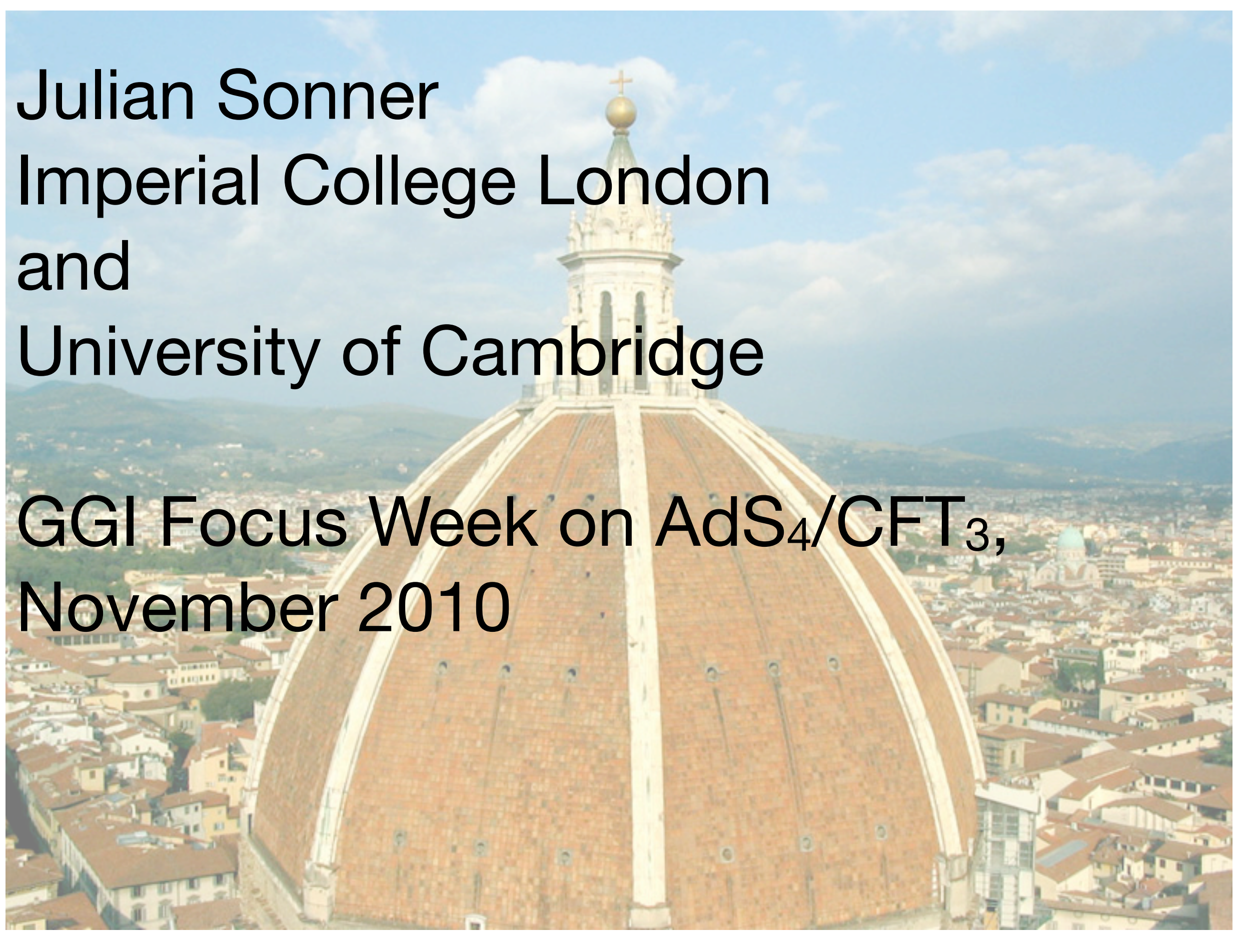
Recent Developments in Top-Down Models of Holographic Superconductivity



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GGI Focus Week on AdS_4/CFT_3,
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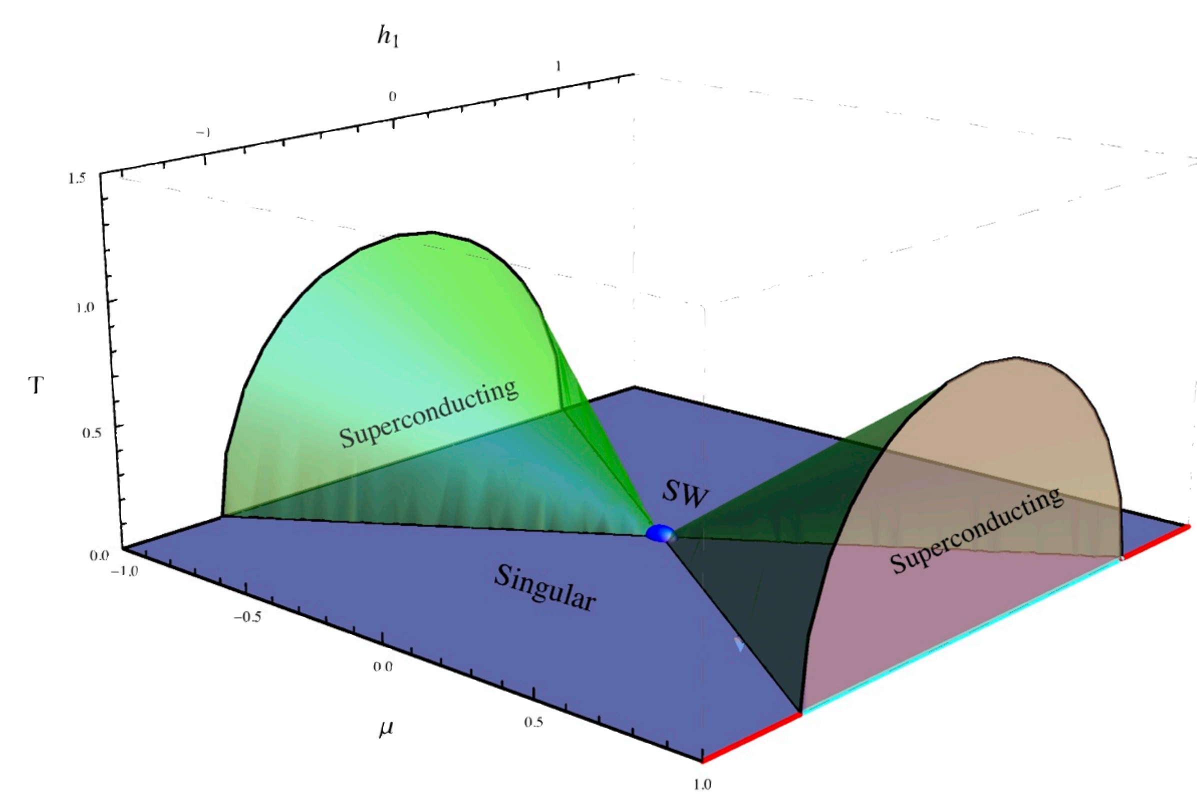


