Bound-state effects in ttbar production at the LHC

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based on works in collaboration with
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GGI workshop, Firenze, 2011.09.28

Outline :

1. Introduction : Top-Quark at hadron colliders
2. Bound-state effects on ttbar production

   at hadron colliders
3. Differential cross-section / Event Generation
4. Summary

1. Introduction

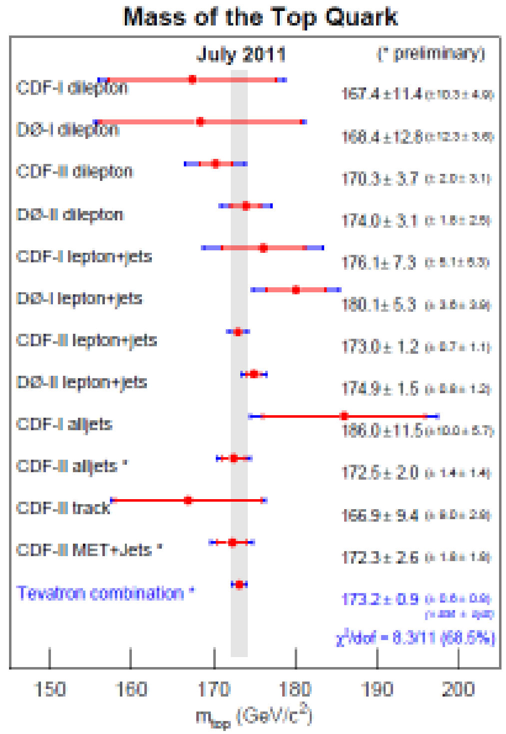
1. Introduction

Properties of the top-quark

- Mass : (CDF and D0 combined, arXiv:1107.5255)

$$m_t = 173.2 \pm 0.6(\text{stat.}) \pm 0.8(\text{syst.})\ [\text{GeV}]$$

  - Heaviest fundamental particle ever found
  - Important input for EW precision test
  - $y_t \sim 1 \Rightarrow$ may be related to the BSM physics

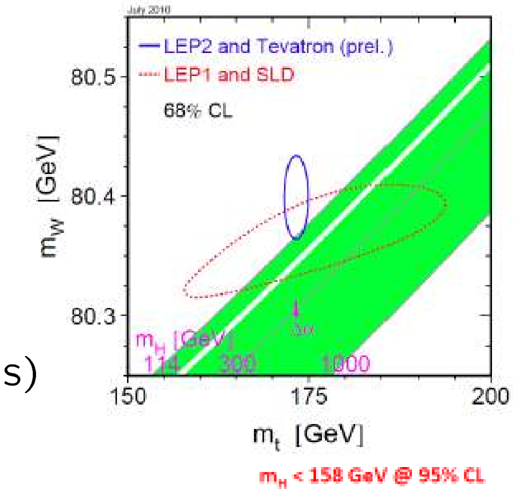


- Width : SM prediction; $\Gamma_t \simeq \frac{G_F m_t^3}{8\sqrt{2}\pi}|V_{tb}|^2 \sim 1.5\,[\text{GeV}]$

  D0 measurement ('10) ; $\Gamma_t = 1.99^{+0.69}_{-0.55}$ GeV

  - decay before hadronization, $\Gamma_t \gg \Lambda_{QCD}$ $(\tau \sim 10^{-25}\text{s})$

    thus spin information is preserved in decay products



1. Introduction

- Precise mass (and width) determination :

What mass of the top-quark is observed ?

pole mass? a parameter in Monte-Carlo?

Fact : Pole mass is not well-defined.
Theoretical prediction using pole mass has bad pert. convergency

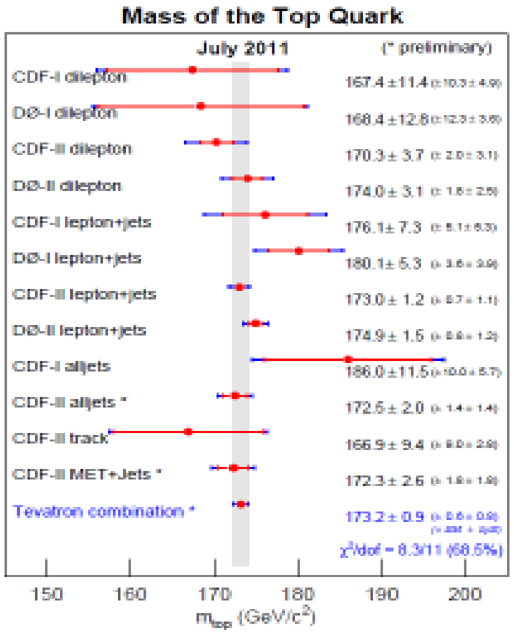
Important : definition of the mass in an IR-safe manner
(short-distance mass)

Langenfeld,Moch,Uwer('09) determination of the $\overline{\mathrm{MS}}$ mass from the total cross-section at Tevatron.

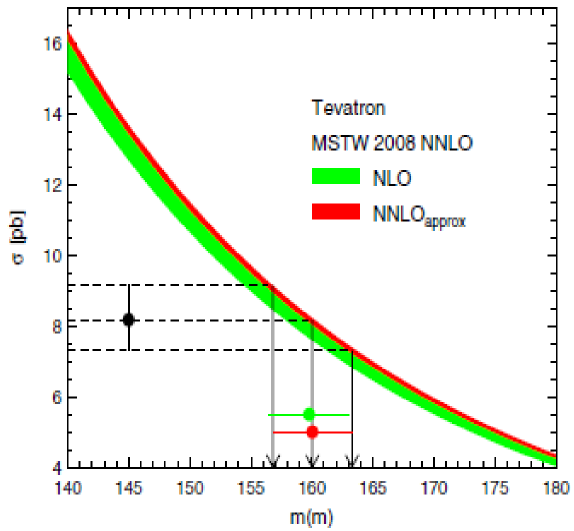
$$\bar{m}(\bar{m}) \;=\; 160.0^{+3.3}_{-3.2}\,[\mathrm{GeV}] \qquad (\sim\mathrm{NNLO})$$

$$\rightarrow \text{ pole mass } m_t = 168.9 \pm 3.5 \text{ GeV}$$

Another choice may be "threshold mass"



PHYSICAL REVIEW D 80, 054009 (2009)



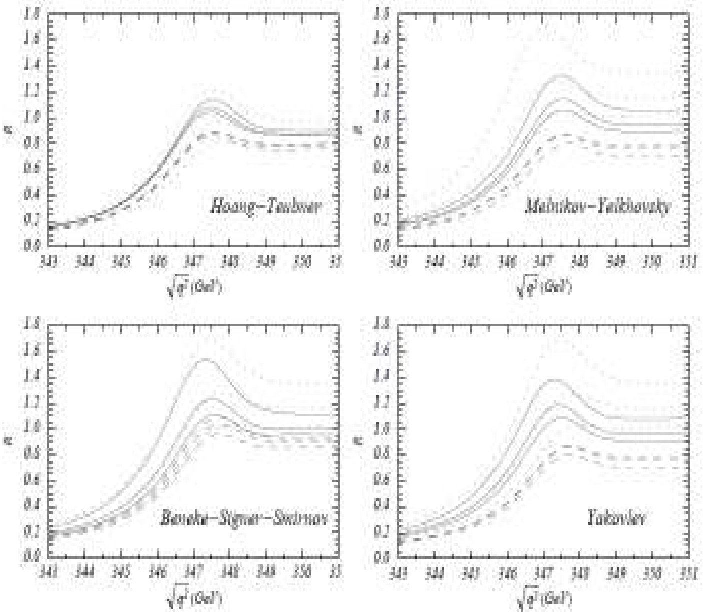
(b)

