Holographic Dimers

Wolfgang Mück

Università di Napoli "Federico II" and INFN, Sezione di Napoli

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Introduction

Idea of Kachru, Karch and Yaida [arXiv/0909.2639]

- Break translation invariance by impurity $\sim$ D5-brane in D3-brane background

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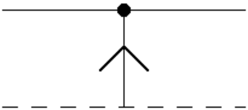

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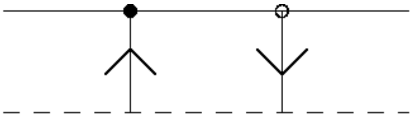

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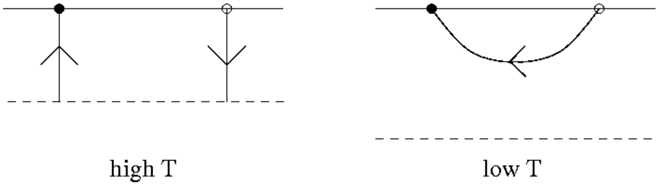
- Break translation invariance by impurity $\sim$ D5-brane in D3-brane background
- At finite temperature, the D5-brane ends at the horizon
- For 2 nearby impurities, take $D5$ and $\overline{D5}$-branes
- They can either stretch to the horizon or connect the impurities
- First order phase transition between the connected and disconnected configurations



Some related work

Rey, Theisen and Yee hep-th/9803135

Brandhuber, Itzhaki, Sonnenschein and Yankielowicz hep-th/9803137
string connecting a quark-antiquark pair at finite temperature

Hartnoll and Kumar hep-th/0603190 D5-brane at finite temperature (Polyakov loop of antisymmetric representation)

Yamaguchi hep-th/0603208, Hartnoll and Kumar hep-th/0605027
circular Wilson loops of antisymmetric representations, D5 brane dual and matrix model calculation, zero temperature

Gomis and Passerini hep-th/0604007 proof of duality between Wilson loops and D-brane configurations, impurity action

Outline

What is a holographic dimer?

Setup

Background AdS_5-Schwarzschild $\times S^5$

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{L^2}dx_i^2 + L^2(d\theta^2 + \sin^2\theta d\Omega_4^2)$$

$$f(r) = \frac{r^2}{L^2}\left(1 - \frac{{r_+}^4}{r^4}\right) \qquad L^4 = 4\pi g_s N\alpha'^2$$

$$F_5 = dC_4 \qquad C_4 = \frac{r^4}{L^4}dt \wedge d^3x + L^4 C(\theta)d\Omega_4$$

$$C(\theta) = \frac{3}{2}\theta - \frac{3}{2}\sin\theta\cos\theta - \sin^3\theta\cos\theta$$

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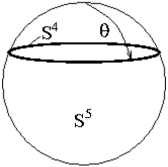
D5-brane wrapping S^4

AdS_5					S^5	
t	x_1	x_2	x_3	r	θ	S^4
τ	$x(\rho)$	0	0	$r(\rho)$	const.	+



Action

D5-brane action (DBI + WZ)

$$I_{D5} = \frac{N}{3\pi^2\alpha'}\int d\rho d\tau \left[-\sin^4\theta\sqrt{r'^2 + \frac{r^2}{L^2}f(r)x'^2 - F_{\tau\rho}^2} + C(\theta)F_{\tau\rho}\right]$$

$$\frac{N}{3\pi^2\alpha'} = T_5 V_4 L^4 \qquad\qquad F_{\tau\rho} = -\partial_\rho A_\tau$$

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Boundary terms at large r_{cutoff}

$$I_b = -\,\mathrm{sgn}\, r' \int d\tau \left(r\pi_r + A_\tau \pi_A \right)$$

$$\pi_r = \frac{\delta I_{D5}}{\delta r'} \qquad\qquad \pi_A = \frac{\delta I_{D5}}{\delta A'_\tau}$$

take care of boundary conditions and cancel bulk divergences
Drukker and Fiol [hep-th/0501109]

