Large-N Gauge Theories
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World-Sheet Theories on Non-Abelian Strings and their Large-N Solution

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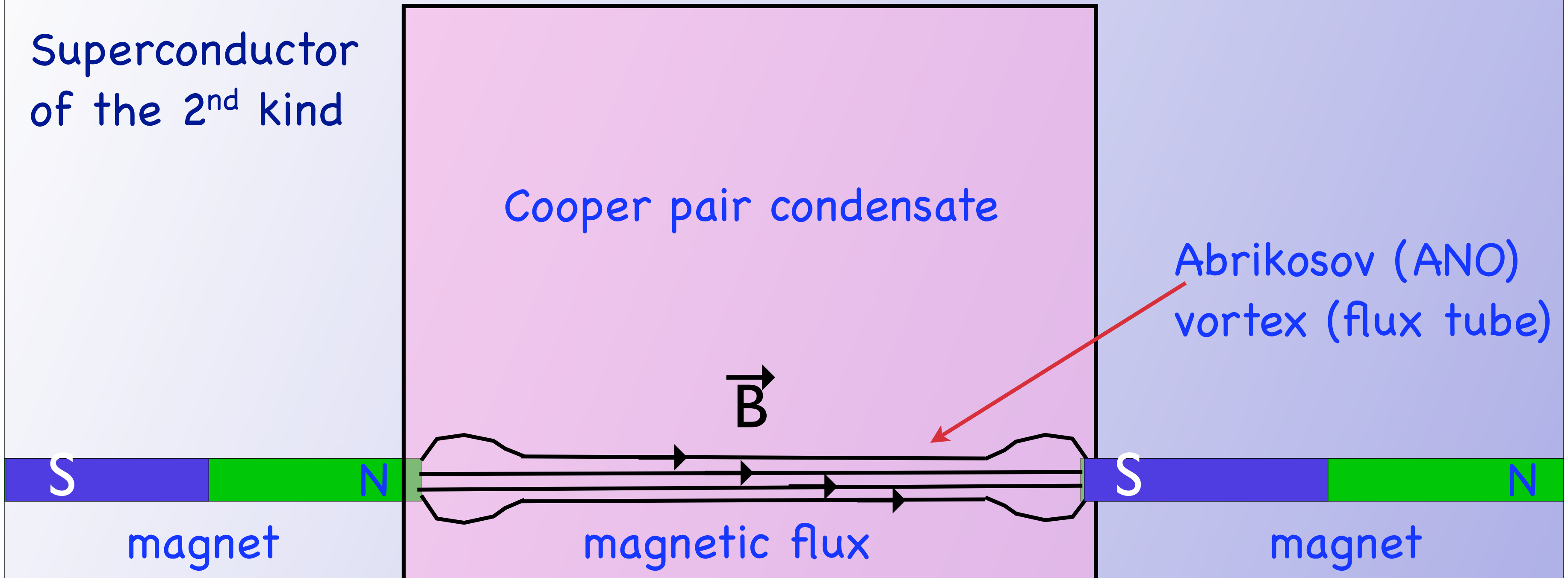
☞ The Meissner effect: 1930s, 1960s

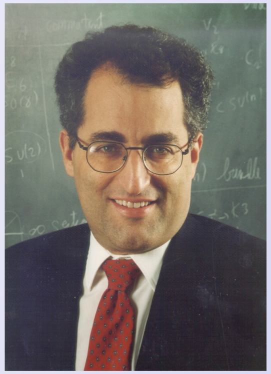
Superconductor
of the 2^{nd} kind



Abelian ☚

DUAL MEISSNER EFFECT (Nambu-'t Hooft-Mandelstam, ~1975)

☺ First demonstration of the dual Meissner effect: Seiberg & Witten, 1994 ☺



- gluons+complex scalar superpartner
- two gluinos
- Georgi-Glashow model built in

N=2 (extended) SUSY → SU(2) → U(1), monopoles →

Monopoles become light → N=1 deform. forces M condensatition →

U(1) broken, electric flux tube formed →

☹ ☹ Dynamical Abelization ... dual Abrikosov string

analytic continuation

☞ Non-Abelian Strings, 2003 → Now

Prototype model

$$S = \int d^4x \left\{ \frac{1}{4g_2^2}\left(F^a_{\mu\nu}\right)^2 + \frac{1}{4g_1^2}\left(F_{\mu\nu}\right)^2 + \frac{1}{g_2^2}|D_\mu a^a|^2 \right.$$

$$+ \quad \mathrm{Tr}\,(\nabla_\mu\Phi)^\dagger\,(\nabla^\mu\Phi) + \frac{g_2^2}{2}\left[\mathrm{Tr}\,\left(\Phi^\dagger T^a\Phi\right)\right]^2 + \frac{g_1^2}{8}\left[\mathrm{Tr}\,\left(\Phi^\dagger\Phi\right) - N\xi\right]^2$$

$$\left. + \quad \boxed{\frac{1}{2}\mathrm{Tr}\,\left|a^aT^a\,\Phi + \Phi\,\sqrt{2}M\right|^2} + \frac{i\,\theta}{32\,\pi^2}\,F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu} \right\},$$

$$\boxed{\Phi = \begin{pmatrix} \varphi^{11} \varphi^{12} \\ \varphi^{21} \varphi^{22} \end{pmatrix} \qquad M = \begin{pmatrix} m & 0 \\ 0 & -m \end{pmatrix}}$$

