Sterile neutrino as Dark Matter

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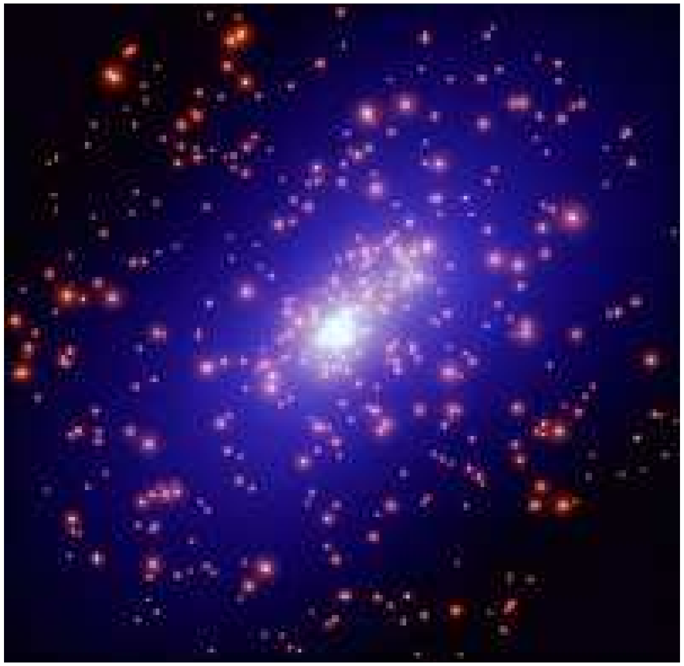
Outline

- Dark Matter in the Universe
- Theory of sterile neutrino
- The minimal set of parameters describing sterile neutrino
- Sterile neutrino as Warm DM
- Production of sterile neutrino in early Universe
- Astrophysical observations of sterile neutrino
  - Present bounds
  - Uncertainties in their determination
  - Program of future search

Dark Matter in the Universe

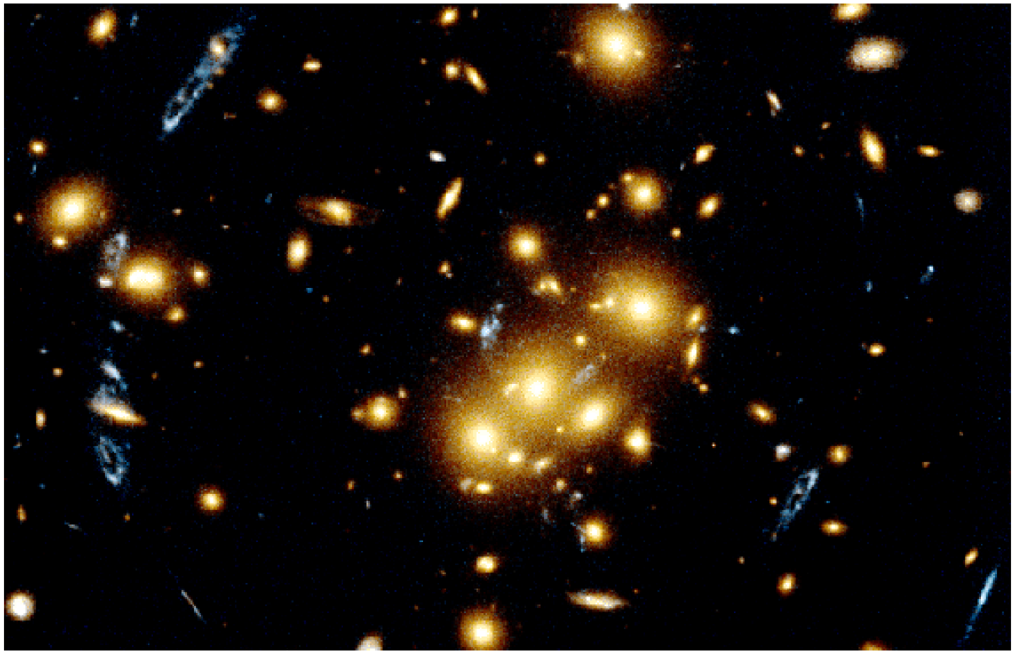
Extensive astrophysical evidence for the presence of the **dark non-baryonic** matter in the Universe

- Rotation curves of stars in galaxies and of galaxies in clusters
- Distribution of (X-ray bright) *intracluster gas*
- Gravitational lensing data



Galaxy cluster CL0024+1654 $(z = 0.39)$

Courtesy of ESA-NASA

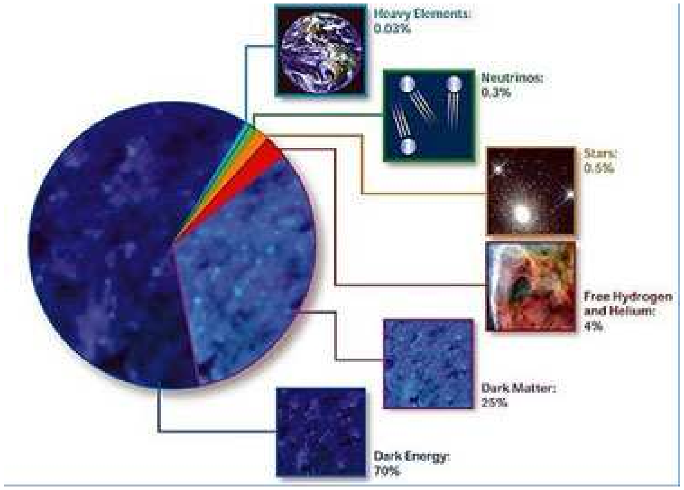


Left: Galaxy cluster CL0024+1654 as a gravitational lense

Courtesy of HST

Composition of the Universe

- Cosmological evidence for DM:
  - ☐ gravitational potential which allows for structure formation from tiny primeval fluctuations
  - ☐ gravitational potential which creates CMB anisotropy



- In the **concordance model**

$$\Omega_\Lambda \simeq 0.74 \qquad \Omega_{\text{DM}} \simeq 0.22$$

$$\Omega_{\text{baryonic}} \simeq 0.04$$

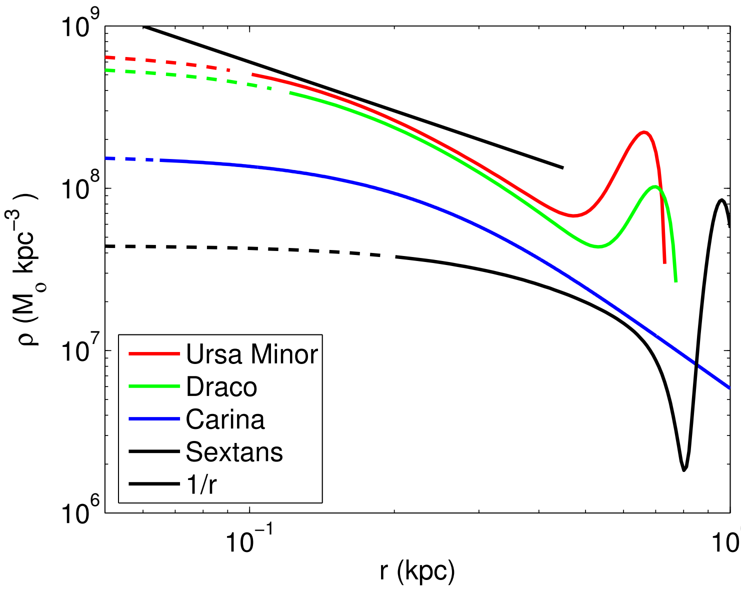
- Currently, there are no SM candidates for the DM
- Any DM candidate must be
  - Produced in the early Universe and have correct relic abundance
  - Very weakly interacting with electromagnetic radiation (“dark”)
  - Stable on cosmological time scales

DM is the physics beyond SM

- Non-baryonic DM candidates include
  - Gravitons, mass $\sim 10^{-21}$ eV
  - Axions – light pseudo-scalars, mass $\sim 10^{-5}$ eV
  - **Sterile neutrinos** mass ~ 10 keV
  - WIMPs – particles with masses ~ 10 GeV $-$ 10^4 GeV
  - WIMPZILLA – particles with mass $\sim 10^{10}$ GeV
- All this requires some **physics beyond the Standard Model**
- After the finding and identification of DM particle, a new elementary particle will appear and we will learn about underlying particle theory

CDM ? . . . HDM ? . . . WDM ? . . .

- **Free-streaming length** of DM particles
- Modern paradigm (Λ**CDM**)
  - DM is ”cold” (**CDM**)
  - Structure formation is *bottom-up* – smaller objects formed first: (stars ⟶ galaxies ⟶ galaxy clusters)
- CDM has its problems:
  - Cuspy profiles
  - Missing satellites problem
- Alternatives? **HDM**? $\lambda_{FS} = 40\left(\frac{M_\nu}{30\ \mathrm{eV}}\right)$ Mpc $\sim H^{-1}$. Top-down structure formation, (superclusters form first). **But!**
  - Too many large galaxy clusters
  - Galaxy formation starting too late



Sterile neutrinos – viable WDM candidate

- **Warm** DM can cure all these problems.
- Particle candidate? Extension of the SM?
- Experiments on neutrino oscillations (Kamland, SNO, super-K) – the most definite signal of physics beyond the SM.
- **Sterile neutrinos**: the simplest and natural ***extension*** of the Minimal SM that describe oscillations. Make leptonic sector of the SM symmetric.
- Break CP and **allow for baryogenesis**
- Sterile neutrino are good WDM candidates, as they:
  - ***Can be intensively produced in the Early Universe***
  - Can have *long life-time*.
  - Can have mass in **keV** range
- Let us see it in details

Asaka,
Shaposhnikov,
PLB **620**, 17
(2005)

