Explaining Fermi Liquid stability with AdS Black holes

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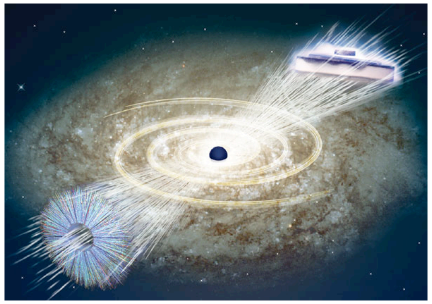


Mihailo Cubrovic, Jan Zaanen, Koenraad Schalm
arxiv/0904.1993
arxiv/1011.XXXX

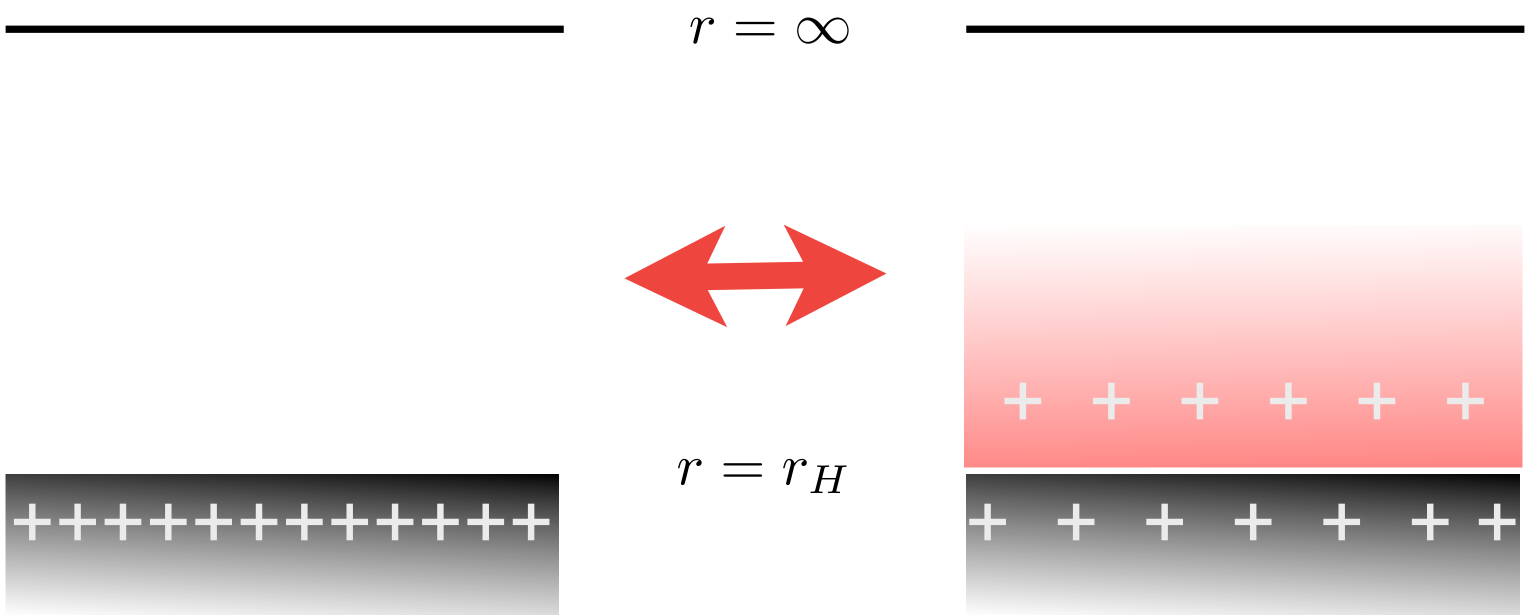
GGI Firenze 2010

AdS/CMT $=$ *Rich* Black Hole physics

- **The origin of rich black hole physics**
  - Instability of charged Black holes [e.g. Gubser @ Strings 2009]

$$S = \int \ldots + \bar{\phi}(A_0)^2\phi + \ldots \quad \Leftrightarrow \quad m_\phi^2 \sim -A_0^2$$

  - The holographic superconductor [Hartnoll, Herzog, Horowitz]



- **AdS/CFT: Ground state stability is BH stability**

$$Z_{CFT}(\phi) = \exp S_{AdS}^{on-shell}(\phi(\phi_{\partial AdS=J}))$$

- **What about Fermions?**

$$S = \int \sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - \bar{\Psi} e^M_A \Gamma^A \left(D_M + igA_M\right)\Psi - m\bar{\Psi}\Psi\right] + S_{bnd}$$

  - No perturbative instability (no superradiance)

- $\boxed{\textbf{Conjecture}}$ **AdS-RN BH with Fermions is metastable.**

  – AdS-RN cannot be the true groundstate

    – *Large groundstate entropy*

$$S_{CFT} = \frac{A_H}{4G_N}$$

$$ds^2_{Near\ horizon} = L_2^2\left(-\omega^2 dt^2 + \frac{d\omega^2}{\omega^2} + dx^i dx^i\right)$$

  – Should exist 1st order transition to AdS Fermi-hair BH

    – Similar indications from [Faulkner et al (unpublished)] , [Hartnoll, Polchinski, Silverstein, Tong] , [Kraus et al.]

- **What is the true dual groundstate ?**
  - "Free Fermi-gas" in the bulk.

PROBLEM:

  - Fermi-Dirac statistics demands non-local behavior in AdS
    [always true for multiple Fermions]

Non-local RG flow?!

- **What is the true dual groundstate ?**

  – "Free Fermi-gas" in the bulk.

