Some entertaining aspects of Multiple Parton Interactions Physics

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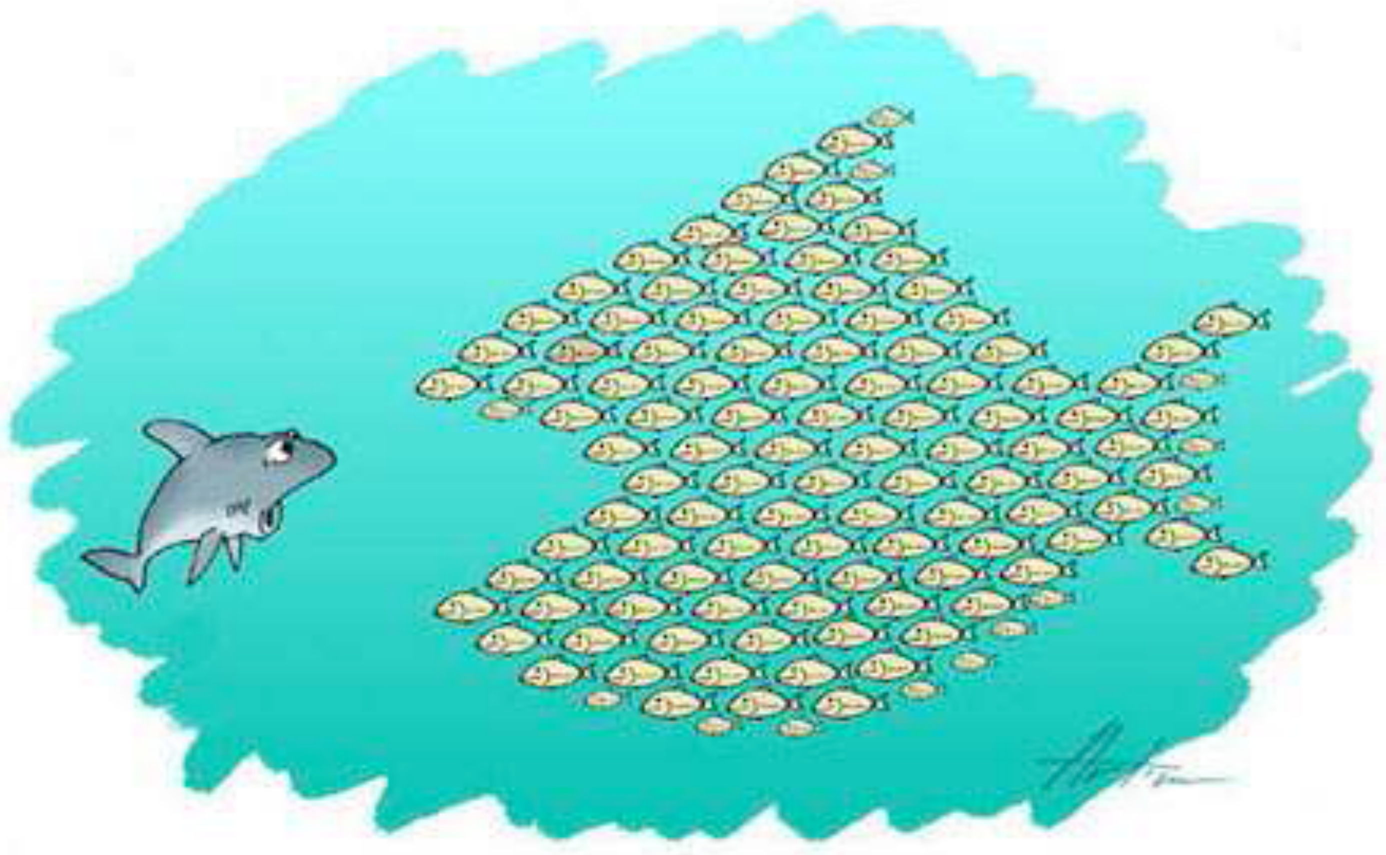
Multi-Parton Interactions



Work in collaboration with B.Blok, L.Frankfurt and M.Strikman

The Four jet production at LHC and Tevatron in QCD.
Phys. Rev. D83 : 071501, 2011; e-Print: **arXiv:1009.2714** [hep-ph]

pQCD physics of multiparton interactions.
e-Print: **arXiv:1106.5533** [hep-ph]

Hadrons are complex subjects. Not only because they are composite :
At high energies the rich QFT-structure of hadron constituents becomes visible.

To resolve the internal structure of interacting objects we probe them with large momentum transfer processes - "hard interactions"

$$\frac{d\sigma}{dQ^2} \propto \frac{1}{Q^4} \qquad Q^2 \gg R^{-2}$$

Hadron structure matters little in such standard (small-cross section) hard interactions.

Colliding ***hadrons*** serve as sources of two colliding ***partons***, in inclusive manner: $D_h^p(x, Q^2)$

The size - and the buildup - of the hadron manifests itself in ***multi-parton collisions***

$$\frac{d\sigma}{dQ^2} \propto \frac{1}{Q^4} \times \frac{1}{R^2 Q^2}$$

This may seem a small ("higher twist") correction to the total cross section. And so it is.

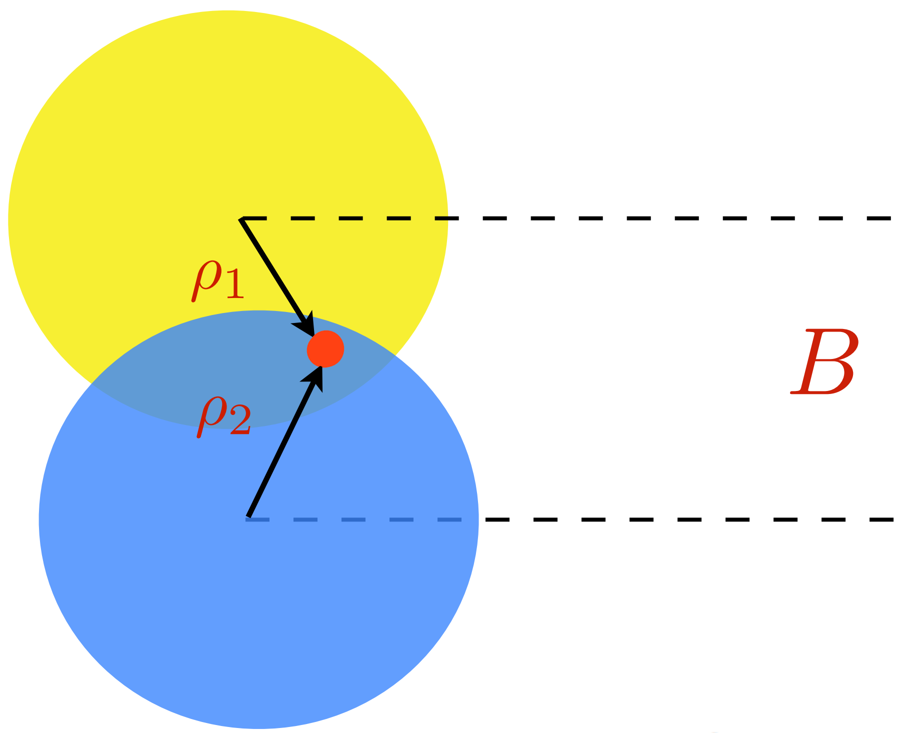
However, in some specific circumstances such eventuality turns out to be dominant !