Dodelson
Widrow'93

νMSM

- Lagrangian: addition of several sterile neutrino (fields N_I, $I = 1, \ldots, \mathcal{N}$) to the *Minimal Standard Model* gives:

$$\mathcal{L}_{\nu MSM} = \mathcal{L}_{MSM} + i\bar{N}^I \partial\!\!\!/ N_I - \left(\bar{L}_\alpha M^D_{\alpha I} N_I + \frac{M_I}{2}\bar{N}^c_I N_I + h.c.\right)$$

Asaka, Shaposhnikov, PLB **620**, 17 (2005)

Asaka, Blanchet, Shaposhnikov, PLB **631**, 151 (2005)

- Majorana masses M_I, Dirac mass matrix $M^D_{\alpha I} \equiv F_{\alpha I}\langle\Phi\rangle$ where $\alpha = \{e, \mu, \tau\}$ – mixing between left-handed L_α and right-handed neutrinos. $F_{\alpha I}$ – Yukawa couplings, Higgs VEV $\langle\Phi\rangle \simeq 174$ GeV.
- The sterile neutrino with $I = 1$ is chosen to be the lightest one.
- Coupling of N_1 is parameterized via **mixing angle θ**:

$$\theta^2 = \frac{1}{M_1^2} \sum_{\alpha=\{e\,\mu\,\tau\}} |M_D|^2_{1\alpha}$$

Parameters of νMSM

- $\mathcal{L}_{\nu MSM} = \mathcal{L}_{MSM} + i\bar{N}^I \partial\!\!\!/ N_I - \left(\bar{L}_\alpha M^D_{\alpha I} N_I + \frac{M_I}{2} \bar{N}^c_I N_I + h.c.\right)$
- νMSM includes **18 new parameters** (3 Majorana masses, 3 Dirac masses, 6 mixing angle and 6 CP-violating phases)
- Dirac masses $M_D \ll M_I$ (Majorana masses). See-saw formula works
- If scales of $M_{2,3} \sim \mathcal{O}(1-20)$ GeV can explain baryon asymmetry of the Universe
- $M_I \sim M_W$. No new energy scales, but Yukawa couplings very small: $F_{\alpha I} < 10^{-10}$
- M_1 can be as low, as ~ 300 eV (Tremaine-Gunn limit on the mass of fermionic DM)

Asaka,
Shaposhnikov,
PLB **620**, 17
(2005)

Back to sterile neutrino properties

How sterile neutrino is produced?

- Sterile neutrino interacts with the rest of the SM matter **only** via coupling with active neutrinos, parametrized by θ
- For a cosmological scenario 18 new parameters of νMSM are not enough
- Acceptable θ can be so small, that the rate of this interaction Γ is much slower than the expansion ($\Gamma \ll H$)
  - ⇒ Sterile neutrino are **not thermalized**
  - ⇒ One must know **initial conditions** of sterile neutrino at temperatures $T \gtrsim 1$ GeV

Therefore:

- Definite prediction of the sterile neutrino abundance **is not possible** as it involves knowledge of physics beyond the SM and even beyond the νMSM

For example, abundance of sterile neutrino can be determined **entirely by initial conditions**

Example I: νMSM coupled with inflaton

- To go beyond SM, one can incorporate inflation into νMSM
- Lagrangian of νMSM can be coupled with **inflaton** field χ in the natural way:

Tkachev, Shaposhnikov PLB **639**, 414 (2006)

$$\mathcal{L}_{\nu\mathrm{MSM}} = \mathcal{L}_{\mathrm{SM}} + i\bar{N}_I \partial\!\!\!/ N_I - F_{\alpha I}\bar{L}_\alpha \Phi N_I - \boxed{\frac{f_I}{2}\chi}\,\bar{N}_I^c N_I + \mathrm{h.c.} - V(\Phi,\chi)$$

SM **without** Higgs potential

- Inflaton coupling generates Majorana mass $\boxed{M_I}$ of sterile neutrino N_I after spontaneous breaking of scale invariance by the inflaton mass term:

$$V(\Phi,\chi) = -\boxed{\frac{1}{2}M_\chi^2\chi^2} + \lambda\left(\Phi^+\Phi - \frac{\alpha}{\lambda}\chi^2\right)^2 + \frac{\beta}{4}\chi^4$$

Production via coupling with the inflaton

$$\mathcal{L}_{\nu\text{MSM}} = \mathcal{L}_{\text{SM}} + i\bar{N}_I \not{\partial} N_I - F_{\alpha I}\bar{L}_\alpha \Phi N_I - \frac{f_I}{2}\chi\, \bar{N}_I^c N_I + \text{h.c.} - V(\Phi,\chi)$$

Tkachev, Shaposhnikov PLB **639**, 414 (2006)

- The lightest sterile neutrino production goes via $\boxed{\chi \to N_1 N_1}$
- Parameters of the model $\Big(\alpha, \beta, \lambda, f_I, \langle\chi\rangle\Big)$ can be chosen so that:
  - □ Conditions for chaotic inflation are satisfied. Inflaton potential is sufficiently flat and gives correct amplitude of scalar perturbations.
  - □ Correct Higgs mass is generated
  - □ Model allows for correct baryogenesis (large reheating temperature)
  - □ Decay of inflaton **produces enough light sterile neutrino** to account for all the DM

For $m_I \sim 300$ MeV correct Ω_s obtained for $M_s \sim 16-20$ keV

For $m_I \sim 100$ GeV correct Ω_s obtained for $M_s \sim 10$ MeV

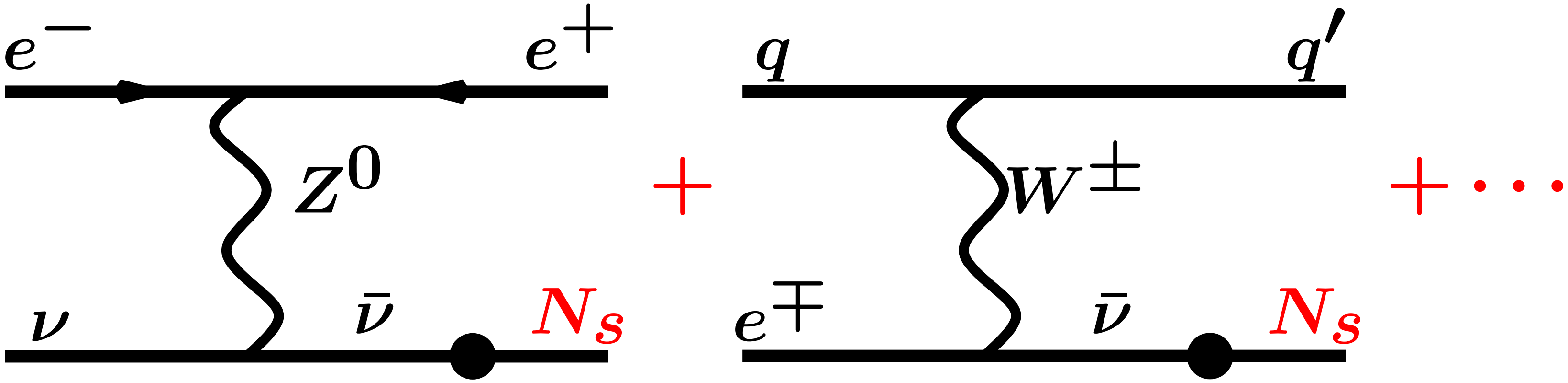
Back to sterile neutrino DM properties

Go to the DW scenario

Sterile neutrino in Early Universe

- Sterile neutrino in the early Universe interact with the rest of the SM matter via ***neutrino oscillations:***

Dodelson Widrow'93



- **Naively**, rate of production

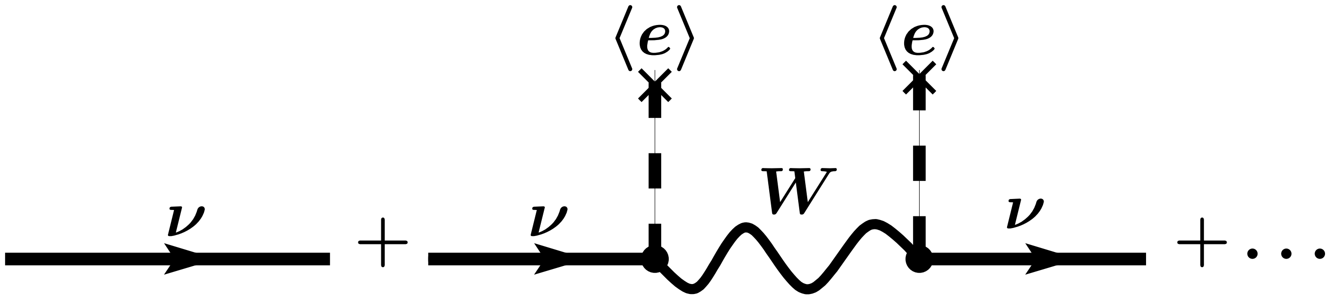
$$\Gamma \sim \sigma n v, \qquad \sigma \sim G_F^2\theta^2 T^2, \quad n \sim T^3$$

$$\frac{\Gamma}{H} \sim G_F^2\theta^2 T^3 M_{\mathrm{Pl}} \gg 1 \quad \text{at} \quad T \sim M_W \qquad (\text{for } \theta \gtrsim 10^{-7})$$

- This estimate is however **wrong** by many orders of magnitude!

Matter effects on oscillations

- The primeval plasma changes properties of the active neutrino



- ... and suppressed oscillation effects:

Nötzhold
Raffelt'88

Barbieri
Dolgov'90

Dodelson
Widrow'93

Dolgov
Hansen'00

$$\sin 2\theta_{\text{media}} = \frac{\sin 2\theta}{1 + c\left(\frac{T}{200\ \text{MeV}}\right)^6 \left(\frac{\text{keV}}{M_s}\right)^2}$$

numeric coefficient $c \sim \mathcal{O}(1)$

- Production is sharply peaked at

$$T_{\text{max}} \simeq 130 \left(\frac{M_s}{\text{keV}}\right)^{1/3} \text{MeV}$$

Example II: Dodelson-Widrow scenario

- Interaction of the sterile neutrino with the rest of the SM particles effectively takes place only ***around temperatures***

$$T_{\text{max}} \simeq 130 \left(\frac{M_s}{\text{keV}}\right)^{1/3} \text{ MeV}$$

- For interesting values of mixing angle θ the interaction rate is not enough to thermalize sterile neutrino
- To compute abundance of sterile neutrino one needs to know initial conditions at temperatures above $\sim$ GeV
- Even if one *ad hoc* assumes **zero initial conditions**, reliable computations are still not possible, as production takes place around QCD transition temperatures T_{QCD}.

