Entropy, Contact Interaction with Horizon and Dark Enenrgy
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1. Introduction. Motivation.

- The mismatch between Bekenstein- Hawking entropy and the entropy of entanglement for vector fields has been a subject of intense discussions for the last couple of years.

- This talk is concentrated just on one specific subtlety first discussed by Kabbat, Strassler, 1995 and Iellici, moretti, 1996

- It has been claimed that for $S=0,\ S=1/2$ fields the one loop correction to the S^{BH} is equal to the entropy of entanglement while for gauge $S=1$ field the S^{BH} has an extra term with "wrong sign" which can be interpreted as the <u>contact interaction with the horizon</u>.

- The unique features of the contact term related to the vector gauge field in the entropy computations can be summarized as follows:

- the contact term being a total derivative can be represented as a surface term determined by the behaviour at the horizon;

- this term makes a <u>negative</u> contribution to the S^{BH} therefore, it can not be identified with entropy of entanglement S which is <u>intrinsically positive</u> quantity;

- this contribution does not vanish even in 2D when the entropy of entanglement S is identically zero as no physical propagating degrees of freedom are present in the system.

- The technical reason for this to happen: One can not use the physical Coulomb gauge in path-integral (when only physical DoF are present in the system) as it breaks down at the origin.

- an alternative description in terms of a covariant gauge (when unphysical degrees of freedom inevitably appear in the system) lead to a negative value for the entropy S^{BH}.

- In this talk I want to argue that:
a). The presence of this ``weird" term is intimately related to presence of the multiple topological sectors in gauge theories;
b). mismatch between S^{BH} and S for gauge fields is due to the <u>same gauge configurations</u> which saturate the contact term with "wrong sign" in topological susceptibility in QCD $\chi(\beta, \theta=0) = -\left.\frac{\partial^2 F_{\rm vac}(\beta,\theta)}{\partial\theta^2}\right|_{\theta=0}$

2. Example: 2D Schwinger model

- We start with 2D example where all computations can be explicitly performed.

- topological susceptibility is defined as follows

$$\chi \equiv \frac{e^2}{4\pi^2} \lim_{k\to 0} \int d^2x e^{ikx} \langle TE(x)E(0)\rangle \,, \qquad \int d^2x \; Q(x) = \frac{e}{2\pi} \int d^2x \; E(x) = k$$

- expression for the topological susceptibility in 2D Schwinger QED model is known exactly:

Contact, non-dispersive term which can not be related to any physical degrees of freedom

Conventional term due to the single physical massive field

- $$\chi_{QED} = \frac{e^2}{4\pi^2} \int d^2x \left[\delta^2(x) - \frac{e^2}{2\pi^2} K_0(\mu|x|)\right], \quad \mu^2 = e^2/\pi$$

- Any physical state contributes to χ with negative sign

$$\chi_{dispersive} \sim \lim_{k\to 0} \sum_n \frac{< 0|\frac{e}{2\pi}E|n >< n|\frac{e}{2\pi}E|0 >}{-k^2 - m_n^2} < 0.$$

- Ward Identities (WI) are satisfied only as a result of cancellation between the contact (non-dispersive) term with sign (+) and real physical contribution with sign (-)

$$\chi = \frac{e^2}{4\pi^2} \int d^2x \left[\delta^2(x) - \frac{e^2}{2\pi^2} K_0(\mu|x|)\right] = \frac{e^2}{4\pi^2}\left[1 - \frac{e^2}{\pi}\frac{1}{\mu^2}\right] = \frac{e^2}{4\pi^2}\left[1-1\right] = 0$$

- The contact term $\chi_{E\&M} \neq 0$ even when no physical degrees of freedom are present in the system

- $$\chi_{E\&M} = \frac{e^2}{4\pi^2} \int d^2x \left[\delta^2(x)\right] \neq 0$$

3.The contact term: where does it come from?