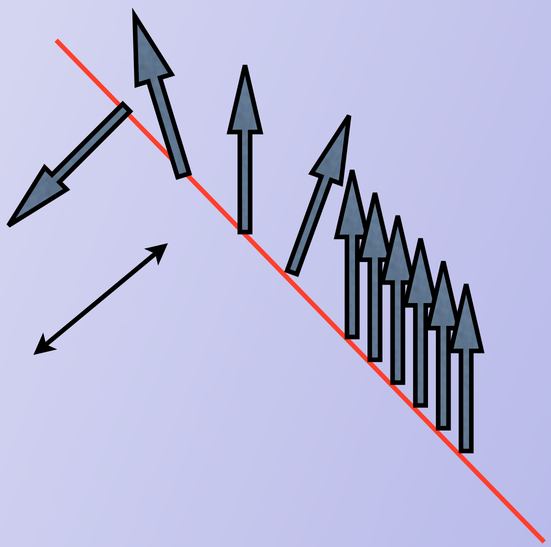
U(2) gauge group, 2 flavors of (scalar) quarks
SU(2) Gluons $A^a{}_\mu$ + U(1) photon + gluinos+ photino

$\Phi = \sqrt{\xi} \times I$

Basic idea:

- **Color-flavor locking in the bulk → Global symmetry G;**
- **G is broken down to H on the given string;**
- G/H coset; **G/H sigma model on the world sheet.**

"Non-Abelian" string is formed if all non-Abelian degrees of freedom participate in dynamics at the scale of string formation



2003: Hanany, Tong
Auzzi et al.
Yung + M.S.

classically gapless excitation

SU(2)/U(1) = CP(1)~O(3) sigma model

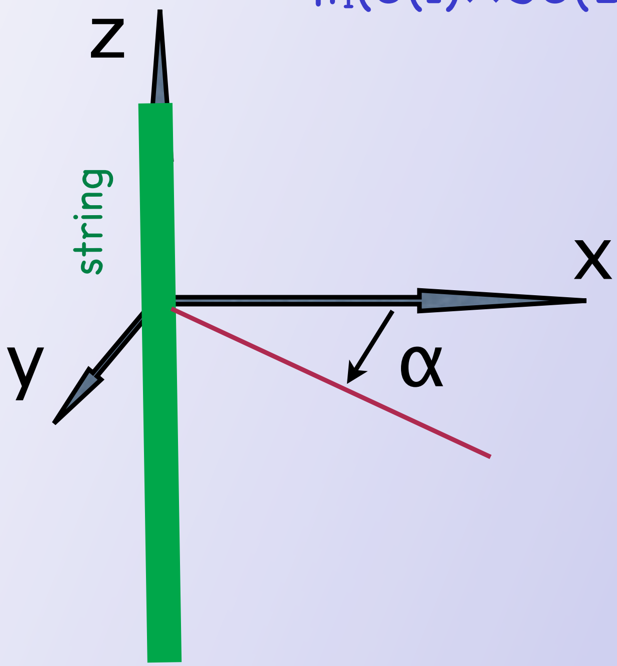
☆ ANO strings are there because of U(1)!

☆ New strings:

$\pi_1(SU(2)\times U(1)) = Z_2$: **rotate by π around 3-d axis in SU(2)**

→ -1; another -1 rotate by π in U(1)

$\pi_1(U(1)\times SU(2))$ nontrivial due to Z_2 center of SU(2)



x_0 ← string center in perp. plane

ANO $\quad \sqrt{\xi}\, e^{i\alpha} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$T=4\pi\xi$

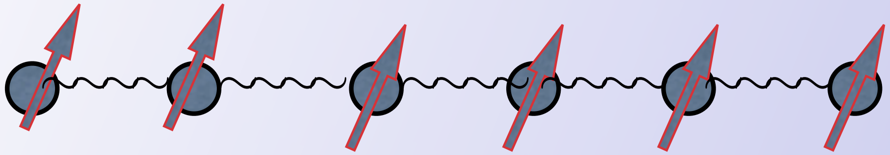
Non-Abelian $\quad \sqrt{\xi} \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & 1 \end{pmatrix}$