Asaka, Laine, Shaposhnikov 2006

- Models with zero initial conditions which used some heuristic ways to treat quark contributions around T_{QCD} are ruled out by direct astrophysical observations (see below)

Sterile neutrinos – viable WDM candidate

- **Warm** DM can cure all problems of CDM and HDM
- Particle candidate? Extension of the SM?
- Experiments on neutrino oscillations (Kamland, SNO, super-K) – the most definite signal of physics beyond the SM.
- **Sterile neutrinos**: the simplest and natural extension of the Minimal SM that describe oscillations. Make leptonic sector of the SM symmetric.
- Break CP and **allow for baryogenesis**
- Sterile neutrino are good WDM candidates, as they:
  - ✓ Can be intensively produced in the Early Universe
  - ✓ Can have mass in **keV** range
  - – ***Can have* long life-time *and be dark enough*?**

Asaka, Shaposhnikov
PLB **620**, 17
(2005)

Dodelson
Widrow'93

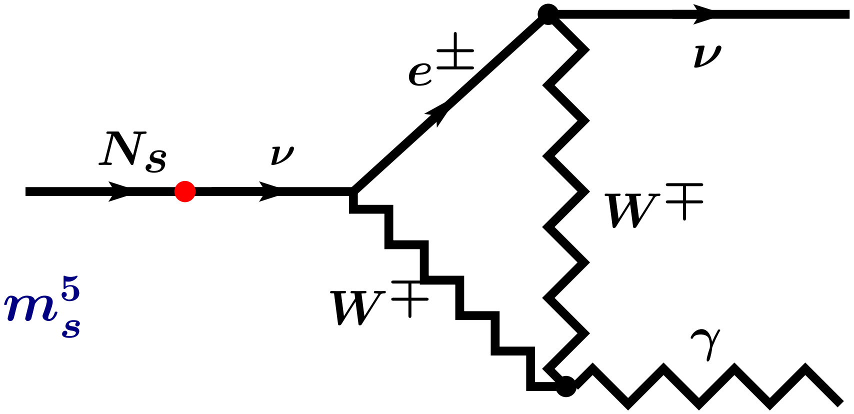
Properties of sterile neutrino

- Dominant decay channel for sterile neutrino (for masses below $\sim$ 1 MeV) is $N_s \to 3\nu$. Life-time $\tau = 5 \times 10^{26}\mathrm{sec} \times \left(\frac{\mathrm{keV}}{M_s}\right)^5 \left(\frac{10^{-8}}{\theta^2}\right)^2$

Wolfenshtein
Pal (1982)

- Subdominant (BR $\sim \frac{1}{128}$) **radiative decay channel**
  - Photon energy: $E_\gamma = \frac{m_s}{2}$
  - Radiative decay width:

$$\Gamma_{\text{rad}} = \frac{9\,\alpha_{\text{EM}}\,G_F^2}{256 \cdot 4\pi^4}\,\sin^2(2\theta)\,m_s^5$$



Barger Phillips
Sarkar (1995)

- Sterile neutrino DM **is not completely dark**
  - Flux from DM decay:

$$F_{\text{DM}} = \frac{E_\gamma}{m_s}\frac{\Gamma_{\text{rad}} M_{\text{DM}}^{\text{fov}}}{4\pi D_L^2} \approx \frac{\Gamma_{\text{rad}}\Omega_{\text{fov}}}{8\pi}\int \rho_{\text{DM}}(r)dr$$

line of sight

$(z \ll 1,\ \Omega_{\text{fov}} \ll 1)$

Dolgov
Hansen (2000

Abazajian
Fuller Tucker
(2001)

Boyarsky et al
(2006)

Back to sterile neutrino DM properties

Sterile neutrino as WDM: summary

- Sterile neutrinos:
  - the simplest and natural extension of the SM that describe neutrino oscillations.
  - Break CP and allow for baryogenesis
- Lightest sterile neutrino is ***good WDM candidates***, as it
  - ✓ Can be intensively produced in the Early Universe

    But there are ***no definite prediction of abundance*** Ω_s as a function of $(M_s, \sin^2(2\theta))$, as it involves in essential way the knowledge of physics beyond the νMSM
  - ✓ Can have mass in **keV** range
  - ✓ Can have cosmologically long life-time
  - ✓ Sterile neutrino DM is dark enough

    But it has ***signature decay*** with a very narrow line
- DM sterile neutrino are parameterized by two numbers: **mass** M_s and **mixing angle** $\sin^2(2\theta)$.

Astrophysical search for sterile neutrino and restrictions on its parameters M_s and θ

Where to look for DM decay line?

■ Extragalactic diffuse X-ray background (XRB)	Dolgov & Hansen, 2000; Abazajian et al., 2001 Mapelli & Ferrara, 2005; **Boyarsky et al. 2005**
■ Clusters of galaxies	Abazajian et al., 2001 **Boyarsky et al. astro-ph/0603368**
■ DM halo of the Milky Way. Signal increases as we increase FoV!	**Boyarsky et al. astro-ph/0603660** Riemer-Sørense et al. astro-ph/0603661 **Boyarsky, Nevalainen, O.R.** (in preparation)
■ Local Group galaxies	**Boyarsky et al. astro-ph/0603660** Watson et al. astro-ph/0605424
■ "Bullet" cluster 1E 0657-56	**Boyarsky, Markevitch, O.R.** (in preparation)
■ Cold nearby clusters	**Boyarsky, Vikhlinin, O.R.** (in preparation)
■ Soft XRB	**Boyarsky, Neronov, O.R.** (in preparation)

Need to find the best ratio between the DM decay *signal* and object's X-ray emission

How to choose the best object?

- Size does not matter: signal from the Milky way halo comparable with that of clusters like Coma or Virgo

$$F_{\rm DM} = \frac{E_\gamma}{m_s}\frac{\Gamma_{\rm rad} M_{\rm DM}^{\rm fov}}{4\pi D_L^2}$$

$$\approx \frac{\Gamma_{\rm rad}\Omega_{\rm fov}}{8\pi}\int \rho_{\rm DM}(r)dr$$

line of sight

- DM flux from e.g. **Draco, Ursa Minor** is 3 times stronger than that of the Milky Way halo.
- ***Dwarfs*** are really dark (M/L ~ 100)
- Continuum X-ray emission from Milky Way is about 2 orders **weaker** than that of a cluster

- The signal is stronger than XRB by a factor $E/\Delta E = 20 \div 50$ for modern X-ray satellites.

Boyarsky, Neronov, O.R. Shaposhnikov, Tkachev

astro-ph/0603660

Constraints from Local Halo...

DM distribution can be conservatively described by isothermal (cored) ***model***

$$\rho_{\text{halo}}(r) = \frac{v_h^2}{4\pi G_N} \frac{1}{r_c^2 + r^2}$$

- Milky Way DM halo **isothermal profile** describes rotation curve for $r \gtrsim 3$ kpc ($v_h \approx 170$ km/sec, $r_c \approx 4$ kpc)

- ***Dwarfs*** (Draco, Ursa Minor): $v_h \approx 22$ km/sec, $r_c \approx 0.1$ kpc

- LMC: $v_h \approx 50$ km/sec, $r_c \approx 1$ kpc

- Although these objects have quite different range of masses ($10^7 - 10^{12}\ M_\odot$) they have similar $\int \rho(r) dr$ – give comparable DM signal

- Assuming NFW (cusped) profile instead of isothermal (cored) one, increases the estimated DM flux by about 30%.

Strategy to optimize signal/noise ratio

- One way to improve S/N ratio is to reduce the noise, i.e. find astrophysical objects with very faint X-ray background
  $\Rightarrow$ Dwarf galaxies

- **But** there is another way to improve S/N ration.

