Modifications of General Relativity and the dark sector

« *Some thoughts* »

Jean-Philippe UZAN

Cosmological models

Theoretical physics → Cosmology ← Astrophysics

Principles
Local law of nature

Phenomena

Extrapolations → models ← Constraints

Gravity=GR
Matter=SU(3)XSU(2)xU(1)

Compatible with local physics
Save the apperances

Its construction relies on 4 hypothesis

1. Theory of gravity [General relativity]
2. Matter [Standard model fields + CDM + Λ]
3. Symmetry hypothesis [Copernican Principle]
4. Global structure [Topology of space is trivial]

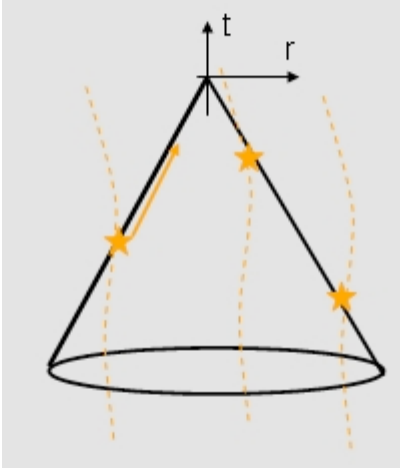
→ New physics with simple cosmological solution

→ Standard physics with more involved solution

In agreement with most (cosmological) data.

Implications of the Copernican principle

Independently of any theory (**H1, H3**), the Copernican principle implies that the geometry of the universe reduces to *a(t).*



Consequences:

- $$1+z=\frac{E_{rec}}{E_{em}}\overset{\text{H2}}{=}\frac{a_0}{a(t)}$$

- $$a(t)=a_0\left[1+H_0(t-t_0)-\tfrac{1}{2}q_0H_0^2(t-t_0)^2+\ldots\right]$$

  so that

$$H^2(z)/H_0^2=1+(q_0+1)z+\mathcal{O}(z^2)$$

$$q_0=\Omega_{m0}/2$$

- **Hubble diagram** gives
  - H_0 at small z
  - q_0

Supernovae data (1998+) show

$\boxed{q_0<0}$ ⟷ The expansion is now **accelerating**

Dark sector called by the observations

<u>Galaxie rotation curves</u>

Taken as a proof of the existence of dark matter
MOND alternative: modification of Newton law in low acceleration

<u>Acceleration of the universe</u>

SNIa
Conclusion depends only on the validity of the Copernican principle

IF CP holds THEN necessity to extend our reference theory

« Dark energy »

Various ways to achieve this.

Gravitation = any long range force that cannot be screened

Some questions

Cosmology requires new d.o.f.: what is their nature (physical vs geometrical)

Is gravitation well described by General Relativity? On which scales?

General Relativity

- in which regimes is it tested?
- can we define classes of universality
- from a theoretical point of views what are the constraints

Cosmological principle: on which scale does it hold?

Matter: are we allowed to describe it by a perfect fluid on cosmological scales?

New physical degrees of freedom

-

nature & signatures

GR

Underlying hypothesis

Equivalence principle

- Universality of free fall
- Local lorentz invariance
- Local position invariance

$$S_{matter}(\psi, g_{\mu\nu})$$

Dynamics

$$S_{grav} = \frac{c^3}{16\pi G}\int\sqrt{-g_*}\,R_*\,d^4x$$

Relativity

$$\boxed{g_{\mu\nu} = g^*_{\mu\nu}}$$

Universality classes of extensions

Cosmological dynamics

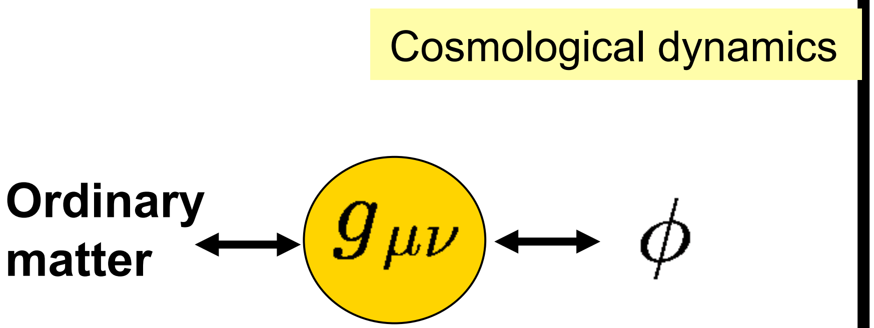
Ordinary matter $\longleftrightarrow$ $g_{\mu\nu}$ $\longleftrightarrow$ ϕ

<u>Ex</u> : quintessence, ...

$S_{\rm de}[{\rm de}; g_{\mu\nu}]$

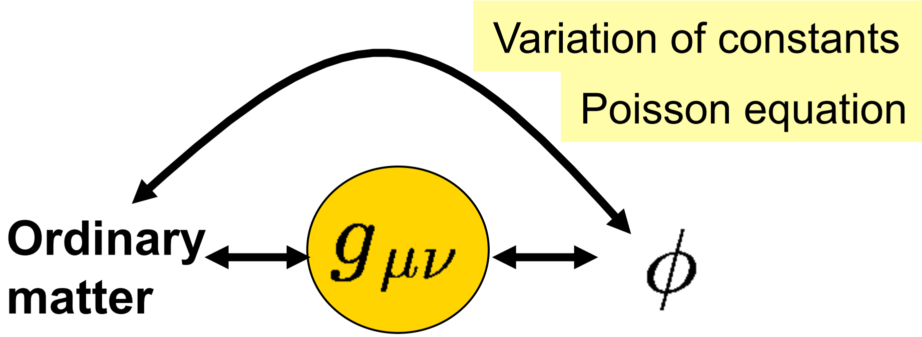
[JPU, Aghanim, Mellier, PRD 05]
[JPU, GRG 2007]

Universality classes of extensions



Ex : quintessence, ...

$S_{\rm de}[\mathrm{de}; g_{\mu\nu}]$



Ex : scalar-tensor, TeVeS .... $S_\varphi[\varphi; g_{\mu\nu}]$

$$S_m[\mathrm{mat}; g_{\mu\nu}] \to S_m[\mathrm{mat}_i; A_i^2(\varphi) g_{\mu\nu}].$$

[JPU, Aghanim, Mellier, PRD 05]
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Variation of constants

Poisson equation

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$S_{\rm em}[A_\mu; g_{\mu\nu}] \to S_{\rm em}[A_\mu, a_\mu; g_{\mu\nu}].$

Ordinary matter $\longleftrightarrow$ $g_{\mu\nu}$

$A_\mu \updownarrow a_\mu$

Ex : axion-photon mixing

Distance duality

[JPU, Aghanim, Mellier, PRD 05]
[JPU, GRG 2007]

Universality classes of extensions

Cosmological dynamics

Ordinary matter $\longleftrightarrow g_{\mu\nu} \longleftrightarrow \phi$

Ex : quintessence, ...

$S_{\rm de}[{\rm de}; g_{\mu\nu}]$

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Variation of constants

Poisson equation

Ordinary matter $\longleftrightarrow \tilde{g}^{(i)}_{\mu\nu}$

Ex : brane induced gravity
multigravity,...

Always need NEW fields

[JPU, Aghanim, Mellier, PRD 05]
[JPU, GRG 2007]

Newtonian limit

new fields

$$\text{Gravity} + \rho_{\text{dark}}(r) + \rho_b(r)$$

Dark sector field theory

$$\text{Newton} + \rho_{\text{DM}}(r) + \rho_b(r)$$

$$\text{MOND} + 0 + \rho_b(r)$$

Newtonian limit

$$T_{\mu\nu}^{\text{dark}} \gg T_{\mu\nu}^{\text{b}}$$

Dark matter

$$v^2(r)$$

$$T_{\mu\nu}^{\text{dark}} \ll T_{\mu\nu}^{\text{b}}$$

Gravity

Extensions

Any of these extensions requires new-degrees of freedom

we always have new matter fields
distinction matter/gravity is a Newtonian notion

Matter: amount imposed by initial conditions

This matter dominates matter content and triggers acceleration (**dark energy**)
This matter clusters and generates potential wells (**dark matter**)

Gravity: ordinary matter « generates » an effective dark matter halo
« induces » an effective dark energy fluid

We would like to determine

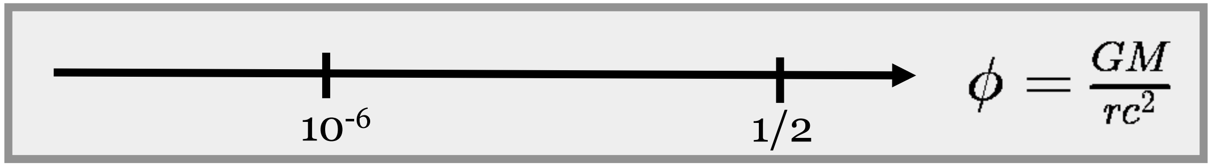
the nature of these degrees of freedom
the nature of their couplings

If they are light and if they couple to ordinary matter
responsible for a long range interaction

Most models contain ΛCDM as a continuous limit.