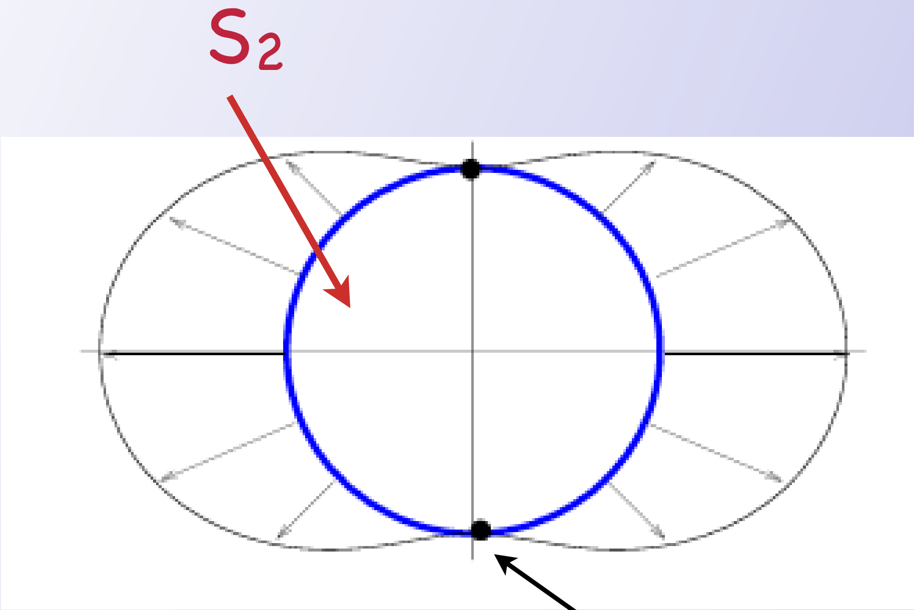
$T_{U(1)} \pm T^3_{SU(2)}$

$T=2\pi\xi$

SU(2)/U(1) ←orientational moduli; O(3) σ model

Worldsheet theory



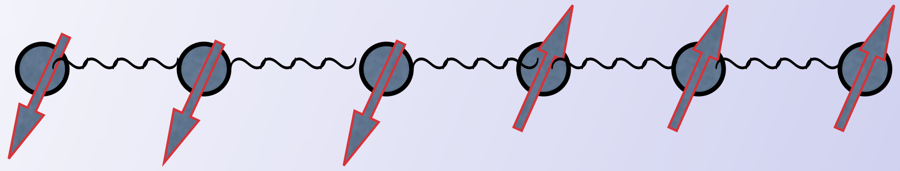


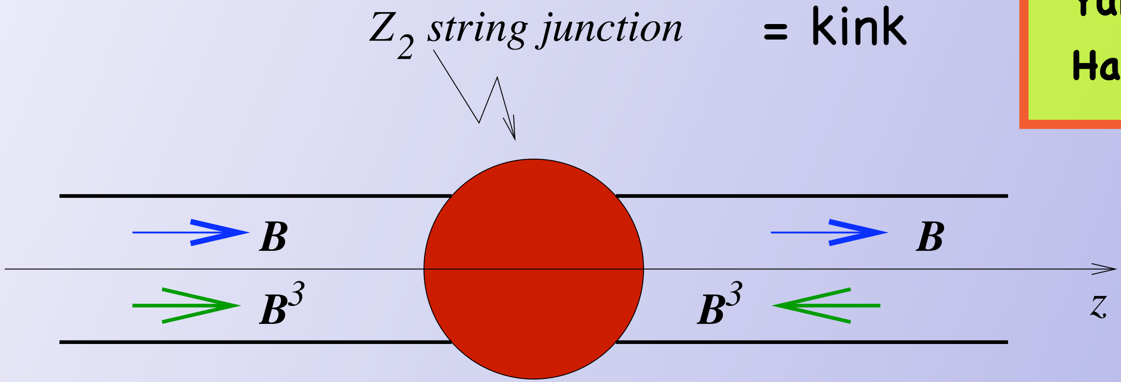
Global SU(2) is gone!
U(1) remains intact

Two vacua= 2 degenerate strings

CP(1) model with twisted mass

$$S=\int d^2x\left\{\frac{2}{g^2}\,\frac{\partial_\mu\bar{\phi}\,\partial^\mu\phi-(\Delta m)^2\bar{\phi}\phi}{(1+\bar{\phi}\phi)^2}+fermions\right\}$$





= kink

Yung + M.S.
Hanany, Tong

Evolution in dimensionless parameter m^2/ξ

✳ Kinks are confined in 4D (attached to strings).

