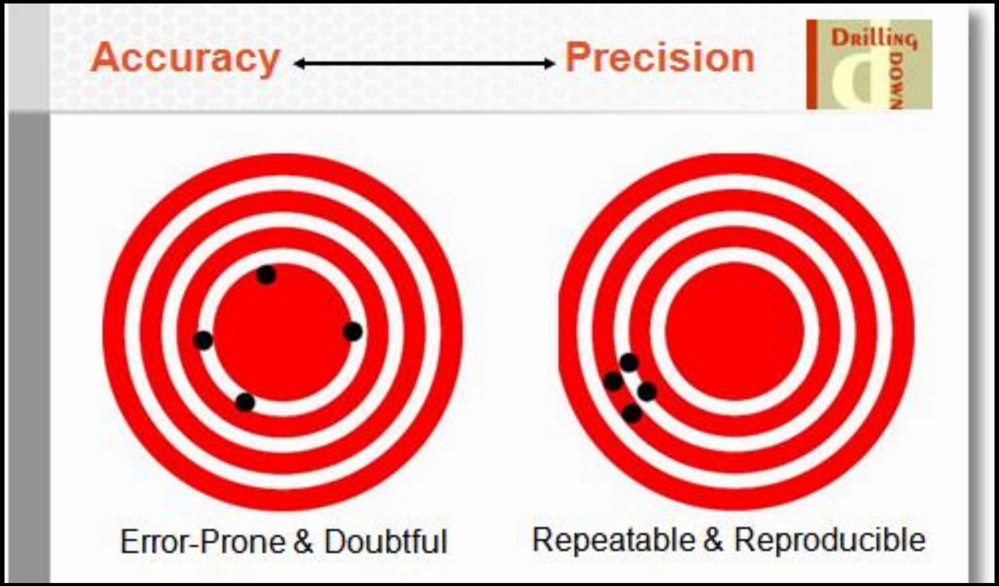
Accuracy cosmology
or, Testing DE with future surveys



Luca Amendola

ITP, University of Heidelberg

Euclid in a nutshell

Simultaneous (i) visible imaging (ii) NIR photometry (iii) NIR spectroscopy

15,000 square degrees

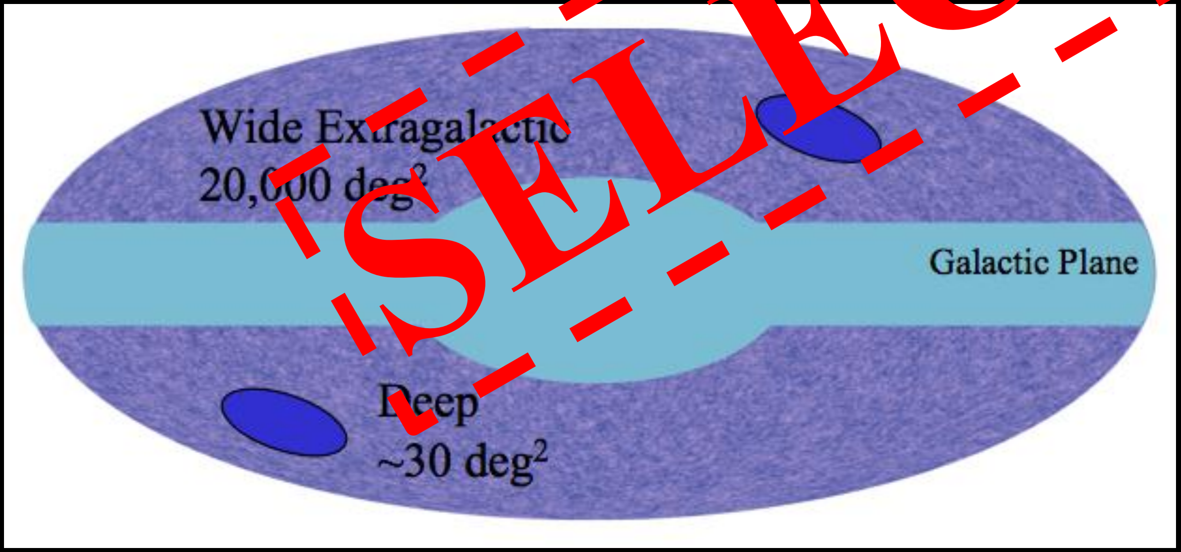
100 million redshifts, 2 billion images

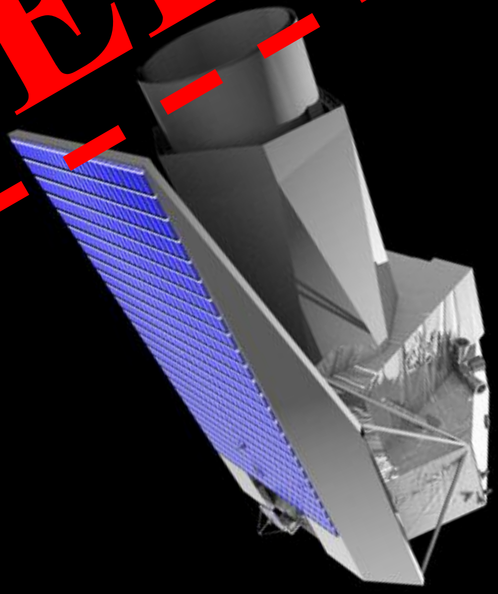
Median redshift z = 1

PSF FWHM ~0.18''

>800 peoples, >10 countries

SELECTED!





Euclid satellite

Euclid twin probes

P(k,z)

15,000 square degrees
70,000,000 galaxy redshifts
$0.5 < z < 2$

Weak lensing

15,000 square degrees
40 galaxy images per sq. arcmin
$0.5 < z < 3$

Precision is nothing without accuracy

1

The power of statistics

(collaboration E. Branchini, C. di Porto, C. Quercellini, V. Pettorino, A. Vollmer)

2

Lensing and supernovae

(collab. V. Marra, M. Quartin, J. Kannulainen)

3

Homogeneity and isotropy

(collab. C. Quercellini, M. Quartin)

Two free functions

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

- Poisson's equation $$\nabla^2\Psi = 4\pi G a^2 Q(k,a)\rho_m\delta_m$$

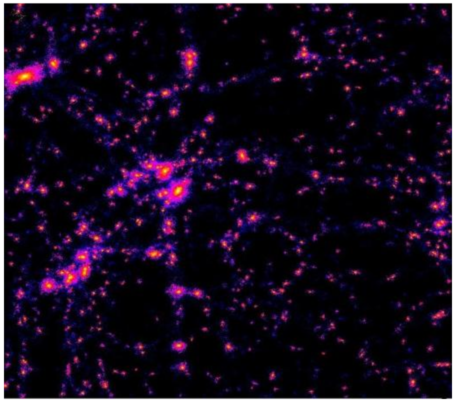
- anisotropic stress $$\eta(k,a) = \frac{\Phi+\Psi}{\Psi}$$

Modified Gravity at the linear level

▪ standard gravity	$Q(k,a)=1$ $\eta(k,a)=0$	
▪ scalar-tensor models	$Q(a)=\frac{G^*}{FG_{cav,0}}\frac{2(F+F'^2)}{2F+3F'^2}$ $\eta(a)=\frac{F'^2}{F+F'^2}$	Boisseau et al. 2000 Acquaviva et al. 2004 Schimd et al. 2004 L.A., Kunz &Sapone 2007
▪ f(R)	$Q(a)=\frac{G^*}{FG_{cav,0}}\frac{1+4m\frac{k^2}{a^2R}}{1+3m\frac{k^2}{a^2R}},\quad \eta(a)=\frac{m\frac{k^2}{a^2R}}{1+2m\frac{k^2}{a^2R}}$	Bean et al. 2006 Hu et al. 2006 Tsujikawa 2007
▪ DGP	$Q(a)=1-\frac{1}{3\beta};\quad \beta=1+2Hr_c w_{DE}$ $\eta(a)=\frac{2}{3\beta-1}$	Lue et al. 2004; Koyama et al. 2006
▪ coupled Gauss-Bonnet	$Q(a)=...$ $\eta(a)=...$	see L. A., C. Charmousis, S. Davis 2006

Reconstruction of the metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$



massive particles respond to Ψ

$$\dot{v} = -Hv - \nabla\Psi$$

massless particles respond to Φ-Ψ

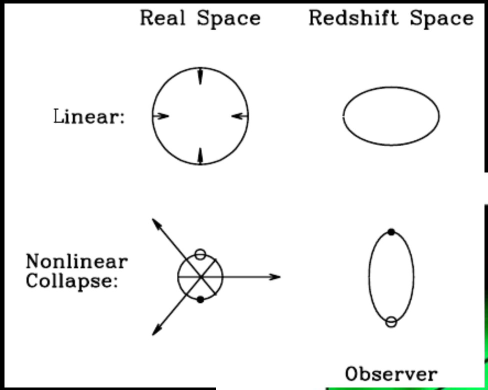
$$\alpha = \int \nabla_{perp}(\Psi - \Phi)dz$$

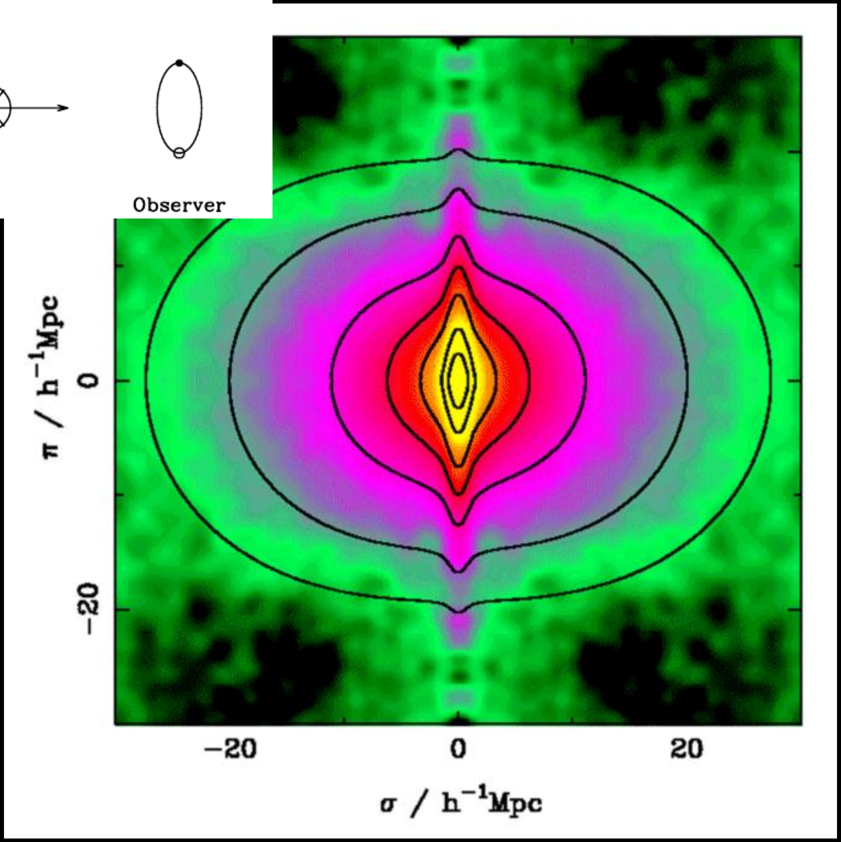
Peculiar velocities

$$z = z_{\mathrm{cosm}} + z_{pec.vel.}$$

Correlation of galaxy velocities:
galaxy peculiar field

$$\nabla v = -\delta'$$





$$P_z(k,\mu) = (1+\beta\mu^2)^2 P_r(k), \quad \mu = \cos\theta$$

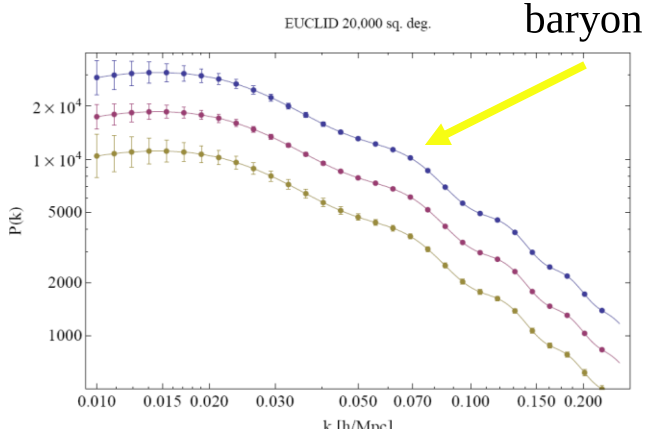
redshift distortion parameter

$$\beta = \frac{\delta'}{\delta b} = \frac{f}{b}$$

Kaiser 1987

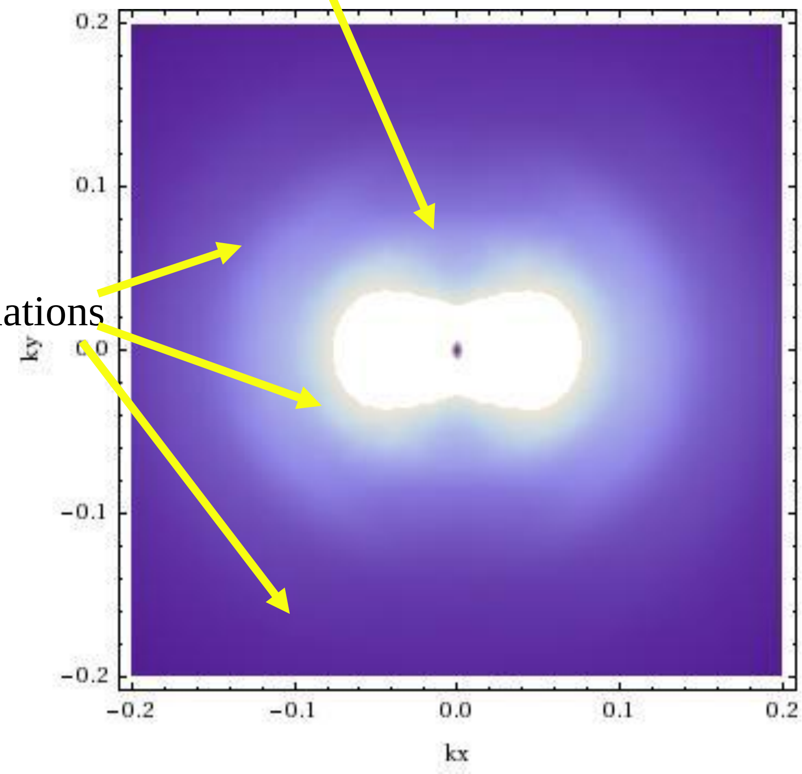
Clustering in redshift and momentum space

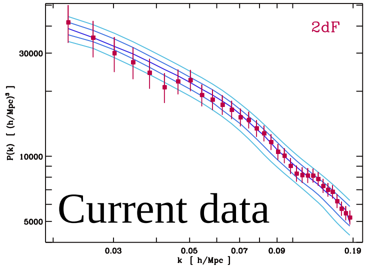
$$P_z(k,\mu,z) = G^2(z)(1+\beta\mu^2)^2 P_r(k)$$



baryon oscillations

redshift distortions

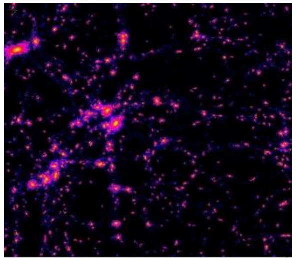


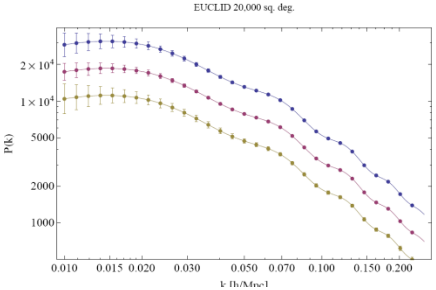


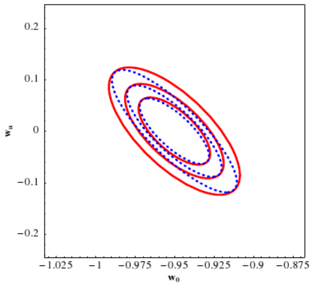
Current data

Firenze 2011

Euclid + Bayes + Fisher







$$P = N \exp\left[-\frac{1}{2}\sum_i \frac{\left[P_i - P_t(\theta_j)\right]^2}{\sigma_i^{\,2}}\right]$$

$$L = N \exp\left[-\frac{1}{2}\sum(\theta_i - \theta^{(F)}{}_i)F_{ij}(\theta_j - \theta^{(F)}{}_j)\right]$$

$$F_{ij} = \frac{1}{8\pi^2}\int_{-1}^{1} d\mu \int_{k_{\min}}^{k_{\max}} k^2 dk \, \frac{\partial \ln P(k,\mu)}{\partial \theta_i}\frac{\partial \ln P(k,\mu)}{\partial \theta_j}\left[\frac{nP(k,\mu)}{nP(k,\mu)+1}\right]^2 V_{survey}$$

Firenze 2011

Euclid beyond w

- Growth rate (mod. gravity)
- Sound speed
- Dark energy coupling
- Early dark energy
- Ultra-light fields
- Neutrino mass and generations
- Non-gaussianity
- ….