  PROBLEM:

  – Fermi-Dirac statistics demands non-local behavior in AdS
  [always true for multiple Fermions]

  SOLUTION:

  – Need some local approximation:

    – Integrate out the fermions [Kraus..]
    – Fluid/Thomas Fermi approximation [de Boer et al, Hartnoll et al.]
    – Single fermion

  In all cases we wish to know how the fields behave at ∂AdS to read off what happens in the CFT

- **In all cases we wish to know how the fields behave at ∂AdS to read off what happens in the CFT.**
  - AdS acts like a confining potential well. What is it the behavior of the Fermi gas at infinity?
    - Consider a spinless fermion in a d-dim harmonic oscillator potential well

$$\frac{-\partial_x^2}{2m} + \frac{m\omega^2 r^2}{2}\Psi = E\Psi \ , \quad \rho(r) = \sum_{E<E_F} \bar{\Psi}_E(r)\Psi_E(r)$$

- **Thomas-Fermi:**

  fluid is confined and has an edge $L^2 = 2/m\omega$

$$\rho_{TF} \propto (\frac{E_F}{\omega} - \frac{r^2}{L^2})^{d/2}$$

- **Exact answer**

  a long range tail: **[Brack, van Zyl]**

$$\rho_{exact} \propto \sum_{i=0}^{E_F/\omega} a_i L_i(4\frac{r^2}{L^2})e^{-2r^2/L^2}$$

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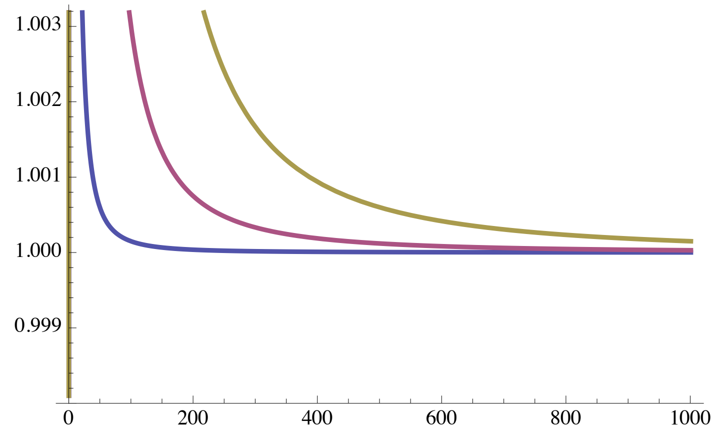
  - Far away, confined composite "gas" should approximate a "point particle"
    - Normalized Ratio of fluid density to single particle density in same $V(r)$ with $n = E_F/2\omega$ for $n = 1, 20, 100$



- **We wish to know how the behavior at ∂AdS to read off what happens in the CFT.**

Conjecture:

Study a single Dirac particle in the presence of a BH

- This "hydrogen atom" captures the dynamics at ∂AdS

Single particle but large Backreaction — if charge is macroscopic

- Expect a Lifschitz $S = 0$ solution for any charge q_F

- **Instead of Ψ, work directly with probability density**

$$J^\mu_\pm \equiv \bar{\Psi}_\pm i\gamma^\mu \Psi_\pm$$

  - Obtain dynamics for composite fields...

$$J^\mu_\pm \equiv \bar{\Psi}_\pm i\gamma^\mu \Psi_\pm \quad , \quad I = \bar{\Psi}_+ \Psi_- \quad , \quad A_0 = \Phi$$

  - Infer "equations of motion" from EOM of $\Psi_\pm$

$$\boxed{\begin{aligned} \left(\partial_z + 2\mathcal{A}_\pm\right) J^0_\pm &= \mp\frac{\Phi}{f} I . \\ \left(\partial_z + \mathcal{A}_+ + \mathcal{A}_-\right) I &= \frac{2\Phi}{f}(J^0_+ - J^0_-) \\ \partial_z^2 \Phi &= -\frac{1}{2z^3\sqrt{f}}(J^0_+ + J^0_-) \end{aligned}}$$

$$\text{Recall} \qquad \mathcal{A}_\pm = -\frac{1}{2z}\left(3 - \frac{zf'}{2f}\right) \pm \frac{mL}{z\sqrt{f}}$$

- **“Entropy Collapse” to a Lifshitz BH**
  [also Hartnoll, Polchinski, Silverstein, Tong]

  – Boundary conditions at the horizon $z=1$...

$$
\begin{aligned}
J^0_\pm &= \mathcal{J}_\pm (1-z)^{-1/2} + \dots \\
I &= I_{hor} (1-z)^{-1/2} + \dots \\
\Phi &= \Phi^{(1)}_{hor} (1-z)\ln(z-1) + \Phi^{(2)}_{hor}(1-z) + \dots .
\end{aligned}
$$

  – $\Phi^{(1)}_{hor}$ corresponds to a “source” on the horizon, (infinite backreaction)

  – Dynamically $\Phi^{(1)}_{hor} = 0 \quad \rightarrow \quad \mathcal{J}_\pm = 0 = I_{hor}$

$\Rightarrow$ “Dirac Hair” requires (mild) backreaction at the horizon

- **A holographic Migdal's relation**
  - Boundary behavior of the Dirac field

$$\begin{aligned}\Psi_+ &= A_+ z^{3/2-m} + B_+ z^{5/2+m} + \dots \\ \Psi_- &= A_- z^{5/2+m} + B_- z^{5/2-m} + \dots\end{aligned}$$

  - Recall that for bosons

$$\Phi = J z^{\Delta} + \langle \mathcal{O} \rangle_J z^{d-\Delta} + \dots$$

  - Spontaneous symmetry breaking [Gubser, Hartnoll, Herzog, Horowitz] :

    solution with $J = 0, \langle \mathcal{O} \rangle \neq 0$.

    $J = 0$ is the quasinormal mode.