✳ ✳ Kinks are confined in 2D:

Kink = Confined Monopole

4D ↔ 2D Correspondence

☛ World-sheet theory ↔ strongly coupled bulk theory inside

Dewar flask

Versions of CP(N-1) models in 2D: non-SUSY, and SUSY

✵ ✵ **N** = (0,2) and (2.2)

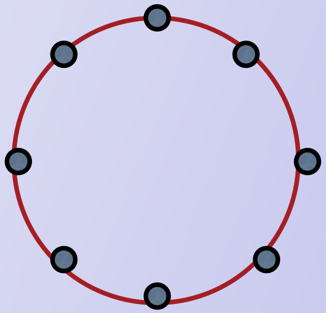
★ Gauged formulation ★ (Witten, 1979)

I. Non-SUSY bulk

$$S^{(1+1)} = \int dt\, dz\, \Big\{ 2\beta\, |\nabla_\alpha\, n|^2 + \frac{1}{4e^2} F^2_{\alpha\gamma} + \frac{1}{e^2} |\partial_\alpha \sigma|^2$$

$$+ \quad 4\beta \left| \left(\sigma - \frac{m_\ell}{\sqrt{2}} \right) n^\ell \right|^2 + 2e^2\beta^2 \left(|n^\ell|^2 - 1 \right)^2 \Big\}$$

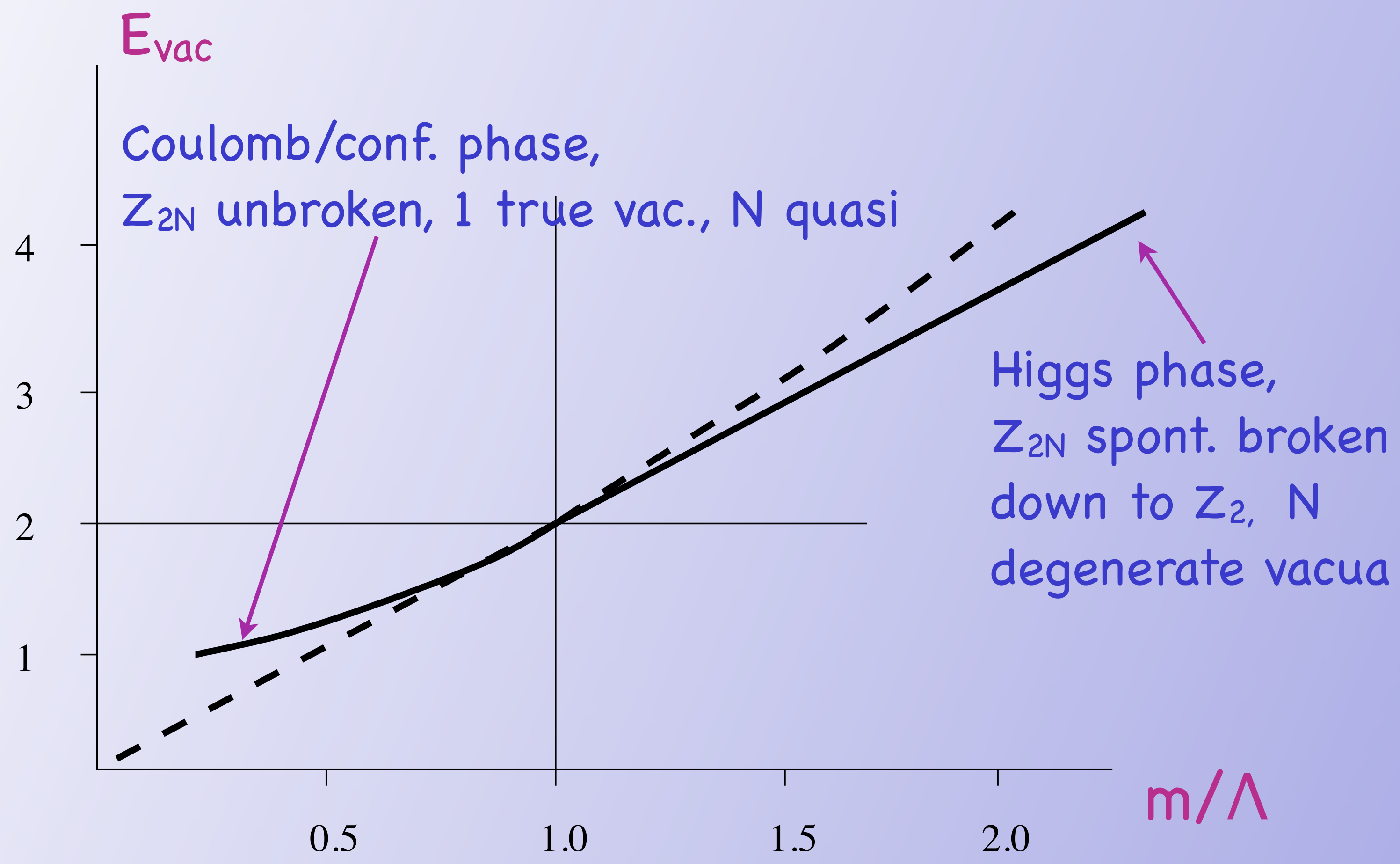
$$\nabla_\alpha = \partial_\alpha - i A_\alpha$$



m~ $e^{2\pi i/N}$, $e^{4\pi i/N}$, ..., $e^{2(N-1)\pi i/N}$, 1

Z_{2N} symmetry

$\boxed{m/\Lambda}$


E_{vac}
Coulomb/conf. phase,
Z_{2N} unbroken, 1 true vac., N quasi
4
3
2
1
Higgs phase,
Z_{2N} spont. broken
down to Z_2, N
degenerate vacua
0.5
1.0
1.5
2.0
m/Λ