outline

1. hydrodynamics of holographic superfluids
   "finite supercurrent solutions in AdS/CFT"

2. ads/cft and an m-theory superconductor
   "top-down approach to holographic superconductivity"

3. fermions in the m-theory superconductor
   "recent progress in top-down models"

4. conclusions and outlook

based on work with Jerome Gauntlett, Toby Wiseman, Dan Waldram and Ben Withers

arXiv:0907.3796 (PRL 103:151601, 2009) [JG, JS, TW]
arXiv:0912.0512 (JHEP 1002:060, 2010) [JG, JS, TW]
arXiv:1004.2707 (PRD 82:026001, 2010) [JS, BW]

and work in progress with JG and DW (to be published)

superfluid hydrodynamics from gravity

[see also work by H^3, Kovtun et al., Amado et al., Bhattarcharya et al., Landau et al.]

holographic superfluid/superconductor

- Superfluidity is associated with **broken** *global* U(1) symmetry

$$\begin{aligned} D\psi &= d\psi - iqA\psi \\ V &= -\frac{2}{\ell^2} + m^2\psi\bar{\psi} \end{aligned}$$

- Note: BCS theory has no ***dynamical*** photon

- Minimal Bulk Theory:

$$S_{\text{bulk}} = \int \sqrt{-g} d^4x \left[R - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D\psi|^2 - V\left(|\psi|\right)\right]$$

- charged scalar has asymptotics

$$\psi(r,x) \sim \psi^{(0)}(x) r^{\Delta - d} + \psi^{(1)}(x) r^{-\Delta} + \cdots$$

- First approximation: everything ***constant*** in field-theory directions. Scalar field real everywhere (gauge choice).

  →scalar develops ***condensate*** at low temperature: $\mathcal{O}_\psi = \langle \psi^{(1)} \rangle \neq 0$

hydrodynamics of the broken phase

- Broken ***global*** U(1) symmetry means there is a Nambu-Goldstone boson
- NGB shows up as a ***hydrodynamical*** mode
- Studied for ***Helium II*** by [Landau and Tisza]. ***Relativistic*** model was developed by [Khalatnikov et al, Son] and has equations + constitutive relations:

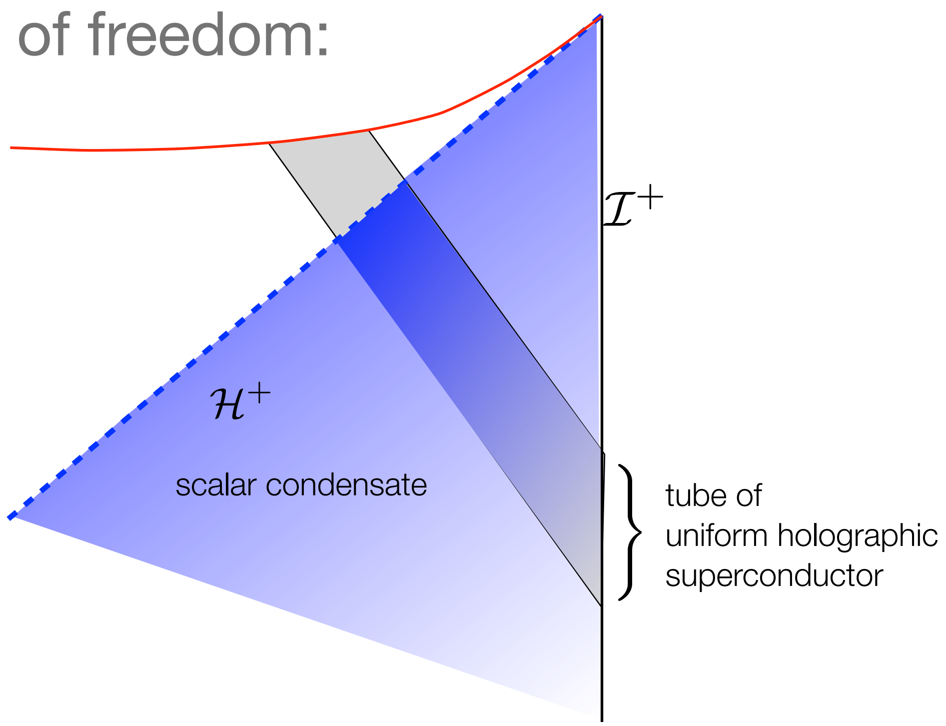
$$
\begin{aligned}
T_{\mu\nu} &= (\epsilon + P)\, u_\mu u_\nu + P\eta_{\mu\nu} + \mu\rho_s v_\mu v_\nu \\
J_\mu &= \rho_n u_\mu + \rho_s v_\mu \,, \\
\partial_\mu J^\mu &= 0 \\
\partial_\mu T^{\mu\nu} &= 0
\end{aligned}
$$