1. Introduction

- In e^+e^- colliders, "Threshold Scan" can be performed

(p,v)NRQCD up to N^nLL/N^nLO

hep-ph/0001286



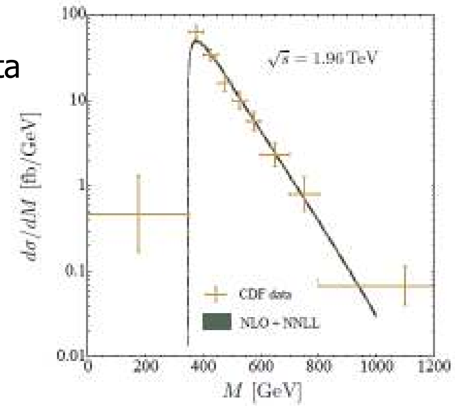
precise determinations of the top-quark mass,
width and strong coupling constant are possible

$\delta m_t \sim 100\,[\mathrm{MeV}] \quad \delta\Gamma_t/\Gamma_t \sim 20\,[\%]$

- Peak of the 1S resonance ⇔ Threshold mass
  (1S mass, PS mass,,,)

Relation between the threshold mass, $\overline{\mathrm{MS}}$ mass and pole mass is well-known to higher-orders

CDF data



- However, at hadron colliders, (partonic) collision energy is not fixed, so one has to reconstruct top-pair invariant mass

It would be also a challenging task to have good precision at the LHC

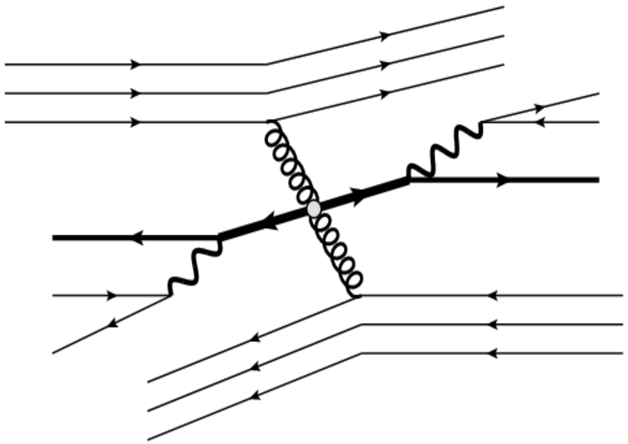
1. Introduction

- Top-quark production at Hadron Colliders

$$d\sigma_{t\bar{t}}(s) = \sum_i \int d\tau \frac{dL_i}{d\tau}(\tau) d\hat{\sigma}_i(\hat{s} = \tau s)$$

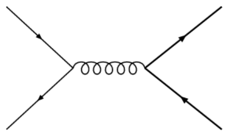
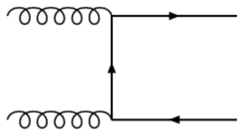
partonic cross-section

partonic luminosity

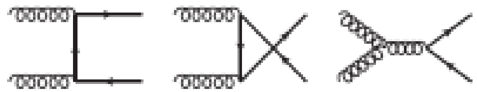


1. Introduction

- Partonic subprocess at Hadron Colliders

			Tevatron	LHC
$q\bar{q} \to t\bar{t}$		Color: Octet |J|=1	85%	10%
$gg \to t\bar{t}$		Color: Singlet, Octet |J|=0,1,2,,,	15%	90%

- Color decomposition in gg process is obtained from the color-matrix in the amplitude



$$\mathcal{M}_{gg\to t\bar{t}} = (T^aT^b)_{ij}\mathcal{M}_1 + (T^bT^a)_{ij}\mathcal{M}_2$$
$$= \frac{1}{2}\{T^a, T^b\}_{ij}\mathcal{M}_S + \frac{1}{2}[T^a, T^b]_{ij}\mathcal{M}_A$$

$$\frac{1}{2}\{T^a, T^b\} = \underline{\frac{1}{2N_c}\delta^{ab}}\delta_{ij} + \frac{1}{2}d^{abc}T^c_{ij}$$

color-singlet

$$\left|\frac{1}{2N_c}\delta_{ab}\delta_{ij}\right|^2 / \left|\frac{1}{2}d^{abc}T^c_{ij}\right|^2 = \frac{2}{N_c^2 - 4}$$

~20-30% of ttbar are color-singlet in gg fusion

1. Introduction

- Perturbative calculations for the partonic cross-section :

$$\hat{\sigma}_i(\hat{s}) = \hat{\sigma}_i^{(0)}(\hat{s}) + \frac{\alpha_s}{\pi}\hat{\sigma}_{ij}^{(1)}(\hat{s};\mu) + \left(\frac{\alpha_s}{\pi}\right)^2 \hat{\sigma}_i^{(2)}(\hat{s};\mu) + \cdots$$

Fixed-Order Calculation :

- NLO : Dawson,Ellis,Nason('88), Beenakker etal.('90),Mangano,Nason,Ridolfi(92)
  (analytic) : Cazkon,Mitov('08)
- Towards Full NNLO correction :
  Korner etal.('06),Dittmaier etal.('07),Cazkon etal.('07),,,
- ElectroWeak correction : Bernreuther,Fuecker,Si('06), Kuhn,Scharf,Uwer('06),Moretti,Nolten,Ross('06)

Resummation :

- Threshold Resummation
  NLL : Bonciani etal.('98),,, ;
  NNLL : Moch,Uwer('08), Ahrens et al ('10), Beneke et al ('11)
- **Coulomb Summation** : Catani,Mangano,Nason,Trentadue('96), Hagiwara,Sumino,HY('08), Kiyo etal('08), Sumino,HY('10)


TOP 2011
4th International Workshop on
TOP QUARK PHYSICS
September 25-30, 2011
Sant Feliu de Guixols, Costa Brava, SPAIN

Talk by P.Uwer at TOP2011

NLO predictions --- Theoretical accuracy

$m_t = 173.1 GeV/c^2, m_t/2 < \mu < 2m_t$ MSTW2008 PDF(90%cl)

	Tevatron	LHC (7 TeV)
σ_{LO} (pb)	$6.66^{+2.95\,+0.34}_{-1.87\,-0.27}$	$122^{+49\,+6}_{-32\,-7}$
σ_{NLO}	$6.72^{+0.36\,+0.37}_{-0.76\,-0.24}$	$159^{+20\,+8}_{-21\,-9}$

NLO

$$\frac{\Delta\sigma}{\sigma} \approx \begin{cases} {}^{+5\%}_{-10\%}\ \text{(scale)}\ {}^{+5\%}_{-4\%}\ \text{(pdf) Tevatron} \\ {}^{+12\%}_{-13\%}\ \text{(scale)}\ \pm 5\%\ \text{(pdf) LHC} \end{cases}$$

