Does Nature Have a Preferred 1/Nc Expansion?

(Of course, some limits do go both ways)

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Large N Gauge Theories
Galileo Galilei Institute

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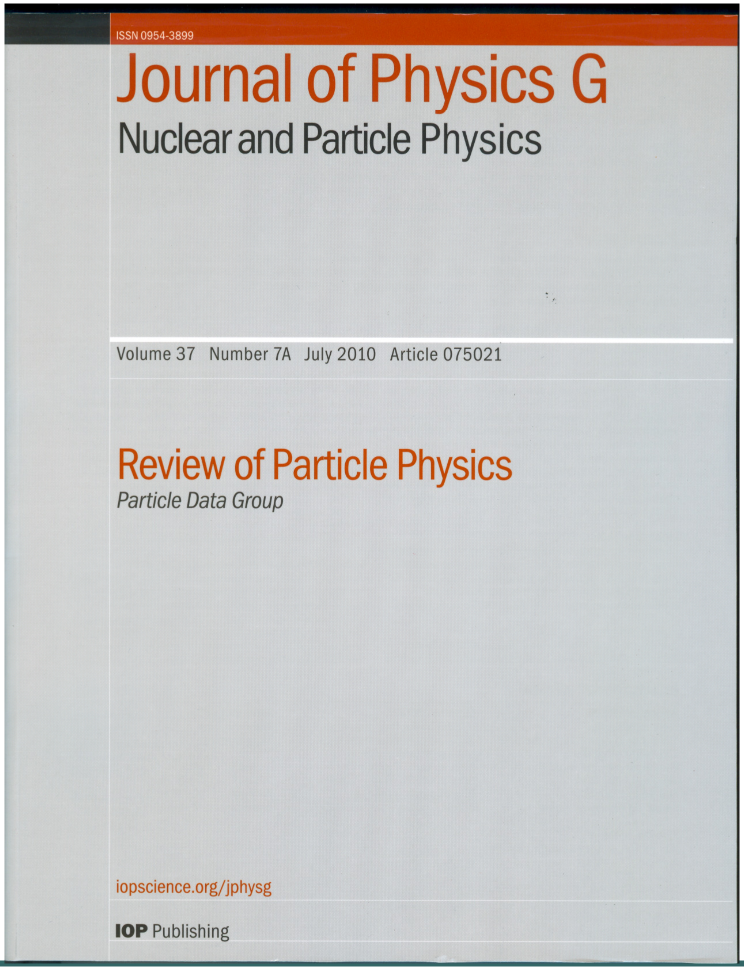
Warning:
I will refer to this textbook



Outline

1) Elements of large N_C phenomenology
2) ’t Hooft large N_C vs. orientifold large N_C
3) How to build a baryon
4) Operator analysis and effective Hamiltonian
5) Probes: Masses vs. magnetic moments
6) Results: Which limit works better?

Just the Facts, Ma'am

- Large N_C QCD means:
  - QCD gauge group is enlarged from SU(3) to SU(N_C)
  - Quarks transform under fundamental representation ★ N_C states, labeled by values of the color quantum number
- It is a well-defined gauge theory limit when $\alpha_s \sim 1/N_C$ ['t Hooft (1974)]
  - Meaningful to perform a $1/N_C$ expansion
  - Color singlets formed from $q\bar{q}$ pairs (color structure $\delta^{\alpha}{}_{\beta}$) or from N_C quarks (color structure $\varepsilon^{\alpha 1, \alpha 2, \ldots, \alpha N_c}$)
- Mesons at large N_C are:
  - free [$O(N_C^0)$ masses]
  - stable [$O(1/N_C)$ widths]
  - scatter weakly [$O(1/N_C^2)$ cross sections] [Veneziano (1976)]

“Classical” Large N_C Baryons

[Witten (1979)]

- Large N_C baryon masses = $O(N_C^{1})$ (N_C fundamental rep quarks!)
  - Much heavier than mesons → Treat semiclassically
- Ground-state band assumed to have totally symmetric spin × flavor wave function, each quark in an orbital *s* wave
  - Supported by phenomenology: For $N_C = 3$ these states fill a single symmetric **56** of spin × flavor SU(6): *N*, *Δ*, *Σ*, *Ω*, etc.
  - Meson-baryon trilinear coupling scales as $N_C^{1/2}$
  - Meson-baryon scattering amplitude scales as N_C^{0}
  - Combinatorics of the N_C quarks plus ’t Hooft scaling ($\alpha_s \sim 1/N_C$) gives $O(N_C^{0})$ potential energy per quark
    - ➢ Hartree approximation holds; size of baryon scales as $O(N_C^{0})$

For $N_c > 3$, multiplets are much bigger

The $N_c = 17$
spin-½ “octet”

```
        1   1
      1   2   1
    1   2   2   1
   1  2   2   2   1
  1  2  2   2   2  1
 1  2  2  2   2  2  1
1  2  2  2  2  2  2  1
1 2  2  2  2  2  2  2 1
1 2 2  2  2  2  2  2 2 1
 1 1 1  1  1  1  1 1 1
```

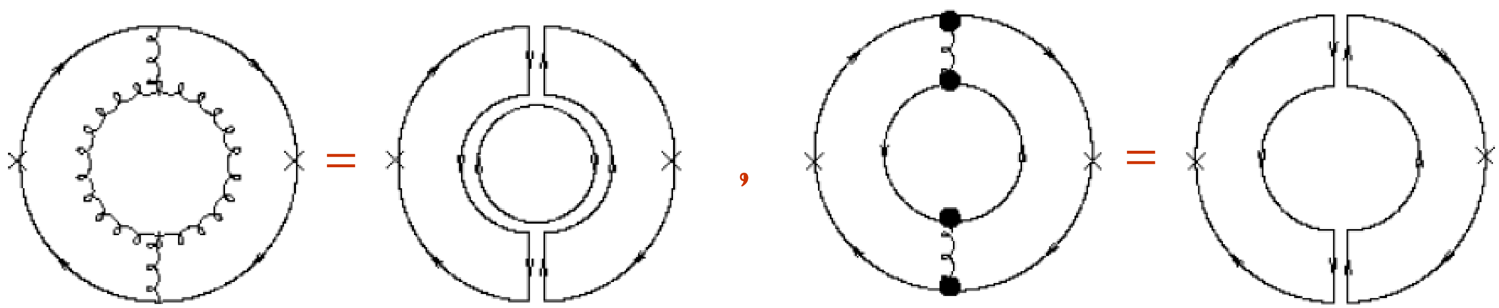
Summary of Hadrons in $1/N_c$

- If N_C is considered large, mesons exist and live long enough to be detected
  - Fact: Unflavored mesons seen all the way up to 2 GeV and beyond; if their lifetimes were sufficiently short, only collections of π's would be seen
- Baryons have N_C quarks (fundamental rep) but don't grow in size with N_C
- Meson-baryon scattering amplitudes don't grow or shrink with N_C
  - Fact: Many distinct baryon resonances (poles in scattering amplitudes) are observed

In Standard ’t Hooft Large N_c, …

- Quarks transform under N_C-dimensional fundamental representation □ of $SU(N_C)$ Yang-Mills gauge group; each one carries a single color fundamental charge r, b, g, …
- ’t Hooft double-line notation: Each quark carries single directed line indicating color charge flow; gluons (in the adjoint) carry two oppositely-oriented lines

  → Suppression of internal quark loops



But when $N_c = 3$, ...

