Heating up the BIon

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Based mainly on the two papers
arXiv:1012.1494 [hep-th] and arXiv:1101.1297 [hep-th]
with Grignani, Marini, Obers and Orselli
Also based on the blackfold papers
Work in progress, JHEP 1004:046 (2010), JHEP 1003:063 (2010), PRL 102: 191301 (2009) and JHEP 0710:110 (2007)
with Emparan, Niarchos and Obers, the 2007 one also with Rodriguez

Motivation:

Subject of talk: New method to describe D-brane probes in thermal backgrounds

Why relevant to large N gauge theories?

→ Through AdS/CFT

Take as example the AdS5/CFT4 correspondence
(AdS_5 x S^5 dual to large N_c $N_Q = 4$ super-Yang-Mills theory)

F-string probes → Wilson loops

D3-brane/D5-brane probes → Wilson loop in large antisym/sym rep

D7-brane probes → Flavors

D3-brane probes on S^3 in AdS_5 or S^5 → Large operators in gauge theory (Giant gravitons)

Generalization to finite temperature gauge theory? AdS_5 → BH in AdS_5

We need to put also the probe branes at finite temperature

→ Method of this talk could be used to find new finite temperature effects in AdS/CFT

<u>Motivation:</u>

Subject of talk: New method to describe D-brane probes in thermal backgrounds

Two definitions of D-branes in string theory:

Open strings with Dirichlet boundary conditions (in some directions)
→ "D-branes are what open strings end on"

A Dp-brane source the Ramond-Ramond (RR) p+2 form field strength in the SUGRA effective low energy description of type IIA or IIB string theory

Polchinski showed that these two definitions define one and the same object

First one: Open string description of D-branes,
Second one: Closed string description of D-branes

→ Open/closed string duality

Three low energy effective description of D-branes:

Open string side:

1) Low energy effective theory for a single D-brane is DBI action (valid for $g_s \ll 1$)

$$I_{\mathrm{DBI}} = -T_{\mathrm{Dp}} \int_{\mathrm{w.v.}} d^{p+1}\sigma \sqrt{-\det(\gamma_{ab} + 2\pi l_s^2 F_{ab})} + T_{\mathrm{Dp}} \int_{\mathrm{w.v.}} e^F \wedge C$$

Fradkin & Tseytlin.
Abouelsaood, Callan, Nappi & Yost

2) In linearized regime with $l_s \rightarrow 0$:

A single D3-brane: U(1) $N_Q = 4$ super-Yang-Mills theory

For a stack of N coincident D3-branes: U(N) $N_Q = 4$ super-Yang-Mills theory

Regime: $\lambda \ll 1$ with $\lambda = g_s N = g_{ym}^2 N$

Witten

Closed string side:

3) For $N \gg 1$ and $\lambda \gg 1$: D-branes backreact on the geometry

Effective description of N flat D3-branes in 10D flat space-time:

Extremal charged brane solution

$l_s \rightarrow 0$ near-horizon limit: Solution is $AdS_5 \times S^5$

Maldacena: Open/closed duality → The AdS/CFT correspondence

All this was for D-branes in zero-temperature string theory backgrounds

What about finite-temperature backgrounds?

DBI action, U(N) SYM theory, SUGRA solution are effective low-energy descriptions, there should be effective descriptions also for non-zero temperature

Two of the three descriptions have obvious generalizations:

2) Linearized regime with $l_s \rightarrow 0$, N coincident D3-branes:
U(N) $N_Q = 4$ Super-Yang-Mills at finite temperature ($\lambda \ll 1$)

3) Non-extremal solution for N coincident flat D-branes in 10D flat space

$l_s \rightarrow 0$ near-extremal limit: (Black hole in AdS_5) x S^5 (the Poincare patch)

2) + 3) $\rightarrow$ AdS/CFT at finite temperature

What about: 1) The DBI action at finite temperature?

Standard method employed the last 10 years:
Use (Wick rotated) classical DBI action in Euclidean background

We are proposing new method that can provide new insights into the thermal physics

But why it is important to understand the DBI action in thermal backgrounds?

→ The DBI action has proven to be a crucial tool in understanding certain non-perturbative string theory phenomena that relies on knowing the full non-linear D-brane dynamics (in zero temperature backgrounds)

In this talk: We want to study the thermal generalization of the BIon solution of the DBI action using a new method that we propose

BIon solution: The first example of a solution of the DBI action that contains full non-linear dynamics of the DBI action → New phenomena introduced

Heating up the Bion is a steppingstone towards more complicated cases of thermal D-brane probes (and other thermal brane probes as well)

Plan for talk:

Motivation

Review of BIon

How to heat up the BIon → Blackfolds

Why the previously used method is not accurate

Solution, phases, dynamics and thermodynamics of the hot BIon

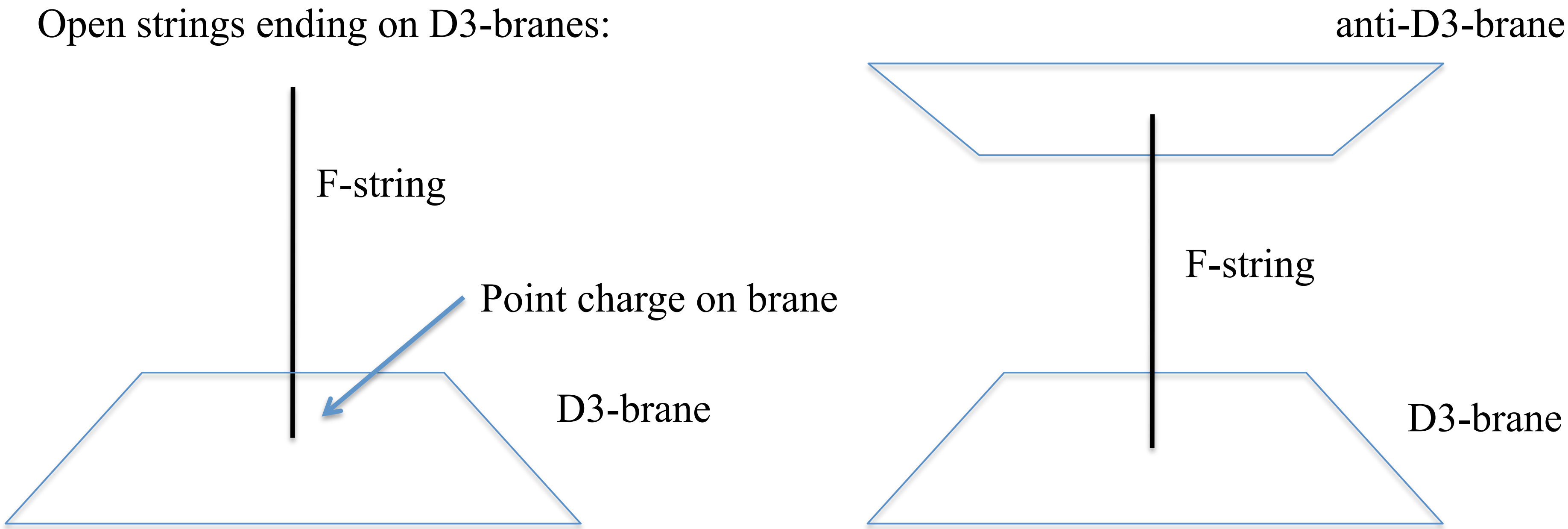
Conclusions and outlook

Review of BIon solution of the DBI action

Callan & Maldacena. Gibbons. Emparan.

