MANY-BODY PHYSICS & STRING THEORY

ĐORĐE MINIĆ
VIRGINIA TECH

1) NON-EQUILIBRIUM STAT PHYSICS & AdS/CFT
2) GAUGE FIELDS, MEMBANES & CMP
3) TURBULENCE

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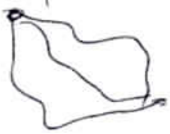
1) ARXIV: 1007.3970 (Đ.M & M. PLEIMLING)
2) ARXIV: 1002.2437 (R.G. LEIGH, A. MAURI, Đ.M & TASSOS PETKOU)
(PRL, 104, 221801 (2010))
3) IN PROGRESS, WITH A.E. STAPLES
(ALSO, ARXIV: 0912:2725, MPL)

JARZYNSKI IDENTITY & AdS/CFT

COMPARE (*)

EQ$_1$



EQ$_2$

AVERAGE

$$\langle \exp(-\beta W) \rangle = \exp(-\beta \Delta G) \quad (*)$$

WORK

THERMODYNAMIC FREE ENERGY

Use JENSEN $\langle e^{A} \rangle \geq e^{\langle A \rangle}$

TO GET $\langle W \rangle \geq \Delta G \rightarrow$ IMPLIED BY SECOND LAW OF THERMOD.

TO (**)

CFT

CFT

$$\langle \exp(\textstyle\int J O) \rangle = Z(\phi) \rightarrow \exp(-S_{bulk}(g, \phi))$$

($JO \rightarrow$ <u>WORK</u>!) $\quad \phi|_{bnd} = J \quad g \rightarrow$ AdS metric $\circlearrowleft$

$$\frac{\eta_{ab} dx^a dx^b + dz^2}{z^2} = ds^2_{AdS}$$

Path-Integral Proof of (*) Hummer & Szabo.

$\vec{x}$ - PHASE SPACE $\quad f(\vec{x}, t) \leftarrow$ PHASE SPACE DENSITY

LIOUVILLE EQ: $\quad \frac{\partial f(x,t)}{\partial t} = L_t f(x,t)$

OPERATOR

Boltzmann $\rightsquigarrow$ stationary solution $\quad L_t e^{-\beta H(x,t)} = 0$

<u>CONSIDER</u>:

$$p(\vec{x}, t) = \frac{e^{-\beta H(\vec{x}, t)}}{\int \mathcal{D}y\, e^{-\beta H(y, 0)}}$$

The distribution $\rho(x,t)$ is stationary

$$\mathcal{L}_t\, p(x,t) = 0$$

& ALSO, BY DEFINITION $\quad \frac{\partial p}{\partial t} = -\beta\left(\frac{\partial H}{\partial t}\right)p$

Thus $\rho(x,t)$ solves the "sink" equation

$$\frac{\partial p}{\partial t} = \mathcal{L}_t\, p - \beta\left(\frac{\partial H}{\partial t}\right)p$$

($t=0 \rightarrow$ equilibrium distribution) ← Feynman-Kac

$$P(x,t) = \left\langle \delta(x-x_t)\, \exp\left(-\beta\int_0^t \frac{\partial H}{\partial t'}(x_{t'},t')\,dt'\right)\right\rangle$$

average over an ensemble of trajectories starting from the equilibrium distribution at $t=0$ & evolving using the Liouville eq.

External work $\quad W_t = \int_0^t \frac{\partial H}{\partial t'}(x_{t'},t')\,dt'$

By definition the difference in free energy.

$$e^{-\beta\Delta G} = \frac{\int dx\, e^{-\beta H(x,t)}}{\int dy\, e^{-\beta H(y,0)}}$$

Thus

$$\boxed{\exp(-\beta \Delta G) = \cancel{\langle \exp(-\beta} \frac{\int dx\, e^{-\beta H(x,1)}}{\int dy\, e^{-\beta H(y,0)}} = \langle \exp(-\beta W) \rangle}$$

Go to QFT & use (Wilson's trick)

$$\beta H(x,t) \to S(\varphi, \Lambda)$$

$$\frac{\partial}{\partial t} \to \Lambda \frac{\partial}{\partial \Lambda} \equiv \frac{\partial}{\partial \tau}$$

RG equation (Scalar field φ in 4d)

$$\frac{\partial e^{-S_I(\varphi,\tau)}}{\partial \tau} = L_\tau e^{-S_I(\varphi,\tau)}$$

$$L_\tau = -\frac{1}{2} \int d^4p\, (2\pi)^4 (p^2+m^2) \frac{\partial K}{\partial \tau} \frac{\partial^2}{\partial \varphi(-p)\, \partial \varphi(p)}$$

where $S = S_0 + S_I$

$$S_0 = \frac{1}{2} \int d^4p\, (2\pi)^{-4}\, \varphi(p)\, \varphi(-p)\, (p^2+m^2)\, K^{-1}\left(\frac{p^2}{\Lambda^2}\right)$$

cut-off function

(regularization scheme is in here!)