- The fundamental (F) and two-index antisymmetric (AS) representations are equivalent:

$$q_{ij} \equiv \varepsilon_{ijk}\, q^k \leftrightarrow \text{(anti-red)(anti-green)} \equiv \text{blue} \leftrightarrow \overline{\begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}} \equiv \square$$

- Equivalent AS representation for arbitrary N_C has $N_C - 1$ indices
- But how do we know that quarks at arbitrary N_C live in the F and not the two-index AS representation? (hence represented by double line, unsuppressed compared to gluons)
- *Sociology*: Because Witten says so
- *Philosophy*: If quarks are not F, then what use is the fundamental representation?
- *Irritability*: Give me just one good reason to even consider it!

Orientifold Large N_c

Armoni, Shifman, Veneziano (ASV):

Nucl. Phys. **B667**, 170 (2003); Phys. Rev. Lett. **91**, 191601 (2003)

- Pure gauge $\mathcal{N} = 1$ SUSY for $U(N_C)$, in which a large number of nontrivial symmetry relations can be obtained, contains gluinos, which transform according to the *adjoint* (color-anticolor) representation
- As $N_C \to \infty$, one obtains a theory exactly equivalent in many sectors by replacing adjoint fermions with AS fermions:
- This "daughter" theory can be found to live on a brane configuration with an orientifold plane (a place where string lacks orientation)

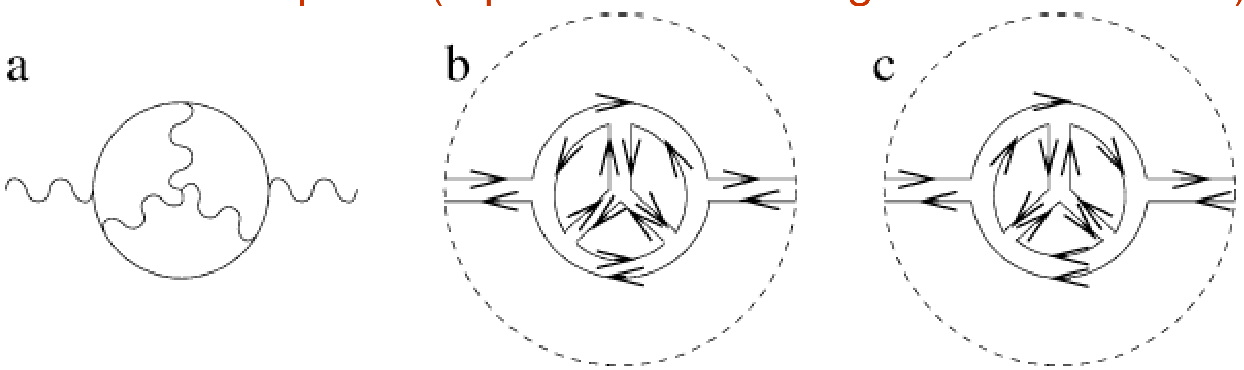


Fig. 2. (a) A typical planar contribution to the vacuum polarization. (b) For $\mathcal{N} = 1$ SYM. (c) For the non-SUSY theory.

The Three Large N_c Limits

$$N_C^{Adj} \leftrightarrow N_C^{AS} \leftrightarrow N_C^{F}$$

SUSY gluodynamics

ASV equivalence

orientifold theory

equivalent at $N_C = 3$

conventional 't Hooft large N_C

Baryons in the Two Limits

- Baryon wave functions comprised of $N_C{}^F$ quarks have been studied since Witten (Nucl. Phys. **B160**, 57 [1979]):

$$B_{\mathrm{F}} \sim \epsilon^{i_1, i_2, \ldots, i_{N_c}} q_{i_1} q_{i_2} \cdots q_{i_{N_c}}$$

- With AS quarks, several constructions are possible (Bolognesi, Phys. Rev. D **75**, 065030 [2007]). For example,
  - Using the same ε invariant with ½N_C of the AS quarks:

$$B_\phi \sim \epsilon^{j_1, j_2, \cdots, j_{N_c}} q_{j_1, j_2} q_{j_3, j_4} \cdots q_{j_{N_c-1}, j_{N_c}}$$

  but B_Φ baryons exist only for even N_C, and (as pointed out by Bolognesi) have other physical problems

The N_C^{AS} Baryon

- Bolognesi instead proposed a construction for baryon wave functions in which all AS quarks [$\frac{1}{2}N_C(N_C - 1)$ in total] are completely antisymmetrized; for $N_C = 3$ it reads:

$$B_\psi \sim (\epsilon_{i_2,j_2,i_1}\epsilon_{i_3,j_3,j_1} - \epsilon_{i_3,j_3,i_1}\epsilon_{i_2,j_2,j_1})q^{i_1,j_1}q^{i_2,j_2}q^{i_3,j_3}$$

  where, again, $q_{ij} \equiv \varepsilon_{ijk}\, q^k$
  - This B_ψ construction reduces to B_F when $N_C = 3$
  - One can build a B_ψ baryon for every integer $N_C \geq 2$
  - The general N_C wave function can be expressed in closed form (RFL, unpublished)
- So let us compute observables for B_F (large N_C^F expansion) and B_ψ (large N_C^{AS} expansion) baryons and compare them

Parametrizing Static Baryon Properties

- The lightest (N, Δ, Σ, etc.) baryons are degenerate as $N_C \to \infty$ (for either the N_C^F or N_C^{AS} limit), and fill a multiplet that reduces for $N_C = 3$ to the old SU(6) **56**-plet
- They differ only in quark flavor content or relative quark spin orientation, whose effects can be parametrized by operators using the basis (again, with either N_C^F or N_C^{AS} quarks):

$$J^i = \sum_\alpha q_\alpha^\dagger \left(\frac{\sigma^i}{2} \otimes \mathbb{1} \right) q_\alpha,$$