bulk geometry of superfluid hydrodynamics

- Can write down **bulk metric** which contains all hydrodynamic modes, including Goldstone, at non-linear level [Ben Withers, JS]

- Generalised boosted black brane solution. Degrees of freedom:

  - Local temperature T
  - normal fluid velocity u^μ
  - energy density ε
  - charge density/ chemical potential ρ / μ
  - pressure P
  - superfluid velocity v^μ
  - superfluid density ρ_s



- For $\rho_s = 0$, $v^\mu = 0$ recover Bhattarcharyya et al. result and its generalisations. In general situation recover local thermodynamic description of superfluid

- v^μ is related to ***gradient*** of Goldstone mode. Can derive conditions on boundary theory to show that it obeys precisely the Khalatnikov-Tisza-Landau equations

m-theory superconductor and fermions

[see also type IIB work by Gubser et al.]

general motivation

- **Fermi** aspects of holographic models are very rich. No top-down results known so far beyond probe-brane setups [Ammon et al.]
  Q: but why would anybody care about **top-down** fermions? Or, in fact, top-down in general?

- No good evidence that AdS/CFT is ***well-defined*** (i.e. there is a well-defined dual CFT) outside of backgrounds that come from ***decoupling limits*** of branes

- Couplings and field content of bottom-up models undetermined vs. ***fully specified*** in top-down aspects: Can definitely answer questions about the nature of ***Fermi surfaces*** etc...

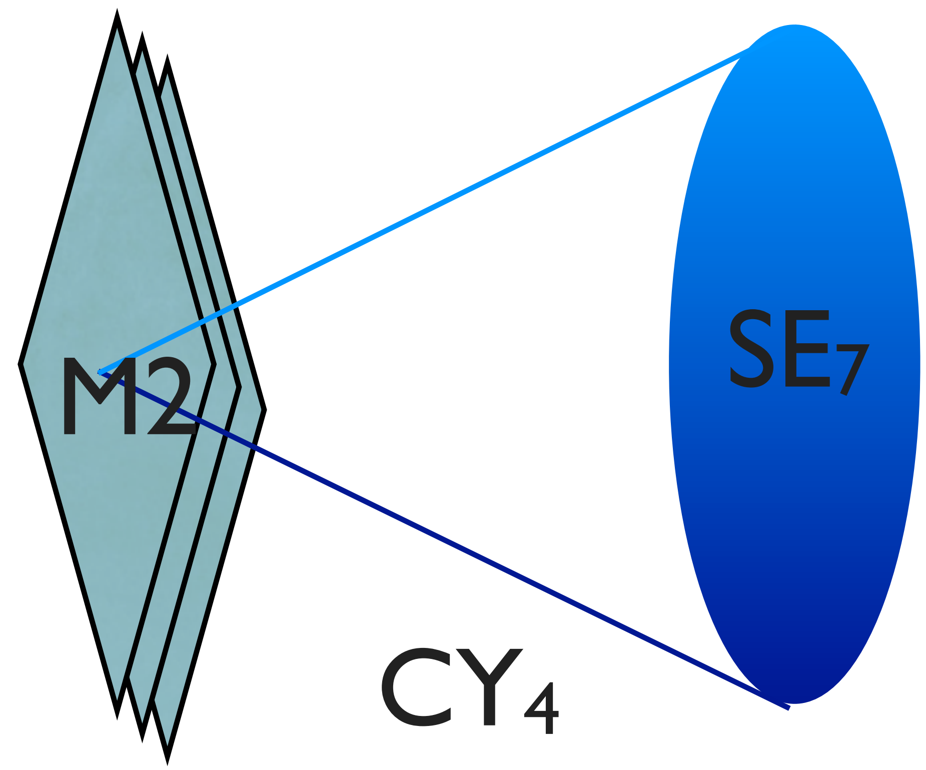
- Bottom-up (as well as top-down) models may be ***unstable***, but lacking a proper definition of the truncated modes makes it impossible to even ask precise questions about stability etc... (see work by Pilch, Warner et al.)

the m-theory setup

- Want: 4D theory with **massive charged** scalar

- Consider M-theory on SE_7

- **Consistent** KK truncation for (anti-) membranes



$$ds^2 = e^{-6U-V} ds_4^2 + e^{2U} ds^2\left(\mathrm{KE}_6\right) + e^{2V}\left(\eta + A_1\right)\otimes\left(\eta + A_1\right)$$

