Phenomenological applications of QCD threshold resummation

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QCD threshold resummation:

- Important applications at LHC: "precision QCD" (see talks of previous weeks)
- Today: discuss a few phenomenological applications towards lower energies: Tevatron, RHIC, fixed target
- Here, focus is to achieve quantitative description of observables

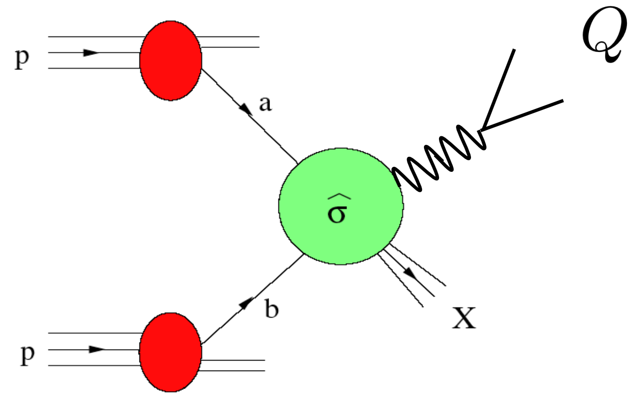
Outline:

- Introduction
- W boson production at RHIC
- Drell-Yan process in πN scattering
- Hadron pair production in pp collisions
- Top quark charge asymmetry at the Tevatron

Focus on phenomenology, less on technical aspects of resummation

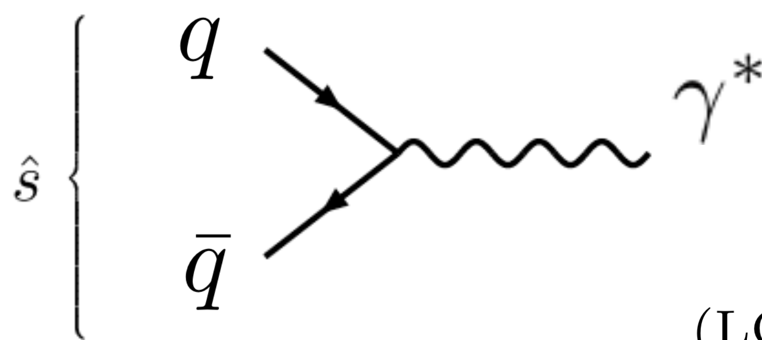
Introduction

The archetype: Drell-Yan



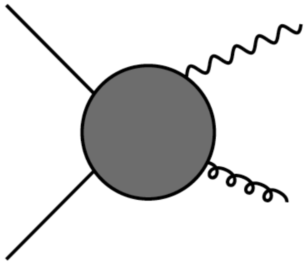
$$Q^2 d\sigma = \sum_{ab} \int dx_a dx_b \, f_a(x_a,\mu) \, f_b(x_b,\mu) \, \omega_{ab} \left(z = \frac{Q^2}{\hat{s}}, \alpha_s(\mu), \frac{Q}{\mu} \right) + \ldots$$

LO :



$$\omega_{ab}^{(\mathrm{LO})} \propto \delta(1-z)$$

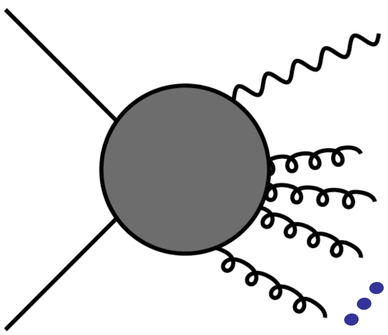
- NLO correction:



$$z \longrightarrow 1:$$

$$\omega_{ab}^{(\mathrm{NLO})} \propto \alpha_s \left(\frac{\log(1-z)}{1-z} \right)_+ + \ldots$$

- higher orders:



$$\omega_{ab}^{(\mathrm{N^kLO})} \propto \alpha_s^k \left(\frac{\log^{2k-1}(1-z)}{1-z} \right)_+ + \ldots$$

"threshold logarithms"

- for z->1 real radiation inhibited

- logs emphasized by parton distributions :

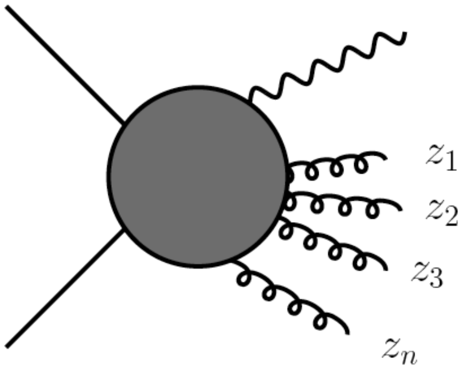
$$d\sigma \sim \int_{\tau}^{1} \frac{dz}{z}\, \mathcal{L}_{q\bar{q}}\left(\frac{\tau}{z}\right) \omega_{q\bar{q}}(z) \qquad \tau = \frac{Q^2}{S}$$

z = 1 relevant,
in particular as $\tau \rightarrow 1$

Large logs can be resummed to all orders

Catani, Trentadue; Sterman; …

- factorization of matrix elements
- and of phase space when integral transform is taken:



$$\delta\left(1 - z - \sum_{i=1}^{n} z_i\right) = \frac{1}{2\pi i}\int_C dN\, \mathrm{e}^{N\left(1-z-\sum_{i=1}^{n} z_i\right)}$$

$$z_i = \frac{2E_i}{\sqrt{\hat{s}}}$$

$\overline{\text{MS}}$ scheme

$$\hat{\sigma}^{\rm res}_{q\bar{q}}(N) \propto \exp\left[2\int_0^1 dy\, \frac{y^N - 1}{1-y} \int_{\mu^2}^{Q^2(1-y)^2} \frac{dk_\perp^2}{k_\perp^2} A_q\left(\alpha_s(k_\perp^2)\right) + \ldots \right]$$

$$A_q(\alpha_s) = C_F \left\{ \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{C_A}{2}\left(\frac{67}{18} - \zeta(2)\right) - \frac{5}{9} T_R n_f \right] + \ldots \right\}$$

- they enhance cross section !

Catani, Mangano, Nason, Trentadue

to NLL (much more is known):

$$\hat{\sigma}^{\mathrm{res}}_{q\bar{q}} \;\propto\; \exp\left\{2\ln\bar{N}\; h^{(1)}(\lambda) + 2h^{(2)}\left(\lambda, \frac{Q^2}{\mu^2}\right)\right\}$$

$$\lambda = \alpha_s(\mu^2)\, b_0\, \log(N\mathrm{e}^{\gamma_E})$$

$$h^{(1)}(\lambda) \;=\; \frac{A_q^{(1)}}{2\pi b_0 \lambda}\left[2\lambda + (1-2\lambda)\ln(1-2\lambda)\right]$$

$$h^{(2)}\left(\lambda, \frac{Q^2}{\mu^2}\right) \;=\; -\frac{A_q^{(2)}}{2\pi^2 b_0^2}\left[2\lambda + \ln(1-2\lambda)\right] + \frac{A_q^{(1)}}{2\pi b_0^3}\left[2\lambda + \ln(1-2\lambda) + \frac{1}{2}\ln^2(1-2\lambda)\right]$$

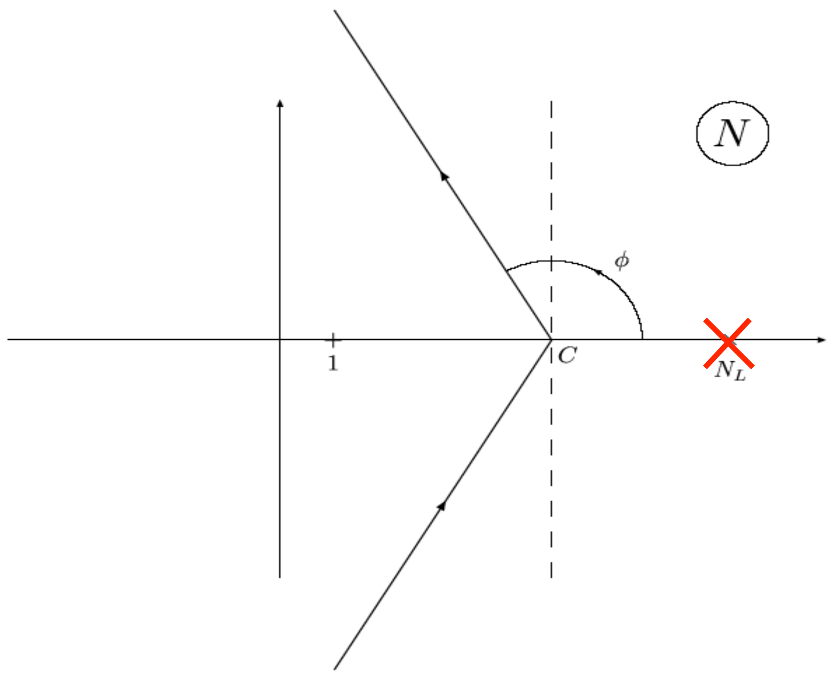
$$+\frac{A_q^{(1)}}{2\pi b_0}\left[2\lambda + \ln(1-2\lambda)\right]\ln\frac{Q^2}{\mu^2} - \frac{A_q^{(1)}\alpha_s(\mu^2)}{\pi}\;\ln\bar{N}\;\ln\frac{Q^2}{\mu^2}$$

Inverse transform:

$$\sigma^{\rm res} = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} dN\,\tau^{-N}\;\tilde{\sigma}^{\rm res}(N)$$

"Minimal prescription"

Catani, Mangano, Nason, Trentadue



"Matching" to NLO: $\sigma = \sigma^{\rm res} + \left[\sigma^{\rm NLO} - (\sigma^{\rm res})_{\rm NLO}\right]$

W boson production at RHIC

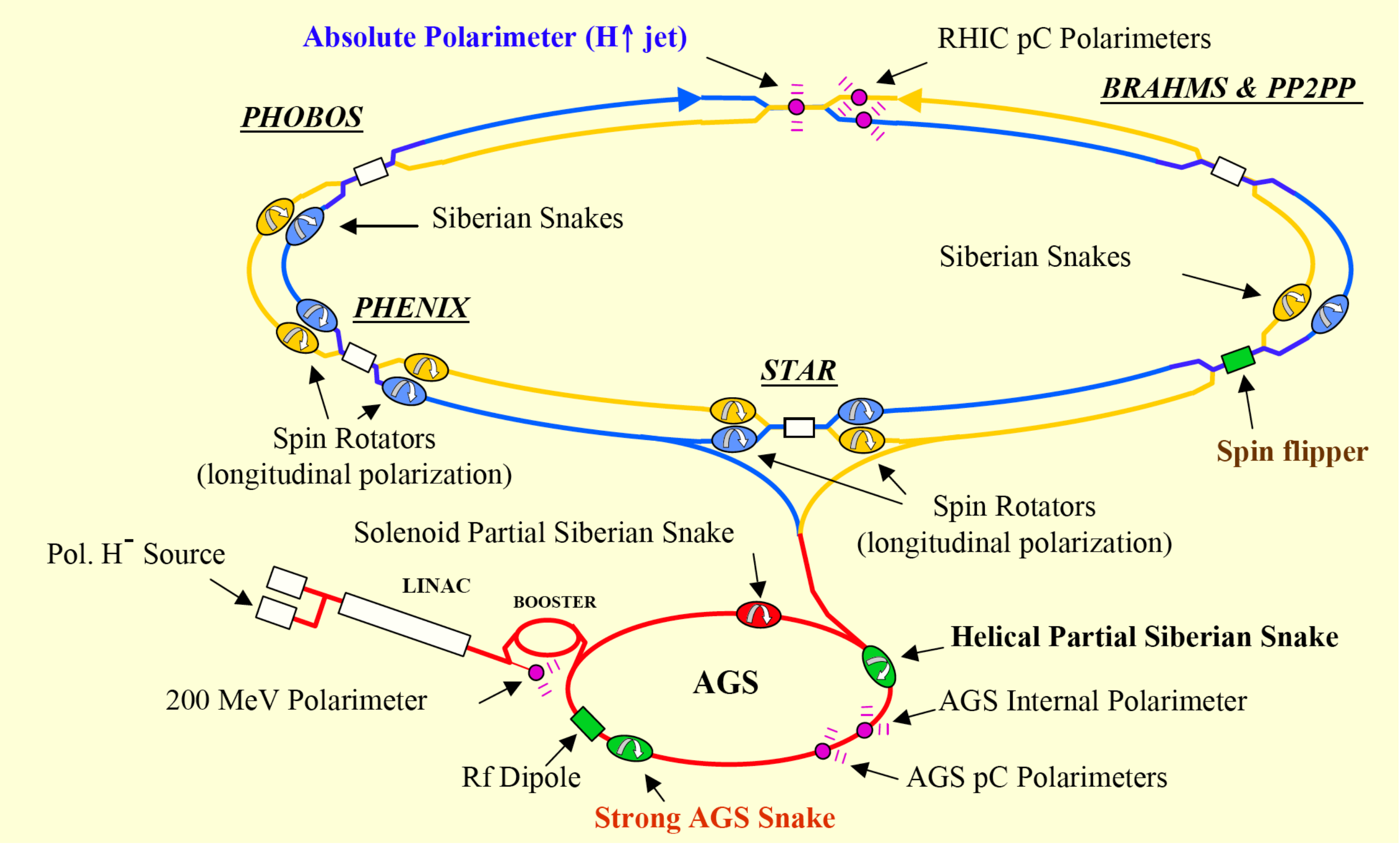
A. Mukherjee, WV

Polarized pp collider RHIC

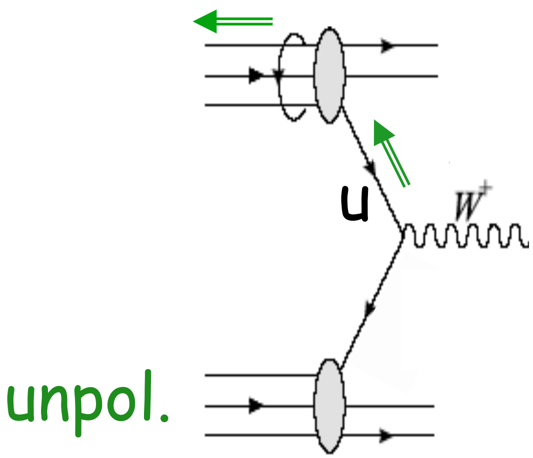
BNL

PHENIX, STAR

$\sqrt{s} = 200,\ 500\ \mathrm{GeV}$



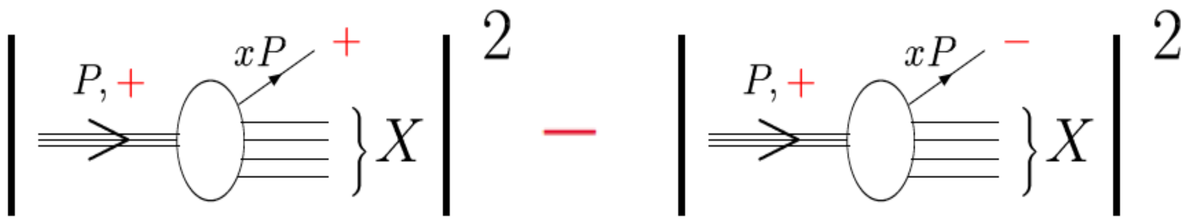
W boson production:



$$\sqrt{s} = 500 \text{ GeV}$$

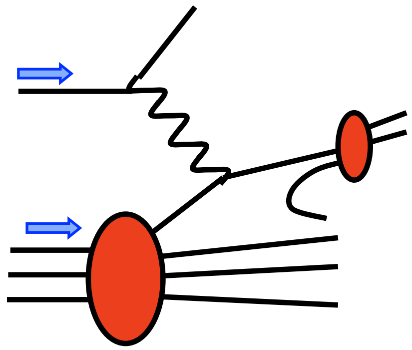
- goal: probe proton's *helicity distributions* $\Delta u, \Delta d, \Delta \bar{u}, \Delta \bar{d}$

$$\Delta q(x) =$$

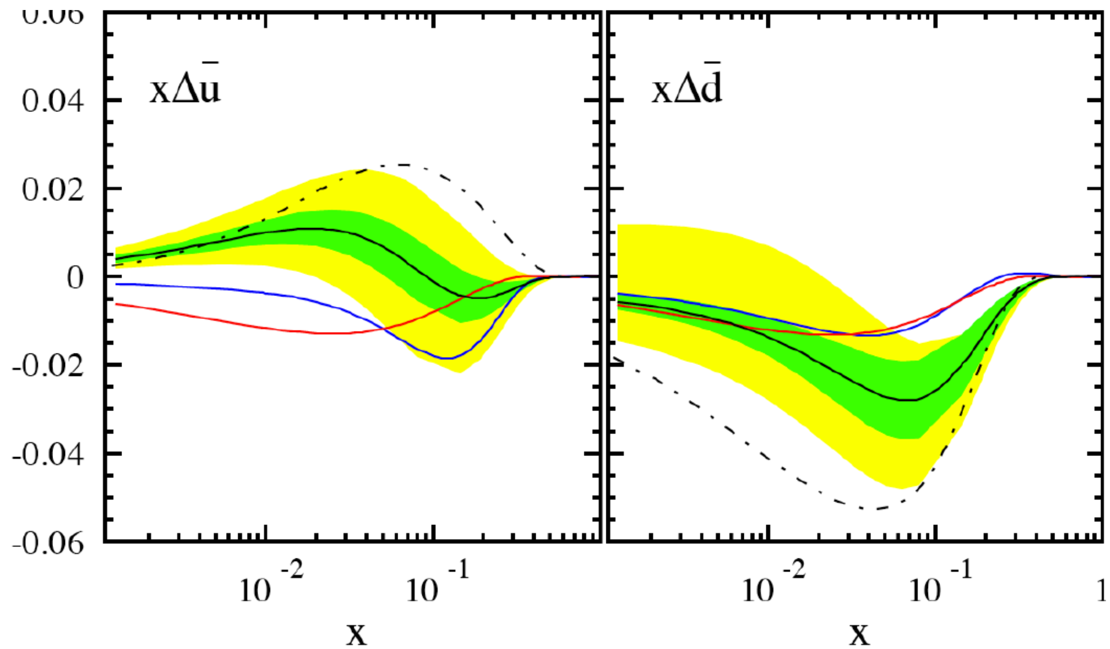


- use Parity Violation: $A_L = \dfrac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} \neq 0$

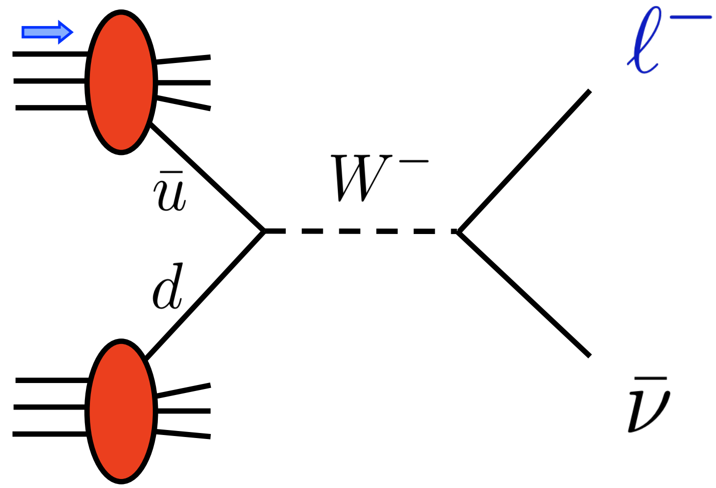
- so far, obtained from SIDIS:



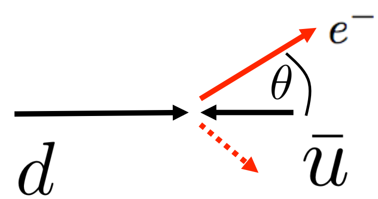
DSSV: de Florian, Sassot, Stratmann, WV



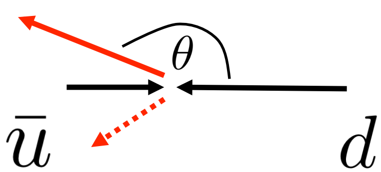
- insight into QCD via models (large-N_c, chiral quark, meson cloud,...)



$$A_L^{e^-} \sim \frac{\Delta\bar{u}(x_1)d(x_2)(1-\cos\theta)^2 - \Delta d(x_1)\bar{u}(x_2)(1+\cos\theta)^2}{\bar{u}(x_1)d(x_2)(1-\cos\theta)^2 + d(x_1)\bar{u}(x_2)(1+\cos\theta)^2}$$



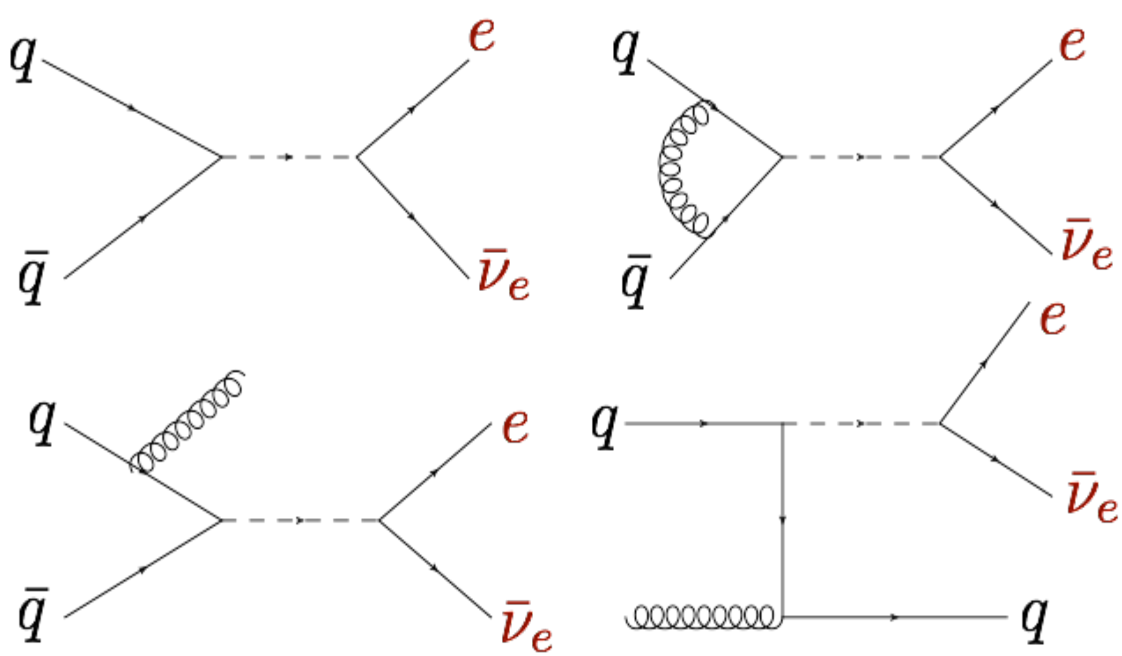
$$\sim (1+\cos\theta)^2$$



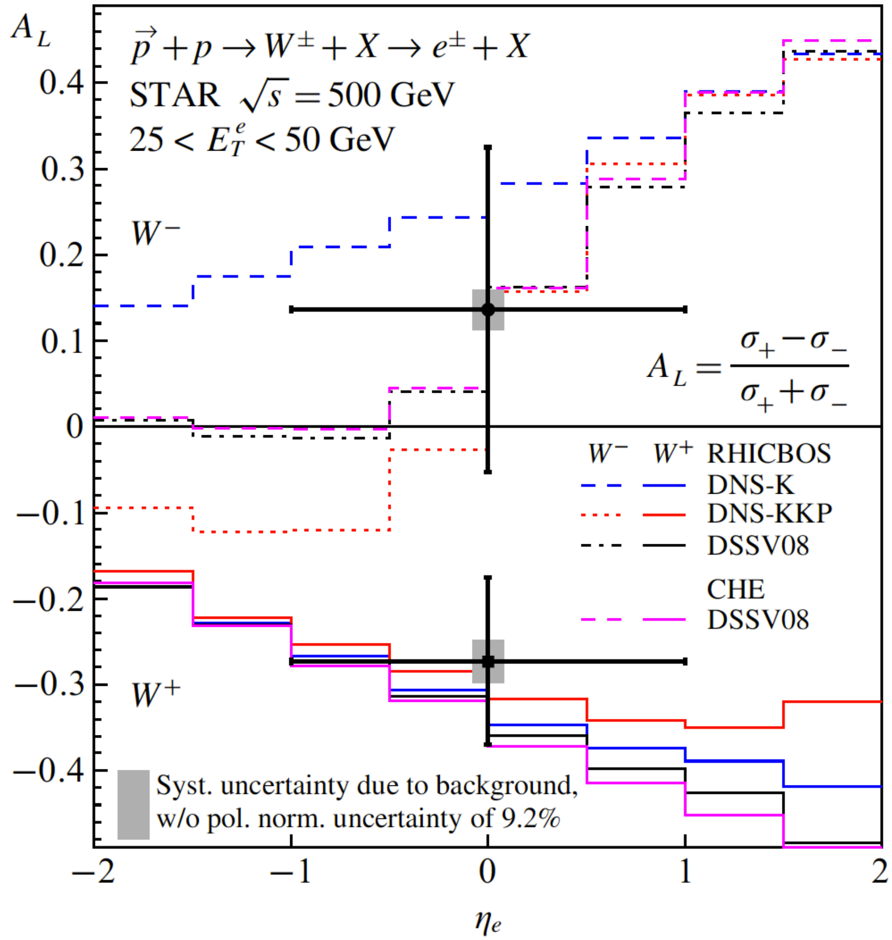
$$\sim (1-\cos\theta)^2$$

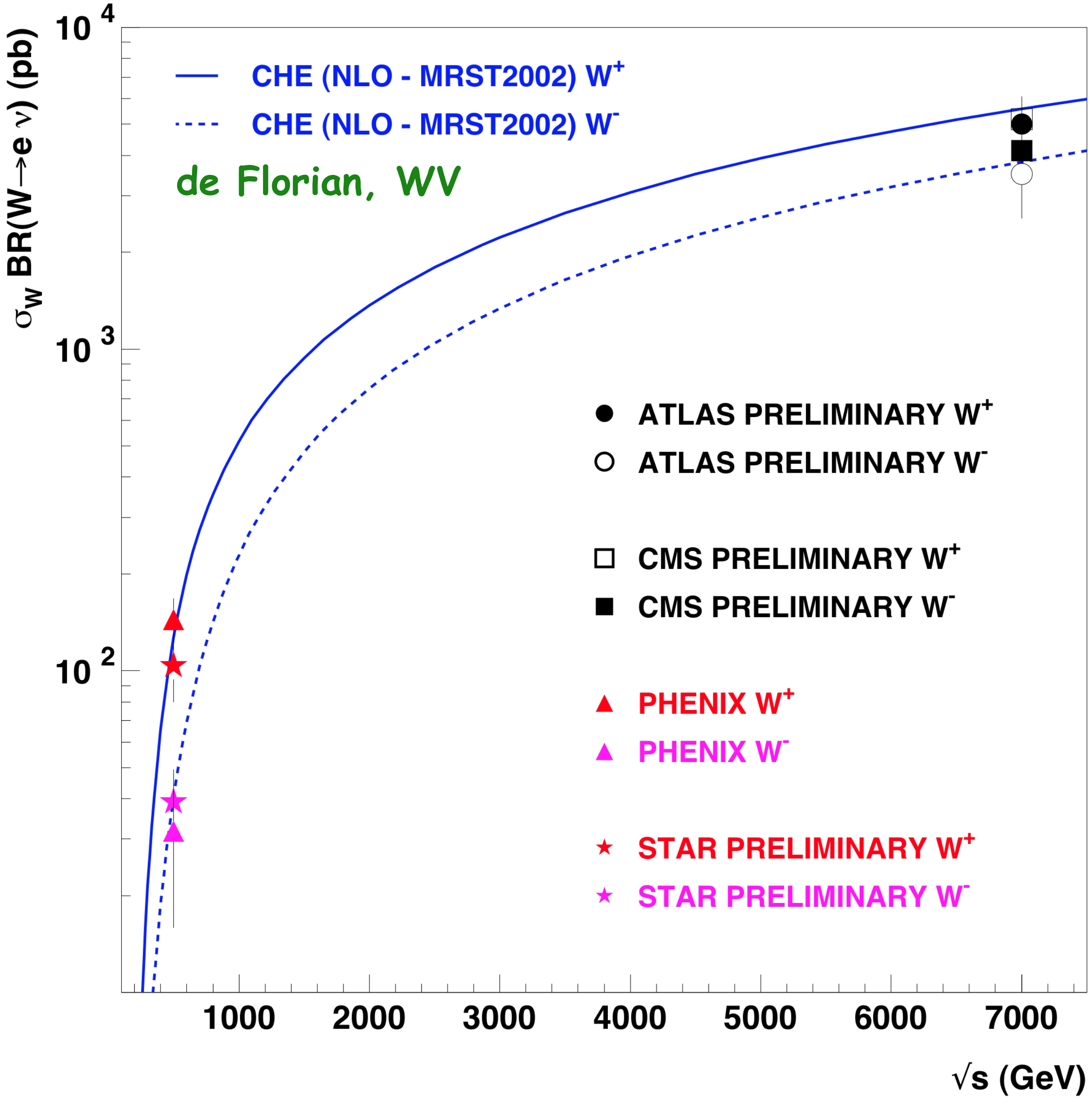
Recent NLO calculation:

de Florian, WV

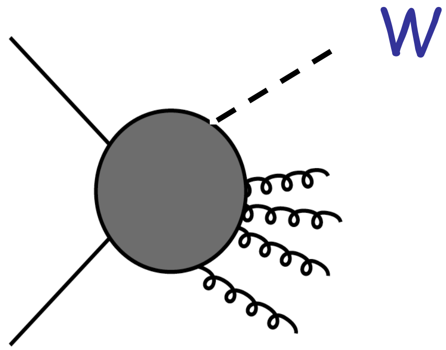


STAR (also Phenix)