Talk by P.Uwer at TOP2011

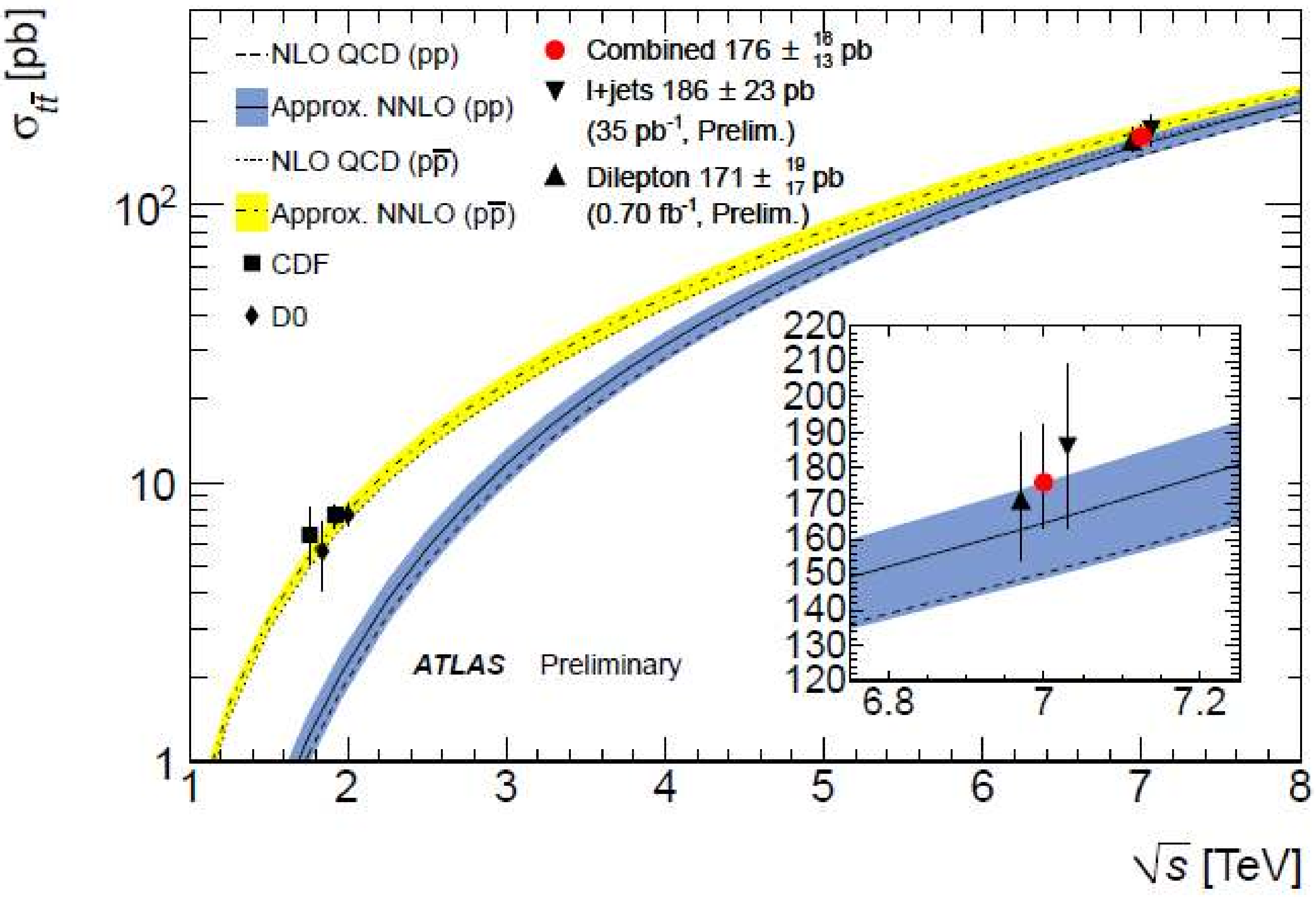
Top-quark pair production in NLO QCD

Experimental accuracy below 10 %

→ need to go beyond NLO accuracy

Possible corrections (percent level):

QCD NNLO	Bound state effects	Mixed weak-QCD corrections	finite width
$\sim \alpha_s^4$	"$\sim (\alpha_s^3 \frac{1}{\beta})\beta$"	$\sim \alpha_s^2 \alpha$	Γ_t / m_t



NLO QCD (pp)
Approx. NNLO (pp)
NLO QCD (pp̄)
Approx. NNLO (pp̄)
CDF
D0
Combined 176 ± 18/13 pb
l+jets 186 ± 23 pb
(35 pb-1, Prelim.)
Dilepton 171 ± 19/17 pb
(0.70 fb-1, Prelim.)
ATLAS Preliminary
σ_tt̄ [pb]
√s [TeV]

2. Bound-state effects on ttbar production at Hadron Colliders

2. Bound-state effects at Hadron colliders

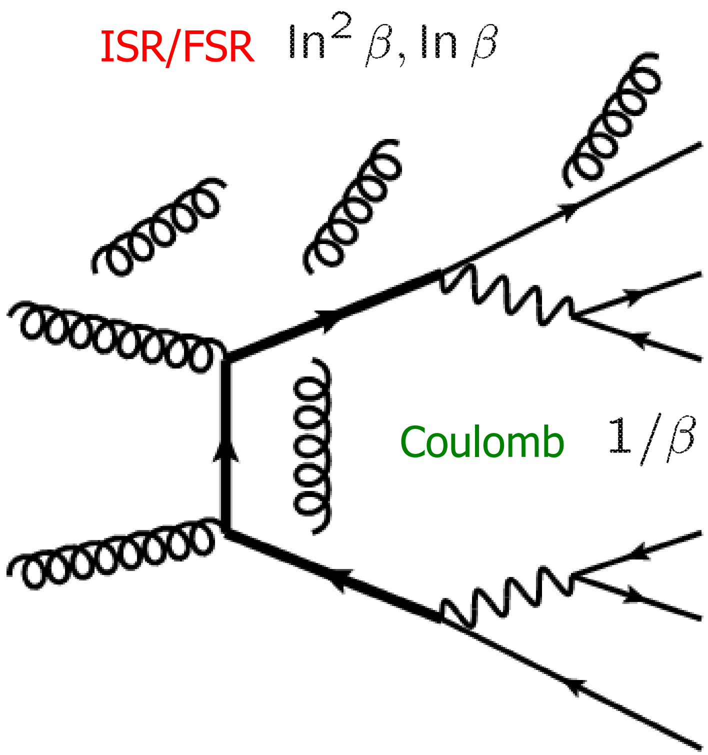
- QCD higher-order correction Nason,Dawson,Ellis

$$\hat{\sigma}_i^{(c),\mathrm{NLO}} \sim \hat{\sigma}_i^{(c),\mathrm{LO}} \frac{\alpha_s}{\pi} \left\{ A_i \ln^2 (8\beta^2) + B_i^{(c)} \ln (8\beta^2) + C_i^{(c)} \frac{\pi^2}{\beta} + D_i^{(c)} + \mathcal{O}(\beta) \right\}$$

Threshold logs : emission of soft and/or collinear gluon in initial-state and final-state

Coulomb singularity : Coulomb gluon exchange between t and t-bar

Hard correction : process dependent virtual corr.