On growth, bias and amplitude

It is sometimes stated that galaxy clustering alone cannot constrain at the same time the growth rate, the σ_8 and the bias since they are degenerate. However, if they are parametrized this is no longer true.

$$P_z(k,\mu,z) = G^2(z)b^2(z)\sigma^2{}_8(1+\frac{f(z)}{b(z)}\mu^2)^2 P_r(k)$$

$$= b^2(z)e^{2\int f(z)d\ln a}\sigma^2{}_8(1+\frac{f(z)}{b(z)}\mu^2)^2 P_r(k)$$

$$= \beta^2(z)e^{2\int \Omega_m{}^{\gamma}(z)d\ln a}\Omega_m{}^{2\gamma}(z)\sigma^2{}_8(1+\beta(z)\mu^2)^2 P_r(k)$$

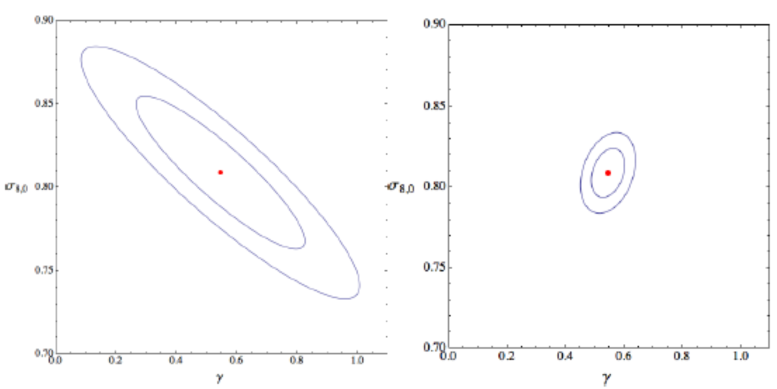
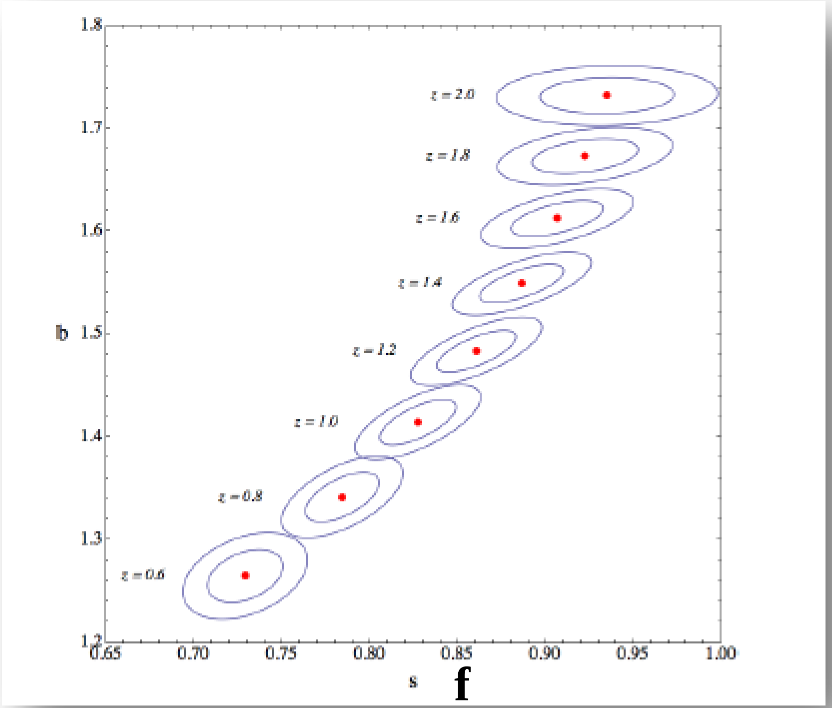


Figure 1: Contour plot of $\gamma - \sigma_8$. Left panel: marginalization over all other parameters; right panel: marginalization after fixing γ_1 and Ω_K

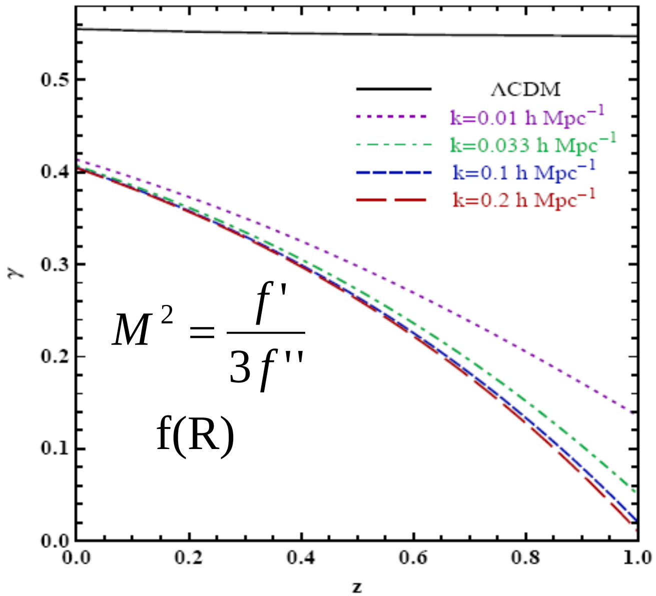
C. Di Porto, E. Branchini, L.A. 2011



Firenze 2011

Growth rate & modified gravity

Tsujikawa et al. 2009



$$\delta'' + \left(1 + \frac{H'}{H}\right)\delta - \frac{3}{2}\Omega_m Q(k,a)\delta = 0$$

$$Q = 1 + \frac{2\beta^2 k^2}{a^2 M^2 + k^2}$$

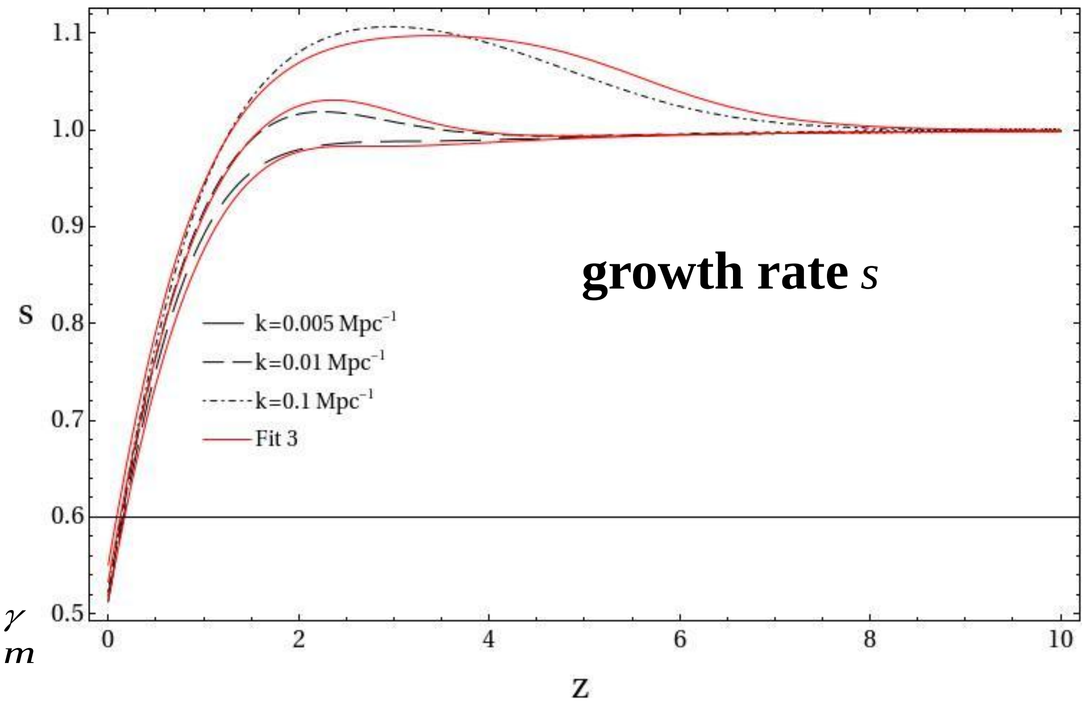
For scalar-tensor and f(R) and DGP etc.

$$s \equiv \frac{d \log \delta}{d \log a} = \Omega_m^\gamma$$

Standard Peebles fit

Peebles fit is not accurate in general !

reconstructing f(R)



growth rate *s*

one-parameter general fit

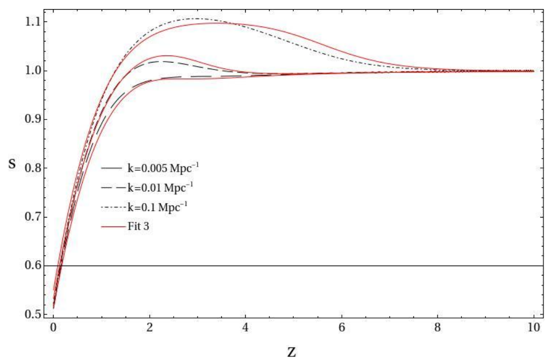
$$s \equiv \frac{d \log \delta}{d \log a} = \tilde{Q}^A \Omega_m^\gamma$$

$$\gamma = \frac{1 + 2\tilde{Q} - 3w}{1 + 4\tilde{Q} - 6w}$$

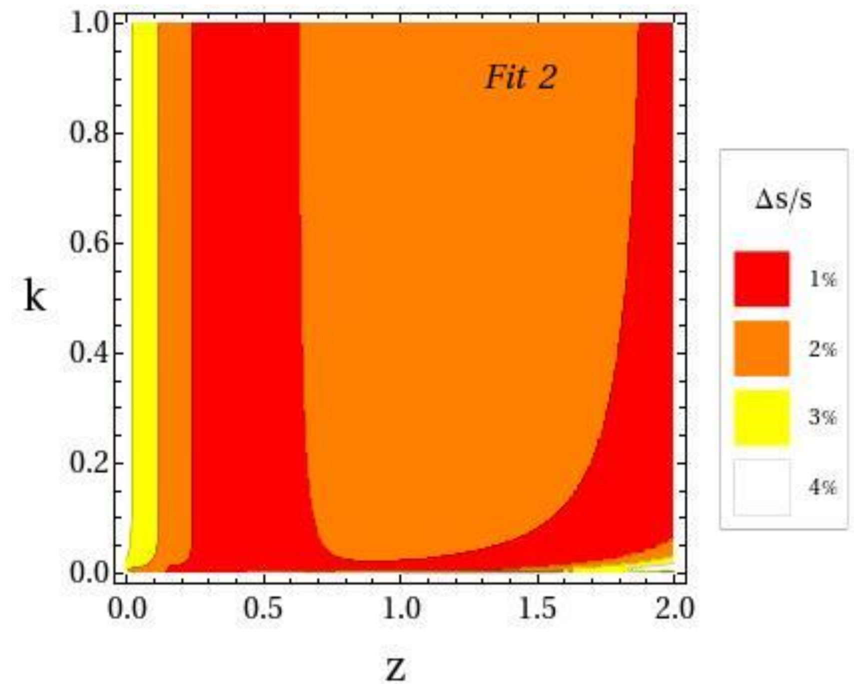
$$\tilde{Q} = \frac{1}{4}(-1 + \sqrt{1 + 24Q})$$

$$A \approx 0.7$$

E.g. f(R)



accuracy



models

$$f(R) = R + \lambda R_c f_1(x)$$

$$f_1 = x^{2n}/(x^{2n}+1)$$

$$f_1 = 1 - e^{-x}$$

$$f_1 = \tanh x$$

$$f_1 = \ldots$$

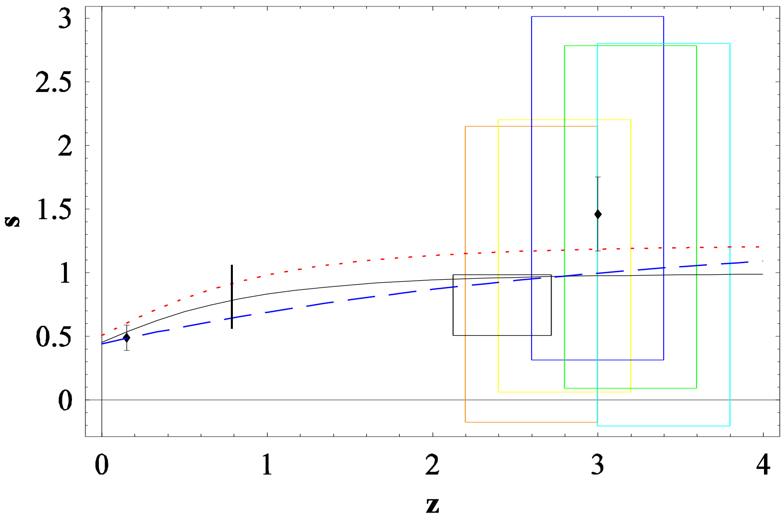
$$s(k,z) = \tilde{Q}^A \Omega_m^\gamma$$