4-jet production in the back-to-back kinematics

2-parton collision

The standard approach to the multi-jet production is the QCD improved parton model.

It is based on the assumption that the cross section of a hard hadron–hadron interaction is calculable in terms of the convolution of parton distributions within colliding hadrons with the cross section of a hard ***two-parton collision***.



$$\sigma_2 = \int d^2\rho_1 d^2 B \; f(x_1, \vec{\rho}_1, p^2) f(x_2, \vec{B} - \vec{\rho}_1, p^2) \; \frac{d\sigma^h}{d\hat{t}} d\hat{t}$$

parton probability density : $$f(x, \vec{\rho}, p^2) = \psi^+(x, \vec{\rho}, p^2) \psi(x, \vec{\rho}, p^2)$$

Result of the impact parameter integration - squaring of the amplitude in the momentum space:

$$\int \frac{d^2 k_\perp}{(2\pi)^2} \psi(x, k_\perp) \int \frac{d^2 k'_\perp}{(2\pi)^2} \psi^\dagger(x, k'_\perp) \times \int d^2\rho \, e^{i\vec{\rho}\cdot(\vec{k}_\perp - \vec{k}'_\perp)} \quad = \int \frac{d^2 k_\perp}{(2\pi)^2} \psi(x, k_\perp) \times \psi^\dagger(x, k_\perp)$$

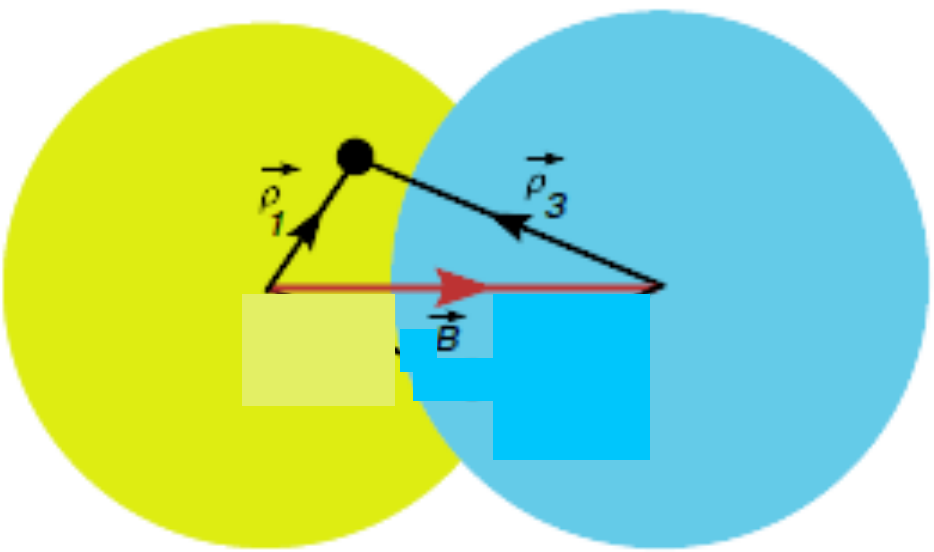
An application of this picture to the processes with production of, e.g., ***four jets*** implies that all jets in the event are produced in a hard collision of ***two*** initial state partons.

Recent data of the CDF and D0 Collaborations provide evidence that there exists a kinematical domain where a more complicated mechanism becomes important :

double hard interaction

of two partons in one hadron

with two partons in the second hadron.



Let us see, what difference does it make to our formulae

multi-partons

Multi-parton wave function

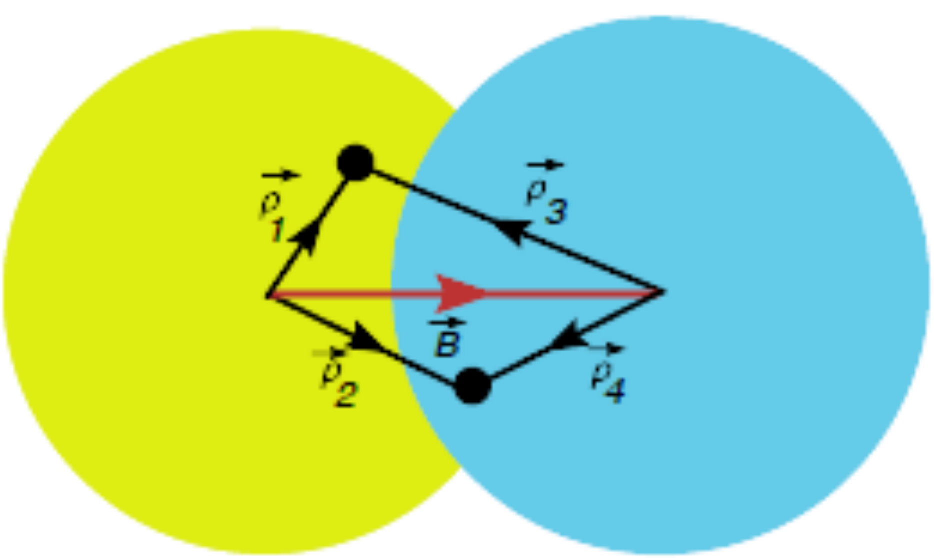
$$\psi_n \; (x_1, \vec{\rho}_1, x_2, \vec{\rho}_2, ...) = \int \prod_{i=1}^{i=n} \frac{d^2 k_i}{(2\pi)^2} \exp(i \sum_{i=1}^{i=n} \vec{k}_i \vec{\rho}_i) \; \psi_n(x_1, \vec{k}_1, x_2, \vec{k}_2, ..)(2\pi)^2 \delta(\sum \vec{k}_i)$$

Inclusive 2-parton probability distribution in the impact parameter space :

$$D \; (x_1, x_2, \vec{\rho}_1, \vec{\rho}_2) = \sum_{n=3}^{n=\infty} \int \prod_{i\geq 3}^{i=n} [dx_i d^2\rho_i] \, \psi_n(x_1, \vec{\rho}_1, x_2, \vec{\rho}_2, ...x_i, \vec{\rho}_i,) \; \psi_n^+(x_1, \vec{\rho}_1, x_2, \vec{\rho}_2, ..., x_i, \vec{\rho}_i, ...)\delta(\sum_{i=1}^{i=n} x_i \vec{\rho}_i)$$