Equations of motion

E.o.m. for θ gives

$$F_{\tau\rho} = \cos\theta \sqrt{r'^2 + \frac{r^2}{L^2} f(r) x'^2}$$

Quantization of fundamental string charge

$$\boxed{n = \frac{N}{\pi}(\theta - \sin\theta\cos\theta) \qquad n = 0, 1, \ldots, N}$$

This also solves the e.o.m. for A_τ

The same formula appears for circular (susy) Wilson loops of antisymmetric representations

Hartnoll and Kumar [hep-th/0603190], Yamaguchi [hep-th/0603208]

Equations of motion

The system now reduces to a Nambu-Goto string with a θ-dependent effective string tension

$$\begin{aligned} T_{eff} &= \frac{N}{3\pi^2\alpha'}[C(\theta)\cos\theta - \sin^5\theta] \\ &= \frac{N}{3\pi^2\alpha'}\left[\frac{3n}{2N}\pi\cos\theta - \sin^3\theta\right] \end{aligned}$$

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$$= \frac{N}{3\pi^2\alpha'}\left[\frac{3n}{2N}\pi\cos\theta - \sin^3\theta\right]$$

E.o.m. for x

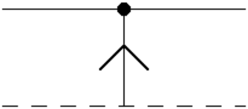
$$\frac{\frac{r^2}{L^2}f(r)x'}{\sqrt{r'^2 + \frac{r^2}{L^2}f(r)x'^2}} = c$$

This implies

$$c^2L^2 = r^2 f(r) \qquad \text{where } r' = 0$$

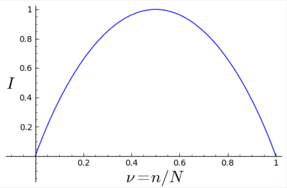
Straight brane solution

The simplest solution is given by $c = x' = 0$, the brane goes straight to the horizon.



Renormalized on-shell action (after Wick rotation)

$$I_{|} = -\frac{N}{3\pi^2\alpha'}\beta r_+ \sin^3\theta = -\frac{N}{3\pi}\sqrt{\lambda}\sin^3\theta$$



inverse temperature

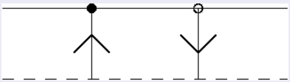
$$\beta = \frac{\pi L^2}{r_+}$$

$I_{|}$ is just $1/2$ the value of the circular Wilson loop.
What is the connection?

Straight brane solution, 2 impurities

2 parallel branes ($\|$)

take a pair of $D5$ and $\overline{D5}$ branes



On-shell action

$$I_{\|} = 2I_{|} = -\frac{2N}{3\pi}\sqrt{\lambda}\sin^3\theta$$

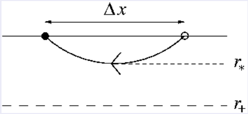
thermodynamics is trivial

$$I = -\beta F \qquad \Rightarrow \qquad S_{\|} = \frac{2N}{3\pi}\sqrt{\lambda}\sin^3\theta, \quad E_{\|} = 0$$

Dimerized configuration

Brane connects impurities (∪)

$$c^2 = \frac{{r_*}^4}{L^4}(1-\gamma) \qquad \gamma = \frac{{r_+}^4}{{r_*}^4} \in (0,1)$$



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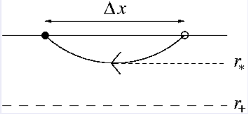

Distance between impurities

$$\Delta x = 2\int_{r_*}^{\infty} dr \left|\frac{dx}{dr}\right|$$

gives

$$\frac{\Delta x}{\beta} = \frac{1}{2\pi}\,\mathrm{B}\left(\frac{3}{4},\frac{1}{2}\right)\gamma^{\frac{1}{4}}\sqrt{1-\gamma}\,\mathrm{F}\left(\frac{1}{2},\frac{3}{4};\frac{5}{4};\gamma\right)$$

$\mathrm{B}\,(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$, $\mathrm{F}\,(a,b;c;x)$ hypergeometric function