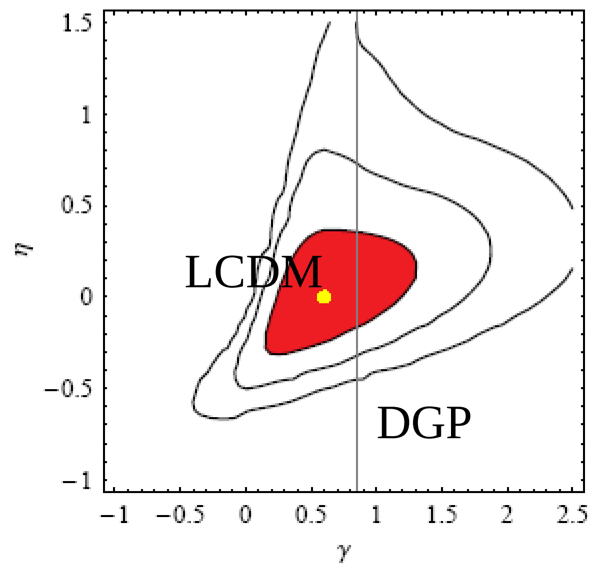
1 or 2 fit parameters

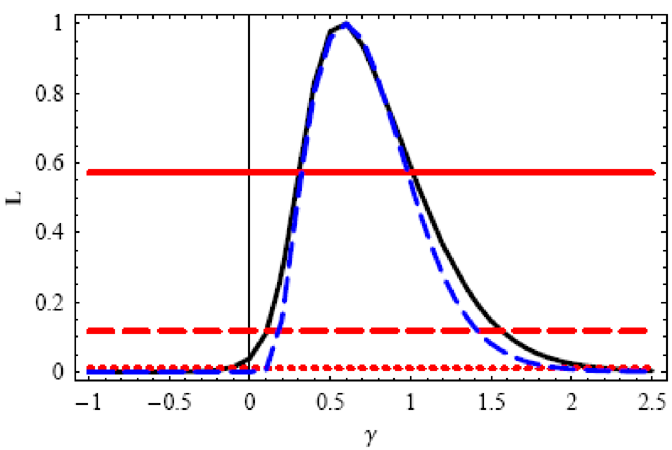
Di Porto, Quartin & LA 2011

Present constraints on gamma

$$s_{fit} \equiv \Omega_m^{\ \gamma}(1+\eta)$$





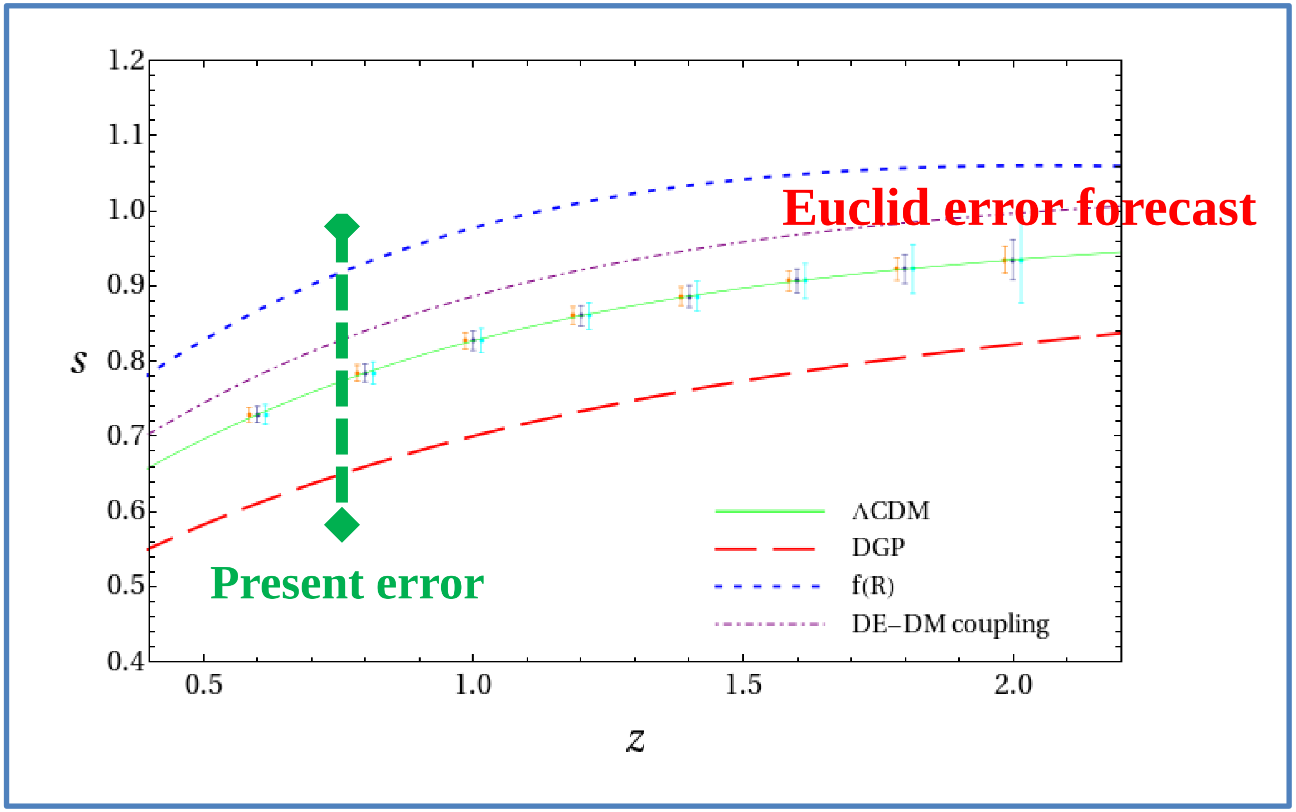


$$\gamma = 0.6 \pm 0.4$$

C. Di Porto & L.A. Phys.Rev. 2007

ze 2011

Euclid's challenge



Growth rate

$$s \equiv \frac{\delta'}{\delta}$$

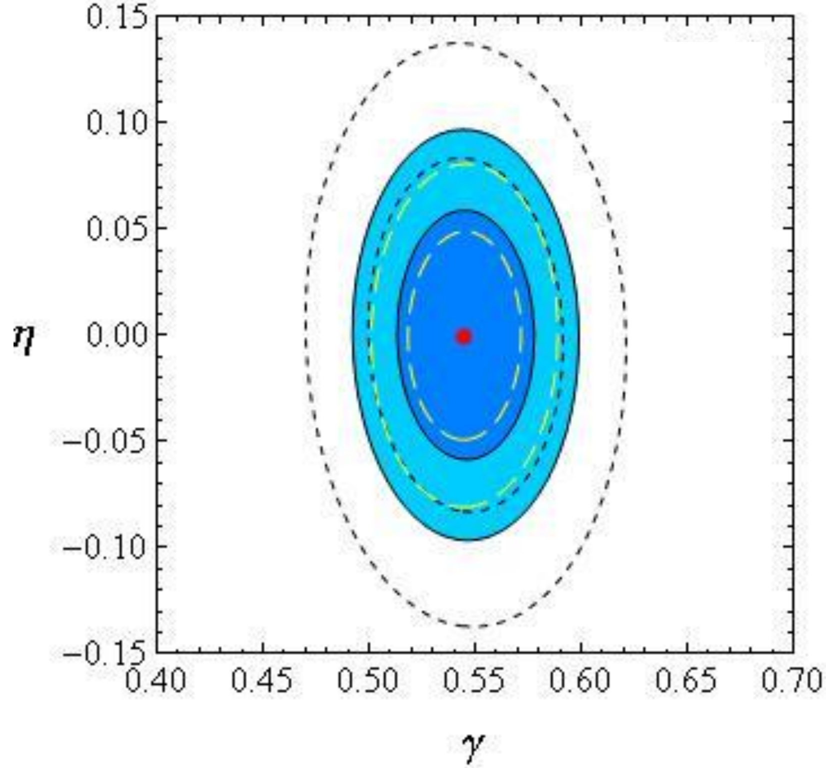
$$P_{g,z}(k,\mu,z) = G^2(z)b^2(z)(1+\frac{\delta'}{\delta b}\mu^2)^2 P_{m,r}(k,z=0)$$

C. Di Porto, L.A., E. Branchini 2011

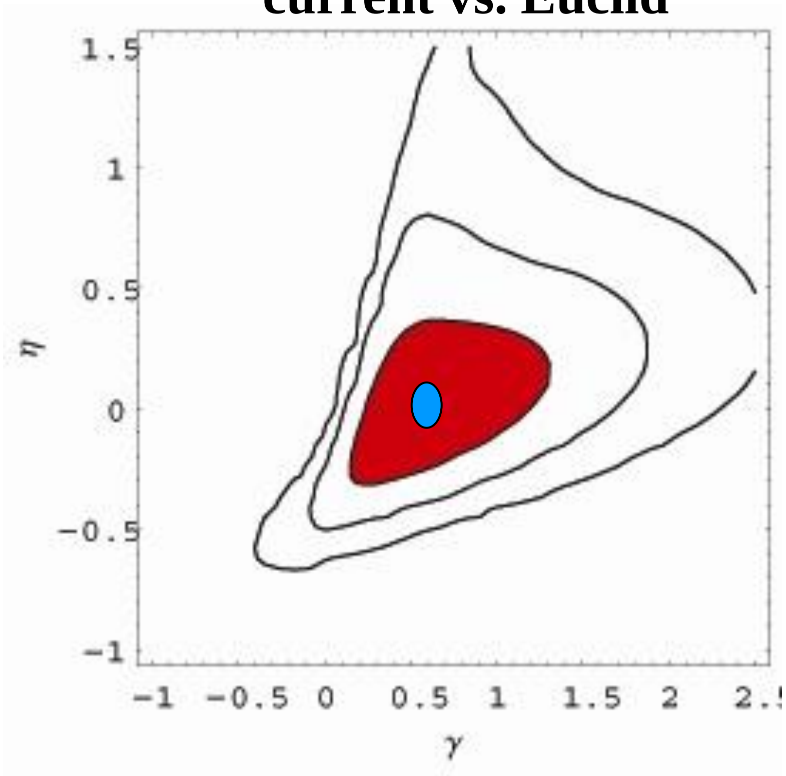
Euclid forecasts, I

$$s_{fit} \equiv \Omega_m^{\ \gamma}(1+\eta)$$

Euclid



current vs. Euclid

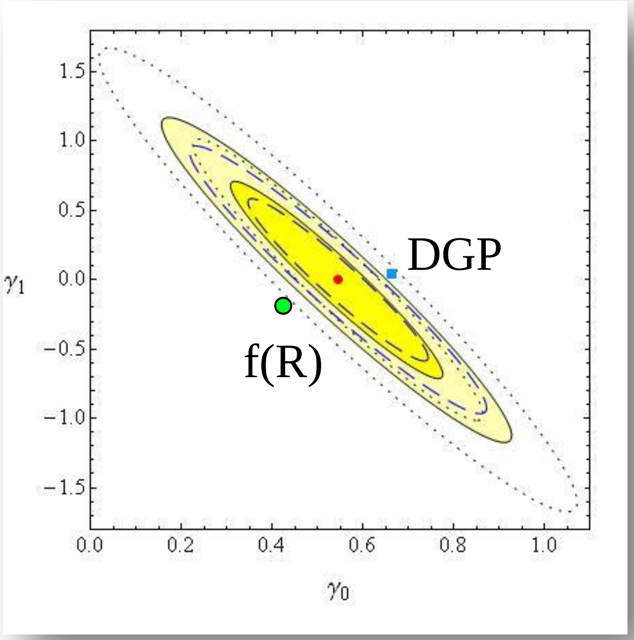


C. Di Porto, L.A., E. Branchini 2011

Euclid forecasts, II

$$s_{fit} \equiv \Omega_m^{\gamma_0+\gamma_1 z/(1+z)}$$

E.g. LCDm and wCDM predicts* negative γ_1, while DGP predicts positive γ_1



*Gannouji, et al. 2009

Euclid forecasts, III

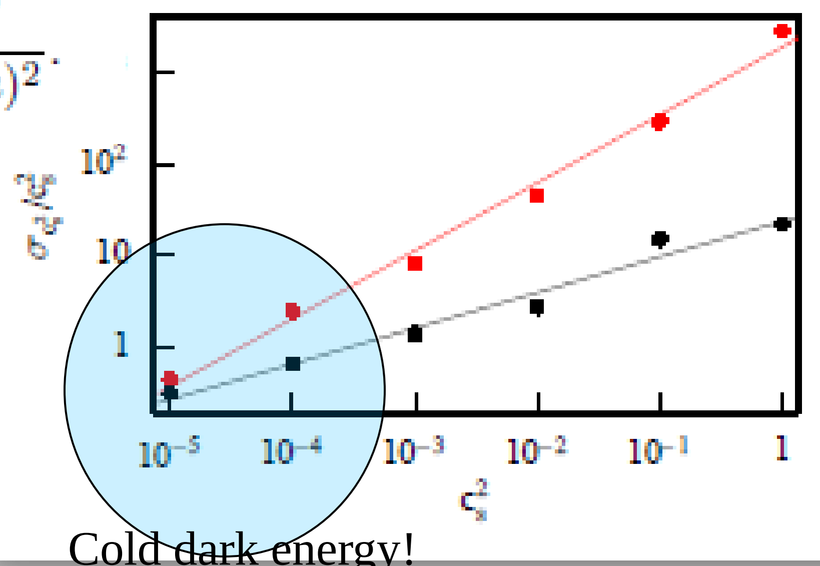
Effects of DE sound speed on matter clustering

$$k^2\Psi = -4\pi G a^2(\rho_m\delta_m + \rho_{DE}\delta_{DE}) = -4\pi G a^2 Q(k,a)\rho_m\delta_m$$

$$Q(k,a) = 1 + \frac{\rho_{DE}\delta_{DE}}{\rho_m\delta_m}$$

$$Q(k,a) = 1 + \frac{1-\Omega_{m,0}}{\Omega_{m,0}}\frac{(1+w)a^{-3w}}{1-3w+\frac{2}{3}\nu(a)^2}.$$

$$\nu(a)^2 = k^2 c_s^2 a / \left(\Omega_{m,0}H_0^2\right)$$

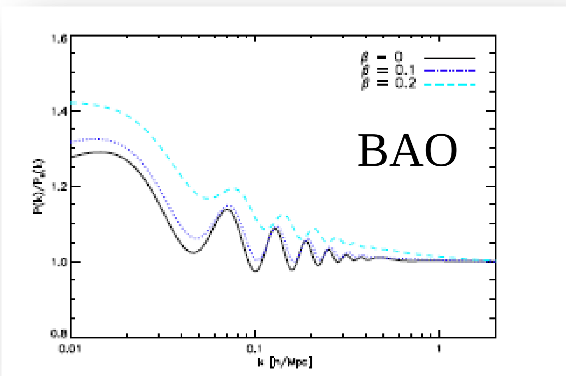


D. Sapone, L.A., M. Kunz 2010

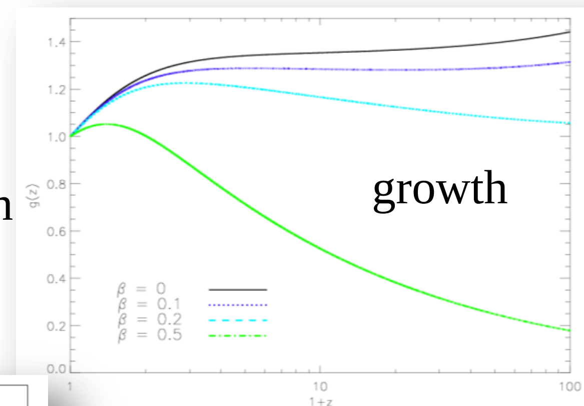
Euclid forecasts, IV

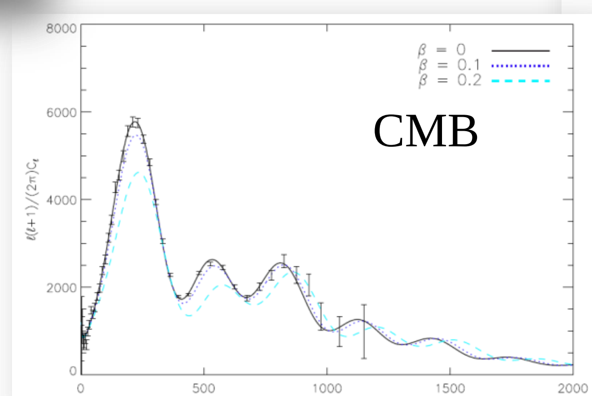
Coupled dark energy

$$T^{\mu}_{(m)\nu:\mu} = \beta T_{(m)} \phi_{;\nu}$$
$$T^{\mu}_{(\phi)\nu:\mu} = -\beta T_{(m)} \phi_{;\nu}$$



