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Holographic d-wave superconductors

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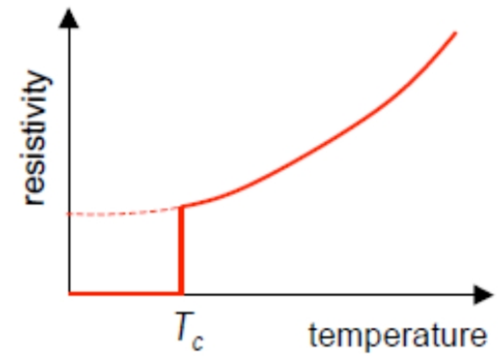
based on 1006.0731
1007.1981

Outline

- Holographic superconductors
- d-wave superconductors
  charged massive spin-2 fields
- Fermionic operators and spectral function
- d-wave gap, Dirac nodes, Fermi arcs
- Future directions

Real superconductors

•Superconductor: a system characterized by a transition to a state with zero resistivity (below T_c)



It can be modeled by spontaneous breaking of $\mathrm{U}(1)$ e.m. Gizburg Landau

•BCS: weakly coupled description. *E.g.* cuprates look strongly coupled

•Phenomenology depends on the nature of the order parameter

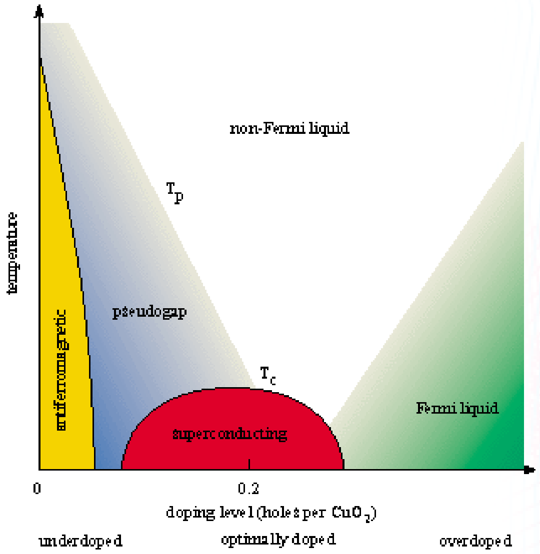
•Cuprates: d-wave superconductors (spin-2 order parameter)

•Interesting phenomenology, ARPES & STM:
d-wave gap, Dirac nodes, Fermi arcs, pseudo-gap, ...

•Motivation:
how far can we go without using the details of the atomic structure, but only "symmetries" and basic features?

Experimental results

- Normal phase: Fermi surface
- Superconducting phase: Fermi surface is gapped
- d-wave: anisotropic gap $\sim |\cos 2\theta|$
- 4 nodes
- Dirac cones at the nodes



- In pseudo-gap phase: nodes open into Fermi arcs

Holographic Superconductors

- Holographic superfluid: a field theory (CFT) at temperature $T \geq 0$ and non-zero chemical potential ρ, with $\mathrm{U}(1)$ global symmetry and charged order parameter ψ

- Holographic superconductor: weakly gauge the $\mathrm{U}(1)$ (photon)

- Study the CFT at strong coupling via AdS/CFT

- Study the behavior of extra operators (not directly involved in condensation), *e.g.* fermionic operators

- Bottom-up approach: focus on subset of fields

AdS/CFT Map

CFT_d, T_{mn}	Gravity (Einstein-Hilbert) in asymptotically AdS_{d+1} $\quad g_{\mu\nu}$
U(1) global, J_m	U(1) gauge symmetry, A_μ
Charged order parameter O of dimension Δ	charged (massive) field ψ of mass m
CFT at T > 0	BH in AdS $\quad ds^2 \underset{z\to 0}{\sim} \frac{z^2}{L^2}\left(-dt^2+d\,\vec{x}_{d-1}^2+dz^2\right)$
Turn on chemical potential source $J_0^{(s)}$ for J_m	$A_m \sim J_m^{(s)} z^{d-\Delta-1} + \langle J_m \rangle\, z^{\Delta-1}$
No source $O^{(s)}$, read off VEV	$O \sim O^{(s)} z^{\#} + \langle O \rangle\, z^{\#}$
Causal CFT	regular & infalling b.c. at horizon

d-wave

- The order parameter is d-wave
  → massive charged spin-2 field in the bulk
  (graviton: massless neutral)

- Various problems could arise:
  wrong number of d.o.f.
  ghosts
  faster than light signals on non-trivial background

- What action?

Spin-2 fields

- *E.g.*: $L = -\partial_\rho \varphi_{\mu\nu} \partial^\rho \varphi^{\mu\nu} - m^2 \varphi_{\mu\nu} \varphi^{\mu\nu}$ in $\mathbb{R}^{d,1}$

- Number of d.o.f.:

symmetric $\varphi_{\mu\nu}$	massive spin-2 particle	*constraints*
$\frac{(d+1)(d+2)}{2}$	$\frac{d(d+1)}{2} - 1$	$d+2$

- The extra modes contain *ghosts*.