Building a holographic Fermi-liquid

- **A holographic Migdal's relation**
  - Boundary behavior of the Dirac field

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  - For fermions the Green's function

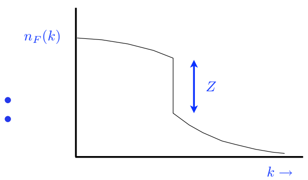
$$G(\omega, k) = \frac{Z}{\omega - v_F(k - k_F)} + \text{reg} = \frac{B_-}{A_+}$$

  - For $A_\pm(k_F) = 0$, $B_-(k_F)$ cannot be a fermionic vev.

    Instead

$$Z \simeq \frac{B_-(k_F)}{A_+(k_F)} = \frac{|B_-(k_F)|^2}{\partial_\omega W}$$

Migdal: n_F :



- **The Green's function (bulk extension)**

$$G(z) = \Psi_-(z) S \Psi_+^{-1}(z)\ , \quad \not{D} + \mathcal{A}_\pm \Psi_\pm = -\not{\mathcal{T}} \Psi_\mp$$

  - Convenient way to solve G directly

$$\partial_z G = (\mathcal{A}_+ - \mathcal{A}_-) G + G \not{\mathcal{T}} G - \not{\mathcal{T}} \Psi_+(z) S \Psi_+^{-1}(z)$$

  - Consider however the combinations $(\Gamma^I = \{\mathbb{1}\ , \gamma^i, \gamma^{ij}, \ldots\})$

$$J_\pm^I(z) = \bar{\Psi}_+^{-1}(z_0) \bar{\Psi}_\pm(z) \Gamma^I \Psi_\pm(z) \Psi_+^{-1}(z_0)$$
$$G^I(z) = \bar{\Psi}_+^{-1}(z_0) \bar{\Psi}_+(z) \Gamma^I \Psi_-(z) S \Psi_+^{-1}(z)$$

If $\mathcal{T}^i$ has only a single nonvanishing component, these are the same equations as before.

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  - For generic ω, k:

$$J_-^I(z_0) = (J_+^{\mathbb{1}}\ )^{-1}\bar{G}\Gamma^I G$$

- **The Green's function (bulk extension)**

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$$G^I(z) = \bar{\Psi}_+^{-1}(z_0)\bar{\Psi}_+(z)\Gamma^I\Psi_-(z)S\Psi_+^{-1}(z)$$

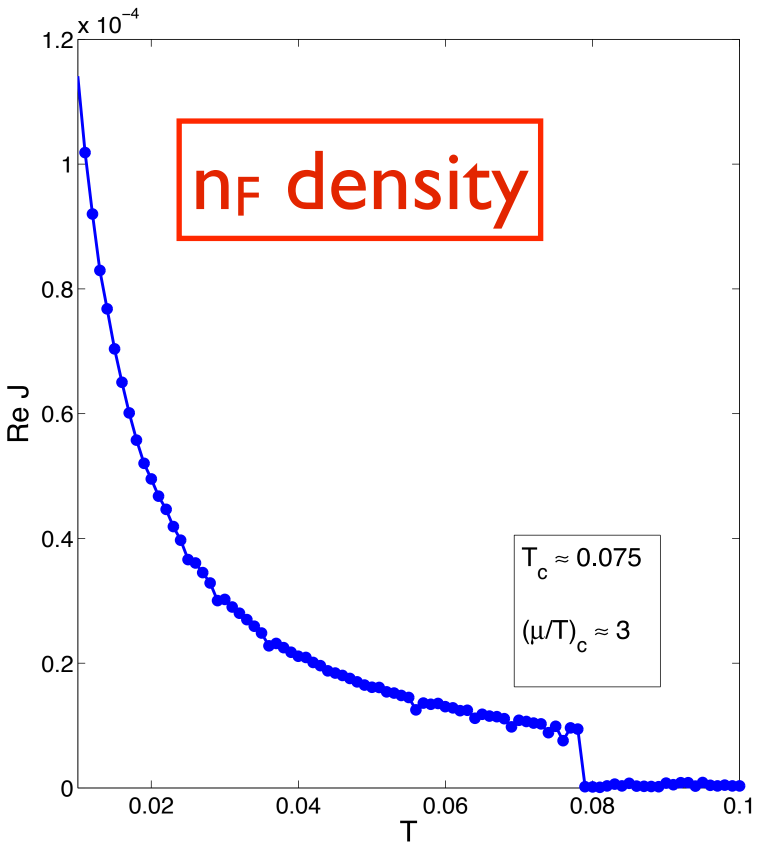
  - For *pole* $\omega(k)$:

$$\begin{aligned} J_-^I &= \Gamma^I G|_{on-shell} \\ \mathrm{Tr}\gamma^0 G_F(\omega(k))|_{on-shell} &= f_{FD}(T, \omega(k))\rho_{states}(\omega(k)) \end{aligned}$$

.