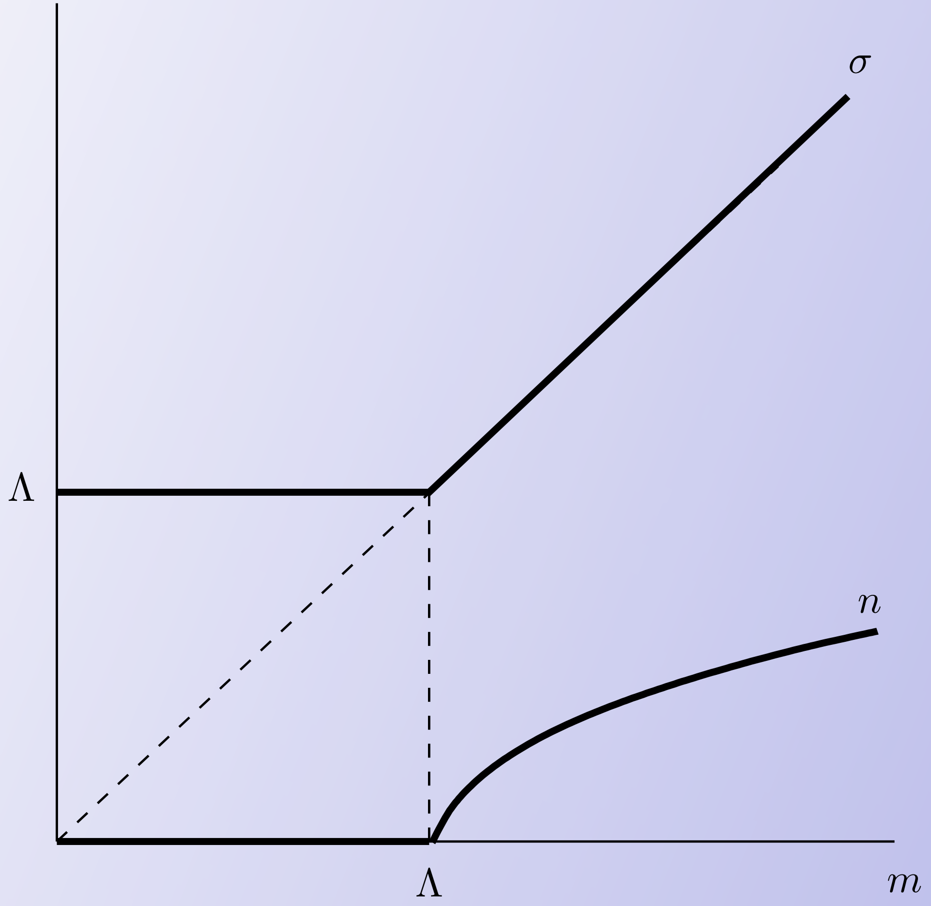
II. **N** = 2 SUSY bulk

N = (2,2) CP(N-1) model

$$\mathcal{L} = \frac{1}{e_0^2}\left(\frac{1}{4}F_{\mu\nu}^2 + |\partial_\mu \sigma|^2 + \frac{1}{2}D^2\right) + i\,D\left(\bar{n}_i n^i - 2\beta\right)$$

$$+ \left|\nabla_\mu n^i\right|^2 + 2\sum_i \left|\sigma - \frac{m_i}{\sqrt{2}}\right|^2 |n^i|^2$$

+ fermions



E_{vac}=0 always, SUSY unbroken,

Z_{2N} always broken, (N degenerate vacua)

Crossover instead of phase transition

Strong-coupling ↔ Higgs regime

III. $\mathcal{N}$ = 1 SUSY bulk

$\mathcal{N}$ = (0,2) CP(N-1) model

Supersymmetry is broken, generally speaking !!!

Phase transitions possible

All phase transitions are of the second kind!

Tong
Yung + M.S.

Break N = 2 down to **N** = 1 in the bulk

Deformation of the bulk: ADD $W = \mu (A^a)^2 + \mu' A^2$

Heterotic deformation the of the World-sheet theory:

(2,2) supersymmetry is broken down to (0,2)

$$L_{heterotic} = \zeta_R^\dagger i\partial_L \zeta_R + \left[\gamma \zeta_R R \left(i\partial_L \phi^\dagger\right) \psi_R + H.c.\right] - g_0^2 |\gamma|^2 \left(\zeta_R^\dagger \zeta_R\right) \left(R \psi_L^\dagger \psi_L\right)$$

at small γ
ζ_R is Goldstino

$$\mathcal{E}_{vac} = |\gamma|^2 \left| \langle R \psi_R^\dagger \psi_L \rangle \right|^2$$