Linearized limit of DBI action for D3-brane:
Bosonic part is simply Maxwell's electrodynamics (plus transverse scalars)

Open strings ending on D3-branes:



Full DBI: Non-linear electrodynamics, describes many coincident F-strings → BIon solution

Setup: A D3-brane with electric flux embedded in 10D flat space-time

$X^\mu(\sigma^a)$ - The embedding of the brane in 10D space, σ^a the world-volume coord's

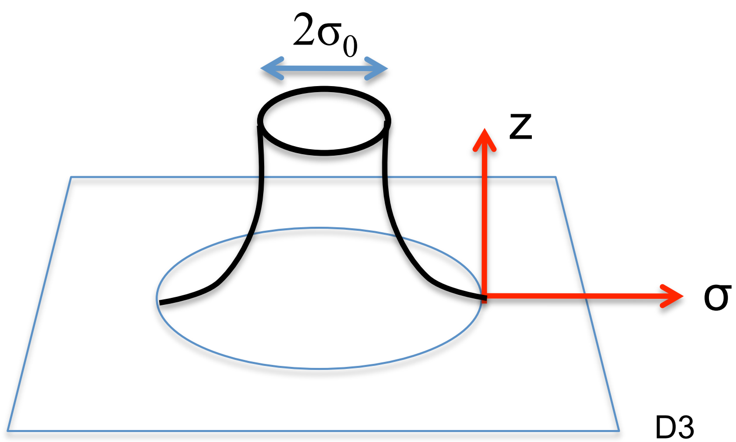
Background metric $$ds^2 = -dt^2 + d\sigma^2 + \sigma^2(d\theta^2 + \sin^2\theta d\phi^2) + dz^2 + \sum_{i=1}^{5} dx_i^2$$

Flat D3-brane defined by: $z = 0$ and $x_i = 0$

We curve the D3-brane in the transverse direction z

Non-trivial profile of brane given by: $z(\sigma)$

We keep the spherical symmetry (no dep. on angles)



Boundary conditions:

For large radius σ the D3-brane becomes flat: $z(\sigma) \to 0$ for $\sigma \to \infty$

Define σ_0 as minimal radius for solution: $z'(\sigma) \to -\infty$ for $\sigma \to \sigma_0$

DBI action for D3-brane: $$I_{\rm DBI} = -T_{\rm D3}\int_{\rm w.v.} d^4\sigma\sqrt{-\det(\gamma_{ab}+2\pi l_s^2 F_{ab})}$$

→ Non-linear electrodynamics (Born-Infeld theory)

(Assuming constant dilaton and no RR fluxes)

Here we introduced the world-volume metric $\gamma_{ab} = g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu$

$g_{\mu\nu}$ - The background metric from previous slide

$X^\mu(\sigma^a)$ - The embedding specified on previous slide

We get

$$\gamma_{ab}d\sigma^a d\sigma^b = -d\tau^2 + \left(1+z'(\sigma)^2\right)d\sigma^2 + \sigma^2\left(d\theta^2+\sin^2\theta d\phi^2\right)$$

To get F-string charge coming out of the D3-brane we should turn on electric field F_{01} along the radial direction σ

Resulting Lagrangian:

$$L = -4\pi T_{\rm D3}\int_{\sigma_0}^{\infty} d\sigma\sigma^2\sqrt{1+z'(\sigma)^2-(2\pi l_s^2 F_{01})^2}$$

We find the solution imposing F-string charge along σ is fixed, and equal to k units charge

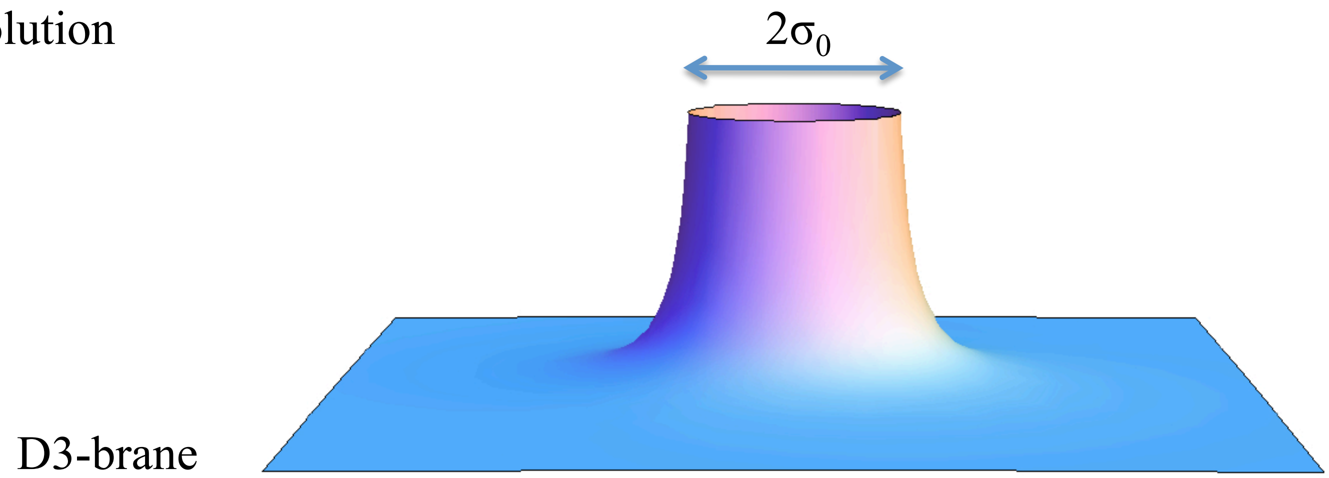
Hamiltonian $$H_{\rm DBI} = 4\pi T_{\rm D3} \int d\sigma \sqrt{1+z'(\sigma)^2} F_{\rm DBI}(\sigma) \ , \quad F_{\rm DBI}(\sigma) \equiv \sigma^2 \sqrt{1+\frac{\kappa^2}{\sigma^4}}$$

where $$\kappa = \frac{k T_{\rm F1}}{4\pi T_{\rm D3}} \ , \quad T_{\rm F1} = \frac{1}{2\pi l_s^2} \ , \quad T_{\rm D3} = \frac{1}{g_s (2\pi)^3 l_s^4}$$

We find the EOM for z(σ): $$\left(\frac{z'(\sigma) F_{\rm DBI}(\sigma)}{\sqrt{1+z'(\sigma)^2}} \right)' = 0$$

Solution (imposing b.c.' s): $$z(\sigma) = \int_\sigma^\infty d\sigma' \left(\frac{F_{\rm DBI}(\sigma')^2}{F_{\rm DBI}(\sigma_0)^2} - 1 \right)^{-\frac{1}{2}} = \int_\sigma^\infty d\sigma' \frac{\sqrt{\sigma_0^4+\kappa^2}}{\sqrt{\sigma'^4 - \sigma_0^4}}$$

We call this the BIon solution



Infinite spike solution:

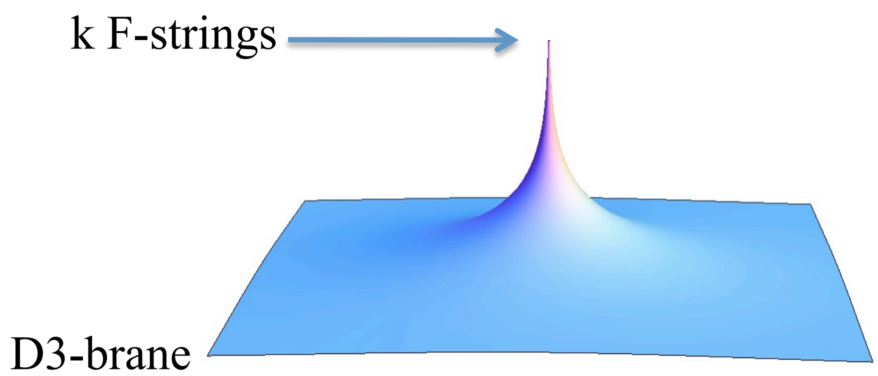
Solution for zero minimal sphere: $\sigma_0 = 0$