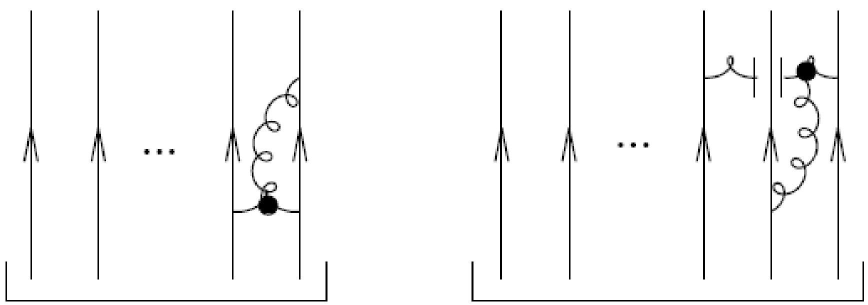
$$T^a = \sum_\alpha q_\alpha^\dagger \left(\mathbb{1} \otimes \frac{\lambda^a}{2} \right) q_\alpha,$$

$$G^{ia} = \sum_\alpha q_\alpha^\dagger \left(\frac{\sigma^i}{2} \otimes \frac{\lambda^a}{2} \right) q_\alpha,$$

The Effective Hamiltonian

Dashen, Jenkins & Manohar; Carone, Georgi & Osofsky; Luty & March-Russell (1994)

- Processes involving the (entangled) interaction of ***n*** quarks are represented by ***n*-body operators**; in N_C^F, typical diagrams are:



2-body 3-body

- Generic ***n***-body operators are suppressed by $1/N_C^{\boldsymbol{n}}$ [N_C^F] or $1/[\frac{1}{2}N_C(N_C-1)]^{\boldsymbol{n}} \sim 1/N_C^{\boldsymbol{2n}}$ [N_C^{AS}], one factor for each *J*, *T*, *G*
- From these operators construct a baryon Hamiltonian that is perturbative in powers of $1/N_C$ [**Effective theory**]

Calculating with the Hamiltonian

- For N_C^F,

$$H = c_0 \, N_C \, 1 + c_1^{(8)} \, N_C^0 \, T^8 + c_J \, J^2 / N_C + \ldots$$

where $T^8 = \sum_{quarks\ \alpha} q_\alpha^+ \frac{\lambda^8}{2} q_\alpha$, $J^2 = \sum_\alpha \sum_\beta \left(q_\alpha^+ \frac{\sigma^i}{2} q_\alpha \right) \left(q_\beta^+ \frac{\sigma^i}{2} q_\beta \right)$

- For N_C^{AS}, just replace each $N_C \rightarrow N_C^2$
- c_k: dimensionless coefficients ($\times \Lambda_{QCD}$), should be of order unity
- Easy to include SU(3) flavor breaking: *e.g.*, $c_1^{(8)} \rightarrow \varepsilon c_1$, $\varepsilon \approx 0.25$
- Since the operators form a complete set, to each one corresponds a unique combination of baryon masses
- Compare to the average multiplet mass (N_C [N_C^F], N_C^2 [N_C^{AS}]) (N_C^F Calculation performed by Jenkins & RFL [1995])

Calculate & tabulate matrix elements

- *e.g.*, an excerpt for a few (magnetic moment) operators in N_c^F:

$\Sigma^0\Lambda$	0	$-\frac{1}{12N_c}\sqrt{(N_c-1)(N_c+3)}$	0	0	0
Σ^-	$\frac{1}{18}$	$-\frac{1}{12N_c}(N_c-1)$	$-\frac{1}{36N_c}(N_c-9)$	$-\frac{1}{6N_c}$	$\frac{1}{12N_c^2}(N_c-9)$
Ξ^0	$-\frac{2}{9}$	$-\frac{1}{18N_c}(N_c+9)$	$\frac{1}{9N_c}(N_c-3)$	$-\frac{1}{3N_c}$	$\frac{1}{6N_c^2}(N_c-3)$
Ξ^-	$-\frac{2}{9}$	$\frac{1}{18N_c}(N_c-9)$	$\frac{1}{9N_c}(N_c-9)$	$-\frac{1}{3N_c}$	$\frac{1}{6N_c^2}(N_c-9)$
Δ^+p	0	0	0	0	0
Δ^0n	0	0	0	0	0
$\Sigma^{*0}\Lambda$	0	$\frac{1}{6\sqrt{2}N_c}\sqrt{(N_c-1)(N_c+3)}$	0	0	0
$\Sigma^{*0}\Sigma^0$	$\frac{\sqrt{2}}{9}$	$\frac{1}{3\sqrt{2}N_c}$	$-\frac{1}{9\sqrt{2}N_c}(N_c-3)$	0	0
$\Sigma^{*+}\Sigma^+$	$\frac{\sqrt{2}}{9}$	$\frac{1}{12\sqrt{2}N_c}(N_c+5)$	$-\frac{1}{9\sqrt{2}N_c}(N_c+3)$	0	0
$\Sigma^{*-}\Sigma^-$	$\frac{\sqrt{2}}{9}$	$-\frac{1}{12\sqrt{2}N_c}(N_c-3)$	$-\frac{1}{9\sqrt{2}N_c}(N_c-9)$	0	0
$\Xi^{*0}\Xi^0$	$\frac{\sqrt{2}}{9}$	$\frac{\sqrt{2}}{9N_c}(N_c+3)$	$-\frac{1}{9\sqrt{2}N_c}(N_c-3)$	0	0
$\Xi^{*-}\Xi^-$	$\frac{\sqrt{2}}{9}$	$-\frac{\sqrt{2}}{9N_c}(N_c-3)$	$-\frac{1}{9\sqrt{2}N_c}(N_c-9)$	0	0

$I = 0$ Baryon Mass Operators

$$
\begin{aligned}
M &= M_0^1 + M_0^8 + M_0^{27} + M_0^{64} \\
M_0^1 &= c_{(0)}^{1,0}\, N_c \mathbb{1} + c_{(2)}^{1,0}\, \frac{1}{N_c} J^2, \\
M_0^8 &= c_{(1)}^{8,0}\, T^8 + c_{(2)}^{8,0}\, \frac{1}{N_c}\{J^i, G^{i8}\} + c_{(3)}^{8,0}\, \frac{1}{N_c^2}\{J^2, T^8\}, \\
M_0^{27} &= c_{(2)}^{27,0}\, \frac{1}{N_c}\{T^8, T^8\} + c_{(3)}^{27,0}\, \frac{1}{N_c^2}\{T^8, \{J^i, G^{i8}\}\}, \\
M_0^{64} &= c_{(3)}^{64,0} \frac{1}{N_c^2}\{T^8, \{T^8, T^8\}\}, \qquad (3.4)
\end{aligned}
$$

Isosinglet Mass Combinations

$$
\begin{aligned}
N_0 &= \frac{1}{2}(p+n)\,, \\
\Sigma_0 &= \frac{1}{3}\left(\Sigma^+ + \Sigma^0 + \Sigma^-\right), \text{ and } \Lambda \\
\Xi_0 &= \frac{1}{2}\left(\Xi^0 + \Xi^-\right), \\
\Delta_0 &= \frac{1}{4}\left(\Delta^{++} + \Delta^+ + \Delta^0 + \Delta^-\right), \\
\Sigma_0^* &= \frac{1}{3}\left(\Sigma^{*+} + \Sigma^{*0} + \Sigma^{*-}\right), \\
\Xi_0^* &= \frac{1}{2}\left(\Xi^{*0} + \Xi^{*-}\right). \text{ and } \Omega
\end{aligned}
$$