2. Bound-state effects at Hadron colliders

- Factorization of each contribution : Beneke,Falgari,Schwinn ('09)

⇒ Resummation of each correction separately

$$\hat{\sigma}_i^{\rm NLO} \sim \hat{\sigma}_i^{\rm LO}\left[1+\frac{\alpha_s}{\pi}\left(A_i \ln^2\beta^2 + B_i^{(c)}\ln\beta^2 + C^{(c)}\frac{\pi^2}{\beta} + D_i^{(c)} + \mathcal{O}(\beta)\right)\right]$$

$$\sim \hat{\sigma}_i^{\rm LO}\left[1+\frac{\alpha_s}{\pi}\left(A_i \ln^2\beta^2 + B_i^{(c)}\ln\beta^2\right)\right]\left[1+\frac{\alpha_s}{\pi}C_i^{(c)}\frac{\pi^2}{\beta}\right]\left[1+\frac{\alpha_s}{\pi}D_i^{(c)}\right] + \mathcal{O}(\beta)$$

- **Threshold resummation**

Laenen,Smith,van Neerven('92),
Kidonakis Sterman('97),Bonciani etal.('98)
Moch,Uwer('08), Caccari etal('08),
Ahrens etal ('10), Beneke et al('11),,,

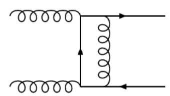
- **Coulomb summation, or Bound-state formation**

Fadin,Khoze,Sjostrand('90)
Catani,Mangano,Nason,Trentadue('96)
Hagiwara,Sumino,HY('08)
Kiyo,Kuhn,Moch,Steinhauser('09)
Sumino,HY('10)

2. Bound-state effects at Hadron colliders

Coulomb corrections to all-orders

- Coulomb singularity

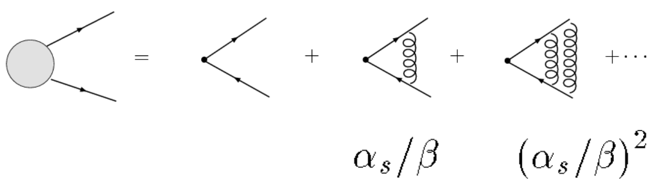


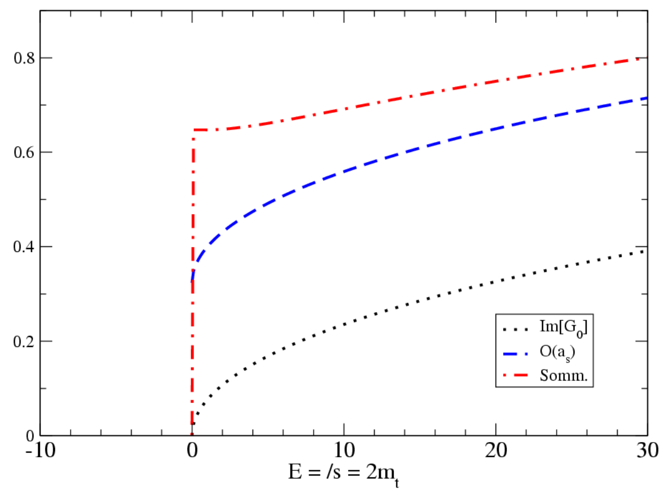
$$\propto C^{(c)}\frac{\alpha_s}{\beta} \qquad \mathcal{O}(1) \text{ for } \beta \simeq \alpha_s$$

color-factor $\begin{cases} \text{singlet} & C^{(1)} = -C_F \\ \text{octet} & C^{(8)} = C_A/2 - C_F \end{cases}$

- Summation of ladder diagrams = Sommerfeld factor Sommerfeld,Sakharov (QED)

$$S(z) = \frac{z}{1-\exp[z]}, \quad z = C^{(c)}\pi\alpha_s/\beta$$





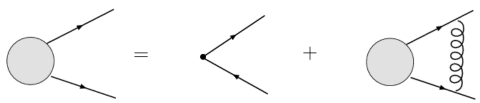
2. Bound-state effects at Hadron colliders

☐ Coulomb corrections to all-orders

- Green's function formalism (NRQCD) Fadin,Khoze('87),,,

Schrodinger's Eq. $$\left[(E+i\Gamma_t)-\left\{-\frac{\nabla^2}{m_t}+V^{(c)}_{QCD}(r)\right\}\right]G^{(c)}(E,\vec{x})=\delta^3(\vec{x})$$

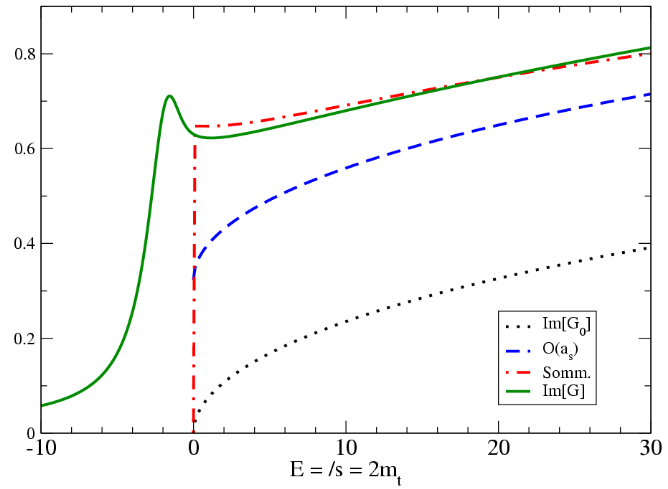
where $E=m_{tt}-2m_t$



finite width effect is incorporated by complex energy
⇒ Off-shellness of top-quarks

$$G(E,\vec{x})=\sum_n\frac{\Psi_n(\vec{x})\Psi_n^*(0)}{E-E_n+i\Gamma_n/2}+\text{continuum}$$

- Large width smears the multiple resonance structure, but only one broad peak can be seen as a remnant.



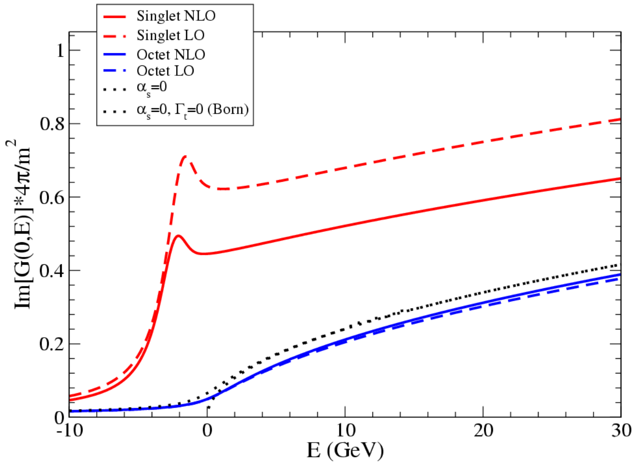
2. Bound-state effects at Hadron colliders

- Perturbative QCD potential (NLO), since an IR cut-off by $r \lesssim \frac{1}{\Gamma_t}$

$$V^{(c)}_{\rm QCD}(r) = C^{(c)}\frac{\alpha_s(\mu_B)}{r} \times \left[1 + \frac{\alpha_s}{\pi} v_1^{(c)}(r) + \cdots\right]$$

$$\begin{cases} \text{singlet} & C^{(1)} = -4/3 \\ \text{octet} & C^{(8)} = 1/6 \end{cases}$$

Singlet is attractive,
but octet is repulsive and small color-factor



- Toponium system : $m_t \gg \mu_B > E_B \simeq \Gamma_t \gg \Lambda_{\rm QCD}$
  - Binding energy : $E_B \simeq m_t \alpha_s^2 \simeq 2\text{GeV}$
  - Bohr radius : $\mu_B \simeq m_t \alpha_s \simeq 20 - 30\text{GeV}$

(typical momentum of the Coulomb gluon)

If $\Gamma_t > E_B$, top-quark decays before bound-state formation

2. Bound-state effects at Hadron colliders

- Cross-section is proportional to the Imaginary part of the Green function by Optical theorem.