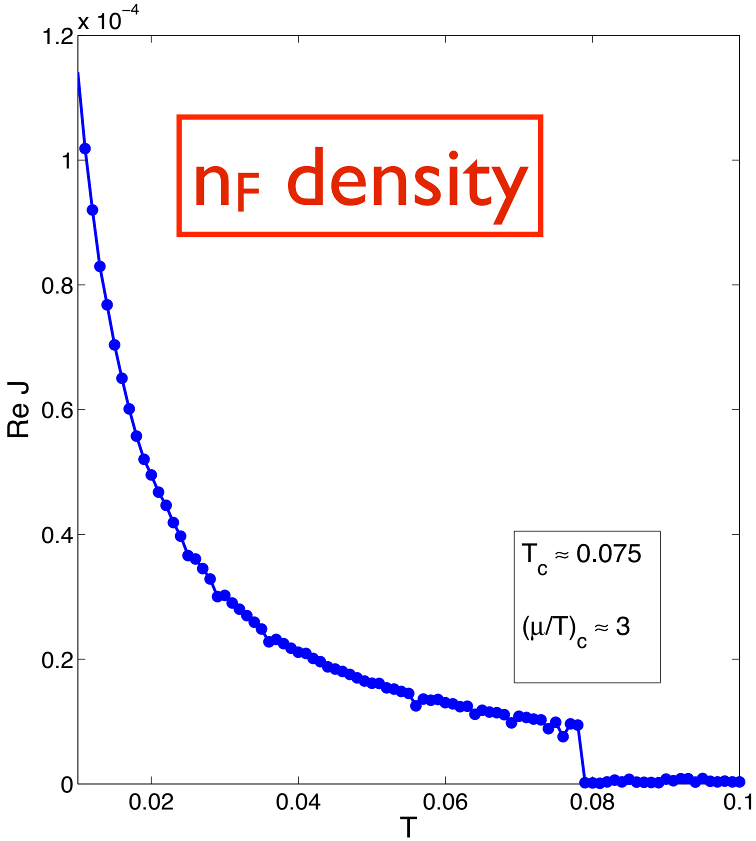
$$\boxed{J_-(\omega(k), k) = n_F(k)}$$

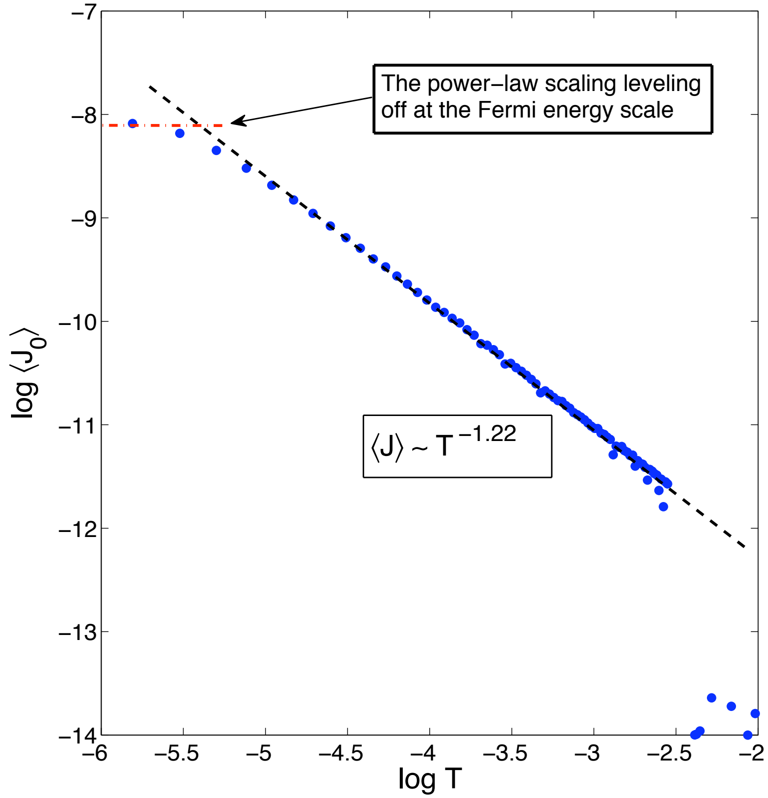
- **“Proof” AdS-RN BH with Fermions is metastable.**



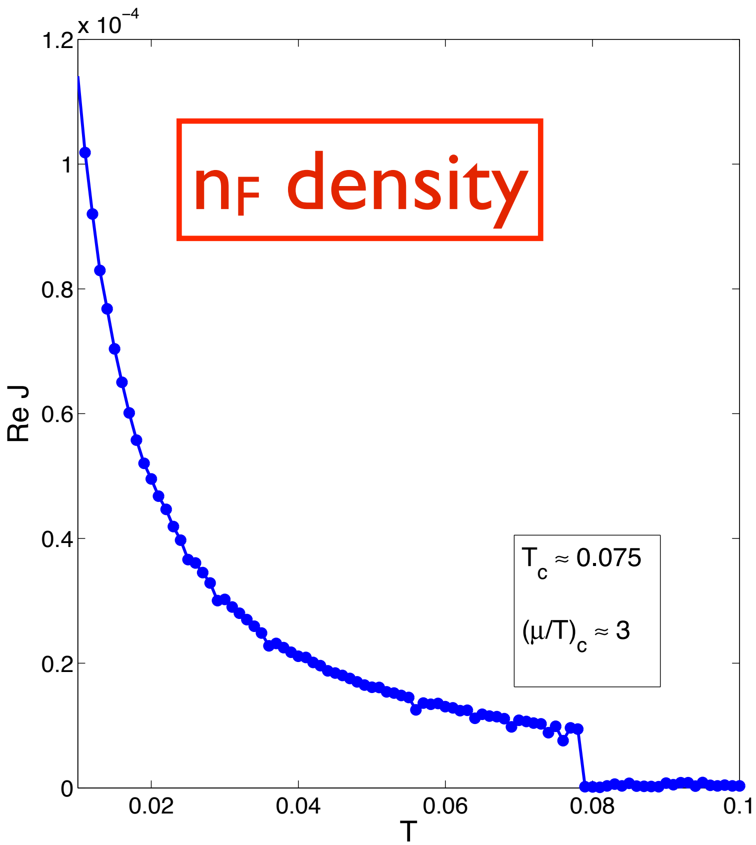
(vdW liquid to FL transition as in He_3.)