Independent impact parameter integration $\Rightarrow$ equality of parton momenta in ψ and $\psi^\dagger$

$$k_\perp = k'_\perp$$



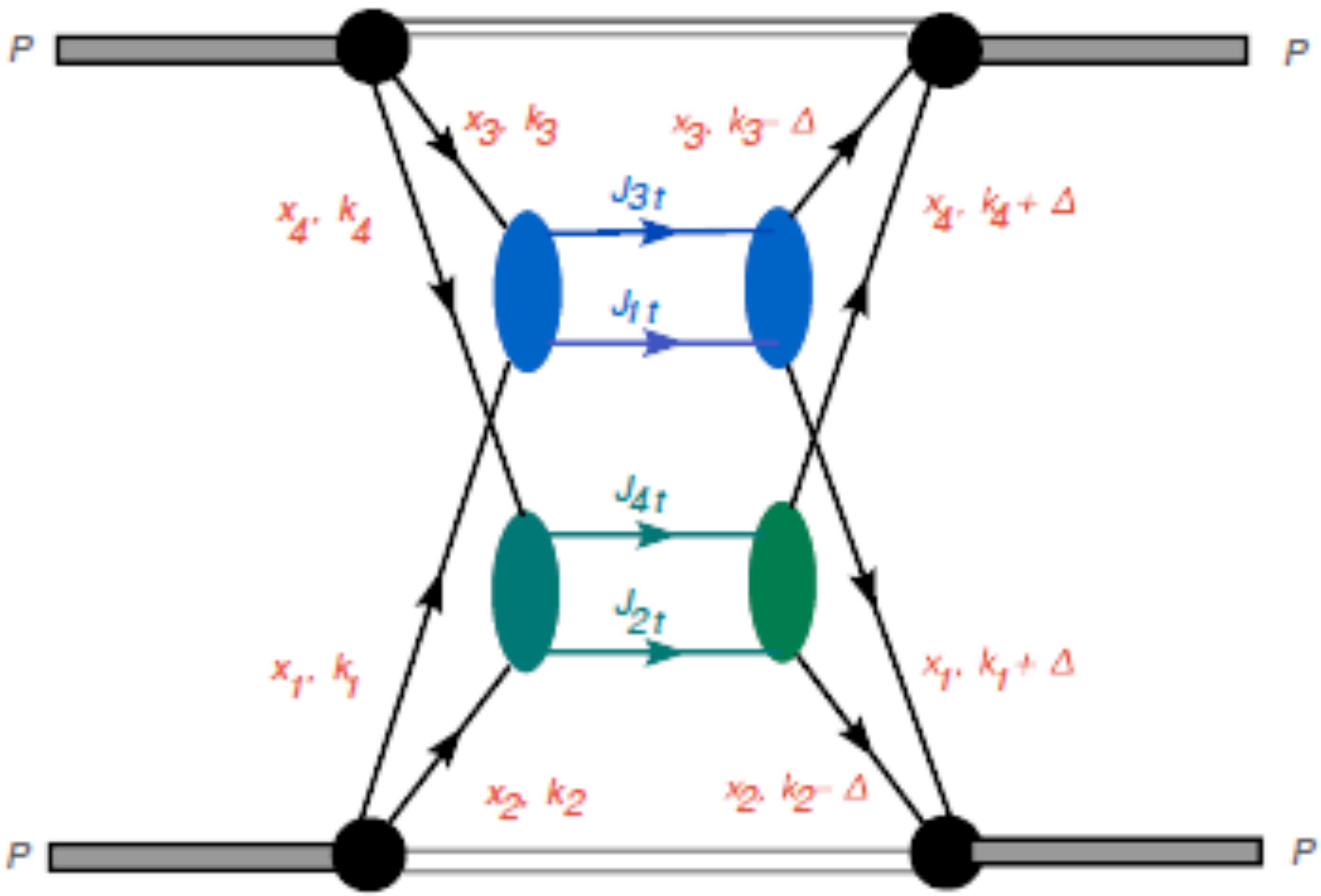
$\rho_1 + \rho_2 \Rightarrow k'_1 - k_1 = -(k'_2 - k_2) \equiv \Delta$

$\rho_3 + \rho_4 \Rightarrow k'_3 - k_3 = -(k'_4 - k_4) \equiv \widetilde{\Delta}$

$(\rho_1 - \rho_2) + (\rho_3 - \rho_4) \Rightarrow \Delta = -\widetilde{\Delta}$

$\delta\big((\rho_1 - \rho_2) - (\rho_3 - \rho_4)\big) \Rightarrow \vec{\Delta}$ ***arbitrary***

4-parton collision



In order to be able to trace the *relative distance between the partons*, one has to use the mixed *longitudinal momentum – impact parameter* representation which, in the momentum language, reduces to introduction of a **mismatch** between the transverse momentum of the parton in the ***amplitude*** and that of the same parton in the ***amplitude conjugated***.

4-parton cross section

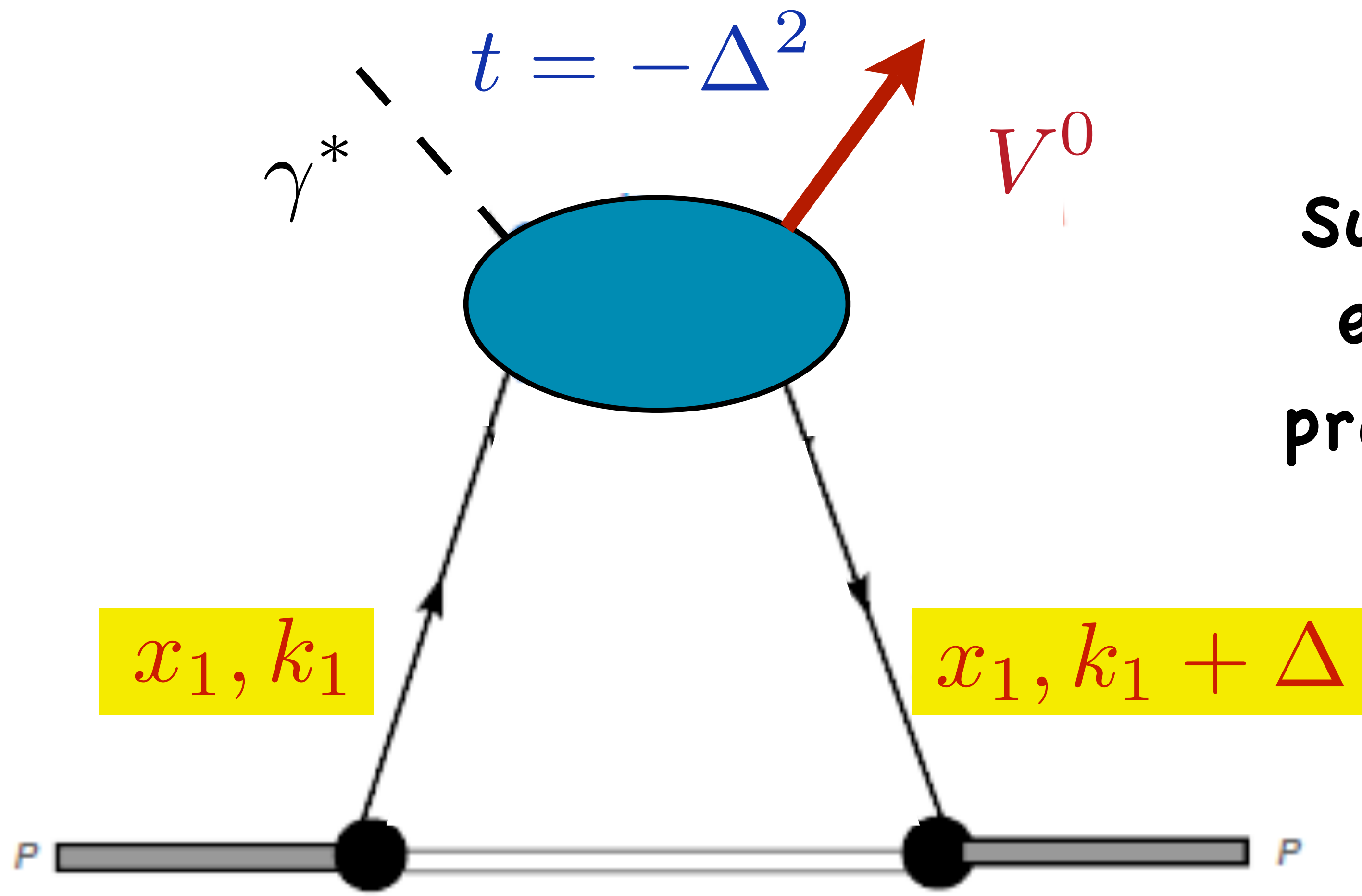
$$\frac{1}{S} = \int \frac{d^2\vec{\Delta}}{(2\pi)^2} D_a(x_1, x_2; \vec{\Delta}) \, D_b(x_3, x_4; -\vec{\Delta})$$