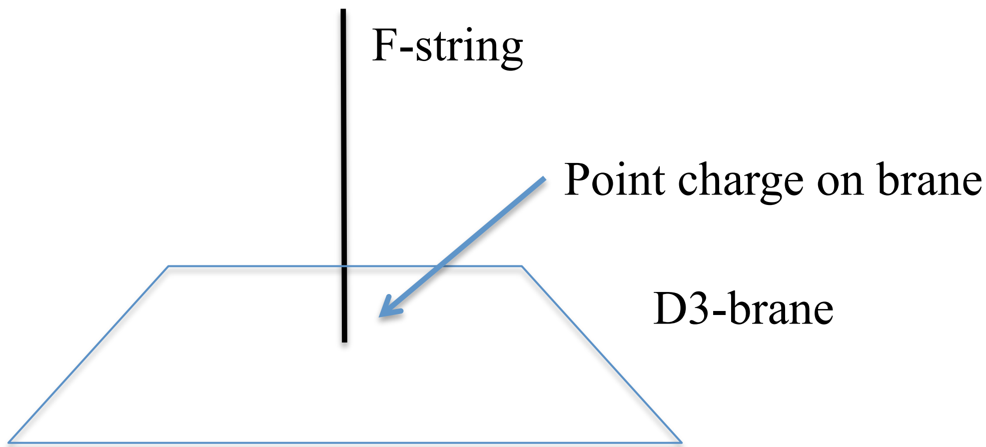
$$z(\sigma) = \frac{\kappa}{\sigma}$$

Measure the string tension at the tip:

$$\left.\frac{dH}{dz}\right|_{\sigma=\sigma_0=0} = kT_{\rm F1}$$

Compare to linearized solution:





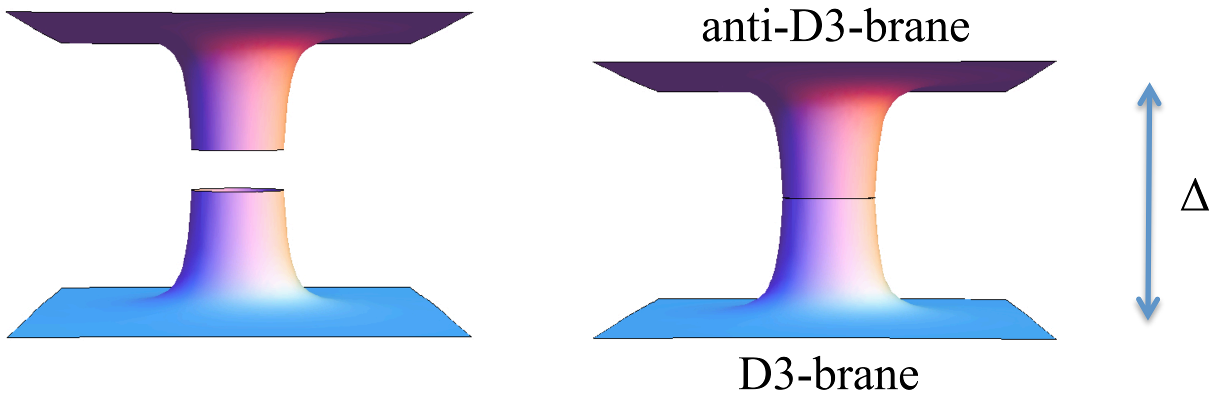
In the BIon solution the singular point at the point charge is resolved

- The F-strings dissolve smoothly into the D-brane
- One can describe a large number of coincident F-strings
- F-strings described in terms of D-brane theory:
  k F-strings is "blown up" to a spherical D-brane

New phenomena from the non-linearity of DBI action

Brane-antibrane-wormhole solution:

Solution for $\sigma_0 > 0$



Need to attach a mirror of the solution
(otherwise D3-brane current not conserved)

Δ is the distance between the D3-branes = $2z(\sigma_0)$

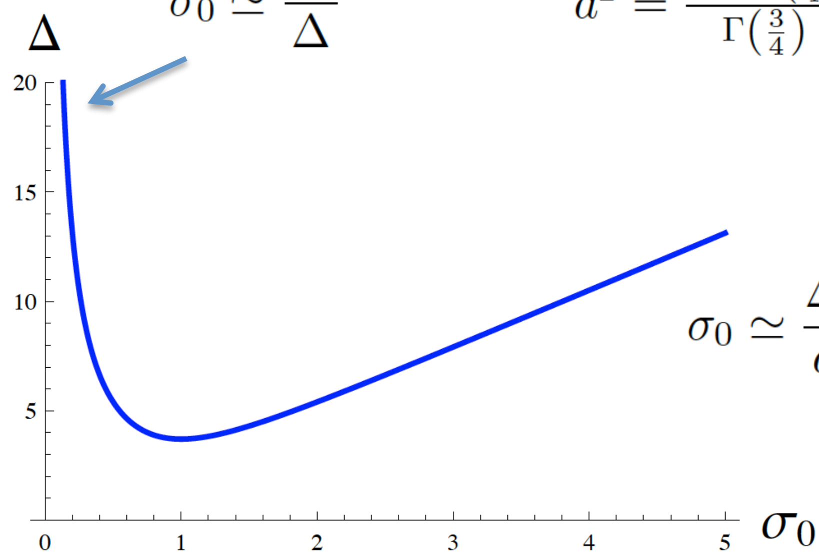
We find:

$$\Delta \equiv 2z(\sigma_0) = \frac{2\sqrt{\pi}\Gamma(\frac{5}{4})\sqrt{\sigma_0^4+\kappa^2}}{\Gamma(\frac{3}{4})\sigma_0}$$

Minimal distance:

$$\Delta_{\rm min} = \frac{2\sqrt{2\pi}\Gamma\left(\frac{5}{4}\right)\sqrt{\kappa}}{\Gamma\left(\frac{3}{4}\right)}$$

$$\sigma_0 \simeq \frac{a\kappa}{\Delta} \qquad a^2 = \frac{2\sqrt{\pi}\Gamma(\frac{5}{4})}{\Gamma(\frac{3}{4})}$$



$$\sigma_0 \simeq \frac{\Delta}{a}$$

How to heat up the BIon?

We want to put the D3-brane with electric flux in background of hot flat space

Take another look at DBI action:

$$I_{\rm DBI} = -T_{\rm D3}\int_{\rm w.v.} d^4\sigma\sqrt{-\det(\gamma_{ab}+2\pi l_s^2 F_{ab})}$$

$$\gamma_{ab} = g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu$$

Varying the target space fields $X^\mu(\sigma^a)$ we get the general EOM's

$$K_{ab}{}^{\rho} T^{ab} = 0$$

Here T^{ab} is the Energy-Momentum tensor for the D3-brane:

$$T^{ab} = -\frac{T_{\rm D3}}{2}\frac{\sqrt{-\det(\gamma+2\pi l_s^2 F)}}{\sqrt{\gamma}}\left[((\gamma+2\pi l_s^2 F)^{-1})^{ab} + ((\gamma+2\pi l_s^2 F)^{-1})^{ba}\right]$$

and $K_{ab}{}^{\rho}$ is the extrinsic curvature tensor for the embedding:

$$K_{ab}{}^{\rho} = \perp^{\rho}{}_{\lambda}(\partial_a\partial_b X^\lambda + \Gamma^\lambda_{\mu\nu}\partial_a X^\mu \partial_b X^\nu)$$

where we defined the projectors $g^{\mu\nu} = h^{\mu\nu} + \perp^{\mu\nu}$, $h^{\mu\nu} = \gamma^{ab}\partial_a X^\mu \partial_b X^\nu$

$K_{ab}{}^{\rho} T^{ab} = 0$ is the extrinsic EOM for any infinitely thin p-brane with EM tensor T^{ab}

I.e. it holds for all probe branes!