B. Surrow (STAR)



$$\frac{M_W}{\sqrt{S}}$$ moderately large

$$\frac{d\sigma}{d\eta} = \mathcal{N} \sum_{i,j} \int_{x_1^0}^1 \frac{dx_1}{x_1} \int_{x_2^0}^1 \frac{dx_2}{x_2} \quad \mathcal{D}_{ij}\left(\frac{x_1^0}{x_1}, \frac{x_2^0}{x_2}, \alpha_s(\mu^2), \frac{M_W^2}{\mu^2}\right) \quad f_i(x_1, \mu^2)\, f_j(x_2, \mu^2)$$

$$x_{1,2}^0 = \frac{M_W}{\sqrt{S}}\, e^{\pm\eta}$$

$$\mathcal{D}_{ij} = \mathcal{D}_{ij}^{(0)} + \frac{\alpha_s}{2\pi}\mathcal{D}_{ij}^{(1)} + \ldots$$

$$\mathcal{D}_{q\bar{q}}^{(0)} = \delta\left(1 - \frac{x_1^0}{x_1}\right)\delta\left(1 - \frac{x_2^0}{x_2}\right)$$

$$\tilde{\sigma}(N,\nu) \equiv \int_0^1 d\tau\, \tau^{N-1} \int_{-\ln\frac{1}{\sqrt{\tau}}}^{\ln\frac{1}{\sqrt{\tau}}} d\eta\, \mathrm{e}^{i\nu\eta} \frac{d\sigma}{d\eta} \qquad \tau = \frac{M_W^2}{S}$$

Introduce

$$z = \frac{M_W^2}{x_1 x_2 S} = \frac{\tau}{x_1 x_2} \qquad \hat{\eta} = \eta - \frac{1}{2}\ln\frac{x_1}{x_2}$$

$$\tilde{\sigma}(N,\nu) = \mathcal{N}\sum_{i,j} \int_0^1 dx_1 x_1^{N+i\nu/2-1} f_i(x_1) \int_0^1 dx_2\, x_2^{N-i\nu/2-1} f_j(x_2) \int_0^1 dz\, z^{N-1} \int_{-\ln\frac{1}{\sqrt{z}}}^{\ln\frac{1}{\sqrt{z}}} d\hat{\eta}\, \mathrm{e}^{i\nu\hat{\eta}}\, \mathcal{D}_{ij}$$

$$\equiv \mathcal{N}\sum_{i,j} f_i^{N+i\nu/2} f_j^{N-i\nu/2}\, \tilde{\mathcal{D}}_{ij}(N,\nu)$$

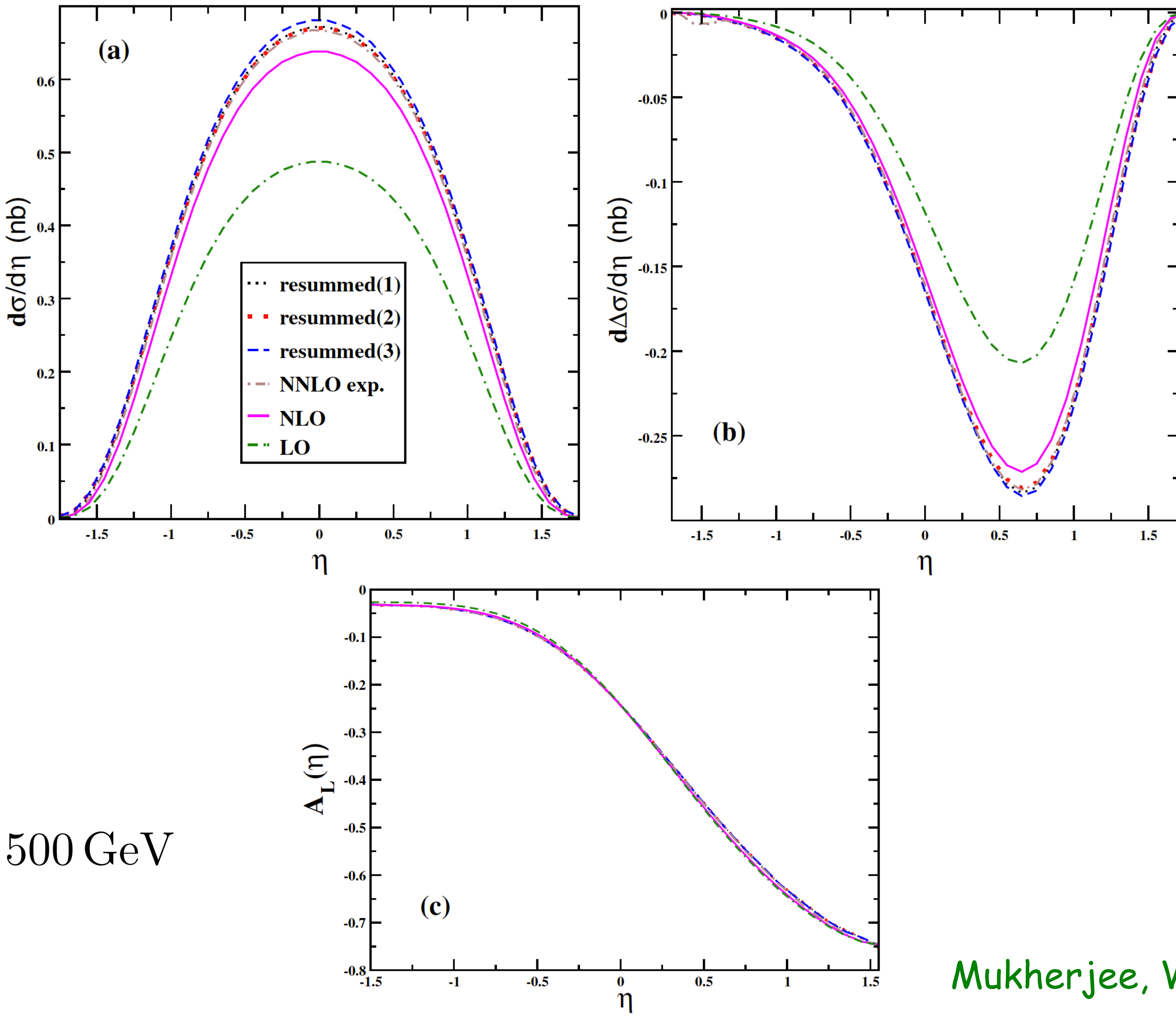
$$\mathcal{D}^{(0)}_{q\bar{q}} = \delta\left(1 - \frac{x_1^0}{x_1}\right)\delta\left(1 - \frac{x_2^0}{x_2}\right) \quad \longleftrightarrow \quad \delta(\hat{\eta})\,\delta(1-z)$$

No dependence on ν near threshold:

$$\tilde{\mathcal{D}}^{(\mathrm{res})}_{q\bar{q}}(N,\cancel{\nu}) = C_q\left(\alpha_s(\mu^2), \ln\frac{M_W^2}{\mu^2}\right)\exp\left\{2\int_0^1 d\zeta\,\frac{\zeta^{N-1}-1}{1-\zeta}\int_{\mu^2}^{(1-\zeta)^2 M_W^2}\frac{dk_T^2}{k_T^2}A_q(\alpha_s(k_T^2))\right\}$$

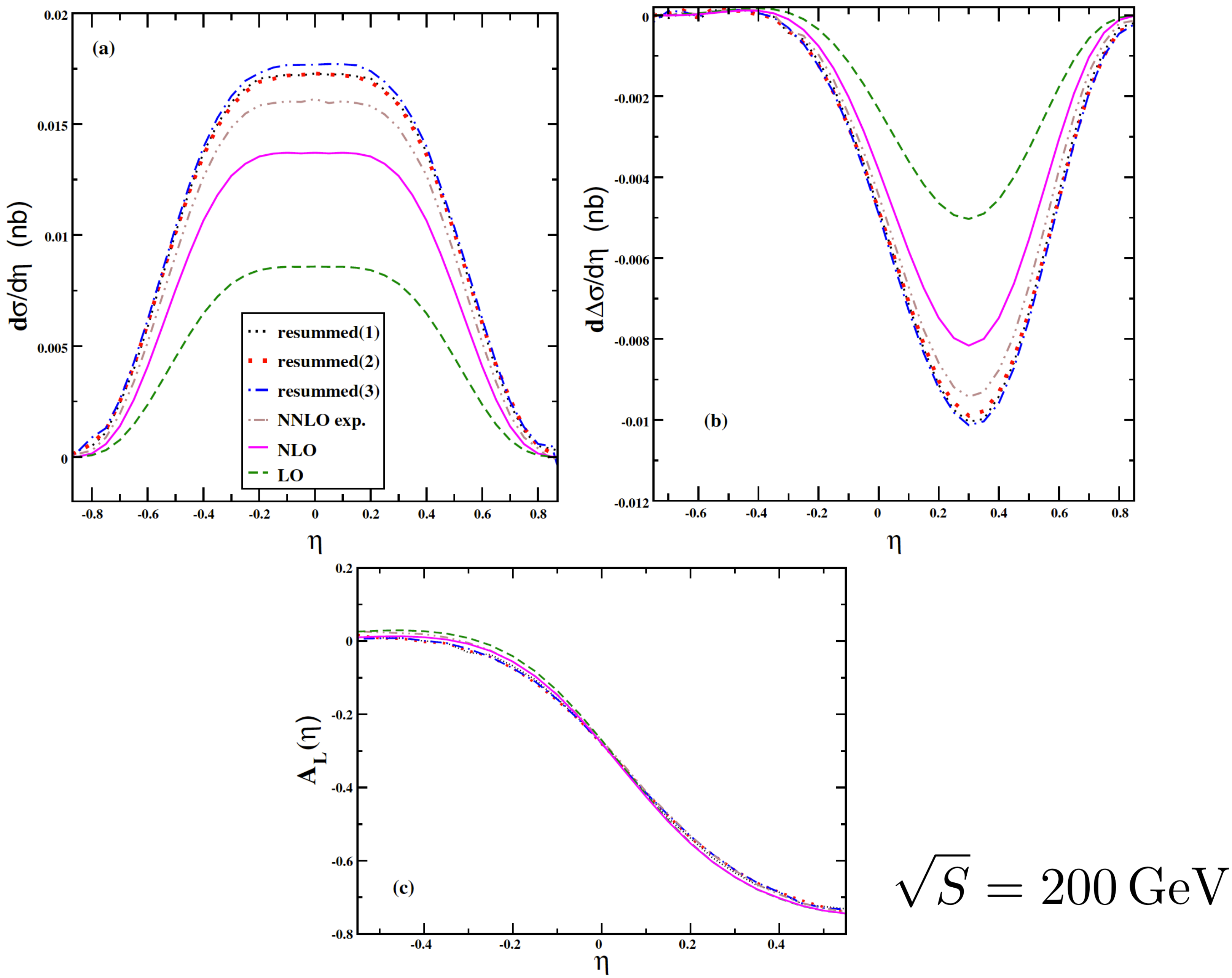
$$C_q\left(\alpha_s(\mu^2), \ln\frac{M_W^2}{\mu^2}\right) = 1 + \frac{\alpha_s}{\pi}C_F\left(-4 + \frac{2\pi^2}{3} + \frac{3}{2}\ln\frac{M_W^2}{\mu^2}\right) + \mathcal{O}(\alpha_s^2)$$

W^+



$$\sqrt{S} = 500\,\mathrm{GeV}$$

Mukherjee, WV


(a)
dσ/dη (nb)
η
resummed(1)
resummed(2)
resummed(3)
NNLO exp.
NLO
LO
(b)
dΔσ/dη (nb)
η
(c)
$A_L(\eta)$
η
$\sqrt{S} = 200\,\mathrm{GeV}$

Drell-Yan process in πN scattering

M. Aicher, A.Schäfer, WV

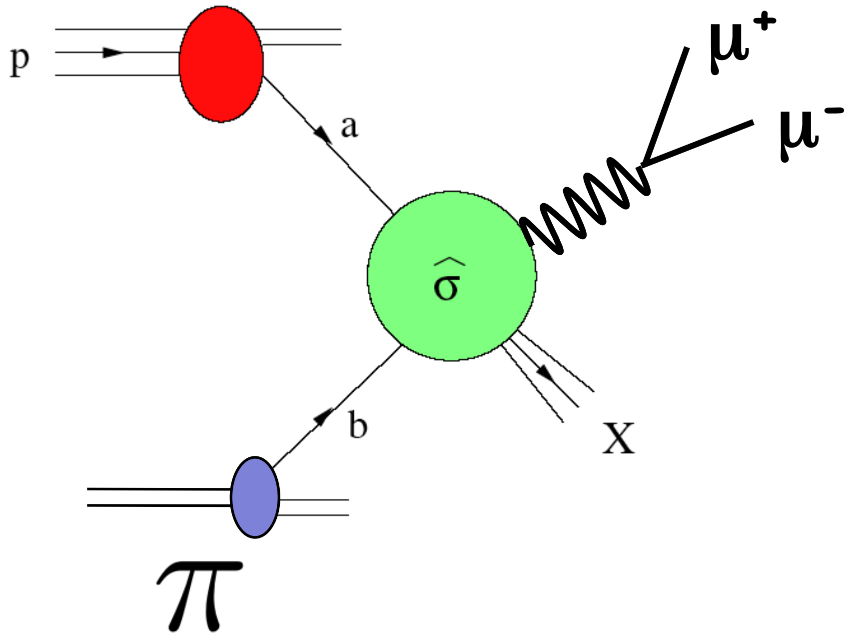
Drell-Yan is key focus in nucleon structure physics:

- in pp, pN: probe of anti-quark distributions
- in πN: probe of pion structure
- probe of spin phenomena: TMDs, Sivers effect

Currently:	E906	ongoing
	RHIC, COMPASS	near-term plans
	J-PARC, FAIR	future possibilities

- Drell-Yan process has been main source of information on pion structure:

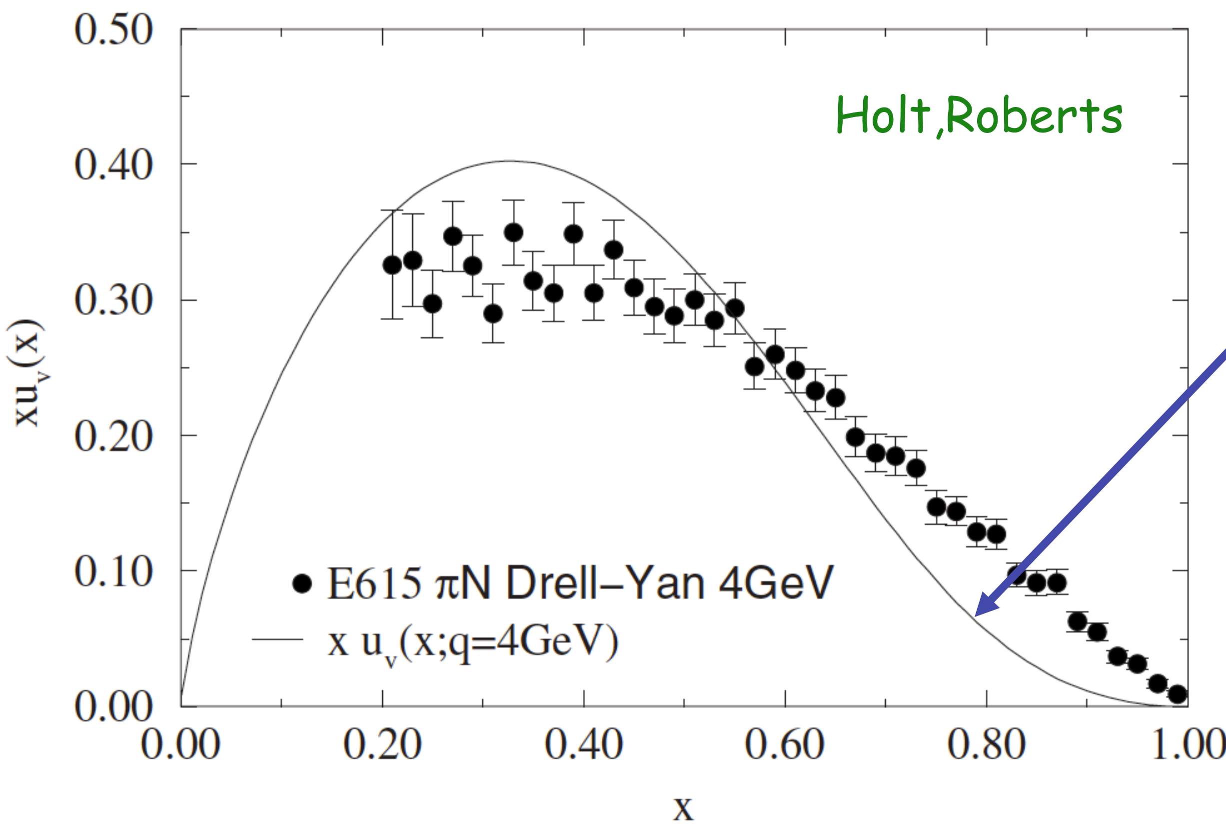
E615, NA10



$$d\sigma = \sum_{ab} \int dx_a \int dx_b\, f_a^{\pi}(x_a, \mu) f_b(x_b, \mu)\; d\hat{\sigma}_{ab}(x_a P_a, x_b P_b, Q, \alpha_s(\mu), \mu)$$

- Kinematics such that data mostly probe valence region: ~200 GeV pion beam on fixed target

- LO extraction of u_v from E615 data: $\sqrt{S} = 21.75\,\mathrm{GeV}$



$\sim\ (1-x)^2$

QCD counting rules

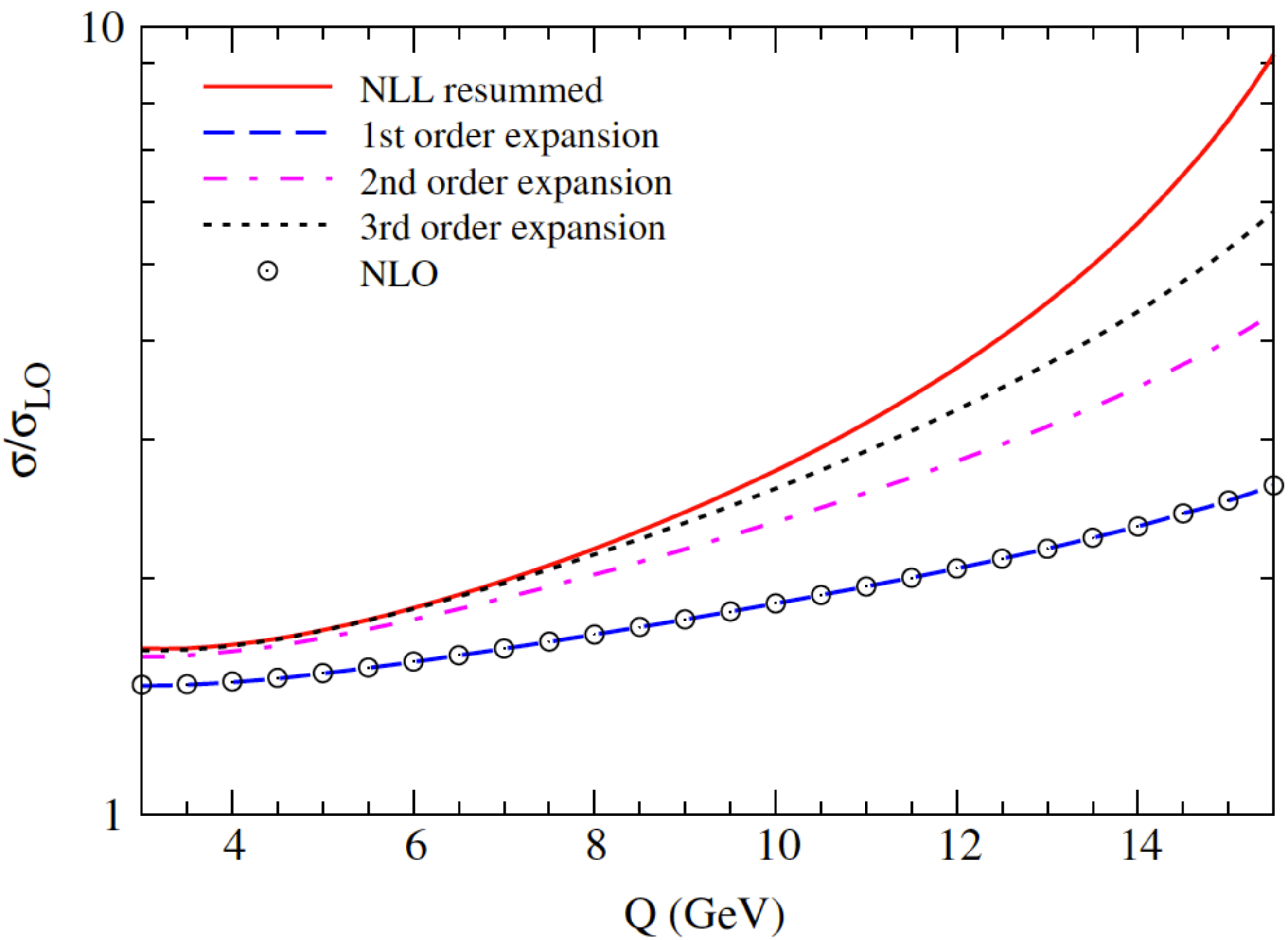
Farrar,Jackson;
Berger, Brodsky; Yuan
Blankenbecler,Gunion,
Nason

Dyson-Schwinger

Hecht et al.

(Compass kinematics)

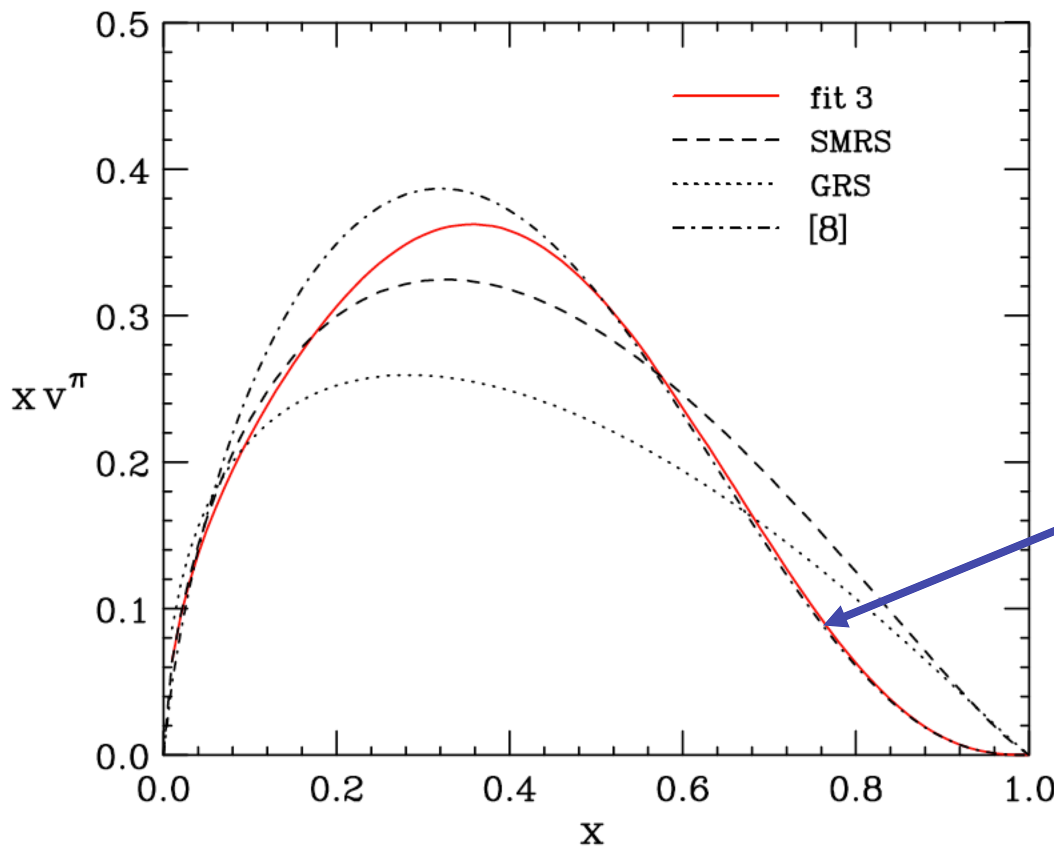
$$\sqrt{S} = 19\,\mathrm{GeV}$$



Aicher,Schäfer, WV
(earlier studies: Shimizu,Sterman,WV,Yokoya)

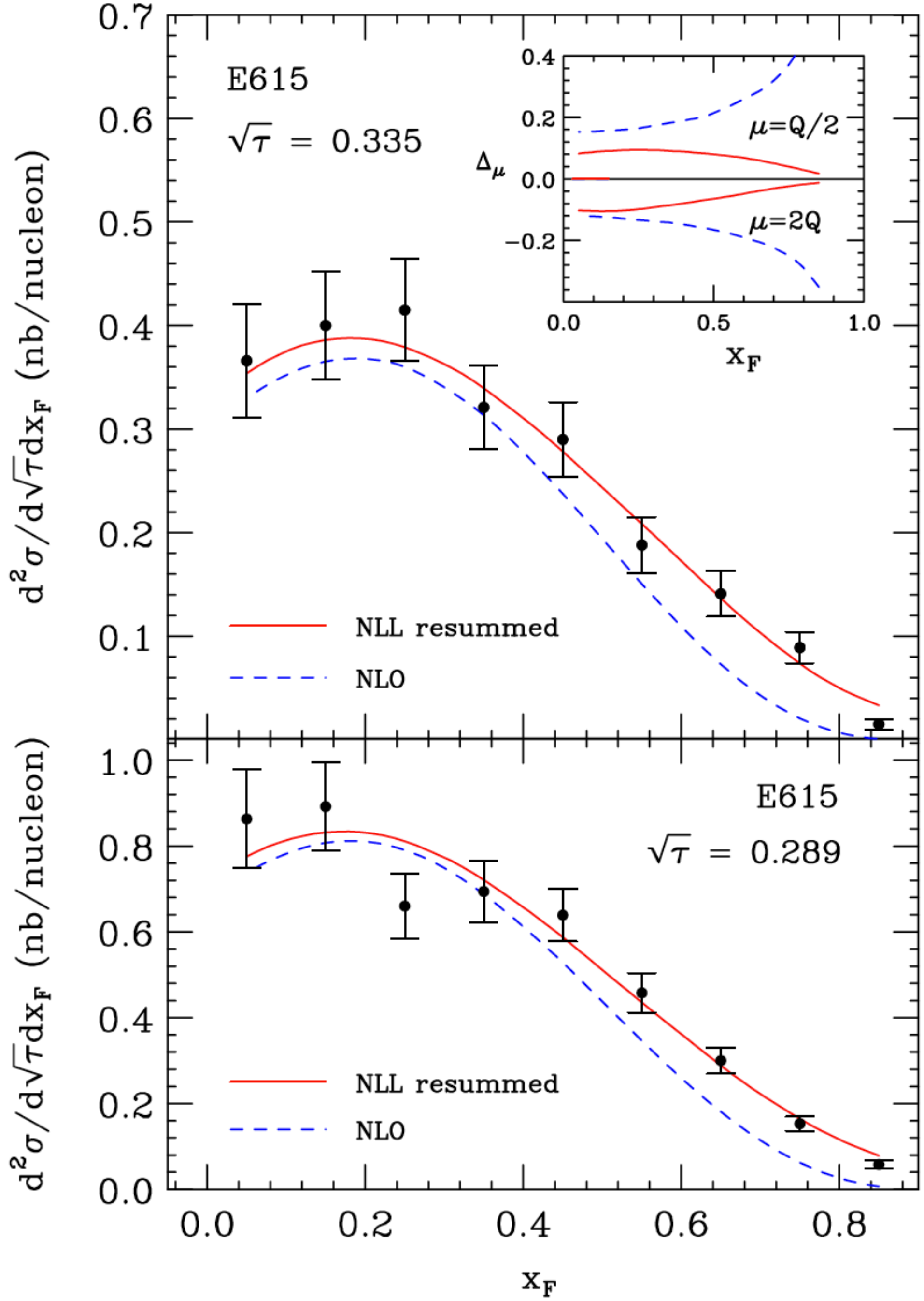
$$xv^\pi(x, Q_0^2) = N_v x^\alpha (1-x)^\beta (1+\gamma x^\delta)$$

Fit	$2\langle xv^\pi \rangle$	α	β	γ	K	χ^2 (no. of points)
1	0.55	0.15 ± 0.04	1.75 ± 0.04	89.4	0.999 ± 0.011	82.8 (70)
2	0.60	0.44 ± 0.07	1.93 ± 0.03	25.5	0.968 ± 0.011	80.9 (70)
3	0.65	0.70 ± 0.07	2.03 ± 0.06	13.8	0.919 ± 0.009	80.1 (70)
4	0.7	1.06 ± 0.05	2.12 ± 0.06	6.7	0.868 ± 0.009	81.0 (70)



$Q = 4$ GeV

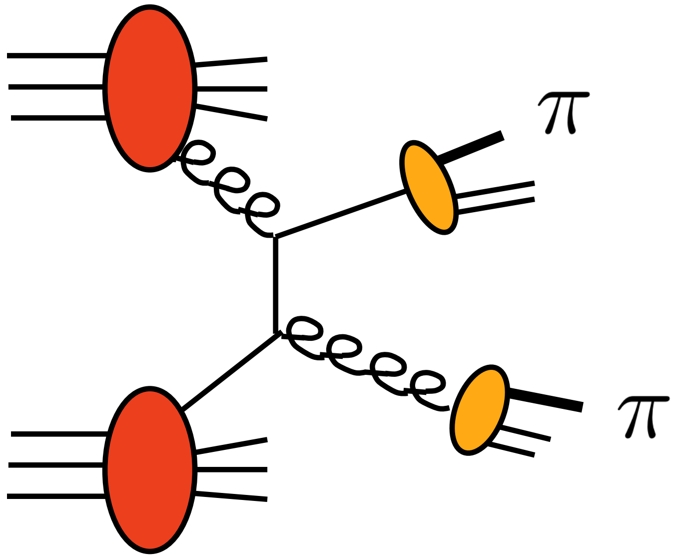
$\sim (1-x)^{2.34}$



E615
$\sqrt{\tau}$ = 0.335
E615
$\sqrt{\tau}$ = 0.289
NLL resummed
NLO
NLL resummed
NLO
Δ_μ
μ=Q/2
μ=2Q
x_F
x_F
$d^2\sigma/d\sqrt{\tau}dx_F$ (nb/nucleon)
$d^2\sigma/d\sqrt{\tau}dx_F$ (nb/nucleon)

Hadron pair production

L. Almeida, G.Sterman, WV



pair mass[2]

$$M^2 = (p_\pi + p'_\pi)^2$$

- in some sense, a generalization of Drell-Yan to "completely hadronic" situation

- data: fixed target (NA24,E711,E706)
  ISR (CCOR)

- typically ok with NLO *only if* small scales are chosen (~ M/3)

Owens, Binoth et al.