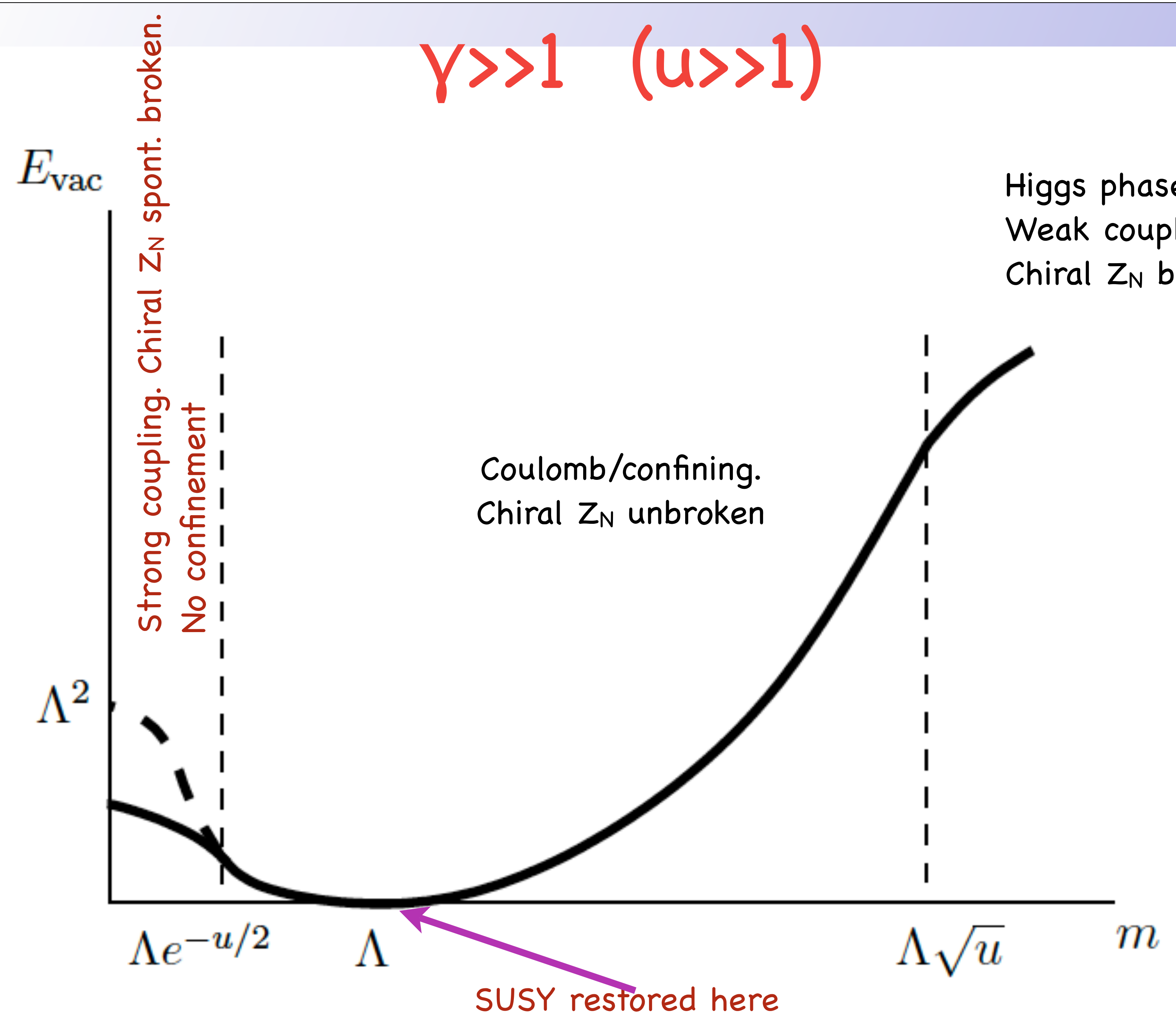
(0,2) supersymmetry is spontaneously broken!