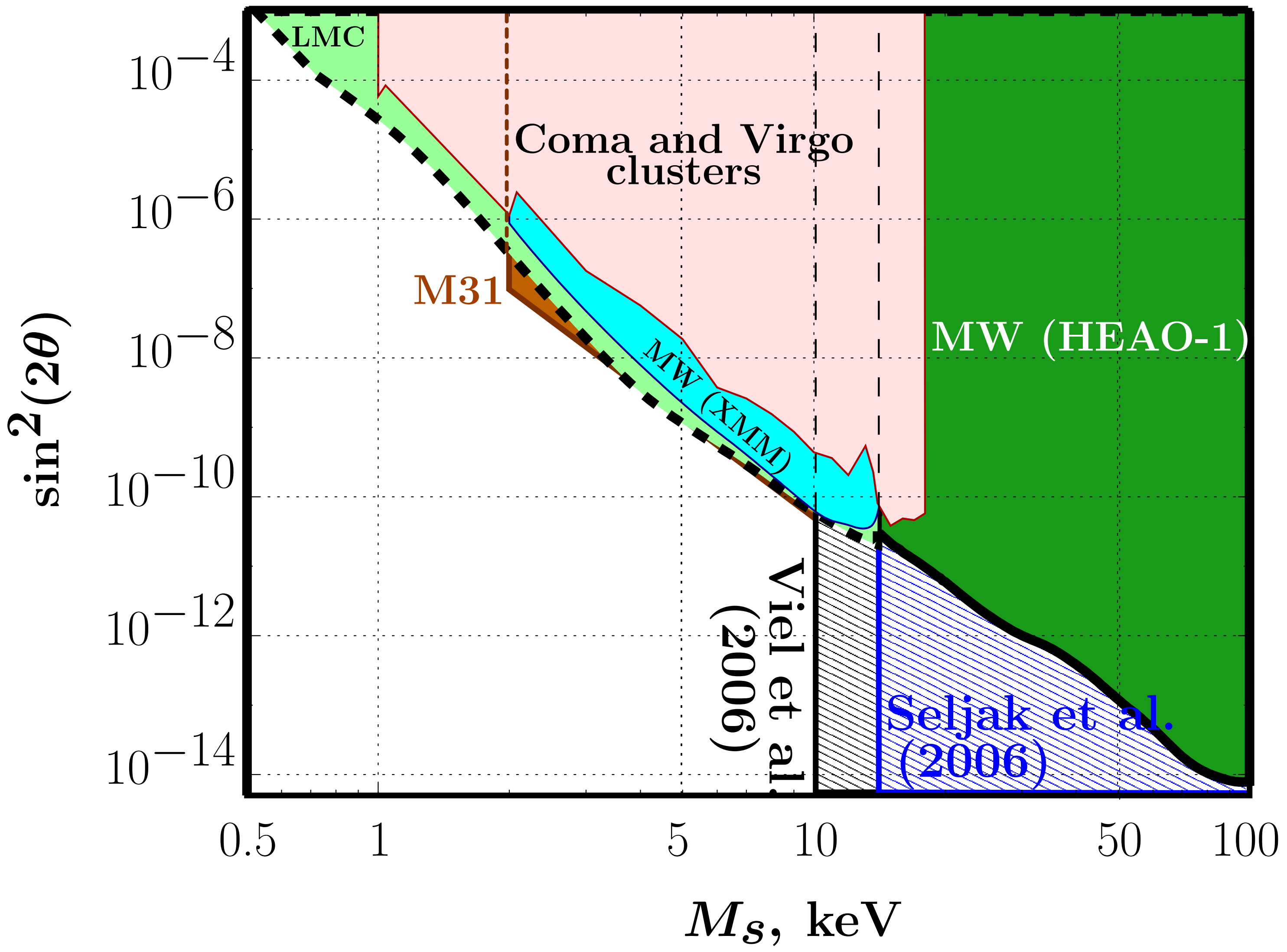
- Galaxy and galaxy clusters can be fairly bright in X-ray. **But** feature we are looking for is **a narrow line**. Astrophysical background can be strong, yet described with the good precision by the power-law. ***Adding a thin line on top of such a power-law...***

- Depending on the data one of these methods (“full flux” and “statistical”) can be used.

- Studies of different objects and types of objects is important, as it reduces the uncertainties of DM modeling

Parameters of sterile neutrino DM

Fine print: all results subject to intrinsic factor ~ 2 uncertainty!



MW (HEAO-1) Boyarsky et al 2005

Coma and Virgo clusters Boyarsky et al 2006a

LMC+MW(XM Boyarsky et al 2006b

MW (Chandra) Riemer-Søorensen et al. 2006

M31 Watson et al. 2006

Ly-α data Vie et al. 2006; Seljak et al. 2006

Dilution of sterile neutrino abundance

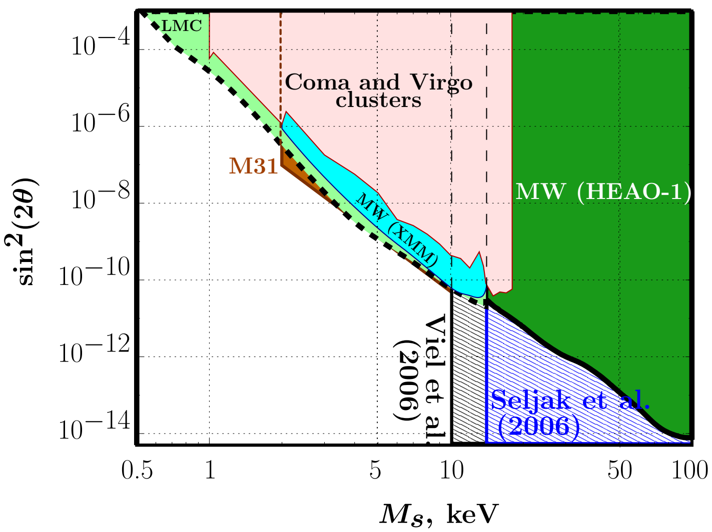
- Do Lyman-α results mean that any mass below $M_s \simeq 10$ keV ($M_s \simeq 14$ keV) are excluded for all θ?
- **No,** the actual result reads: $M_{\text{lower limit}} = \frac{\langle p_a \rangle}{\langle p_s \rangle} M_{\text{Ly}-\alpha}$
- νMSM also contains two heavy sterile neutrino $N_{2,3}$ with masses $M_{2,3} \sim \mathcal{O}(1{-}10)$ GeV.
- Their Yukawa couplings can be chosen such that they are thermalized at $T_D \sim \mathcal{O}(20)$ GeV and decay at $T \sim \mathcal{O}(1)$ MeV (after the lightest sterile neutrino has been produced)

Asaka et al. PLB 638 (2006)

- This leads to the **entropy production** $S \sim \mathcal{O}(1{-}100)$.
- Entropy production leads to the dilution DM sterile neutrino abundance: $\Omega_{\text{DM}} \longrightarrow \frac{\Omega_{\text{DM}}}{S}$
- It also leads to momentum distribution and $\langle p_s \rangle$ red-shifting by $S^{1/3}$
- Therefore $M_{\textbf{lower limit}} = \frac{M_{\textbf{Ly}-\alpha}}{S^{1/3}}$

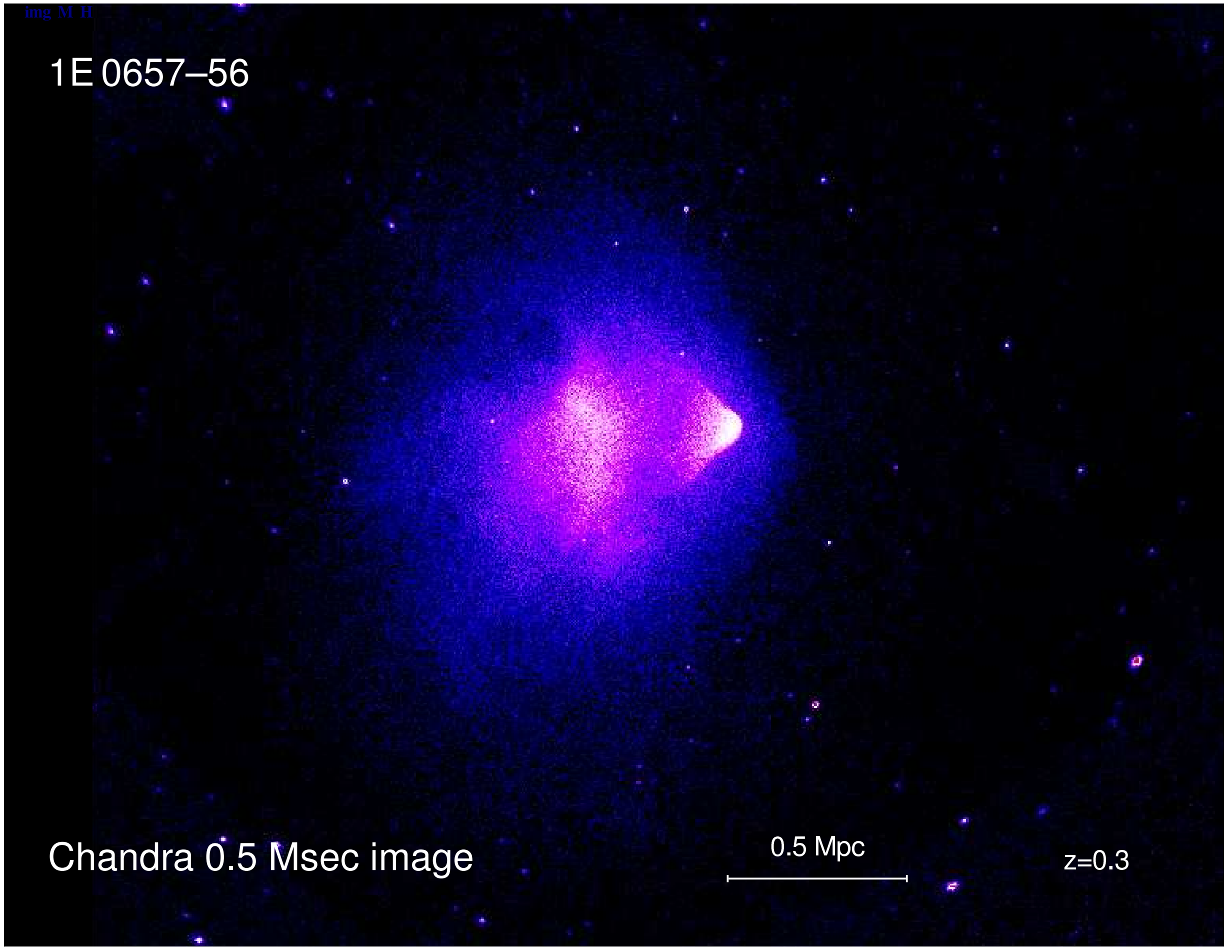
Uncertainties

- All these restrictions subject to uncertainties of the DM determination
- The uncertainty of the DM mass determination is typically **factor of 2**
- DM decay flux for different DM profiles differs by about 30%.