In which regime

Usually, we distinguish *weak-strong field* regimes



Corrective terms in the action have to be compared to *R*:

$$S = \frac{1}{16\pi G}\int [R + \Delta R]\sqrt{-g}d^4x$$

Also discussed in distinguishing *large-small distances*

Static configuation:

these limits are related because main dependance is (M,r)

acceleration may also be the best parameter (e.g. rotation curves)

Cosmology:

<u>background level</u>: R increases with z

<u>perturbation</u>: always in weak field

but at late time, we can have high curvature corrections

Parameter space

Tests of general relativity on astrophysical scales are needed

- galaxy rotation curves: low acceleration
- acceleration: low curvature

Solar system:

$$\frac{R}{\phi^3} = \frac{c^4}{G^2 M_\odot^2}$$

Cosmology:

$$R = 3H_0^2\{\Omega_m(1+z)^3 + 4\Omega_\Lambda\}$$

Dark energy:

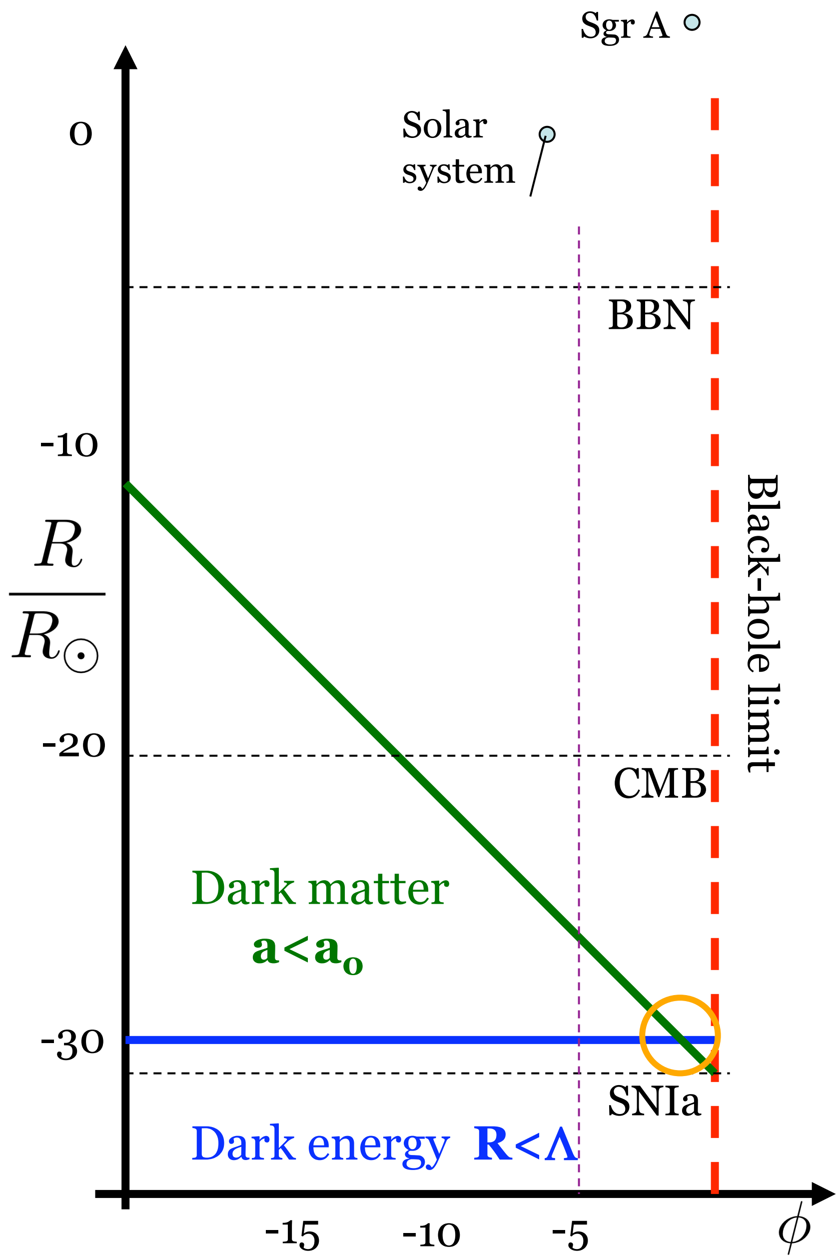
$$R < R_\Lambda = 12H_0^2\Omega_\Lambda$$

Dark matter:

$$a < a_0 \sim 10^{-8}\mathrm{cm.s}^{-2}$$

$$a^2 = \phi R < a_0^2$$

[Psaltis, 0806.1531]



Solar system

Metric theories are usually tested in the PPN formalism

$$ds^2 = (-1 + 2U + 2(\beta - \gamma)U^2)dt^2 + (1 + 2\gamma U)dr^2 + r^2 d\Omega^2$$

Light deflection

$$\Delta\theta = 2(1+\gamma)\frac{GM}{bc^2}$$

Perihelion shift of Mercury

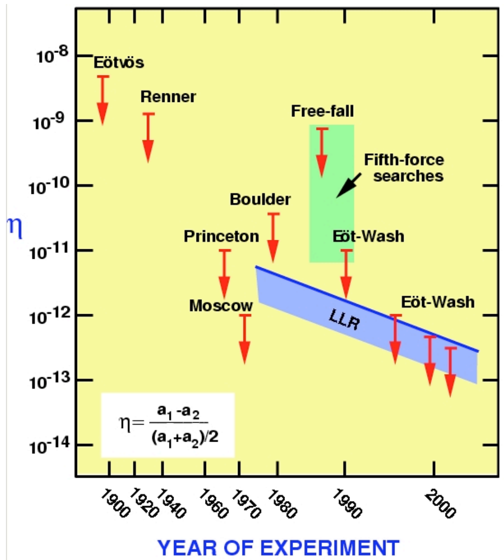
$$\Delta\varphi = \frac{2\pi GM}{a(1-e^2)}(2 + 2\gamma - \beta)$$

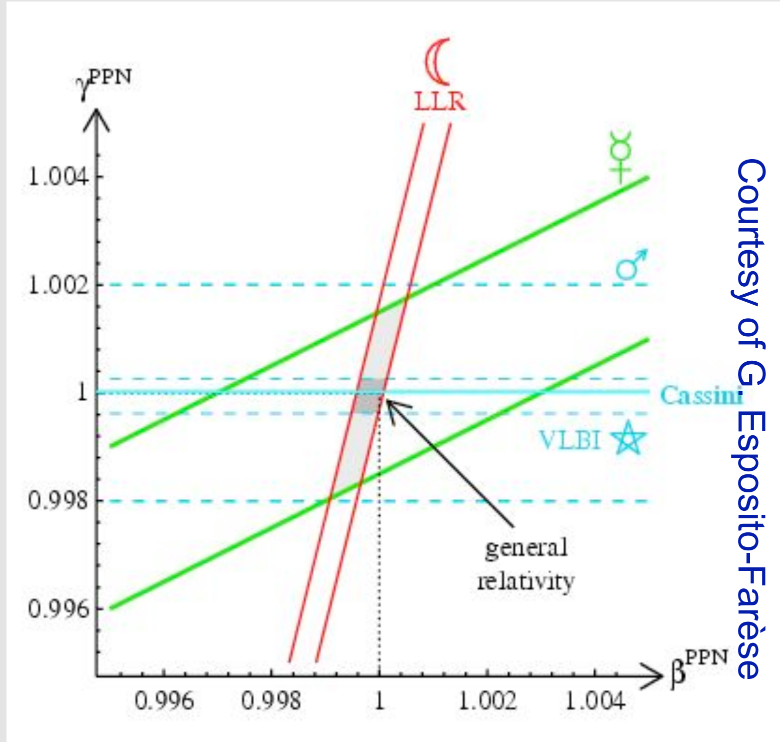
Nordtvedt effect

$$\delta r \sim 13.1(4\beta - \gamma - 3)\cos(\omega_0 - \omega_s)t \quad (\mathrm{m})$$