- We compute the contact term in 2D photo-dynamics

$$\chi = \frac{e^2}{4\pi^2 \mathcal{Z}} \sum_{k\in\mathbb{Z}} \int \mathcal{D}A \int d^2x E(x)E(0) e^{-\frac{1}{2}\int d^2x E^2}.$$

- We saturate different topological sectors of the theory by "instantons" defined on two dimensional Euclidean torus with total area "V":

$$A_\mu^{(k)} = -\frac{\pi k}{eV}\epsilon_{\mu\nu}x^\nu, \quad eE^{(k)} = \frac{2\pi k}{V}, \quad \frac{1}{2}\int d^2x E^2 = \frac{2\pi^2 k^2}{e^2 V}.$$

- This gaussian integral can be easily evaluated:

$$\chi = \frac{e^2}{4\pi^2}\cdot V \cdot \frac{\sum_{k\in\mathbb{Z}} \frac{4\pi^2 k^2}{e^2 V^2} \exp(-\frac{2\pi^2 k^2}{e^2 V})}{\sum_{k\in\mathbb{Z}} \exp(-\frac{2\pi^2 k^2}{e^2 V})} \to \quad \frac{e^2}{4\pi^2}\cdot V \cdot \frac{4\pi^2}{e^2 V^2}\cdot \frac{e^2 V}{4\pi^2} \to \frac{e^2}{4\pi^2}$$

-

- The obtained expression $\chi \to e^2/4\pi^2$ is finite in large volume limit and coincides with contact term from exact expression.

- It has "wrong sign" in comparison with any physical contributions;

- the topological sectors with very large $k \sim \sqrt{e^2 V}$ saturates the series;

- the final result is sensitive to the boundaries, infrared regularization;

- the contact term $\chi \to e^2/4\pi^2$ does not vanish in a trivial 2D model when no any propagating degrees of freedom are present in the system!

4. The ghost as a tool to describe the contact term

- We want to describe the same contact term without explicit summation over topological sectors "k". We use Kogut Susskind, 1975 ghost fields instead.

- The effective Lagrangian describing the low energy physics (including ghosts) is given by

$$\mathcal{L} = \frac{1}{2}\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 - \frac{1}{2}\mu^2\hat{\phi}^2 + m| < \bar{q}q > |\cos 2\sqrt{\pi}\left[\hat{\phi} + \phi_2 - \phi_1\right] .$$

- The ghost field ϕ_1 is always paired up with ϕ_2 in every gauge invariant matrix element. Condition which enforces this statement is the Gupta - Bleuler like condition, similar to 4d QED:

$$(\phi_2 - \phi_1)^{(+)} \left|\mathcal{H}_{\text{phys}}\right\rangle = 0$$

-

- The contact term in this framework is precisely represented by the KS- ghost contribution replacing the standard procedure of summation over different topological sectors.

$$
\begin{aligned}
\chi(x) &\equiv \left\langle T\frac{e}{2\pi}E(x), \frac{e}{2\pi}E(0)\right\rangle = \left(\frac{e}{2\pi}\right)^2 \frac{\pi}{e^2}\int \frac{d^2p}{(2\pi)^2}p^4 e^{-ipx}\left[-\frac{1}{p^2+\mu^2}+\frac{1}{p^2}\right] \\
&= \left(\frac{e}{2\pi}\right)^2\left[\delta^2(x) - \frac{e^2}{2\pi^2}K_0(\mu|x|)\right], \text{ where } \frac{e}{2\pi}E = (\frac{e}{2\pi})\frac{\sqrt{\pi}}{e}\left(\Box\hat{\phi} - \Box\phi_1\right)
\end{aligned}
$$