Effects on CMB, growth rate, BAO peaks, etc



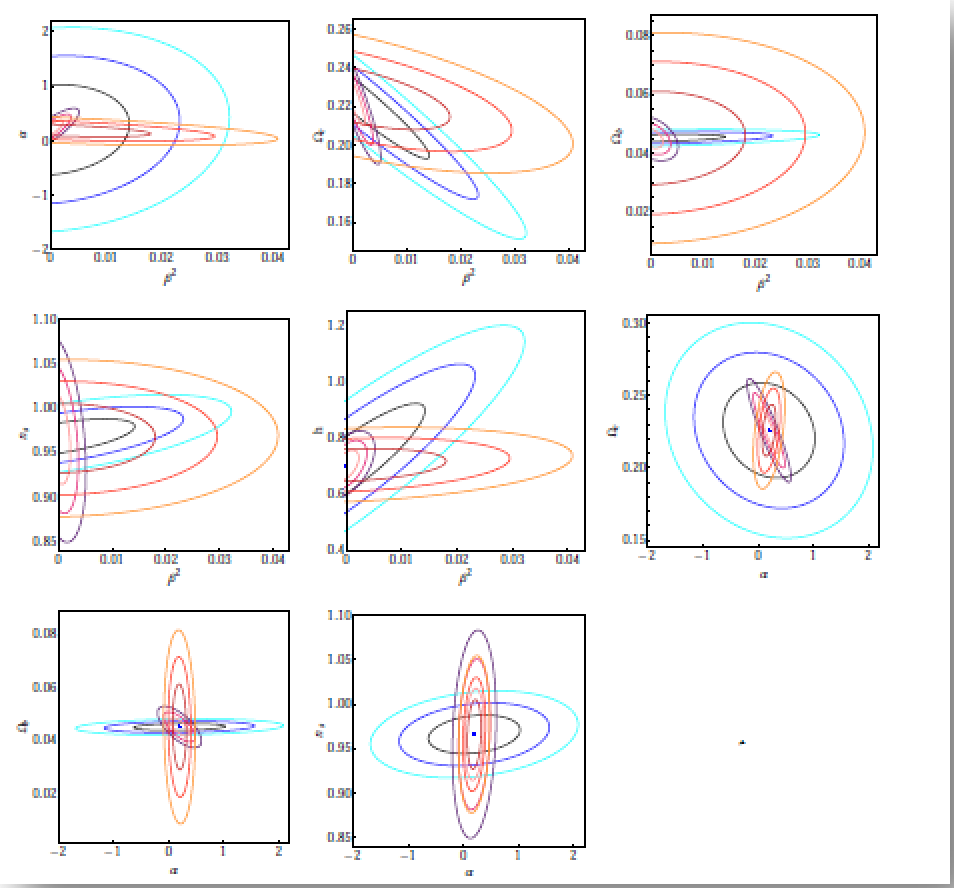


Euclid forecasts, IV

Coupled dark energy

$$T^{\mu}_{(m)\nu:\mu} = \beta T_{(m)}\phi_{;\nu}$$
$$T^{\mu}_{(\phi)\nu:\mu} = -\beta T_{(m)}\phi_{;\nu}$$

CMB+P(k)+WL



Euclid forecasts, IV

Coupled dark energy

$$T^{\mu}_{(m)\nu:\mu} = \beta T_{(m)} \phi_{;\nu}$$
$$T^{\mu}_{(\phi)\nu:\mu} = -\beta T_{(m)} \phi_{;\nu}$$

Parameter	σ_i CMB	σ_i $P(k)$	σ_i WL
β^2	0.0094	0.0015	0.012
α	0.55	0.12	0.083
Ω_c	0.022	0.010	0.012
h	0.15	0.036	0.039
Ω_b	0.00087	0.0022	0.010
n_s	0.014	0.034	0.026

Combined constraint on coupling $\sigma_{\beta^2} = 0.0003$

Precision is nothing without accuracy

1

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(collaboration C. di Porto, A. Vollmer)

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Lensing and supernovae

(collab. V. Marra, M. Quartin, J. Kannulainen)

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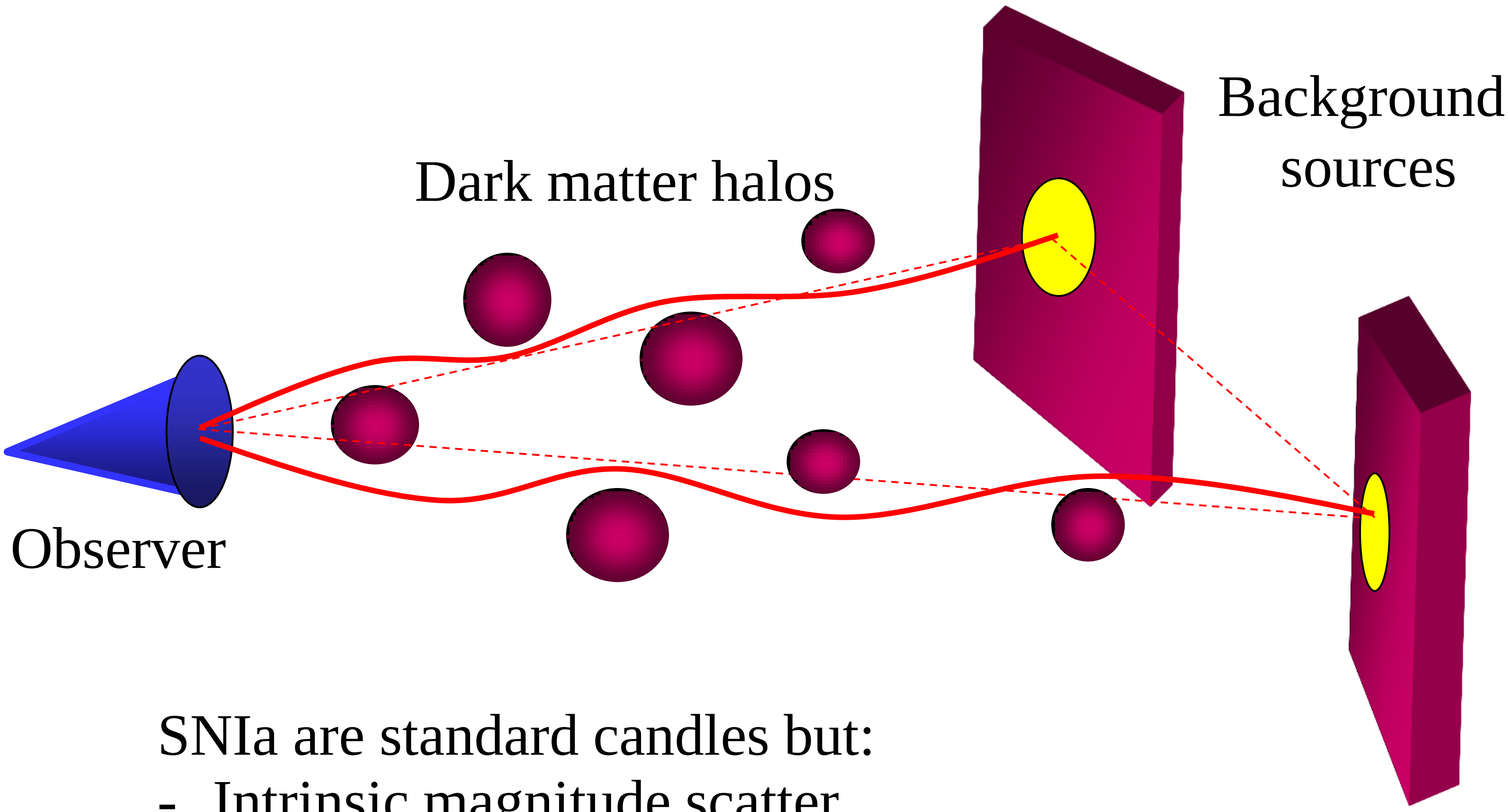
(collab. C. Quercellini, M. Quartin)

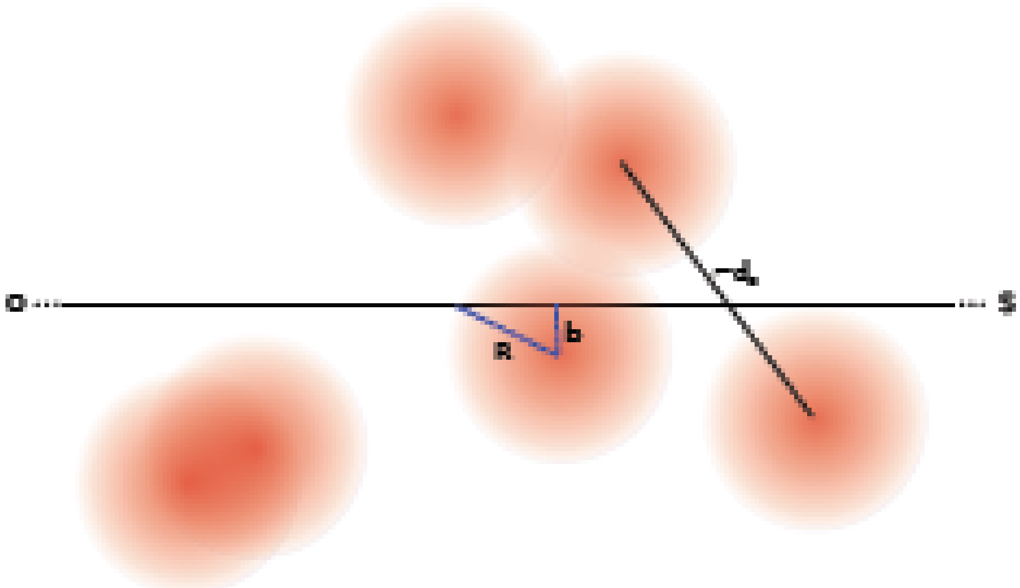
Lensing of distant sources



SNIa are standard candles but:
- Intrinsic magnitude scatter
- Lensing magnitude scatter

Lensing of distant sources

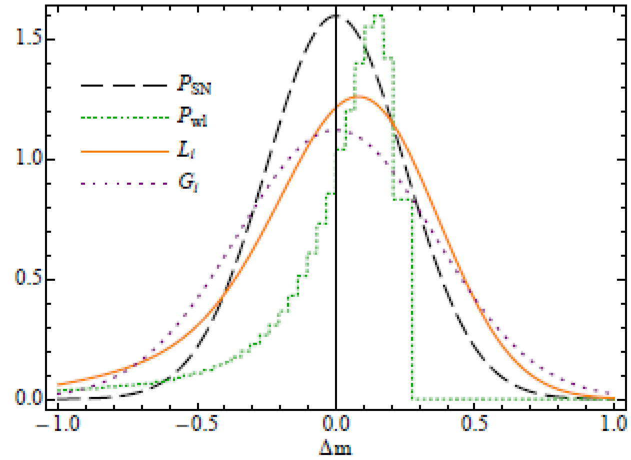
Lens parameters:
Halo mass and concentration



Marra & Kainulainen 2009

Biasing the estimates

Distribution of SN deviations from the mean in presence of lensing



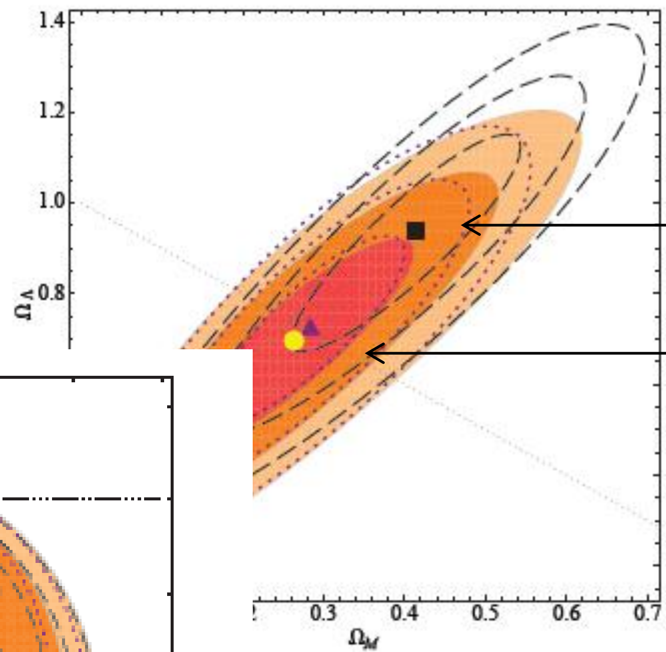
The lensing distribution depends on Redshift and on cosmological model!

$\rho_c\,\Omega_M = M\,n_c$. For numerical values we explored the range $\lambda_c = (5.4,\,9.0,\,12.6)\,h^{-1}$Mpc and correspondingly $M = (0.44,\,2.0,\,5.6)\,10^{14}h^{-1}\Omega_M\,M_\odot$ for $z_{\rm vir} = 0.8$, and $z_{\rm vir} = (0,\,0.8,\,1.6)$ for $\lambda_c = 12.6\,h^{-1}$Mpc. The numerical value of R_p depends on the background matter density at $z_{\rm vir}$. For the ΛCDM model the previous range of $z_{\rm vir}$ corresponds to $R_p \simeq (0.9,\,0.7,\,0.5)\,h^{-1}$Mpc.

Biasing the SN estimates

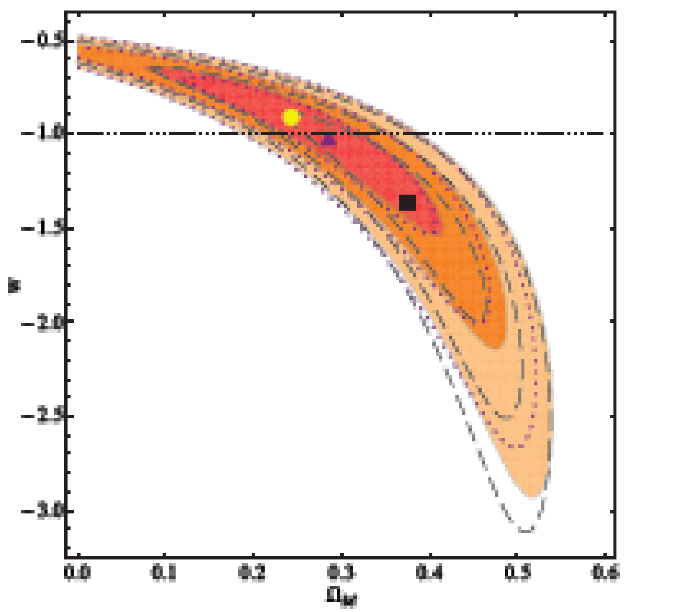
Union data set Kowalsky et al. 2008

Non-linear/non-gaussian effects can significantly bias the estimates of cosmological parameters



No lensing

Lensing



$$(\Omega_M^*,\, w^*) = (\Omega_M^V - 0.13\, M^{0.47},\, w^V + 0.45\, M^{0.33})$$
$$(\Omega_M^*,\, \Omega_\Lambda^*) = (\Omega_M^V - 0.15\, M^{0.29},\, \Omega_\Lambda^V - 0.25\, M^{0.26})$$

where $M \leq 1$ is in units of $5.6 \cdot 10^{14} h^{-1} \Omega_M M_\odot$.