S - effective parton interaction area

$$\boxed{\frac{d\sigma(x_1, x_2, x_3, x_4)}{d\hat{t}_1 \, d\hat{t}_2} = \frac{d\sigma^{13}}{d\hat{t}_1} \frac{d\sigma^{24}}{d\hat{t}_2} \times \frac{1}{S}}$$

D is a ***generalized double parton distribution*** - a new object we know little about.

Can it be modeled, for lack of anything better ?



G P D

Such an amplitude describes exclusive photo-(/electro-) production of vector mesons at HERA !

Generalized parton distribution :

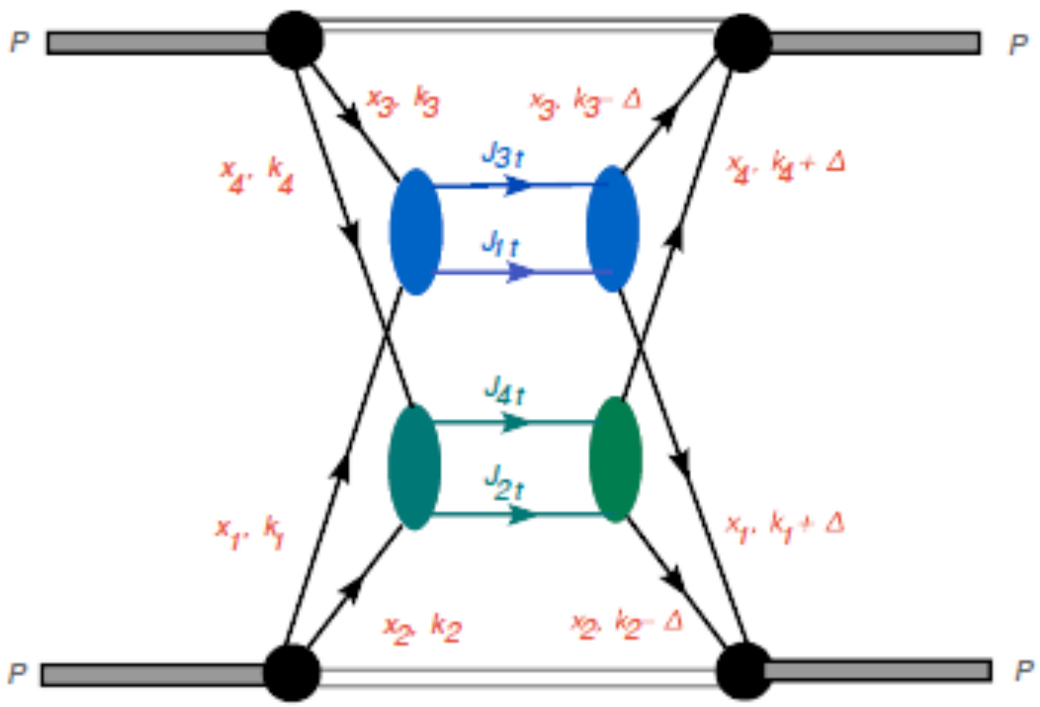
$$\boxed{G_N(x, Q^2, \vec{\Delta}) \;=\; G_N(x, Q^2) F_{2g}(\Delta)}$$

G - the usual 1-parton distribution (determining DIS structure functions)

F - the two-gluon form factor of the nucleon

the dipole fit : $$F_{2g}(\Delta) \simeq \frac{1}{\left(1 + \Delta^2 / m_g^2\right)^2}$$

$m_g^2(x \sim 0.03, Q^2 \sim 3\mathrm{GeV}^2)$
$\simeq 1.1\mathrm{GeV}^2$



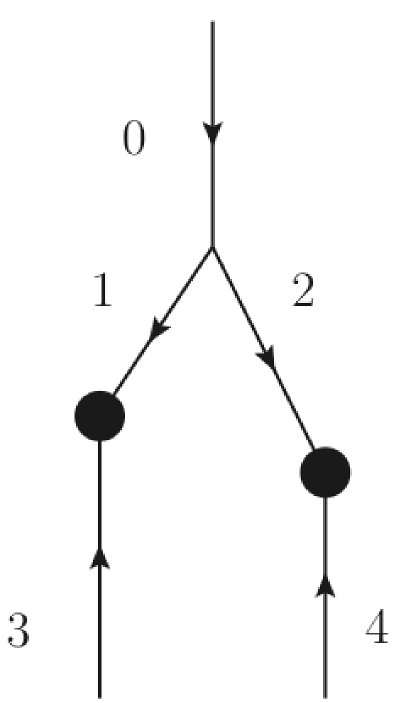
If partons were ***uncorrelated***, we could write

$$D(x_1, x_2, p_1^2, p_2^2, \vec{\Delta}) = G(x_1, p_1^2, \vec{\Delta}) G(x_2, p_2^2, \vec{\Delta})$$

and use the dipole fit to get the estimate

$$\frac{D(x_1, x_2, -\vec{\Delta}) D(x_3, x_4, \vec{\Delta})}{D(x_1) D(x_2) D(x_3) D(x_4)} \simeq F_{2g}^4(\Delta)$$