Scale of SU(3) flavor breaking

- One of many possible measures:

$$\epsilon \equiv \frac{1}{3}\sum_{i=1}^{3} \frac{B_i - N_0}{(B_i + N_0)/2} \approx 0.25$$

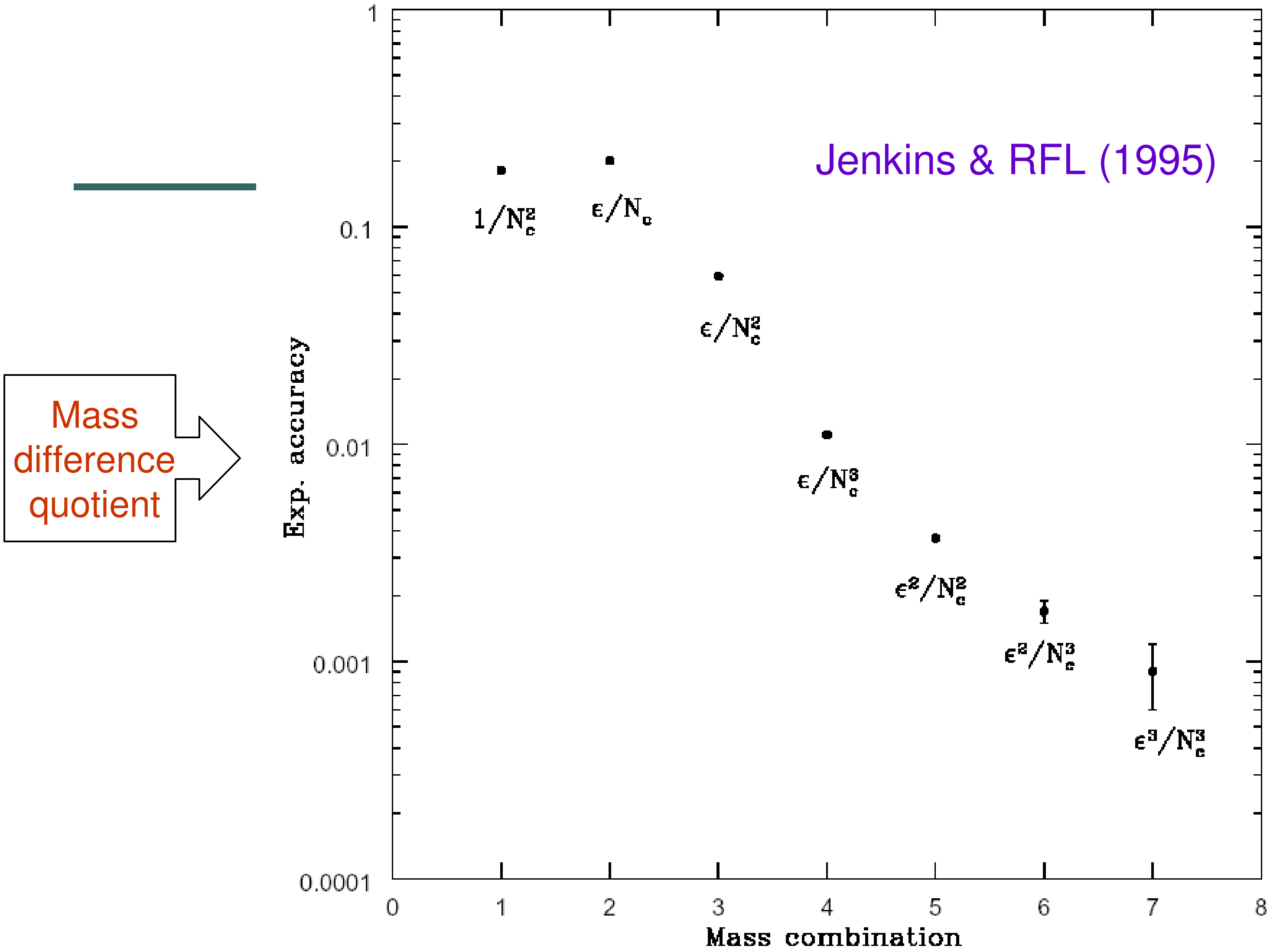
with $B_i = \Sigma_0, \Lambda, \Xi_0$

- Any other reasonable definition should give $\varepsilon \approx 0.20$–0.30

The $I = 0$ Mass Combinations Special to $1/N_c$

	Mass Combination	Large N_c^{F} suppression	Large N_c^{AS} suppression
M_1	$5(2N_0 + 3\Sigma_0 + \Lambda + 2\Xi_0) - 4(4\Delta_0 + 3\Sigma_0^* + 2\Xi_0^* + \Omega)$	$1/N_c$	$1/N_c^2$
M_2	$5(6N_0 - 3\Sigma_0 + \Lambda - 4\Xi_0) - 2(2\Delta_0 - \Xi_0^* - \Omega)$	ϵ	ϵ
M_3	$N_0 - 3\Sigma_0 + \Lambda + \Xi_0$	ϵ/N_c	ϵ/N_c^2
M_4	$(-2N_0 - 9\Sigma_0 + 3\Lambda + 8\Xi_0) + 2(2\Delta_0 - \Xi_0^* - \Omega)$	ϵ/N_c^2	ϵ/N_c^4
M_5	$35(2N_0 - \Sigma_0 - 3\Lambda + 2\Xi_0) - 4(4\Delta_0 - 5\Sigma_0^* - 2\Xi_0^* + 3\Omega)$	ϵ^2/N_c	ϵ^2/N_c^2
M_6	$7(2N_0 - \Sigma_0 - 3\Lambda + 2\Xi_0) - 2(4\Delta_0 - 5\Sigma_0^* - 2\Xi_0^* + 3\Omega)$	ϵ^2/N_c^2	ϵ^2/N_c^4
M_7	$\Delta_0 - 3\Sigma_0^* + 3\Xi_0^* - \Omega$	ϵ^3/N_c^2	ϵ^3/N_c^4

Cherman, Cohen & RFL, Phys. Rev. D **80**, 036002 [2009]:
Compare these results for N_C^F and N_C^{AS}



Mass difference quotient
Jenkins & RFL (1995)
Exp. accuracy
Mass combination
1
0.1
0.01
0.001
0.0001
0
1
2
3
4
5
6
7
8
1/N_c^2
ε/N_c
ε/N_c^2
ε/N_c^3
ε^2/N_c^2
ε^2/N_c^3
ε^3/N_c^3