A generalization of the geodesic equation for point particles

Idea: To heat up the D3-brane, we should replace the EM tensor with that of a "heated D3-brane"

How to do this? Such a EM tensor is not known for a single D3-brane

However, we can use the EM tensor for the non-extremal D3-brane SUGRA solution

Corresponds to a finite-temperature D3-brane with $N \gg 1$ and $\lambda \gg 1$

Note: We switch from an open string description (the DBI action) to a closed string description of the D3-brane, via SUGRA solutions for the D3-brane

Moreover, by using the EM tensor as read off from the SUGRA D3-brane solution in the probe brane EOM we are employing the so-called Blackfold method

Blackfolds

Emparan, TH, Niarchos, Obers & Rodriguez
Emparan, TH, Niarchos & Obers

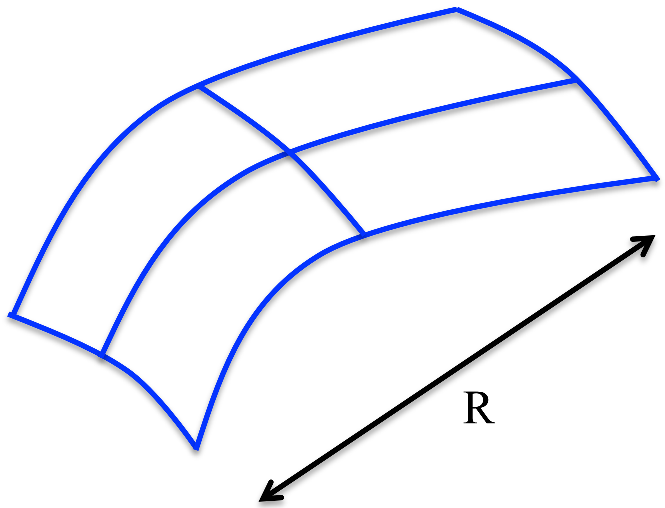
General idea:

We have an effective theory describing black hole dynamics with widely separate scales

Think of a **black** p-brane curved along a submani**fold** (=Blackfold) in a background space-time

R: Curvature length scale (i.e. inverse curvature)

r_0: Horizon scale ("thickness") of black p-brane



In $r_0 << R$ regime: We can make an effective description of the geometry near the horizon of the black brane with an effective Energy-Momentum tensor T_{ab} → EM tensor of effective fluid

Effective theory at large scale R: Dynamics governed by conservation of T_{ab}

Leading order in r_0/R: Probe approximation

Higher orders in r_0/R: Backreaction can systematically be taken into account

But wait, wasn' t there some other method to probe thermal backgrounds?

Indeed: The Euclidean DBI probe method

This method has been used in many works on AdS/CFT in the last 10 years

Recipe: Take a thermal background, go to Euclidean section (by Wick rotation), and put the Wick rotated classical DBI action in this background, and solve the EOM' s

Idea of method: The D-brane is heated up by setting the radius of the Euclidean time circle of the brane equal to the Euclidean time circle of the background

However, in general this method does not give an accurate description

Reason for this: The EOM' s that are solved are independent of the radius of the Euclidean time circle, and by Wick rotation back to Lorentzian signature they are given by

$$K_{ab}{}^{\rho} T^{ab} = 0$$

with T^{ab} given by the **extremal** D3-brane

Putting the probe brane in a thermal background is like putting the matter on the brane in a heat bath of that temperature

Thus, the Euclidean DBI probe method is inaccurate because there are DOF' s on the brane that gets heated up, which in turn changes the EM tensor

In more detail: Consider putting a D3-brane without electric and magnetic fluxes on the brane in a thermal background

The extremal EM tensor is (locally) Lorentz invariant, thus in appropriate coordinates

$$T_{ab} = -T_{\mathrm{D3}}\eta_{ab}$$

But since the thermal background is like a heat bath the electromagnetic DOF' s on the brane gets heated up

For a single D3-brane at small temperatures this is described by N=4 SYM (QED + SUSY):
It is a gas of photons + superpartners on the brane

$$T_{ab} = -T_{\mathrm{D3}}\eta_{ab} + T^{(\mathrm{NE})}_{ab} \ , \quad T^{(\mathrm{NE})}_{00} = \rho \ , \quad T^{(\mathrm{NE})}_{ii} = p \ , \ i = 1,2,3$$

Equation of state: $\rho = 3p = \frac{\pi^2}{2} T^4$

Note also: For non-zero temperature the local Lorentz invariance on the brane is broken

<u>We conclude</u>: The Euclidean DBI probe method is not accurate!

(Of course, in some regimes it can be a good approximation)

Instead for the method we propose at small temperatures (N ≫ 1 and λ ≫ 1)

$$T^{00} = NT_{\mathrm{D3}} + \frac{3\pi^2}{8} N^2 T^4 \ , \quad T^{ii} = \gamma^{ii}(-NT_{\mathrm{D3}} + \frac{\pi^2}{8} N^2 T^4) \ , \ i = 1,2,3$$

Clearly the EM tensor is changed for T > 0 and has the form of a gas of gluons (λ ≫ 1)

Back to heating up the BIon:

We need the EM tensor for a non-extremal D3-brane

However, we also had the electric flux F_{01} turned on

→ The brane is locally an D3-F1 bound state (1/2 BPS for extremal brane)

EM tensor can be read off from the flat non-extremal D3-F1 brane bound state solution

$$T^{00} = \frac{\pi^2}{2} T_{\mathrm{D3}}^2 r_0^4 (5 + 4\sinh^2\alpha) \ , \quad T^{11} = -\gamma^{11} \frac{\pi^2}{2} T_{\mathrm{D3}}^2 r_0^4 (1 + 4\sinh^2\alpha)$$

$$T^{22} = -\gamma^{22} \frac{\pi^2}{2} T_{\mathrm{D3}}^2 r_0^4 (1 + 4\cos^2\zeta \sinh^2\alpha) \ , \quad T^{33} = -\gamma^{33} \frac{\pi^2}{2} T_{\mathrm{D3}}^2 r_0^4 (1 + 4\cos^2\zeta \sinh^2\alpha)$$

with $\quad \gamma_{ab} d\sigma^a d\sigma^b = -d\sigma_0^2 + \gamma_{11} d\sigma_1^2 + \gamma_{22} d\sigma_2^2 + \gamma_{33} d\sigma_3^2$

Number of D3-branes in bound state = N: $\quad \cos\zeta r_0^4 \cosh\alpha \sinh\alpha = \dfrac{N}{2\pi^2 T_{\mathrm{D3}}}$

Imposing number of F-strings = k: $\quad \dfrac{k}{N} = \dfrac{T_{\mathrm{D3}}}{T_{\mathrm{F1}}} \displaystyle\int_{V_{23}} d\sigma^2 d\sigma^3 \sqrt{\gamma_{22}\gamma_{33}} \tan\zeta$

Temperature: $\quad T = \dfrac{1}{\pi r_0 \cosh\alpha}$

The three variables r_0, α and ζ can then be found in terms of k, N and T

Extremal limit: T → 0

We find the EM tensor

$$T^{00} = \frac{NT_{\mathrm{D3}}}{\cos\zeta}\ , \quad T^{11} = -\gamma^{11}\frac{NT_{\mathrm{D3}}}{\cos\zeta}\ , \quad T^{ii} = -\gamma^{ii}NT_{\mathrm{D3}}\cos\zeta\ , \ i = 2, 3$$

Here we should identify $\sin\zeta = 2\pi l_s^2\sqrt{\gamma^{11}}F_{01}$

For N=1 this is the EM tensor for the DBI action

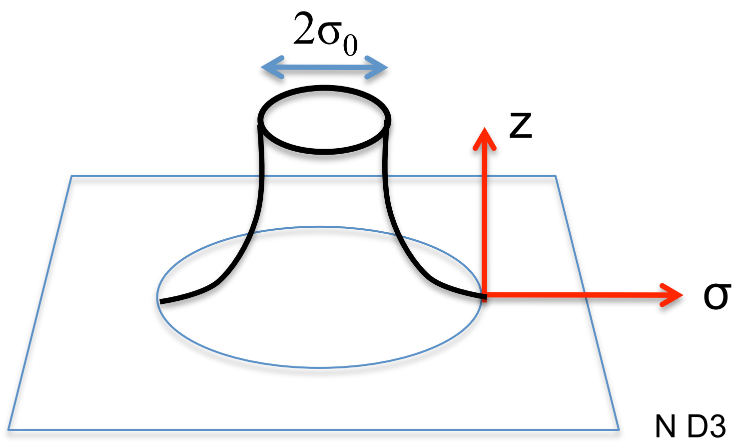
Because of SUSY
→ N =1, λ ≪ 1 regime behaves essentially the same as N ≫ 1, λ ≫ 1 regime,
only difference is trivial N factor which can be absorbed in the 3-brane tension