Fierz-Pauli action

- Fierz-Pauli action (unique quadratic and 2 derivatives):

$$L_{\mathrm{FP}} = -\left|\partial_\rho \varphi_{\mu\nu}\right|^2 + 2\left|\varphi_\mu\right|^2 - 2\varphi^\mu \partial_\mu \varphi + \left|\partial_\mu \varphi\right|^2 - m^2\left(\left|\varphi_{\mu\nu}\right|^2 - \varphi^2\right)$$

where $\varphi_\mu \equiv \partial^\nu \varphi_{\nu\mu}$ and $\varphi \equiv \varphi^\mu_\mu$

- Get the equations:

$$\left.\begin{aligned} 0 &= (\Box - m^2)\varphi_{\mu\nu} \\ 0 &= \varphi_\nu \\ 0 &= \varphi \end{aligned}\right\} \; d+2 \text{ constraints}$$

- Correct number of d.o.f., no ghosts, causal propagation

Fierz-Pauli action

$$L_{\mathrm{FP}} = -\left|\partial_\rho \varphi_{\mu\nu}\right|^2 + 2\left|\varphi_\mu\right|^2 - 2\varphi^\mu \partial_\mu \varphi + \left|\partial_\mu \varphi\right|^2 - m^2\left(\left|\varphi_{\mu\nu}\right|^2 - \varphi^2\right)$$

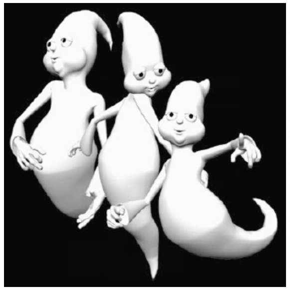
- Coefficients uniquely fixed by either:
  - require d+2 constraint equations
  - use Stückelberg formalism and require not higher derivative terms

$$\varphi_{\mu\nu} \rightarrow h_{\mu\nu} + \frac{1}{m}\left(\partial_\mu B_\nu + \partial_\nu B_\mu\right) - \frac{1}{m^2}\partial_\mu \partial_\nu X$$

$$\delta h_{\mu\nu} = \partial_\mu \lambda_\nu + \partial_\nu \lambda_\mu$$
$$\delta B_\mu = \partial_\mu \lambda - m \lambda_\mu$$
$$\delta X = 2\mathrm{m}\lambda$$

  - require no ghosts nor tachyons in the propagator

Charged Spin-2 field on background



- Covariant derivative: $\partial_\mu \;\rightarrow\; D_\mu = \partial_\mu + \Gamma_\mu - iq\,A_\mu$

  → get the problems back!

  $$[D_\mu, D_\nu] = R^a_{\mu\nu} L^a - i\,q\,F_{\mu\nu}$$

  Solved by coupling to curvatures $R_{\mu\nu\rho\lambda}$ & $F_{\mu\nu}$

- Write down most general quadratic 2-derivative action (up to dim d+1 operators)

  Require d+2 constraint equations

- Metric: background must be Einstein (vacuum)

  → probe limit

Buchbinder Gitman Pershin

- $F_{\mu\nu}$: background can be generic

Federbush

Charged spin-2 field on background

- Action:

$$L_{\text{spin 2}} = -|D_\rho \varphi_{\mu\nu}|^2 + 2|\varphi_\nu|^2 + |D_\mu \varphi|^2 - (\varphi^{*\nu} D_\nu \varphi + \text{c.c.}) - m^2(|\varphi_{\mu\nu}|^2 - |\varphi|^2)$$
$$+ 2 R_{\mu\nu\rho\lambda} \varphi^{*\mu\rho} \varphi^{\nu\lambda} - \frac{1}{d+1} R|\varphi|^2 - i q F_{\mu\nu} \varphi^{*\mu\lambda} \varphi_\lambda^\nu$$

- Einstein background → *probe limit*

$$L_{\text{tot}} = R - \Lambda - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + L_{\text{spin 2}}$$

- Large q and small ρ: matter & gauge fields do not backreact on the metric

$$\varphi_{\mu\nu} = \tilde{\varphi}_{\mu\nu}/q$$
$$A_\mu = \tilde{A}_\mu/q$$
$$\rho = \tilde{\rho}/q$$
$$L_{\text{mat}} = \tilde{L}_{\text{mat}}/q$$

Charged Spin-2 field on background

$$L_{\text{spin }2} = -|D_\rho \varphi_{\mu\nu}|^2 + 2|\varphi_\nu|^2 + |D_\mu \varphi|^2 - (\varphi^{*\nu} D_\nu \varphi + \text{c.c.}) - m^2(|\varphi_{\mu\nu}|^2 - |\varphi|^2)$$
$$+ 2 R_{\mu\nu\rho\lambda} \varphi^{*\mu\rho} \varphi^{\nu\lambda} - \frac{1}{d+1} R|\varphi|^2 - i\, q\, F_{\mu\nu} \varphi^{*\mu\lambda} \varphi_\lambda^\nu$$

- $F_{\mu\nu}$ → new problem: faster than light signals

Velo

$$v_{\max} \simeq \left(1 + \frac{q|F_{\mu\nu}|}{m^2}\right)$$

at large momenta
(hyperbolic for small $F_{\mu\nu}$)

- Argyres-Nappi: causal action on 26-dimensional flat spacetime & constant $F_{\mu\nu}$.
  Each term is non-linear function of $F_{\mu\nu}$.