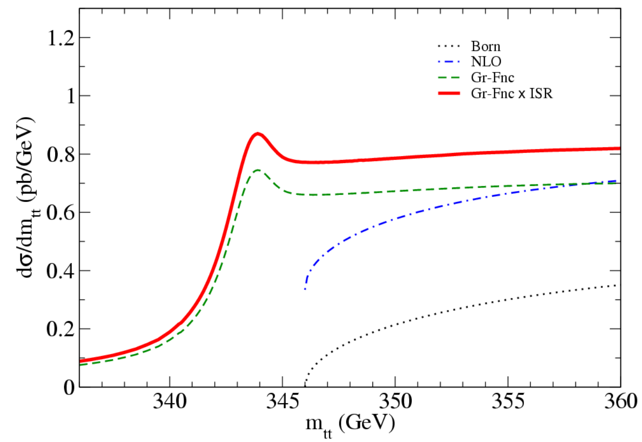
$$\hat{\sigma}_{tt}^{(c)} \to \hat{\sigma}_{tt,\text{Born}}^{(c)} \cdot \text{Im}[G^{(c)}(E, \vec{x} = \vec{0})]$$

- Combining Initial-state/Final-state radiation effects,

$$\frac{d\sigma}{dm_{tt}}(s, m_{tt}^2) = \hat{\sigma}_i^{(c)}(m_{tt}^2) \cdot K_i^{(c)} \int_{\tau_0}^{1} \frac{dz}{z} F_i^{(c)}(z) \frac{d\mathcal{L}_i}{d\tau}(\tau_0/z)$$

$$F_i^{(c)}(z) = \delta(1-z) + \frac{\alpha_s}{\pi}\left[A_i\left\{\left(\frac{\ln(1-z)}{1-z}\right)_+ - \left(\frac{1}{1-z}\right)_+ \ln\left(\frac{\mu_F}{2m_t}\right)\right\} + D_{tt}^{(c)}\left(\frac{1}{1-z}\right)_+ + k_i^{(c)}\delta(1-z)\right]$$

gg->tt, color-singlet at the LHC



NLL : Kiyo et al. ('08)

- Color-dependent hard correction : $K_i^{(c)} = 1 + \frac{\alpha_s}{\pi} h_i^{(c)}$

  extracted from the Quarkonium cross-section at NLO

  Petrelli,Cacciari,Greco,Maltoni,Mangano ('98)
  +Non-decoupling term HSY('08), Czakon,Mitov('08)

$$h_{gg}^{(1)}\left(\frac{\mu_R}{m_t}\right) = C_A\left(1+\frac{\pi^2}{12}\right) + C_F\left(-5+\frac{\pi^2}{4}\right) + \beta_0 \ln\left(\frac{\mu_R}{2m_t}\right),$$

$$h_{gg}^{(8)}\left(\frac{\mu_R}{m_t}\right) = C_A\left(3-\frac{\pi^2}{24}\right) + C_F\left(-5+\frac{\pi^2}{4}\right) + \beta_0 \ln\left(\frac{\mu_R}{2m_t}\right),$$

$$h_{q\bar{q}}^{(8)}\left(\frac{\mu_R}{m_t}\right) = C_A\left(\frac{59}{9}-\frac{\pi^2}{4}+\frac{2\ln 2}{3}\right) + C_F\left(-8+\frac{\pi^2}{3}\right) - \frac{5}{9}n_q - \frac{8}{9} + \beta_0 \ln\left(\frac{\mu_R}{2m_t}\right).$$

2. Bound-state effects at Hadron colliders

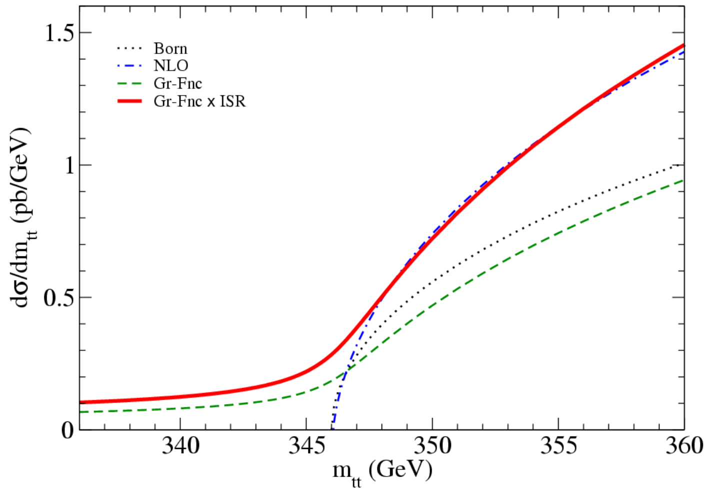
m_t=173 GeV, Γ_t=1.5 GeV, CTEQ6M

☐ ttbar invariant-mass distributions

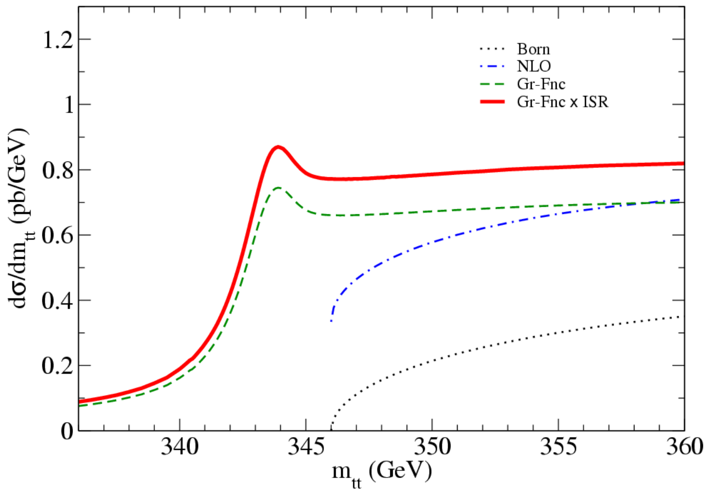
Black : Born
Blue : O(as) corr. (NLO)
Green : Gr-Fnc. without ISR
Red : Gr-Fnc. with ISR

ISR : up to O(a_s) (soft/collinear)