The choice of K

RG $\rightarrow$ $$\frac{\partial Z[J]_\tau}{\partial \tau} = 0$$

where $$Z[J]_\tau = \int \mathcal{D}\varphi\, e^{-S_\tau + \int J\varphi} \equiv \langle e^{\int J\varphi} \rangle_\tau$$

Conformal fixed point (CFT)

$$\mathcal{L}_\tau\, e^{-S_I(\varphi,\tau)} = 0$$

Define $$P(\varphi,\tau) = \frac{e^{-S(\varphi,\tau)}}{\int \mathcal{D}\phi\, e^{-S(\phi,\tau_0)}}$$

$$\left(\mathcal{L}_\tau P = 0 \,, \quad \frac{\partial P}{\partial \tau} = -\left(\frac{\partial S}{\partial \tau}\right) P \,; \quad \frac{\partial P}{\partial \tau} = \mathcal{L}_\tau P - \left(\frac{\partial S}{\partial \tau}\right) P \right)$$

$$\Rightarrow P(\varphi,\tau) = \left\langle \delta(\varphi - \varphi_\tau)\, \exp\left(-\int_0^\tau \frac{\partial S}{\partial \tau'}(\varphi_{\tau'},\tau')\, d\tau'\right) \right\rangle$$

External work: $$W_\tau = \int_0^\tau \frac{\partial S}{\partial \tau'}(\varphi_{\tau'},\tau')\, d\tau'$$

Free energy difference

$$e^{-\Delta G} = \frac{\int \mathcal{D}\varphi\, e^{-S(\varphi,\tau)}}{\int \mathcal{D}\phi\, e^{-S(\varphi,\tau_0)}}$$

RG form of Jarzynski:

$$\exp(-\Delta G) = \frac{\int \mathcal{D}\varphi\, e^{-S(\varphi,\tau_1)}}{\int \mathcal{D}\varphi\, e^{-S(\varphi,\tau_0)}} = \langle \exp(-W_{\tau_1}) \rangle$$

Work $W_\tau = -\int J\varphi$

$\Rightarrow$ AdS-like formula

$$\langle \exp(\textstyle\int J\varphi) \rangle = \exp(-\Delta G)$$

If (!)

1) CFT geometrized   Conformal Group

$\Rightarrow$ AdS grav

$$ds^2 = dr^2 + e^{Ar} ds^2_{CFT}$$

$\underline{\tau = Ar}$!!

2) Invariance under RG scheme

$\Rightarrow$ τ reparametrization / $\underline{r}$ reparametrization

(gravity in the r-direction!)

Finally, gravitational energy is given in term of boundary data (r-reparametrization $\Rightarrow H_r = 0$)

$\Delta G = \underline{\Delta S_{bulk}}$ $\Longleftarrow$

Generalized Jarzynski
(⇒ Generalized AdS/CFT ?)

Entropy $\phi(x,\alpha) = -\log \rho_{ss}(x,\alpha)$

ss → steady state

α - parameter.

Generalized Jarzynski

⇒) $\left\langle \exp\left(-\int_0^t dt' \frac{d\alpha}{dt'} \frac{\partial \phi(x,\alpha)}{\partial \alpha}\right)\right\rangle = 1$

generalized velocity.

"entropic force"

Usual Jarzynski $\Phi = -\beta(G-W)$

AdS/CFT generalized !!

$$\left\langle \exp\left(-\int_0^T dT' \frac{d\alpha}{dT'} \frac{\partial \tilde{\Phi}}{\partial \alpha}\right)\right\rangle = 1$$

Usually $\tilde{\Phi} = -(S_{bulk} + \int J O)$

In general $\tilde{\Phi}$ - entropy ($\tilde{\Phi} = -\log \tilde{\rho}_{ss}(\Phi,\alpha)$)

General dynamic system

Dissipative dynamic $\frac{dp_a}{dt} = -k \frac{\partial F}{\partial p_a}$, $\frac{dq_a}{dt} = p_a$

$F \sim \sum_{ij} a_{ij} p^i p^j$

Hyperbolic dynamics $\frac{dx}{dt} = g(x) \Rightarrow x(t) = f^t x(0)$

$(x(t) = e^{\ell t} x_0)$ $\underline{\underline{\ell > 0}}$ Lyapunov

Bowen - Ruelle - Sinai theorem

(on the attractor of the dynamics !!)

$$\lim_{T \to \infty} \frac{1}{T+t} \int_{-T}^{t} d\sigma\, O(f^{T+t} x) = \int \rho(dy)\, O(y)$$

ρ - BRS measure

(and Gibbsian in general) $\underline{\underline{\text{observable}}}$

Entropy

$$\boxed{S(\rho) = \int \rho(dx)\, (-\mathrm{Div}_x\, g)}$$

$$(S(\rho) \geq 0)$$

Linear response theory $\rightarrow$ entropy production.