There's *no way* N_C^{AS} can give results that good. And yet, …

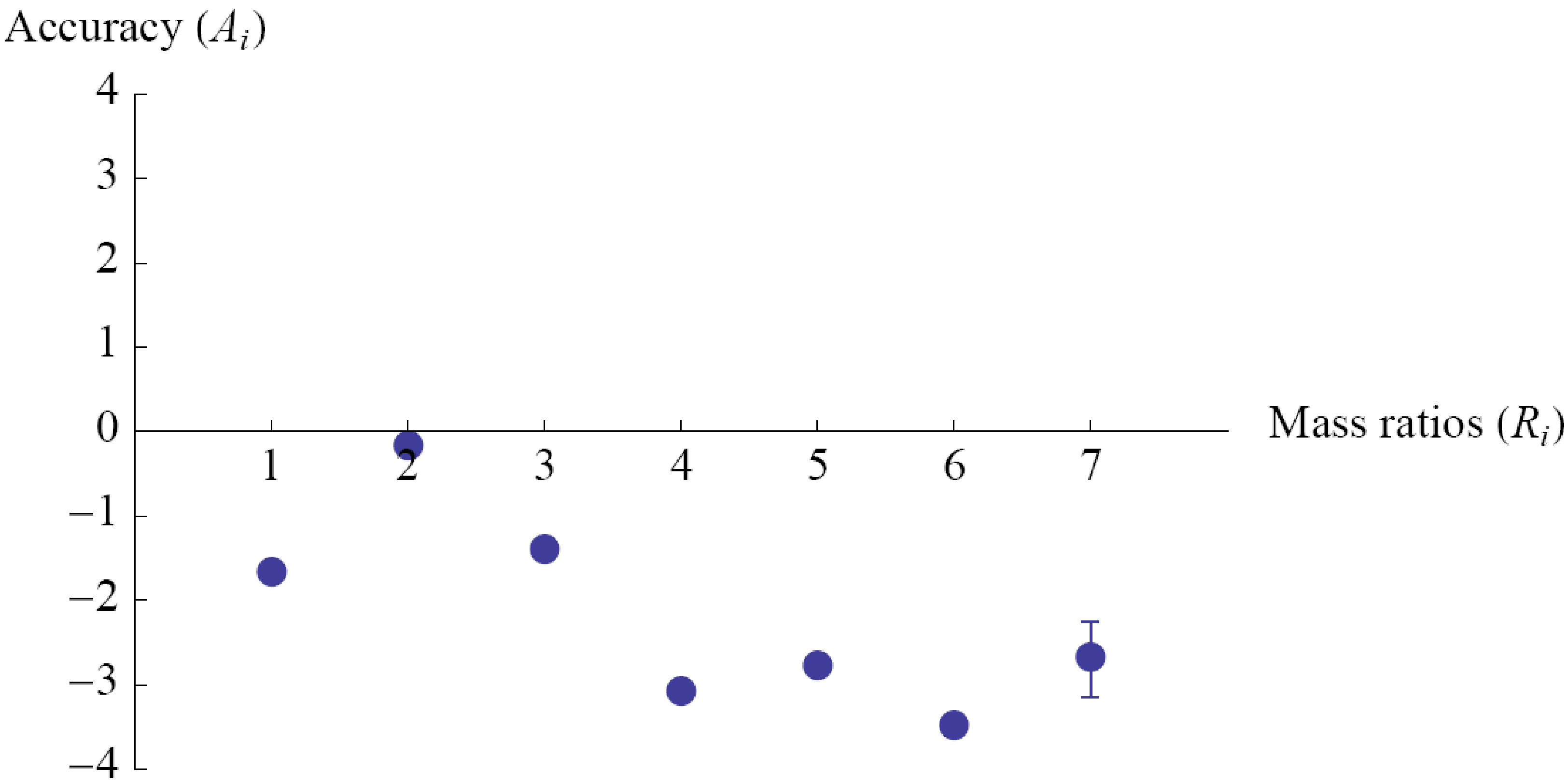
- Take each M_i and form M_i', the same combination with all "–" signs turned to "+" (Note that M_i' is $O(N_C)$ [N_C^F], $O(N_C^2)$ [N_C^{AS}])
- Define the scale-independent ratios $R_i \equiv M_i / (\frac{1}{2} M_i')$

  e.g., $M_3 = N_0 - 3\Sigma_0 + \Lambda + \Xi_0$

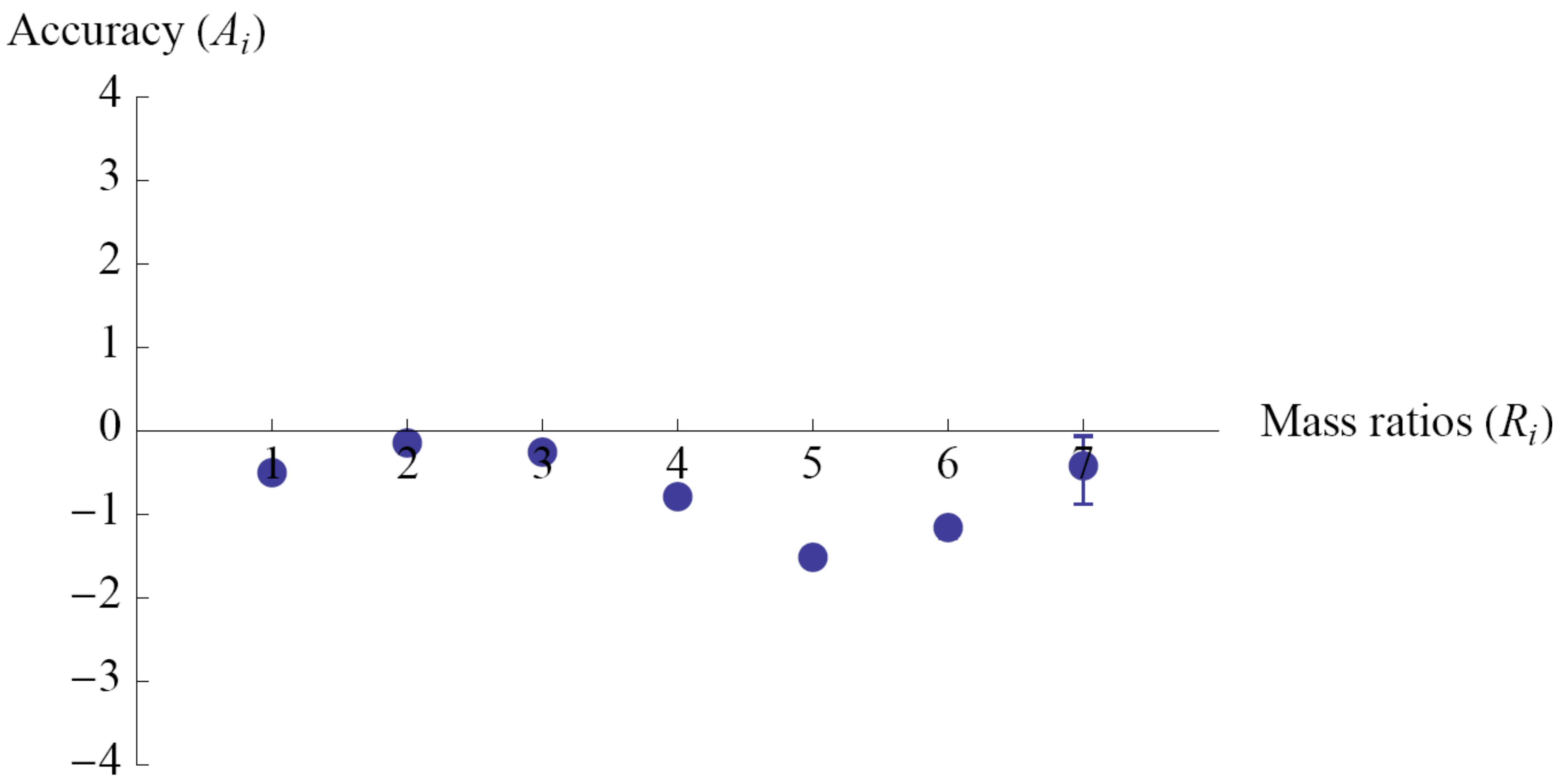
  $\rightarrow R_3 = (N_0 - 3\Sigma_0 + \Lambda + \Xi_0)/[\frac{1}{2} (N_0 + 3\Sigma_0 + \Lambda + \Xi_0)]$
- Compute the corresponding suppression factors S_i by replacing the masses in M_i and M_i', with their N_C and ε scalings