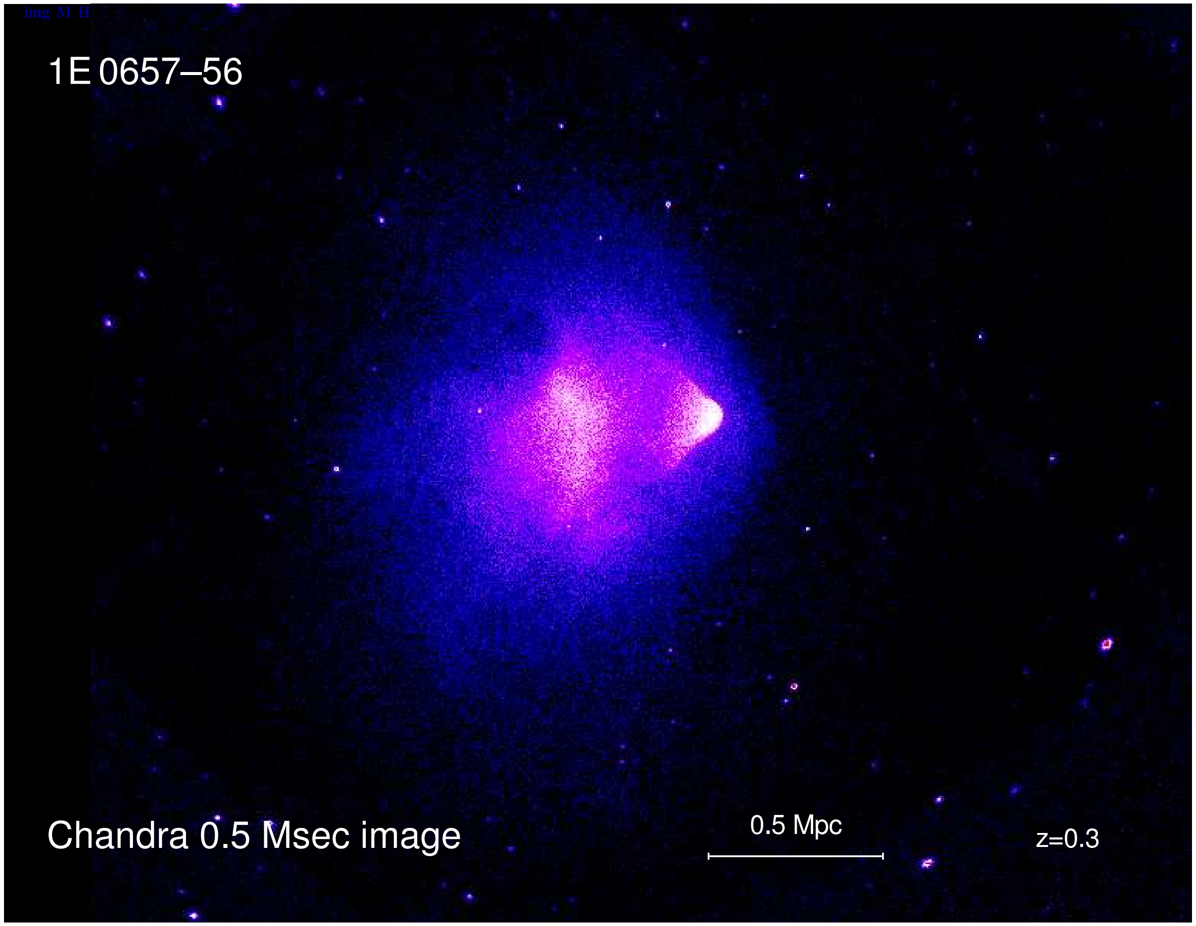
- Paper of Abazajian-Koushiappas (2006) misinterpreted results of Van der Marel et al. (ApJ **124** (2002) on LMC DM mass and results of Boyarsky et al. on LMC)
- Various ways of DM determination
  - Velocity distribution
  - X-ray hydrostatic equilibrium
  - Gravitational lensing
- It is important to study *various* astrophysical objects, with DM mass determined via different methods

"Bullet" cluster



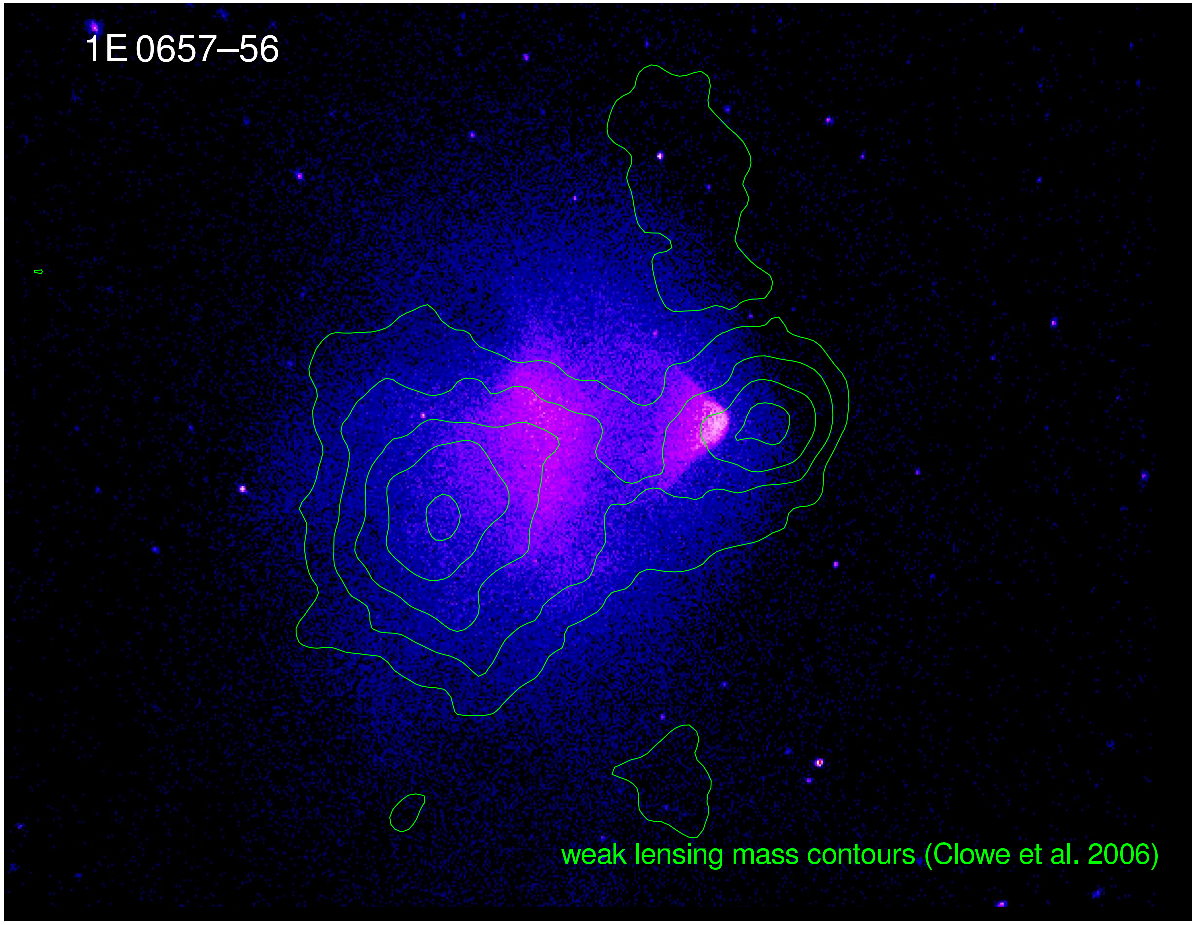
Cluster 1E 0657-56
Red shift $z = 0.296$
Distance $D_L = 1.5$ Gpc

Merging system in the plane of the sky



- ★ Subcluster (right) passed through nearly the center of the main cluster.
- ★ DM and galaxies behave as nearly collisionless gas.
- ★ Gas from the subcluster has been stripped away (shock wave with Mach number $M = 3.2$ and $T_{\rm shock} \sim 30$ keV)

