Lifshitz Geometries in String and M-Theory

Jerome Gauntlett

Aristomenis Donos

Aristomenis Donos, Nakwoo Kim, Oscar Varela

AdS/CMT

Attempt to use the AdS/CFT correspondence to study strongly coupled condensed matter systems.

- One focus: systems with strongly coupled "quantum critical points" - phase transitions at zero temperature.
- Another focus: superconductors (superfluids) [Gubser; Hartnoll,Herzog,Horowitz].

  In fact some superconductors ("heavy fermions", high Tc cuprates) are associated with quantum critical points.

- If a critical point has full relativistic conformal invariance in the far IR, one can aim to find AdS solutions of string or M-theory that describe the system.

- The dual description of the CM system at finite temperature is given by asymptotically AdS black hole (brane) solutions.

- One would also like to study CM systems at finite charge density (finite chemical potential). Want a bulk gauge field, dual to a global symmetry current, and to construct electrically charged AdS black holes.

- Leads to novel black hole solutions with interesting thermodynamic instabilities

- The CM critical points may exhibit a scaling that is not isotropic

$$t \to \lambda^z t, \qquad x^i \to \lambda x^i, \qquad z \neq 1$$

where z is the "dynamical exponent". Note: $i = 1, \ldots, d-1$

- Lifshitz(z) geometries [Kachru, Liu, Mulligan]

$$\boxed{ds^2 = -r^{2z}dt^2 + r^2(dx^i dx^i) + \frac{dr^2}{r^2}}$$

$$t \to \lambda^z t, \qquad x^i \to \lambda x^i, \qquad r \to \lambda^{-1} r$$

Call these $Lif_{d+1}(z)$ geometries and so $Lif_D(z=1) = AdS_D$

- Schrodinger(z) geometries [Son; Balasubramanian, McGreevy]

$$\boxed{ds^2 = -r^{2z}dt^2 + r^2(2d\xi dt + dx^i dx^i) + \frac{dr^2}{r^2}}$$

$$t \to \lambda^z t, \qquad x^i \to \lambda x^i, \qquad r \to \lambda^{-1} r, \qquad \xi \to \lambda^{2-z}\xi$$

These geometries have an extra holographic coordinate, ξ

∂_ξ dual to particle number

$-t\partial_i + x^i\partial_\xi$ dual to non relativistic boosts

When z=2 there are also special conformal transformations - Schrodinger algebra

For both classes we also would like to construct black holes that asymptote to these geometries.

Holographic dictionary not yet well understood.
[Guica,Skenderis,Taylor,van Rees]

Most work has been carried out in "Bottom Up" models.
Find solutions in a simple theory of gravity with a few other degrees of freedom e.g. a vector, plus one or two scalar fields.

Advantages:

- Simple
- Models should exist somewhere in string landscape?
- Might capture some new universal behaviour?

Disadvantages:

- Does the model arise in string theory? Is there any well defined dual (conformal) field theory?
- If phenomenological model is viewed just as an approximation to a model that can be embedded in string theory, it might not capture e.g. interesting low temperature behaviour.

Alternative "Top Down" approach - construct explicit solutions of D=10 or D=11 supergravity (also brane probe limits)

Advantages:

- One is studying bone-fide dual field theories
- One can often study infinite classes of solutions - universality
- Can find new phenomenon

(eg superconducting domes [JPG,Sonner,Wiseman])

Disadvantages:

- Hard! and harder to find solutions relevant for physical systems?

We have been pursuing Top Down supergravity constructions.

Schrodinger(z) Solutions

First examples were Bottom Up.

First Top Down constructions used either duality transformations and/or consistent KK truncations. [Maldacena,Martelli,Tachikawa] [Adams,Balasubramanian,McGreevy][Herzog,Rangamani,Ross]

Led to infinite examples of (supersymmetric) solutions: [Yoshida,Hartnoll][Donos,JPG]

TypeIIB: uses D=5 (Sasaki-)Einstein spaces to construct solutions dual to d=3 field theories with Sch(z) symmetry with $z \geq 3/2$

D=11: uses D=7 (Sasaki-)Einstein spaces to construct solutions dual to d=2 field theories with Sch(z) symmetry with $z \geq 5/4$

Lifshitz(z) Solutions

Bottom Up: examples are well studied

Top Down: Can they be realised in String/M-theory?