Having found T^{ab} we are now ready to write down $K_{ab}{}^{\rho} T^{ab} = 0$

The background is hot flat space → This means the metric is just 10D Minkowski

We take the same embedding as for the BIon solution of DBI

Thus, $K_{ab}{}^{\rho}$ is the same, only T^{ab} changes



We can write the resulting EOM as

$$\left(\frac{z'(\sigma) F(\sigma)}{\sqrt{1 + z'(\sigma)^2}} \right)' = 0$$

where we defined the function

$$F(\sigma) = \sigma^2 \frac{4 \cosh^2 \alpha - 3}{\cosh^4 \alpha}$$

From demanding the number of D3-branes is N, and the number of F-strings is k, we have furthermore the constraints

$$\frac{\sinh \alpha}{\cosh^3 \alpha} = \frac{\pi^2}{2} \frac{N T^4}{T_{\mathrm{D3}}} \sqrt{1 + \frac{\kappa^2}{\sigma^4}} \qquad \text{and} \qquad \cos \zeta = \frac{1}{\sqrt{1 + \frac{\kappa^2}{\sigma^4}}}$$

where we defined $\kappa \equiv \dfrac{k T_{\mathrm{F1}}}{4\pi N T_{\mathrm{D3}}}$

From the constraints we find the inequality: $\dfrac{\pi^2}{2}\dfrac{NT^4}{T_{\rm D3}}\dfrac{1}{\cos\zeta} = \dfrac{\sinh\alpha}{\cosh^3\alpha} \leq \dfrac{2\sqrt{3}}{9}$

This gives: $\bar{T}^4 \equiv \dfrac{9\pi^2 N}{4\sqrt{3}T_{\rm D3}} T^4 \leq \cos\zeta \leq 1$

The upper bound $\bar{T} = 1$ corresponds to the maximal temperature of the non-extremal D3-brane

From the bound $\dfrac{\bar{T}^4}{\cos\zeta} = \bar{T}^4\sqrt{1+\dfrac{\kappa^2}{\sigma^4}} \leq 1$ we find $\sigma \geq \sigma_{\rm min} \equiv \dfrac{\sqrt{\kappa}\bar{T}^2}{(1-\bar{T}^8)^{1/4}}$

In particular: $\sigma_0 \geq \sigma_{\rm min} \equiv \dfrac{\sqrt{\kappa}\bar{T}^2}{(1-\bar{T}^8)^{1/4}}$

The minimal sphere radius has a lower bound for a given temperature $\bar{T} > 0$

This is already a qualitative difference with the zero-temperature BIon – there σ_0 can be arbitrarily small

We should now solve the constraint:

$$\frac{9}{2\sqrt{3}}\frac{\sinh\alpha}{\cosh^3\alpha} = \bar{T}^4\sqrt{1+\frac{\kappa^2}{\sigma^4}}$$

Using that the LHS is less or equal to 1 we can define:

$$\cos\delta(\sigma) \equiv \bar{T}^4\sqrt{1+\frac{\kappa^2}{\sigma^4}}$$

In terms of this angle the constraint can be written

$$\frac{4\cos^2\delta}{27}\cosh^6\alpha - \cosh^2\alpha + 1 = 0$$

This has two solutions (with $\cosh\alpha \geqslant 1$)

$$\cosh^2\alpha = \frac{3}{2}\frac{\cos\frac{\delta}{3}+\sqrt{3}\sin\frac{\delta}{3}}{\cos\delta}$$

→Branch connected to the extremal solution
This is the branch we consider in this talk

$$\cosh^2\alpha = \frac{3}{2}\frac{\cos\frac{\delta}{3}-\sqrt{3}\sin\frac{\delta}{3}}{\cos\delta}$$

→ Branch connected to the neutral solution

From the EOM the finite temperature generalization of the BIon solution is now

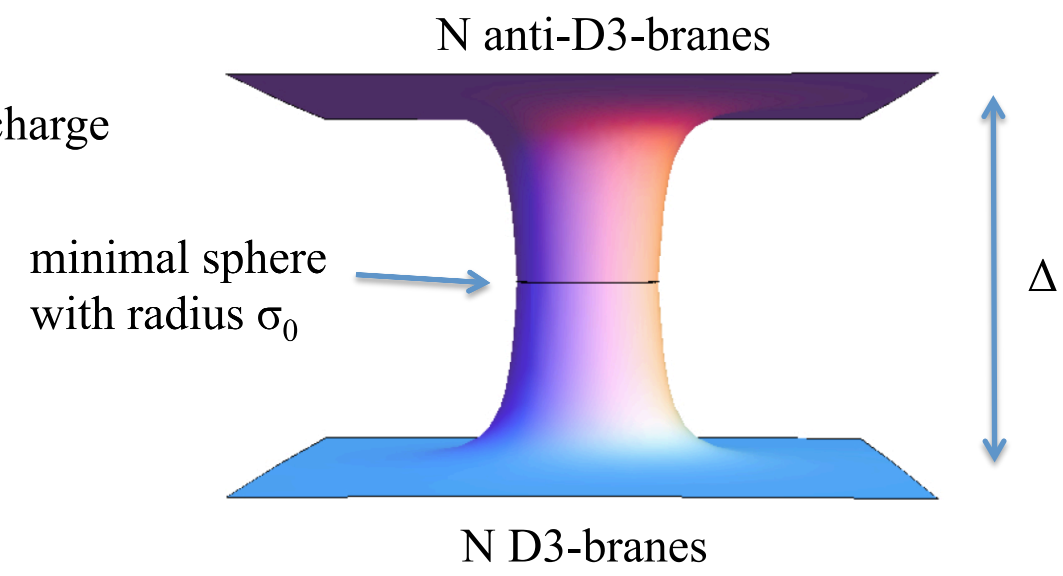
$$z(\sigma) = \int_{\sigma}^{\infty} d\sigma' \left(\frac{F(\sigma')^2}{F(\sigma_0)^2} - 1 \right)^{-\frac{1}{2}} \quad , \quad F(\sigma) = \sigma^2 \frac{4\cosh^2\alpha - 3}{\cosh^4\alpha}$$