Contact term is saturated by the KS ghost

- This unphysical ghost scalar field does not violate unitarity or any other important properties of the theory as consequence of Gupta-Bleuler-like condition on the physical Hilbert space

$$
\mathrm{N} = \sum_k \left(b_k^\dagger b_k - a_k^\dagger a_k\right), \qquad \langle \mathcal{H}_{\mathrm{phys}}|\mathrm{N}|\mathcal{H}_{\mathrm{phys}}\rangle = 0
$$

$$
\left[b_k, b_{k'}^\dagger\right] = \delta_{kk'}, \left[a_k, a_{k'}^\dagger\right] = -\delta_{kk'} \ , (a_k - b_k)\,|\mathcal{H}_{\mathrm{phys}}\rangle = 0
$$

5. Accelerating Rindler Space-time

- The result $\langle \mathcal{H}_{\rm phys}|\mathrm{N}|\mathcal{H}_{\rm phys}\rangle = 0$ obviously implies that no entropy due to the fluctuation of "fictitious particles" may be produced in Minkowski space.

- We want to see how this simple conclusion changes when a horizon is present in the system. A Rindler observer in (R,L) wedge will measure the number density of unphysical states using the density operator

$$\mathrm{N}^{(R,L)} = \sum_k \left(b_k^{(R,L)\dagger} b_k^{(R,L)} - a_k^{(R,L)\dagger} a_k^{(R,L)} \right)$$

- For accelerating Rindler observer the exact cancellation (similar to Minkowski space) holds:

$$\left\langle \mathcal{H}_{\rm phys}^{(R,L)} | \mathrm{N}^{(R,L)} | \mathcal{H}_{\rm phys}^{(R,L)} \right\rangle = 0, \quad \left(a_k^{(R,L)} - b_k^{(R,L)} \right) \left| \mathcal{H}_{\rm phys}^{(R,L)} \right\rangle = 0 \,,$$

- However, if the system is prepared as the Minkowski vacuum state, a Rindler observer will observe the following number density in mode "k

$$\left\langle 0|\mathrm{N}^{(R,L)}|0\right\rangle = \left\langle 0\right| \left(b_k^{(R,L)\dagger} b_k^{(R,L)} - a_k^{(R,L)\dagger} a_k^{(R,L)}\right) \left|0\right\rangle = \frac{2}{(e^{2\pi\omega/a} - 1)}.$$

- No cancellation occurs in Rindler space as a result of mixture of positive and negative frequency modes (Bogolubov's coefficients).

- As a result of this mixture, the vacuum state defined by a particular choice of the annihilation operators will be filled with particles in a different system (Unruh effect). This statement holds for real and for "<u>fictitious" particles.</u>

Technical reason for non-cancellation:

- The ground state for Minkowski observer is defined as usual

$$a_k|0>=0\,,\quad b_k|0>=0\,,\quad \forall k\,.$$

- The vacuum for R-Rindler observer is defined as

$$a_k^L|0_R>=0\,,\quad a_k^R|0_R>=0\,,\quad b_k^L|0_R>=0\,,\quad b_k^R|0_R>=0\,,\quad \forall k\,.$$

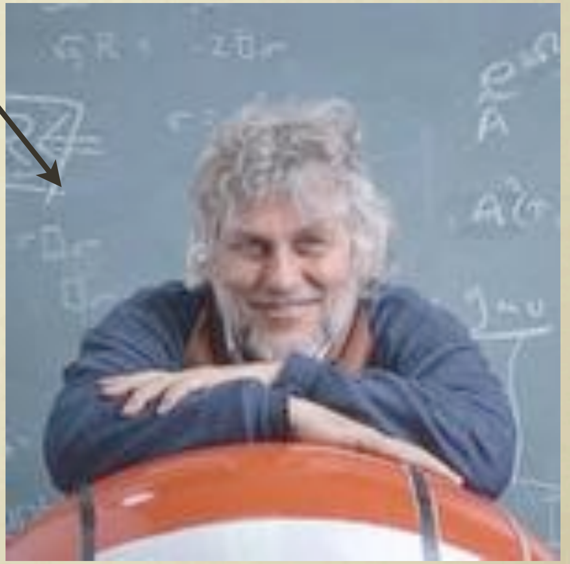
- The Bogolubov's coefficients are known to mix positive and negative frequency modes:

$$a_k^L=\frac{e^{-\pi\omega/2a}a^{1\dagger}_{-k}+e^{\pi\omega/2a}a_k^2}{\sqrt{e^{\pi\omega/a}-e^{-\pi\omega/a}}}\qquad a_k^R=\frac{e^{-\pi\omega/2a}a^{2\dagger}_{-k}+e^{\pi\omega/2a}a_k^1}{\sqrt{e^{\pi\omega/a}-e^{-\pi\omega/a}}}$$

$$b_k^L=\frac{e^{-\pi\omega/2a}b^{1\dagger}_{-k}+e^{\pi\omega/2a}b_k^2}{\sqrt{e^{\pi\omega/a}-e^{-\pi\omega/a}}}\qquad b_k^R=\frac{e^{-\pi\omega/2a}b^{2\dagger}_{-k}+e^{\pi\omega/2a}b_k^1}{\sqrt{e^{\pi\omega/a}-e^{-\pi\omega/a}}}.$$

- No cancellation between the KS ghost and its partner could occur as a result of opposite sign (-) in commutation relations and negative sign (-) in Number density operator.

- if we had started with a conventional scalar field we would derive a well-known formula for Plank spectrum for radiation at $T = a/(2\pi)$ observed by a Rindler observer in Minkowski vacuum which is conventional Unruh effect



The cancellation fail to hold for the accelerating Rindler observer because the properties of the operator which selects the positive frequency modes with respect to Minkowski time t and observer's proper time η are not equivalent.

- WE CONJECTURE THAT THE “WRONG SIGN” IN S^{BH} COMPUTATIONS WITH “WEIRD” FEATURE LISTED ABOVE AND “WRONG SIGN” IN CONTACT TERM IN χ IS A RESULT OF THE SAME PURE GAUGE (NONTRIVIAL) CONFIGURATIONS.

- WE INTERPRET $\left\langle 0|\mathrm{N}^{(R,L)}|0\right\rangle \sim (e^{2\pi\omega/a}-1)^{-1}$ IN THE PRESENCE OF HORIZON AS A RESULT OF FORMATION OF THE SQUEEZED STATE WHICH CAN BE COINED AS THE “GHOST CONDENSATE” RATHER THAN A PRESENCE OF “FREE PARTICLES” AT $T=a/2\pi$ PREPARED IN A SPECIFIC MIXED STATE.

$$|0>=\prod_k \frac{1}{\sqrt{(1-e^{-2\pi\omega/a})}} \exp\left[e^{-\pi\omega/a}\left(b_k^{R\dagger}b_{-k}^{L\dagger}-a_{-k}^{R\dagger}a_k^{L\dagger}\right)\right]|0^R>\otimes|0^L>$$

- MISMATCH BETWEEN S^{BH} AND ENTROPY OF ENTANGLEMENT IS DUE TO THE SAME TOPOLOGICAL CONFIGURATIONS WHICH SATURATE THE NON-DISPERSIVE CONTACT TERM IN TOPOLOGICAL SUSCEPTIBILITY χ.

6.The Veneziano Ghost in 4d.

- One can not perform explicit analytical computations with summation over topological sectors (similar to 2D case). However, one can use an effective Veneziano-ghost based computations similar to KS ghost computations.

- In fact, one can explicitly demonstrate that the low energy lagrangian for $U_A(1)$ degrees of freedom in 4d QCD is **identical** to the 2d Kogut- Susskind lagrangian:

Veneziano ghost' s partner

Veneziano ghost

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu\eta'\partial^\mu\eta' + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 \\ &- \frac{1}{2}m_{\eta'}^2\eta'^2 + m_q|<\bar{q}q>|\cos\left[\theta + \frac{\eta' + \phi_2 - \phi_1}{f_{\eta'}}\right]\end{aligned}$$

- The negative sign for the Veneziano ghost appears in the Lagrangian as a result of induced term