Li, Nishioka, Takayangi: No-go theorem for SUGRA solutions
Hartnoll, Polchinski, Silverstein, Tong: Three schematic constructions.
Balasubramanian, Narayan: Recent examples with z=2

New results:
Donos, JPG

Infinite $Lif_4(z = 2)$ solutions of Type IIB and D=11SUGRA

Infinite $Lif_3(z = 2)$ solutions of D=11 SUGRA

Donos, JPG, Kim, Varela:

Single $Lif_4(z \sim 39)$ solution of D=11 SUGRA

Plan

- Review construction of Schrodinger(z) solutions of type IIB/D=11 supergravity using (Sasaki-)Einstein spaces
- Construction of $Lif_4(z=2)$ solutions of type IIB and $Lif_3(z=2)$ solutions of D=11both using (Sasaki-)Einstein spaces
- A new consistent KK truncation of D=11 SUGRA on $\Sigma_3 \times S^4$ where $\Sigma_3 = H^3, S^3, R^3$
- Construction of $Lif_4(z \sim 39)$ solution of D=11 SUGRA
- Other AdS/CMT applications of new KK truncation: interesting new black holes

Type IIBSolutions

$$
\begin{aligned}
ds^2_{10} &= \Phi^{-1/2}\left[2dx^+dx^- + h(dx^+)^2 + dx_1^2 + dx_2^2\right] + \Phi^{1/2}ds^2(M_6) \\
F_5 &= dx^+ \wedge dx^- \wedge dx_1 \wedge dx_2 \wedge d\Phi^{-1} + *_M d\Phi \\
G_3 &= dx^+ \wedge W \\
P_1 &= g dx^+
\end{aligned}
$$

$ds^2(M_6)$ Ricci-flat Metric - non compact

Φ, h, W Defined on M_6 and also depend on x^+ and $g = g(x^+)$

$$
\begin{aligned}
\nabla^2_M \Phi &= 0 \\
dx^+ \wedge dW = d *_M W &= 0 \\
\nabla^2_M h + 4g^2\Phi + |W|^2_M &= 0
\end{aligned}
$$

- Supersymmetry: if $M_6 = CY_3$ and W is $(1,1)$ and primitive then solution preserves two supersymmetries

- We are interested in the case when M_6 is a cone

$$ds^2(M_6) = dr^2 + r^2 ds^2(E_5)$$

If $M_6 = CY_3$ then $E_5 = SE_5$

- We are also interested in the case where $\Phi = r^{-4}$

- If $h = W = g = 0$ then we have $AdS_5 \times E_5$

Type IIB Schrodinger(z) Solutions

- Take $h = r^{2z-2} f$, $W = d[r^z \omega]$ where f, ω are defined on the Einstein space E_5 and independent of x^+. $g = 0$

- Take $x^+ = t$ to be dual time coordinate

$$\boxed{ds^2_{10} = r^2 \left[2dtdx^- + dx_1^2 + dx_2^2\right] + f\, r^{2z} (dt)^2 + \frac{dr^2}{r^2} + ds^2(E_5)}$$

$$\boxed{\begin{array}{c} (d^\dagger d + dd^\dagger)\omega = z(z+2)\omega, \qquad d^\dagger \omega = 0 \\ \nabla^2_E f + 4(z^2-1) f + z^2 |\omega|^2_E + |d\omega|^2_E = 0 \end{array}}$$

$\omega = 0, \quad z \geq 3/2$ $\qquad\qquad$ $\omega \neq 0, \quad z \geq 2$

$z = 2$ achieved when ω is dual to a Killing Vector

$Lif_4(z=2)$ Solutions in type IIB and D=11

- Take $h = r^{-2} f$ with f, W just defined on E_5 (like z=0 in the Sch case) and they can also depend on x^+. Also $g = g(x^+)$

- Now take $x^- = t$ to be the dual time coordinate

$$\boxed{\begin{aligned} ds^2_{10} &= r^2 \left[2dx^+dt + dx_1^2 + dx_2^2\right] + \frac{dr^2}{r^2} + f\,(dx^+)^2 + ds^2\,(E_5) \\ &= -\frac{r^4}{f}dt^2 + r^2\,\left(dx_1^2 + dx_2^2\right) + \frac{dr^2}{r^2} + f\left(dx^+ + \frac{r^2}{f}dt\right)^2 + ds^2\,(E_5) \end{aligned}}$$

We then take x^+ to be a periodic coordinate and we have obtained infinite class of $Lif_4(z=2)$ solutions if $f > 0$ and

$$\boxed{\begin{aligned} dx^+ \wedge dW = d *_E W &= 0 \\ \nabla^2_E f - 4f + 4\,|g|^2 + |W|^2_E &= 0 \end{aligned}}$$