1) GAUGE FIELDS, MEMBRANES & THE "ENTROPY" PUZZLE
(N^2 vs. $N^{3/2}$ (N^3))

STRINGS $\longleftrightarrow$ GAUGE FIELDS (A_μ)

$$W(C) = \exp\left(i \oint_C A_\mu dx^\mu\right)$$

NAMBU-GOTO (NG) $\sim \int d^2\sigma \sqrt{\det g_{2\times 2}}$

$$\det g_{2\times 2} \Rightarrow \{X^\mu, X^\nu\}^2$$

WHERE $\{A,B\} \equiv \epsilon^{ab}\partial_a A \partial_b B$ $\rightsquigarrow$ Poisson

DEFORMATION $\{A,B\} \rightsquigarrow [\hat{A},\hat{B}]$ COMMUTATOR (LIE BRACKET)

$\hat{A}$ - SQUARE MATRIX

MEMBRANES $\longleftrightarrow$ X (?) ($B_{\mu\nu}$)

$$W(S) = \exp\left(i \oint_S B_{\mu\nu} d\sigma^{\mu\nu}\right)$$

NON-ABELIAN ?

NAMBU-GOTO $\sim \int d^3\sigma \sqrt{\det g_{3\times 3}}$

$$\det g_{3\times 3} \Rightarrow \{X^\mu, X^\nu, X^\lambda\}^2$$

WHERE $\{A,B,C\} = \epsilon^{abc}\partial_a A \partial_b B \partial_c C$ $\rightsquigarrow$ NAMBU

DEFORMATION

$$\{A,B,C\} \rightsquigarrow [\hat{A},\hat{B},\hat{C}] = \hat{A}[\hat{B},\hat{C}] + \hat{B}[\hat{C},\hat{A}] + \hat{C}[\hat{A},\hat{B}]$$

(TRIPLES: CUBIC MATRICES)

NOTE: $\hat{A}_{ij} \rightsquigarrow N^2$

$\hat{\hat{A}}_{ijk} \rightsquigarrow N^3$

FROM AdS / CFT

$AdS_5 \rightsquigarrow$ 4d $\mathcal{N}=4$ SYM SU(N) N D3 BRANES

$AdS_4 \rightsquigarrow$ 3d $\mathcal{N}=8$ M MEMBRANE THEORY

$\uparrow$

IR LIMIT OF

$\mathcal{N}=8$ 3d SYM

COUNTING DEGREES OF FREEDOM

gauge theory, YM $\rightarrow N^2$

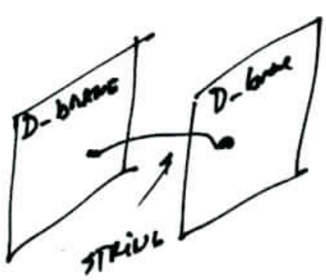
$U(N)$ $\left[\ \Phi\ \ \cdot\ \right]$ N D3 branes

X theory, membranes $\rightsquigarrow N^{3/2}$

N 2-branes $\uparrow$ FROM AdS_4/CFT_3

ALSO, 5-BRANES E.M. DUALS OF 2-BRANES

N^3 FROM AdS_7/CFT_6



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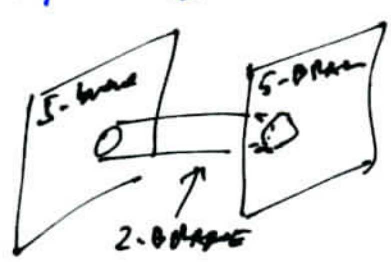

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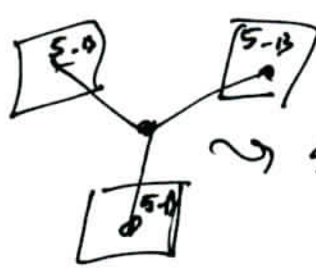


$\rightsquigarrow$ STRING JUNCTION

A technical clue:

VECTOR $O(M)$ MODEL AT LARGE M IN $4-\epsilon$ DIMENSIONS

EUCLIDEAN ACTION $I_E = \int \frac{1}{2}(\partial_\mu \phi^i \partial_\mu \phi^i) - V(\phi^i\phi^i)$

$$[\phi] = \frac{2-\epsilon}{2} \quad \text{IN } 4-\epsilon \text{ DIMENSIONS}$$

HIGHEST MARGINALLY RELEVANT OPERATOR

$$V \sim \phi^{2\frac{(4-\epsilon)}{2-\epsilon}}$$

$$\left(\begin{array}{ll} 3d \sim \phi^6 & ; \quad [A,B,C]^2 \rightsquigarrow \phi^6 \\ 4d \sim \phi^4 & ; \quad [A,B]^2 \sim \phi^4 \\ 6d \sim \phi^3 & \end{array} \right)$$

LET ρ BE LAGRANGE MULTIPLIER FOR ϕ^2

$$V(\rho) \sim \rho^{\frac{4-\epsilon}{2-\epsilon}}$$

EFFECTIVE ACTION

$$Z(J) = e^{W(J)} \quad ; \quad \Gamma(\phi) = \int J^i\phi^i - W(J)$$

$$\text{WHERE} \quad \frac{\delta\Gamma}{\delta\phi^i} = J^i$$

SADDLE POINT $\Gamma = \frac{1}{2}\mathrm{Tr}\log(-\partial^2+\sigma_x) - \frac{1}{2}\int J^i(-\partial^2+\sigma_x)^{-1}J^i + \int[V(\rho_x) - \frac{1}{2}\rho_x\sigma_x]$

$$\sigma_x \sim \frac{\partial V}{\partial \rho} \sim \rho^{\frac{2}{2-\epsilon}}$$

$$\Rightarrow V_{eff} = \frac{\Gamma}{\Omega} \Rightarrow \boxed{V_{eff} \sim \sigma^{\frac{4-\epsilon}{2}}}$$

SPACE-TIME VOLUME

$\left\{ \begin{array}{l} \text{If } \sigma \sim N \\ 3d \sim N^{3/2} \\ 4d \sim N^2 \\ 6d \sim N^3 \end{array} \right.$

$3d \leftarrow 4d \leftarrow 6d$

ϵ - expansion !!

6d - master theory

LET'S MAKE THIS INTUITION PRECISE

2) GAUGE FIELDS, MEMBRANES & NEW VECTOR MODELS IN $d=3,4$

$d=4$ YM : $V_{SU(2)} = \frac{1}{4} \Phi^I_a \Phi^J_b \Phi^I_e \Phi^J_f \epsilon^{abc} \epsilon^{ef}{}_c$

$a,b,c = 1,2,3$

GENERALIZE ($U(N)$: $\epsilon_{abc} \to f_{abc}$)

VECTOR MODEL GENERALIZATION

$$V_N^{(4)} = \frac{g}{2(N-2)!} \Phi^I_a \Phi^J_b \Phi^I_e \Phi^J_f \epsilon^{abc\ldots} \epsilon^{ef}{}_{c\ldots}$$

$a,b = 1,2,\ldots N$ (IN PRINCIPLE N_f FLAVORS $I,J,\ldots$)

EFFECTIVE POTENTIAL A LA HUBBARD–STRATONOVICH

DEFINE : $\rho_{ab} = \Phi_a^I \Phi_b^J$ $N \times N$ SYMMETRIC MATRIX

IMPOSE BY $\sigma_{ab}(\Phi^I_a \Phi^I_b - \rho_{ab})$

$$\Rightarrow V_N^{(4)}(\rho) \equiv g \det{}^{(N)}_{2\times 2}(\rho) \equiv \frac{g}{2}\left[(\mathrm{tr}\rho)^2 - \mathrm{tr}\rho^2\right]$$

$\det^{(N)}_{2\times 2}$: 2×2 SUBDETERMINANT OF ρ

$$V_{eff} = -\frac{1}{64\pi^2}\left[\sum_{a=1}^{N} \bar{\sigma}_a^2 \ln\frac{\bar{\sigma}_a}{\Lambda^2} + \frac{g}{8\pi^2}\sum_{a<b}^{N} \bar{\sigma}_a \bar{\sigma}_b \ln\frac{\bar{\sigma}_a}{\Lambda^2}\ln\frac{\bar{\sigma}_b}{\Lambda^2}\right]$$