Charged spin-2 field on background

- Argyres-Nappi action:

$$\mathcal{L}_{\mathrm{GF}} = \mathrm{tr}[H^* h^* H \cdot (\mathcal{P} H \mathcal{P}) h] - \mathrm{tr}[H^* h^* H](\mathcal{P} H \mathcal{P}) \mathrm{tr}(H h H^*)$$
$$+ 2[\mathrm{tr}(H^* h^* H h) - \mathrm{tr}(H^* h^* H) \mathrm{tr}(H h H^*)] - 2(\mathcal{P}^* H^* h^*) \cdot H \cdot (\mathcal{P} H h)$$
$$+ [\mathcal{P}^* \mathrm{tr}(H^* h^* H)] \cdot H \cdot (\mathcal{P} H h) + (\mathcal{P}^* H^* h^*) \cdot H \mathcal{P}\, \mathrm{tr}(H h H^*) + 4\mathrm{i}\, \mathrm{tr}(H^* h^* H \epsilon h).$$

- We think of our action as first terms in expansion in

$$\frac{q\left|F_{\mu\nu}\right|}{m^2} \ll 1 \qquad \rightarrow \qquad \Delta \gg 1$$

AdS/CFT

- Field/operator correspondence: $\varphi_{\mu\nu} \leftrightarrow O_{mn}$

$$\varphi_{mn} \underset{z\to 0}{\sim} \langle O_{mn} \rangle\, z^{\Delta-2} + O_{mn}^{(s)}\, z^{d-\Delta-2} \qquad m^2 L^2 = \Delta(\Delta - d) \qquad \Delta > d$$

- Ansatz:

$$ds^2 = \frac{L^2}{z^2}\left(-f(z)\,dt^2 + d\,\vec{x}_d^2 + \frac{dz^2}{f(z)}\right) \qquad f(z) = 1 - \left(\frac{z}{z_h}\right)^d \qquad T = \frac{d}{4\pi z_h}$$

$$A = \phi(z)\; dt$$

$$\varphi_{xy} = \frac{L^2}{2z^2}\psi(z) \qquad \Rightarrow D^\mu \varphi_{\mu\nu} = \varphi = 0$$

- Equations: same as s-wave!

- Boundary conditions: chemical potential ρ

  → critical temperature T_c

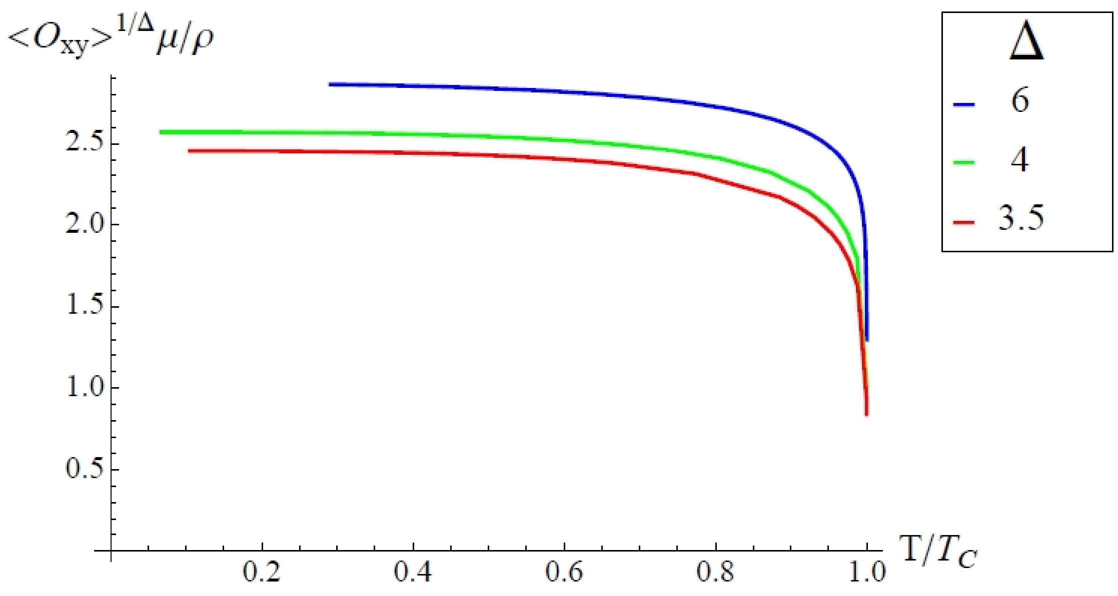
AdS/CFT

- There exist a critical temperature T_c
- $T > T_c$: normal state (charged BH)

$$A = \mu\left[1 - \left(\frac{z}{z_h}\right)^{d-2}\right] dt \qquad \varphi_{\mu\nu} = 0$$

- $T < T_c$: superconducting phase (condensate) $\quad \varphi_{xy} \neq 0$

d = 2+1:



- Compute conductivity σ_{mn}: in d = 2+1 isotropic at leading order

Fermions

- In AdS/CFT only gauge-invariant operators: study fermionic operators

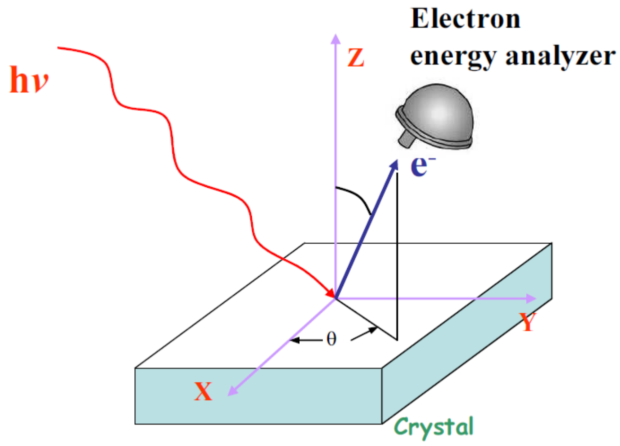
  bulk spinor Ψ $\leftrightarrow$ composite fermionic operator O_Ψ

  (from p.o.v. of weakly gauged $\mathrm{U}(1)$ "composite electron")

- Compute retarded Green's function & spectral function:

$$G_R(t,\vec{x}) = i\Theta(t)\langle\{O_\Psi(t,\vec{x}), O_\Psi^+(0)\}\rangle \qquad \rho(\omega,\vec{k}) = \mathrm{Tr}\ \mathrm{Im}\ G_R(\omega,\vec{k})$$