Connected brane - dimerized configuration

On-shell action

$$I_{\cup} = -S_{\|}\gamma^{-1/4} \lim_{r_c \to \infty} \left[\int_{1}^{r_c/r_*} du \sqrt{\frac{u^4 - \gamma}{u^4 - 1}} - \frac{r_c}{r_*} \right]$$

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To get a doable integral, integrate by parts, so that the boundary term is cancelled in the limit. Do not simply absorb the divergent term into the integrand. Result is

$$I_{\cup} = S_{\parallel} \frac{1}{4} \mathrm{B}\left(\frac{3}{4}, \frac{1}{2}\right) \gamma^{-1/4} \mathrm{F}\left(-\frac{1}{2}, -\frac{1}{4}; \frac{1}{4}; \gamma\right)$$

Thermodynamics

introduce dimensionless variables

$$\mathcal{T} = \frac{\Delta x}{\beta} \qquad \mathcal{F} = \frac{\Delta x}{S_{\parallel}} F$$

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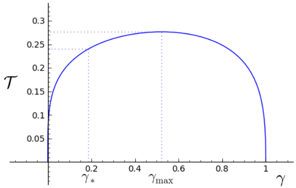
2 dimer solutions exist for

$$\mathcal{T} < \mathcal{T}_{max} = 0.27665$$
$$\gamma_{max} = 0.52147$$



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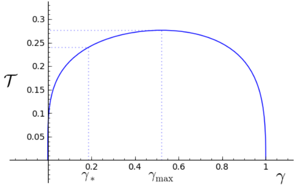
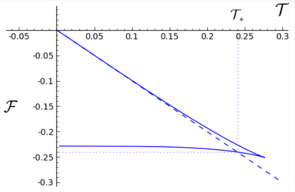


compare to $\mathcal{F}_{\parallel} = -\mathcal{T}$



1^{st} order phase transition

$$\mathcal{T}_* = 0.24004$$
$$\gamma_* = 0.18555$$

Thermodynamics

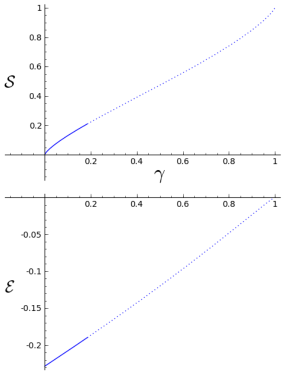
Entropy

$$\mathcal{S}_{\cup} = \frac{1}{2}\,\mathrm{B}\left(\frac{7}{4},\frac{1}{2}\right)\gamma^{\frac{3}{4}}\,\mathrm{F}\left(\frac{1}{2},\frac{3}{4};\frac{9}{4};\gamma\right)$$

Energy

$$\mathcal{E} = -\frac{\pi}{2}\gamma^{-\frac{1}{2}}\sqrt{1-\gamma}\,\mathcal{T}^2$$

latent heat at phase transition
$\Delta\mathcal{E} = 0.18962$



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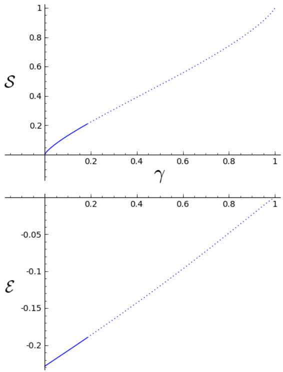


Landau theory of phase transition

take γ as order parameter, $\gamma = 1$ is disconnected solution

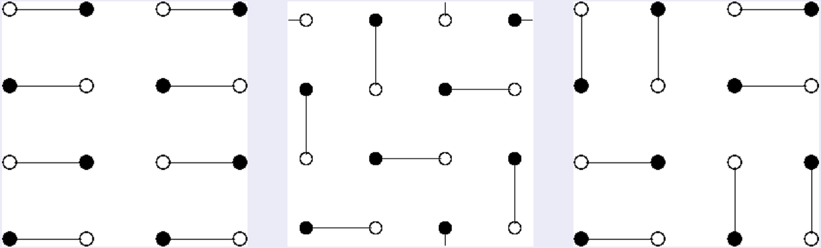
$$\mathcal{F}(\gamma, \mathcal{T}) = -\mathcal{S}(\gamma)\mathcal{T} + \mathcal{E}(\gamma)$$

Applications

Lattices and glasses

Kachru, Karch and Yaida [arXiv/0909.2639]

On a square lattice, there is a plethora of possible configurations



Of course, one could just play around with various configurations. To seriously study the interaction between the dimers that could lead to long-range order, one needs to go beyond the probe approximation.