The "interaction area" : $\Rightarrow$ $$\int \frac{d^2\Delta}{(2\pi)^2} F_g^2(\Delta^2) \times F_g^2(\Delta^2) = \frac{m_g^2}{7\pi}$$



Another mechanism : 2 partons from a short-range PT correlation

No Δ —dependence from the upper side ! $\Rightarrow$ $$\int \frac{d^2\Delta}{(2\pi)^2} F_g^2(\Delta^2) = \frac{m_g^2}{3\pi}$$

3-> 4 contribution vs. **4-> 4** is enhanced by a factor

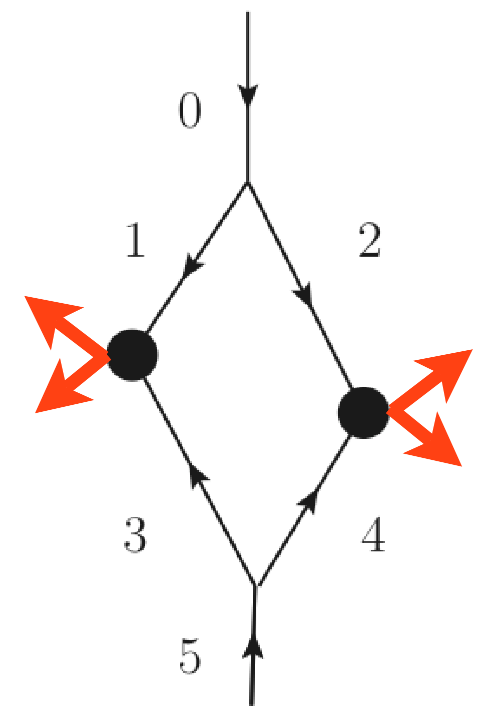
$$2 \times \frac{7}{3} \simeq 5$$

2 -> 4 processes

What if *both parton pairs* originate from PT splittings ?

No Δ —dependence whatsoever.   The integral diverges...?..

This is NOT an amplitude of a *4-parton collision*
but a one-loop correction to the *2-parton collision*



***4-parton interaction* is a "higher twist" effect**

hard 2-parton scattering : $$\frac{d\sigma^{(2\to 2)}}{d\hat{t}} \propto \frac{\alpha_s^2}{Q^4}$$

plus two additional jets : $$\frac{d\sigma^{(2\to 4)}}{d\hat{t}_1 d\hat{t}_2} \propto \frac{\alpha_s^4}{Q^6}$$

4 jets from 4-parton scattering : $$\frac{d\sigma^{(4\to 4)}}{d\hat{t}_1 d\hat{t}_2} \propto R^{-2} \cdot \left(\frac{\alpha_s^2}{Q^4}\right)^2 \propto \frac{\alpha_s^4}{R^2 Q^8}$$

extra $\frac{m_g^2}{Q^2}$

Always a *small contribution* to the *total 4-jet production cross section*

End of story?..   Not at all

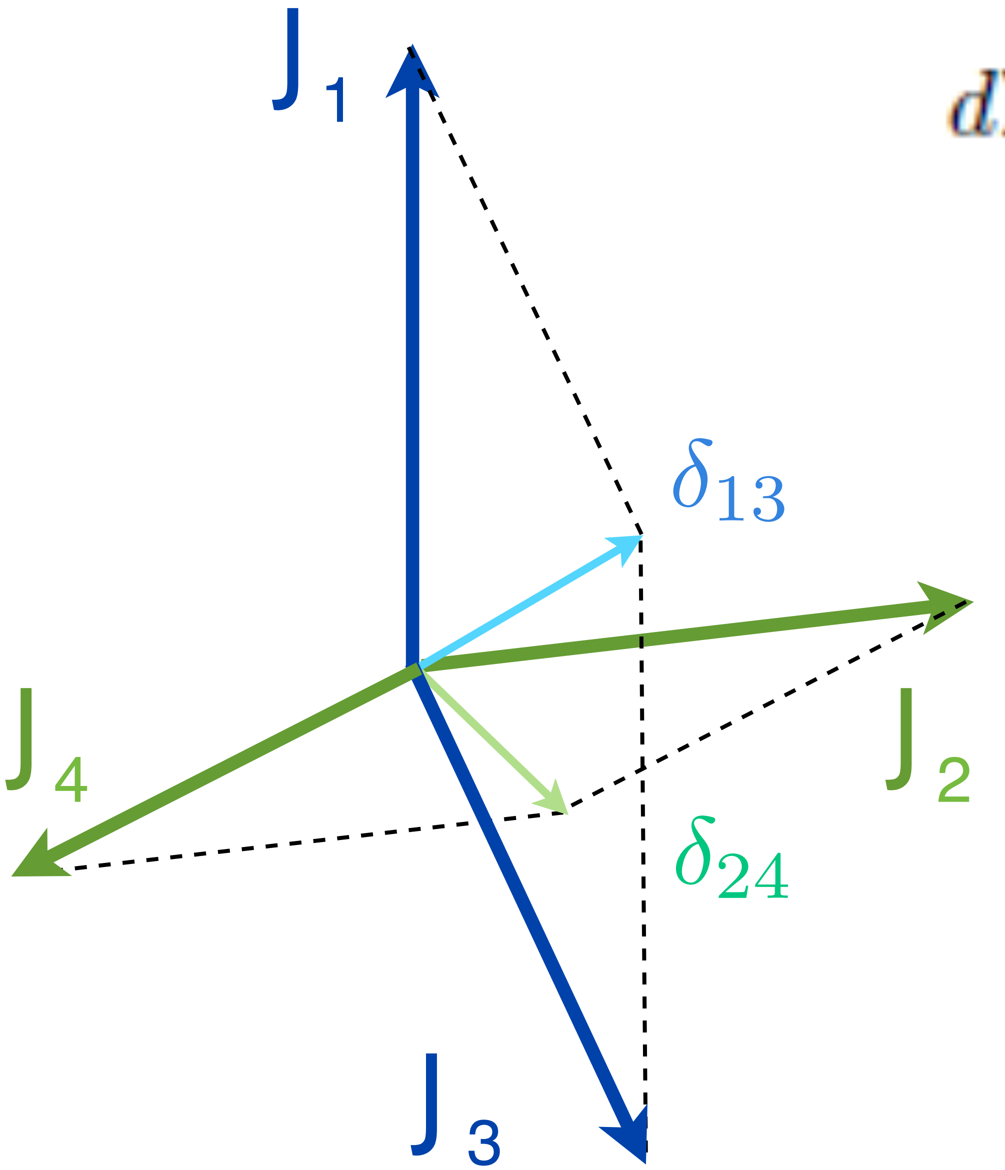
What distinguishes "double hard collisions" is the differential jet spectrum