  e.g., in N_C^F, $M_3 \sim \varepsilon N_C^0$, $M_3' \sim N_C \rightarrow S_3 = \varepsilon / N_C$
- How good is the expansion? Define *accuracy* $A_i \equiv \ln(|R_i|/S_i)$
  A perfect prediction has $|R_i| = S_i \rightarrow A_i = 0$
  A poor prediction has $|R_i|/S_i > N_C$ or $< 1/N_C$
  Since $\ln(3) \approx 1$, the figure of merit is whether all A_i turn out to lie in a band of < 2 units wide around zero

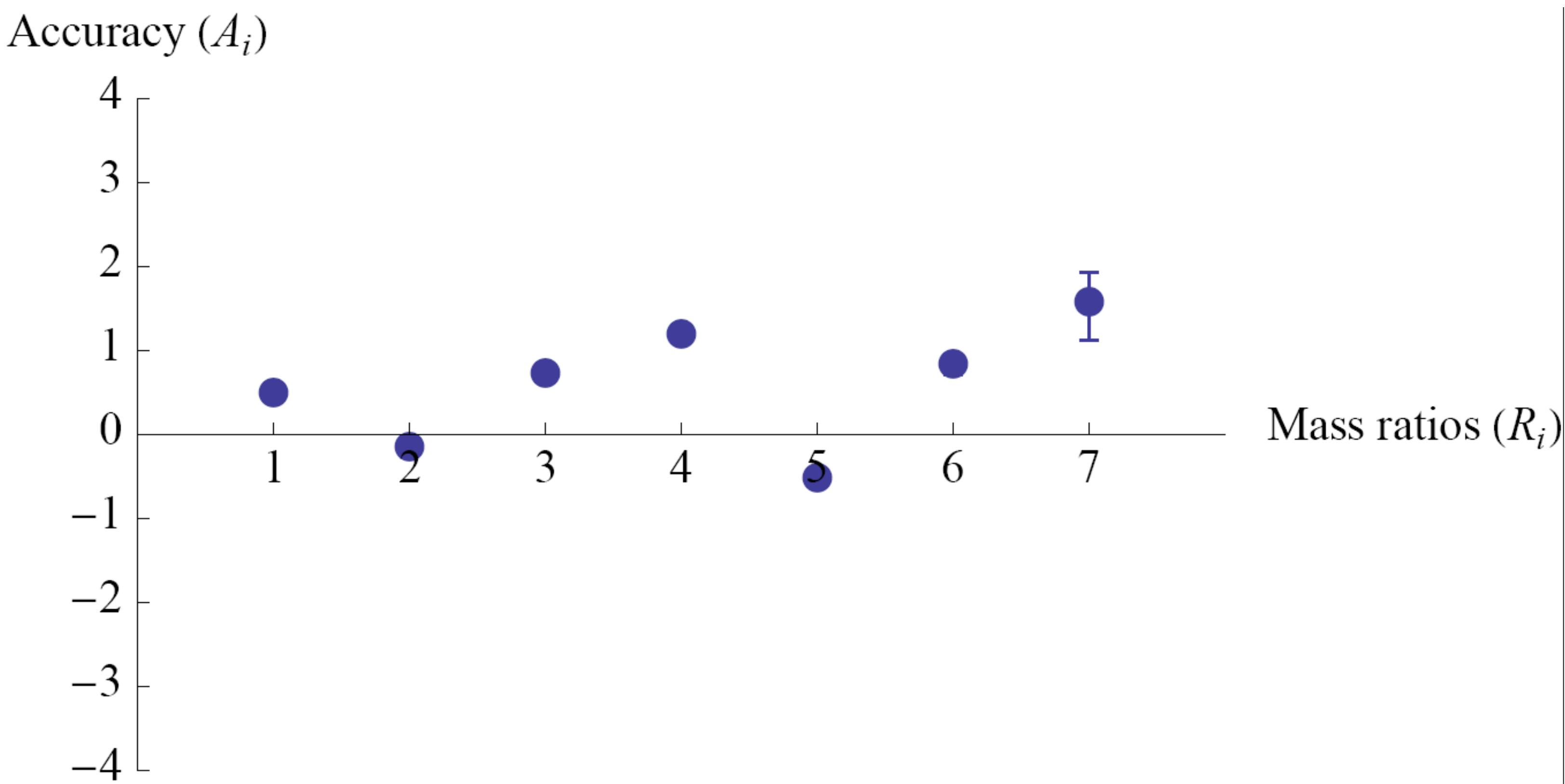
SU(3) Breaking Only, ε = 0.25



Large N_c^F Limit, ε = 0.25



Large N_c^{AS} Limit, ε = 0.25



News Flash:

The baryon mass spectrum demands a $1/N_c$ expansion, but does not strongly prefer $1/N_c^F$ to $1/N_c^{AS}$

What about the $I \neq 0$ splittings?

$$N_1 = (p - n),$$
$$\Sigma_1 = (\Sigma^+ - \Sigma^-),$$
$$\Xi_1 = (\Xi^0 - \Xi^-),$$
$$\Delta_1 = (3\Delta^{++} + \Delta^+ - \Delta^0 - 3\Delta^-),$$
$$\Sigma_1^* = (\Sigma^{*+} - \Sigma^{*-}),$$
$$\Xi_1^* = (\Xi^{*0} - \Xi^{*-}),$$
$$\Lambda\Sigma^0$$
$$\Sigma_2 = (\Sigma^+ - 2\Sigma^0 + \Sigma^-),$$
$$\Delta_2 = (\Delta^{++} - \Delta^+ - \Delta^0 + \Delta^-),$$
$$\Sigma_2^* = (\Sigma^{*+} - 2\Sigma^{*0} + \Sigma^{*-}).$$
$$\Delta_3 = (\Delta^{++} - 3\Delta^+ + 3\Delta^0 - \Delta^-)$$

BUT:
- Δ and Σ^* isospin splittings are poorly known
- $\Lambda\Sigma^0$ not directly measured

Eliminating them leaves just two $I = 1$ combinations at $O(1/N_C^F)$ and none at higher order → Can just choose isospin violation parameter ε' to soak up extra N_C in N_C^{AS}

Only one $I = 2$ and no $I = 3$ combinations remain

→ *No decisive prediction*

Can the lattice tell us something?