Fermi—non-Fermi liquid transitions

Kachru, Karch and Yaida [arXiv/1009.3268], see also Sachdev [arXiv/1006.3794]

Couple impurity operators to conduction electrons

$$S = S_{strong} + \sum_{J,J'} \int dt\, c_J^\dagger [\delta_{JJ'}(i\partial_t + \mu) + t_{JJ'}] c_{J'}$$

$$+ g \sum_J \int dt\, (c_J^\dagger \mathcal{O}_J^F + c.c.)$$

for D5-branes

$$\mathcal{O}_J^F = \chi_J^\dagger \lambda_{\mathcal{N}=4}(J) \chi_J$$

λ is gaugino field, χ are probe fermions (see later)

Fermi—non-Fermi liquid transitions

Green function for c, schematically

$$\mathcal{G}_g(\mathbf{k},\omega) \sim \frac{1}{\omega - v|\mathbf{k}-\mathbf{k}_F| - g^2\mathcal{G}_\mathcal{O}}$$

Phases are generically distinguished by low-frequency behaviour of $\mathcal{G}_\mathcal{O}$

undimerized phase spectrum is not gapped

$\mathcal{G}_\mathcal{O} \sim \omega^{2\Delta-1}$ ($\omega \ln \omega$ for $\Delta = 1$)

c's form a non-Fermi liquid

dimerized phase spectrum is gapped

$\mathcal{G}_\mathcal{O} \to$ const.

c's form a Fermi liquid with shifted $\mathbf{k}_F$

Hints at a field theory description

Dual field theory for D5-branes

Gomis and Passerini [hep-th/0604007]

A single D5-brane with charge n is dual to a defect operator in the representation

$$\Gamma_n = \left.\begin{array}{|c|}\hline \\ \hline \\ \hline \vdots \\ \hline \\ \hline \end{array}\right\} n$$

Impurity action

$$S = S_{\mathcal{N}=4} + \sum_J \int dt \left[i\chi_J^\dagger \partial_t \chi_J + \chi_J^\dagger (A_0 + \phi)\chi_J + \mu_J(\chi_J^\dagger \chi_J - n_J) \right]$$

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Can one reproduce impurity entropy?

does not match the degeneracy of Γ_n

$$\ln d_n = -N[\nu \ln \nu + (1-\nu)\ln(1-\nu)] \qquad \nu = n/N \text{ fixed}$$

Overscreened multichannel $SU(N)$ Kondo model

Parcollet, Georges, Kotliar, Sengupta [PRB 58, 3794 (1998)]

Action

$$
\begin{aligned}
S = &- \int_0^\beta d\tau d\tau' \sum_{i\alpha} c^\dagger_{i\alpha}(\tau) \mathcal{G}_0^{-1}(\tau - \tau') c_{i\alpha}(\tau') \\
&+ \int_0^\beta d\tau \sum_\alpha \left[f^\dagger_\alpha \partial_\tau f_\alpha + i\mu \left(f^\dagger_\alpha f_\alpha - \nu \right) \right] \\
&+ \frac{J}{N} \int_0^\beta d\tau \sum_{i\alpha\beta} c^\dagger_{i\alpha} c_{i\beta} \left(f^\dagger_\beta f_\alpha - \nu \delta_{\alpha\beta} \right)
\end{aligned}
$$