Differences w.r.t. Drell-Yan:

- color structure of hard scattering

- fragmentation -> only part of parton pair mass is converted to observed pair mass M

Define

$$\bar{\eta} = \frac{1}{2}(\eta_1 + \eta_2) \qquad \Delta\eta = \frac{1}{2}(\eta_1 - \eta_2)$$

$$M^4 \frac{d\sigma^{H_1 H_2 \to h_1 h_2 X}}{dM^2 d\Delta\eta d\bar{\eta}} = \sum_{abcd} \int_0^1 dx_a dx_b\, dz_c\, dz_d\, f_a^{H_1}(x_a) f_b^{H_2}(x_b)\, z_c D_c^{h_1}(z_c) z_d D_d^{h_2}(z_d)$$

$$\times\, \omega_{ab\to cd}\left(\hat{\tau}, \Delta\eta, \hat{\eta}, \alpha_s(\mu), \frac{\mu}{\hat{m}}\right)$$

where

$$\hat{\tau} = \frac{\hat{m}^2}{\hat{s}} \qquad \hat{m}^2 = \frac{M^2}{z_c z_d}$$

$$\hat{\eta} = \bar{\eta} - \frac{1}{2}\ln\frac{x_a}{x_b}$$

$$M^4 \frac{d\sigma^{H_1 H_2 \to h_1 h_2 X}}{dM^2 d\Delta\eta d\bar{\eta}} = \sum_{abcd} \int_0^1 dx_a dx_b\, dz_c\, dz_d\, f_a^{H_1}(x_a) f_b^{H_2}(x_b)\, z_c D_c^{h_1}(z_c) z_d D_d^{h_2}(z_d)$$

$$\times\, \omega_{ab\to cd}\left(\hat{\tau}, \Delta\eta, \hat{\eta}, \alpha_s(\mu), \frac{\mu}{\hat{m}}\right)$$

Take moments :

$$\int_{-\infty}^{\infty} d\bar{\eta}\, \mathrm{e}^{i\nu\bar{\eta}} \int_0^1 d\tau\, \tau^{N-1} M^4 \frac{d\sigma^{H_1 H_2 \to h_1 h_2 X}}{dM^2 d\Delta\eta d\bar{\eta}}$$

$$= \sum_{abcd} \tilde{f}_a^{H_1}(N+1+i\nu/2) \tilde{f}_b^{H_2}(N+1-i\nu/2) \tilde{D}_c^{h_1}(N+2) \tilde{D}_d^{h_2}(N+2)$$

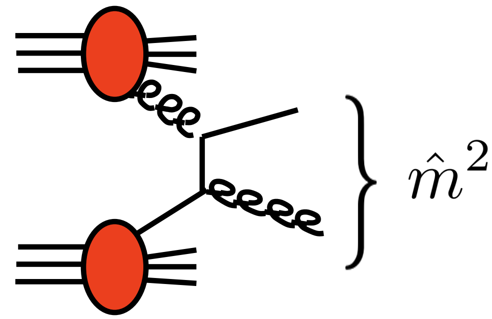
$$\times \int_{-\infty}^{\infty} d\hat{\eta}\, \mathrm{e}^{i\nu\hat{\eta}} \int_0^1 d\hat{\tau}\, \hat{\tau}^{N-1}\;\; \omega_{ab\to cd}\left(\hat{\tau}, \Delta\eta, \hat{\eta}, \alpha_s(\mu), \frac{\mu}{\hat{m}}\right)$$

-> works only at LO

Instead, write

$$M^4 \frac{d\sigma^{H_1 H_2 \to h_1 h_2 X}}{dM^2 d\Delta\eta d\bar{\eta}} = \sum_{cd} \int_0^1 dz_c\, dz_d\, z_c\, D_c^{h_1}(z_c)\, z_d\, D_d^{h_2}(z_d)\, \Omega_{cd}\left(\tau', \frac{\mu}{\hat{m}}\right)$$

$$\tau' = \frac{\hat{m}^2}{S}$$



$$\Omega_{cd}\left(\tau', \frac{\mu}{\hat{m}}\right) = \sum_{ab} \int_0^1 dx_a\, dx_b\, f_a^{H_1}(x_a)\; f_b^{H_2}(x_b)\; \omega_{ab\to cd}\left(\hat{\tau} = \frac{\tau'}{x_a x_b}, \frac{\mu}{\hat{m}}\right)$$

$$\int_{-\infty}^{\infty} d\bar{\eta}\, \mathrm{e}^{i\nu\bar{\eta}} \int_0^1 d\tau'\; (\tau')^{N-1}\, \Omega_{cd}\left(\tau', \frac{\mu}{\hat{m}}\right)$$

$$= \sum_{ab} \tilde{f}_a^{H_1}(N+1+i\nu/2, \mu)\, \tilde{f}_b^{H_2}(N+1-i\nu/2, \mu)$$

$$\times \int_{-\infty}^{\infty} d\hat{\eta}\, \mathrm{e}^{i\nu\hat{\eta}} \int_0^1 d\hat{\tau}\, \hat{\tau}^{N-1}\, \omega_{ab\to cd}\left(\hat{\tau}, \Delta\eta, \hat{\eta}, \alpha_s(\mu), \frac{\mu}{\hat{m}}\right)$$

$$\tilde{\omega}_{ab\to cd}\left(N,\nu,\Delta\eta,\alpha_s(\mu),\frac{\mu}{\hat{m}}\right) \equiv \int_{-\infty}^{\infty} d\hat{\eta}\, \mathrm{e}^{i\nu\hat{\eta}} \int_0^1 d\hat{\tau}\, \hat{\tau}^{N-1}\, \omega_{ab\to cd}\left(\hat{\tau},\Delta\eta,\hat{\eta},\alpha_s(\mu),\frac{\mu}{\hat{m}}\right)$$

$$\omega_{ab\to cd} = \left(\frac{\alpha_s}{\pi}\right)^2 \left[\omega^{\mathrm{LO}}_{ab\to cd} + \frac{\alpha_s}{\pi}\omega^{\mathrm{NLO}}_{ab\to cd} + \ldots\right]$$

LO:

$$\omega^{\mathrm{LO}}_{ab\to cd}\left(\hat{\tau},\Delta\eta,\hat{\eta}\right) = \delta\left(1-\hat{\tau}\right)\,\delta\left(\hat{\eta}\right)\,\omega^{(0)}_{ab\to cd}(\Delta\eta)$$

NLO:

$$\omega^{\mathrm{NLO}}_{ab\to cd}\left(\hat{\tau},\Delta\eta,\hat{\eta},\mu/\hat{m}\right) = \delta\left(\hat{\eta}\right)\left[\omega^{(1,0)}_{ab\to cd}(\Delta\eta,\mu/\hat{m})\,\delta(1-\hat{\tau}) + \omega^{(1,1)}_{ab\to cd}(\Delta\eta,\mu/\hat{m})\left(\frac{1}{1-\hat{\tau}}\right)_+ \right.$$
$$\left. +\omega^{(1,2)}_{ab\to cd}(\Delta\eta)\left(\frac{\log(1-\hat{\tau})}{1-\hat{\tau}}\right)_+\right] + \omega^{\mathrm{reg,NLO}}_{ab\to cd}\left(\hat{\tau},\Delta\eta,\hat{\eta},\mu/\hat{m}\right)$$

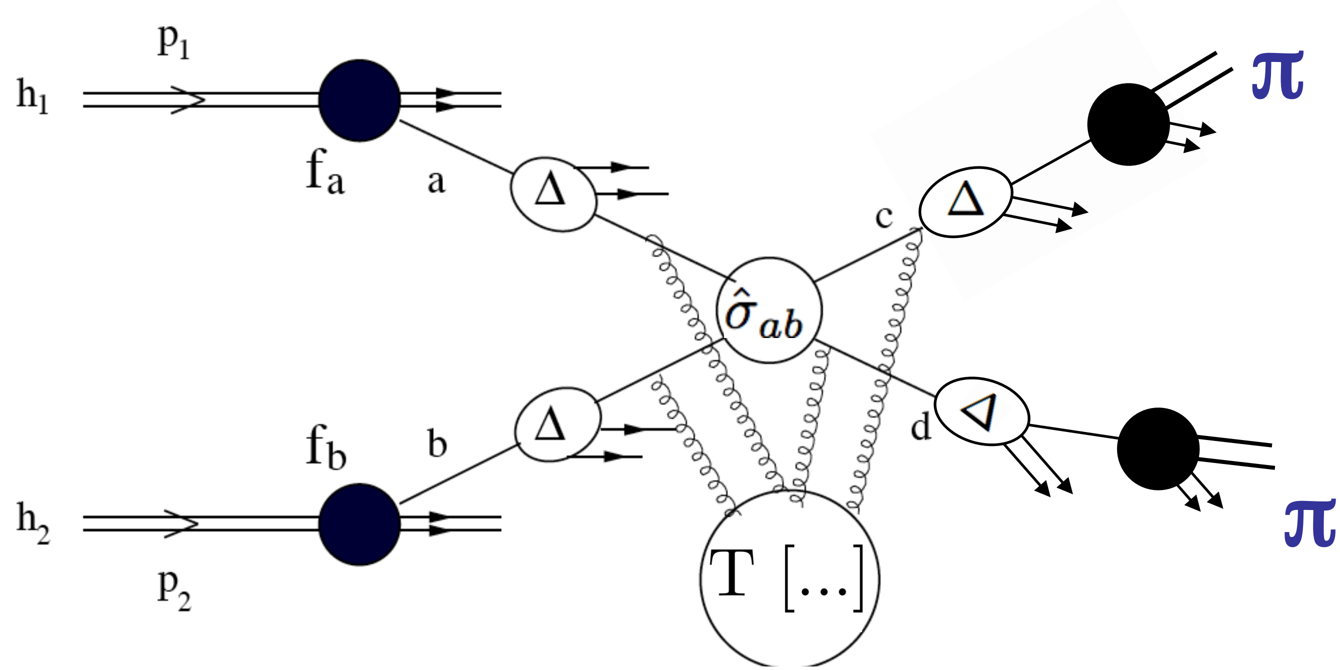
true to all orders

like DY

$$\tilde{\omega}^{\rm resum}_{ab\to cd}\left(N, \Delta\eta, \alpha_s(\mu), \frac{\mu}{\hat{m}}\right) = \Delta_a^{N+1}\left(\alpha_s(\mu), \frac{\mu}{\hat{m}}\right) \Delta_b^{N+1}\left(\alpha_s(\mu), \frac{\mu}{\hat{m}}\right)$$

$$\times \Delta_c^{N+2}\left(\alpha_s(\mu), \frac{\mu}{\hat{m}}\right) \Delta_d^{N+2}\left(\alpha_s(\mu), \frac{\mu}{\hat{m}}\right)$$

$$\times \,{\rm Tr}\left\{H \mathcal{S}_N^\dagger S \mathcal{S}_N\right\}_{ab\to cd}\left(\Delta\eta, \alpha_s(\mu), \frac{\mu}{\hat{m}}\right)$$



- matrix problem

$$\mathrm{Tr}\left\{H\mathcal{S}_N^\dagger S\mathcal{S}_N\right\}_{ab\to cd}$$

Kidonakis,Oderda,Sterman
Bonciani,Catani,Mangano,Nason
Banfi,Salam,Zanderighi
Dokshitzer,Marchesini