- $$\int d^4x\, q^2 \sim \int d^4x\, (\Box\Phi)^2$$ where $$q = \partial_\mu K^\mu = \frac{g^2}{64\pi^2}\epsilon_{\mu\nu\rho\sigma}G^{a\mu\nu}G^{a\rho\sigma} = \Box\Phi$$

- 4-derivative operator can be represented as a superposition of the physical state and the ghost

$$\frac{1}{\Box\Box + m_{\eta'}^2\Box} = \frac{1}{m_{\eta'}^2}\left(\frac{1}{-\Box - m_{\eta'}^2} - \frac{1}{-\Box}\right)$$

- One can compute the topological susceptibility using the Veneziano ghost with following result:

Non-dispersive contact term from Ghost

Conventional physical contribution

$$\chi_{QCD} \equiv \int d^4x\, \langle T\{q(x), q(0)\}\rangle = \frac{f_{\eta'}^2 m_{\eta'}^2}{4} \cdot \int d^4x\, \left[\delta^4(x) - m_{\eta'}^2 D^c(m_{\eta'}x)\right],$$

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- The negative sign in the Lagrangian does not lead to any problems (unitarity, causality...) when auxiliary GB-like conditions on the physical Hilbert space are imposed:

$$(\phi_2 - \phi_1)^{(+)} \left|\mathcal{H}_{\rm phys}\right\rangle = 0\,.$$ *positive frequency part enters this condition!*

$$(Gubta - Bleuler \text{ in } QED): \quad (\partial_\mu A^\mu)^+ |\mathcal{H}_{\rm phys} >= 0;$$

- the expectation value for any physical state vanishes in Minkowski space as a result of GB condition.

- However, as in 2d QED the number density of "fictitious" particles (Veneziano ghost) start to fluctuate in the presence of the horizon in the Rindler space (similar to 2d computations)

$$\left\langle 0|\mathrm{N}^{(R,L)}|0\right\rangle = \left\langle 0\right| \left(b_k^{(R,L)\dagger} b_k^{(R,L)} - a_k^{(R,L)\dagger} a_k^{(R,L)}\right) \left|0\right\rangle = \frac{2}{(e^{2\pi\omega/a} - 1)},$$