Shapiro time delay

$$\delta t \propto (1+\gamma)$$





$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

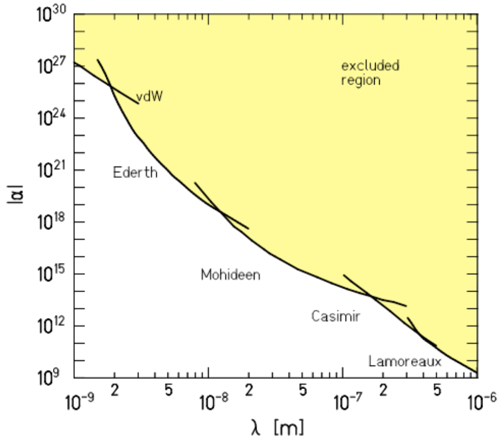
$$|2\gamma - \beta - 1| < 3 \times 10^{-3}$$

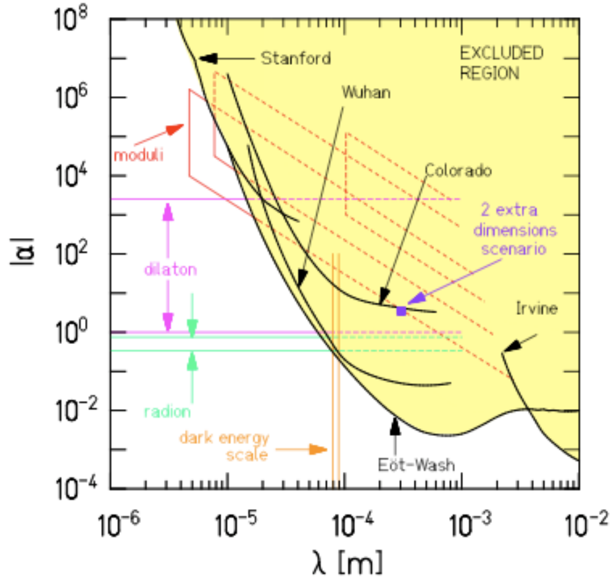
Fifth force

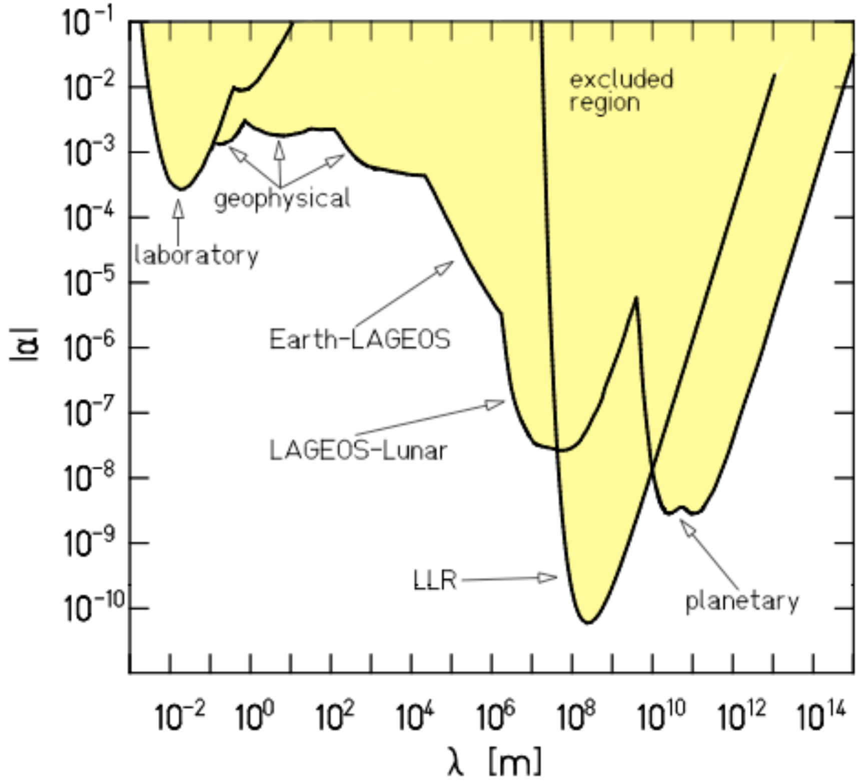
The PPN formalism cannot be applied if the modification of General relativity has a range smaller than the Solar system scale.

Fifth force experiments

Adelberger et al., Ann. Rev. Nucl. Part. Sci., 53 77 (2003)
Adelberger et al., Prog. Part. Nucl. Phys 62, 102 (2009)







Modifying GR

The number of modifications are numerous.

I restrict to field theory.

We can require the followin constraints:

- Well defined **mathematically**
  - *full Hamiltonian should be bounded by below*
    - *-no ghost ($E_{kinetic} > 0$)*
    - *-No tachyon ($m^2 > 0$)*
  - *Cauchy problem well-posed*

- In agreement with existing **experimental** data
  - *Solar system & binary pulsar tests*
  - *Lensing by « dark matter » - rotation curve*
  - *Large scale structure – CMB – BBN - …*

- Not pure fit of the data!

Design

The regimes in which we need to modify GR to explain DE and DM are different.

DM case: *we need a force ~ 1/r*

a priori easy:

- consider $V(\varphi) = -2a^2 e^{-b\varphi}$ [Not bounded from below]
- static configuration: $\Delta\varphi = V'(\varphi)$ and thus $\varphi = (2/b)\ln(abr)$

But:

The constant $(2/b)$ has to be identified with $M^{1/2}$!!

[see PRD76 (2007) 124012]

DE case:

Coincidence problem

ST: 2 free functions that can be determine to reproduce
$H(z)$ and $D_+(z)$.

	bgd	bgd + Newt. pert.	bgd + Newt. pert. + Solar syst.
DGP vs quintessence	Y	N	N
DGP vs scalar-tensor	Y	?	N

First example: higher-order gravity...

At quadratic order

$$S_g = \frac{c^3}{16\pi G}\int (R + \alpha C^2_{\mu\nu\rho\sigma} + \beta R^2 + \gamma GB)\sqrt{-g}\, d^4x$$

- $GB = R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2$ does not contribute to the field eqs.
- $\alpha C^2_{\mu\nu\rho\sigma}$ theory contains a ghost [Stelle, PRD**16** (1977) 953]

$$\frac{1}{p^2+\alpha p^4} = \underbrace{\frac{1}{p^2}}_{\text{massless graviton}} - \underbrace{\frac{1}{p^2+\alpha^{-1}}}_{\text{massive degrees of freedom with } m^2=1/\alpha}$$

massless graviton

massive degrees of freedom with $m^2=1/\alpha$

carries negative energy

$\alpha<0$: it is also a tachyon.

- βR^2 equivalent to positive energy massive scalar d.o.f

...and beyond

These considerations can be extended to $f(R, R_{\mu\nu}, R_{\mu\nu\alpha\beta})$

[Hindawi et al., PRD**53** (1996) 5597]

Generically contains massive spin-2 ghosts but for f(R)

These models involve generically higher-order terms of the variables.

the Hamiltonian is then generically non-bounded by below

[Ostrogradsky, 1850]
[Woodard, 0601672]

Argument does not apply for an infinite number of derivative

non-local theories may avoide these arguments

Only allowed models of this class are *f(R)*.

Reconstruction of the cosmological dynamics [see Amendola, Dunsby talks]

$f(R)$ and scalar-tensor theories

We consider the theory $S_g = \int f(R)\sqrt{-g}\,d^4x$

Introducing a Lagrange parameter to rewrite it as

$$S_g = \int\{f(\phi) + (R-\phi)f'(\phi)\}\sqrt{-g}\,d^4x$$

The field equation for ϕ reads $(R-\phi)f''(\phi) = 0$

The field equations of the 2 theories are identical.

The theory is thus equivalent to the ST:

$$S_g = \int\{f'(\phi)R - (\phi f'(\phi) - f'(\phi))\}\sqrt{-g}\,d^4x$$

Einstein frame: $\varphi = \frac{\sqrt{8}}{2}\ln f'(\phi) \quad V(\varphi) = \frac{\phi f'(\phi) - f(\phi)}{4f'^2(\phi)} \quad A(\varphi) = e^{\phi/\sqrt{8}} \quad g^*_{\mu\nu} = A^2 g_{\mu\nu}$

Generalisation:

$$f(R, \nabla^2 R, \ldots, (\nabla^2)^n R)$$

[Teyssandier, Tourrenc, JMP **24** (1983) 2793]
[Wands, CQG **11** (1994) 269]...