$\sigma_a \rightsquigarrow$ EIGENVALUE OF σ_{ab}

$\bar{\sigma}_a$ — SADDLE POINT VALUE

NOW, COMPARE TO THE YM EFFECTIVE POTENTIAL (VIA THE BACKGROUND FIELD METHOD)

6

$$I_{YM} = \int d^4x \, \mathrm{Tr}\left(\frac{1}{2} \partial_\mu \Phi^I \partial_\mu \phi^I - \frac{1}{4} [\phi^I, \phi^J]^2 \right)$$

$$\Phi^I = \bar{\Phi}^I + \varphi^I \quad ; \quad \Phi^I = \Phi^I_\alpha T^\alpha$$ α - ADJOINT OF $U(N)$

CARTAN $\quad \bar{\Phi}^I = \mathrm{diag}(\Phi_a^I)$

$$(\alpha = 1, \dots N^2 \; ; \; a = 1, \dots, N)$$

EFFECTIVE ACTION (AFTER INTEGRATING OUT φ^I)

ZAREMBO

$$\boxed{V_{eff} = \frac{(Vol)_4}{4\pi^2} \sum_{a<b} |\bar{\Phi}_a^I - \bar{\Phi}_b^I|^4 \ln \frac{|\Phi_a^I - \Phi_b^I|^2}{\Lambda^2}}$$

COMPARE TO OUR VECTOR MODEL GENERALIZATION

$$V_{eff} = -\frac{1}{64\pi^2} \left[\sum_{a=1}^{N} \bar{\sigma}_a^2 \ln \frac{\bar{\sigma}_a}{\Lambda^2} + \frac{1}{8\pi^2} \sum_{a<b} \bar{\sigma}_a \bar{\sigma}_b \ln \frac{\bar{\sigma}_a}{\Lambda^2} \ln \frac{\bar{\sigma}_b}{\Lambda^2} \right]$$

$$\sigma_a \longleftrightarrow |\Phi_a^I - \Phi_b^I|^2$$

NOTE, THE EFFECTIVE POTENTIAL DEPENDS ON 2-BODY INTERACTIONS IN BOTH CASES.

VECTOR MODEL: $V(\rho) \sim \rho_1 \rho_2 + \rho_1 \rho_3 + \dots$

THUS $\underline{N^2}$ TERMS (N^2 DEGREES OF FREEDOM)

CAPTURES THE CORRECT COUNTING OF THE YM THEORY

STRINGS $\Leftrightarrow$ 2-BODY INTERACTIONS

GENERALIZE TO $\underline{3d}$!

IN 3d THE ANALOG OF $SU(2)$ YM FOR MEMBRANES IS BGL $SO(4)$ THEORY

(AT PRESENT NO REAL N MEMBRANE THEORY WITH TRIPLES)

$$V_N^{(3)} = \frac{g}{6(N-3)!} \epsilon^{abcd\ldots} \epsilon^{efg} \Phi_a^I \Phi_b^J \Phi_c^K \Phi_e^I \Phi_f^J \Phi_g^K$$

(EXACT FOR $SO(4)$ BGL ; THIS GENERALIZATION FOR $O(N)$)

ONCE AGAIN HUBBARD - STRATONOVICH

$$\rho_{ab} = \Phi_a^I \Phi_b^I \rightsquigarrow \begin{array}{l} N \times N \\ \text{SYMMETRIC MATRIX} \end{array}$$

IMPOSE BY $\sigma_{ab} (\Phi_a^I \Phi_b^I - \rho_{ab})$

$$V_N^{(3)} \equiv g \det{}^{(N)}_{3\times 3} (\rho) \equiv \frac{g}{6} \left[(\operatorname{tr}\rho)^3 - 3 (\operatorname{tr}\rho)(\operatorname{tr}\rho^2) + 2 \operatorname{tr}\rho^3 \right]$$

$\downarrow$

3x3 SUBDETERMINANT OF $N \times N$ ρ_{ab} MATRIX

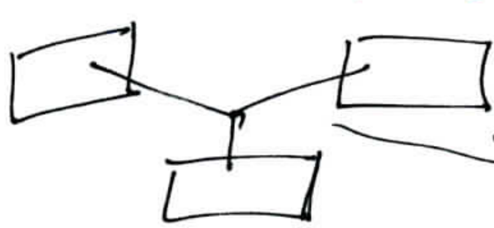
COMPUTE V_{eff} (AT SADDLE POINT):

$$V_{eff} = \frac{1}{24\pi} \left(\sum_a^N \bar{\sigma}_a^{3/2} - \frac{6g}{(4\pi)^2} \sum_{a<b<c}^N (\bar{\sigma}_a \bar{\sigma}_b \bar{\sigma}_c)^{1/2} \right)$$

NO LOGS! (AS COMPARED TO 4d!)