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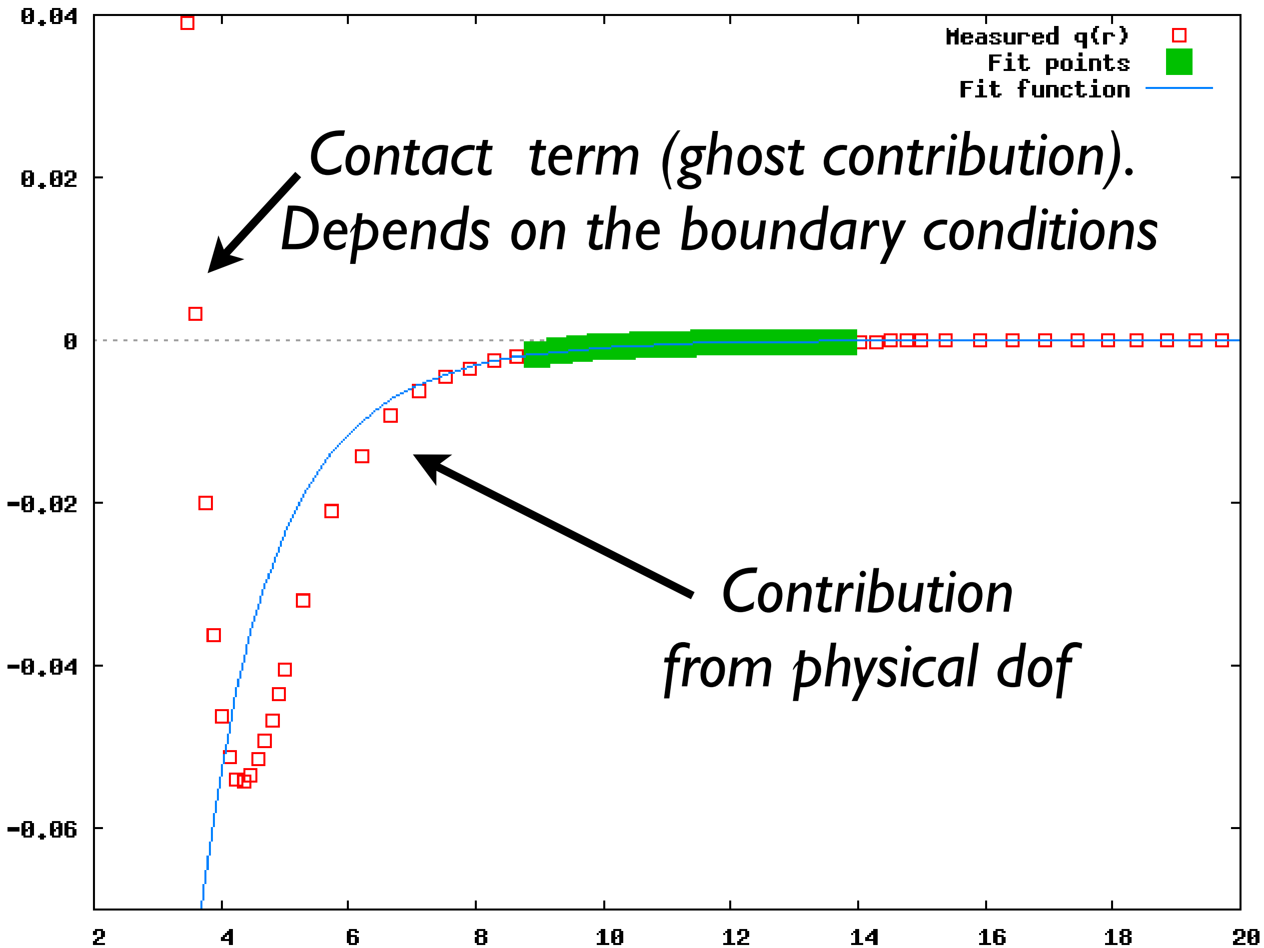

The topological susceptibility $\chi(r)$ as a function of r. Wrong sign for $\chi(r \to 0) \sim \delta(r)$ is well established phenomenon; it has been tested on the lattice (plot above is from C. Bernard et al, LATTICE 2007). This contribution is not related to any physical degrees of freedom, and can be interpreted as a contact term (Witten, 79) or as the ghost contribution (Veneziano, 79). It is related to necessity to sum over different k-topological sectors of the theory.

7.Mismatch between BH entropy and entropy of entanglement

- The surface term with a ``wrong sign" in entropy computations (Kabat,1995) and the contact term with ``wrong sign" in topological susceptibility are both originated form the same physics, and both related to the same gauge configurations.

- This unambiguously identifies the nature of the well known mismatch between computations of the black hole entropy and entropy of entanglement.

- Similar mismatch occurs also in different system (Frolov, 1997). The degeneracy of the vacuum state is achieved by non-minimally coupling with a scalar field. Degeneracy emerges in that system as a result of dynamics of the ``soft modes" at the horizon.

8. Contact interaction and dark energy

- We speculate that the source of the observed dark energy (DE) may also be related to the gauge configurations which are responsible for the mismatch between S^{BH} entropy and the entropy of entanglement in the presence of causal horizon in FRW Universe.

- We adopt the paradigm that the relevant definition of the energy which enters the Einstein equations is the difference $\Delta E \equiv (E - E_{\rm Mink})$ similar to the Casimir effect (Zeldovich, 1967+many others after)

- How does ΔE scale with Hubble constant H? Obviously $\Delta E \to 0$ when $H \to 0$. But how does it vanish?