Scalar-tensor theories

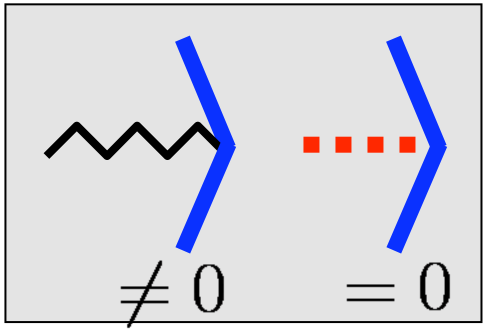
spin 2

spin 0

$$S = \frac{c^3}{16\pi G}\int\sqrt{-g}\{R - 2(\partial_\mu\phi)^2 - V(\phi)\}$$
$$+ S_m\{\text{matter}, \tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}\}$$

Maxwell electromagnetism is conformally invariant in d=4

$$S_{em} = \frac{1}{4}\int\sqrt{-\tilde{g}}\,\tilde{g}^{ab}\tilde{g}^{cd}F_{ac}F_{bd}\mathrm{d}^d x$$
$$= \frac{1}{4}\int\sqrt{-g}\,g^{ab}g^{cd}F_{ac}F_{bd}A^{d-4}(\phi)\mathrm{d}^d x$$



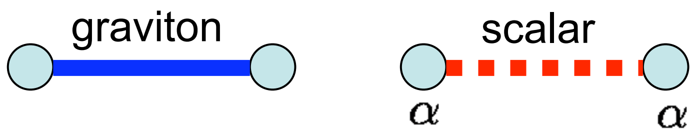
Light deflection is given as in GR

$$\delta\theta = \frac{4GM}{bc^2}$$

What is the difference?

The difference with GR comes from the fact that massive matter feels the scalar field

$$G_{\mathrm{cav}} = G(1+\alpha^2) \qquad \alpha = \mathrm{d}\ln A/\mathrm{d}\phi$$



Motion of massive bodies determines $G_{\mathrm{cav}}M$ **not** GM.

Thus, in terms of observable quantities, light deflection is given by

$$\delta\theta = \frac{4G_{\mathrm{N}}M}{(1+\alpha^2)bc^2} \leq \frac{4GM}{bc^2}$$

which means

$$M_{\mathrm{lens}} \leq M_{\mathrm{rot}}$$

Cosmological features of ST theories

Close to GR today

assume light scalar field

Can be attracted toward GR during the cosmological evolution.

[Damour, Nordtvedt]

Dilaton can also be a quintessence field

[JPU, PRD 1999]

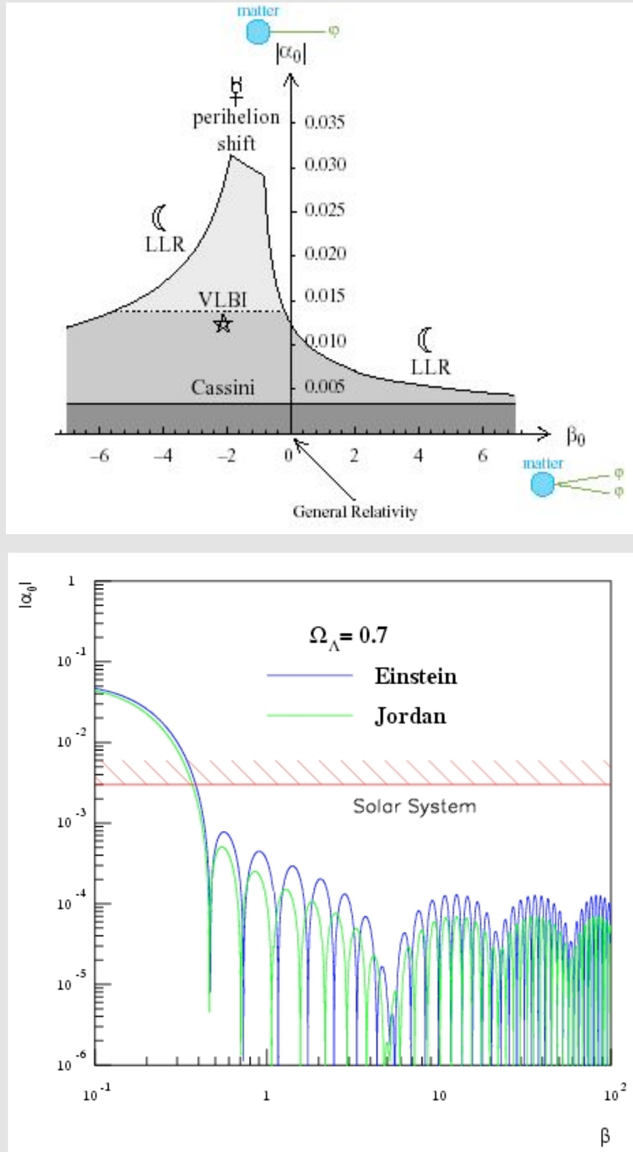
Equation of state today

$$3\Omega_{\rm de0}(w_0+1) \simeq 2(1-\beta_0)\phi_0'^2 - 2\alpha_0\phi_0''$$

[Martin, Schimd, JPU, 0510208]

Cosmological predictions computable (BBN, CMB, WL,...]

[Schimd et al., 2005; Riazuelo JPU, 2000, Coc et al., 2005]



[Coc et al, 0601299]

Example of varying fine structure constant

It is a priori « **easy** » to design a theory with varying fundamental constants

Consider

$$S = \int \left\{ \frac{1}{16\pi G} R \boxed{- 2(\partial_\mu \phi)^2 - V(\phi)} - \frac{1}{4} \boxed{B(\phi)} F_{\mu\nu}^2 \right\} \sqrt{-g}\, d^4x$$

But that may have dramatic implications.

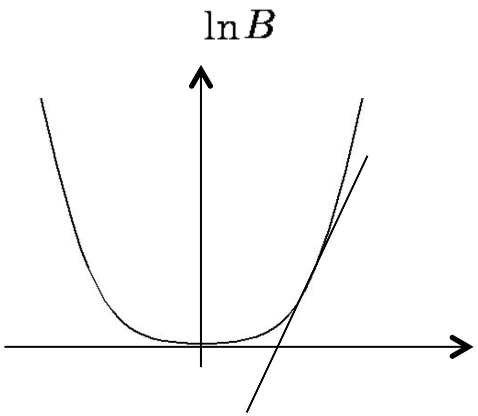
$$m_A(\phi) \supset 98.25\alpha \frac{Z(Z-1)}{A^{1/3}} \text{MeV} \longrightarrow f_i = \partial_\phi \ln m_i \sim 10^{-2} \frac{Z(Z-1)}{A^{4/3}} \alpha'(\phi)$$

Violation of UFF is quantified by

$$\eta_{12} = 2 \frac{|\vec{a}_1 - \vec{a}_2|}{|\vec{a}_1 + \vec{a}_2|} = \frac{f_{\text{ext}} |f_1 - f_2|}{1 + f_{\text{ext}}(f_1 + f_2)/2}$$

It is of the order of

$$\eta_{12} \sim 10^{-9} \underbrace{\mathrm{X}_{1,2,\text{ext}}(A, Z)}_{\mathcal{O}(0.1-10)} \times (\partial_\phi \ln B)_0^2$$



Requires to be close to the minimum

Equivalence principle and constants

Action of a test mass:

$$S = -\int m_A[\alpha_i] c \sqrt{-g_{\mu\nu} v^\mu v^\nu} dt \quad \text{with} \quad \begin{aligned} v^\mu &= dx^\mu/dt \\ u^\mu &= dx^\mu/d\tau \end{aligned}$$

Dependence on some constants

$$\delta S = 0$$

$$f_{A,i}$$

$$a_A^\mu = -\sum_i \left(\boxed{\frac{\partial \ln m_A}{\partial \alpha_i}} \frac{\partial \alpha_i}{\partial x^\beta} \right) (g^{\beta\mu} + u^\beta u^\mu)$$

(NOT a geodesic)

$$g_{00} = -1 + 2\Phi_N/c^2$$

(Newtonian limit)

$$\mathbf{a} = \mathbf{g}_N + \delta \mathbf{a}_A$$

$$\delta \mathbf{a}_A = -c^2 \sum_i f_{A,i} \left(\nabla \alpha_i + \dot{\alpha}_i \frac{\mathbf{v}}{c^2} \right)$$

Anomalous force
Composition dependent

[Dicke 1964,...]