We can again explore two possibilities: $\sigma_0 = 0$ and $\sigma_0 > 0$

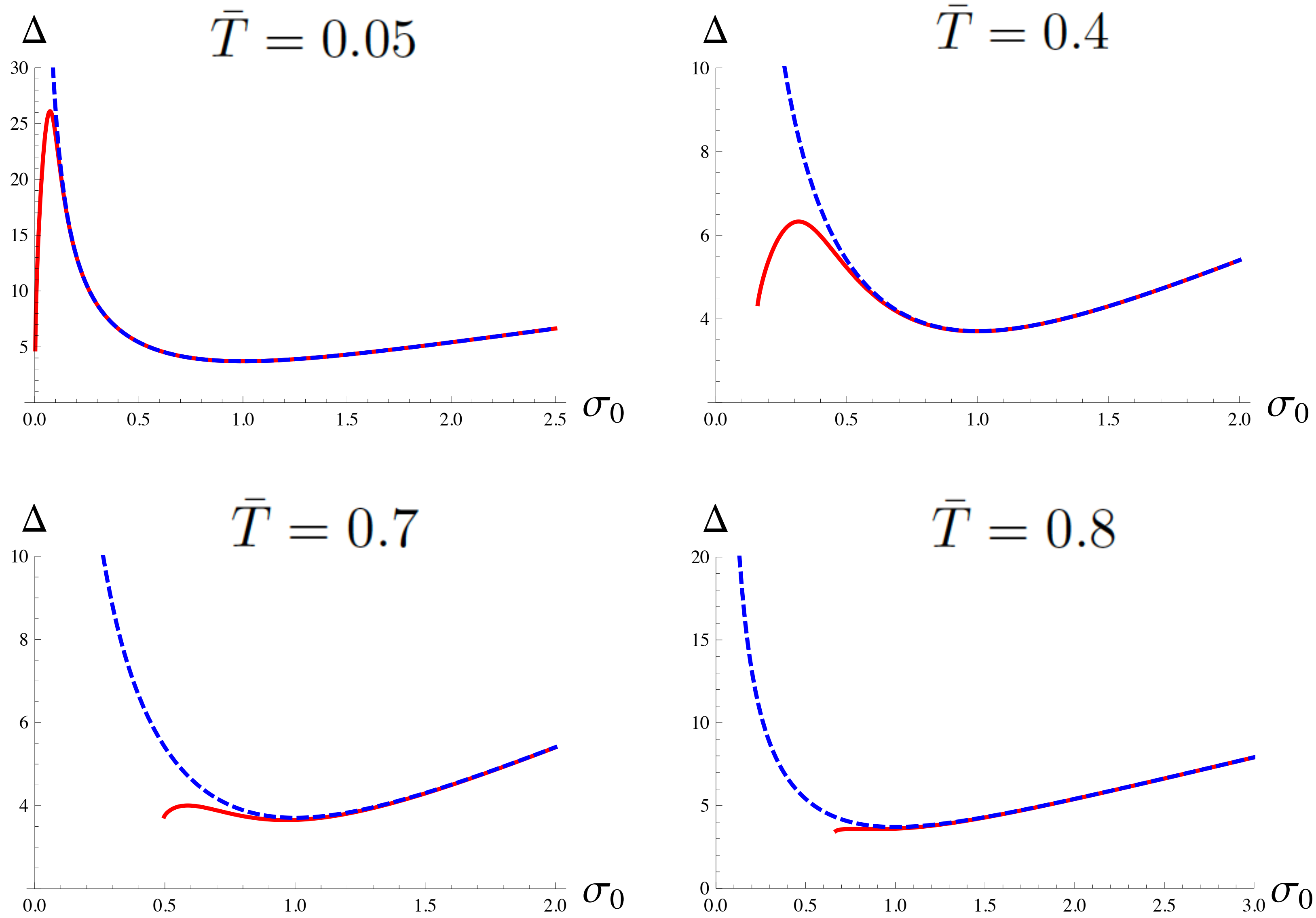
However, $\sigma_0 = 0$ is not possible due to the lower bound on σ_0

→We should consider the brane-antibrane-wormhole with the mirror of the solution (unless we find a case with $z(\sigma_0) = \infty$)

Configuration is thus N D3-branes connected to N anti-D3-branes through a throat with electric flux corresponding to k units of F-string charge



We can now make σ_0-Δ diagrams for various values of $\bar{T} \leq 1$ (we set κ=1)



Blue line is the zero-temperature curve

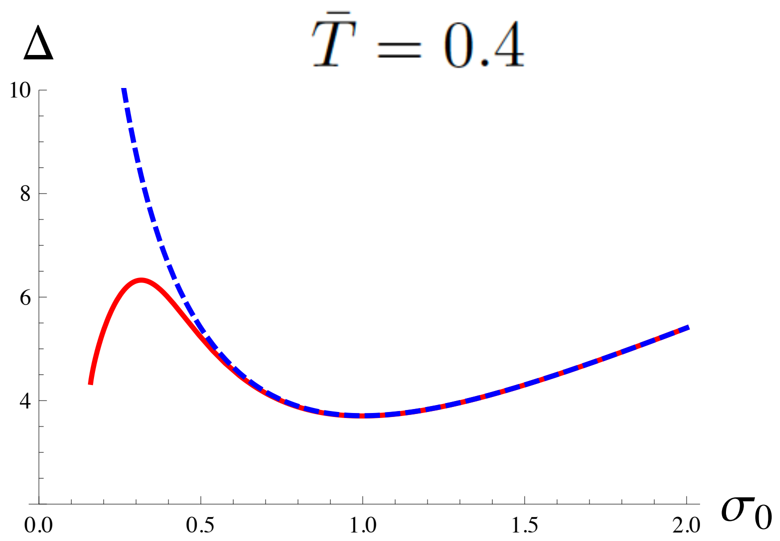
We see that:

1)Not possible to have $z(\sigma_0) = \infty$ for $\sigma_0 < \infty$

2)Three branches instead of two

3)Local maximum for Δ

Goes like $T^{-2/3}$ for small T



Thus, we conclude

I) Non-zero temperature BIon qualitatively different than zero temperature BIon, even for small T

Note here that the "Euclidean DBI probe" method would have given the "blue curve", i.e. the same σ_0-Δ diagram as the zero temperature BIon

II) There is no immediate generalization of the infinite spike BIon solution to non-zero temperature

Can one observe the new thermal physics in the limit $\bar{T} \to 0$?

Put for simplicity $N_{D3} = 1$ (like for DBI probe)

Perturbative corrections to a D3-brane EM tensor small for $\bar{T} << 1$: $\bar{T} \sim g_s^{\frac{1}{4}} l_s T$

But we have

$$\frac{\Delta_{\min}}{l_s} \sim g_s^{\frac{1}{2}} k^{\frac{1}{2}}$$ ← Comes from perturbative part

$$\frac{\Delta_{\max}}{l_s} \sim g_s^{\frac{1}{3}} k^{\frac{1}{2}} (l_s T)^{-\frac{2}{3}}$$ ← Comes from non-perturbative part

We can find regimes where Δ_{max} survives while Δ_{min} goes to zero

Specifically for $g_s = \lambda/N$ (AdS/CFT dictionary, N is from background flux, $R^4 = \lambda l_s^4$):

$$\bar{T} \sim N^{-\frac{1}{4}} RT \qquad \frac{\Delta_{\min}}{R} \sim k^{\frac{1}{2}} N^{-\frac{1}{2}} \lambda^{\frac{1}{4}} \qquad \frac{\Delta_{\max}}{R} \sim k^{\frac{1}{2}} N^{-\frac{1}{3}} \lambda^{\frac{1}{4}} (RT)^{-\frac{2}{3}}$$

→ Δ_{max} can scale like N^0 given the appropriate scaling of k with N

Thermodynamics of the hot BIon:

We can compute the mass, entropy and chemical potentials

They satisfy the 1st law of thermodynamics, and the Smarr relation

$$dM = TdS + \mu_{\mathrm{D3}} dN + \mu_{\mathrm{F1}} dk \ , \quad 4(M - \mu_{\mathrm{D3}} N - \mu_{\mathrm{F1}} k) = 5TS$$

We can also find the free energy

$$\mathcal{F} = M - TS = \frac{4T_{\mathrm{D3}}^2}{\pi T^4} \int_{\sigma_0}^{\infty} d\sigma \sqrt{1 + z'(\sigma)^2} F(\sigma)$$

The free energy can be used as an action for the hot BIon: It is easily seen that varying it with respect to z(σ) we find the EOM

$$\left(\frac{z'(\sigma) F(\sigma)}{\sqrt{1 + z'(\sigma)^2}} \right)' = 0$$

Easier than using $K_{ab}{}^{\rho} T^{ab} = 0$ → This is true more generally in the blackfold formalism

Provides a way to generalize the DBI action for non-zero temperature

But we can also use the thermodynamics to understand which of the three phases is the thermodynamically preferred one

What is a good ensemble?