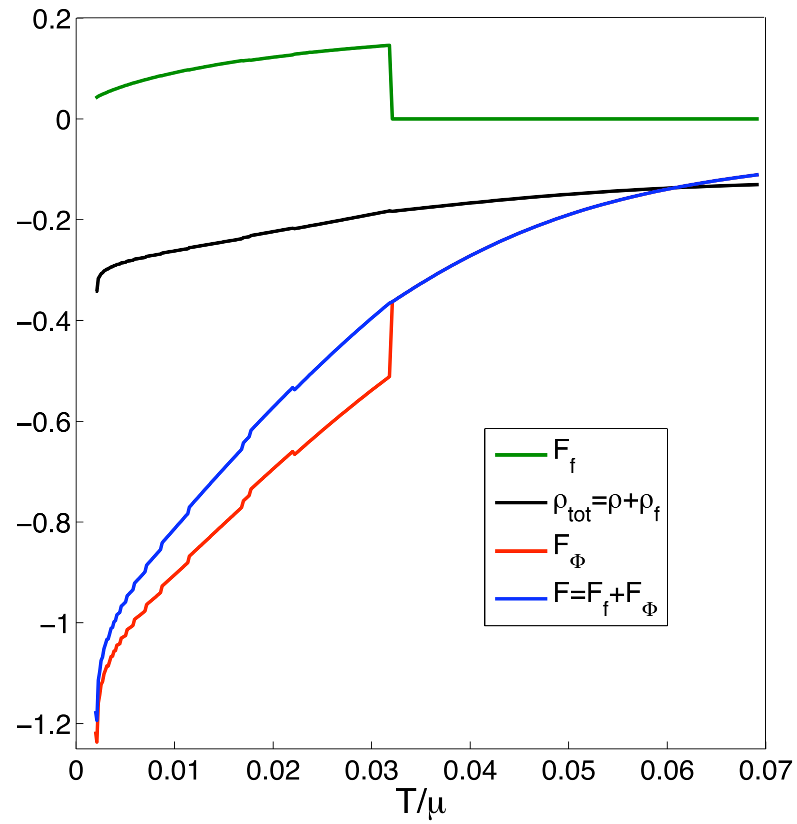
- **"Proof" AdS-RN BH with Fermions is metastable.**





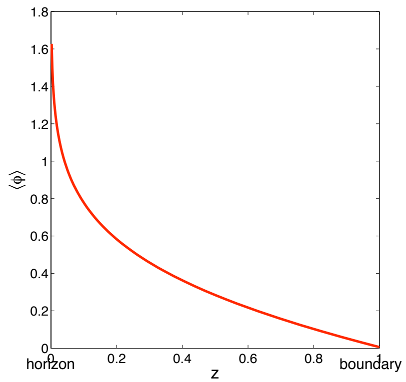
- **“Proof”** **AdS-RN BH with Fermions is metastable.**

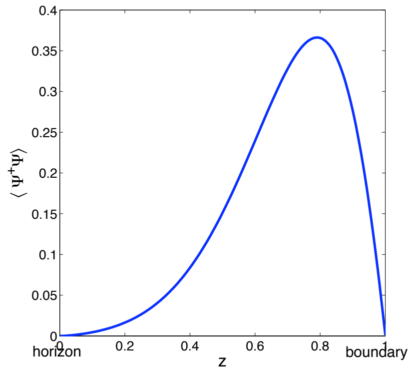




- **Luttinger theorem Check**
  - Radial profile of "hair"

    [CSZ preliminary]

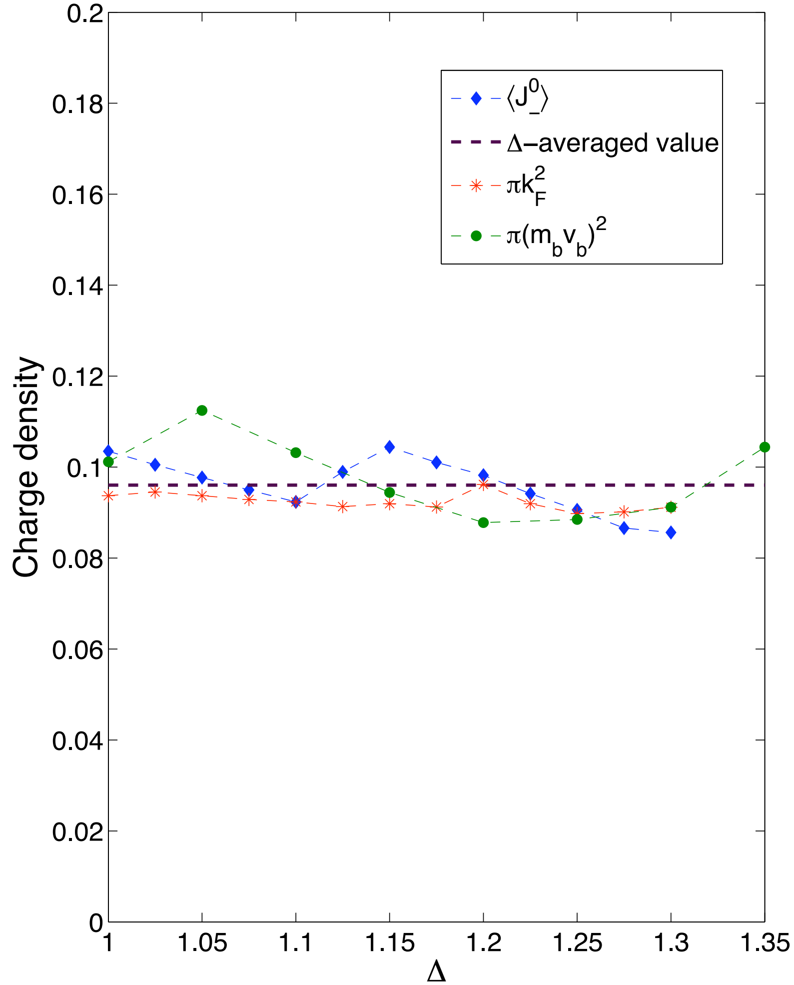




Bose Hair

Fermi Hair

  - Three ways to measure k_F



Can we understand FL stability from AdS/CFT?