Equivalence principle and constants

In general relativity, any test particle follow a geodesic, whic does not depend on the mass or on the chemical compositio

Imagine some constants are space-time dependent

1- Local position invariance is violated.

2- Universality of free fall has also to be violated

Mass of test body = mass of its constituants + binding energy

In Newtonian terms, a free motion implies $\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = \vec{0}$

But, now

$$\frac{d\vec{p}}{dt} = \vec{0} = m\vec{a} + \underbrace{\frac{dm}{d\alpha}\dot{\alpha}\vec{v}}_{m\vec{a}_{\text{anomalous}}}$$

Varying constants

The new fields can make the constants become dynamical.

The constant has to be replaced by a dynamical field or by a function of a dynamical field

This has 2 consequences:

1- the equations derived with this parameter constant will be modified

one cannot just make it vary in the equations

2- the theory will provide an equation of evolution for this new parameter

The field responsible for the time variation of the « constant » is also responsible for a long-range (composition-dependent) interaction

i.e. at the origin of the deviation from General Relativity.

In most extensions of GR (e.g. string theory), one has varying constants.

Importance of unification

Unification $$\alpha_i^{-1}(E) = \alpha_{GUT}^{-1} + \frac{b_i}{2\pi}\ln\frac{M_{GUT}}{E}$$

Variation of α is accompanied by variation of other coupling constants

QCD scale $$\Lambda_{\mathrm{QCD}} = E\left(\frac{m_c m_b m_t}{E^3}\right)^{2/27}\exp\left[-\frac{2\pi}{9\alpha_s(E)}\right]$$

Variation of Λ_{QCD}/M_p from α_S and from Yukawa coupling and Higgs VEV

Theories in which EW scale is derived by dimensional transmutation $$v \sim \exp\left[-\frac{8\pi^2}{h_t^2}\right]$$

Variation of Yukawa and Higgs VEV are coupled

String theory All dimensionless constants are dynamical – their variations are all correlated.

These effects cannot be ignored in realistic models.

String (inspired) models

In the framework of string theory, all dimensionless constants are expected to be dynamical.

From a phenomenological point of view

$$S = \int d^4x \sqrt{-g}\,(B_g R - B_\phi(\partial\phi)^2 - \tfrac{1}{4}B_{F_i}F_i^2 \; - B_\psi \psi D\bar{\psi} - V)$$

Little is known about these functions, the computation of which requires to go beyond tree-level.

$$B_i = e^{-\phi} + c_0^{(i)} + c_1^{(1)}e^{\phi} + \ldots$$

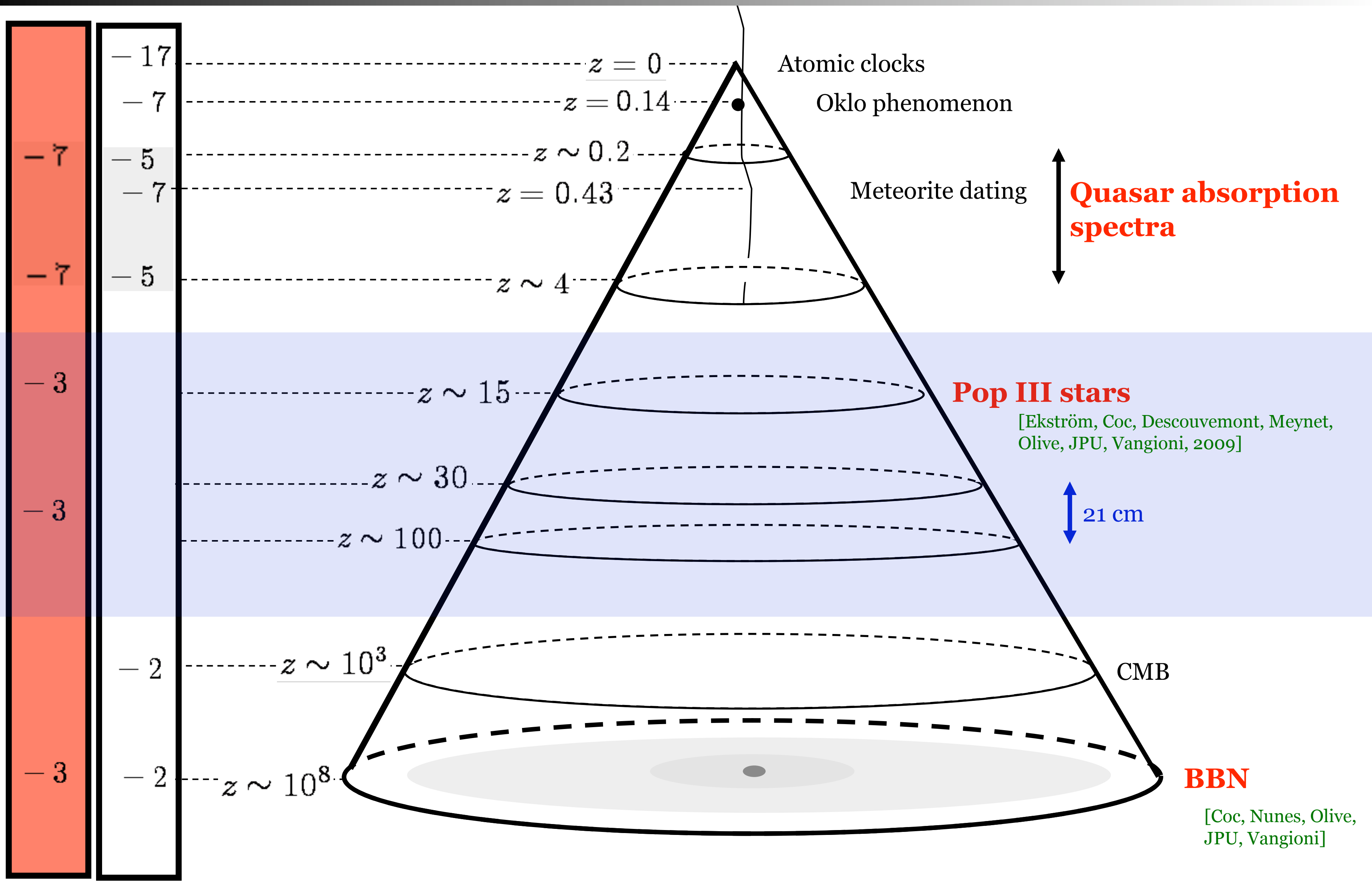
Damour, Polyakov (1994)

For the attraction mechanism toward GR to exist, they must have a minimum at a common value.

Composition idependent	Composition dependent
$\lvert\gamma - 1\rvert, \beta - 1, \dot{G}$	$\eta, \dot{\alpha}, \dot{\mu}$

$B \simeq -\frac{1}{2}\kappa(\phi - \phi_m)^2$ all deviations are proportional to $(\phi_0 - \phi_m)^2$