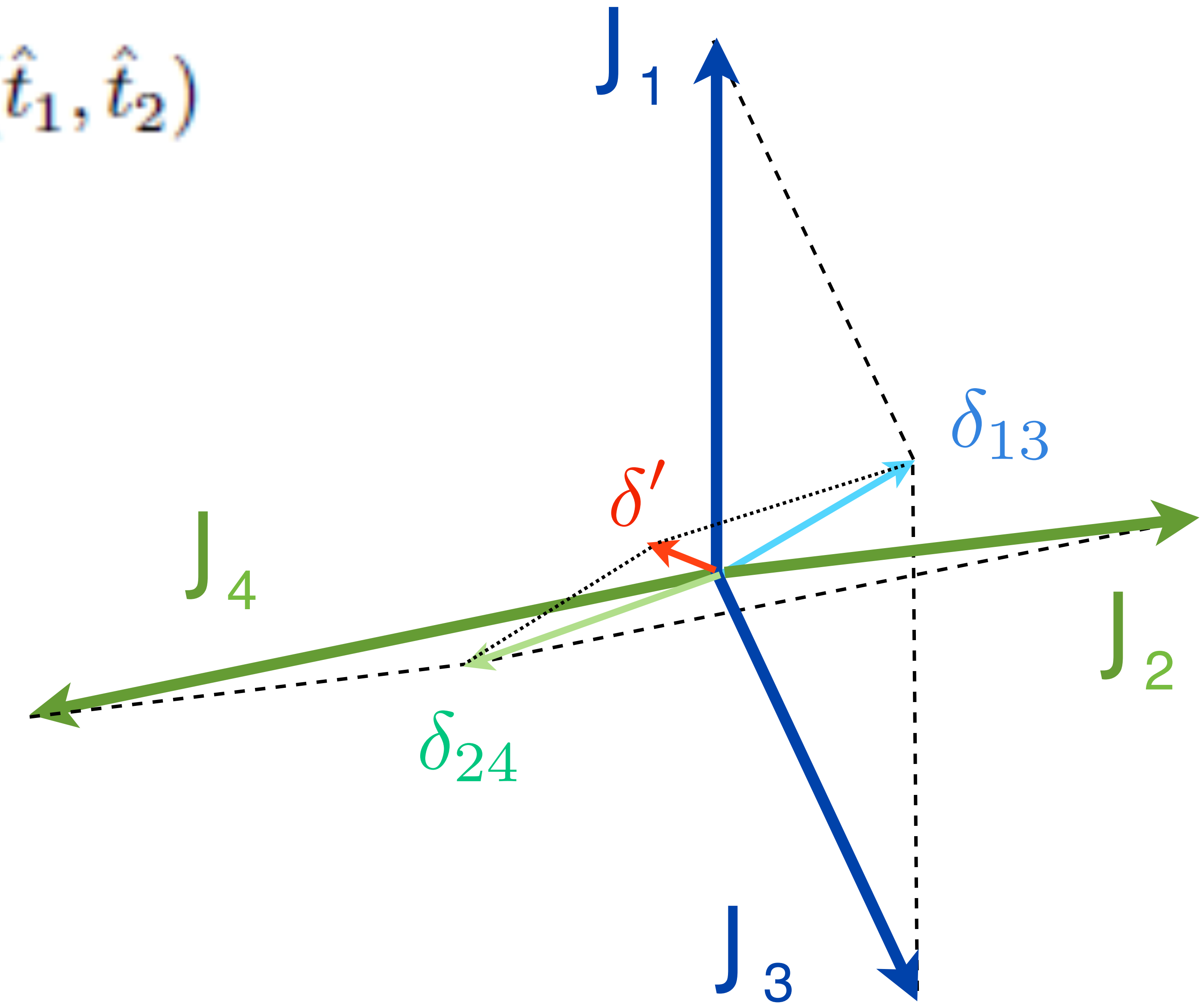
back-to-back kinematics

$\delta_{13}^2, \delta_{24}^2 \ll J_{i\perp}^2$

$\delta'^2 \ll \delta_{13}^2 \simeq \delta_{24}^2 \ll J_{i\perp}^2$

$d\Sigma(\hat{t}_1, \hat{t}_2)$

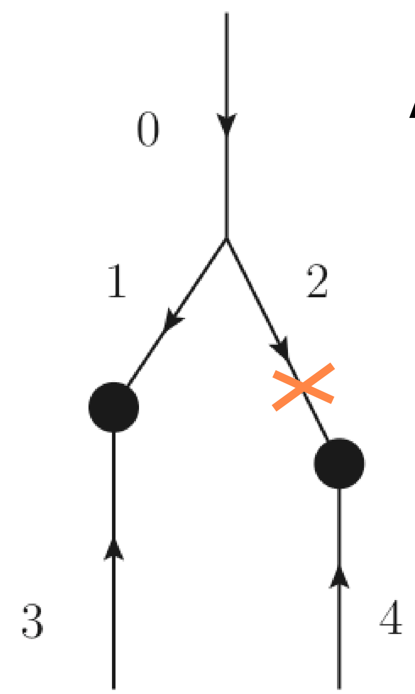




$$d\sigma^{(4\to4)} \propto \frac{\alpha_s^2}{\delta_{13}^2\,\delta_{24}^2}\, d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma$$

$$d\sigma^{(3\to4)} \propto \frac{\alpha_s^2}{\delta'^2\,\delta^2}\, d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma$$

Underwater stones of the MPI analysis

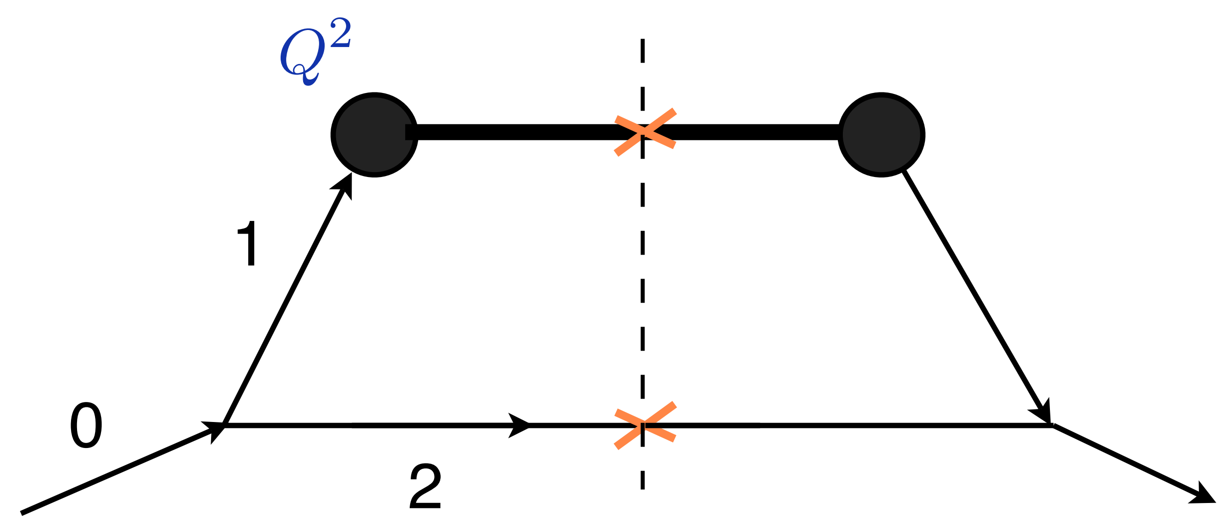


A tree Feynman diagram.   Momenta of internal parton lines are fixed ...

not anymore

Singularities in the physical region of parton momenta !