. Kannulainen, V. Marra,
ırtin, Phys Rev Lett 2010

Firenze 2011

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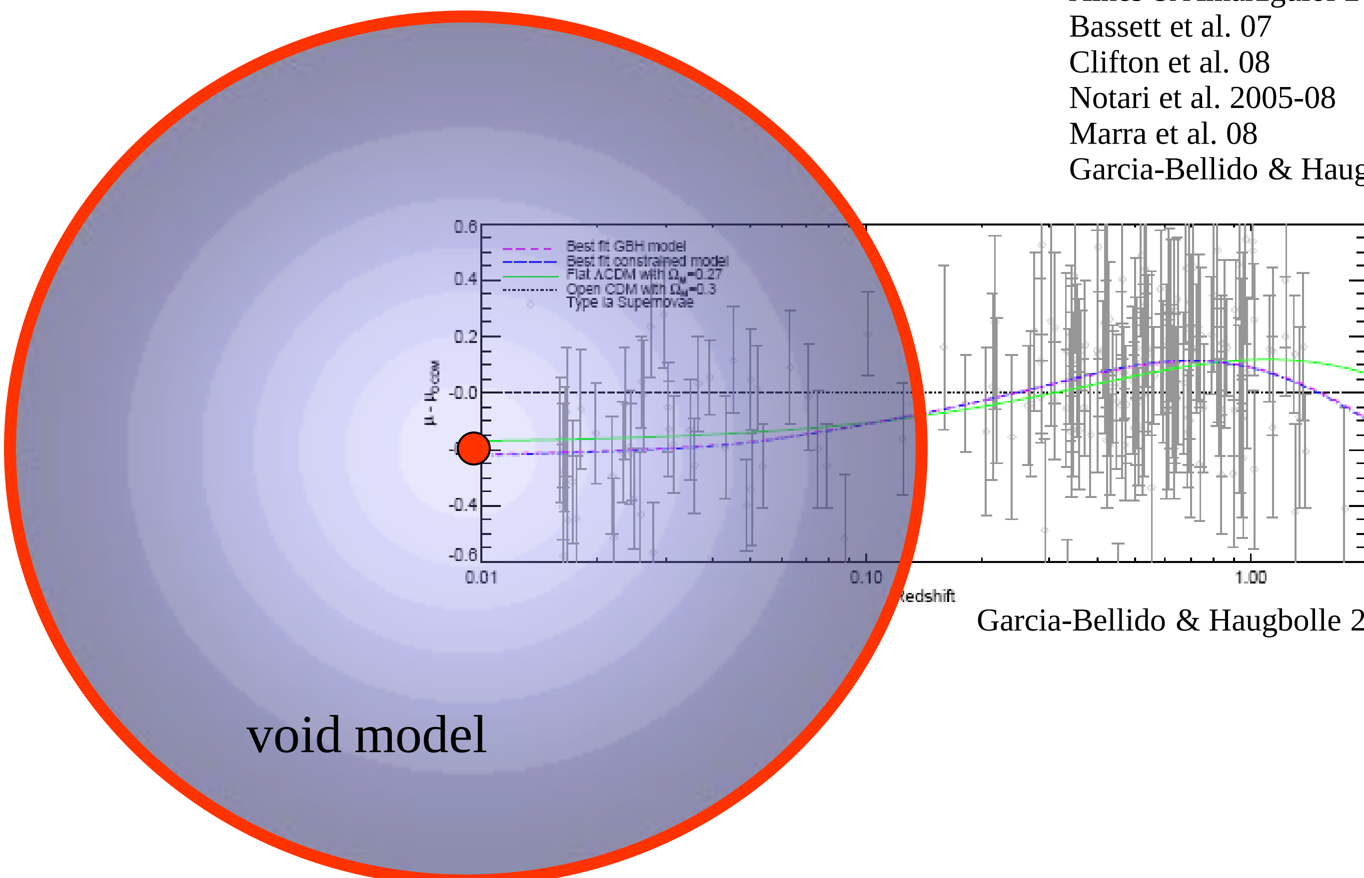
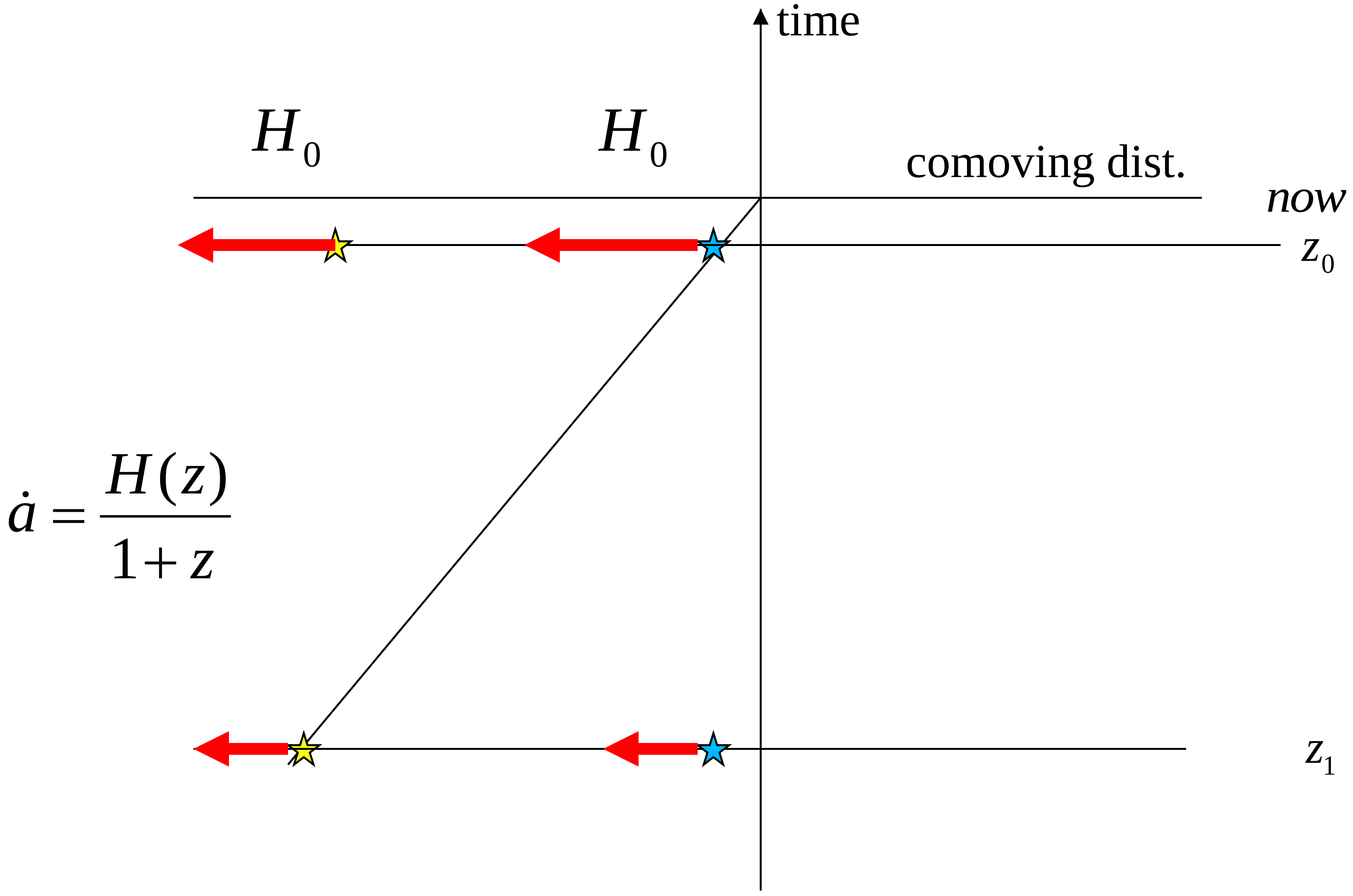
Cosmic Degeneracy

Tomita 2001
Celerier 2001
Alnes & Amarzguioi 2006,07
Bassett et al. 07
Clifton et al. 08
Notari et al. 2005-08
Marra et al. 08
Garcia-Bellido & Haugbolle 2008

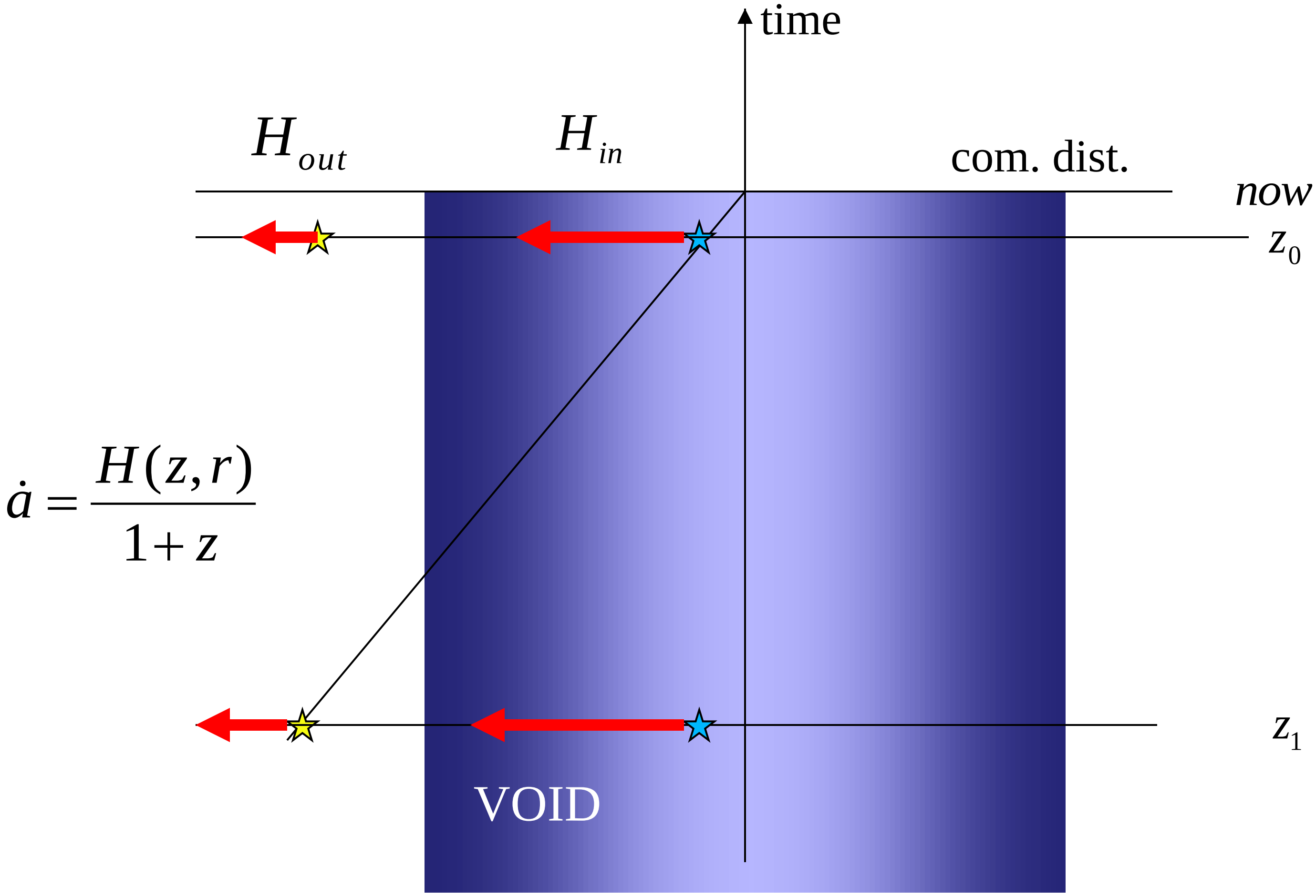


Garcia-Bellido & Haugbolle 2008

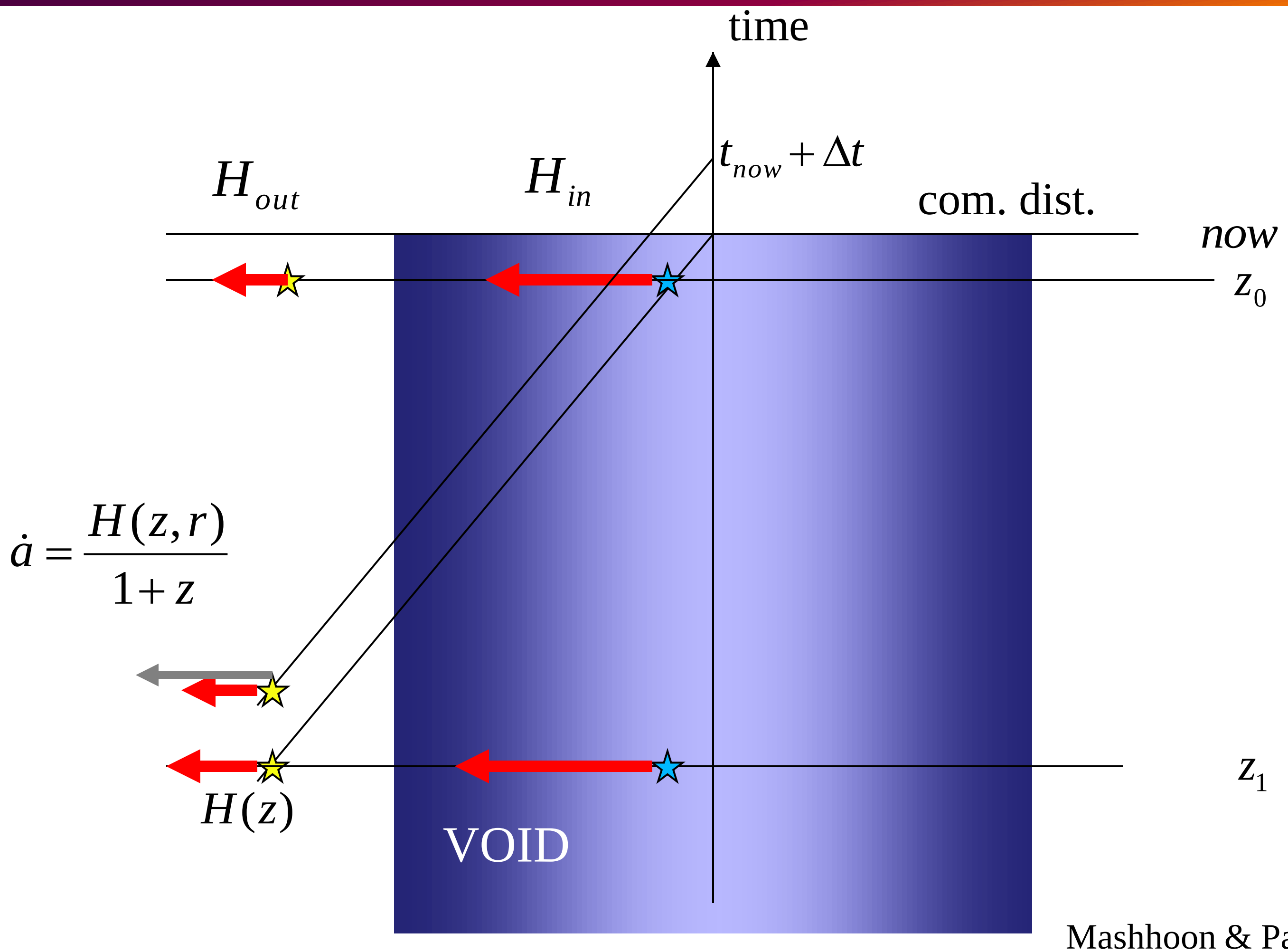
One null cone



One null cone



Two null cones are better than one!



Mashhoon & Partovi 1985
Uzan, Clarkson & Ellis 2007
Quartin, Quercellini, L.A. 2009

Sandage 1962

$H(z_1)$ $H(z_2)$

$$\Delta v \approx \pm 1 cm/\sec/\,year$$

VOLUME 136 SEPTEMBER 1962 NUMBER 2

THE CHANGE OF REDSHIFT AND APPARENT LUMINOSITY OF GALAXIES DUE TO THE DECELERATION OF SELECTED EXPANDING UNIVERSES

ALLAN SANDAGE
Mount Wilson and Palomar Observatories
Carnegie Institution of Washington, California Institute of Technology
(With an Appendix by G. C. McVITTIE, University of Illinois Observatory, Urbana)

Received February 2, 1962; revised April 13, 1962

ABSTRACT

The redshift and apparent luminosity of any given galaxy are not constant with time for most models of the expanding universe. Redshifts decrease with time because of the braking action of the gravitational field in all exploding models, except for the one where the matter density is zero. Apparent luminosities decrease with time, except for the oscillating model in the contracting phase and for galaxies with very large $\Delta\lambda/\lambda_0$ values, because the distances between galaxies are increasing Redshifts increase with time for every galaxy in the steady-state model.