Physical systems: new and future



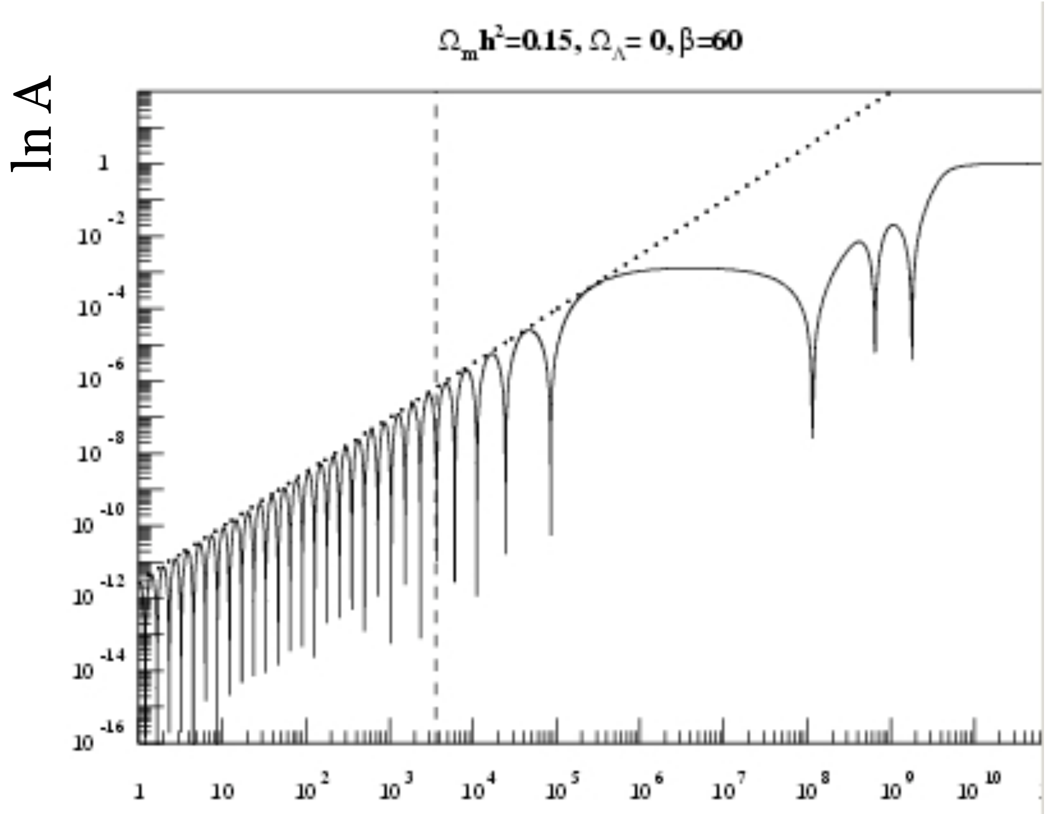
JPU, Liv. Rev. Relat., arXiv:1009.5514

Screening & decoupling mechanisms

To avoid large effects, one has various options:

- *Least coupling principle*: all coupling functions have the same minimum and the theory can be attracted toward GR

[Damour, Nordtvedt & Damour, Polyakov]



$$V = \mathrm{cst}, \quad A = \exp\left(\frac{1}{2}\beta\phi^2\right)$$

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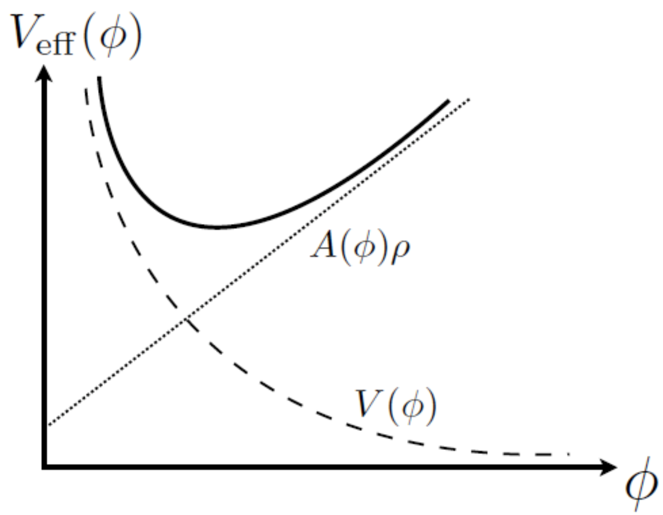
- *Chameleon mechanism*: Potential and coupling functions have different minima.



$$m^2_{\rm min} = V_{,\phi\phi}(\phi_{\rm min}) + A_{,\phi\phi}(\phi_{\rm min})\rho$$

The field can become massive enough to evade existing constraints.

[Khoury, Weltmann, 2004]
[Ellis et al., 1989]

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- *Chameleon mechanism*: Potential and coupling functions have different minima. [Khoury, Weltmann, 2004]

- *Symmetron mechanism*: similar to chameleon but VEV depends on the local density. [Pietroni 2005; Hinterbichler,Khoury, 2010]

$$\left.\begin{aligned} V(\phi) &= -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 \\ A(\phi) &= 1 + \frac{1}{2M^2}\phi^2 + \mathcal{O}(\phi^4/M^4) \end{aligned}\right\} \quad V_{\rm eff}(\phi) = \frac{1}{2}\left(\frac{\rho}{M^2} - \mu^2\right)\phi^2 + \frac{1}{4}\lambda\phi^4$$

Symmetry is restored at high density.

Environmental dependence

Disformal coupling

$$\tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu} + B(\phi)\partial_\mu\phi\partial_\nu\phi$$

Bekenstein, gr-qc/9211017

Bekenstein, Sanders, 9311062

<u>Preferred direction</u>
(radial for spherical system)

It was extended by Bekenstein (TeVeS theory…)

$$\tilde{g}_{\mu\nu} = A^2(\varphi) g_{\mu\nu} + B(\varphi) V_\mu V_\nu$$

Dynamical unit timelike vector

This is at the basis of the construction of TeVeS theories and many other bimetric theories:

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,\{R - 2f(\partial_\mu\varphi\partial^\mu\varphi)\} + S_M(\psi, \tilde{g}_{\mu\nu})$$

« k-essence » can extract
MOND behaviour or acceleration

Matter coupled to ϕ

Necessary
for lensing

Problems

Gravitational waves and bimetric

In models involving 2 metrics (scalar-tensor, TeVeS,...), gravitons and standard matter are coupled to different metrics.

In GR:

photons and gravitons are massless and follow geodesics of the same spacetime

$$\delta T_{\gamma g} = T_\gamma - T_g = 0$$

In bi-metric:

photons and gravitons follow geodesics of two spacetimes

(not in scalar-tensor theories)

$$\delta T_{\gamma g} \neq 0$$

Example:

TeVeS model. Observable=SN1987a

$$\delta T_{\gamma g} = -\ 5.3\,\mathrm{days}$$

Kahya & Woodard, 0705.0153

Testing General Relativity with large scale structure

Cosmological effects

How do these modifications influence the cosmology ?

Community seems to reach a state of thermal equilibrium of
How to test deviation from ΛCDM.

Original idea of 2001

On sub-Hubble scales, in weak field

(typical regime for the large scale structure)

$$\Delta\Phi = 4\pi G\rho a^2\delta$$

<u>Weak lensing</u>

$$\delta\theta = \frac{2}{c^2}\int \nabla_\perp \Phi \, \mathrm{d}\lambda$$

$$\langle \Phi(\theta)\Phi(\theta + n)\rangle$$

Distribution of the gravitational potential

<u>Galaxy catalogues</u>

$$n_{gal}(\mathbf{x})$$

$$\xi(r) = \langle \delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r})\rangle$$

Distribution of the matter

[JPU, Bernardeau (2001)]

Compatible?

Can we construct a post-ΛCDM formalism for the interpretation the large scale structure data?

Post-ΛCDM

Restricting to low-*z* and sub-Hubble regime

$$ds^2 = a^2(\eta)[-(1+2\Phi)d\eta^2 + (1-2\Psi)\gamma_{ij}dx^i dx^j]$$

Background

$$H^2/H_0^2 = \Omega_m^0(1+z)^3 + (1-\Omega_m^0-\Omega_\Lambda^0)(1+z)^2 + \boxed{\Omega_{\rm de}(z)}$$

w

Sub-Hubble perturbations

η,R,...

Q

$$\Delta(\Phi-\Psi) = \boxed{\pi_{\rm de}}$$

$$-k^2\Phi = 4\pi G_N \boxed{F(k,H)}\rho a^2\delta + \boxed{\Delta_{\rm de}}$$

$$\delta' + \theta = 0$$

$$\theta' + \mathcal{H}\theta = -\Delta\Phi + \boxed{S_{\rm de}}$$

Interacting DE

Numbers?
Functions?