Return to a good old single hard interaction picture :



In DIS we trace the fate of **1** but *integrate* over "histories" of the accompanying parton **2**.

Now we want #**2** to enter 2nd hard interaction.

In the above picture it does it "*in the next room*".   Or on the Moon, for that matter ...

In fact, partons **3** and **4** ***cannot be*** represented by plainly independent ***plane waves***: they belong to ***one hadron***, and therefore, are ***localized within the hadron pancake***...

Remedy: introduce wave packet smearing (longitudinal momentum fraction integral).

Importantly, this has to be done at the ***amplitude level*** !

$k_{3+} + k_{4+}$ fixed by hard scattering kinematics

$k_{3+} - k_{4+}$ arbitrary

Drell-Yan process

Massive lepton pair production cross section

$$\frac{d\sigma}{dq^2\, dq_\perp^2} = \frac{d\sigma_{\rm tot}}{dq^2} \quad \times \frac{\partial}{\partial q_\perp^2}\left\{ D_a^q\left(x_1, q_\perp^2\right) D_b^q\left(x_2, q_\perp^2\right) S_q^2\left(q^2, q_\perp^2\right) \right\}$$

Quark form factor :

$$S_q(Q^2,\kappa^2) \;=\; \exp\left\{ -\int_{\kappa^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_0^{1-k/Q} dz\, P_q^q(z) \right\}$$

Gluon form factor :

$$S_g(Q^2,\kappa^2) \;=\; \exp\left\{ -\int_{\kappa^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_0^{1-k/Q} dz\, \left[z P_g^g(z) + n_f P_g^q(z) \right] \right\}$$

Parton splitting probabilities

$$P_q^q(z) = C_F \frac{1+z^2}{1-z}, \qquad P_q^g(z) = P_q^q(1-z),$$

$$P_g^q(z) = T_R\left[z^2 + (1-z)^2\right], \qquad P_g^g(z) = C_A \frac{1+z^4+(1-z)^4}{z(1-z)}$$

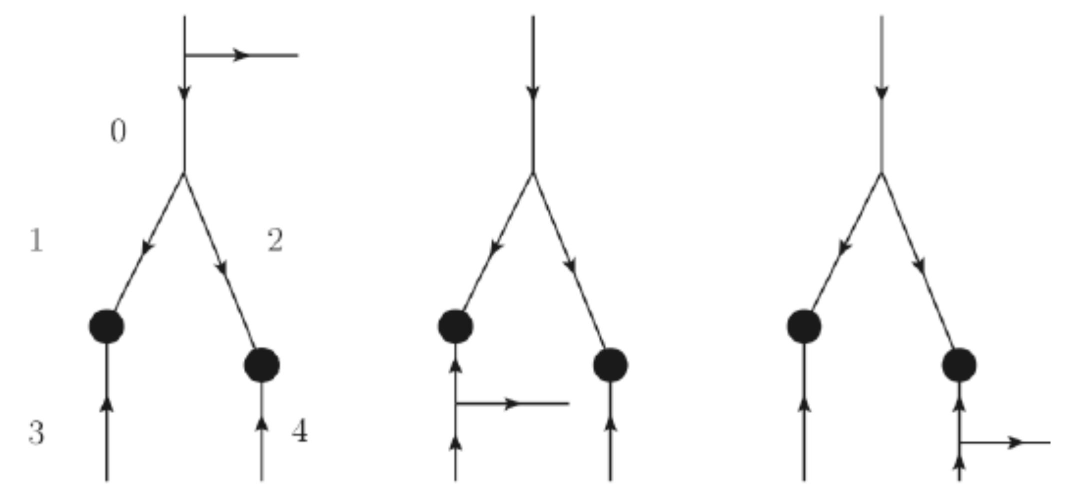
Generalization of the DDT-formula for back-to-back 4-jet production spectrum

$$\pi^2 \frac{d\sigma^{(4\to4)}}{d^2\delta_{13}\, d^2\delta_{24}} = \frac{d\sigma_{\rm part}}{d\hat{t}_1\, d\hat{t}_2} \cdot \frac{\partial}{\partial\delta_{13}^2}\frac{\partial}{\partial\delta_{24}^2}\Big\{ {}_{[2]}D_a^{1,2}(x_1,x_2;\delta_{13}^2,\delta_{24}^2) \times {}_{[2]}D_b^{3,4}(x_3,x_4;\delta_{13}^2,\delta_{24}^2)$$
$$\times\; S_1\left(Q^2,\delta_{13}^2\right) S_3\left(Q^2,\delta_{13}^2\right) \times S_2\left(Q^2,\delta_{24}^2\right) S_4\left(Q^2,\delta_{24}^2\right) \Big\}$$

Not forgetting the Δ—integration and short-range correlations :

$${}_{[2]}D_a \times {}_{[2]}D_b \;+\; {}_{[2]}D_a \times {}_{[1]}D_b \;+\; {}_{[1]}D_a \times {}_{[2]}D_b$$

Additional 3 -> 4 contribution :



$$\frac{\pi^2\, d\sigma_2^{(3\to4)}}{d^2\delta_{13}\, d^2\delta_{24}} = \frac{d\sigma_{\rm part}}{d\hat{t}_1\, d\hat{t}_2} \cdot \frac{\alpha_s(\delta^2)}{2\pi\,\delta^2} \sum_c P_c^{1,2}\left(\frac{x_1}{x_1+x_2}\right)$$

$$S_1(Q^2,\delta^2)\, S_2(Q^2,\delta^2)\, \frac{\partial}{\partial\delta'^2}\left\{ S_c(\delta^2,\delta'^2)\frac{G_a^c(x_1+x_2;\delta'^2,Q_0^2)}{x_1+x_2} S_3(Q^2,\delta'^2) S_4(Q^2,\delta'^2) \times {}_{[2]}D_b^{3,4}(x_3,x_4;\delta'^2,\delta'^2)\right\}$$