- Charged scalar comes from **4-form**

- 4D spectrum: χ, A_1, H_2, H_3, U, V, h, g, ε

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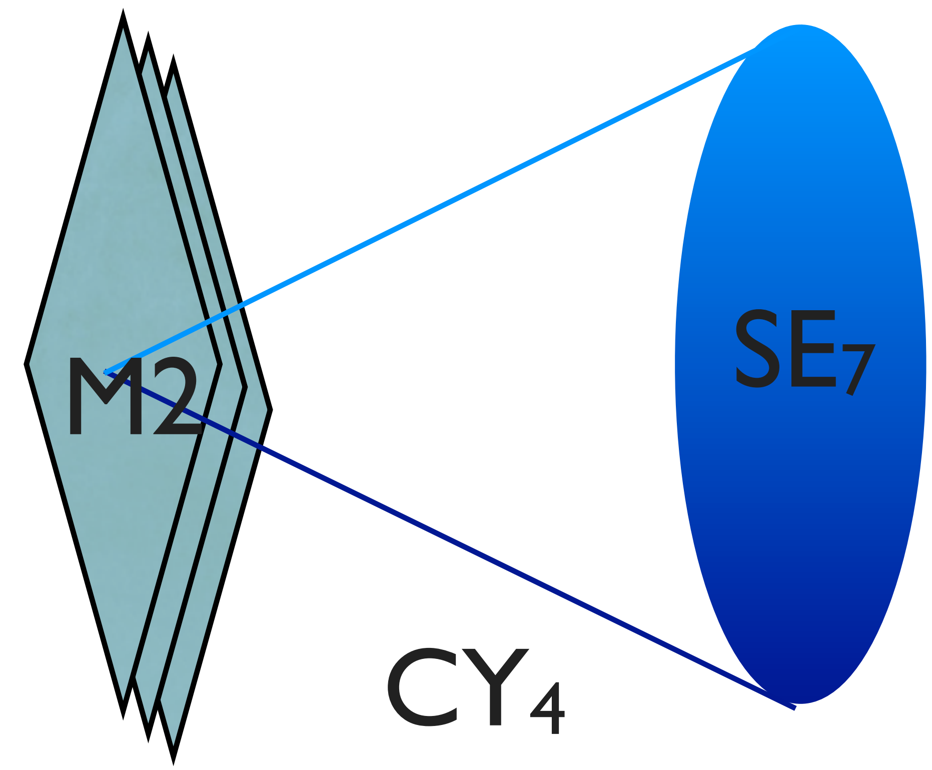

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orientation of SE_7

a further truncation

- For **anti**-M2 branes (ε=-1) there exists a further truncation
- This type of solution is also known as **skew-whiffed**

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\Big[R - \frac{(1-h^2)^{3/2}}{1+3h^2}F_{\mu\nu}F^{\mu\nu} - \frac{3}{2(1-\frac{3}{4}|\chi|^2)^2}|D\chi|^2 - \frac{3}{2(1-h^2)^2}(\nabla h)^2 - \frac{24(-1+h^2+|\chi|^2)}{(1-\frac{3}{4}|\chi|^2)^2(1-h^2)^{3/2}}\Big] + \frac{1}{16\pi G}\int \frac{2h(3+h^2)}{(1+3h^2)}F\wedge F$$

- At **leading order** and with h=0 reproduce original bottom-up model of s-wave superconductor
- Higher-order corrections to potential **modify IR** behaviour significantly

top-down means you have to eat what's on the plate!

- In addition to gravity and U(1) charged scalar we also have a **neutral** scalar h with conformal dimension $\Delta_h = 2$

- Solutions have additional **relevant** parameter: Dirichlet data for h

$$g \leftrightarrow T\,, \quad \chi \leftrightarrow \mathcal{O}_\chi\,, \quad h \leftrightarrow \mathcal{O}_h$$

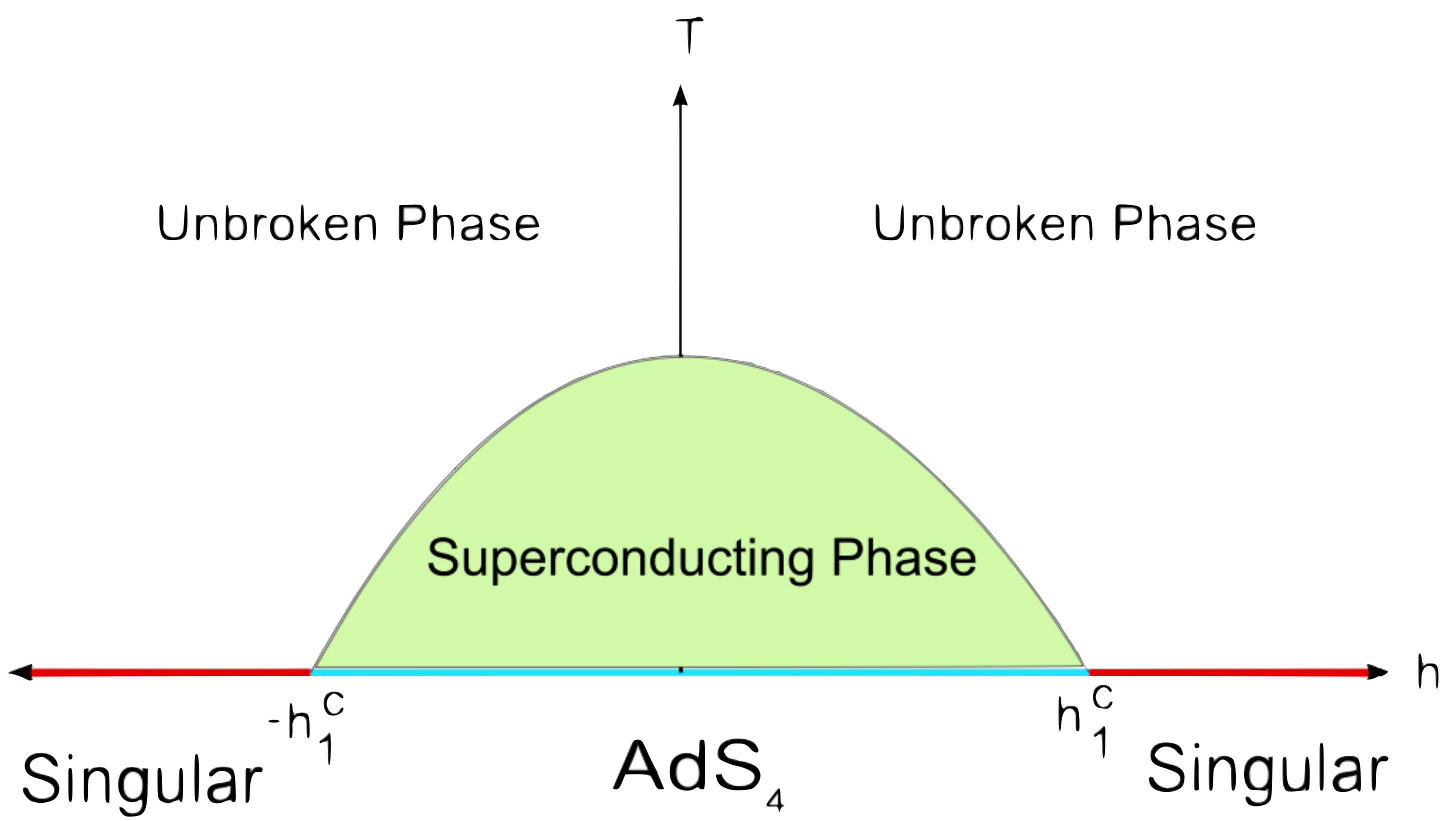
- M-theory has determined **exact** spectrum and all **couplings**