NUMBER OF TERMS IN $V_{eff} \sim \underline{\underline{N^3}}$

$V_{eff}(\rho) \sim \rho_1 \rho_2 \rho_3 + \rho_1 \rho_2 \rho_4 + \ldots$ $\quad$ <u>3-BODY INTERACTION</u>



$\longrightarrow$ STRING JUNCTIONS

3) APPLICATIONS TO CONDENSED MATTER

$d=4$ POTENTIAL

$$V_4 = \epsilon_{abc_1\ldots c_N}\,\epsilon_{ef}{}^{c_1\ldots c_N}\,\Phi_a^I\,\Phi_b^J\,\Phi_e^I\,\Phi_f^J$$

$$a,b,c = 1,2,\ldots N$$
$$I,J = 1,2\ldots N_f$$

WRITE $\epsilon_{ab}{}^{c_1\ldots c_N} \equiv (T^A)_{ab}$, $A = 1,2\ldots \frac{1}{2}N(N-1)$

$$[T^A, T^B]_{ab} = f^{AB}{}_C\,(T^C)_{ab}$$

SO $V_4 = \delta_{AB}\left(\Phi_a^I\,(T^A)_{ab}\,\Phi_b^J\right)\left(\Phi_e^I\,(T^A)_{ef}\,\Phi_f^J\right)$

OR $V_4 = \delta_{AB}\,S_{IJ}^A\,S_{IJ}^B$

$\downarrow$

GENERALIZED SPINS (AREAS)

E.G. $(S^1)^{IJ} = \phi_2^{[I}\phi_3^{J]}$ & CYCLIC

$d=3$ POTENTIAL

$$\epsilon_{abc}{}^{c_1\ldots c_N} = (T^A)_{abc}\ ;\quad A = 1,2,\ldots,\frac{1}{3!}N(N-1)(N-2)$$

SO $V_3 = \delta_{AB}\left[(T^A)_{abc}\,\Phi_a^I\,\Phi_b^J\,\Phi_c^K\right]\left[(T^B)_{efg}\,\Phi_e^I\,\Phi_f^J\,\Phi_g^K\right]$

& $V_3 \equiv \delta_{AB}\,S_{IJK}^A\,S_{IJK}^B$

$\downarrow$

GENERALIZED SPINS (VOLUMES)

E.G. $(S^1)^{IJK} = \phi_2^{[I}\,\phi_3^{J}\,\phi_4^{K]}$

$[I,J,K] \Rightarrow 6$ TERMS

$d=4$, COMPARE TO SCHWINGER BOSONS

$$S^i = \frac{1}{2} a^+_\alpha (\sigma^i)_{\alpha\beta} a_\beta$$

↑ PAULI

WHERE $[a_i, a_j^+] = \delta_{ij}$.

DEFINE $S_\pm = S^1 \pm i S^2$; $N = S^3$

⇓ ⇓

$$[N, S_\pm] = \pm S_\pm \ , \quad [S_+, S_-] = 2N$$

NOW, TAKE
$N_f = 2$
$(I, J = 1, 2)$

$$(S^A)^{IJ} \rightarrow S^A = \phi^1_a \epsilon^A{}_{ab} \phi^2_b$$

$$[S^A, S^B] = f^{AB}{}_C \, S^C$$

PROVIDED $[\phi^1_a, \phi^2_b] = \delta_{ab}$

& other comm. $= \phi$

SIMILARLY IN $D=3$

$N_f = 3$; $(S^A)^{IJK} \Rightarrow S^A = \phi^1_a \phi^2_b \phi^3_c \epsilon^A{}_{abc}$

BUT $[\phi^1_a, \phi^2_b, \phi^3_c] = \Delta_{abc}$

$$\Delta_{abc} = \begin{pmatrix} 1, & a=b=c \\ \phi, & \text{otherwise} \end{pmatrix}$$

NOTE: ϕs in (HEISENBERG)-NAMBU ALGEBRA

ϵs in TRIPLE (LIE) ALGEBRA
(G_3)

NOTE, 2-BODY INTERACTIONS & HOPPING

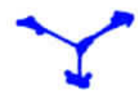
2 HOPPING : $t^{(2)} \sum_{\langle i,j \rangle} c_i^{+} c_j$ $\quad [c_i, c_j^{+}] = \delta_{ij}$

3 HOPPING : $t^{(3)} \sum_{\langle i,j,k \rangle} A_i B_j C_k$ $\quad [A_i, B_j, C_k] = \delta_{ijk}$

RECALL (TAKHTAJAN)

HEISENBERG-NAMBU $[A, B, C] = 1$

CAN BE REALIZED WITH



OPERATORS A, B C $\rightsquigarrow$ ROOTS OF SU(3)

(ORDINARY HEISENBERG $\rightsquigarrow$ a, a^{+} ROOTS SU(2))

$\longleftrightarrow$

PHYSICS: 3-BODY INTERACTIONS
DOWN BY PHASE SPACE ?