Effective interaction areas for 4 -> 4 and 3 -> 4 collisions

$$\boxed{\frac{d\sigma(x_1,x_2,x_3,x_4)}{d\hat{t}_1\,d\hat{t}_2} = \frac{d\sigma^{13}}{d\hat{t}_1}\frac{d\sigma^{24}}{d\hat{t}_2} \quad \times \frac{1}{S}}$$

$$S_4^{-1}(x_1,x_2,x_3,x_4;Q^2) = \int \frac{d^2\Delta}{(2\pi)^2}\Big\{{}_{[2]}D_a(x_1,x_2;Q^2,Q^2;\vec{\Delta})\,{}_{[2]}D_b(x_3,x_4;Q^2,Q^2;-\vec{\Delta})$$

$$+\;{}_{[2]}D_a(x_1,x_2;Q^2,Q^2;\vec{\Delta})\,{}_{[1]}D_b(x_3,x_4;Q^2,Q^2;-\vec{\Delta}) + {}_{[1]}D_a(x_1,x_2;Q^2,Q^2;\vec{\Delta})\,{}_{[2]}D_b(x_3,x_4;Q^2,Q^2;-\vec{\Delta})\Big\}$$

$$S_3^{-1}(x_1,x_2,x_3,x_4;Q^2) = \sum_c \int \frac{d^2\Delta}{(2\pi)^2} P_c^{1,2}\left(\frac{x_1}{x_1+x_2}\right)\int^{Q^2}\frac{d\delta^2}{\delta^2}\frac{\alpha_s(\delta^2)}{2\pi}\prod_{i=1}^{4}S_i(Q^2,\delta^2)$$

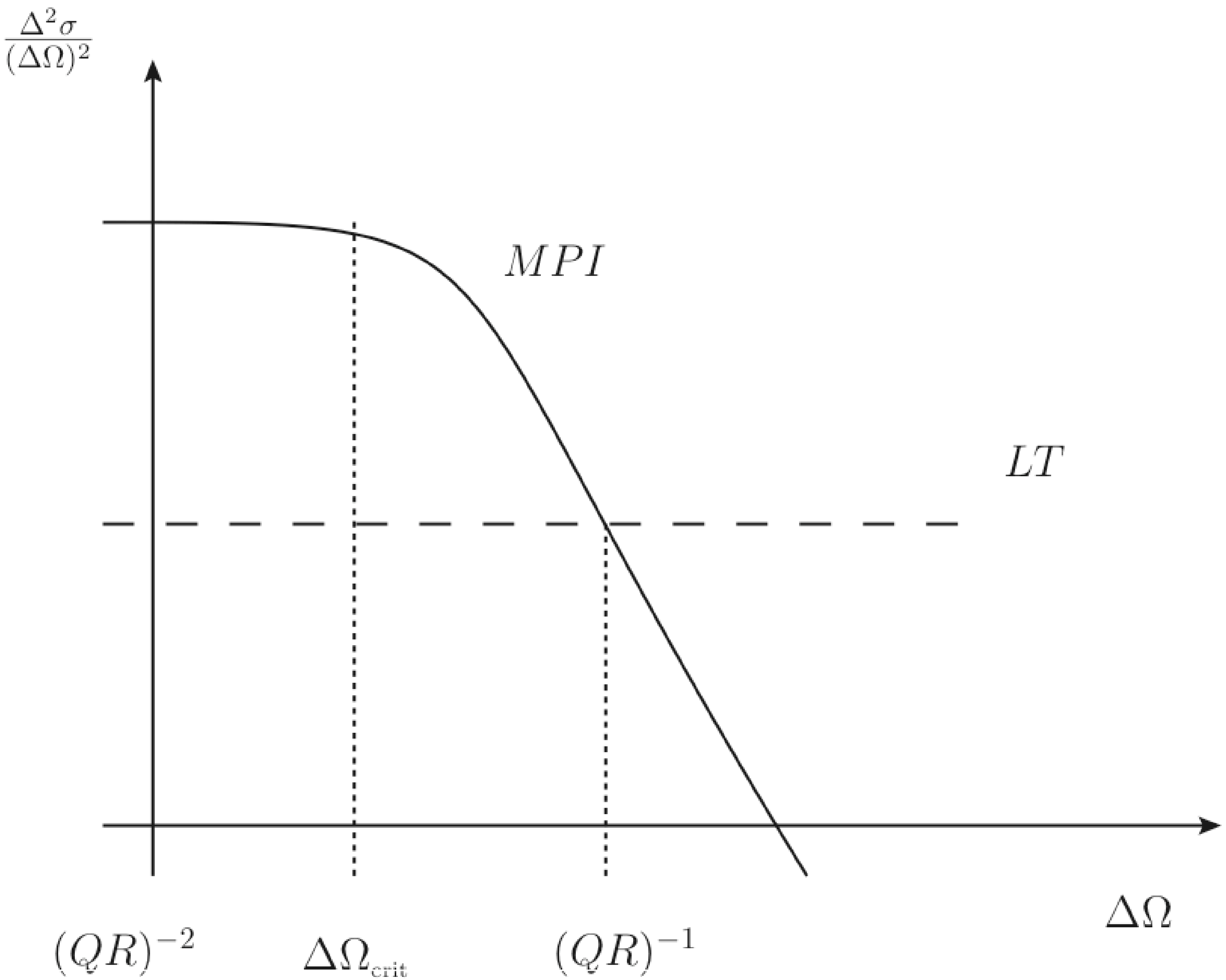
$$\times\,\frac{G_a^c(x_1+x_2,\delta^2,Q_0^2)}{x_1+x_2}\,{}_{[2]}D_b^{3,4}(x_3,x_4;\delta^2,\delta^2;\vec{\Delta}) \qquad + \;\; (a\leftrightarrow b;1,2\leftrightarrow 3,4)$$

domain of 4-parton interaction dominance

2 -> 4 processes produce “hedgehogs”

4 -> 4 and **3 -> 4** produce two pairs of ***anti-collimated*** jets

4-parton interactions dominate when the backward jet solid angles are taken small



Flattening of the spectrum due to *multiple gluons radiation* (Sudakov form factor) effect

$$\boxed{\Delta\Omega_{\rm crit} \simeq \left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)^{1-\exp\{-\beta_2/2\Sigma_C\}}}$$

for collision of two *gluons*

$$\Delta\Omega_{\rm crit} \simeq \left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)^{0.78}$$

punchline

- Multi-parton collisions is a dominant source of 4 jets in the back-to-back kinematics

- 4->4 and 3->4 parton subprocesses are both **enhanced** in the back-to-back region, while perturbative parton splittings generate effectively 2->4, which is **not**

- To describe multi-parton collisions one has to introduce and explore a new object - Generalized Double-Parton Distributions

$$ {}_{[2]}D_h^{a,b}(x_1, x_2; q_1^2, q_2^2; \vec{\Delta}\,) $$

→ the parameter $\vec{\Delta}$ encodes the information about the impact-parameter-space correlation between the two partons from one hadron

- experimentally observed enhancement of a 4-jet cross section indicates the presence of short range two-parton correlations in the nucleon parton wave function, as determined by the range of integral over $\vec{\Delta}$

- the quest for understanding the nature of ${}_2$GPDs calls for new ideas