- h F ^ F **axionic** coupling such that nontrivial profile for h induces **parity** breaking

behaviour for **μ** = 1

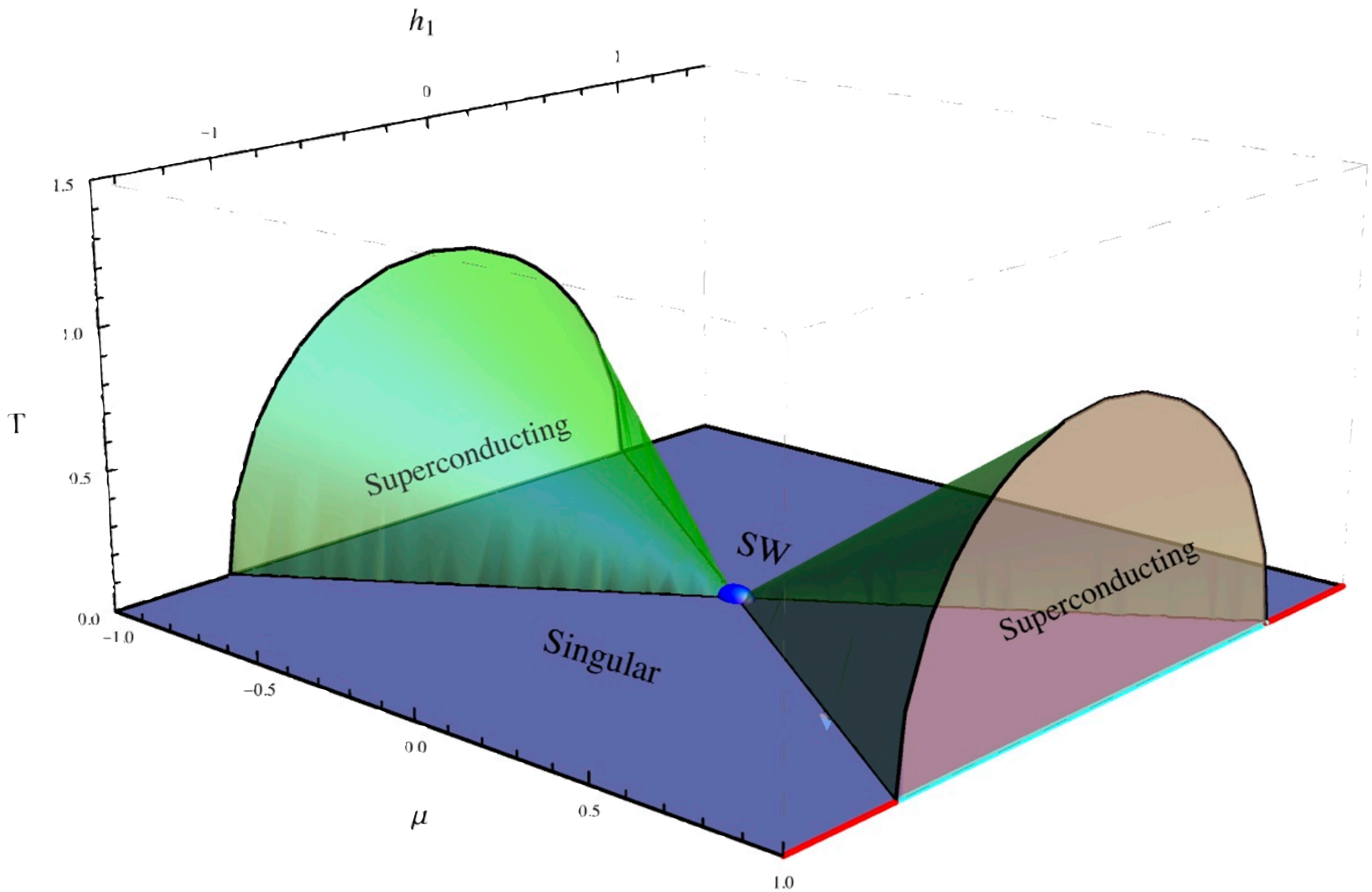
- Under the **dome** ($|h_1|<|h_1^c|$) have superconducting broken phase solution with **regular** T=0 limit
  → T=0 solution has **emergent IR** conformal symmetry corresponding to emergent quantum-critical scaling