Note: $g = W = 0 \Rightarrow f = 0$ Also: can have $f = 1$

- Solutions preserve two supersymmetries if $E_5 = SE_5$

- For $SE_5 = T^{1,1}$ have constructed explicit solutions, including a solution with $f = 1$

- Have also constructed explicit solutions for $SE_5 = Y^{p,q}$

No x^+ dependence, set $g = 0$ T-dualise and then uplift to D=11

$$\boxed{\begin{aligned} ds^2 &= f^{1/3}\left[-\frac{r^4}{f}\,dt^2 + r^2\left(dx_1^2 + dx_2^2\right) + \frac{dr^2}{r^2}\right] + f^{-2/3}\left[Dy_1^2 + Dy_2^2\right] + f^{1/3}ds^2\left(E_5\right) \\ F_4 &= dt \wedge d\left(r^4 dx_1 \wedge dx_2 - \frac{r^2}{f}\,Dy_1 \wedge Dy_2\right) \end{aligned}}$$

Have set $W = dA^{(1)} + i\,dA^{(2)}$ and defined $Dy_i = dy_i - A_i$

$$\boxed{\nabla_E^2 f - 4f + |W|_E^2 = 0} \qquad \boxed{dW = d^\dagger W = 0}$$

- Avoids no-go theorem of Li, Nishioka, Takayangi because it has a more general type of four-form flux than they considered

- When $E_5 = SE_5$ the two susies anticommute to give a (Lifshitz) time translation. Implies stable? (Contrast with Schrodinger case)

$Lif_3(z=2)$ Solutions in D=11

- Similar construction in D=11 supergravity using seven dimensional Einstein spaces
- Schrodinger(z) solutions with $z \geq 5/4$
- $Lif_3(z=2)$ solutions

$$
\begin{aligned}
ds^2 &= \frac{1}{4}\left[-\frac{r^4}{f}dt^2 + r^2 dx^2 + \frac{dr^2}{r^2}\right] + \frac{f}{4}\left[dx^+ + \frac{r^2}{f}dt\right]^2 + ds^2\left(E_7\right) \\
G &= \frac{3}{8}r^2\, dx^+ \wedge dt \wedge dx \wedge dr + \frac{1}{2}dx^+ \wedge V
\end{aligned}
$$

$$
\begin{aligned}
dx^+ \wedge dV = d *_E V &= 0 \\
\nabla^2_E f - 8f + |V|^2_E &= 0
\end{aligned}
$$

Consistent KK Truncations

- Consider KK reduction of some high D dimensional theory on some internal manifold M. Obtain a low d dimensional theory with an infinite number of fields.

- A consistent KK truncation is one where we can keep a finite set of fields such that any solution of the low d theory involving these fields uplifts to an exact solution of the high D theory.

- E.g. KK reduction on S^1 - keep ALL $U(1)$ invariant modes, the usual g, A_μ, ϕ .The modes which are truncated are charged and can't source these neutral modes

- Note that in this case it so happens that the modes that are kept are massless and the ones that are discarded are massive, so one can also view the truncation in an effective field theory sense

- Usually consistent KK truncations don't exist.

Some general results known in context of AdS/CFT

Conjecture-Theorem (in many cases):
Consider most general supersymmetric $AdS \times M$ solutions of D=10/11 SUGRA.
Can always consistently KK reduce on M keeping the supermultiplet containing the graviton.
In the dual CFT language keep the fields dual to the superconformal current multiplet $(T_{ab}, J_a, \dots)$
[JPG,Varela]

For $AdS_4 \times SE_7$ solutions of D=11 SUGRA and
$AdS_5 \times SE_5$ solutions of type IIB SUGRA we can do a lot more by keeping breathing mode multiplets.
[JPG, Kim,Varela,Waldram][Cassani,Dall'agata,Faedo][Liu,Szepiotowski,Zhao]
[JPG,Varela][Skenderis,Taylor,Tsimpis]

Have been used to study top-down holographic superconductivity
[JPG,Sonner,Wiseman][Gubser,Herzog,Pufu,Tesileanu]

Consistent Truncation from Wrapped M5-branes and Lifshitz(z) Solutions of D=11 SUGRA

- Calabi-Yau (M_6, J, Ω) with a SLag 3-cycle Σ_3

$$Vol(\Sigma_3) = Re(\Omega)|_{\Sigma_3}$$

- An M5-brane can wrap Σ_3 and preserve supersymmetry.