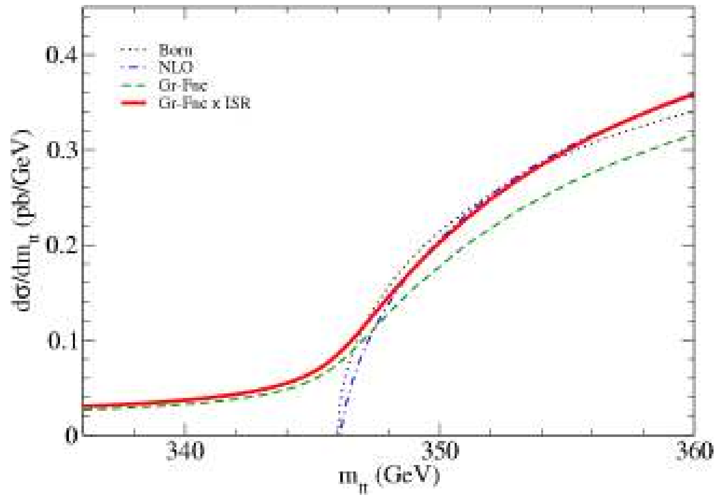
gg->tt, color-octet



gg->tt, color-singlet



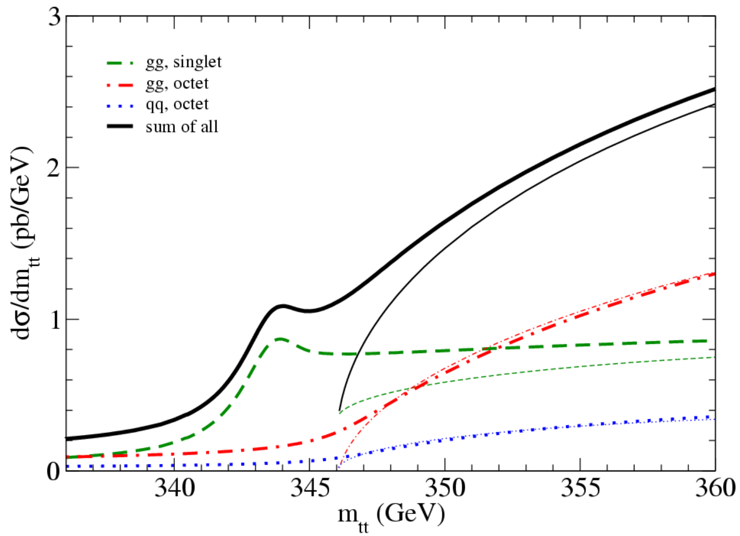
qq->tt, color-octet



2. Bound-state effects at Hadron colliders

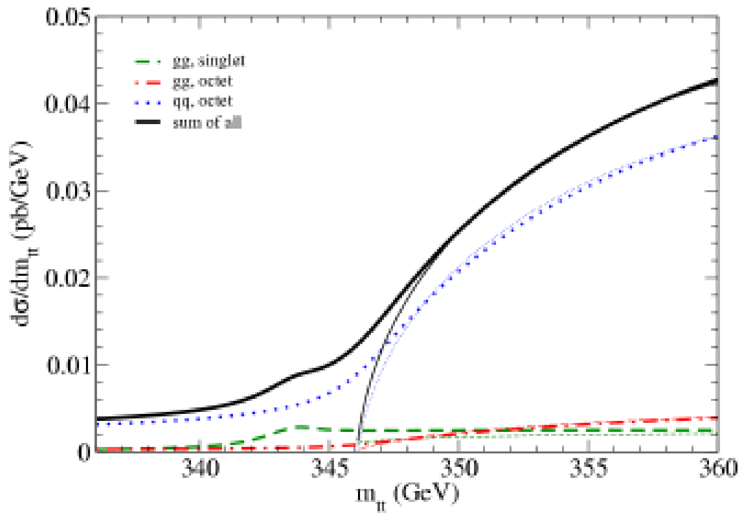
◆ In total at the LHC :

- BS effects deform the invariant-mass distribution near threshold
- Form a broad resonance peak below threshold (observable in principal)
- Enhance the total cross-section by 10 pb ~ O(1%)



◆ On the other hand at the Tevatron :

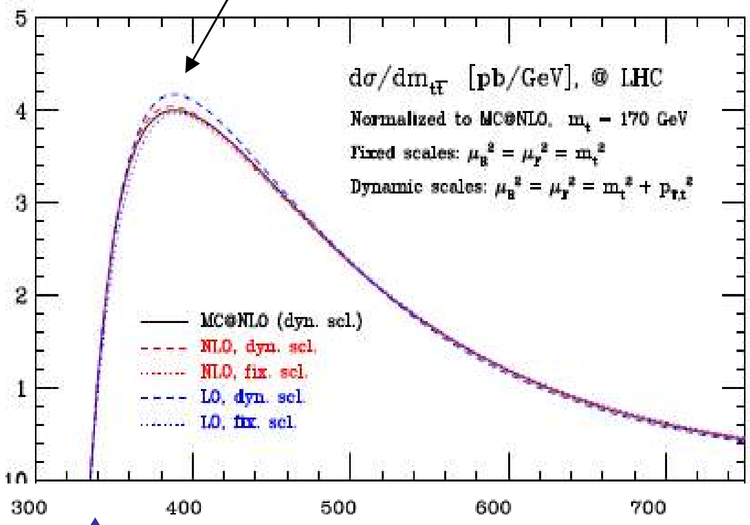
- Resonance can't be seen due to the color-octet dominance
- Correction is small due to the small color-factor.

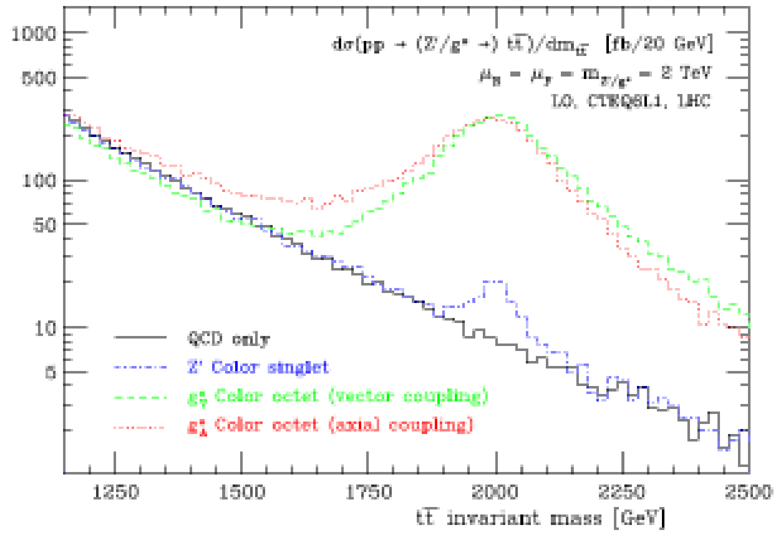


- ttbar Invariant-mass distribution : $m_{tt}^2 = (p_t + p_{\bar{t}})^2$

peak region $\beta \sim 0.5$

Frederix,Maltoni('07)





threshold region $\beta \leq 0.3$ (m_{tt} < 360 GeV)

High-energy region $\beta \sim 1$

(Boosted top, **NP resonances**, FB asymmetries?,,)

3. Differential Cross-section /

Event Generation with BS effects

3. Differential Cross-section

□ Coulomb correction in differential cross-sections

- Differential distributions are useful for the analysis with kinematical cuts
- Full differential distribution is required in Event-Generation by Monte-Carlo simulation
- Method to include BS effects is developed in e^+e^- collider study

Jezabek,Kuhn,Teubner('92)
Sumino,Fujii,Hagiwara,Murayama,Ng('93),,,

- Our Aim : take into account the "Leading-order" contribution in both region :