- **Outside** the dome have h-deformed $RNAdS_4$ black hole



full phase diagram

- New h-deformed black holes. For any non-zero h, their zero-T limit is **singular** (but singularity may be cloaked by dome); μ→ ∞ limit approaches **AdS-Schwarzschild**



fermionic spectral functions

[see also Faulkner et al., Liu et al...., Cubrovic et al., Denef et al., Policastro]

you have to eat what's on the plate (ii)

- **G-structure** reductions on SE_7 manifolds lead to **consistent** truncations with fully specified Fermi content [Andrianopoli et al., JG, JS, DW, Leigh et al.]

- On the menu (in addition to bosonic fields):
  - spin 1/2 fields with various couplings (2-Fermi, 4-Fermi)
  - spin 3/2 **gravitino** coupled to other fermions (who ordered that?)

- Gravitino is generically coupled in an essential way to other Fermi fields and has itself **Pauli couplings** to flux
  - related to **consistent** propagation of spin-3/2 on general backgrounds

- Spin-3/2 degrees of freedom in models of condensed-matter physics
  Does this kill this class of top-down models immediately?

- Dual Fermionic currents are **composite**. Non-analytic features can come from any of the fundamental fermions (spectral decomposition)

holographic fermi surfaces (and other features)

- What is a good definition of a **Fermi surface** in a strongly interacting system?
- Luttinger (1960): consider the one-particle retarded 2pt correlation function

$$G_R(\omega = 0, k_L) \to 0, \infty$$

- Faulkner et al. : Compute this correlation function in **holographic** setup
- One finds a **variety** of possible behaviours:
  -**non-Fermi** liquids whose (non-)analytic behaviour is governed by emergent IR CFT (QCP?)
  -can get **different** scaling behaviours near k_L (e.g. 'marginal Fermi liquid)

comments

- Very interesting new physical playground for non-Fermi liquids. May point to concepts such as ‘**semi-holographic** models’
- **Scaling properties** defined by emergent IR CFT. **Location** of Fermi surface from whole geometry (including UV part)
- Only considered the simplest cases with single spin-1/2 bulk fields (later additional coupling to condensate were included as well)
- Arbitrary bottom-up couplings lead to **landscape** of non-Fermi liquids. Top-down gives **definite answer** for given model.
- Every probe sees a **different** Fermi surface. Fermi surface should be property of the system. In top-down there isn’t really a dichotomy between `probe’ and `the system’ (all couplings are **uniquely** specified)

swallowing the gravitino

- Start by studying the **simplest** case of gravitino Fermi physics: N=2 gauged supergravity in d=4 (susy truncation on SE_7)

$$S = \int d^4x\sqrt{-g}\Big[R - \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{6}{\ell^2} - 2\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho + \frac{2}{\ell}\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu - i\mathcal{F}^{\mu\nu}\bar{\psi}_\rho\gamma_\mu\gamma^{\rho\sigma}\gamma_\nu\psi_\sigma + \dots\Big]$$

- **Dirac** gravitino has charge q=-m under the U(1) gauge field

- Omitted 4-Fermi terms - enough for **linear response**

- Supersymmetric case, bosonic operator does not condense. Ground-state is the Reissner-Nordstrom BH at T=0

symmetries of the problem

- Planar AdS_4RN has R x SO(2) symmetry. Can reduce whole symmetry class to **effective 2d problem**. Advantage: can talk about Lorentz irreps of Spin(1,1) which are all one-dimensional

$$\psi_{\hat{\mu}}(x^m, \mathbf{x}) = \int \frac{d^2k}{(2\pi)^2} H^{-1} \chi_{\hat{\mu}}(x^m, \mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}$$

- χ_μ satisfies effective 2d equations with 2d gauge invariance. Can use gauge transformations to isolate the two independent polarization modes

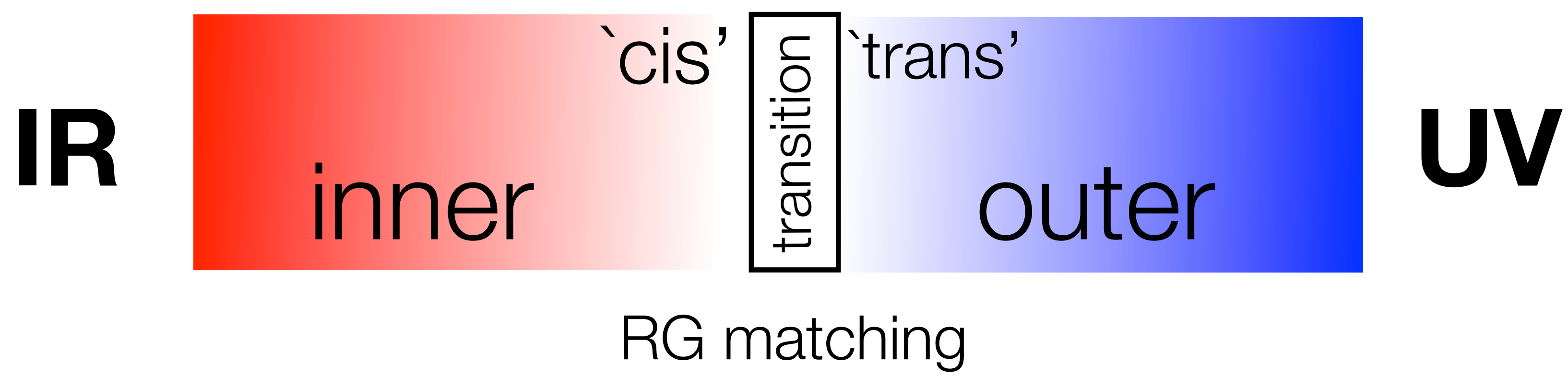
- Fermi surface has **sharp** definition only at T=0, so want to work there