- Direct connection with ARPES



- Green's function used to detect Fermi surface in normal phase

Liu, McGreevy, Vegh
Cubrovic, Zaanen, Schalm

in the following $d = 2+1$

Fermionic action

- What action?
  Write down all terms up to dimension 5 (on background):

$$L_\Psi = i\bar{\Psi}(\Gamma^\mu D_\mu - m_\zeta)\Psi + \begin{matrix} \eta^* \varphi^*_{\mu\nu} \overline{\Psi^c}\,\Gamma^\mu D^\nu \Psi + \text{h.c.} \\ \varphi^*_{\mu\nu} \varphi^{*\mu\nu} \overline{\Psi^c}(c_1 + c_2 \Gamma_5)\Psi + \text{h.c.} \end{matrix} + i|\varphi_{\mu\nu}|^2 \bar{\Psi}(c_3 + i c_4 \Gamma_5)\Psi$$

- We use:

$$L_\Psi = i\bar{\Psi}(\Gamma^\mu D_\mu - m_\zeta)\Psi + \eta^* \varphi^*_{\mu\nu} \overline{\Psi^c}\,\Gamma^\mu D^\nu \Psi + \text{h.c.} \qquad D_\mu = \partial_\mu + \omega_\mu - i\frac{q}{2}A_\mu$$

- Majorana-like term:
  considered for s-wave
  it gives rise to a gapped Fermi surface

Faulkner, Horowitz, McGreevy, Roberts, Vegh

Retarded Green's function

- 2-point function: $G_R(t,\vec{x}) = i\,\Theta(t)\langle\{O_\Psi(t,\vec{x}), O_\Psi^+(0)\}\rangle$

- EOM
  probe
  asymptotic
  infalling b.c.

$$0 = (\Gamma^\mu D_\mu - m_\zeta)\Psi + 2\mathrm{i}\,\eta\,\varphi_{\mu\nu}\Gamma^\mu D^\nu \Psi^c$$

$$\Psi = e^{-i\omega t + i\vec{k}\cdot\vec{x}}\Psi^{(\omega,\vec{k})}(z) + e^{i\omega t - i\vec{k}\cdot\vec{x}}\Psi^{(-\omega,-\vec{k})}(z)$$

$$\Psi = \begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} \qquad \Psi_\alpha \underset{z\to 0}{\sim} \begin{pmatrix}O(z)\\ (\sigma_1 S)_\alpha\end{pmatrix} e^{-m_\zeta L z} + \begin{pmatrix}R_\alpha\\ O(z)\end{pmatrix} e^{m_\zeta L z}$$

$$R_\alpha^{(\omega,\vec{k})} = M_\alpha{}^\beta S_\beta^{(\omega,\vec{k})} + \tilde{M}_\alpha{}^\beta S_\beta^{(-\omega,-\vec{k})c} \qquad G_R(\omega,\vec{k}) = -i M \gamma^t$$

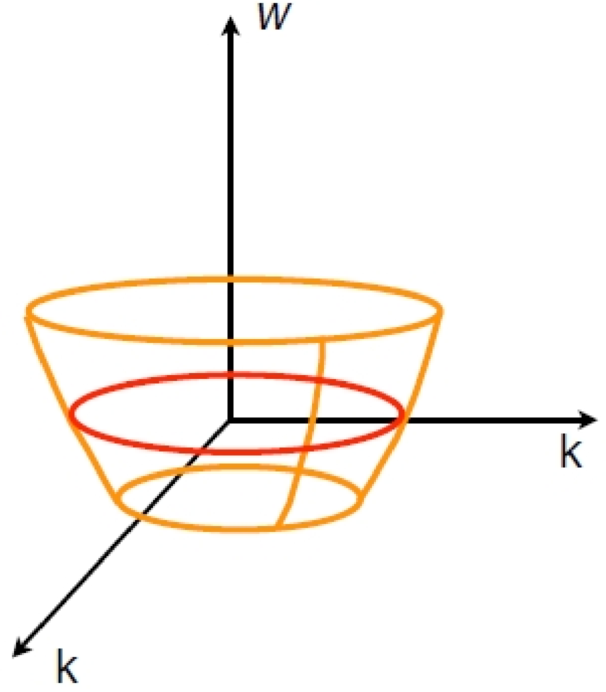
- Spectral function (density of states): $\rho(\omega,\vec{k}) = \mathrm{Tr}\ \mathrm{Im}\ G_R(\omega,\vec{k})$
  sharp peaks → dispersion relation $\omega(\vec{k})$ of quasi-normal modes

The gap – *E.g.* s-wave

- Peaks in spectral func. → $\omega(k)$ quasi-normal modes

- $\eta = 0$: Fermi surface

$$\begin{aligned} 0 &= D_{(1)}\Psi_1 \\ 0 &= D_{(2)}\Psi_2 \end{aligned} \quad \Rightarrow \quad \omega = E(\vec{k})$$