The theory and numerical results of the deceleration are presented for four selected world models. For a galaxy with redshift $z = \Delta\lambda/\lambda_0 = 0.4$ at the present epoch, the change of redshift with time is found to be $dcz/dt = -11 \times 10^{-6}$ km/sec year for the oscillating model in the expanding phase at $q_0 = +1$; $dcz/dt = -5.9 \times 10^{-6}$ km/sec year for the Euclidean model; $dcz/dt = -4\,3 \times 10^{-6}$ km/sec year for the hyperbolic model at $q_0 = 0\,3516$; and $dcz/dt = +9\,2 \times 10^{-6}$ km/sec year for the steady-state model. These all assume that $H^{-1} = 13 \times 10^9$ years at the present epoch With present optical techniques

IV. CONCLUSION

1. The foregoing considerations show that an "ideal" deceleration test exists between the exploding and the steady-state models in the sense that the *sign* of the effect is reversed. However, for the test to be useful, it would seem that a precision redshift catalogue must be stored away for the order of 10^7 years before an answer can be found because the decelerations are so small by terrestrial standards.

2. For all models, except the oscillating case, it will become more and more difficult to obtain observational information from the universe because the apparent luminosities of galaxies decrease with time. Indeed, if the oscillating case is excluded, there will be a time in the very distant future when most galaxies will recede beyond the limit of easy observation and when data for extragalactic astronomy must be collected from ancient literature.

Loeb 1998

Direct Measurement of Cosmological Parameters from the Cosmic Deceleration of Extragalactic Objects

Abraham Loeb

Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138

ABSTRACT

The redshift of all cosmological sources drifts by a systematic velocity of order a few m s^{-1} over a century due to the deceleration of the Universe. The specific functional dependence of the predicted velocity shift on the source redshift can be used to verify its cosmic origin, and to measure directly the values of cosmological parameters, such as the density parameters of matter and vacuum, Ω_M and Ω_Λ, and the Hubble constant H_0. For example, an existing spectroscopic technique, which was recently employed in planet searches, is capable of uncovering velocity shifts of this magnitude. The cosmic deceleration signal might be marginally detectable through two observations of $\sim 10^2$ quasars set a decade apart, with the HIRES instrument on the Keck 10 meter telescope. The signal would appear as a global redshift change in the Lyα forest templates imprinted on the quasar spectra by the intergalactic medium. The deceleration amplitude should be isotropic across the sky. Contamination of the cosmic signal by peculiar accelerations or local effects is likely to be negligible.

Subject headings: cosmology: theory

submitted to *ApJ Letters*, Feb. 10th, 1998

The Sandage effect

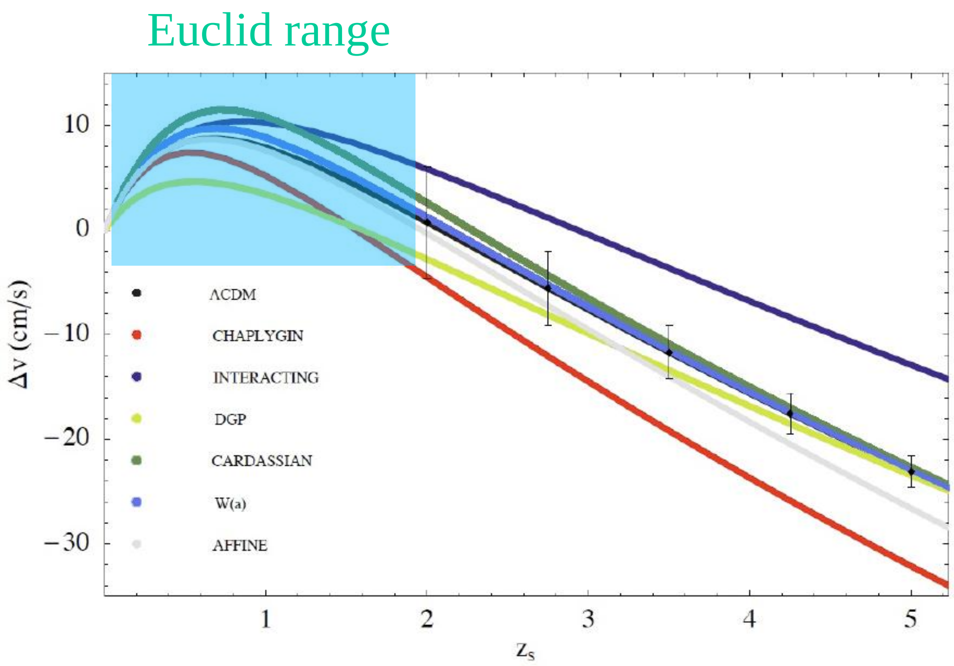
$$H_0 \Delta t \approx 10^{-9} \quad (10 yrs)$$

$$10^{-9} c \approx 30 cm / \sec$$

$$\Delta z \approx \frac{a(t_0 + \Delta t_0)}{a(t_s + \Delta t_s)} - \frac{a(t_0)}{a(t_s)}$$

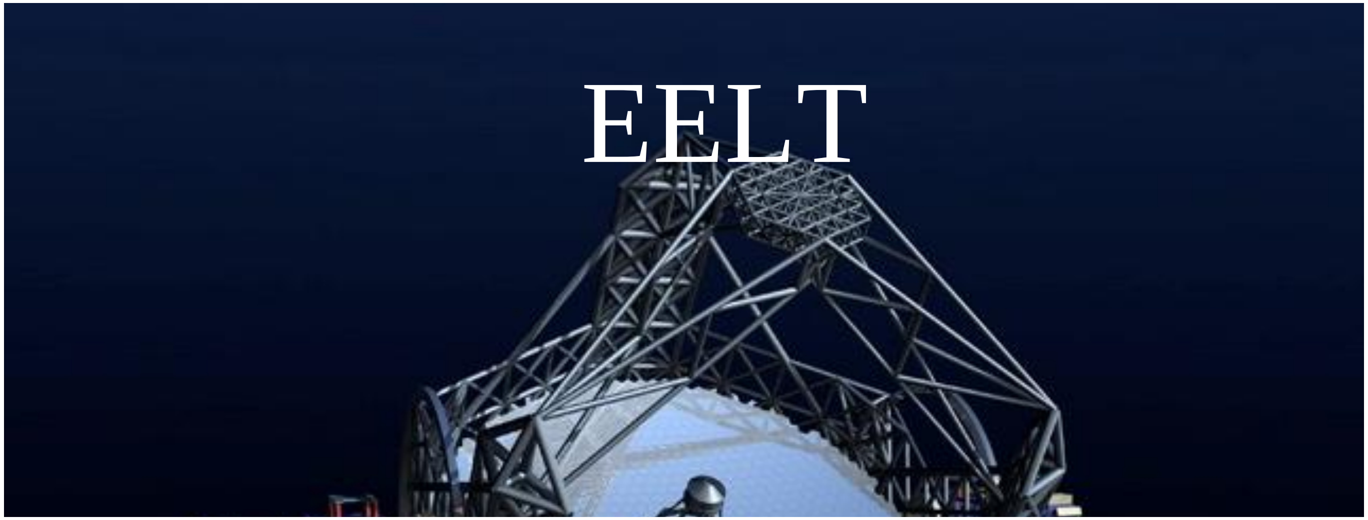
$$\Delta z = H_0 \Delta t_0 (1 + z - \frac{H(z)}{H_0})$$

$$\Delta v = \frac{c \Delta z}{1+z} |_{1yr} \approx 1 cm / \sec$$



Corasaniti, Huterer, Melchiorri 2007
Balbi & Quercellini 2007

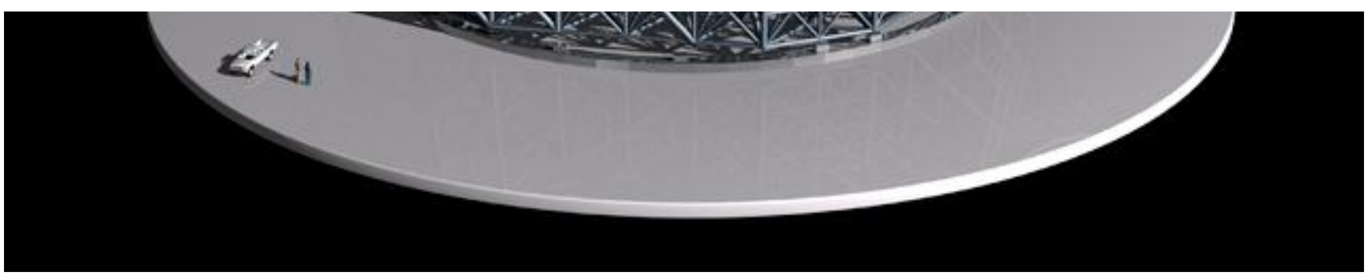
EELT



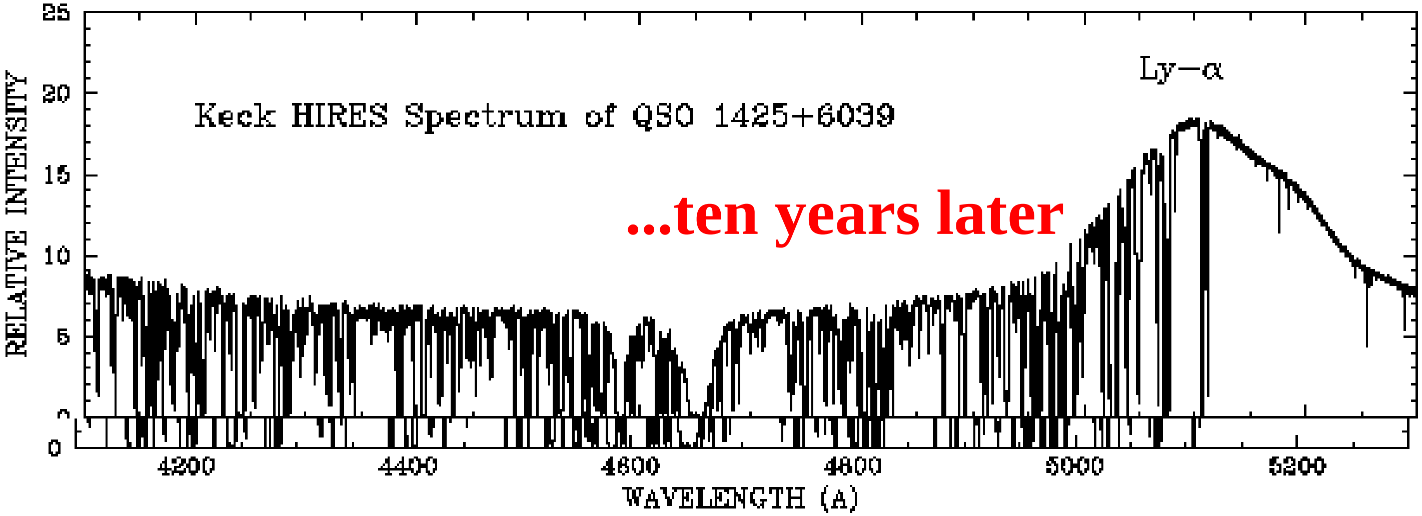
The Extremely Large Telescope

IS THE ESSENTIAL NEXT STEP IN MANKIND'S DIRECT OBSERVATION OF THE NATURE OF THE UNIVERSE.

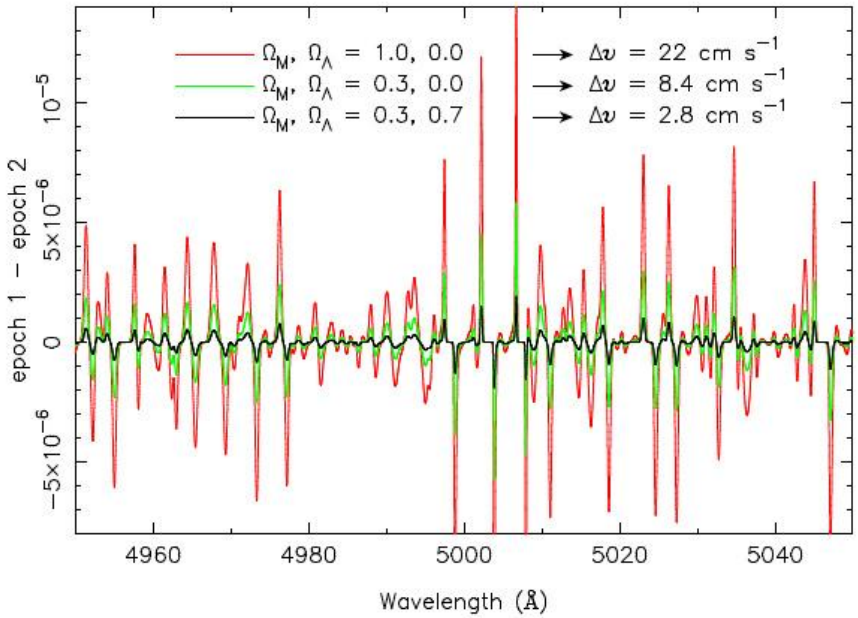
IT WILL PROVIDE THE DESCRIPTION OF REALITY WHICH WILL UNDERLIE OUR DEVELOPING UNDERSTANDING OF ITS NATURE.

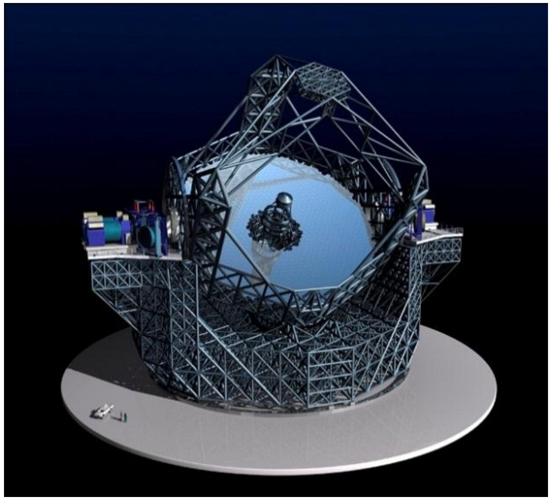


CODEX at EELT



CODEX at EELT





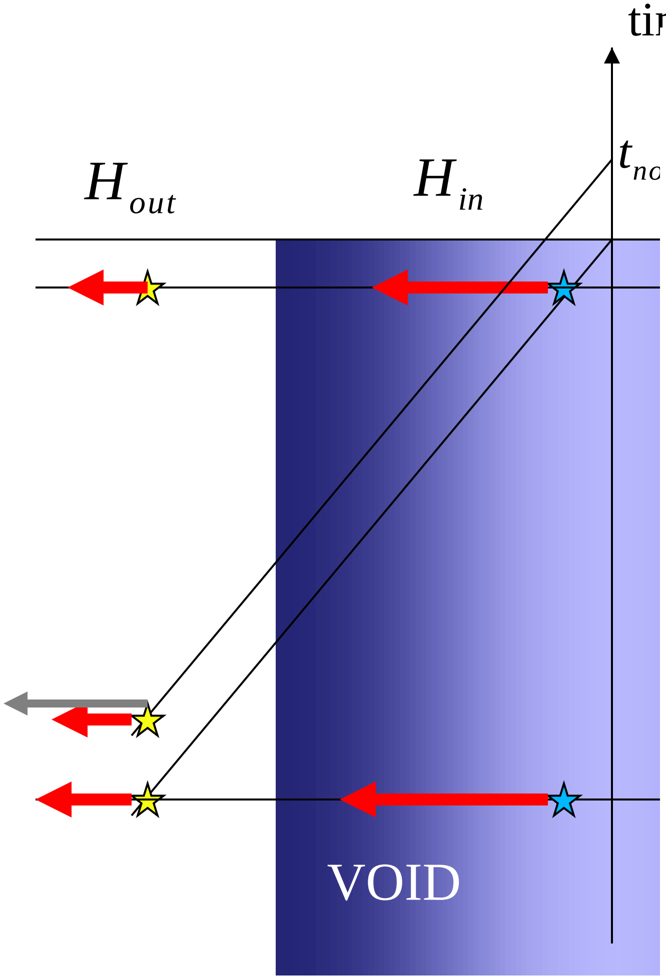
- large colleting area
- high resolution spetrographs
- stable, low-peculiar motion