Fixed temperature (static solution in thermal equilibrium)

Fixed separation distance Δ (is set at infinity $\sigma = \infty$)

Fixed number of D3-branes N and F-strings k (can both be measured for large σ)

Thus the right thermodynamics potential is the free energy

$$\mathcal{F} = M - TS = \frac{4T_{\text{D3}}^2}{\pi T^4} \int_{\sigma_0}^{\infty} d\sigma \sqrt{1 + z'(\sigma)^2} F(\sigma)$$

However for large σ we have $F(\sigma) = \sigma^2 g(\bar{T}) + \mathcal{O}(\kappa/\sigma^2)$

Therefore the free energy formula diverges

Writing the free energy for a given solution in the σ_0-Δ diagram as $\mathcal{F}(\bar{T}, \Delta(\sigma_0); \sigma_0)$

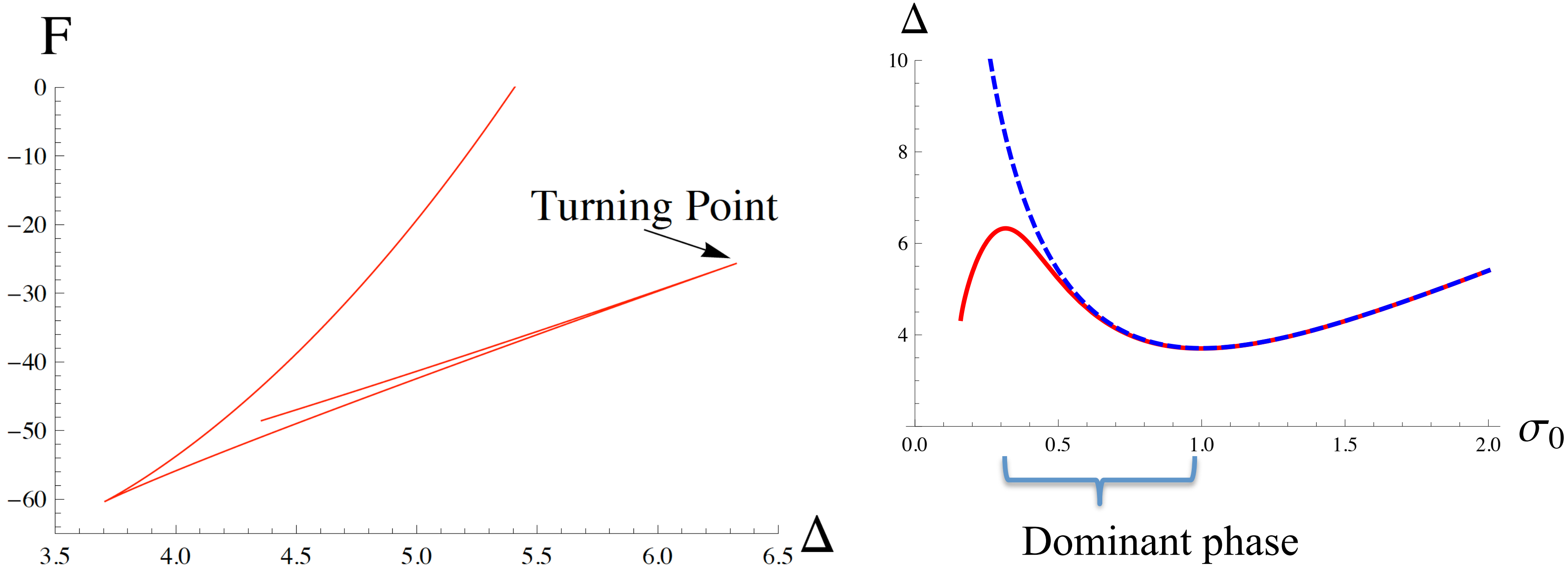
We define the free energy with respect to a particular solution $(\sigma_0,\Delta) = (\sigma_{\text{cut}},\Delta(\sigma_{\text{cut}}))$ as

$$\delta\mathcal{F}(\bar{T}, \Delta(\sigma_0); \sigma_0) \equiv \mathcal{F}(\bar{T}, \Delta(\sigma_0); \sigma_0) - \mathcal{F}(\bar{T}, \Delta(\sigma_{\text{cut}}); \sigma_{\text{cut}})$$

→ This is convergent

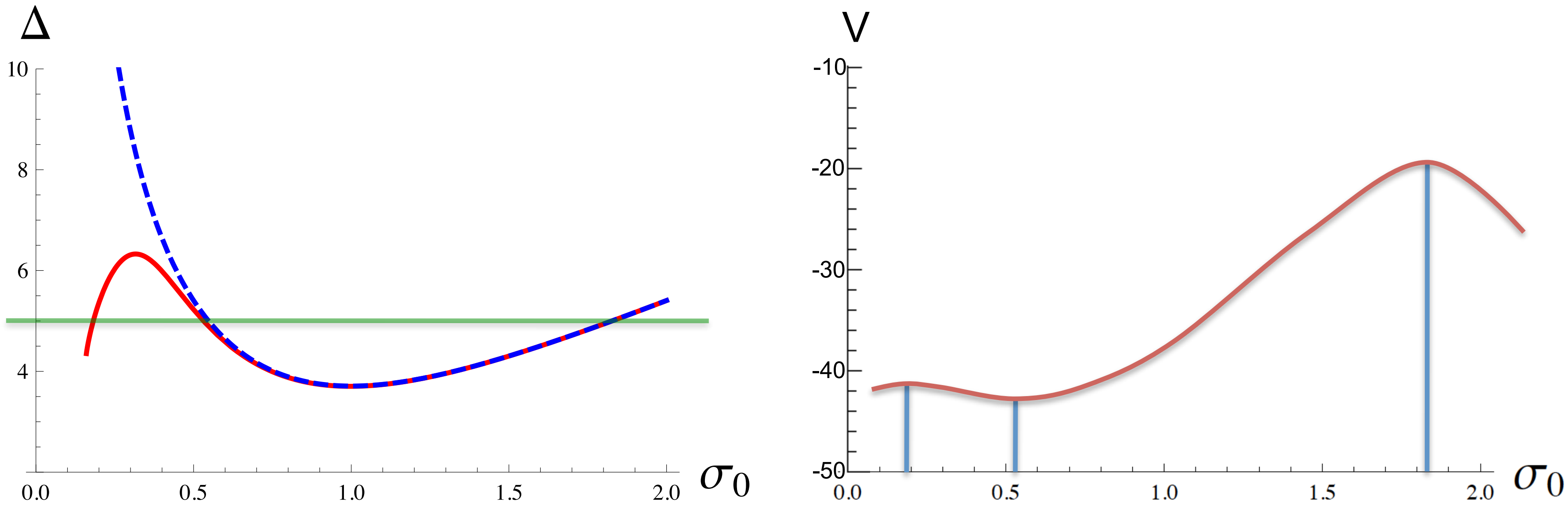
Comparison of phases: Dominant phase is the one with least free energy

Example: $\bar{T} = 0.4$



Dynamics of phases

Consider for given temperature $\bar{T} = 0.4$ and separation $\Delta = 5$:



Extrema of potential: The static solutions, value of potential set to free energy value

Decay channels of unstable equilibria:

Increasing σ_0: Time dependent solution with growing σ_0, everything vanishes

Decreasing σ_0: Either the dominant solution with minimal free energy
Or σ_0 decreases further towards zero, in case the F-string charge annihilate, leaving the brane and antibrane without the wormhole

Validity of the brane probe approximation (in blackfold formalism):

Thickness scale of blackfold is roughly the charge radius r_c, which is the characteristic length scale of the brane geometry associated with the D3-brane charge for the extremal brane
(since we are in the branch connected to the extremal solution)

Charge radius r_c:

$$r_{\mathrm{c}}^4 \sim \frac{N}{T_{\mathrm{D3}}}\sqrt{1+\frac{\kappa^2}{\sigma^4}}$$