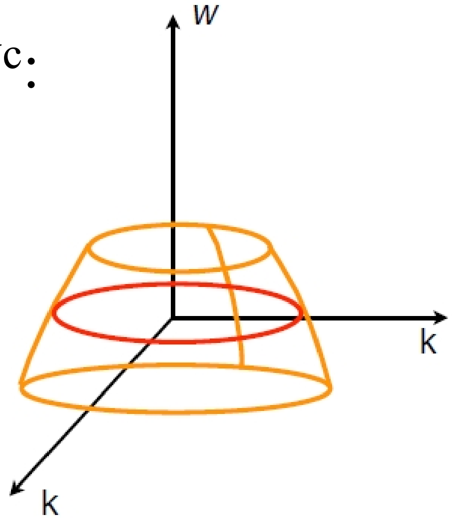


The gap – *E.g.* s-wave

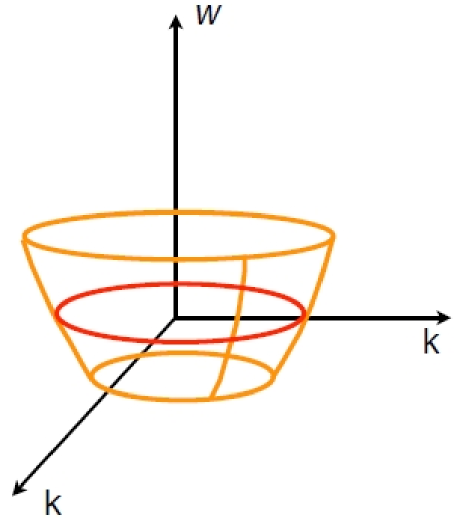
- Peaks in spectral func. → $\omega(k)$ quasi-normal modes
- $\eta \neq 0$: gap

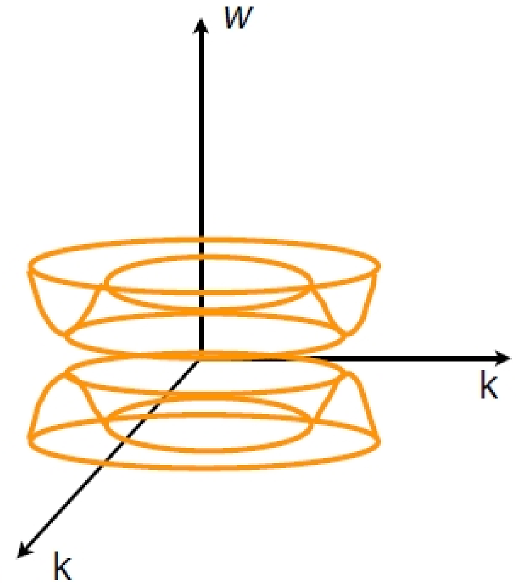
$$0 = D_{(1)}\Psi_1 + \eta\Psi_2^* \qquad \Rightarrow \qquad \Psi: \quad \omega = E(\vec{k})$$
$$0 = D_{(2)}\Psi_2 + \eta\Psi_1^* \qquad\qquad\quad \Psi^c: \quad \omega = -E(-\vec{k})$$

Ψ^c:



Ψ:

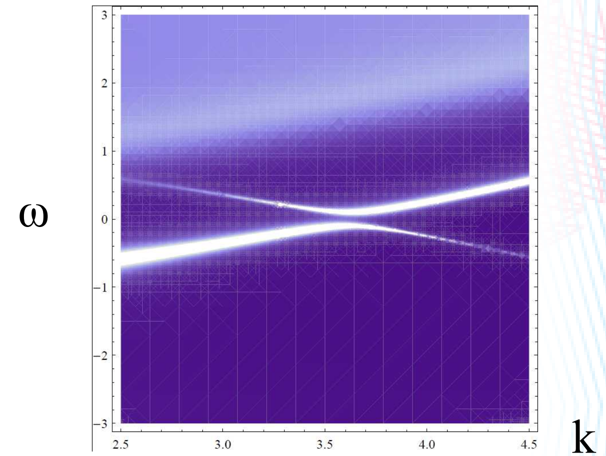




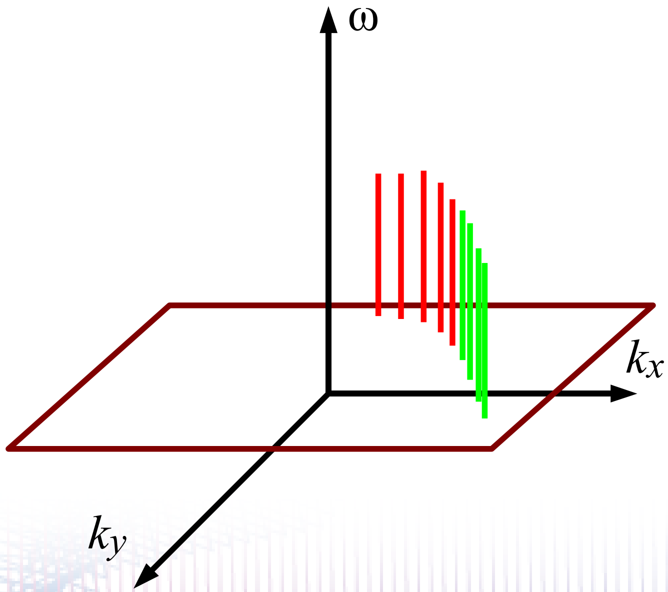
d-wave spectral function

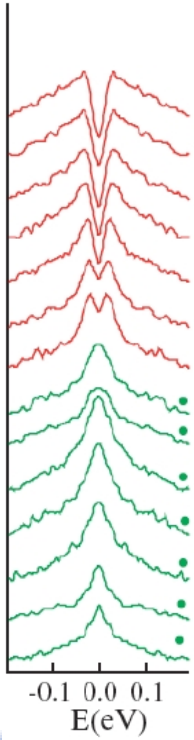
Coupling: $\varphi_{\mu\nu}\overline{\Psi^c}\Gamma^{\mu}D^{\nu}\Psi$