Sumino,HY ('10)

- Threshold region : $(\alpha_s/\beta)^n$ but not α_s^n, β^n
- High-energy region : β^n but not α_s^n
- plus some NLO effects (K-factor, width, QCD potential,,)   note, $\Gamma_t/m_t \sim \alpha_W \sim \alpha_s^2$

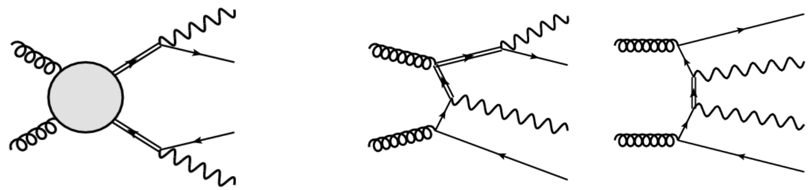
3. Differential Cross-section

Coulomb correction in differential cross-sections

- gg/qq to bWbW process to take into account the off-shellness of top-quarks

- Divide resonant part and non-resonant part of the amplitude (gauge dependent)

$$\mathcal{M}^{(c)}(I \to bWbW) = \mathcal{M}^{(c)}_{t\bar{t}} + \mathcal{M}^{(c)}_{\mathrm{nr}}$$



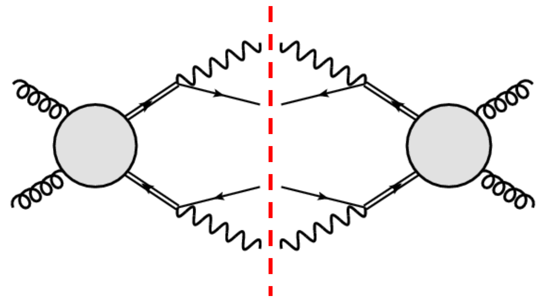
- Double resonant part of the amplitude :

$$\mathcal{M}_{t\bar{t}} = \bar{D}_{t\to bW} \cdot \frac{i}{\not{p}_t - m_t + i\Gamma_t} \cdot P_{i\to t\bar{t}} \cdot \frac{i}{-\not{p}_{\bar{t}} - m_t + i\Gamma_t} \cdot D_{\bar{t}\to\bar{b}W}$$

3. Differential Cross-section

- A correction factor to the resonant part by the Momentum-Space Green Functions

$$\mathcal{M}_{t\bar{t}}^{(c)} \to \mathcal{M}_{t\bar{t}}^{(c)} \times \frac{\tilde{G}^{(c)}(E + i\Gamma_t, \vec{p})}{\tilde{G}_0(E + i\Gamma_t, \vec{p})}$$



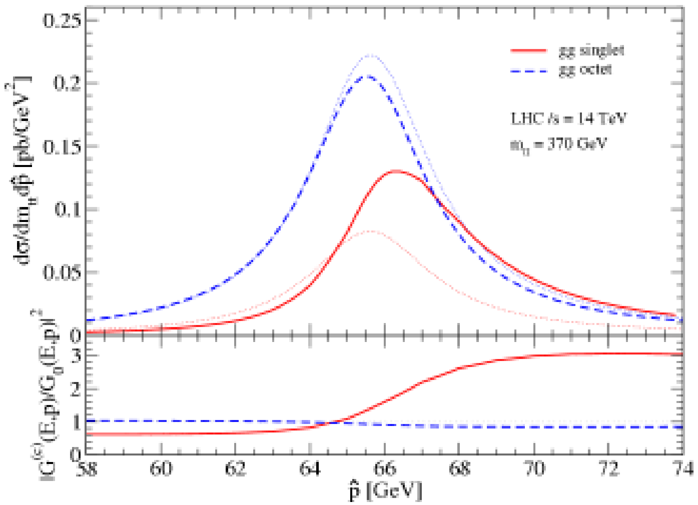
(for S-wave only |p| dependence)

- Top-quark momentum distribution (of the tt cm frame) is affected by the Coulomb correction.

$$\frac{d\sigma}{d|\vec{p}|} \propto |\tilde{G}(E, \vec{p})|^2$$

color-singlet : $\delta p > 0$

color-octet : $\delta p < 0$



3. Event Generation

“Toponium@LHC” Sumino,HY (‘10)

http://madgraph.kek.jp/~yokoya/TopBS

☐ Event Generator (LO + all-order Coulomb) :

- Full gg/qq→bWbW plus W-decays Matrix-Elements (6-bodys)
- Color-decomposition in gg→bWbW process
- Bound-state correction to the double-resonant amplitudes
- Color-dependent K-factors to reproduce NLO m_{tt} dist. near threshold

- Difference from General-purpose Monte-Carlo’s :

  MadGraph/MadEvent, Sherpa,,, (PYTHIA, HERWIG,,,)

  LO(Tree-level), ○non-resonant effects, off-shell effect,,,

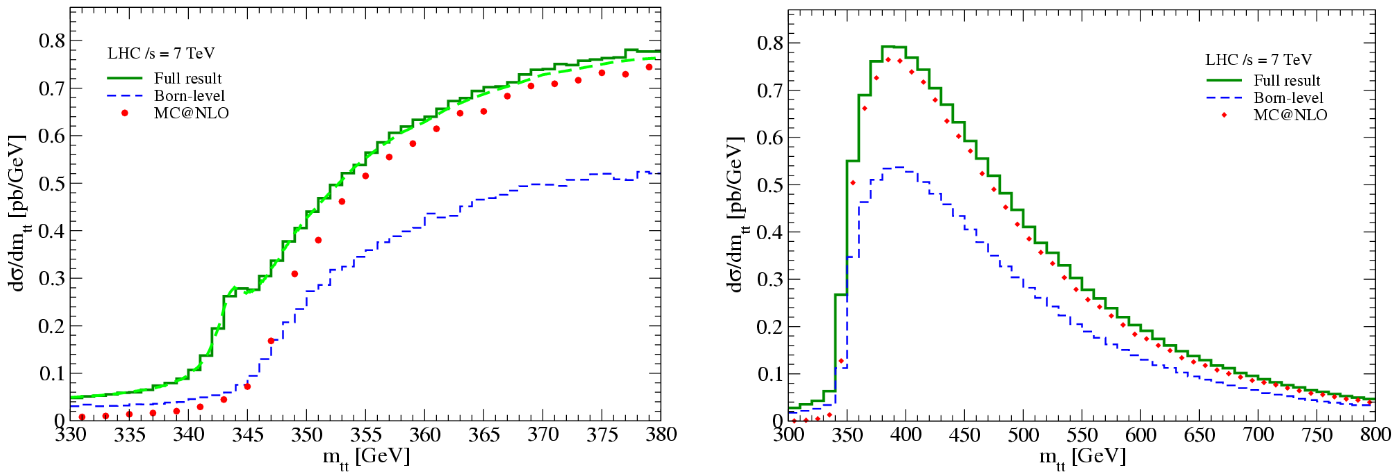
  MCFM, MC@NLO,,,

  NLO (α_s/β term), ×non-resonant effects, Breit-Wigner,,,

3. Event Generation

☐ Some Examples (at partonic-level)

(1) ttbrar invariant-mass (m_{tt}) distribution :