(COUNTER) EXAMPLES:

1) $\frac{5}{2}$ STATE IN FQHE (TOPOLOGICAL QUANTUM COMPUTING)
(EXACT 3-BODY WAVEFUNCTION)

2) OPTICAL LATTICES
(3-BODY INTERACTION RELEVANT!)

CONJECTURE: 3-LATTICE OF ANYONIC QUASIPARTICLES IN FQHE

4) APPLICATIONS TO TURBULENCE

NAVIER-STOKES $\rho(\partial_t v_i + v_j \partial_j v_i) = -\partial_i p + \nu \partial_j^2 v_i$

FULLY DEVELOPED TURBULENCE $\Rightarrow \nu = 0$

WANT TO COMPUTE $Z[J] = \langle \exp(-\int j v) \rangle$

WILSON LOOP (MIGDAL) $W(C) = \exp(-\frac{1}{\gamma} \int dx_i v_i)$

$\omega_{ij} \equiv \partial_i v_j - \partial_j v_i$

$W(C) = \exp(-\frac{1}{\gamma} \int d\sigma_{ij} \omega_{ij})$

MIGDAL (NAVIER-STOKES): $i\gamma \partial_t W(C) = H_C W(C)$ $(\gamma \equiv \hbar_{eff})$

$$H_C \equiv \nu^2 \int_C dx_j \left(i \partial_k \frac{\delta}{\delta \sigma_{kj}} + \int d^3 y_i \frac{y_k - x_k}{4\pi |y-x|^3} \frac{\delta^2}{\delta\sigma_{kj}(x) \delta\sigma_{ik}(y)} \right)$$

KOLMOGOROV: (in 3+1 & 2+1) $\langle v_a v_b \partial_a v_b \rangle \sim \text{const}$ ($v \sim \ell$ from N-S)

$\frac{v^2}{t} \sim \epsilon \Rightarrow v \sim (\epsilon \ell)^{1/3}$ $\nu \sim \frac{1}{3}$

$\langle v^i(\ell) v^j(0) \rangle \sim \ell^{2/3} \delta^{ij}$

OBSERVED!

$\Downarrow$

$$\boxed{W_{KOL} \sim \exp\left(-\frac{\alpha}{\nu} \epsilon^{1/3} A^{2/3}\right)}$$

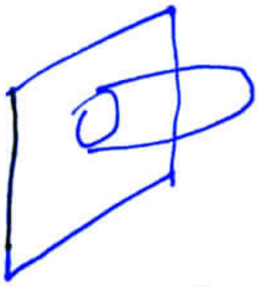
KRAICHNAN: $\langle v_a \omega_b \partial_a \omega_b \rangle \sim \text{const}$ $\vec{\omega} = \vec{\nabla} \times \vec{v}$

$\frac{\omega^2}{t} \sim \text{const} \Rightarrow v \sim \frac{\ell}{t_0}$ $\nu = 1$

$$\boxed{W_{KR} \sim \exp\left(-\frac{\alpha}{\nu t_0} A\right)} \Rightarrow \boxed{\text{Area law}}$$

As in QCD string.

BUT TURBULENCE CONFORMAL $\Rightarrow$ dual AdS_4



$$S_{NG} = -\frac{1}{2\pi L'}\int d^2\sigma \sqrt{\det(g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu)}$$