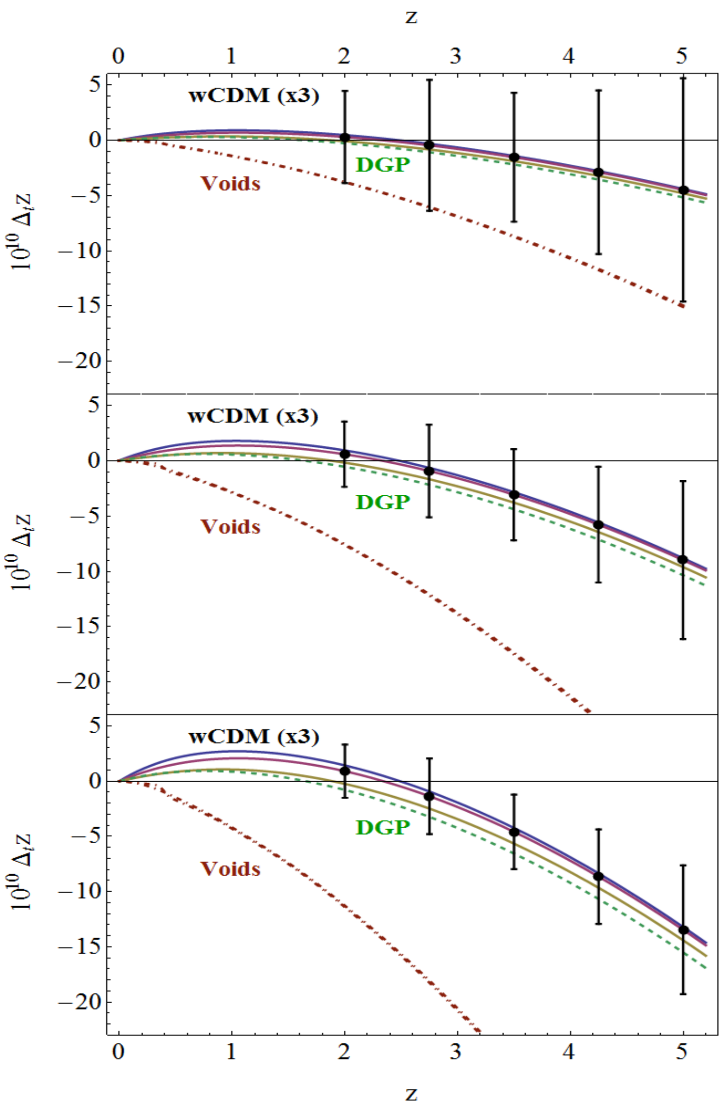
targets: Lyman-alpha lines

$$\sigma = 2\left(\frac{2350}{S/N}\right)\left(\frac{30}{N_{QSO}}\right)^{1/2}\left(\frac{5}{1+z}\right)^{1.8} cm/s$$

Liske et al. 2008

Two null cones are better than one!





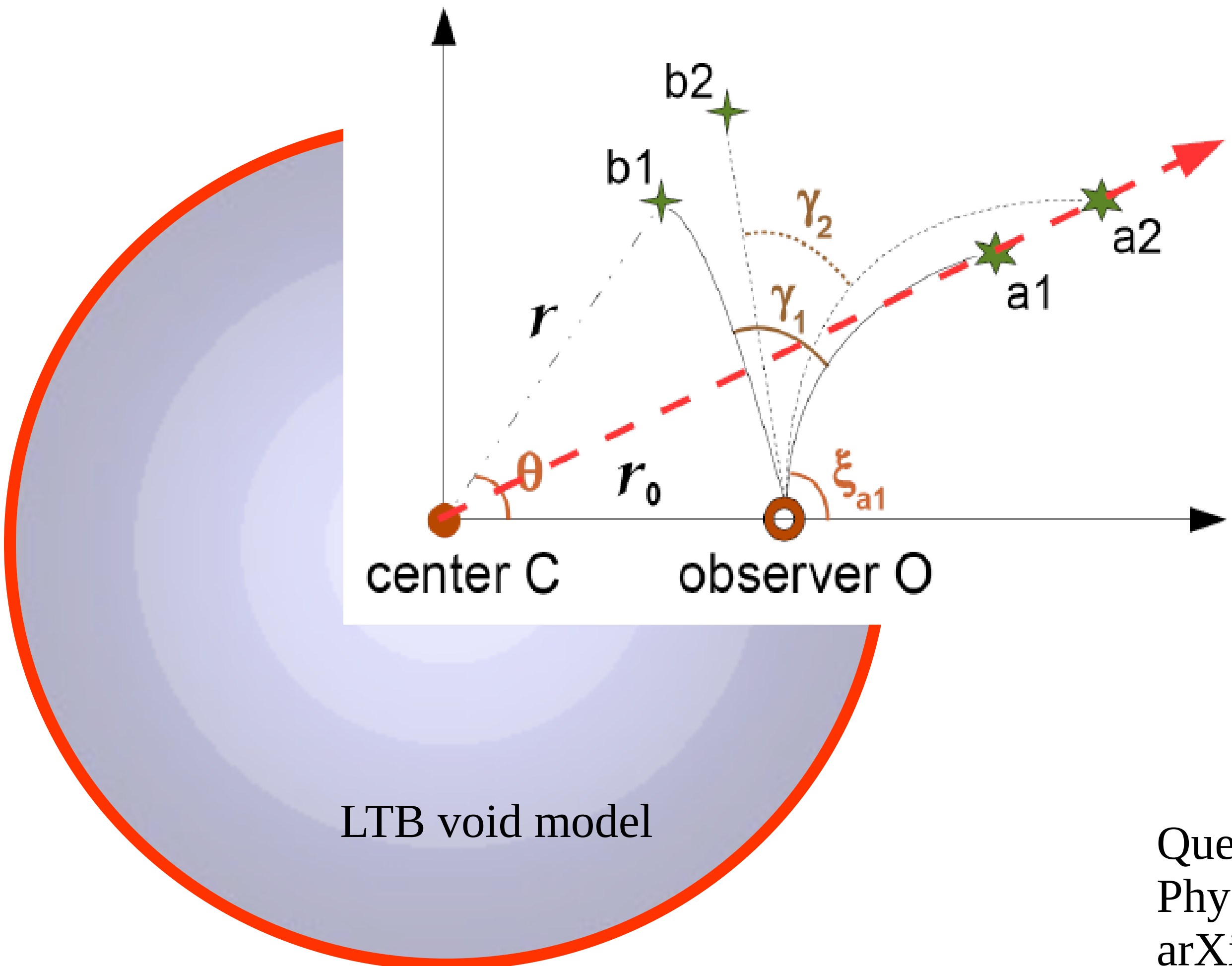
M. Quartin & L. A. 2009

Evolution

Ptolemaic system, I century

LTB void model, XXI century

Cosmic Parallax



$$H_0 \Delta t \approx 10^{-9}$$

$$10^{-9}\,rad \approx 200\,\mu as$$

astrometric satellites
GAIA, SIM, Jasmine etc:
1-100 μas

Quercellini, Quartin & LA,
Phys. Rev. Lett. 2009
arXiv 0809.3675

Lemaître-Tolman-Bondi models

$$R' \equiv \frac{\partial R}{\partial r}$$

- LTB metrics describe void models

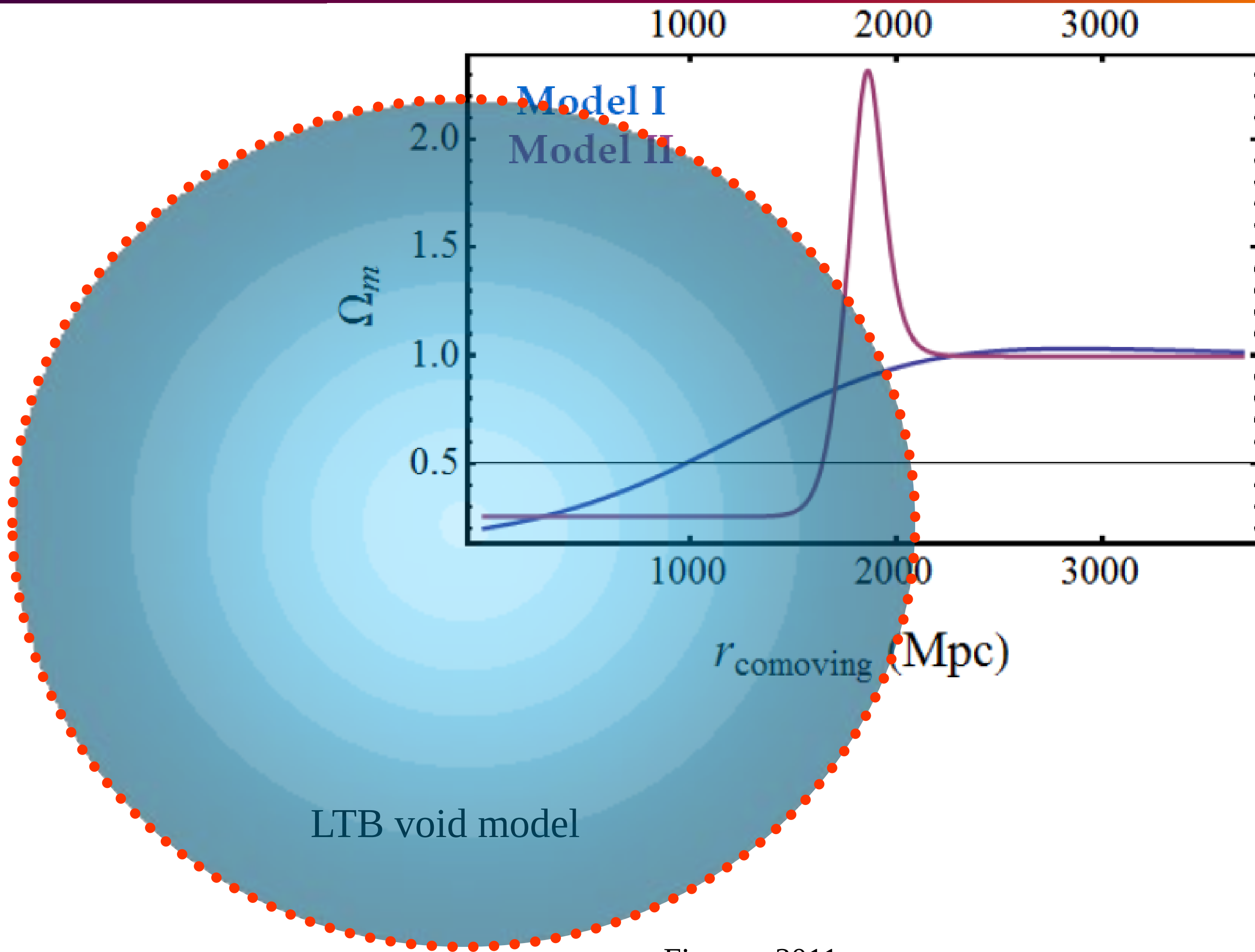
$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \frac{[R'(t,r)]^2}{1+\beta(r)}\mathrm{d}r^2 + R^2(t,r)\mathrm{d}\Omega^2$$

- Exact solution in a matter-dominated era

$$R = (\cosh\eta - 1)\frac{\alpha}{2\beta} + R_{\mathrm{lss}}\left[\cosh\eta + \sqrt{\frac{\alpha+\beta R_{\mathrm{lss}}}{\beta R_{\mathrm{lss}}}}\sinh\eta\right]$$

$$\sqrt{\beta}t = (\sinh\eta - \eta)\,\frac{\alpha}{2\beta} + R_{\mathrm{lss}}\left[\sinh\eta + \sqrt{\frac{\alpha+\beta R_{\mathrm{lss}}}{\beta R_{\mathrm{lss}}}}\,(\cosh\eta - 1)\right]$$

LTB models

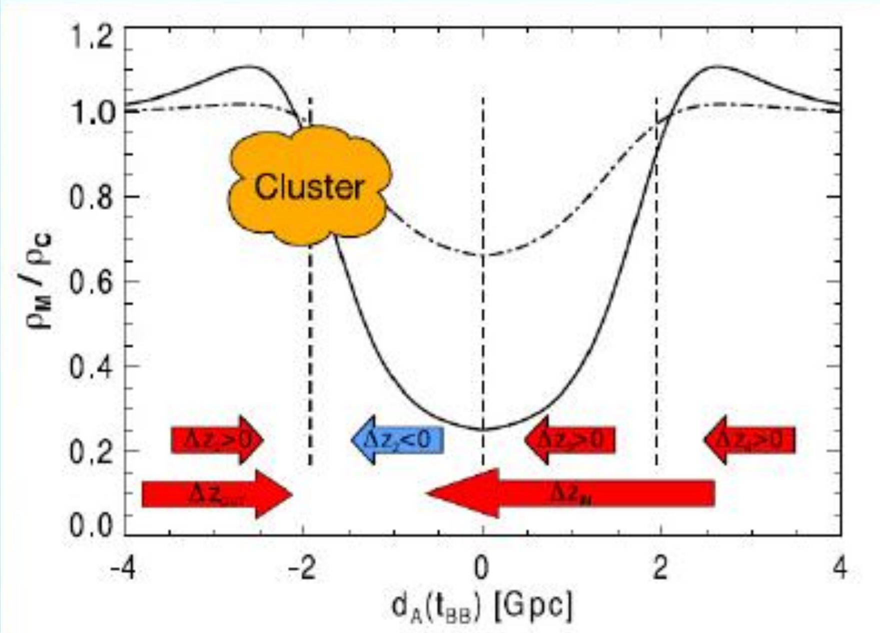


1000
2000
3000
Model I
Model II
2.0
1.5
1.0
0.5
Ω_m
1000
2000
3000
$r_{\rm comoving}$ (Mpc)
LTB void model

Constraints on Void Models

- Large voids (> 1.5 Gpc) are in conflict with
  - CMB blackbody spectrum
    - *Caldwell & Stebbins: 0711.3459 (PRL)*
  - Kinematic Sunyaev-Zeldovich effect from large clusters
    - *García-Bellido & Haugbolle: 0807.1326 (JCAP)*

- Sharp transitions could be in conflict with SDSS LRG or SNe distribution (no excess at $z \approx .3$)



Estimating the Cosmic Parallax

- Calculating the Cosmic Parallax require solving the full LTB geodesic equations
- Simple*, non-consistent* estimate → flat FRW universe with H(t) → H(t, r)
- Assume 2 sources initially separated by ΔX & $\Delta\theta$.