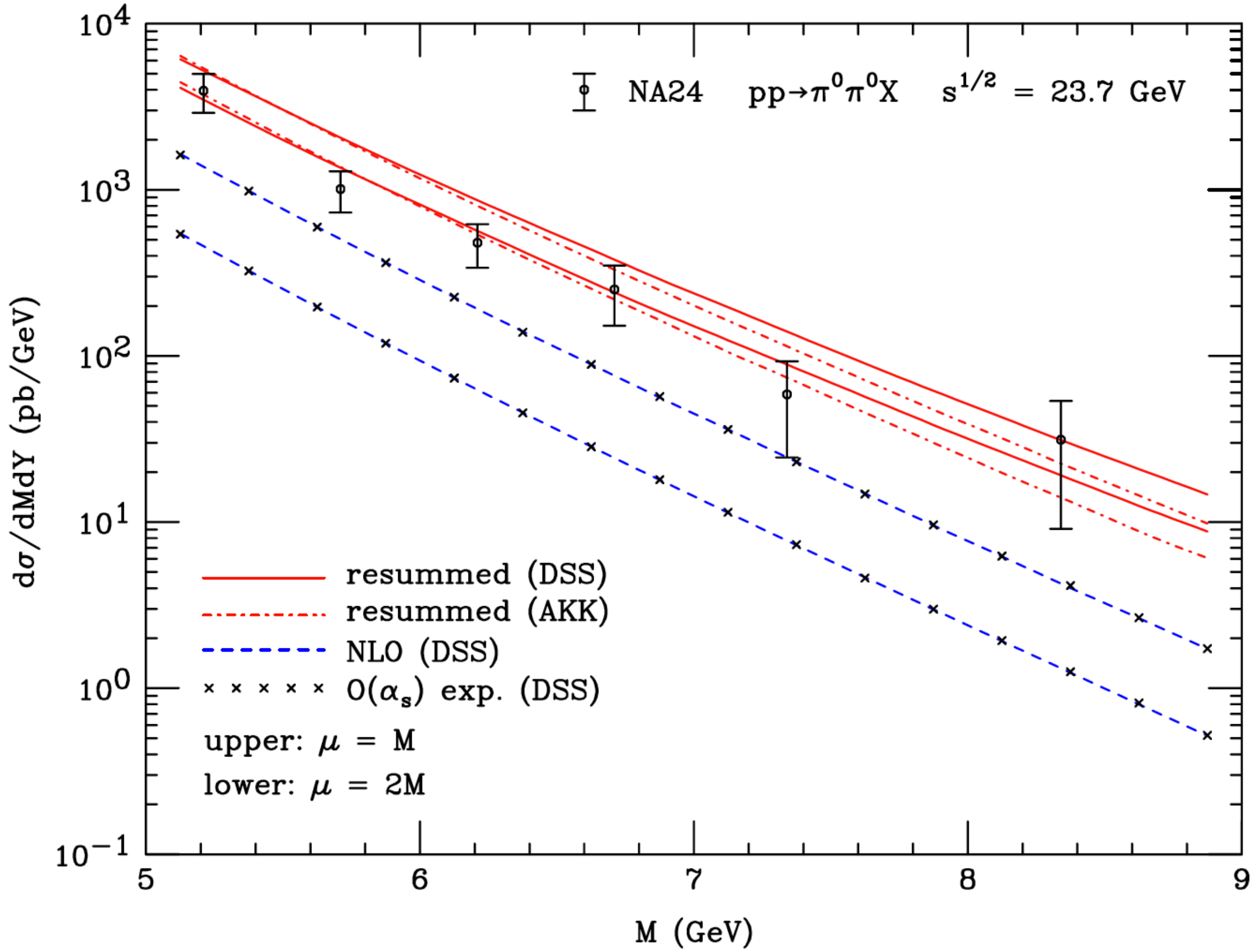
this part depends on scattering angle !

$$H_{ab\to cd}\left(\Delta\eta,\alpha_s(\mu),\frac{\mu}{\hat{m}}\right) = H^{(0)}_{ab\to cd}(\Delta\eta) + \frac{\alpha_s(\mu)}{\pi}H^{(1)}_{ab\to cd}\left(\Delta\eta,\frac{\mu}{\hat{m}}\right) + \mathcal{O}(\alpha_s^2)$$

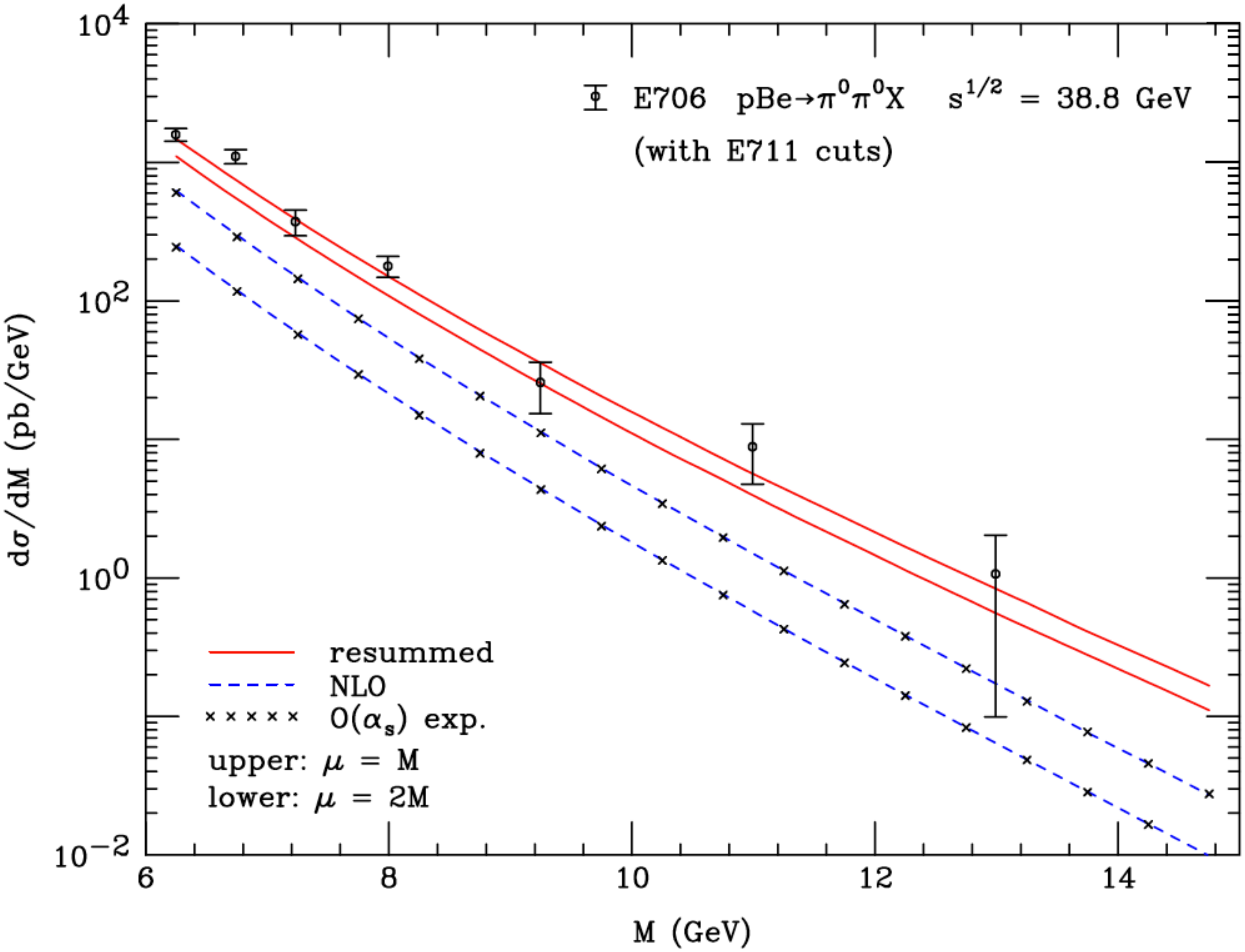
$$S_{ab\to cd}\left(\Delta\eta,\alpha_s,\frac{\mu}{\hat{m}}\right) = S^{(0)}_{ab\to cd} + \frac{\alpha_s}{\pi}S^{(1)}_{ab\to cd}\left(\Delta\eta,\frac{\mu}{N\hat{m}}\right) + \mathcal{O}(\alpha_s^2)$$

$$\mathcal{S}_{N,ab\to cd}\left(\Delta\eta,\alpha_s(\mu),\frac{\mu}{\hat{m}}\right) = \mathcal{P}\exp\left[\frac{1}{2\pi}\int_{\hat{m}^2}^{\hat{m}^2/\bar{N}^2}\frac{dq^2}{q^2}\alpha_s(q^2)\,\Gamma^{(1)}_{ab\to cd}(\Delta\eta)\right]$$

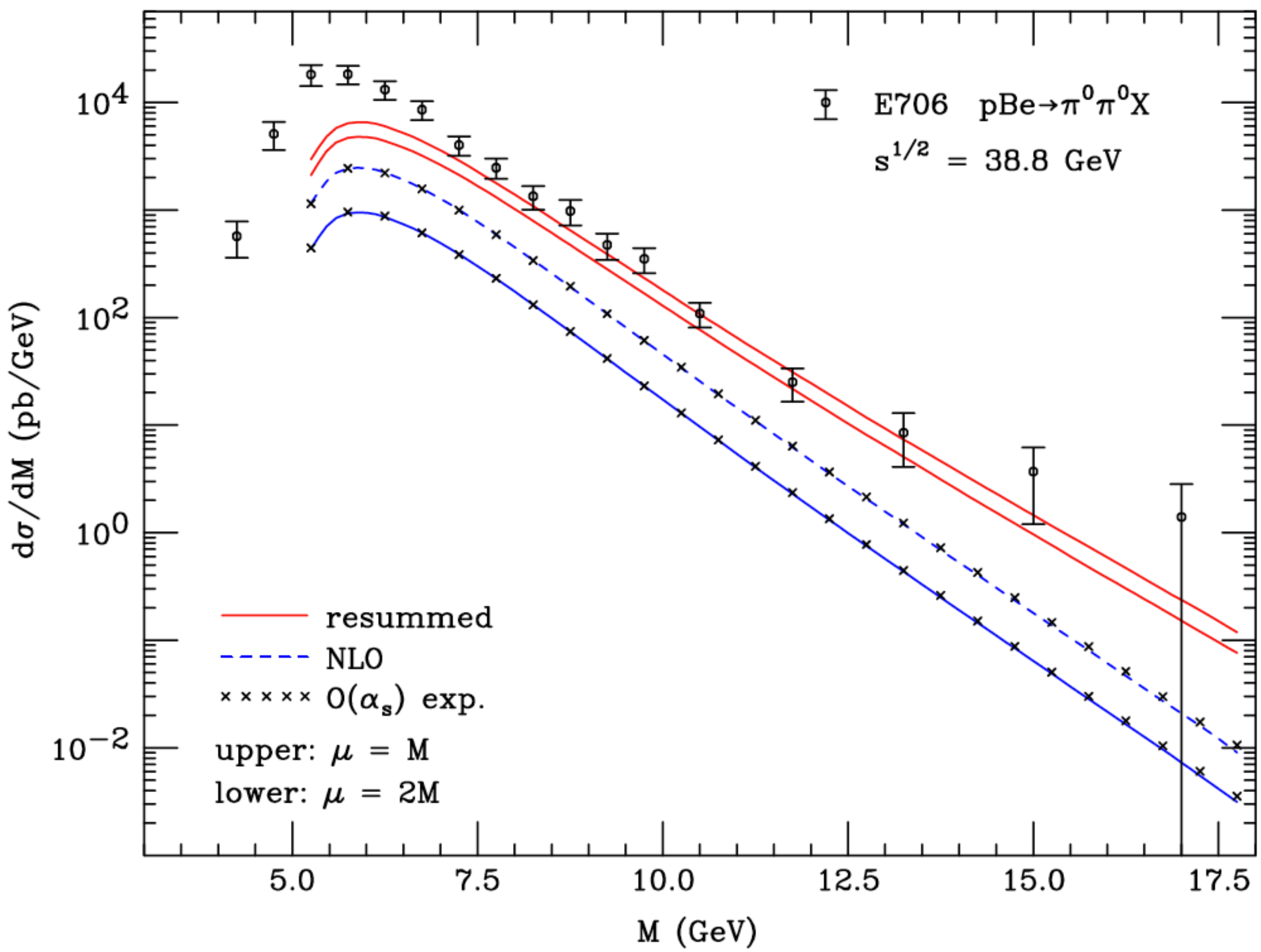
- algebra done numerically



NA24 pp→π⁰π⁰X s^{1/2} = 23.7 GeV
resummed (DSS)
resummed (AKK)
NLO (DSS)
O(α_s) exp. (DSS)
upper: μ = M
lower: μ = 2M
dσ/dMdY (pb/GeV)
M (GeV)

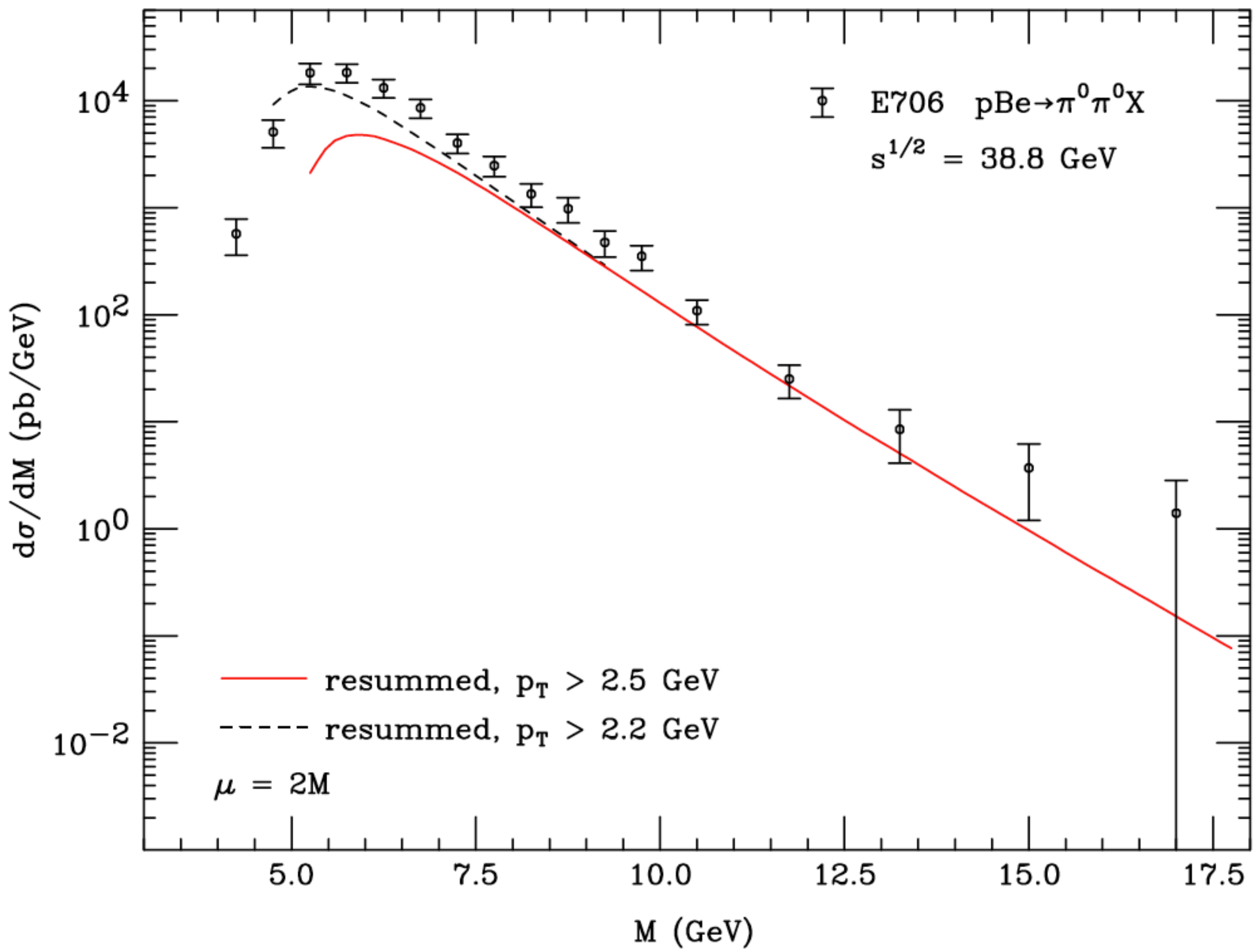


E706 pBe→$\pi^0\pi^0$X $s^{1/2}$ = 38.8 GeV
(with E711 cuts)
resummed
NLO
$O(\alpha_s)$ exp.
upper: μ = M
lower: μ = 2M
$d\sigma/dM$ (pb/GeV)
M (GeV)
10^4
10^2
10^0
10^{-2}
6
8
10
12
14