- The only generator which describes the threshold enhancement and resonance
- Effectively, well reproduce MC@NLO results at large m_{tt} by taking the scales as

$\mu = m_t \quad (\mu = \sqrt{m_t^2 + p_T^2}$ in MC@NLO)

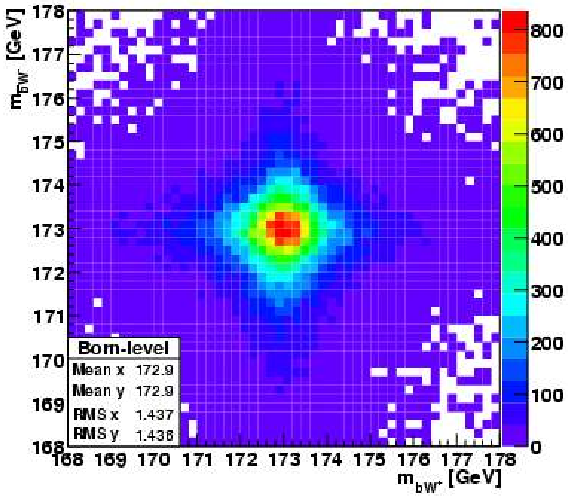
3. Event Generation

◻ Some Examples (at partonic-level)

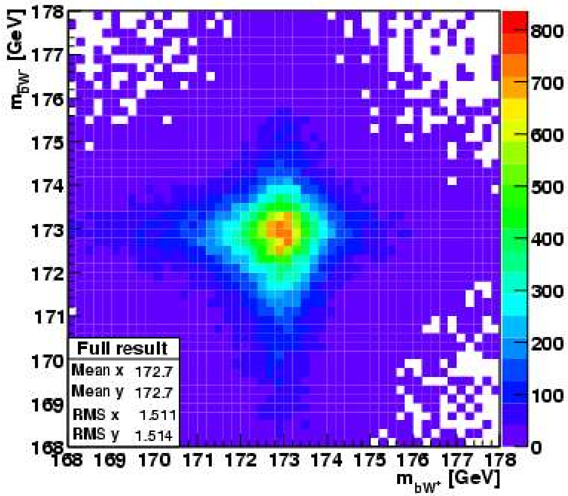
(2) (bW)-(bW) double invariant-mass distribution of top-quarks ; $m_{bW} = (p_b+p_W)^2$

limiting for the events with m_{tt}<370 GeV (10% of the total event)

Born



Coulomb



$\delta m \simeq -200\,[\mathrm{MeV}]$

correlated deviation : one top-quark is still on-shell,
but the other invariant-mass is reduced

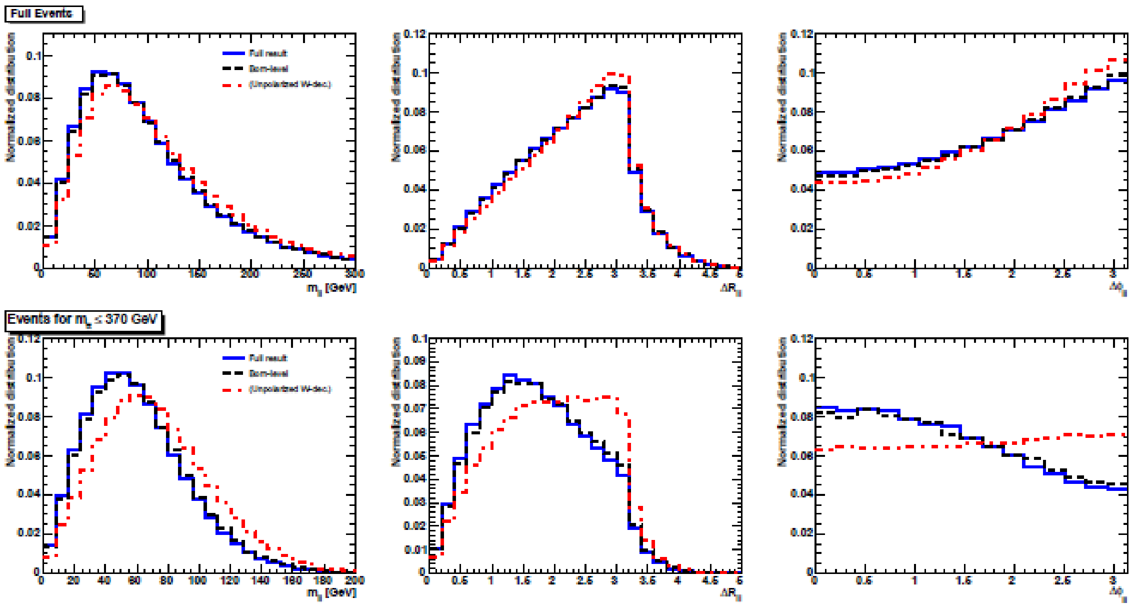
3. Event Generation

☐ Some Examples (at partonic-level)

(3) lepton angular distributions (di-lepton case)

- Unfortunately, small bound-state effects in final lepton angular distributions
- bWbW MEs plus W-decay by parton-shower give wrong distribution (no W polarization)

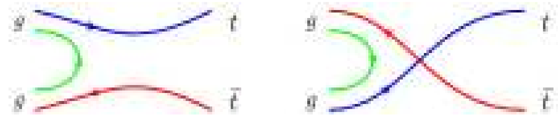
⇒ use 6-body MEs



3. Event Generation

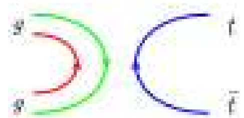
Color-flow assignment

in gluon-fusion process,



$$\mathcal{M}_{gg\to t\bar{t}} = (T^aT^b)_{ij}\mathcal{M}_1 + (T^bT^a)_{ij}\mathcal{M}_2$$
$$= \frac{1}{2}\{T^a,T^b\}_{ij}\mathcal{M}_S + \frac{1}{2}[T^a,T^b]_{ij}\mathcal{M}_A$$

symmetric part : $\frac{1}{2}\{T^a,T^b\} = \underline{\frac{1}{2N_c}\delta^{ab}\delta_{ij}} + \frac{1}{2}d^{abc}T^c_{ij}$



color-singlet

ratio of the amplitudes squared : $\left|\frac{1}{2N_c}\delta_{ab}\delta_{ij}\right|^2 \Big/ \left|\frac{1}{2}d^{abc}T^c_{ij}\right|^2 = \frac{2}{N_c^2-4}$

this is zero in large-N limit, but not in QCD

our color-singlet events have correct color-flow assignment in the LHEF record

4. Summary

Bound-state effects in the top-quark production at hadron colliders are studied

- At the LHC, gluon-fusion process dominates
  and substantial amount of ttbar pair is color-singlet.

- The bound-state effects are calculated for the m_{tt} distribution
  up to NLO (Green's func., gluon radiation, hard-correction)

- Large corrections in m_{tt} dist. near threshold is predicted,
  and a broad resonance below the threshold may be seen

- Correction to the total cross-section ~ 1%

4. Summary

- Differential cross-sections are also calculated including BS effects, non-resonant amp's as well as decays of Ws
  - incorporate momentum-space Green functions for color-singlet and octet
  - smooth interpolation to the high-energy region
  - non-resonant diagrams can be also taken into account
  - develop an Event-Generator to describe the threshold ttbar