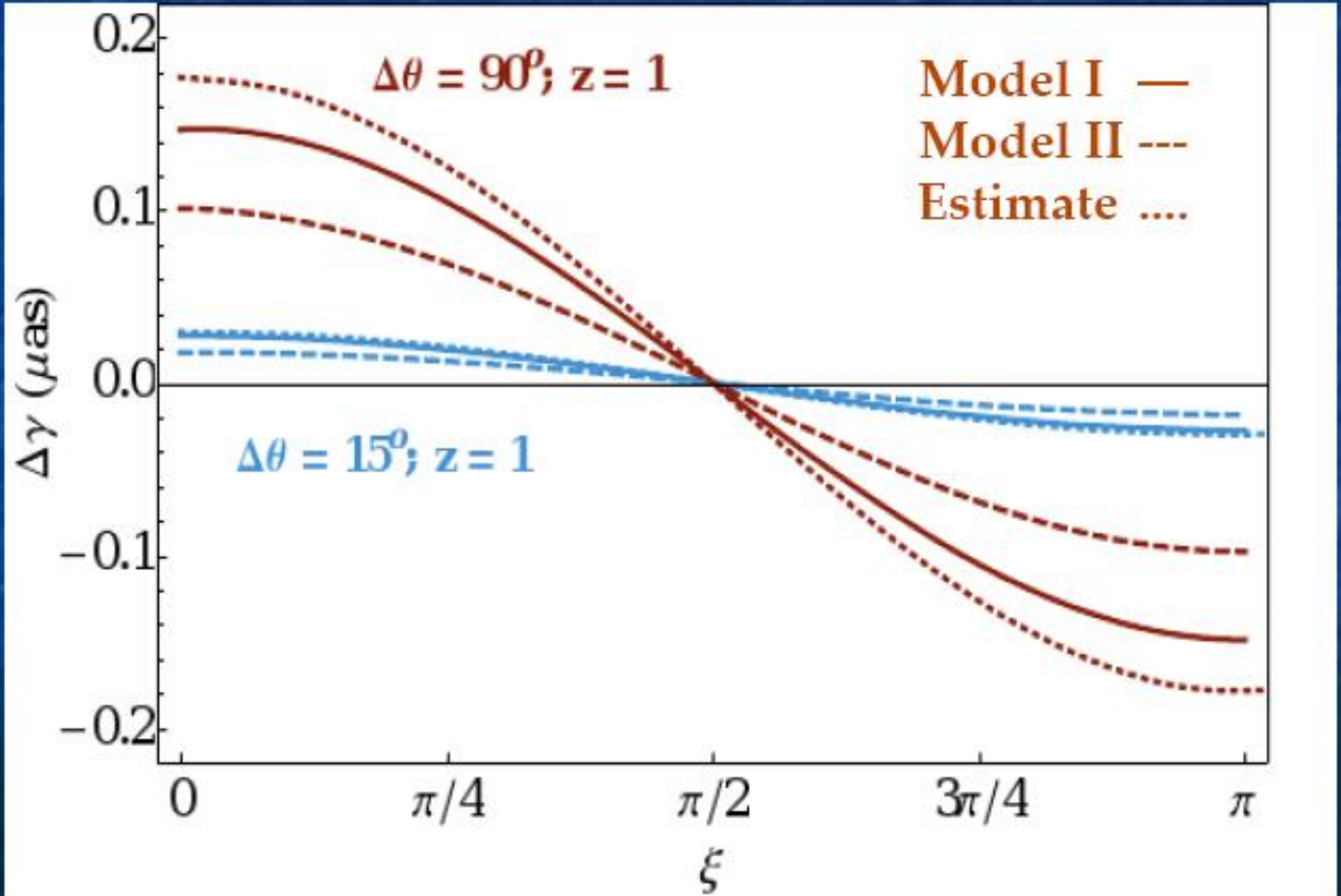
$$\Delta_t \gamma \simeq \Delta t \left(\overline{H}_{\rm obs} - \overline{H}_X \right) \frac{X_{\rm 0bs}}{X} \left(\cos\theta \, \Delta\theta + \sin\theta \, \frac{\Delta X}{X} \right)$$

distance to the void center

“physical” distance

Results

- Actual effect → need to solve the LTB geodesic eqs.
- $\Delta_t\gamma$ in 10 yrs for a pair of quasars at z=1 (typical for Gaia)



Gaia: Complete, Faint, Accurate

	Hipparcos	Gaia
Magnitude limit	12	20 mag
Completeness	7.3 – 9.0	20 mag
Bright limit	0	6 mag
Number of objects	120 000	26 million to V = 15 250 million to V = 18 1000 million to V = 20
Effective distance	1 kpc	50 kpc
Quasars	None	5×10^5
Galaxies	None	$10^6 – 10^7$
Accuracy	1 milliarcsec	7 μarcsec at V = 10 10-25 μarcsec at V = 15 300 μarcsec at V = 20
Photometry	2-colour (B and V)	Low-res. spectra to V = 20
Radial velocity	None	15 km/s to V = 16-17
Observing	Pre-selected	Complete and unbiased

Cosmic Parallax with Gaia

- SNe → off-center distance $X_0 \leq 150$ Mpc.
- CMB dipole → off-center dist. $X_0 \leq 15$ Mpc.

Alnes & Armazguioui
astro-ph/0607334
astro-ph/0610331

- Assuming:
  - $X_0 = 15$ Mpc (aggressive);
  - Astrometric precision of 30 μas;
  - Nominal Gaia duration ($\Delta t = 5$ years)
- Gaia can detect the Cosmic Parallax at 1σ if

  \# sources ≥ 450,000 (conservative)

Noise and Sistematics

- Most obvious source of noise → peculiar velocities

$$\Delta_t \gamma_{\rm pec} = \left(\frac{v_{\rm pec}}{500\,\frac{\rm km}{\rm s}} \right) \left(\frac{D_A}{1\,{\rm Gpc}} \right)^{-1} \left(\frac{\Delta t}{10\,{\rm years}} \right) \mu as$$

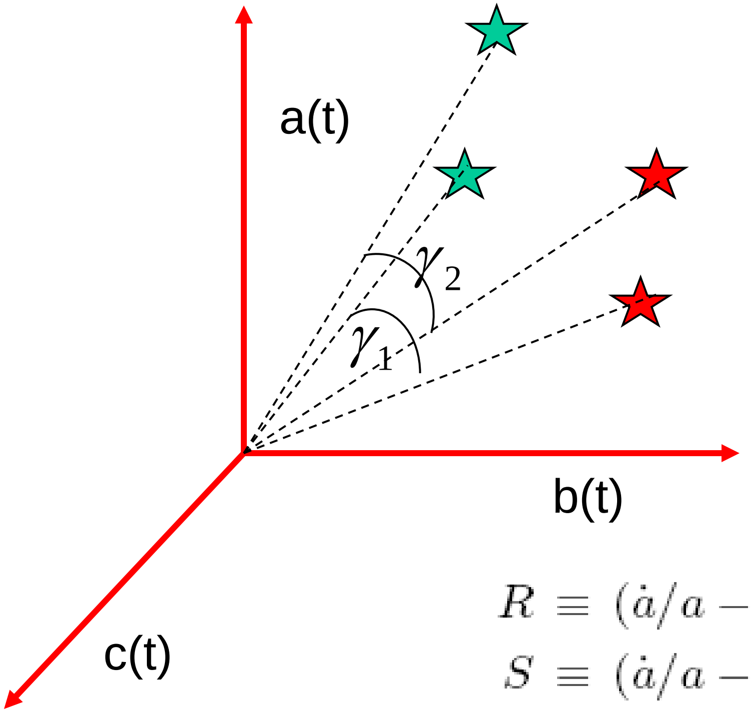
- Most serious source of noise → changing aberration due to acceleration of the solar system

Gaia predicts ≈ 4 μas effect, of which 90% could be subtracted → 0.4 μas spurious dipole

Kovalevsky 2003

Not only LTB

Bianchi I



$$R \equiv (\dot{a}/a - \dot{b}/b)/H = \Sigma_x - \Sigma_y \,,$$
$$S \equiv (\dot{a}/a - \dot{c}/c)/H = 2\Sigma_x - \Sigma_y \,.$$

Current limits on anisotropy

$$R = \frac{\Delta H}{H} \leq 10^{-4}$$ at z = 1000

$$\frac{\Delta H}{H} \leq 10^{-8}$$ at z = 0 in a ΛCDM universe

$$\frac{\Delta H}{H} \leq ?$$ at z = 0 in anisotropic dark energy

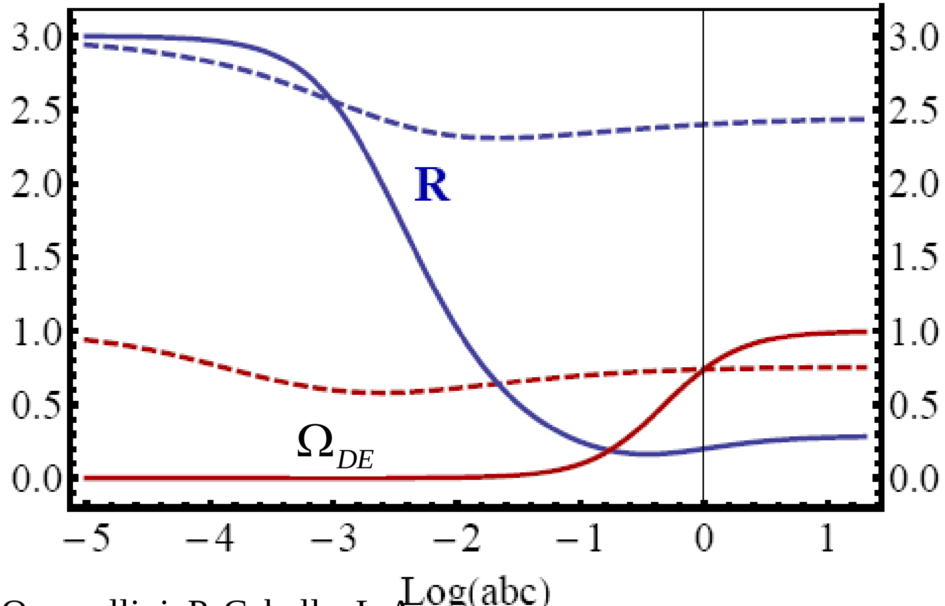
Anisotropic dark energy

Mota & Koivisto 2008,
Barrow, Saha, Bruni, Rodrigues and many others..

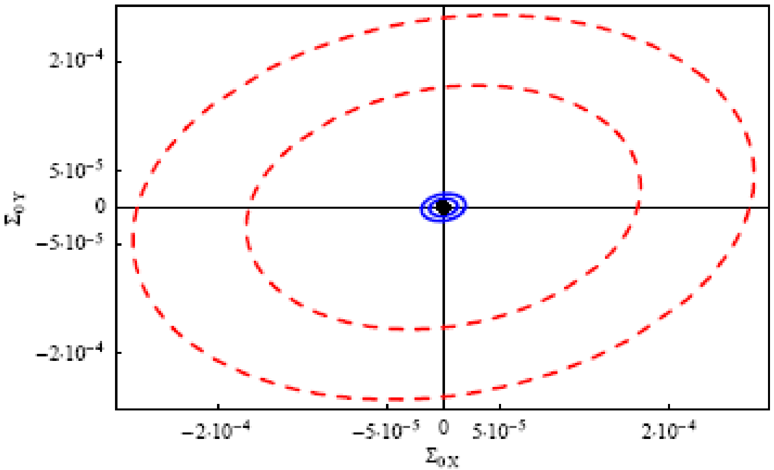
$$T^{\mu}_{(\mathrm{DE})\nu} = \mathrm{diag}(-1, w, w+3\delta, w+3\gamma)\rho_{\mathrm{DE}},$$

$$R \equiv (\dot{a}/a - \dot{b}/b)/H = \Sigma_x - \Sigma_y ,$$
$$S \equiv (\dot{a}/a - \dot{c}/c)/H = 2\Sigma_x - \Sigma_y .$$

Experiment	N_s	σ_{acc}	Δt
Gaia	500,000	10μas	5yrs
Gaia+	1,000,000	1μas	10yrs



C. Quercellini, P. Cabella, L.A.,
M. Quartin, A. Balbi 2009



$$R = \frac{\Delta H}{H} \leq 10^{-4} \quad , \text{ at any } z$$

Firenze 2011

Precision is nothing without accuracy

The power of statistics

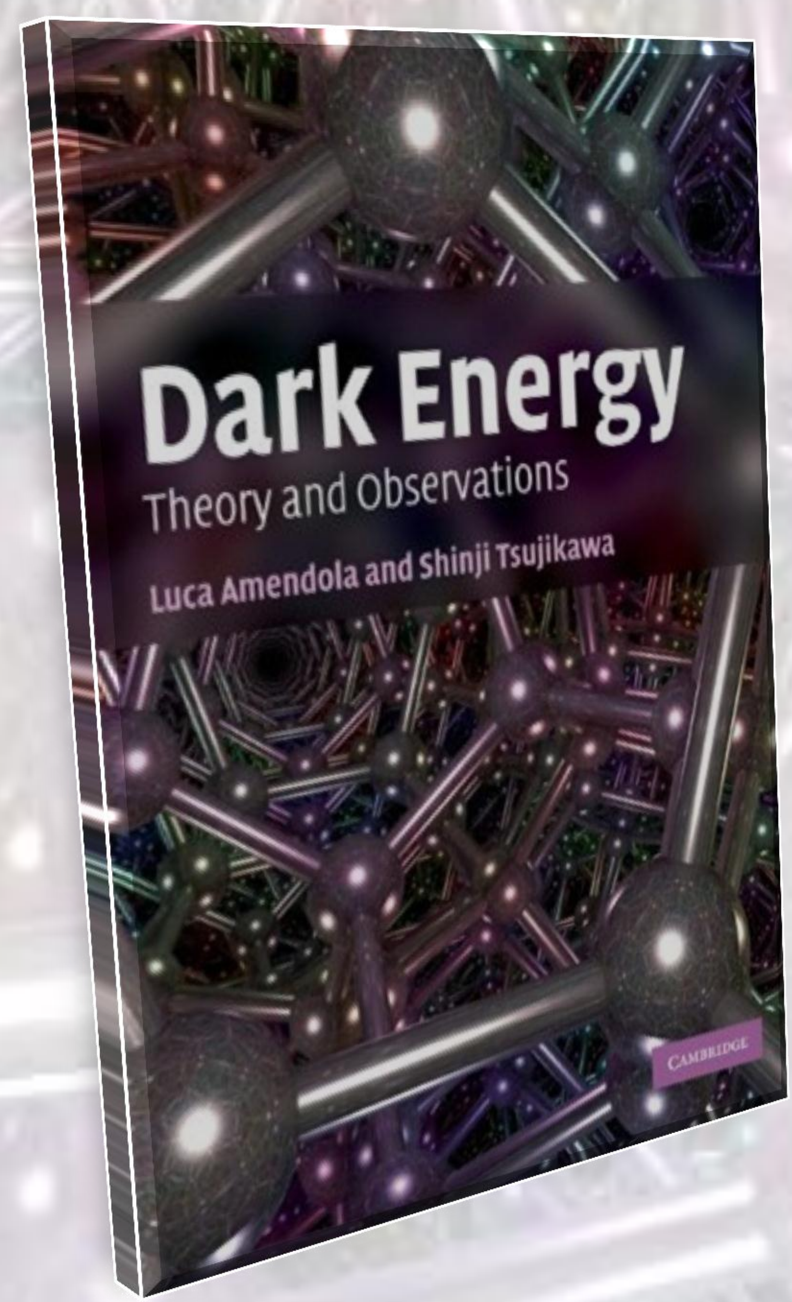
Growth factor AND bias to 2-3% in every redshift bin.

Lensing and supernovae

Lensing effects are non-Gaussian and cosmology dependent.
They can alter significantly the parameter estimates.

Homogeneity and isotropy

Radial inhomogeneities and departures from isotropy are not yet ruled out. Their existence should be disproven or confirmed with next generation experiments.



Cambridge University Press 2010

Generalized density/velocity

Standard relation

$$\theta \equiv \nabla v = -\delta'$$

$$\theta = -f\delta, \qquad f \equiv \frac{\delta'}{\delta}$$

Generalized relation

$$\theta = -F\delta$$

$$F = f + \frac{9\lambda^2\Omega_c}{2}(f - 1 + \beta\phi')(\frac{2}{3}\beta^2 + 1)$$