Testing ΛCDM

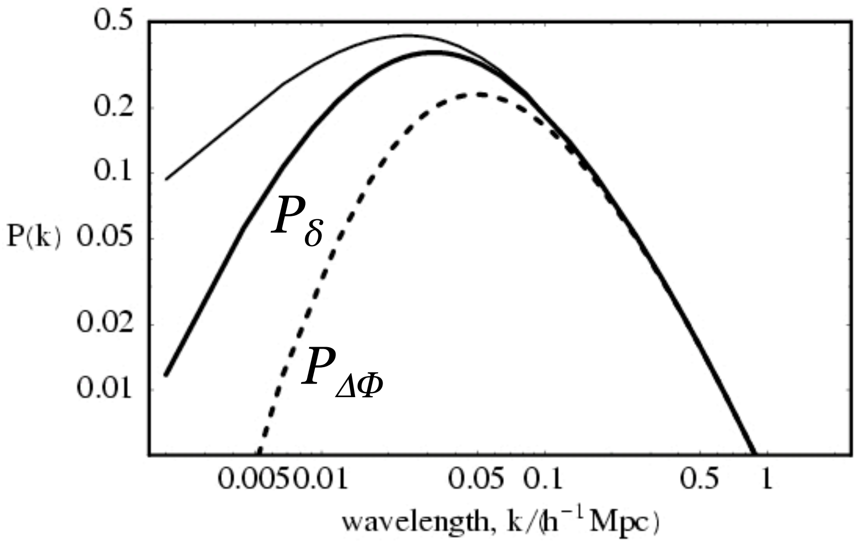
$$(F, \pi_{\rm de}, \Delta_{\rm de}, S_{\rm de}) = (1,0,0,0)$$

[JPU, astro-ph/0605313; arXiv:0908.2243]
[Schimdt, JPU, Riazuelo, astro-ph/0412120]

Data and tests

DATA	OBSERVABLE
Weak lensing	$\kappa \propto \Delta(\Phi + \Psi)$
Galaxy map	$\delta_g = b\,\delta$
Velocity field	$\theta = \beta\,\delta$
Integrated Sachs-Wolfe	$\Theta_{SW} \propto \dot{\Phi} + \dot{\Psi}$

Various combinations of these variables have been considered



JPU and Bernardeau, Phys. Rev. D **64** (2001)

Data and tests

Large scale structure $\delta_g = \frac{\delta n_g}{n_g}$ $\delta_g = b_1\delta + b_2\delta^2$

$$P^z_{gg}(k,\mu) = P_{gg}(k) + 2\frac{\mu^2}{aH}P_{g\theta_g}(k) + \frac{\mu^4}{a^2H^2}P_{\theta_g\theta_g}(k)$$

Lensing

-weak lensing: $P_{\Phi+\Psi,\Phi+\Psi}$

-galaxy-galaxy lensing: $P_{g,\Phi+\Psi}$

In a ΛCDM, all these spectra are related

$$P_{g\theta_g} = aH\frac{f}{b}P_{gg} \qquad P_{\theta_g\theta_g} = a^2H^2\frac{f^2}{b^2}P_{gg}$$

One needs to control the biais.

Biais

velocity map

$$\langle \delta_g \theta \rangle = b\beta \langle \delta^2 \rangle$$

Galaxy map

ΛCDM

$$\langle \delta_g \kappa \rangle \propto b \langle \delta \Delta (\Phi + \Psi) \rangle \propto 8\pi G \rho a^2 b \langle \delta^2 \rangle$$

weak lensing

The ratio of these 2 quantities is independent of the bias

Zhang et al, arXiv:0704.1932

Assume - no velocity bias ($S_{DE}=0$)
- no clustering of DE ($\Delta_{DE}=0$)

Origin of the rigidity

In the linear regime, the growth of density perturbation is then dictated by

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_{\rm mat}\delta = 0$$

This implies a *rigidity* between the growth rate and the expansion history

Bertschinger, astro-ph/0604485,
JPU, astro-ph/0605313

It can be considered as an equation for *H(a)*

Chiba & Takahashi, astro-ph/0703347

$$(H^2)' + 2\left(\frac{3}{a} + \frac{\delta''}{\delta'}\right)H^2 = 3\frac{\Omega_0 H_0^2\delta}{a^5\delta'}$$

$$\frac{H^2}{H_0^2} = 3\Omega_{m0}\frac{(1+z)^2}{\delta'(z)^2}\int_z \frac{\delta}{1+z}(-\,\delta')\mathrm{d}z$$

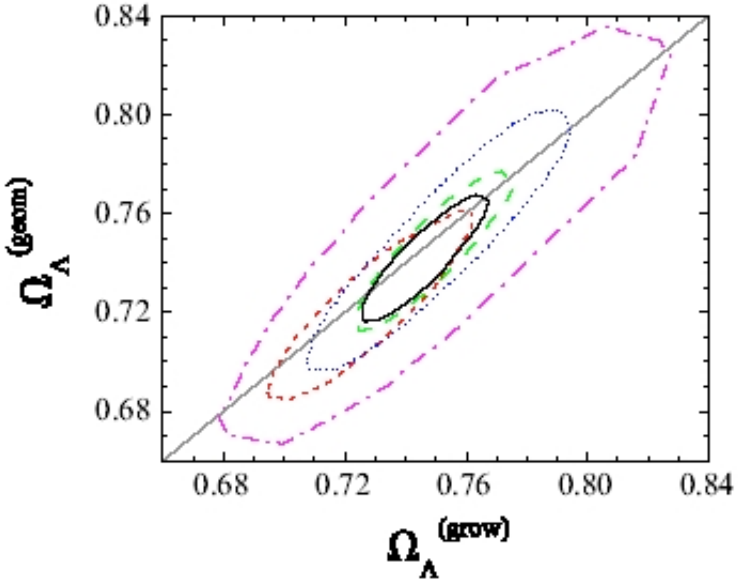
H(a) from the background (geometry) and growth of perturbation have to agree.

Growth factor: example

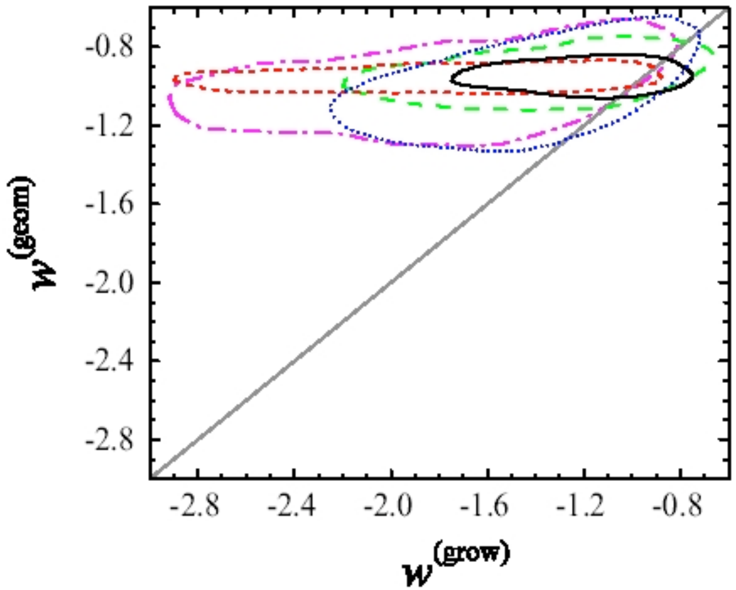
SNLS – WL from 75 deg^2 CTIO – 2dfGRS – SDSS (luminous red gal)
CMB (WMAP/ACBAR/BOOMERanG/CBI)

Wang *et al.*,arViv:0705.0165

Flat ΛCDM model



Flat *w* = constant



Consistency check of any DE model within GR with <u>non clustering</u> DE
Assume Friedmannian symmetries!

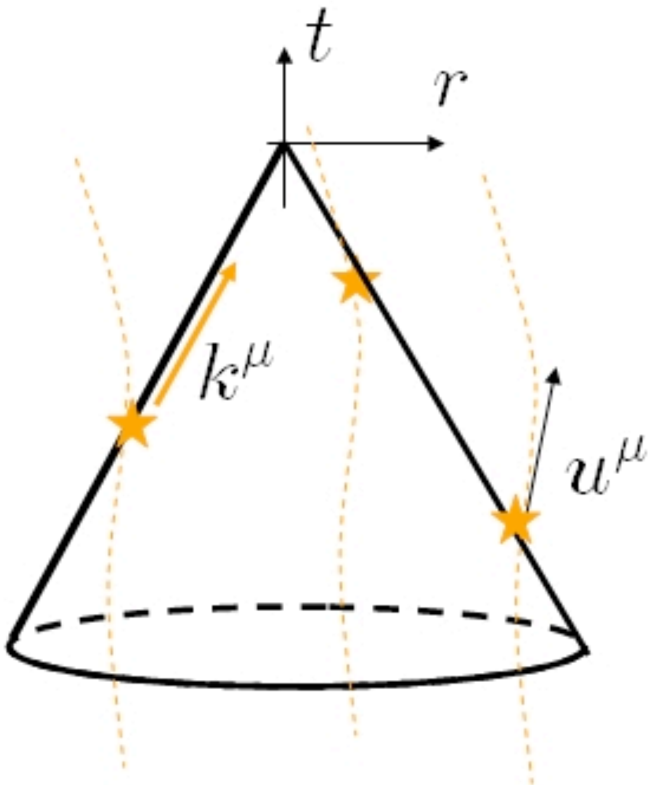
To go beyond we need a parameterization of the possible deviations