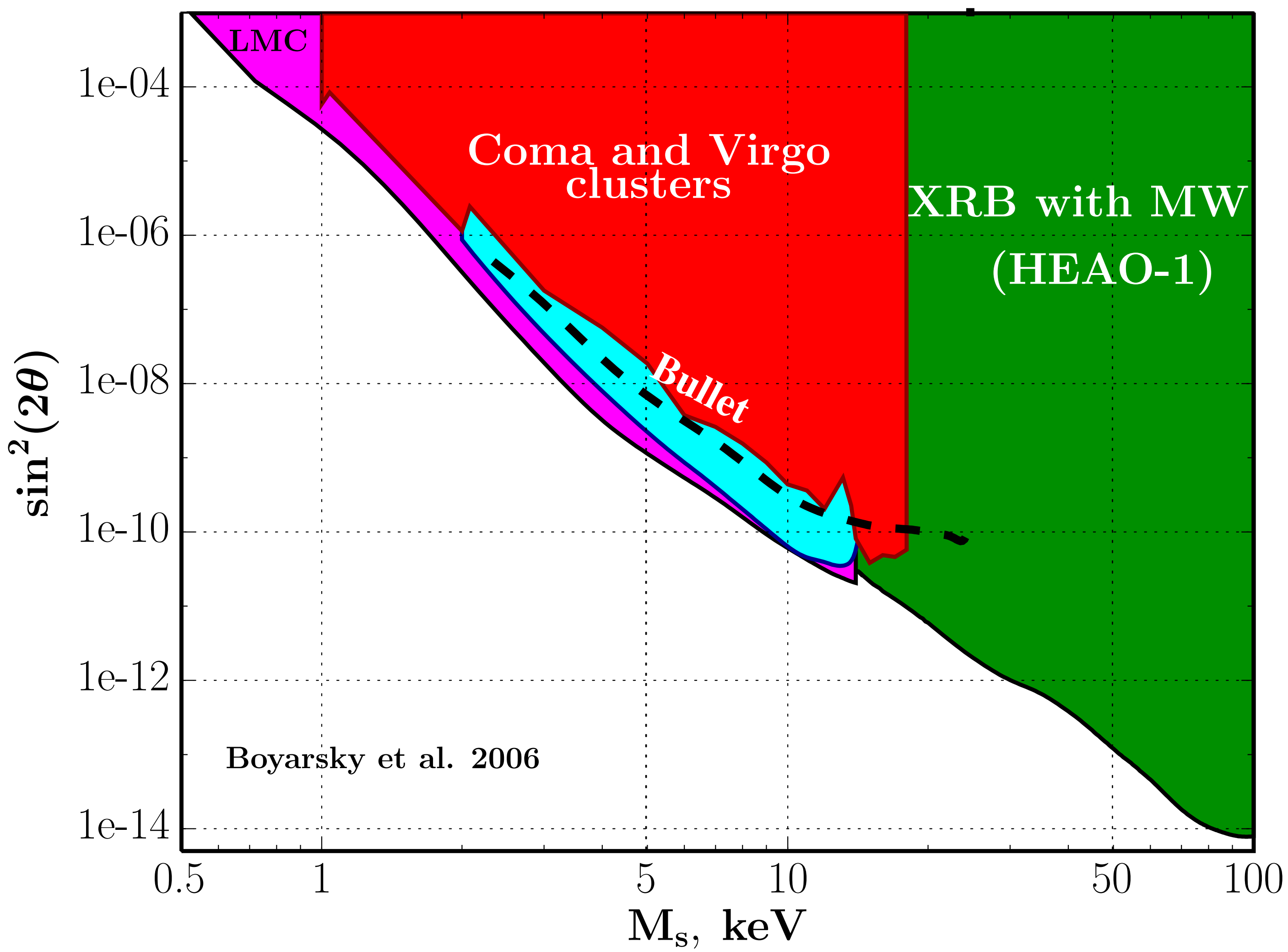
Merging system in the plane of the sky



- ⋆ Subcluster (right) passed through nearly the center of the main cluster.
- ⋆ DM and galaxies behave as nearly collisionless gas.
- ⋆ Gas from the subcluster has been stripped away (shock wave with Mach number $M = 3.2$ and $T_{\rm shock} \sim 30$ keV)
- ⋆ The mass of the DM is determined via weak gravitational lensing

- ⋆ Velocity distributions agree with weak lensing data

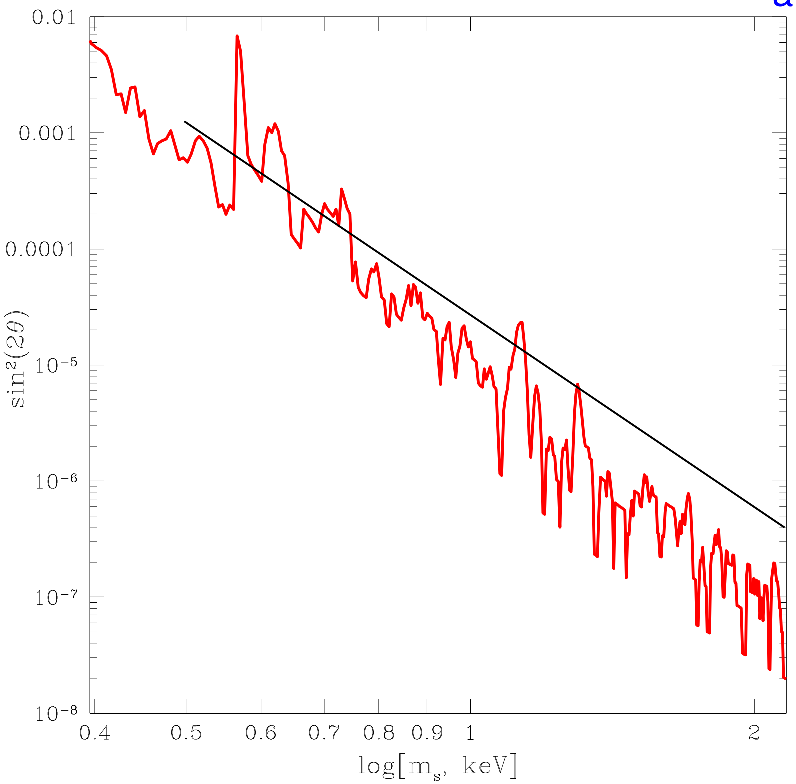
Restrictions, including 1E 0657-06



Soft XRB with calorimeter data

McCammon et
al. 2002

- Launch took place in 1999
- Calorimeter with the FoV ~ 1 sr
- Energy range from 60 eV $- 1$ keV
- Spectral resolution $\Delta E \sim 10$ eV
- Flight time only 10^2 **seconds**
- Provides modest improvement over existing LMC data (*XMM-Newton* observation with exposure time $\sim 1.8 \times 10^4$ **seconds**)



- Provides restrictions in the energy range down to Tremaine-Gunn limit (i.e. down to $M_s \sim 300$ eV)
- Demonstrates potential of non-imaging large-FoV calorimeters

Boyarsky,
Neronov, O.R.
in progress

Summary

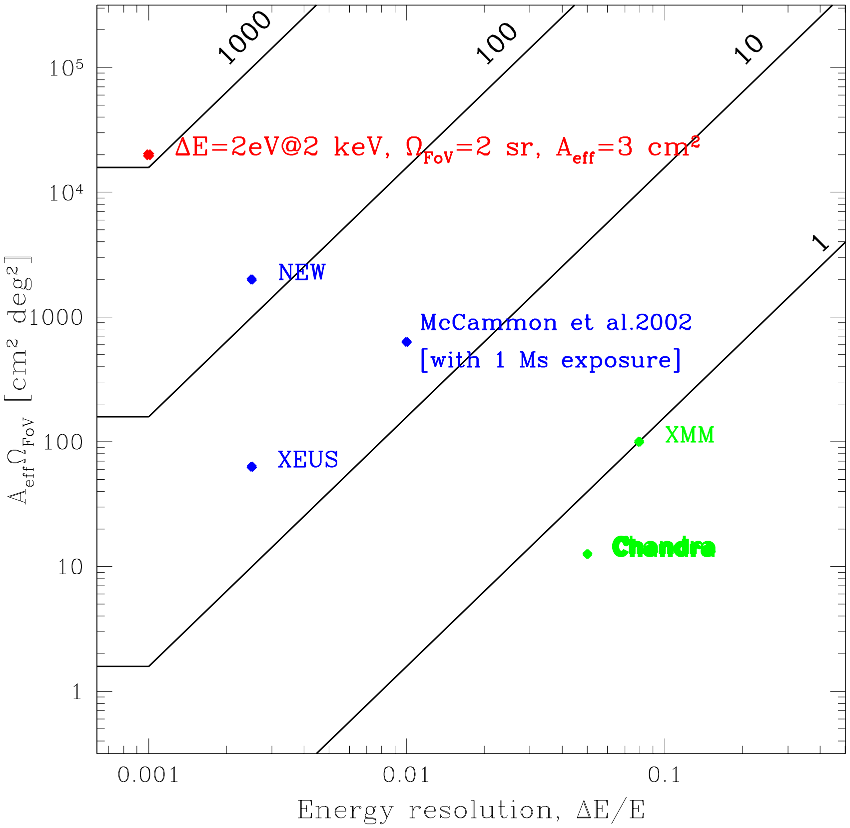
- Sterile neutrino with the mass in **keV** range is a viable DM candidate
- It can be described by two parameters mass M_s and mixing angle $\sin^2(2\theta)$.
- νMSM is enough to reliably compute abundance of DM. Mass and mixing angle should be treated as **independent parameters**
- Sterile neutrino possesses radiative decay channel and one can put restrictions on its decay width from astrophysical observations
- Study of various DM dominated objects allow to reduce uncertainties of DM modeling.
- Preferred objects are either those with the smallest X-ray background for a given $\int \rho(r)dr$ or those, whose continuous X-ray emission is described by a featureless spectra (like power-law)

Modern astrophysical missions

- Over the past year the bounds has been improved by several orders of magnitude
- New types of objects were analyzed and new search strategies has been developed
- Further improve constraints via reduction of the statistical errors due to prolonged observations (especially important for dark objects). Search for other “exotic” like the bullet cluster
- Study soft X-ray – closing the window of large mixing angle and small (down to the Tremaine-Gunn limit) masses
- Chandra and XMM-Newton cover range of masses $\mathbf{1}$ keV $\lesssim M_S \lesssim$ $\mathbf{20}$ keV. For higher masses one can use non-imaging missions (e.g. INTEGRAL)
- It is very hard to detect and identify DM decay line with missions, whose spectral resolution is at least order of magnitude above the line’s width

Future missions

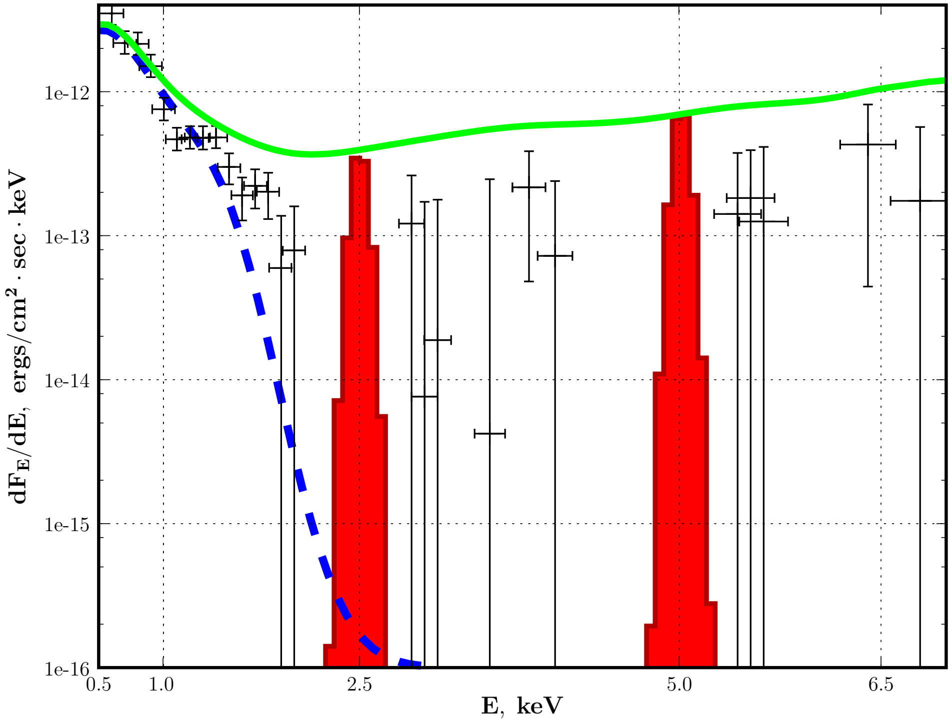
- New data from *Chandra* and *XMM-Newton* can hardly improve constraints by more than a factor of 10
  - ☐ Improvement of spectral resolution is needed (width of DM line is $\Delta E/E \sim 10^{-3}$ in the MW halo).
  - ☐ Bigger FoV – better statistics. This is mostly important for the case of MW halo



- Future missions like *XEUS* or *Constellation X* will have better spectral resolution but very small FoV
- For the DM search one does not need imaging capabilities
- A promising mission being developed right now is NEW by SRON
- When planning for new missions – take into account DM search!