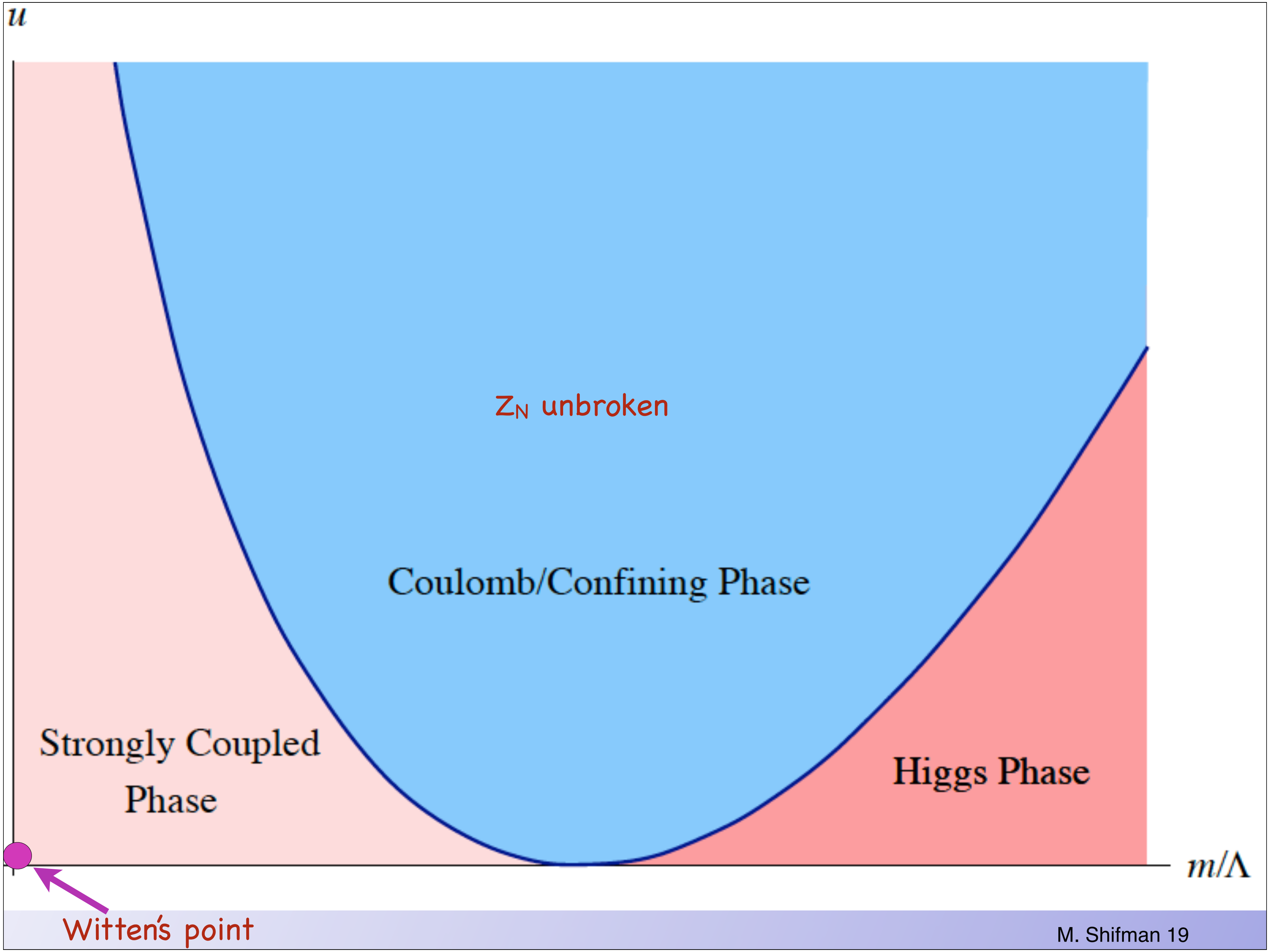
At large N heterotic CP(N-1) is also solvable (a là Witten) and presents a treasure trove of various phases

We have two parameters, γ and m, and a nontrivial phase diagram

With this choice of mass parameters we have Z_N symmetry, and phases with broken/unbroken Z_N.
SUSY is spontaneously broken

$\gamma \gg 1 \quad (u \gg 1)$





u
Z_N unbroken
Coulomb/Confining Phase
Strongly Coupled
Phase
Higgs Phase
m/Λ
Witten's point

IV. **N** = 1 or 2 SUSY bulk, Hanani - Tong model

@ Semilocal Strings

☆ Obtained from string/D brane consideration

☆ ☆ From field theory we get zn model: DIFFERNENT

☆ ☆ ☆ Large-N limit the same!!!

$$\mathcal{L}^{het}_{\mathbb{WCP}^{N_F-1}} = |\nabla_\mu n_i|^2 + |\tilde{\nabla}_\mu \rho_j|^2 \, .$$

$$- \sum_{i=0}^{N-1} |\sigma - m_i|^2 \, |n_i|^2 - \sum_{j=0}^{\tilde{N}-1} |\sigma - \mu_j|^2 \, |\rho_j|^2 - D \left(|n_i|^2 - |\rho_j|^2 - r_0 \right)$$

$$- \, 2|\omega|^2 |\sigma|^2$$

$$\nabla_\mu n_i = (\partial_\mu - iA_\mu) n_i , \quad \tilde{\nabla}_\mu \rho_j = (\partial_\mu + iA_\mu) \rho_j$$

$N_F = N + \tilde{N}$

$$m_k = m \, e^{2\pi i \frac{k}{N}} \, , \qquad k = 0, \ldots, N-1$$

$$\mu_l = \mu \, e^{2\pi i \frac{l}{\tilde{N}}} \, , \qquad l = 0, \ldots, \tilde{N} - 1 \, .$$