Brane probe description valid for whole solution for given T, N, k and σ_0 when:

1) The brane thickness scale is much smaller than the radius of the minimal sphere:
$r_c(\sigma_0) << \sigma_0$

2) The brane thickness scale is much smaller than the length scale of the curvature of the embedding:
$r_c(\sigma_0) << L_{curv}(\sigma_0) \sim \sigma_0$

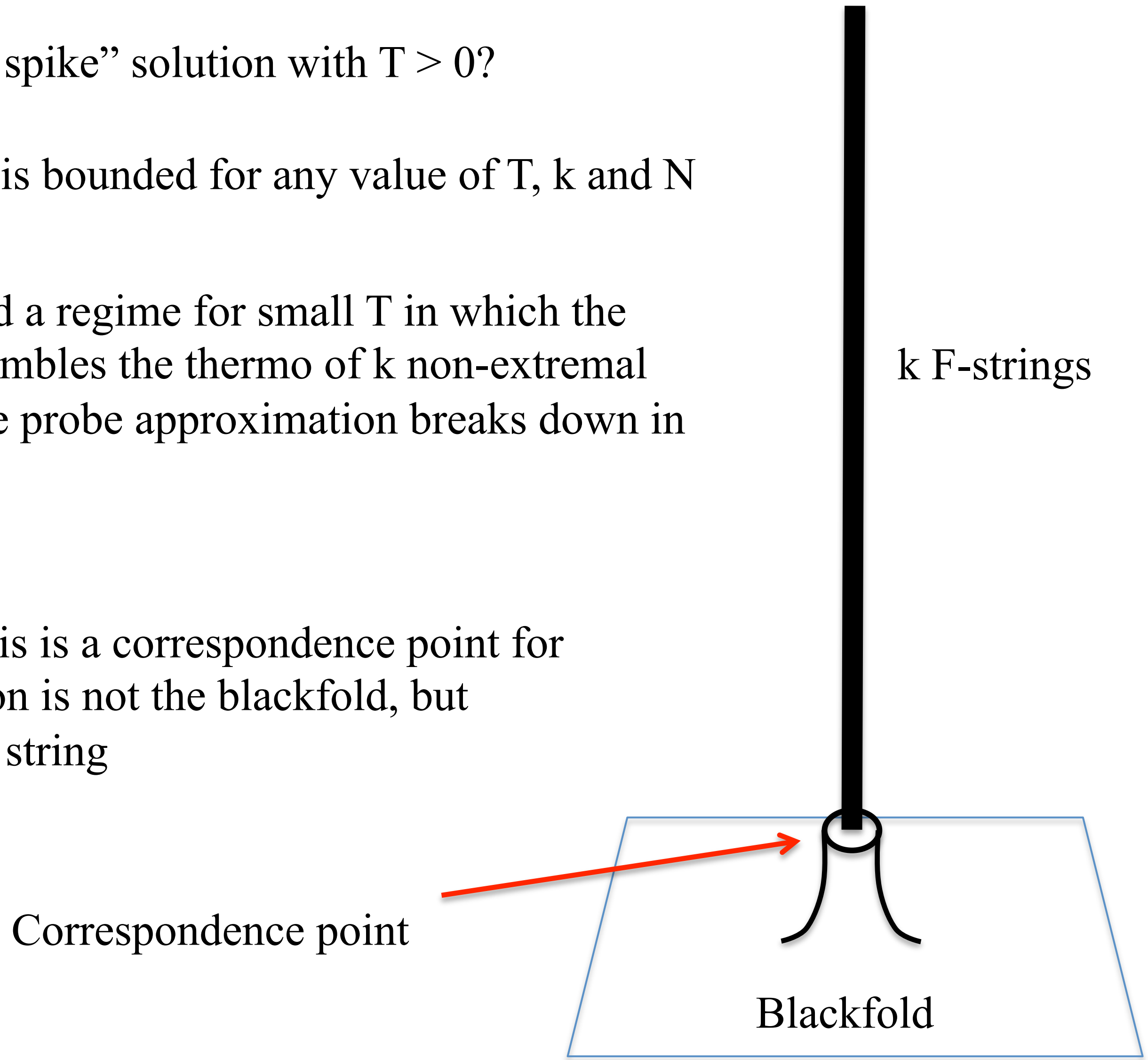
Generalization of the infinite spike solution for non-zero temperature?

Can we have an "infinite spike" solution with $T > 0$?

At first sight no, since Δ is bounded for any value of T, k and N

However, if we try to find a regime for small T in which the local thermo at $\sigma=\sigma_0$ resembles the thermo of k non-extremal F-strings, we find that the probe approximation breaks down in this regime

Natural to suggest that this is a correspondence point for which the right description is not the blackfold, but instead the straight black string



New method for thermal brane probes could be particularly important for black hole backgrounds (instead of thermal flat space):

Why? The Tolman law

$$T_{\rm local} = \frac{T}{R_0}$$

T: Temperature at infinity

T_{local}: Local temperature

R_0: The local red-shift

Probe brane should locally be in thermal equilibrium with T_{local}

But near a black hole: $R_0 \rightarrow 0$

Thus, a probe brane (in a static embedding of the background) is heated up to arbitrarily large temperatures near a black hole horizon!

The Euclidean DBI probe method thus gets increasingly inaccurate as we approach the black hole horizon

Conclusions:

- Proposed new method for D-brane probes in thermal backgrounds

  Argument why previous method is inaccurate

- Method employs blackfold technique of making probe brane with EM tensor read off from SUGRA branes

  Different regime than DBI action

  DBI: N = 1 and $\lambda \ll 1$
  Blackfold: $N \gg 1$ and $\lambda \gg 1$
  with $\lambda = g_s N$

  Finite temperature physics found classically since we use closed string description

- We found thermal generalization of BIon solution to hot flat space

  Qualitatively different than the zero temperature case
  Shows explicitly that we get different results using the new method

  Interesting new thermal physics, and also new dynamics

  Would be interesting to know qualitative arguments for the three equilibria

New low energy description of D-branes, with or without temperature:

Open string side:

1) Low energy effective theory for a single D-brane is DBI action (valid for $g_s \ll 1$)

$$I_{\mathrm{DBI}} = -T_{\mathrm{Dp}} \int_{\mathrm{w.v.}} d^{p+1}\sigma \sqrt{-\det(\gamma_{ab} + 2\pi l_s^2 F_{ab})} + T_{\mathrm{Dp}} \int_{\mathrm{w.v.}} e^F \wedge C$$

Fradkin & Tseytlin.
Abouelsaood, Callan,
Nappi & Yost

Can describe general dynamical D-brane probes

2) In linearized regime with $l_s \rightarrow 0$: Regime: $\lambda \ll 1$ with $\lambda = g_s N = g_{ym}^2 N$

A single D3-brane: U(1) $N_Q = 4$ super-Yang-Mills theory

For a stack of N coincident D3-branes: U(N) $N_Q = 4$ super-Yang-Mills theory

Witten

Closed string side:

3) For $N \gg 1$ and $\lambda \gg 1$: Non-dynamical flat D-branes described by SUGRA brane solution

4) For $N \gg 1$ and $\lambda \gg 1$:

General dynamical D-brane probe described using blackfolds

Includes both extremal and non-extremal D-branes

So, we found a closed string analogue of 1)

Outlook:

- Would be interesting to examine previously found results for thermal probes in finite temperature AdS/CFT

  Polyakov loop (D3 & D5)
  Melting phase transition
  Meson spectrum
  Quark-Antiquark potential

  We have learned from heating up the BIon: Even if corrections from thermal gas on brane seems to be suppressed perturbatively, the dynamics of the brane system can change (e.g. maximal separation Δ)

  The Tolman law suggests modifications to the brane dynamics close to an event horizon

- Thermal version of the DBI action in the regime $N \gg 1$ and $\lambda \gg 1$

- Possible to find thermal DBI action directly from open string theory?