The End

...Dwarfs are really dark...



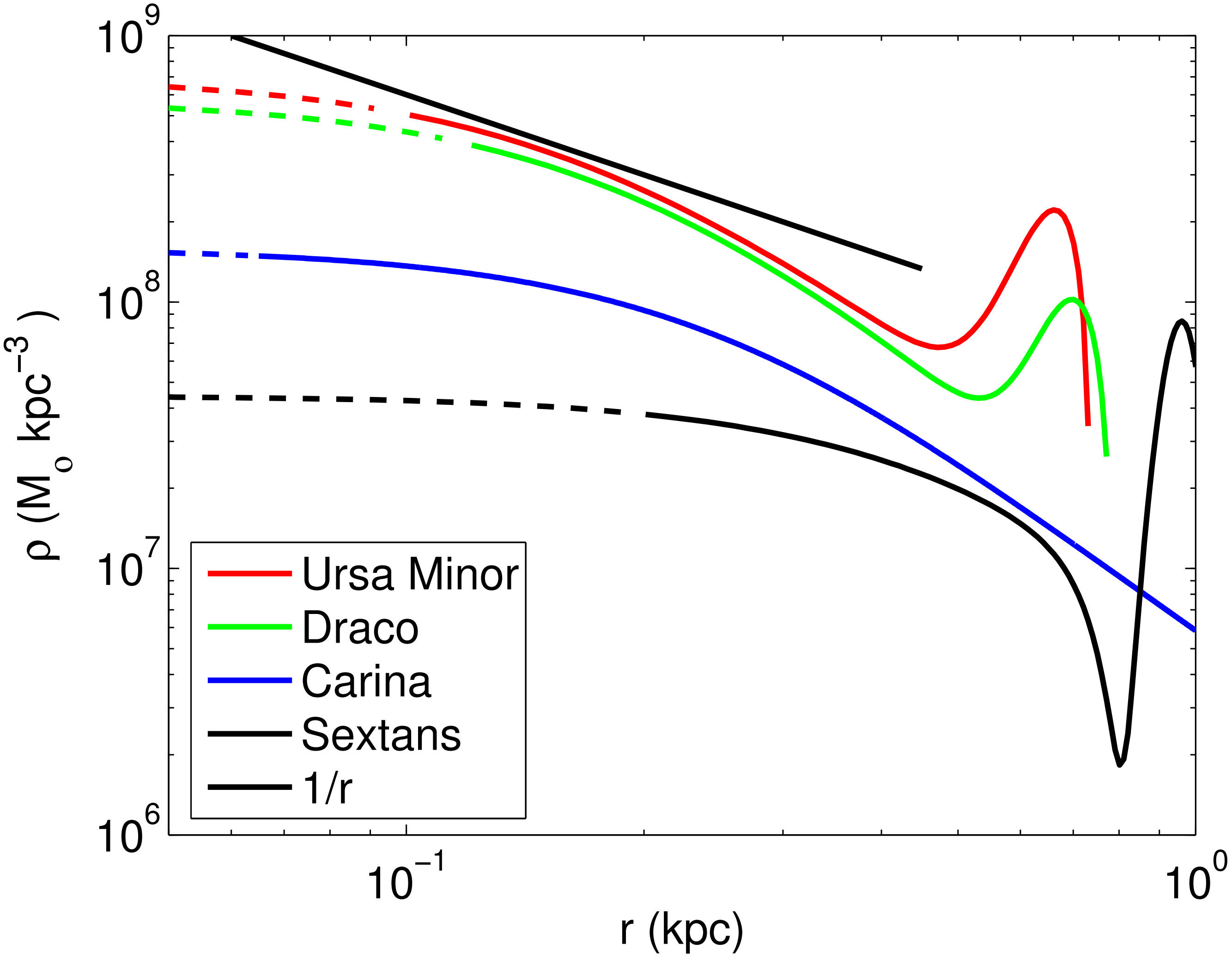
X-ray emission from LMC

[Boyarsky et al. astro-ph/0603660]

Back to preferred targets

- X-ray emission from LMC is **zero within statistical uncertainty** for $\boldsymbol{E \gtrsim 2}$ keV.
- LMC is fairly "bright" (mass-to-light ratio $\sim \mathbf{3}$)
- X-ray emission should be **much smaller** for dwarfs like Ursa Minor or Draco (M/L $\sim$ $\mathbf{100}$).

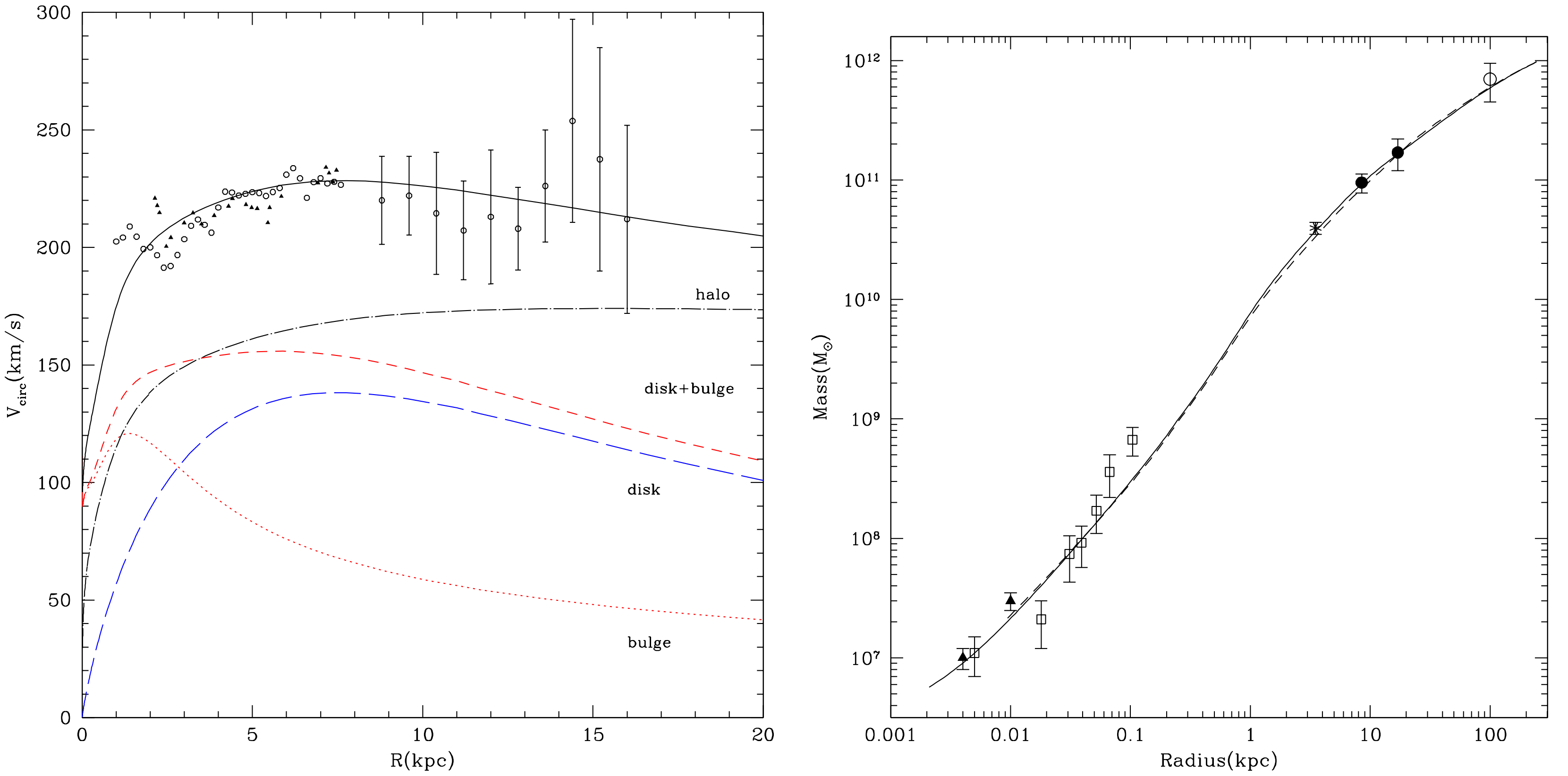
Dwarf DM profile



Wilkinson et al, astro-ph/0602186

Back to DM profiles

Milky Way DM halo



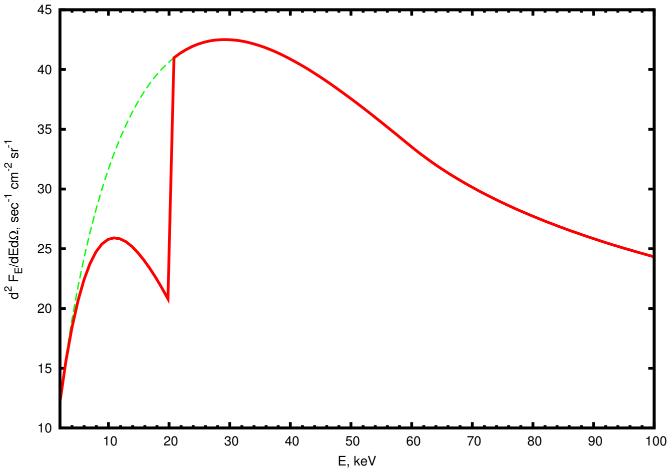
Klypin et al. ApJ 573, (2002) 597

Uncertainties of the mass determination for the MW DM halo (for $r > r_\odot$) are within 30%

Back to DM profiles

How to look for DM decay line?