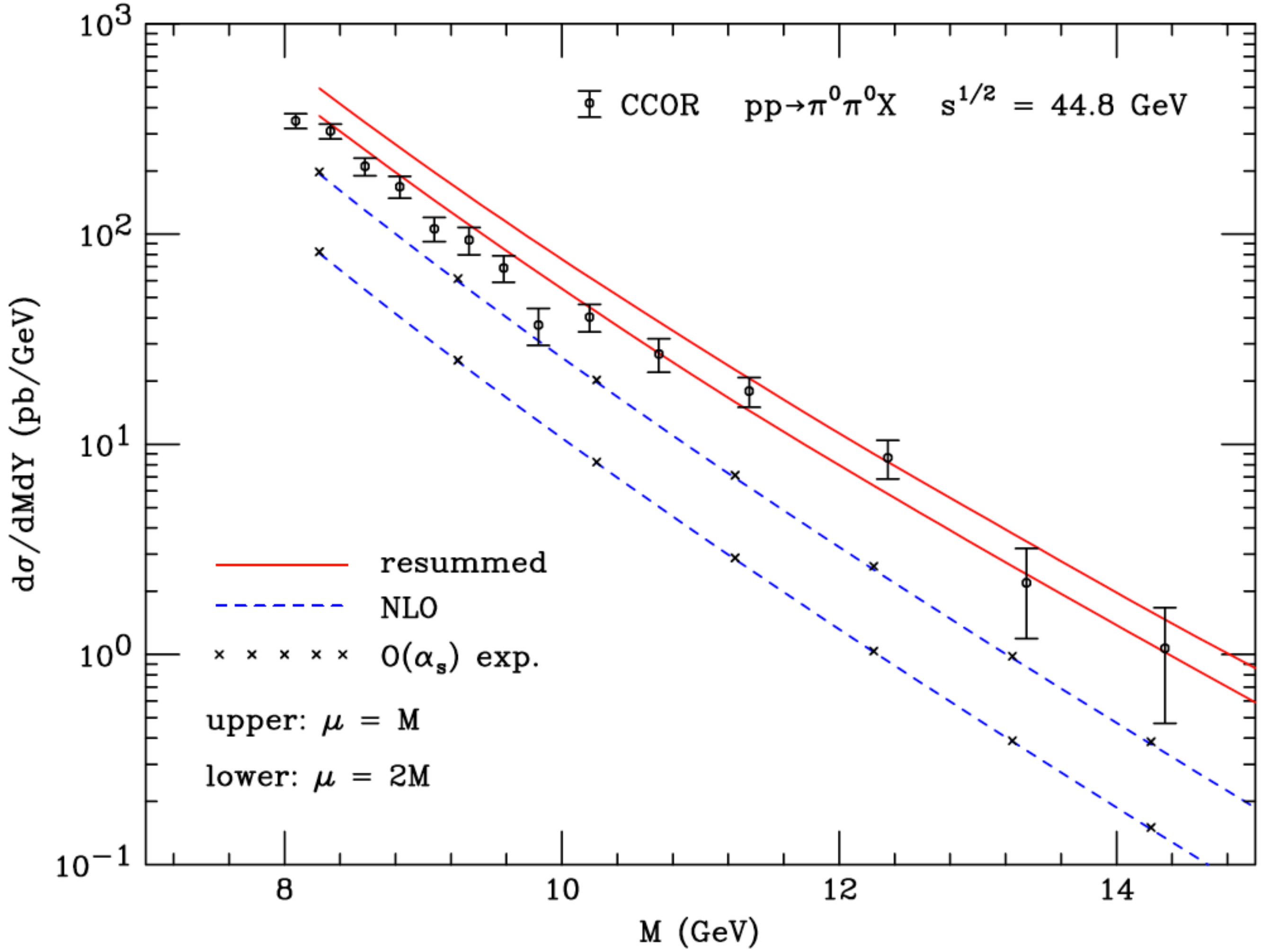


$$p_{T,i} > 2.5 \text{ GeV} \;\equiv p_T^{\rm cut}$$

$|M - 2p_T^{\rm cut}|$ sets new scale



E706 pBe→$\pi^0\pi^0$X
$s^{1/2}$ = 38.8 GeV
resummed, p_T > 2.5 GeV
resummed, p_T > 2.2 GeV
μ = 2M
$d\sigma/dM$ (pb/GeV)
M (GeV)
10^4
10^2
10^0
10^{-2}
5.0
7.5
10.0
12.5
15.0
17.5

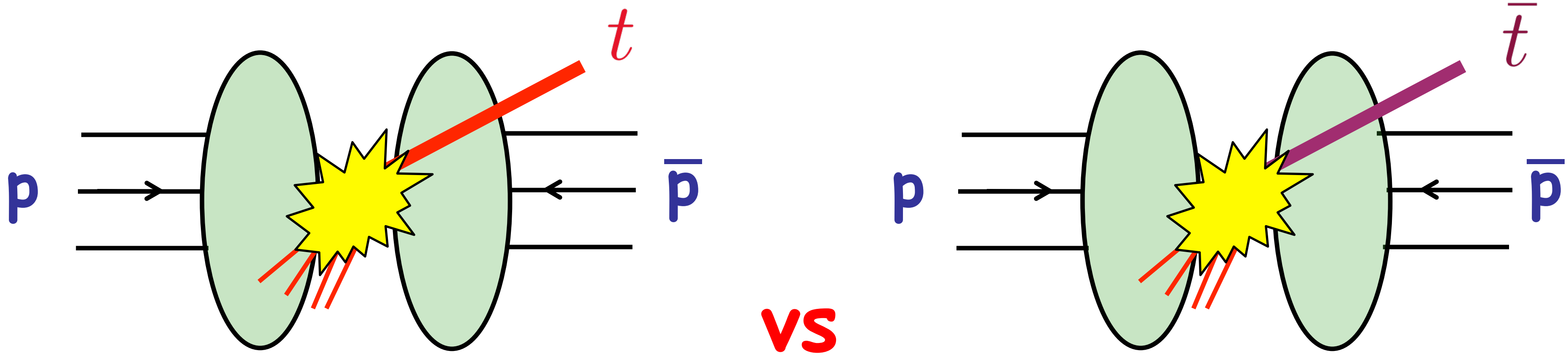


CCOR pp→π⁰π⁰X s^{1/2} = 44.8 GeV
resummed
NLO
O(α_s) exp.
upper: μ = M
lower: μ = 2M
dσ/dMdY (pb/GeV)
M (GeV)
10³
10²
10¹
10⁰
10⁻¹
8
10
12
14

Top quark charge asymmetry

L. Almeida, G.Sterman, WV

Charge asymmetry:



Differential in rapidity y :

$$A_{\rm ch}(y) = \frac{N_t(y) - N_{\bar{t}}(y)}{N_t(y) + N_{\bar{t}}(y)}$$

Integrated:

$$A_{\rm ch} = \frac{N_t(y>0) - N_{\bar{t}}(y>0)}{N_t(y>0) + N_{\bar{t}}(y>0)}$$

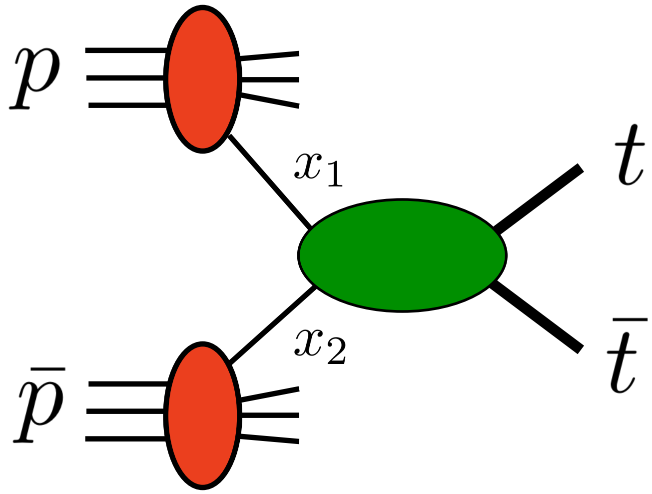
in $p\bar{p}$:

charge asymmetry leads to forward-backward asym.:

$$A_{\mathrm{FB}} = \frac{N_t(y>0) - N_t(y<0)}{N_t(y>0) + N_t(y<0)}$$

$$= A_{\mathrm{ch}}$$

- also: $A_{\mathrm{FB}}^{t\bar{t}} = \frac{N(\Delta y>0) - N(\Delta y<0)}{N(\Delta y>0) + N(\Delta y<0)}$ $\qquad \Delta y \equiv y_t - y_{\bar{t}}$



$$y = \hat{y} + \frac{1}{2}\log\frac{x_1}{x_2}$$

$$y_t - y_{\bar{t}} = \hat{y}_t - \hat{y}_{\bar{t}}$$

$$A_{\rm ch} = A_{\rm FB}$$

$$\propto \int dx_1 dx_2 \left[q_1^p\, \bar{q}_2^{\bar{p}} - \bar{q}_1^p\, q_2^{\bar{p}} \right] \left(\hat{\sigma}_{q\bar{q}\to t}(\hat{y}) - \hat{\sigma}_{q\bar{q}\to \bar{t}}(\hat{y}) \right)$$

$$q\, q - \bar{q}\, \bar{q}$$

- Less diluted for $\Delta y \equiv y_t - y_{\bar{t}}$

Tevatron :

- **D0:** not corrected for acceptance or reconstruction

$$A_{\mathrm{FB}}^{t\bar{t}} = \frac{N(\Delta y > 0) - N(\Delta y < 0)}{N(\Delta y > 0) + N(\Delta y < 0)} = \big(8 \pm 4\,(\mathrm{stat}) \pm 1\,(\mathrm{syst})\big)\%$$

SM expectation (MC@NLO): ~ 1%

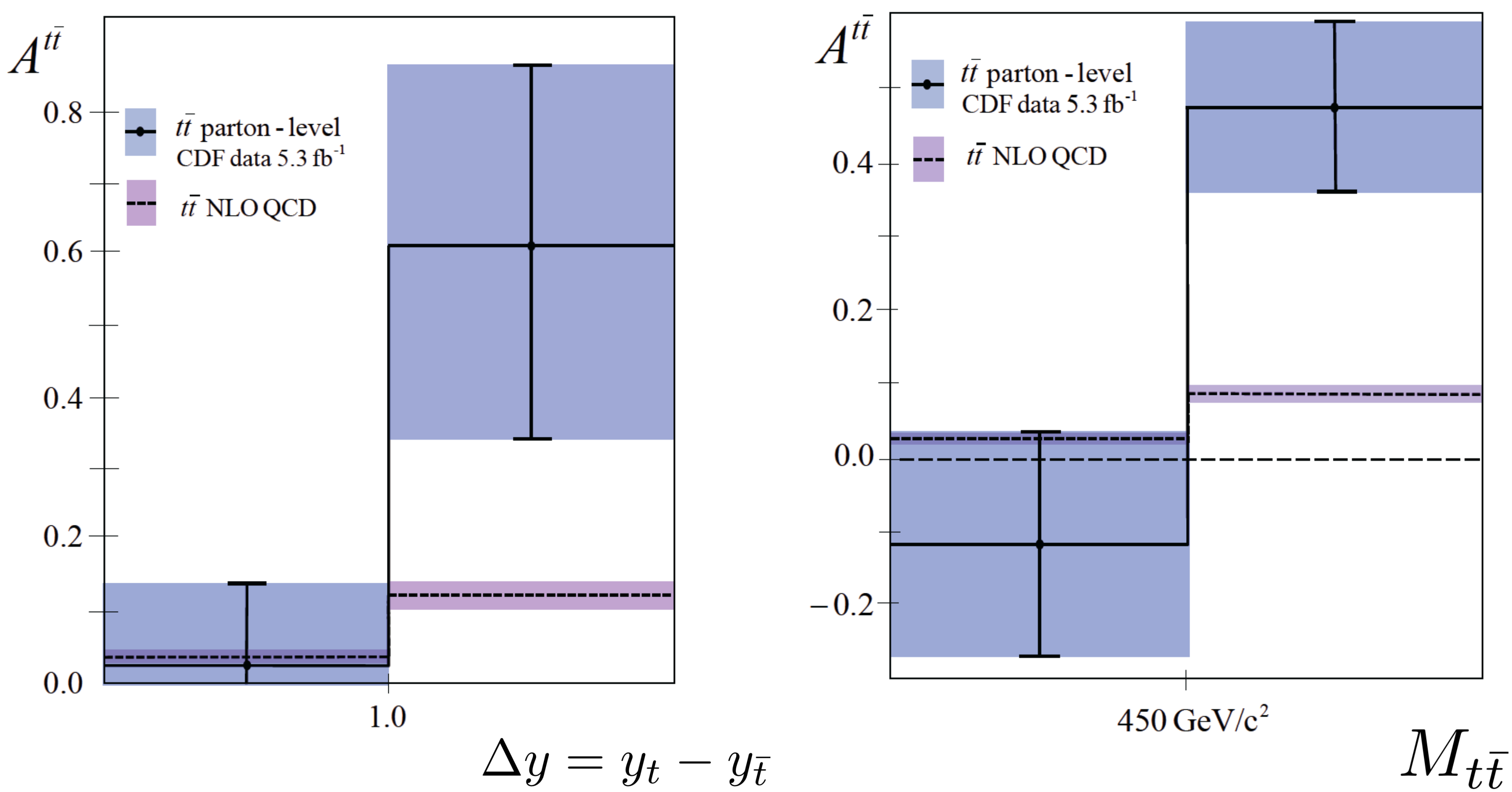
- **CDF:** fully corrected

$$A_{\mathrm{FB}}^{t\bar{t}} = \frac{N(\Delta y > 0) - N(\Delta y < 0)}{N(\Delta y > 0) + N(\Delta y < 0)} = \begin{cases} 0.158 \pm 0.075 & \ell + \mathrm{jets} \\ 0.42 \pm 0.15 \pm 0.05 & 2\ell \end{cases}$$