- *E.g.*: spectral func. at θ = fixed



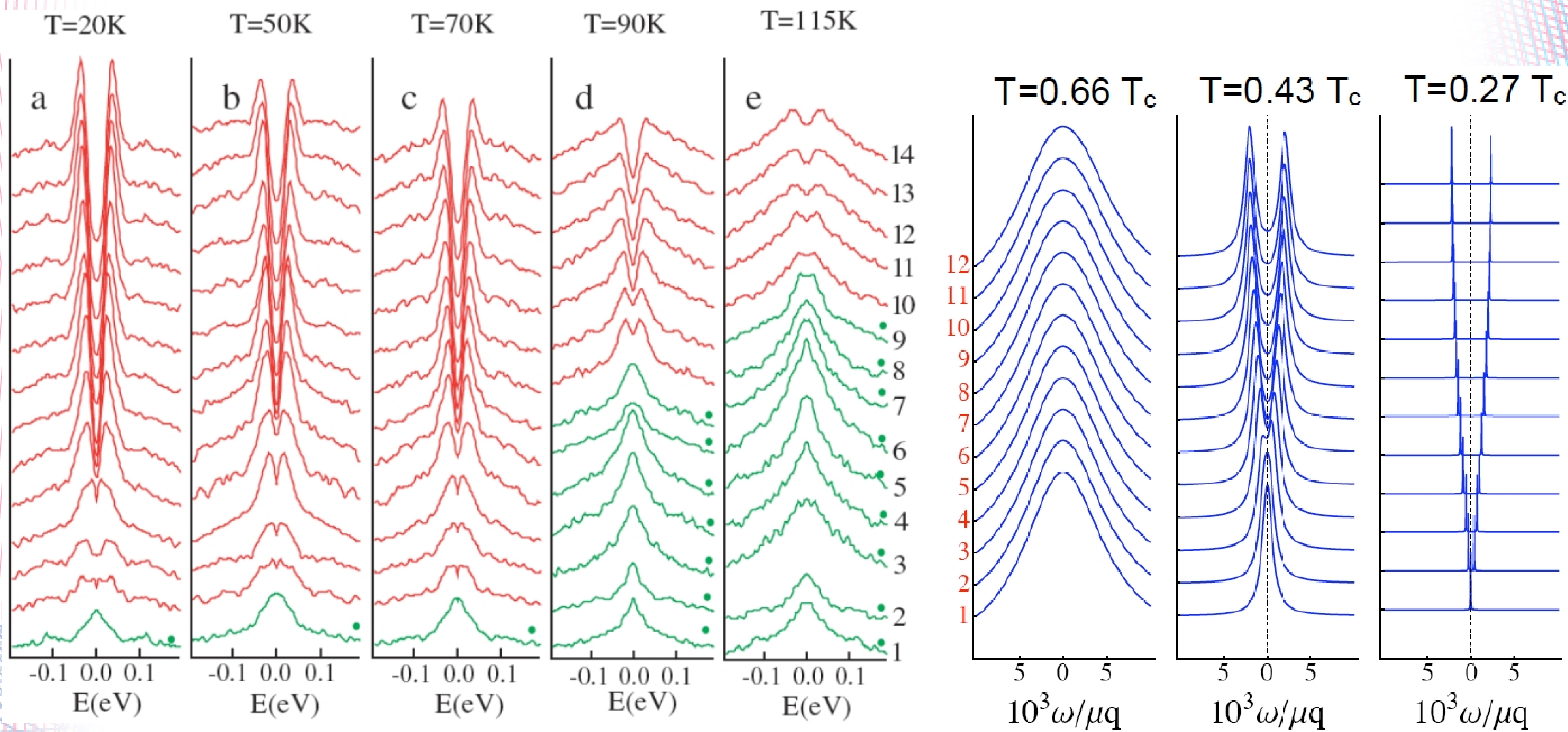
- Exp procedure: for every θ
  1) identify Fermi momentum
  2) draw EDC





EDC's

- Compare Energy Distribution Curves with exp's:

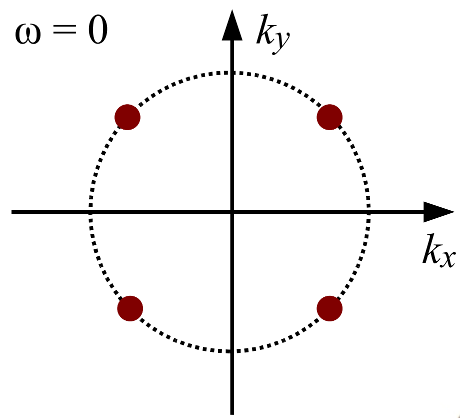


Kanigel et al, PRL 99 (2009) 157001

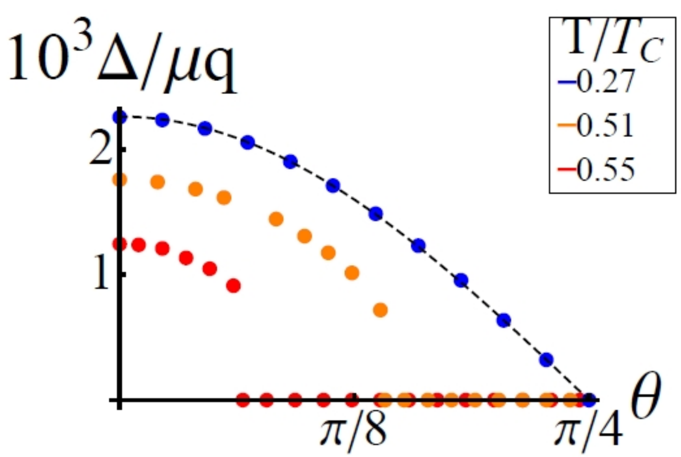
Underdoped $Bi_2Sr_2CaCuO_8$

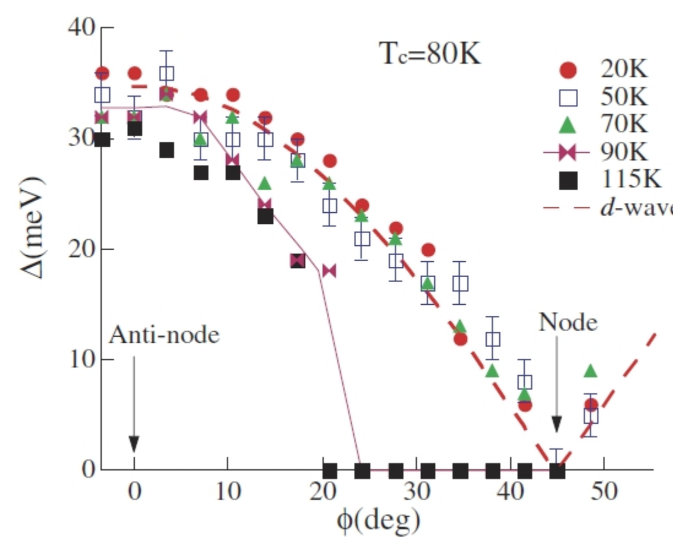
d-wave gap and Dirac cones

- $T < T_{\text{gap}}$: Fermi surface gapped everywhere but at four nodes $\theta = \pi/4$.



$Bi_2Sr_2CaCuO_8$
Kanigel et al, PRL 99 (2009) 157001



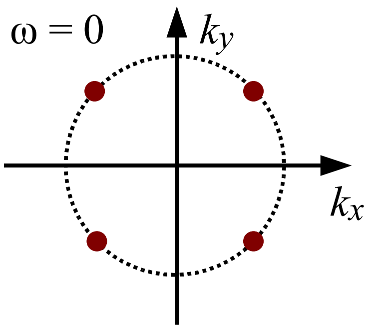


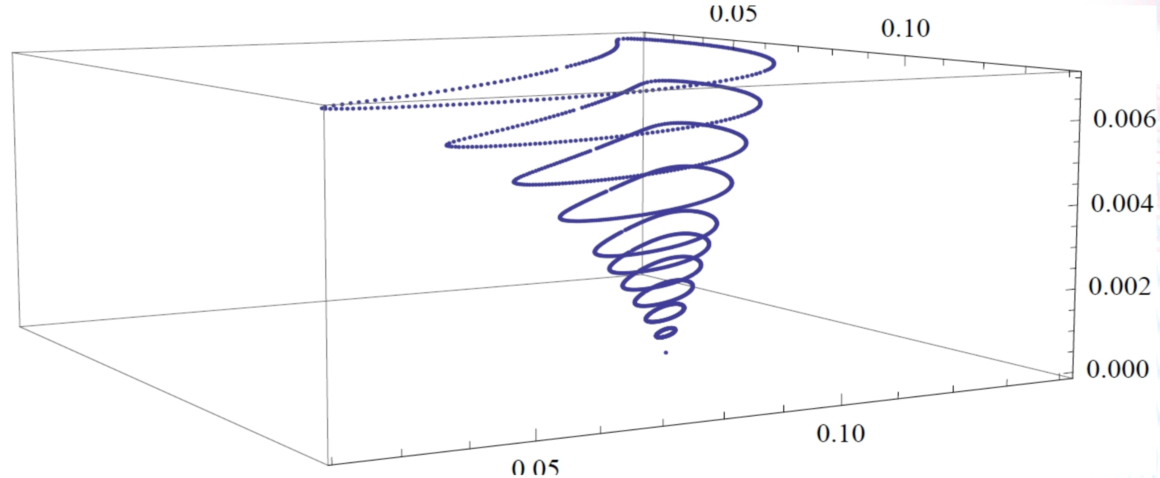
- In both cases, gap fit by $\Delta(\theta)=\Delta_0\,|\cos(2\theta)|$

d-wave gap and Dirac cones

- Nodes:

  Dirac cones





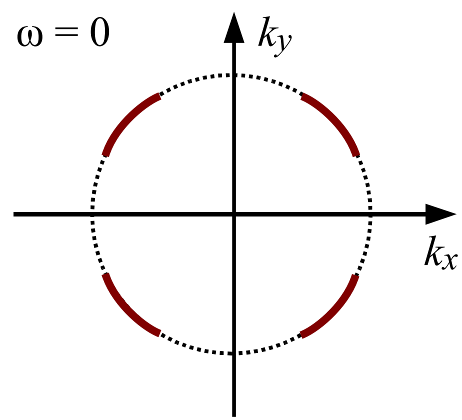
- Define Fermi velocities: $v_\perp = \dfrac{\partial \omega}{\partial k_\perp}$ $\quad v_\parallel = \dfrac{\partial \omega}{\partial k_\parallel}$