- (Failed) strategy: Put equations on a computer, sit back, enjoy plots. The reason is an **essential** singularity in the problem

divide et impera - matched asymptotic expansion

[see also N. Macchiavelli et al.]

- Exact nature of problem is the essentially singular behaviour at zero T and small frequencies but also suggests solution: matched asymptotic expansion in parameter ω.



- DIVIDE ET IMPERA!
  -**Inner region** defined by $z = \omega L_2^3/(r-r_+)$ gives perturbative expansion in small ω of IR CFT. Can solve leading order exactly
  -**Outer region** defined by $r-r_+ >> L_2$. Can set up perturbative series in ω. Numerics necessary even at leading order

- Fermi momentum is **exactly** determined in **zeroth**-order outer region numerics (often need more work for scaling properties)

the emergent infra-red cft

- Geometry of inner region is **$AdS_2 x R^2$**: Corresponds to some CFT1 which governs the long-wavelength modes in this system [see MIT group]

- Remarkably (?) can solve gravitino equation **exactly** in this CFT. Ingoing/ outgoing bulk solutions are given by confluent hypergeometrics

- Extracting IR retarded (advanced) **Green function** has eigenvalues

$$\mathcal{G}_R(\omega, \alpha(s), \nu) \quad = \quad e^{-i\pi\nu} \frac{\Gamma(-2\nu)}{\Gamma(2\nu)} \frac{\Gamma(\frac{1}{2} - \alpha(s) + \nu)}{\Gamma(\frac{1}{2} - \alpha(s) - \nu)} \left(2\omega L_{(2)}\right)^{2\nu}$$

- s labels the Spin(1,1) rep. corresponding to polarization state of gravitino

- Scaling in frequency governed by effective 1d scaling dimension ν, which depends on k, but is **always real** (not so in MIT analysis)

the supercurrent correlation function

- Similar expression for **advanced** correlator. For real ν find phase from:

$$\frac{\mathcal{G}_R(\omega,\alpha,\nu)}{\mathcal{G}_A(\omega\,\alpha,\nu)} = \frac{e^{-2i\pi\nu} + e^{2\pi i s} e^{-\sqrt{2}\pi q L_{(2)}}}{e^{2i\pi\nu} + e^{2\pi i s} e^{-\sqrt{2}\pi q L_{(2)}}}$$

- implies that any **fermionic** rep is phase equivalent to every other fermionic rep (independent of value of spin) and similar for bosons
  ⇒ **analytical** properties of Green function solely depend on whether field is boson or fermion

- Full result from **matching** the IR confluent hypergeometrics to UV expressions. Note: Cannot solve UV part of the problem analytically, but numerics are now easier to control

fermi surface (analytical expressions)

- After matching to the **outer-region** the full correlation function has helicity eigenvalues

$$G_{Rss}(\omega, \alpha, \nu) = \frac{b_{(1)} + \mathcal{G}_R(\omega, \alpha(s))b_{(2)}}{a_{(1)} + \mathcal{G}_R(\omega, \alpha(s))a_{(2)}}$$

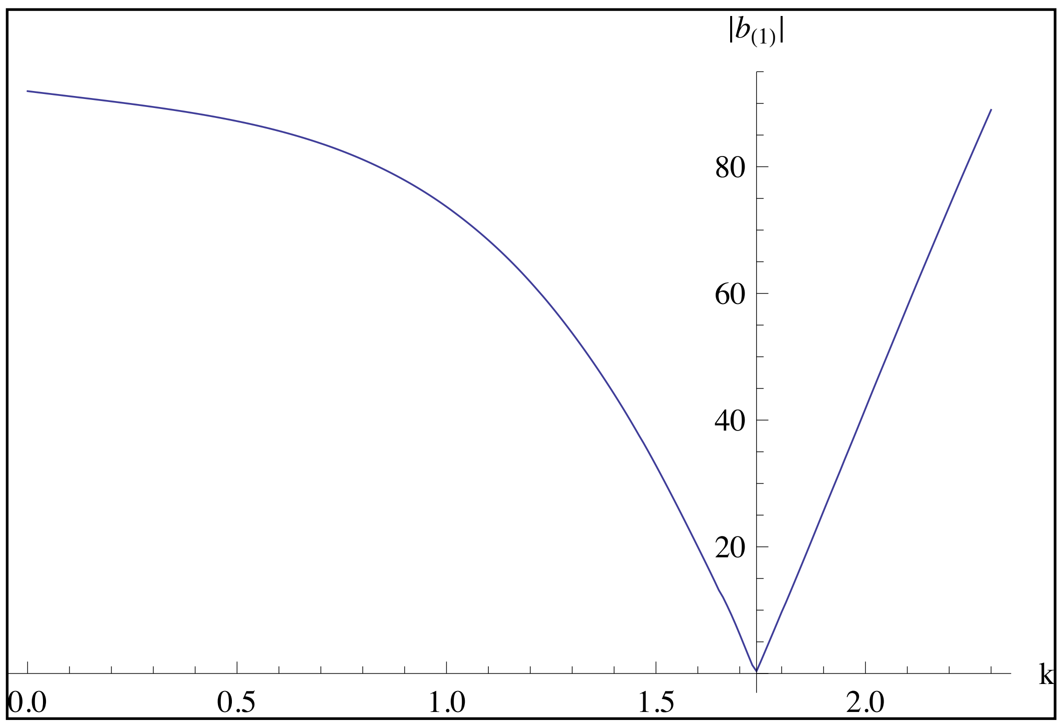
- This is susy quantisation. Also have **alternative** quantisation, which reverses the Green function G → 1/G (there are caveats)