SM expectation: ~ 6%

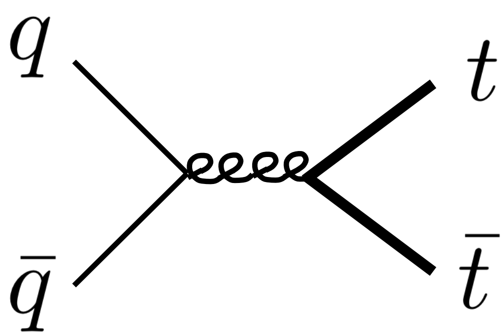
$$A_{\mathrm{FB}} = \frac{N_t(y > 0) - N_t(y < 0)}{N_t(y > 0) + N_t(y < 0)} = 0.150 \pm 0.055$$

SM expectation: ~ 4%



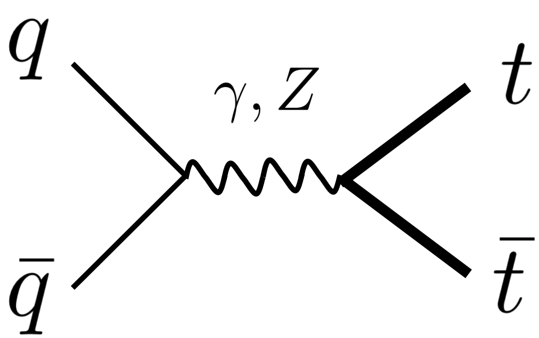
$A^{t\bar{t}}$
0.8
0.6
0.4
0.2
0.0
$t\bar{t}$ parton - level
CDF data 5.3 fb^{-1}
$t\bar{t}$ NLO QCD
1.0
$\Delta y = y_t - y_{\bar{t}}$
$A^{t\bar{t}}$
0.4
0.2
0.0
−0.2
$t\bar{t}$ parton - level
CDF data 5.3 fb^{-1}
$t\bar{t}$ NLO QCD
450 GeV/c^2
$M_{t\bar{t}}$

- Tevatron: ~85% of $t\bar{t}$ cross section is from $q\bar{q}$



LO symmetric in $t, \bar{t}$: no A_{ch}

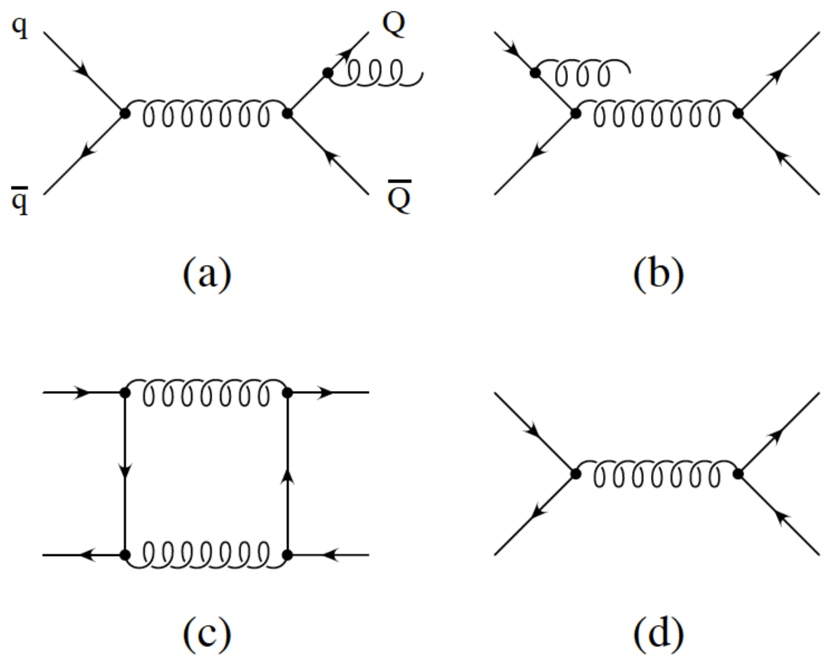
- electroweak:



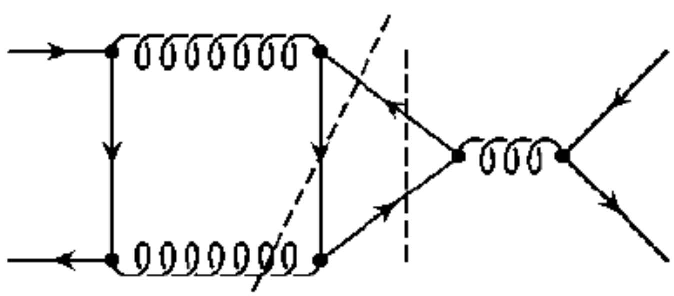
tiny

(no interference with QCD $q\bar{q} \to t\bar{t}$)

- however, at $\mathcal{O}(\alpha_s^3)$:



Brown,Sahdev,Mikaelian '79
Halzen,Hoyer,Kim '87
Kühn,Rodrigo '98
QED:
Berends,Gaemers,Gastmans '73
Putzolu '61

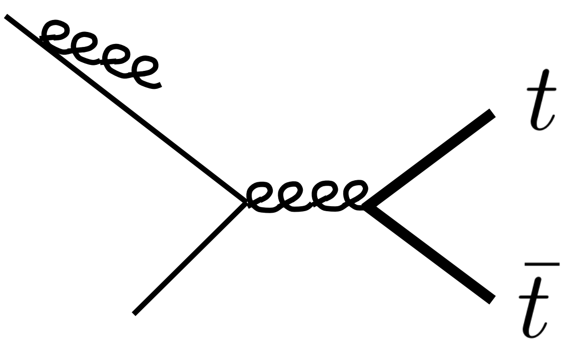


- in QCD, effect involves color factor $d_{abc}\, d^{abc}$

- diagrams are subset of full NLO, and therefore also included there

Beenakker et al.,
Ellis,Dawson,Nason,
MCFM (Campbell,Ellis,et al.)
MC@NLO (Frixione et al.)

- however, for *asymmetric* part, they are LO
- as a result, loops are UV-finite
- diagrams also collinear-finite:



- single IR divergence that cancels between real & virtual

Stability of this prediction ?

Why (might need to) worry:

- only LO
- NLO gives ~30% correction to $t\bar{t}$ cross section, significant scale uncertainty
- NLO for *charge-asymmetric* part not available (would be part of NNLO for full cross sec.)

-> investigate higher orders of perturbation theory

- similar to dihadron resummation:

$$\sigma^{\rm res}_{q\bar{q}}(N,\theta) \;\propto\; \underbrace{\Delta_q(N)\,\Delta_{\bar{q}}(N)}_{\text{like Drell-Yan}}\;{\rm Tr}\left[H_{q\bar{q}}(\theta)\;\underbrace{{\rm e}^{-\int_{M_{t\bar{t}}}^{M_{t\bar{t}}/N}\frac{d\mu}{\mu}\,\mathbf{\Gamma}^\dagger(\alpha_s,\theta)}\;S_{q\bar{q}}\;{\rm e}^{\int_{M_{t\bar{t}}}^{M_{t\bar{t}}/N}\frac{d\mu}{\mu}\,\mathbf{\Gamma}(\alpha_s,\theta)}}_{\text{depends on scattering angle}}\right]$$

- roughly:

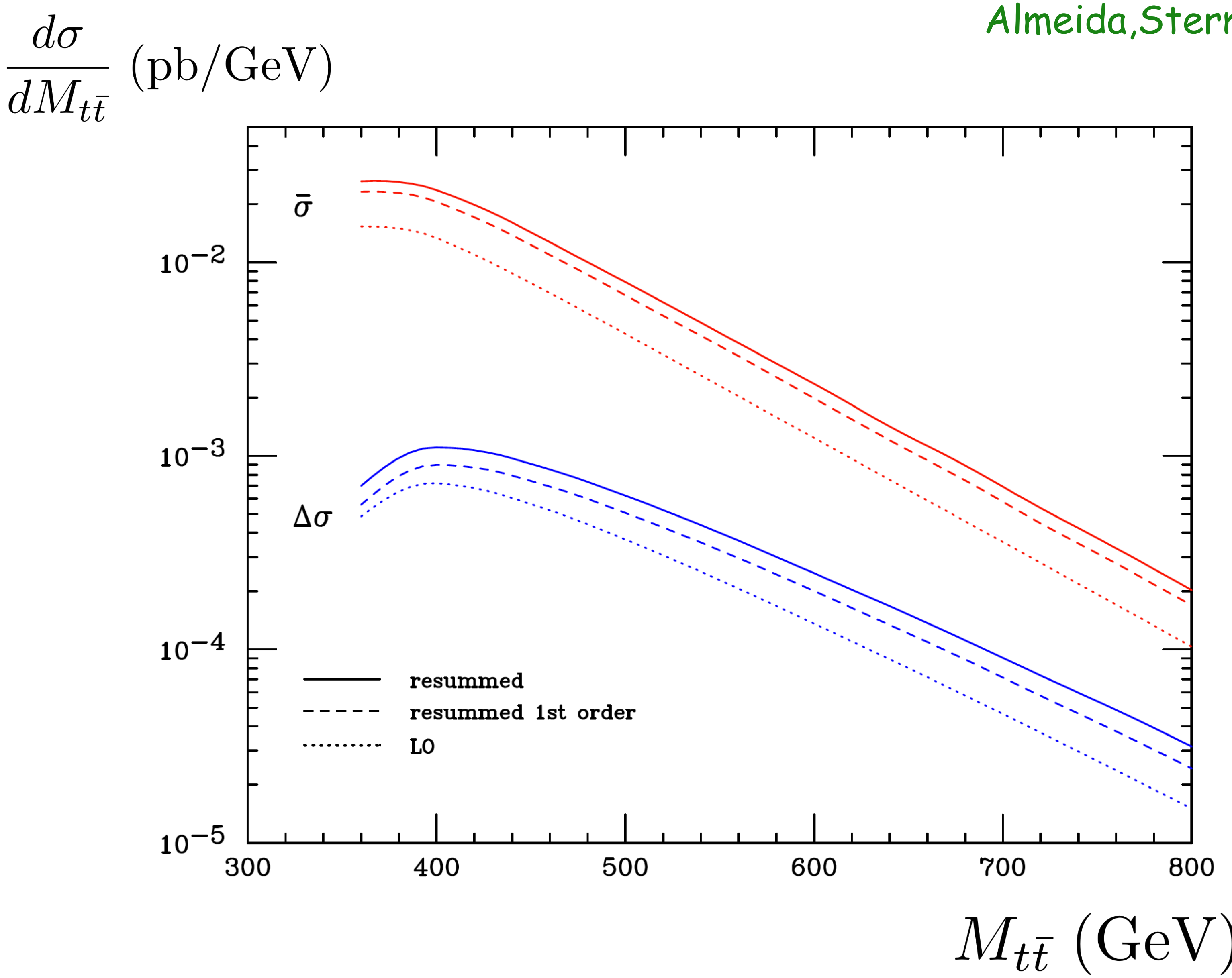
Almeida,Sterman,WV

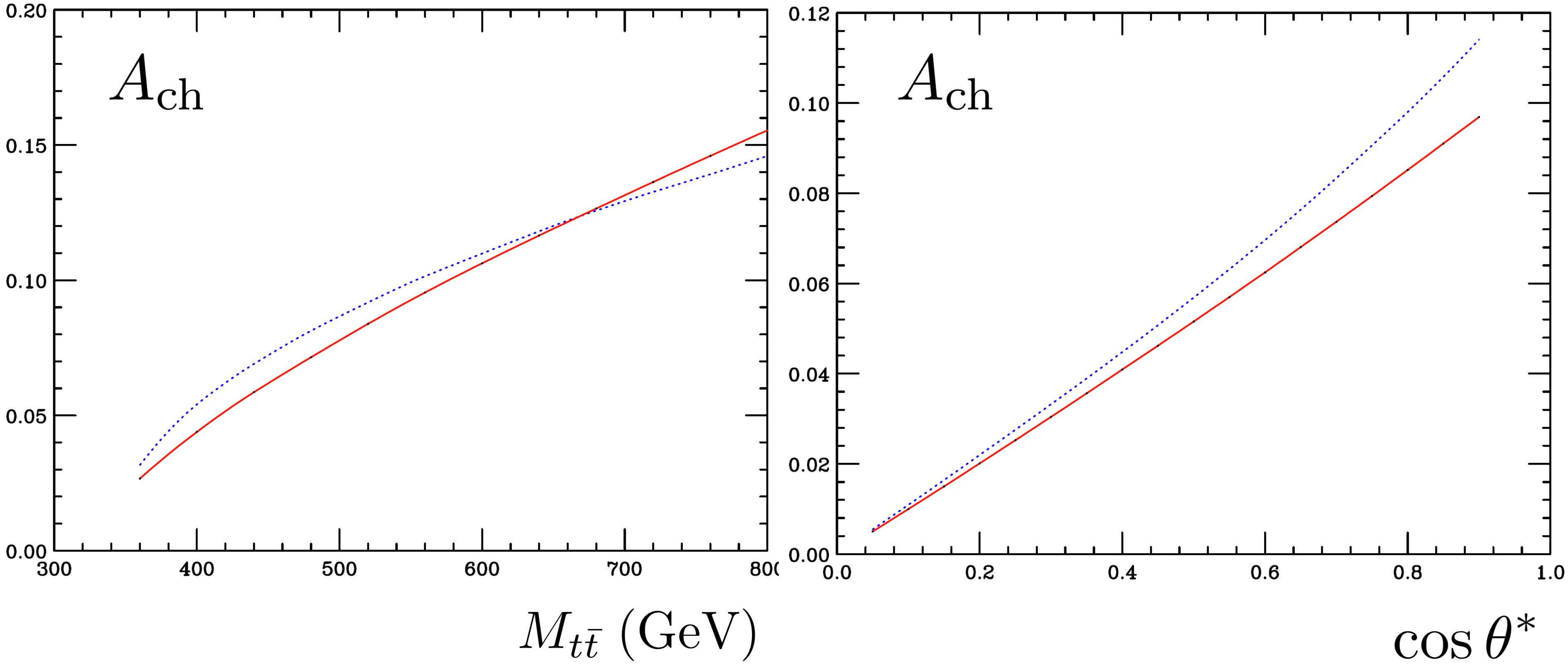
$$\hat{\sigma}^{(\rm res)}_{q\bar{q}}(N,\theta) = \hat{\sigma}^{(\rm Born)}_{q\bar{q}}(\theta)\;(\Delta_q(N))^2\left\{1+\frac{\beta\cos\theta(8C_F-3C_A)\ln(1-2\lambda)}{\pi b_0}\right\}\;{\rm e}^{-\frac{C_A}{2\pi b_0}\ln(1-2\lambda)}$$

$$\lambda = \alpha_s b_0 \log(N)$$

- leading-log part cancels in A_{FB}

Almeida,Sterman,WV





- general trend is like CDF data, but less pronounced
- stability of results confirmed to NNLL

Ahrens,Ferroglia,Neubert,
Pecjak,Yang