$\downarrow$ AdS_4 metric

MALDACENA

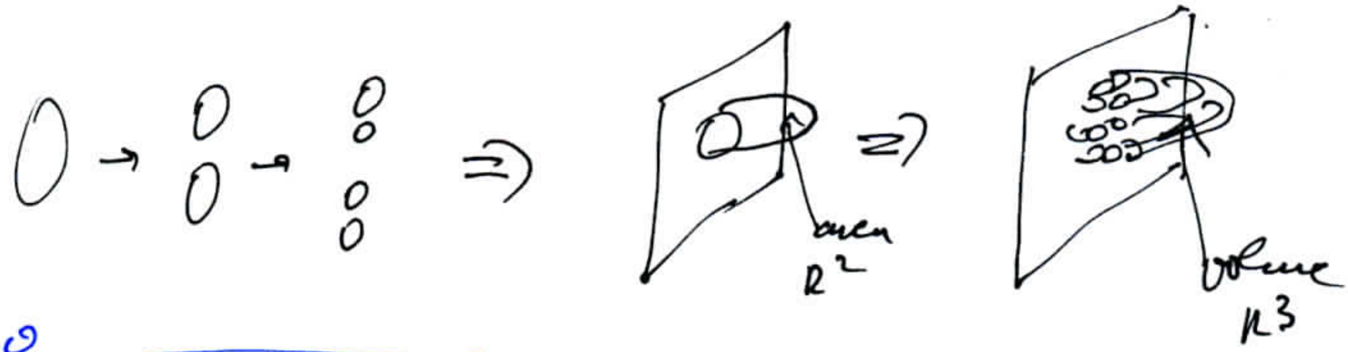
COMPUTE & SUBTRACT INFINITE PART

$$\langle W(A)\rangle = \exp(-S_{NG}) = \exp\left(-\frac{L^2_{AdS_4}}{L'}\right)$$

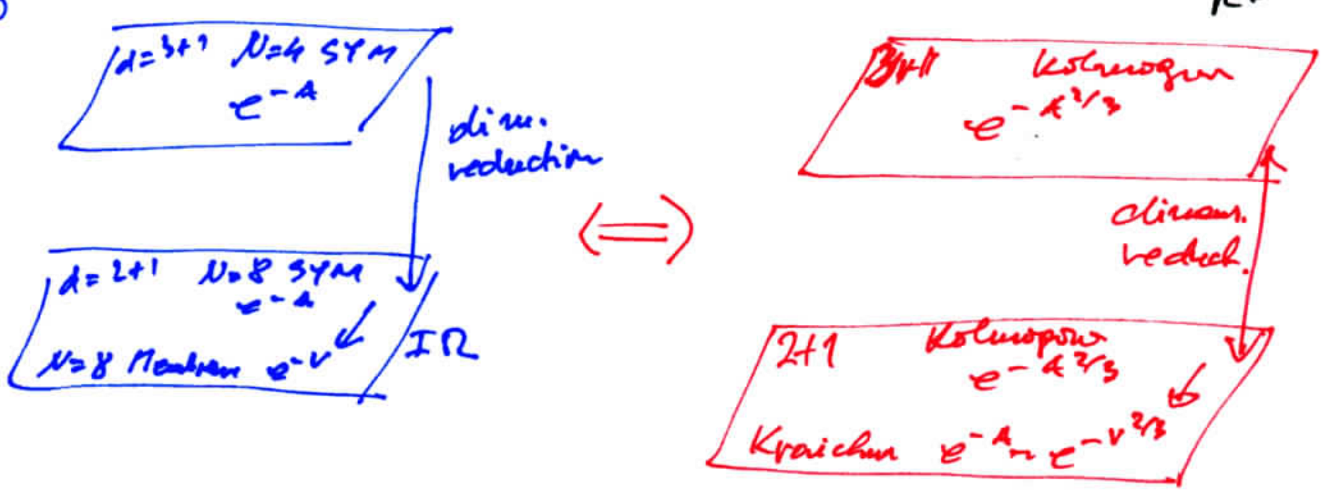
$$\underline{W(A) \sim \exp(-fA)}$$

Note: gauge theory $W(C) \sim e^{-A}$ — Kolmogorov $W(C) \sim e^{-A^{2/3}}$

membrane $W(S) \sim e^{-V}$ — Kraichnan $W(C) \sim e^{-A} \sim e^{-V^{2/3}}$



Also



PROPOSAL:

KRAICHNAN in 2+1 = STRING THEORY in AdS_4

$$\langle e^{-\int j \mathcal{O}(v)} \rangle = e^{-S_{eff}(\phi)}$$

$$J = \phi_{BND}$$

($\varkappa$ - expansion parameter $\nu \sim \hbar_{eff}$)

Also,

Kolmogorov $\leftarrow$ Kraichnan

$e^{-A^{2/3}}$ $\qquad$ $e^{-A} \sim e^{-V^{2/3}}$

(string) $\Longleftarrow$ (membrane)

transition

↓ weak coupling. $\qquad$ ↓ strong coupling

Finally, in

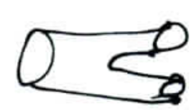
$$\partial_t v_i + \underbrace{v_j \partial_j v_i}$$

↓

from a cubic vertex

(3-body interaction i.e. membrane

OR STRING FIELD TH.



LESSON FOR AdS/CMP

⇒ CONFORMAL BOOTSTRAP OF MANY-BODY CFTs.