Absolutely!

PHYSICAL REVIEW D **81,** 014502 (2010)

Lattice test of $1/N_c$ baryon mass relations

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$1/N_c$ baryon mass relations are compared with lattice simulations of baryon masses using different values of the light-quark masses, and hence different values of $SU(3)$ flavor-symmetry breaking. The lattice data clearly display both the $1/N_c$ and $SU(3)$ flavor-symmetry breaking hierarchies. The validity of $1/N_c$ baryon mass relations derived without assuming approximate $SU(3)$ flavor symmetry also can be tested by lattice data at very large values of the strange quark mass. The $1/N_c$ expansion constrains the form of discretization effects; these are suppressed by powers of $1/N_c$ by taking suitable combinations of masses. This $1/N_c$ scaling is explicitly demonstrated in the present work.

Can the lattice tell us something?

But:

- Preliminary calculations (RFL, unpublished) using lattice simulation results (LHP Collaboration) again show that $1/N_c^F$ and $1/N_c^{AS}$ work comparably well!*
- * Perhaps not so surprising—If the lattice simulations are good, they should give numbers close to experimental data

If the baryon masses won't say…

- What's the next most abundant set of well-measured baryon data?
  - Magnetic dipole moments

(Other possibilities: Axial current couplings, charge radii, *etc.* are much more sparse)

Magnetic moments: How many?

RFL & R. TerBeek, PRD 83, 016009 (2011)

- Observables: **27**
  9 (octet, incl. $\Sigma^0\Lambda$) + **10** (decuplet), + **8** (octet-decuplet transitions)
- Measured: **11**
  8 (octet minus Σ^0) + **2** (decuplet: Δ^{++}, Ω^-) + **1** transition ($\Delta^+ p$)
- Independent operators: **27**
  [RFL & Martin, PRD **70**, 016008 (2004)]

$O(N_c^1)$	G^{33}
$O(N_c^0)$	$J^3,\ G^{38},\ \frac{1}{N_c}T^3G^{33},\ \frac{1}{N_c}N_sG^{33},\ \frac{1}{N_c^2}\frac{1}{2}\{J^iG^{i3},G^{33}\}$
$O(N_c^{-1})$	$\frac{1}{N_c}T^3J^3,\ \frac{1}{N_c}N_sJ^3,\ \frac{1}{N_c}T^3G^{38},\ \frac{1}{N_c}N_sG^{38},\ \frac{1}{N_c^2}\frac{1}{2}\{J^2,G^{33}\},\ \frac{1}{N_c^2}(T^3)^2G^{33},\ \frac{1}{N_c^2}N_s^2G^{33},$
	$\frac{1}{N_c^2}T^3N_sG^{33},\ \frac{1}{N_c^2}J^iG^{i3}J^3,\ \frac{1}{N_c^2}\frac{1}{2}\{J^iG^{i8},G^{33}\},\ \frac{1}{N_c^2}\frac{1}{2}\{J^iG^{i3},G^{38}\}$
$O(N_c^{-2})$	$\frac{1}{N_c^2}J^2J^3,\ \frac{1}{N_c^2}N_s^2J^3,\ \frac{1}{N_c^2}(T^3)^2J^3,\ \frac{1}{N_c^2}T^3N_sJ^3,\ \frac{1}{N_c^2}\frac{1}{2}\{J^2,G^{38}\},\ \frac{1}{N_c^2}(T^3)^2G^{38},\ \frac{1}{N_c^2}N_s^2G^{38},$
	$\frac{1}{N_c^2}T^3N_sG^{38},\ \frac{1}{N_c^2}J^iG^{i8}J^3,\ \frac{1}{N_c^2}\frac{1}{2}\{J^iG^{i8},G^{38}\}$

The Single-Photon Ansatz

- Each quark in any magnetic moment operator couples proportionally to its electric charge:

$$Q = T^Q \equiv T^3 + \frac{1}{\sqrt{3}} T^8 \qquad G^{iQ} \equiv G^{i3} + \frac{1}{\sqrt{3}} G^{i8}$$

- Only 4 indpt. operators otherwise conserving SU(3) flavor exist:

$$\mathcal{O}_1 \equiv G^{3Q}, \ \mathcal{O}_2 \equiv \frac{1}{N_c} Q J^3, \ \tilde{\mathcal{O}}_3 \equiv \frac{1}{N_c^2} \frac{1}{2} \{J^2, G^{3Q}\}, \ \mathcal{O}_4 \equiv \frac{1}{N_c^2} J^i G^{iQ} J^3$$

- SU(3) flavor breaking enters as *s* quark number N_s or spin J_s:

$$\varepsilon \mathcal{O}_5 \equiv \varepsilon q_s J_s^3, \ \varepsilon \mathcal{O}_6 \equiv \frac{\varepsilon}{N_c} N_s G^{3Q}, \ \varepsilon \mathcal{O}_7 \equiv \frac{\varepsilon}{N_c} Q J_s^3$$

O(εN$_c^0$)

$$\varepsilon \mathcal{O}_8 \equiv \varepsilon q_s \frac{N_s}{N_c} J^3, \ \varepsilon \mathcal{O}_9 \equiv \varepsilon \frac{N_s}{N_c^2} Q J^3, \ \varepsilon \mathcal{O}_{10} \equiv \frac{\varepsilon}{N_c^2} \frac{1}{2} \{\mathbf{J} \cdot \mathbf{J}_s, G^{3Q}\},$$

O(εN$_c^{-1}$)

$$\varepsilon \mathcal{O}_{11} \equiv \frac{\varepsilon}{N_c^2} J_s^j G^{jQ} J^3, \ \varepsilon \mathcal{O}_{12} \equiv \frac{\varepsilon}{N_c^2} \frac{1}{2} \{J^j G^{jQ}, J_s^3\} .$$

Magnetic Moments: How to handle the N_c's

- Denominator N_c's come from 't Hooft scaling→
  - In going from $1/N_c^F$ to $1/N_c^{AS}$, just replace $1/N_c^1$ → $1/N_c^2$