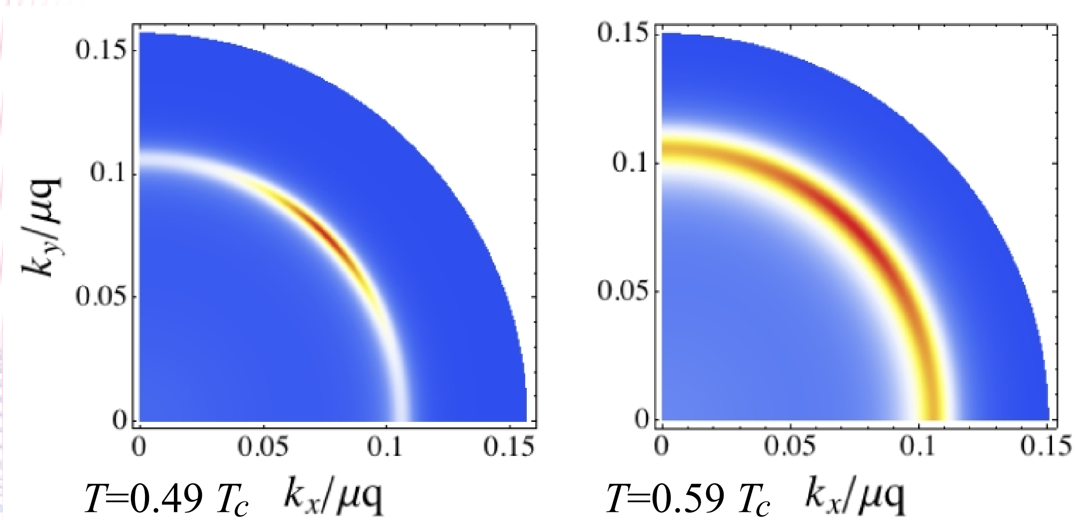
- The ratio $v_\perp / v_\parallel$ is linear in η.

  Experimental value $v_\perp / v_\parallel \approx 15$ - 25 can be accomodated.

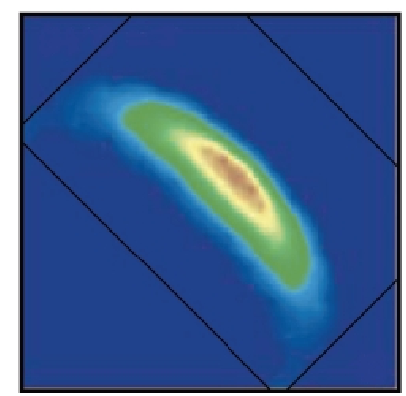
Fermi arcs

- $T_{\text{gap}} < T < T_{\text{arc}}$: Fermi arcs





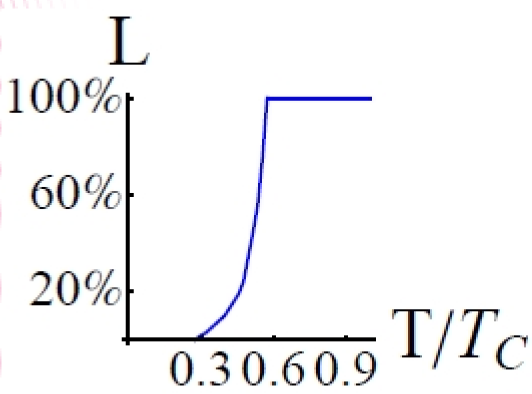
ω = 0

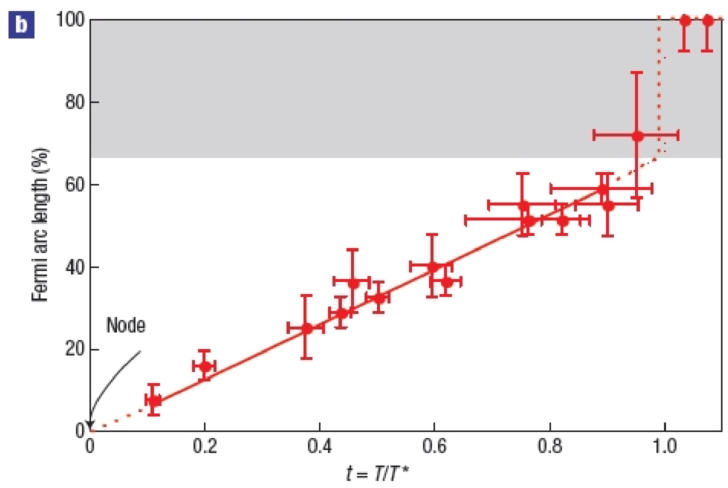


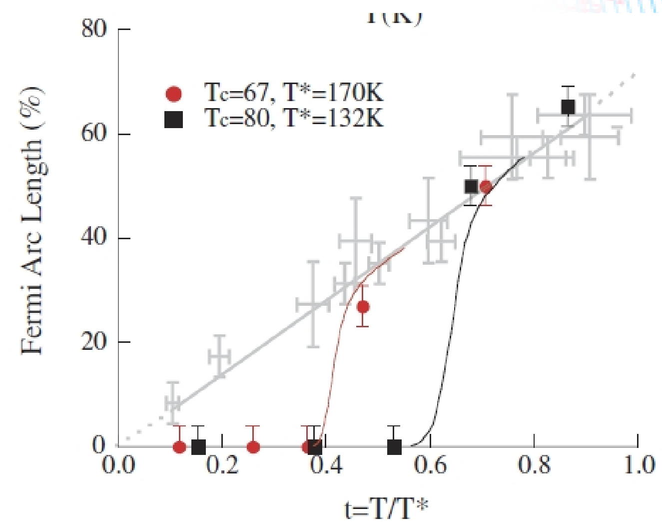
Na-CCOC
Shen et al, Science 307 (2005) 901

Fermi arcs length

- Experimentally length linear with temperature







Kanigel et al, Nature Phys 2 (2006) 447

- Arcs in the *pseudo-gap* phase

→ still more to understand

Future directions

- Improve the action (maybe along Argyres-Nappi)

- Fully consistent model (beyond probe approx):
  KK decomposition, *e.g.* $AdS_d \times S^1$

- Pseudo-gap phase

- Introduce non-relativistic scaling
  Introduce inhomogeneities (arcs)

- Complex ansatz (Hall conductivity,
  chiral d+id superconductivity, boundary currents)