- **Stability is determined by the Free Energy:**
  - Landau-Ginzburg

$$\mathcal{F}(\phi) = \int d^d x \frac{1}{2}(\partial_i \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{4}\lambda \phi^4 + \ldots \Big|_{extremum}$$

  - Wilson
    The quantum-mechanical partition function/path integral at low-energies is expressed in terms of fluctuations of ϕ

$$Z(\beta, \phi) = \int \mathcal{D}\varphi e^{-\beta \mathcal{F}(\phi+\varphi)}$$

  - The control parameter in the Free energy is the boson vev $\langle \phi \rangle$.

- **Since Fermions cannot have a vev, there is no such understanding of the (stability) of the Fermi Liquid.**

$$\langle\Psi\rangle = 0 \quad \text{[by definition]}$$

  - Perturbatively OK [Shankar, Polchinski]

    Calculation assumes FL groundstate;
    no global, ab initio explanation of the groundstate. (local vs global minimum)

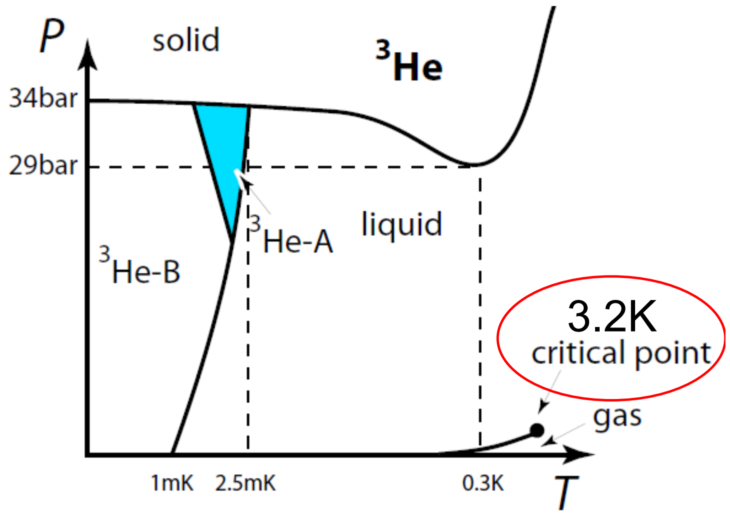
  - Experimentally OK...

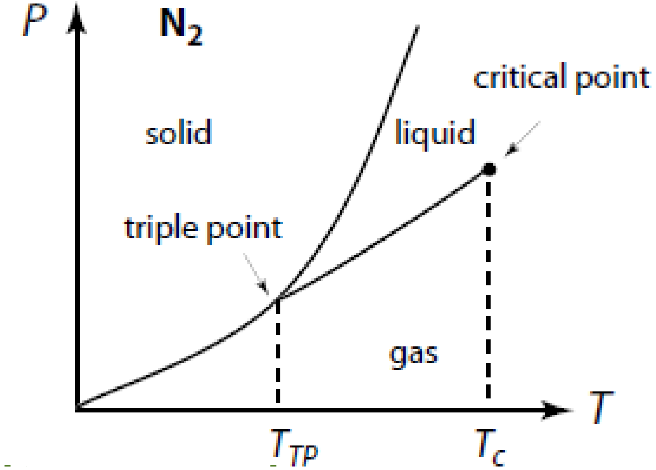
    $\boxed{\text{This is the mystery. Experimentally FL extremely robust}}$

- **Normal FL picture: dressed electrons**
- **There are many systems where this picture fails.**

  – $\boxed{\text{He}^3}$ [Landau]

  – Liquid at $3.2K$



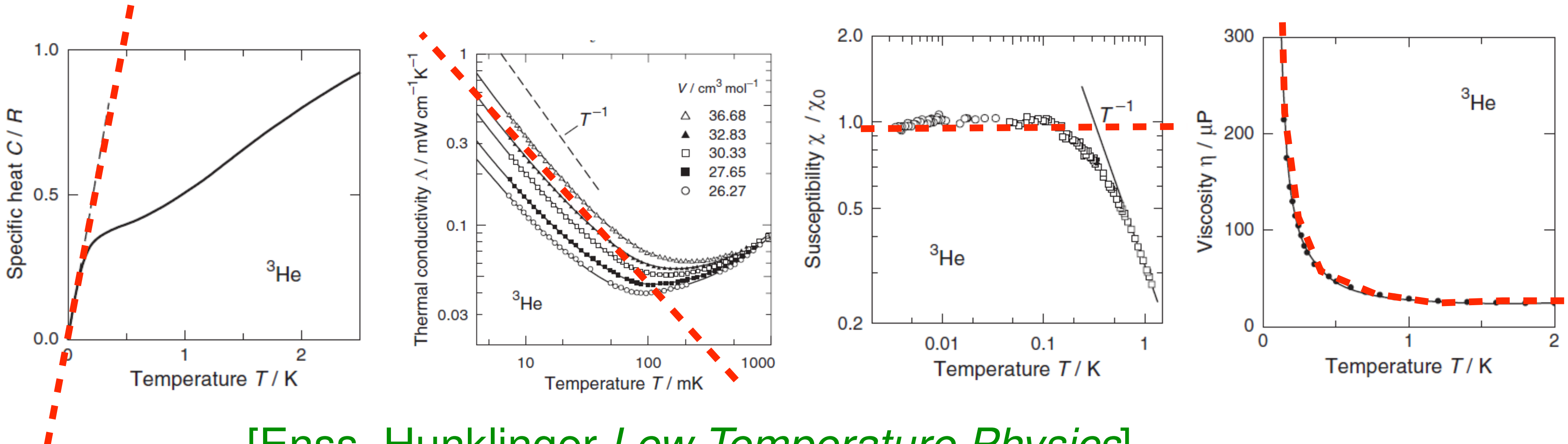


  – strongly coupled van der Waals liquid: $r_{interparticle} \ll r_{He_3}$: no notion of individual He_3 atoms(!)