    [Scaling arguments alone cannot distinguish, *e.g.*, $1/N_c$ from $1/(N_c - 2)$]

- Numerator N_c's come from combinatorics→
  - In going from $1/N_c^F$ to $1/N_c^{AS}$, leave $N_c(N_c-1)/2$ as is

    [Counting quarks properly in each state is essential to obtaining correct electromagnetic behavior: *e.g.*, $Q_p - Q_n = 1$, *etc.*]

Operator Demotion

- If two operators X_1, X_2 give the same $O(1/N_c)$ matrix elements for each observable but give different ones at $O(1/N_c^2)$, $X_1 - X_2$ is called a *demoted* operator of $O(1/N_c^2)$
(More accurate and incisive accounting of $1/N_c$ corrections)
- For magnetic moments: $\frac{1}{2}\varepsilon\mathcal{O}_8 + \varepsilon\mathcal{O}_9$, $-\frac{1}{3}\mathcal{O}_{11} + \mathcal{O}_{12}$, $\mathcal{O}_{10} - \mathcal{O}_{12}$ demoted to $O(\varepsilon N_c^{-2})$, hence neglected;
$\varepsilon\mathcal{O}_{13} \equiv \frac{1}{2}\varepsilon\mathcal{O}_5 + \varepsilon\mathcal{O}_7$ demoted to $O(\varepsilon N_c^{-1})$
- Left with **9** operators: **1** at $O(N_c)$ (G^{3Q}),
1 at $O(N_c^0)$, **2** at $O(\varepsilon N_c^0)$, **2** at $O(N_c^{-1})$, **3** at $O(\varepsilon N_c^{-1})$
- Since 11 observables, can perform least-squares fit to the 9 operator coefficients

Magnetic moment fit parameters

- After the demotions, the 9 surviving operators are: $O_{1,2,3,4,5,6,8,10,13}$ (these include all explicit and implicit N_c factors)
- Isolate SU(3)-breaking parameter ε and set overall scale μ_0 of baryon magnetic moments to make leading coefficient d_1 = O(1)

$$\mu_z = \mu_0 \sum_{n=1}^{9} d_{i_n}\, \varepsilon^{k_{i_n}} \mathcal{O}_{i_n}$$

- Fit to $d_{1,2,3,4,5,6,8,10,13} \rightarrow$ are they all O(1)?

The Goldilocks fits

No $1/N_c$ factors: This fit's too soft!

$d_1 = +0.995 \pm 0.116$	$d_2 = -0.029 \pm 0.138$	$d_3 = +0.150 \pm 0.075$
$d_4 = +0.051 \pm 0.121$	$d_5 = -1.708 \pm 1.593$	$d_6 = -0.085 \pm 0.420$
$d_8 = +0.535 \pm 0.829$	$d_{10} = -0.420 \pm 0.845$	$d_{13} = +0.178 \pm 0.420$

$1/N_c^{AS}$: This fit's too hard!

$d_1 = +0.976 \pm 0.023$	$d_2 = -0.188 \pm 0.176$	$d_3 = +12.846 \pm 1.553$
$d_4 = +5.289 \pm 2.743$	$d_5 = -1.474 \pm 0.223$	$d_6 = -1.147 \pm 0.491$
$d_8 = +4.841 \pm 1.046$	$d_{10} = -36.332 \pm 12.322$	$d_{13} = +1.218 \pm 0.490$

$1/N_c^{F}$: This fit's just right

$d_1 = +0.992 \pm 0.044$	$d_2 = -0.078 \pm 0.148$	$d_3 = +1.363 \pm 0.272$
$d_4 = +0.461 \pm 0.489$	$d_5 = -1.652 \pm 0.566$	$d_6 = -0.288 \pm 0.438$
$d_8 = +1.588 \pm 0.865$	$d_{10} = -3.727 \pm 2.852$	$d_{13} = +0.499 \pm 0.438$

News Flash:

Baryon magnetic moments, despite being a smaller data set than masses, strongly prefer $1/N_c^F$ to $1/N_c^{AS}$ or to no $1/N_c$ expansion

Using the magnetic moment fit, one can predict all the rest

TABLE VIII: Best fit values for the 16 unknown magnetic moments in units of μ_N using the $1/N_c^{\mathrm{F}}$ expansion.

$\mu_{\Delta^+} = +3.09 \pm 0.16$	$\mu_{\Delta^0} = +0.00 \pm 0.10$	$\mu_{\Delta^-} = -3.09 \pm 0.16$	$\mu_{\Sigma^{*+}} = +2.62 \pm 0.35$
$\mu_{\Sigma^{*0}} = -0.06 \pm 0.32$	$\mu_{\Sigma^{*-}} = -2.73 \pm 0.35$	$\mu_{\Xi^{*0}} = -0.12 \pm 0.33$	$\mu_{\Xi^{*-}} = -2.37 \pm 0.39$
$\mu_{\Sigma^0} = +0.65 \pm 0.11$	$\mu_{\Delta^0 n} = +3.51 \pm 0.11$	$\mu_{\Sigma^{*0}\Lambda} = +2.65 \pm 0.32$	$\mu_{\Sigma^{*0}\Sigma^0} = +1.21 \pm 0.31$
$\mu_{\Sigma^{*+}\Sigma^+} = +2.69 \pm 0.32$	$\mu_{\Sigma^{*-}\Sigma^-} = -0.26 \pm 0.31$	$\mu_{\Xi^{*0}\Xi^0} = +2.30 \pm 0.33$	$\mu_{\Xi^{*-}\Xi^-} = -0.26 \pm 0.31$

Taking stock

- How can less data tell us more?
  - Sole leading mass operator, $\mathbb{1}$, gives same mass to all baryons
  - Sole leading magnetic moment operator, G^{3Q}, gives different values even for isospin multiplets (*e.g.*, $\mu_n = -\frac{2}{3}\,\mu_p$)
- What would it take to do better?
  - In the masses: Better decuplet (Δ, Σ^*, Ξ^*) isospin splittings
  - In the magnetic moments: Better values for $\mu_{\Sigma^0\Lambda}$, measurements of a few octet-decuplet transitions (*e.g.*, $\Sigma^*\Sigma$)
- What if both results persist?
  - Resolved for philosophical discussion: Could different observables obey different $1/N_c$ expansions, or is there a unique choice obeyed by all?

Conclusions

- The baryon mass spectrum demands a $1/N_c$ expansion (as has been known for 16 years), but does not strongly prefer one based on fundamental representation quarks $1/N_c^F$ to two-index antisymmetric representation quarks, $1/N_c^{AS}$
- Baryon magnetic moments, despite being a smaller data set than masses, strongly prefer the $1/N_c^F$ expansion to $1/N_c^{AS}$ or to no $1/N_c$ expansion
- Just a few additional data points in either set would greatly sharpen these conclusions
- Then we can argue about what the $1/N_c$ expansion really *means*