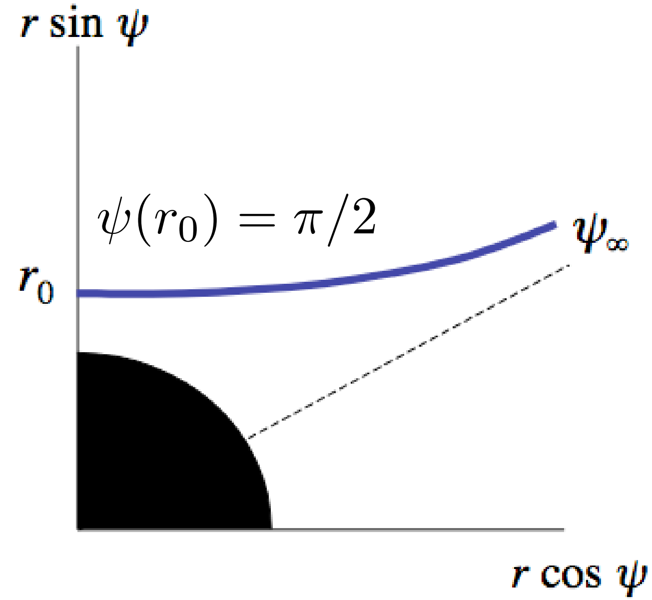
The DBI term contributes to both the longitudinal and Hall current.

In principle, we should find solutions for $a_0(r)$, $a_i(r)$ at fixed $J^0$, and plug in to get $J^i$.

But there are shortcuts.

# Gapped embedding

We require $a'_\mu(r_0) = \text{finite}$, otherwise there are sources (strings) which lead to a gapless embedding.




Evaluating the currents at $r = r_0$:

$$J^\mu_{DBI} = 0 \qquad J^\mu_{CS} = 2c(r_0)\epsilon^{\mu\nu\lambda}F_{\nu\lambda}$$

This is a quantum Hall state, with a quantized filling fraction:

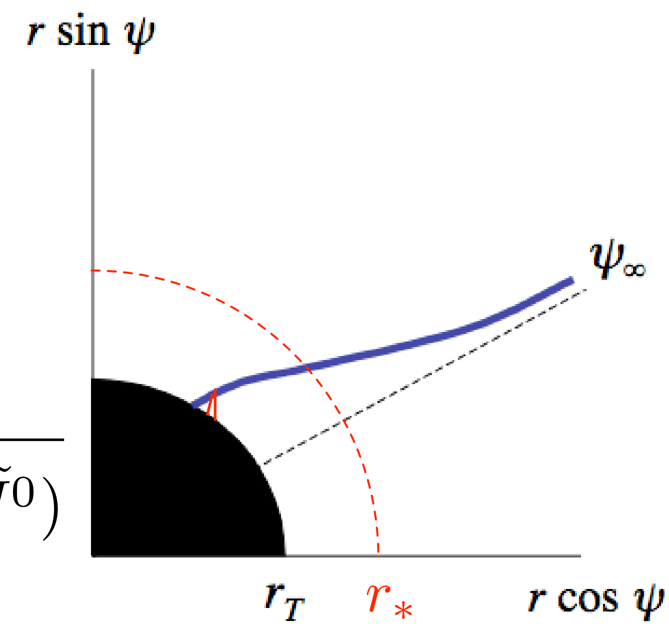
$$\nu \equiv \frac{J^0}{B} = 2c(r_0)$$

and

$$\sigma_{xx} = 0 \quad , \quad \sigma_{xy} = \nu$$

# Gapless embedding

Sources are allowed at $r_T$ : $J^0 = 2c(r_T) + \tilde{J}^0$




## Pseudo-horizon:

$$S_{DBI}(\vec{E}, B, \vec{J}, \tilde{J}_0) \sim \int dr \sqrt{X(r; \vec{E}, \tilde{J}^0) Z(r; \vec{J}, \tilde{J}^0)}$$

$X(r_*) = 0$ for some $r_* > r_T$

Requiring the action to be real gives $\vec{J}(\vec{E}, \tilde{J}^0)$, which in the linear response approximation gives:

Karch,O'Bannon

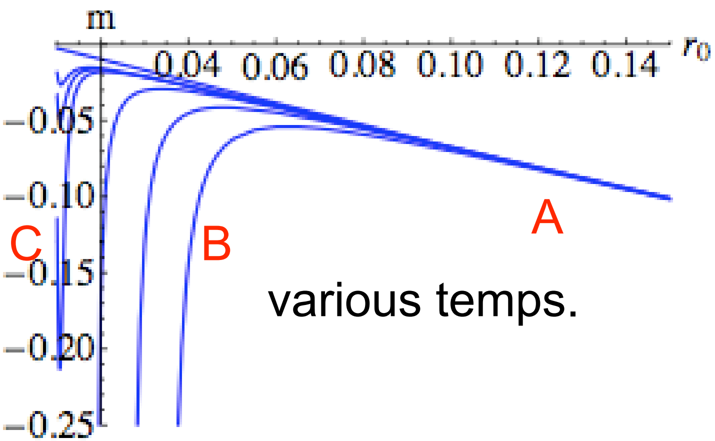
$$\sigma_{xx} = \frac{r_T^2}{B^2 + r_T^4} \sqrt{\tilde{J}_0^2 + 4\cos^4\psi(r_T)\left(f_1^2 + 4\sin^4\psi(r_T)\right)(B^2 + r_T^4)}$$

$$\sigma_{xy} = \frac{B}{B^2 + r_T^4} \tilde{J}_0 + 2c(r_T)$$

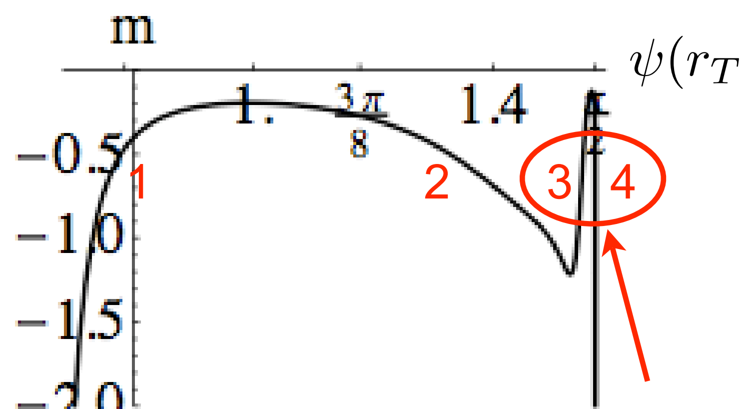
This is a conducting state.

# (Numerical) analysis of solutions

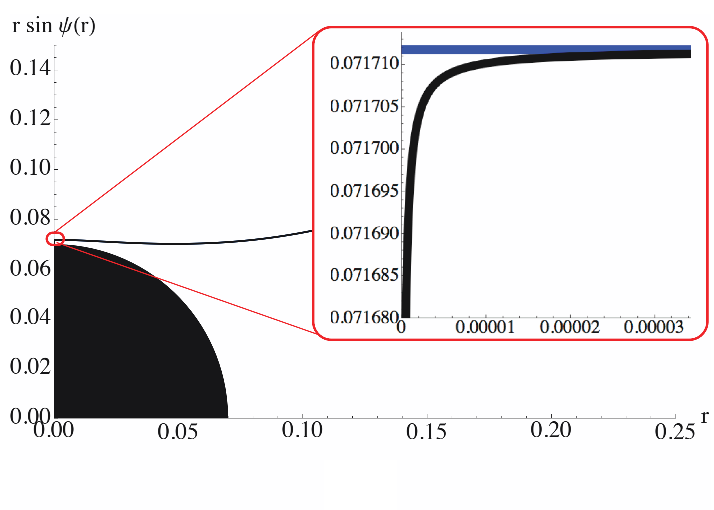
## Gapped embeddings:




## Gapless embeddings: (small $\tilde{J}^0$ )

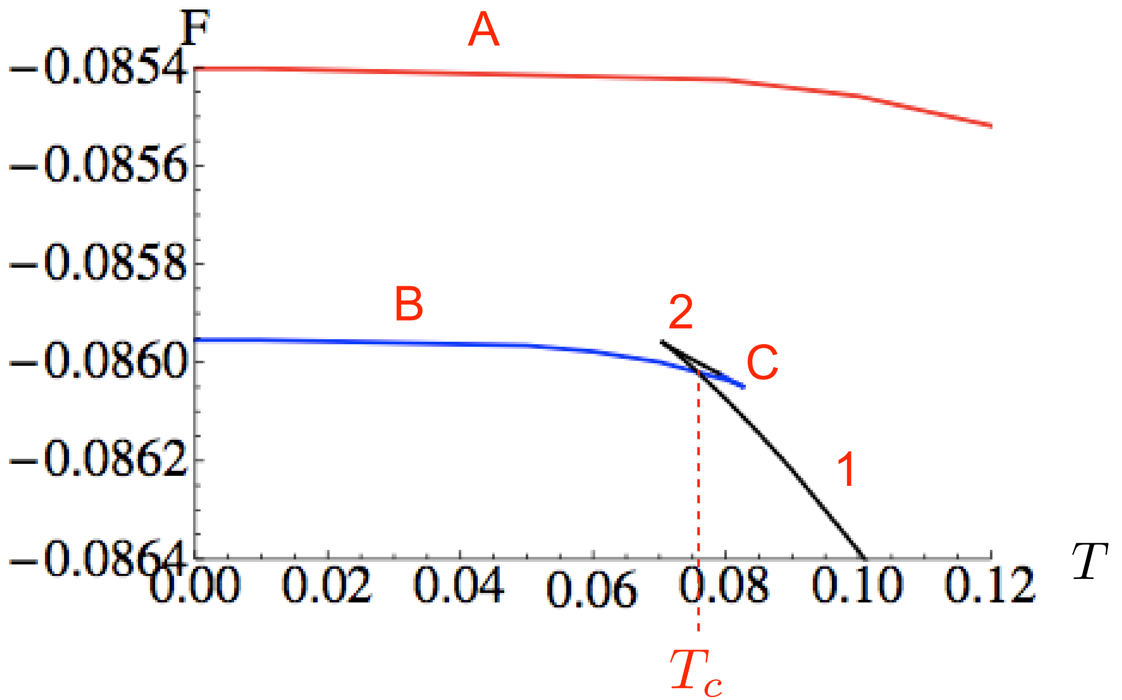



“Spiky” solutions, evolve smoothly into gapped solutions as $\tilde{J}^0 \to 0$.



# Phase diagram

Free energy: $F = S_{on-shell} + \mu J^0 \ , \quad \mu = a_0(\infty)$




1st order QHF/conductor phase transition at $T\!=\!T_c$.

# 4. Conclusions

Holographic model of a QHF of strongly interacting charged fermions in 2+1 dimensions:

- States with $\sigma_{xx} = 0$ and quantized $\sigma_{xy}$ (but not rational).
- Smoothly evolve into conducting states as $J^0$ varies relative to $B$.
- First order conductor/QHF transition at finite temperature.
- quasiparticles, fractional charge? partially wrapped 5-brane (partial baryon), whole baryon = "electron".
- transitions between different QH states? QHF/conductor transition is "1/2 way there", but we need to make the flux $f_1$ dynamical.

# General lesson: bulk ingredients for a holographic QHF

1. $U(1)$ gauge field $A_\mu(x_\mu, r)$

2. Axion-like field $\theta(x_\mu, r)$, with coupling $\theta\, F \wedge F$ $\longrightarrow$ $\sigma_{xy} \neq 0$

3. Dilaton-like field $\phi(x_\mu, r)$ with coupling $V(\phi)\,|F|^2$
   such that $V(\phi(r_0)) = 0$ (dilaton wall).
   $\longrightarrow$ mass gap for charged states, $\sigma_{xx} = 0$

*Note that in our model, 2 and 3 are achieved with a single scalar field.

String theory provides many more possibilities.

Happy hunting!