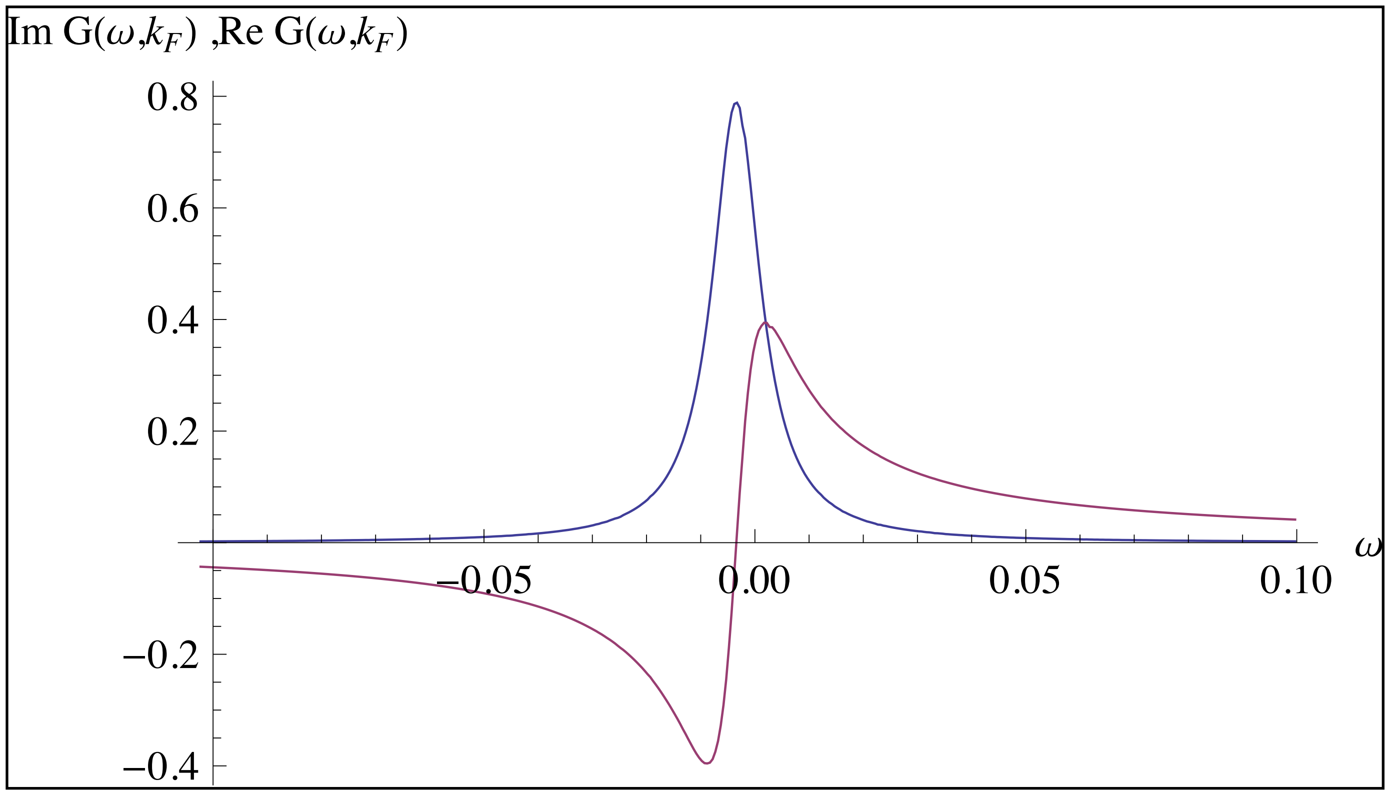
- Scaling around special features of G is governed by **IR** conformal dimension ν of dual operator

- Exact numerical value of k_F and type of Fermi surface (if any) determined by UV numerics...

fermi surface (preliminary results)

- Location of Fermi surface is **gauge invariant**, even though $b_{(1)}$ is explicitly gauge variant (good check on numerics)





$$k_F \approx 1.72...$$

$$\nu_K \approx 1.04...$$

- **`Irrelevant'** Fermi liquid with stable quasi-particle excitation at k_F

conclusions

conclusions

- Embedding of holographic superconductors in **string/m-theory** is possible. Suggests new parameters, such as additional **relevant** operators and the dome. There are several open questions: **stability**, exact phase structure, genericity of consistent truncations

- Top-down sometimes offers more satisfying **conceptual** explanations of holographic models (e.g. no dichotomy between probe and system, no ambiguity in couplings etc.) and more control

- So far, however, at the expense of realism. In a sense we have **toy models** of the type of systems which can be studied in bottom-up contexts with more ease and more realistic features. Does holographic model building make sense?

- First fully top-down computation of Fermion **spectral functions**: Irrelevant Fermi liquid. What happens upon coupling to condensate?

thank you