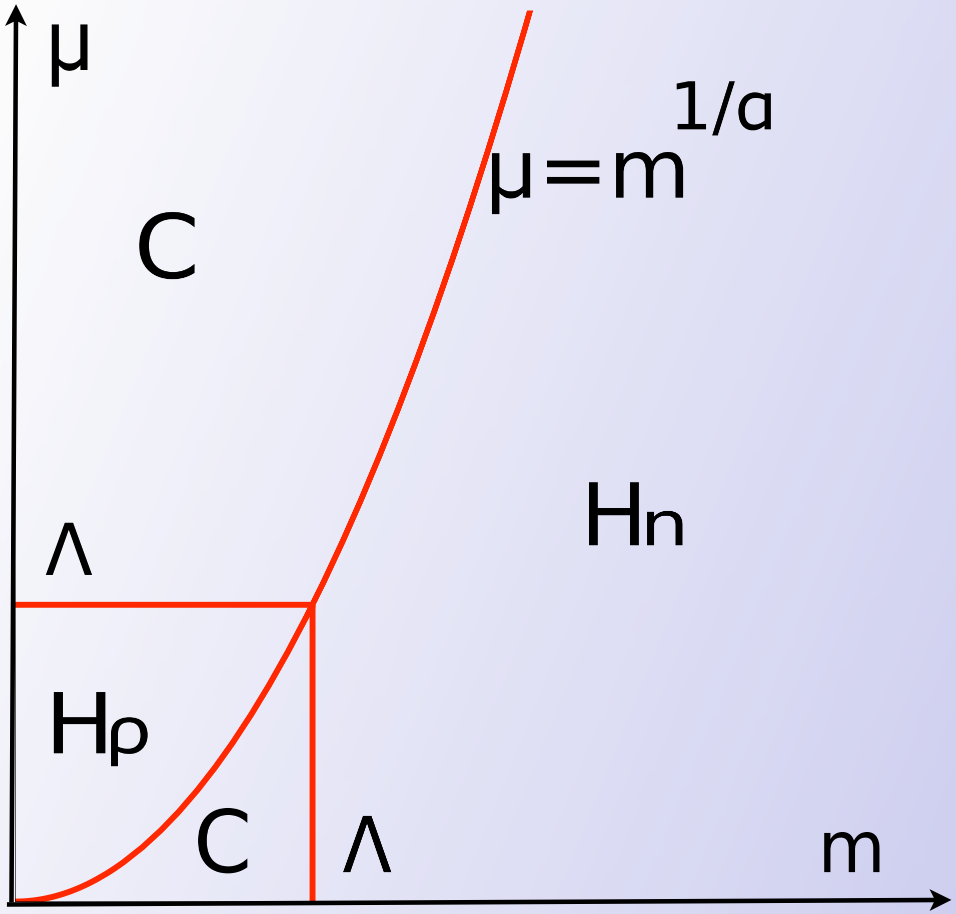


Figure 4: Phase Diagram of the weighted $(2,2)$ $\mathbb{CP}^{N-1}$ model in the large-N approach. There are four domains with different VEVs for σ: two Higgs branches $\mathbf{H}\rho$ and $\mathbf{H}n$, and two Coulomb branches $\mathbf{C}$. In the Coulomb phase $\mathbf{C}$ $r = 0$. The curve $\mu/\Lambda = (m/\Lambda)^{1/\alpha}$ together with horizontal and vertical lines starting from $\mu = \Lambda$ and $m = \Lambda$ respectively separates the $\mathbf{C}$ phases from the Higgs phases. In $\mathbf{H}n$ $r > 0$ and in $\mathbf{H}\rho$ $r < 0$. On the super-conformal line $\mu/\Lambda =(m/\Lambda)^{1/\alpha}$ a new branch described by a super-conformal theory opens up.

V. $\mathcal{N} = 2$ SUSY bulk, zn Model (MS+Vinci+Yung)

$$S_{\rm exact} = \int d^2x \left\{ |\partial_k (z_j n_i)|^2 + |\nabla_k n_i|^2 + \frac{1}{4e^2} F_{kl}^2 + \frac{1}{e^2}\,|\partial_k \sigma|^2 \right.$$

$$\left. + \quad |m_i - \tilde{m}_j|^2\, |z_j|^2 |n_i|^2 + \left|\sqrt{2}\sigma + m_i\right|^2 |n_i|^2 + \frac{e^2}{2}\left(|n_i|^2 - r\right)^2 \right\},$$

$$\boxed{i = 1, ..., N\,, \qquad j = 1, ..., \tilde{N}\,, \qquad \nabla_k = \partial_k - iA_k\,.}$$

z_j of the opposite charge compared to n_i and unconstrained

Derived from the bulk theory in the limit $\ln(\xi L^2) \gg 1$

P. Koroteev , W. Vinci, A. Yung+ MS: work in progress:

- ☞ At $N \to \infty$ HT = zn
- ☞ BPS sectors the same at any N
- ☞ New type of renormalizability

Instead of conclusions

$4D \leftrightarrow 2D$ Correspondence

brings fruits and a treasure trove of novel 2D models with intriguing dynamics!