- Naive answer: $\Delta E \sim \exp(-\Lambda_{QCD}/H) \sim \exp(-10^{41})$. Such a naive expectation formally follows from the dispersion relations which dictate that a sensitivity to very large distances must be exponentially suppressed when the mass gap is present in the system.

- However! The naive argument (on $\Delta E \sim \exp -(\Lambda_{QCD} L)$) fails because the correction is related to non-dispersive term, not related to absorptive physical spectral function.

- It may lead to power like scaling $\Delta E \sim H + \mathcal{O}(H)^2$. If true, the difference between two metrics (FRW and Minkowski) would lead to an estimate which amazingly close to observations

$$\Delta E \sim H \Lambda_{QCD}^3 \sim (10^{-3} \mathrm{e}V)^4$$

- There are a number of arguments supporting the power like (rather than $\Delta E \sim \exp -(\Lambda_{QCD} L)$) behaviour

$$\Delta E \sim H + \mathcal{O}(H)^2$$

- An explicit computation in exactly solvable two-dimensional QED with result $\Delta E \sim L^{-1}$. Model has a single massive physical state. Still, the Casimir -like effect occurs in this massive theory.

- Power like behaviour $\Delta E \sim L^{-1}$ is also supported by recent lattice results (Holdom, 2011). The approach by Holdom is based on physical Coulomb gauge when nontrivial topological structure is represented by the so-called Gribov copies.

- Another support: explicit computation in Rindler space-time in 4D QCD in the limit $\Lambda_{QCD}^4 \gg a^4 \gg m|\langle \bar{q}q \rangle|$ with known Bogolubov's coefficients (AZ, 2010, Ohta 2011).

- With this extra contribution $\Delta E \sim H\Lambda_{QCD}^3$ the Friedman equation reads

$$H^2 = \frac{8\pi G}{3}\left(\alpha H \Lambda_{QCD}^3 + \rho_M\right), \quad \rho_M = \frac{\rho_{M0}}{a^3},$$

- Equation of state will approach (-1) from above, and the universe is dragged into a de- Sitter state at asymptotically large $a(t) \gg a_\star$

$$a(t) \sim \exp(H_\infty t), \quad H_\infty = \frac{8\pi G}{3}\alpha\Lambda_{QCD}^3, \quad \omega \equiv \frac{p}{\rho} = -1 + \frac{1}{2}\left(\frac{a_\star}{a}\right)^3, \quad a \gg a_\star$$

- A comprehensive phenomenological analysis of this model has been recently performed (Cai 2010) with conclusion that this model is consistent with all presently available data.

9. Fine tuning without ``fine tuning''.

- A number of fine tuning issues such as coincidence problem, drastic separation of scales, etc may find a simple and universal explanation within this framework, without new fields, new interactions, new symmetries...

- For example, vacuum energy is determined by the deviation from Minkowski flat space-time,

$$\Delta E = [E(L,H) - E(L=\infty, H=0)] \sim H\Lambda^3_{QCD} \sim (10^{-3}\mathrm{e}V)^4$$

- Why does it happen now?

$$3H^2M^2_{PL} \sim \Delta E \implies \tau \sim H^{-1} \sim \frac{M^2_{PL}}{\Lambda^3_{QCD}} \sim 10 \text{ Gyr}$$

- Typical wavelengths contributing to the "ghost condensate" is $k \sim H^{-1} \sim 10^{10} yr$, which is a property of the Bogolubov's coefficients. This type of matter (large wavelengths) is drastically different from anything else in the Universe as it does not clump.

- The sign of $\Delta E \equiv (E - E_{\rm Mink})$ is normally $\Delta E < 0$ in QFT. The Casimir effect is the well known example of this subtraction procedure as some modes can not be accommodated in a system with a nontrivial geometry/boundaries. In our case $\Delta E > 0$. This is consistent with accelerating universe if DE= $\Delta E > 0$.

- The source for DE and for $(S^{BH} - S)$ is the same and related to the contact term saturated by the topological sectors of a gauge theory.