- Worldvolume of M5 is $\mathbb{R}^{1,2} \times \Sigma_3$. It preserves supersymmetry because R-symmetry currents are switched on. In the IR we obtain a d=3 QFT with N=2 susy.

- Via AdS/CFT we know that if $\Sigma_3 = H_3/\Gamma$ then this QFT is actually an N=2 SCFT that is dual to an $AdS_4 \times H^3/\Gamma \times S^4$ solution of D=11.

[Maldacena,Nunez][JPG, Kim,PakisWaldram]

- Construct D= 11 solutions that asymptote to $AdS_7 \times S^4$ in the UV with

$$ds^2(AdS_7) = \frac{dr^2}{r^2} + r^2 \left[dx^\mu dx^\mu + ds^2(H^3/\Gamma)\right]$$

and in the IR to $AdS_4 \times H^3/\Gamma \times S^4$

This describes an RG flow "across dimensions" to the N=2 SCFT in d=3

Note: the metric is a warped product and the S^4 is fibred over H^3/Γ

Programme: study this N=2 SCFT at finite temperature and charge density (with respect to the abelian R-symmetry) using a consistent KK truncation of D=11 SUGRA.

There exists a consistent KK truncation of D=11 SUGRA on $\Sigma_3 \times S^4$ where $\Sigma_3 = H^3/\Gamma, S^3, T^3$ Donos, JPG, Kim, Varela

Step 1: Reduce D=11 SUGRA on S^4 to get D=7 SO(5) gauged SUGRA.

Step 2: Reduce D=7 SO(5) gauged SUGRA on Σ_3 keep breathing mode multiplet

Obtain:

D=4 N=2 gauged supergravity (metric, vector)
+1 Vector multiplet (1 vector + 2 scalars)
+2 Hypermultiplets (8 scalars)

Scalar parametrise the coset $\dfrac{SU(1,1)}{U(1)} \times \dfrac{G_{2(2)}}{SO(4)}$

When $\Sigma_3 = H_3/\Gamma$ this D=4 theory has a susy AdS_4 solution which uplifts to the $AdS_4 \times H_3/\Gamma \times S^4$ solution

This D=4 theory has a susy $Lif_4(z)$ solution which uplifts to $Lif_4(z) \times H^3/\Gamma \times S^4$

with $z \sim 39$

This avoids no-go theorem due to fibration structure.

Can also use the D=4 truncated theory to study the N=2 SCFT dual to M5-branes wrapping SLag H^3/Γ cycles at finite T and μ

The usual D=4 AdS-RN black hole (brane) of Einstein-Maxwell theory describes the SCFT at high temperatures.
Are there additional branches of black hole solutions as one lowers the temperature?

Study (some) linearised fluctuations within the D=4 theory.
Find two new branches of black hole solutions:

I. Branch of black holes carrying charged scalar hair - new top down holographic superconducting black holes

II. Branch of black holes with no charged scalar hair - top down analogues of black holes with neutral hair c.f. [Horowtiz,Roberts][Goldstein, Kachru,Prakash,Trivedi]

Second branch appears at a higher temperature.

Final Comments

New Top Down solutions relevant for AdS/CMT.

Lifshitz(z) solutions with z=2: $Lif_4(z=2)$ solutions of type IIB and D=11 SUGRA. $Lif_3(z=2)$ solutions of D=11

- Closely associated with Schrodinger(z) solutions.
- Dual interpretation?
- Can we construct finite temperature Lifshitz/Schrodinger solutions?
- Can we construct solutions that interpolate between Lifshitz, Schrodinger and AdS?
- Do the new Lifshitz solutions arise as the ground states of some class of holographic superconductors? [Gubser,Nellore]

New consistent KK truncation of D=11 SUGRA on $\Sigma_3 \times S^4$ with $\Sigma_3 = H^3, S^3, R^3$ to an N=2 D=4 gauged SUGRA

- Rich set of solutions:

$AdS_4 \times H^3/\Gamma \times S^4$ with N=2 susy and also N=0

$Lif_4(39) \times H^3/\Gamma \times S^4$ with N=?

- Are these related via holographic flows?
- Have initiated a study of the N=2 SCFT at finite T and μ
- Shown there are new branches of black holes -- can we construct (numerically) the fully back reacted black holes? What is the phase structure? Is there holographic superconductivity? What are the ground states?
- Does the consistent truncation extend to other N=2 susy solutions of D=11 SUGRA of the form $AdS_4 \times N_7$ corresponding to M5-branes wrapping other SLag 3-cycles?