  – stays liquid due to quantum-fluctuations.

The mystery of He^3

- **Normal FL picture: dressed electrons**
- **There are many systems where this picture fails.**
  - $\boxed{\text{He}^3}$ [Landau]
    - Liquid at $3.2K$
    - Becomes a regular Fermi Liquid at $T = 0.32K$ ($E_F = 4.9K$)



[Enss, Hunklinger *Low Temperature Physics*]

- $r_{interparticle} \ll r_{He_3}$ cannot think of these as dressed He_3 atoms(!)
  Direct liquid to FL transition

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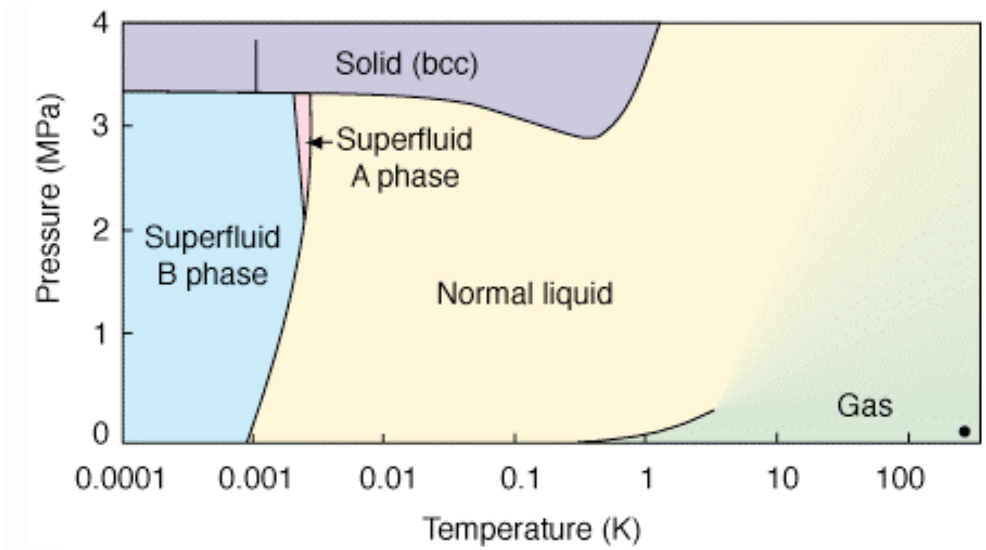
- **There are many systems where this picture fails.**
  - $\boxed{\text{He}^3}$ [Landau]
    - Liquid at $3.2K$
    - Becomes a regular Fermi Liquid at $T = 0.32K$ ($E_F = 4.9K$)
    - (Exotic) Superfluid at $T = 10^{-3}K$ [Lee, Osheroff, Richardson, Nobel 1996]



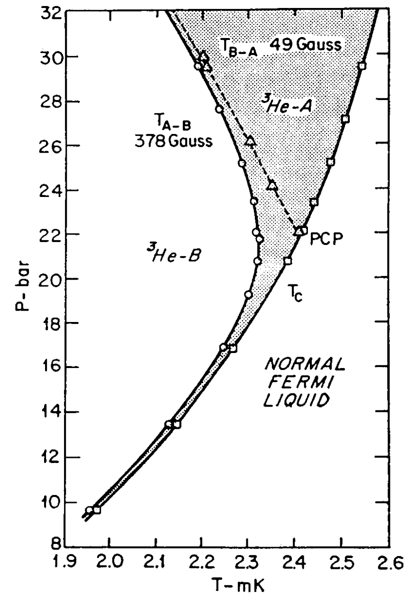
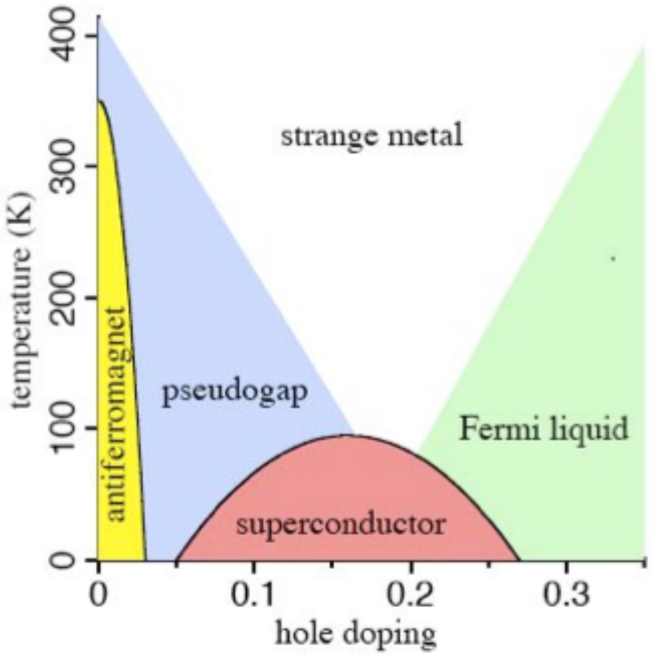
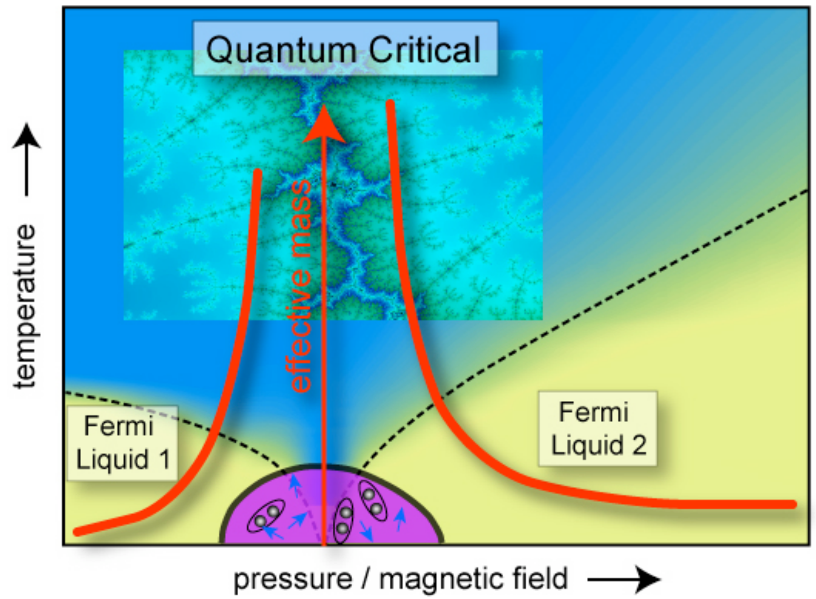