c: heat bath of conduction electrons with Green function $\mathcal{G}_0$

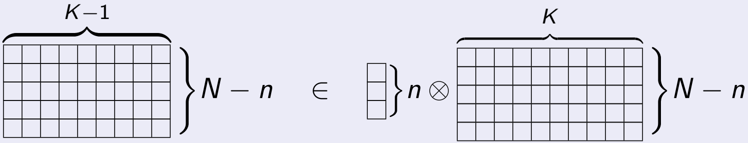
f: impurity fermions

channel index $i = 1, 2, \ldots, K$

$SU(N)$ index $\alpha, \beta = 1, 2, \ldots, N$ $\quad \nu = n/N$

Overscreened multichannel $SU(N)$ Kondo model

Strong coupling ground state



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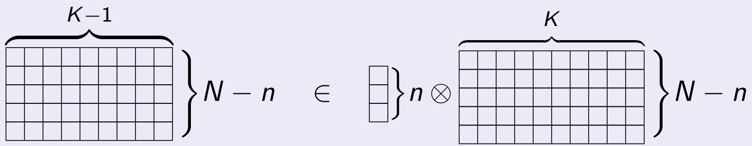


Field theory calculation of impurity entropy

- saddle point approximation in large-N limit
- find chemical potential μ as a function of temperature
- use thermodynamic relation

$$\frac{\partial S}{\partial \nu} = -\frac{\partial \mu}{\partial T}$$

- integrate with respect to ν

Overscreened multichannel $SU(N)$ Kondo model

Spectral asymmetry

for $\omega \to 0$, $\operatorname{Im}\omega > 0$

$$\mathcal{G}_f^R(\omega) \sim h(\gamma,\theta)\frac{e^{-i\pi\Delta_f - i\theta}}{\omega^{1-2\Delta_f}} \qquad \theta \in (-\pi\Delta_f, \pi\Delta_f)$$

breaks symmetry $\mathcal{G}(\beta-\tau) = \mathcal{G}(\tau)$

$$2\Delta_f = \frac{1}{1+\gamma} \qquad \gamma = \frac{K}{N}$$

θ is related to ν

$$\theta = 2\pi\Delta_f\left(\frac{1}{2} - \nu\right)$$

Overscreened multichannel $SU(N)$ Kondo model

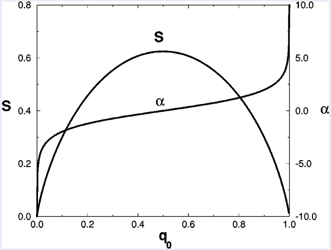
Impurity entropy

$$S = \frac{N}{2\pi\Delta_f}\left[f\left(2\pi\Delta_f\right) - f\left(2\pi\Delta_f\nu\right) - f\left(2\pi\Delta_f(1-\nu)\right)\right]$$

with

$$f(x) = \int_0^x du \, \ln \sin u$$

This matches precisely the degeneracy of the strong coupling ground state.



picture from PRB 58, 3794 (1998)
$q_0 = \nu$

Heisenberg spin glass

Georges, Parcollet, Sachdev [PRB 63, 134406 (2001)]

Hamiltonian

$$H = \sum_{i<j} J_{ij} S_i^a S_j^a$$

with Gaussian distributed J_{ij}

Mean field description

single site model

$$S = S_B - \frac{J^2}{2N} \int_0^\beta d\tau d\tau' \, Q^{ab}(\tau - \tau') S^a(\tau) S^b(\tau')$$

with

$$Q^{ab}(\tau - \tau') = \left\langle S^a(\tau) S^b(\tau') \right\rangle$$

Heisenberg spin glass

Anti-symmetric S

representation of S by Abrikosov fermions f_α with

$$\sum_\alpha f_\alpha^\dagger f_\alpha = \nu N$$

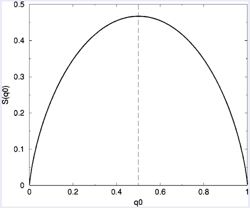
Calculation of entropy

Relation to spectral asymmetry angle is

$$\frac{\theta}{\pi} + \frac{1}{4}\sin\theta = \frac{1}{2} - \nu \qquad \theta \in (-\pi/4, \pi/4)$$

Get entropy from

$$\frac{\partial S}{\partial \nu} = \ln \frac{\sin(\pi/4 - \theta)}{\sin(\pi/4 + \theta)}$$



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