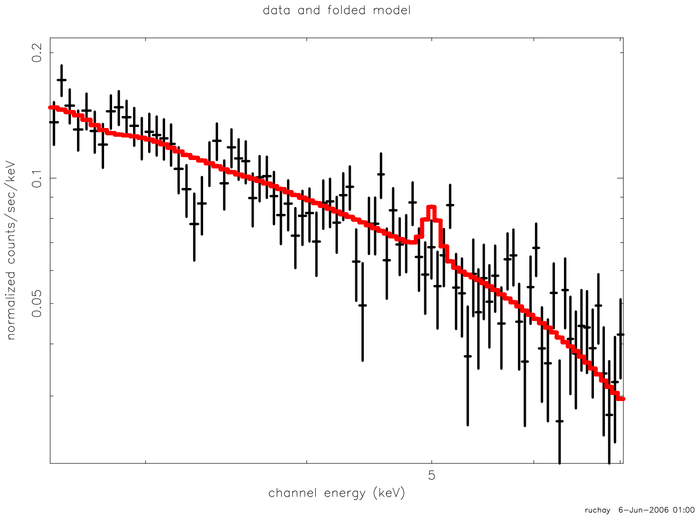
- Possible solutions
  (i) Assume that all flux in the energy bin equal to $\Delta E_{\text{spectral}}$ comes from DM
  - Implies for existence of unnatural features in a spectrum



(ii) Add a thin line against existing power-law spectrum. Allow fit to be worsened by several sigma

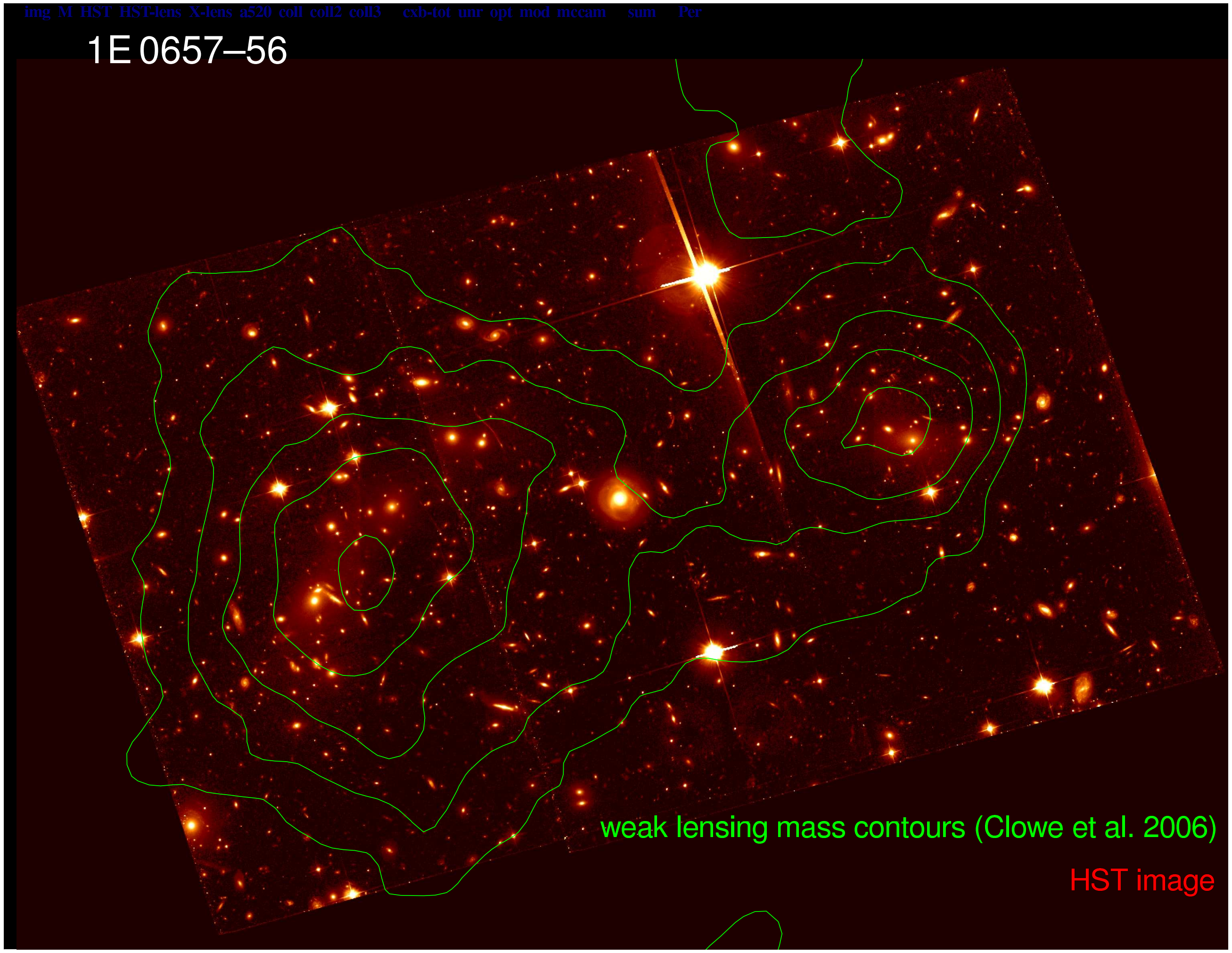
- Works best in data is described by a power law-like spectrum

Boyarsky, Neronov, O.R. Shaposhnikov, 2005

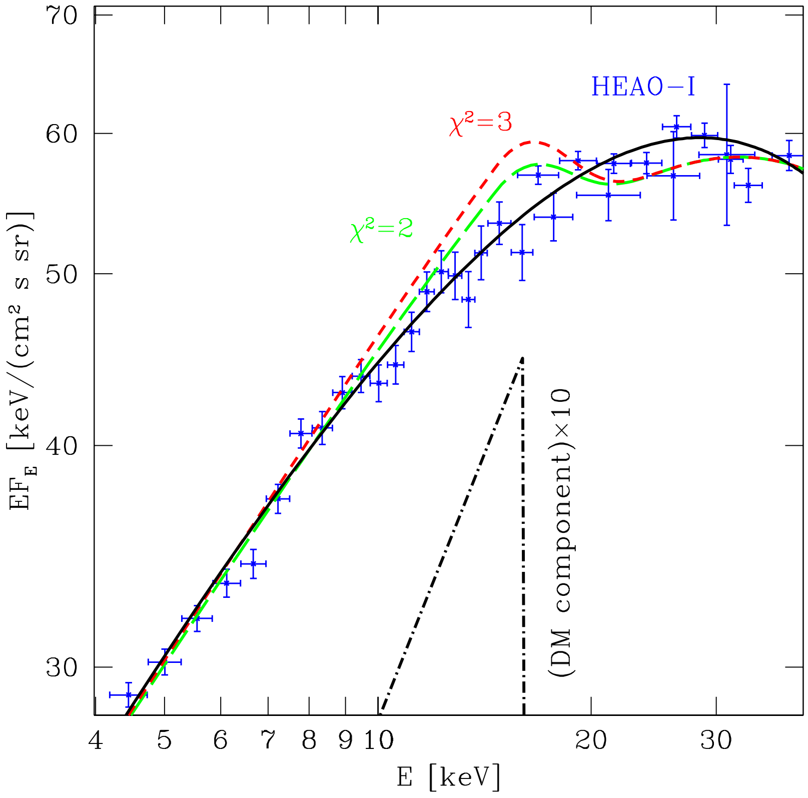


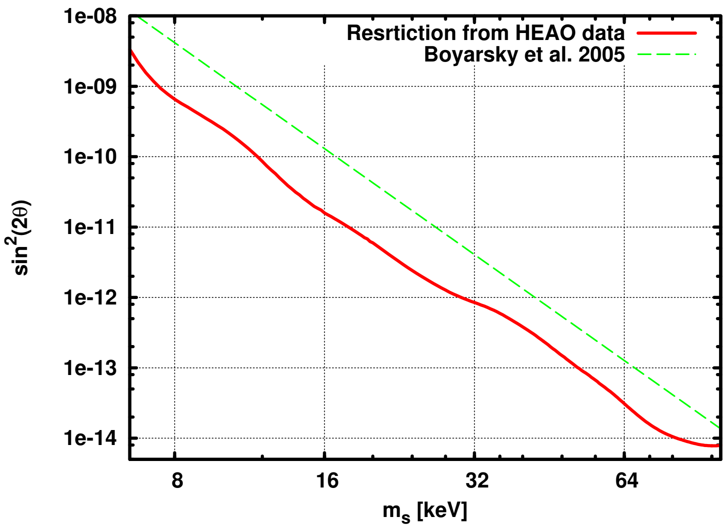
Back to strategy to look for DM decay line

Weak lensing mass contours



Constraints from diffuse X-ray bgnd





DM decay signal accumulates over various red shifts

$$\frac{d^2 F_E}{d\Omega\, dE} = \frac{\Gamma_{\mathrm{rad}} n^0_{\mathrm{DM}}}{4\pi H_0} \frac{1}{\sqrt{\Omega_\Lambda + \Omega_M \left(\frac{m_s}{2E}\right)^3}}$$

Boyarsky, Neronov, O.R., Shaposhnikov, astro-ph/0512509

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