FIG. 11. Experimental data of Paulson, Kojima and Wheatley (1974). At the lowest magnetic field, the A phase is not present below the polycritical point PCP at about 22 bar. In a larger magnetic field, the B phase is suppressed in favor of the A phase even at the lowest pressure, and the polycritical point disappears.

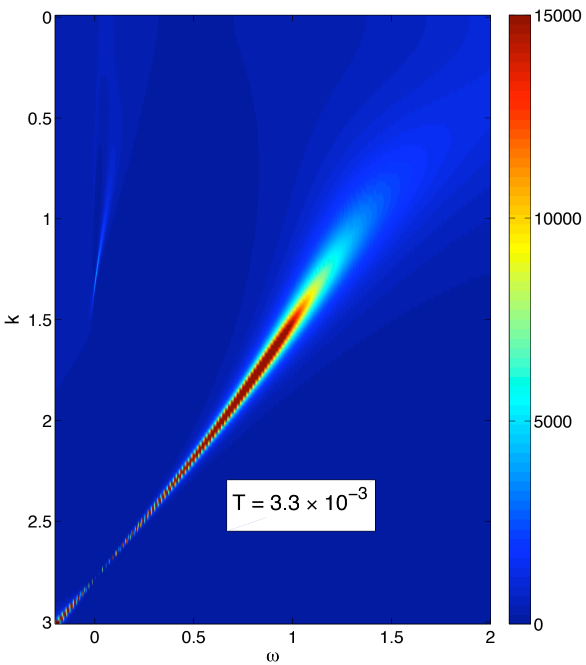
- **Normal FL picture: dressed electrons**
- **There are many systems where this picture fails.**
  - $\boxed{\text{He}^3}$ [Landau]
  - 2D electron gas in MOSFETs,
  - High T_c superconductors, Heavy Fermion systems

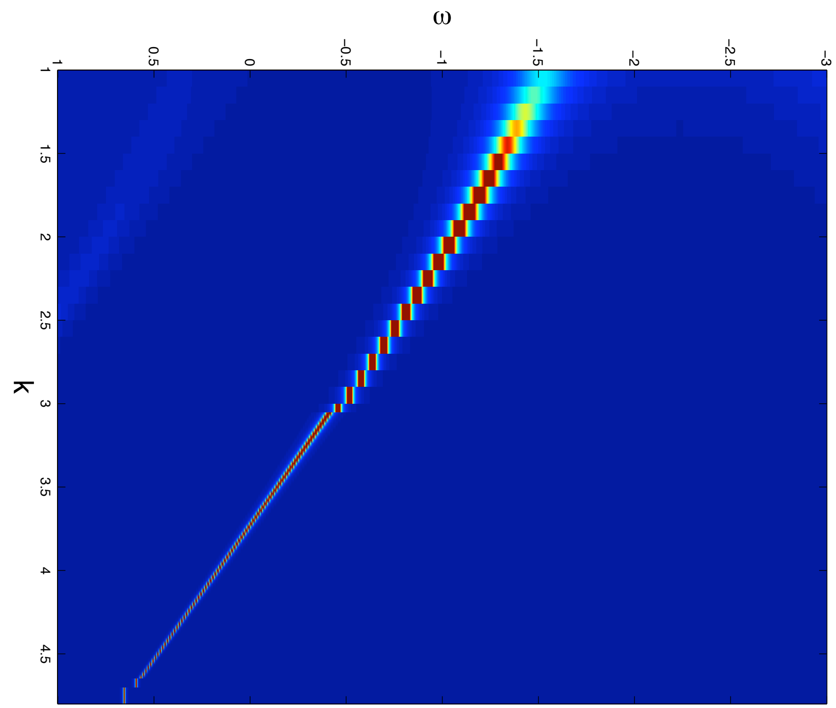




Gravitational stability of the Landau Fermi Liquid

- **Only Landau Fermi-liquid excitations in AdS Dirac Hair BH**





$B_- = 0$

$B_- \neq 0$

"Proves" Robustness of the Landau FL

Conclusion and Outlook

- Dirac instability of AdS RN explains FL stability analogous to order parameter.

- Single fermion approximation.
  - Effectiveness (qualitative)
  - Reliability (quantitative)
  - Applicability (small $\mu \sim q\Delta E$ and/or electron star asymptotics)
- Probing other phases of fermionic matter (a new tool)

$\boxed{\textbf{Thank you.}}$