New geometrical degrees of freedom

Test of the Copernican principle

Redshift: $$1 + z = \frac{\lambda_{\mathrm{rec}}}{\lambda_{\mathrm{em}}} = \frac{a_0}{a}$$

t_o



Time drift of the redshifts

An interesting observable is the time drift of the redshift

Homogeneous and isotropic universe

$$\dot{z} = H_0(1+z) - H(z)$$

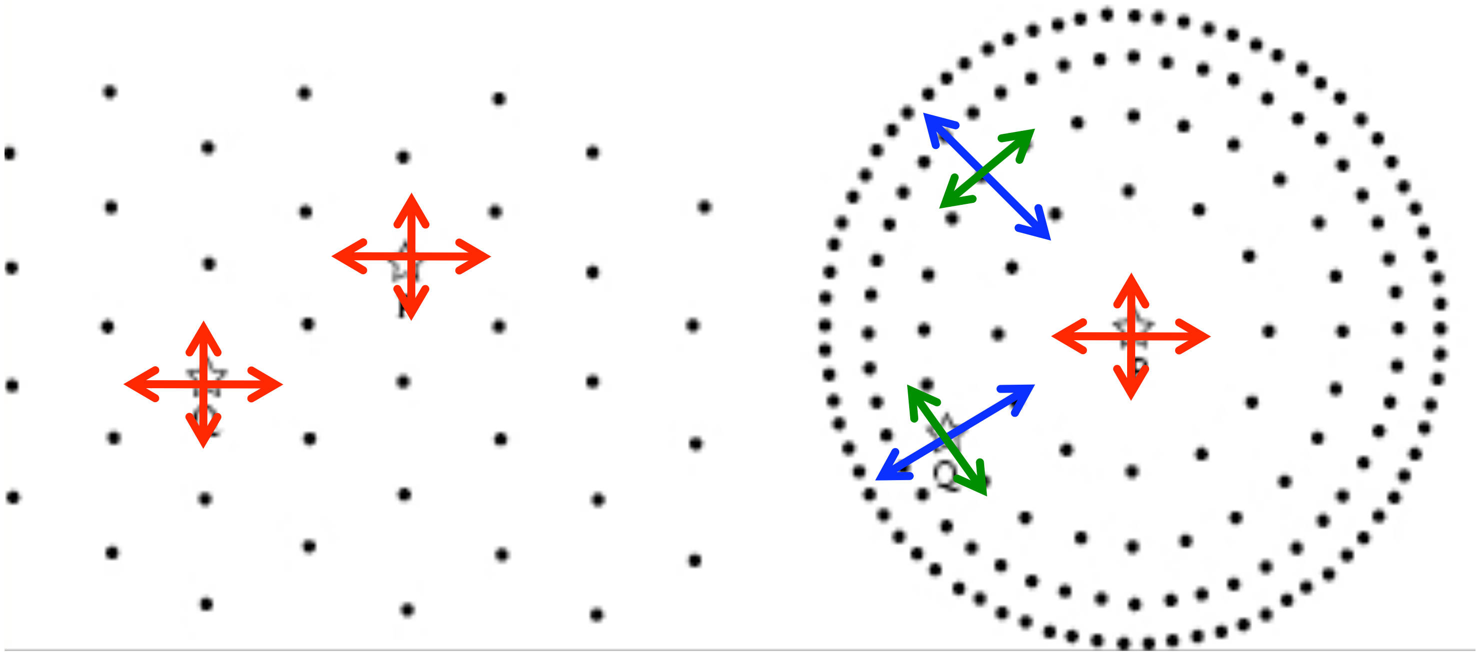
[Sandage1962, McVittie 1962]

Typical order of magnitude (z~4)

$$\delta z \sim -5\times 10^{-10} \quad \mathrm{on} \quad \delta t \sim 10\,\mathrm{yr}$$

Measurement of H(z)

Differences



Time drift and homogeneity

FL

$$H_{||} = H_{\perp}$$

$$\dot{z} = H_0(1+z) - H(z)$$

LTB

$$H_{||} \neq H_{\perp}$$

$$\dot{z} = (1+z)H_0 - H_{\perp}(z)$$

By combining distance measurements (D_A or D_L), one can test whether

$$H_{||} = H_{\perp}$$

We have information off the past light-cone.

ELT

At a redshift of *z=4*, the typical order of magnitude is

$$\delta z \sim -5 \times 10^{-10} \qquad \text{sur} \qquad \delta t \sim 10\,\text{ans}$$

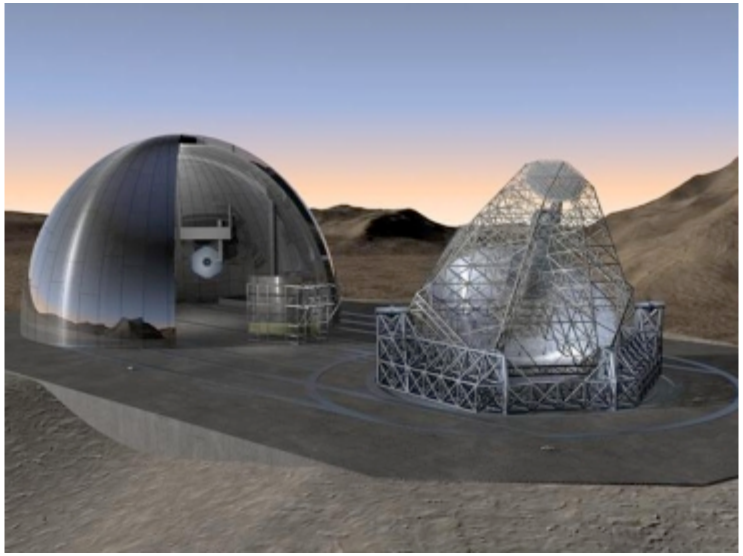
Variance [JPU, Bernardeau, Mellier, PRD (2007)]

Beyond what we can measure today **BUT**

ELT project:

- 40 meters of diameter
- ultrastable high resolution spectrograph (CODEX)
- 25 yrs ?
- 10 yrs of observation !

[see, Pasquini et al. (2005)]



How sensitive can such a test be?

« Popular » universe model: Lemaître-Tolman-Bondi

- spherically symetric but inhomogeneous spacetime
- i.e. spherical symetry around one worldline only : *center*

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \frac{X^2(r,t)}{1+2E(r)}\mathrm{d}r^2 + R^2(r,t)\mathrm{d}\Omega^2$$

Two expansion rates, a priori different

[for an off-center observer, the universe does not look isotropic]

$$H_\perp \equiv \frac{\dot{R}}{R}, \qquad H_\parallel \equiv \frac{\dot{X}}{X} = \frac{\dot{R}'}{R'}$$

The solution depends on 2 arbitrary functions of *r*

$$3 - 1 = 2$$

$$\boxed{\begin{array}{l} E(r) \\ t_B(r) \\ M(r) = mr^3 \\ R(r,t) \end{array}} \xrightarrow{\text{FL limit}} \boxed{\begin{array}{l} -kr^2 \quad k = \text{cst.} \\ 0 \\ m = \text{cst.} \\ a(t)r \end{array}}$$

How sensitive can such a test be?

R can be interpreted as the angular diameter distance so that, evaluated on the past light-cone:

$$R[t_*(z), r_*(z)] = D_A(z)$$

This allows to fix one of the free functions IF $D_A(z)$ is known.

There exist a class of LTB models reproducing the FL-$D_A(z)$, i.e. the FL-$D_L(z)$, observation.

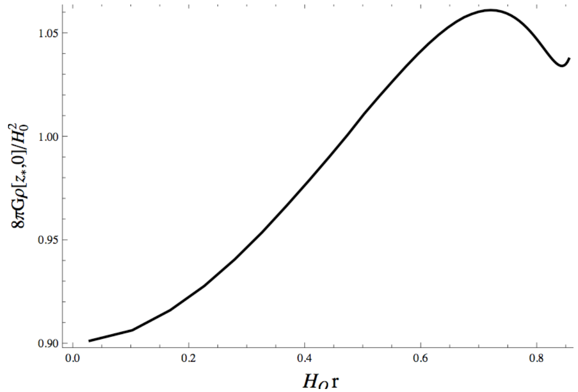
Full reconstruction requires an extra set of independent data.

In that class of models, we have $\dot{z} = (1+z)H_0 - H_\perp(z)$

- $D_A(z)$ and $\delta z(z)$ allow to fully reconstruct the LTB
- Give access to $H_{||}$ and $H_\perp$

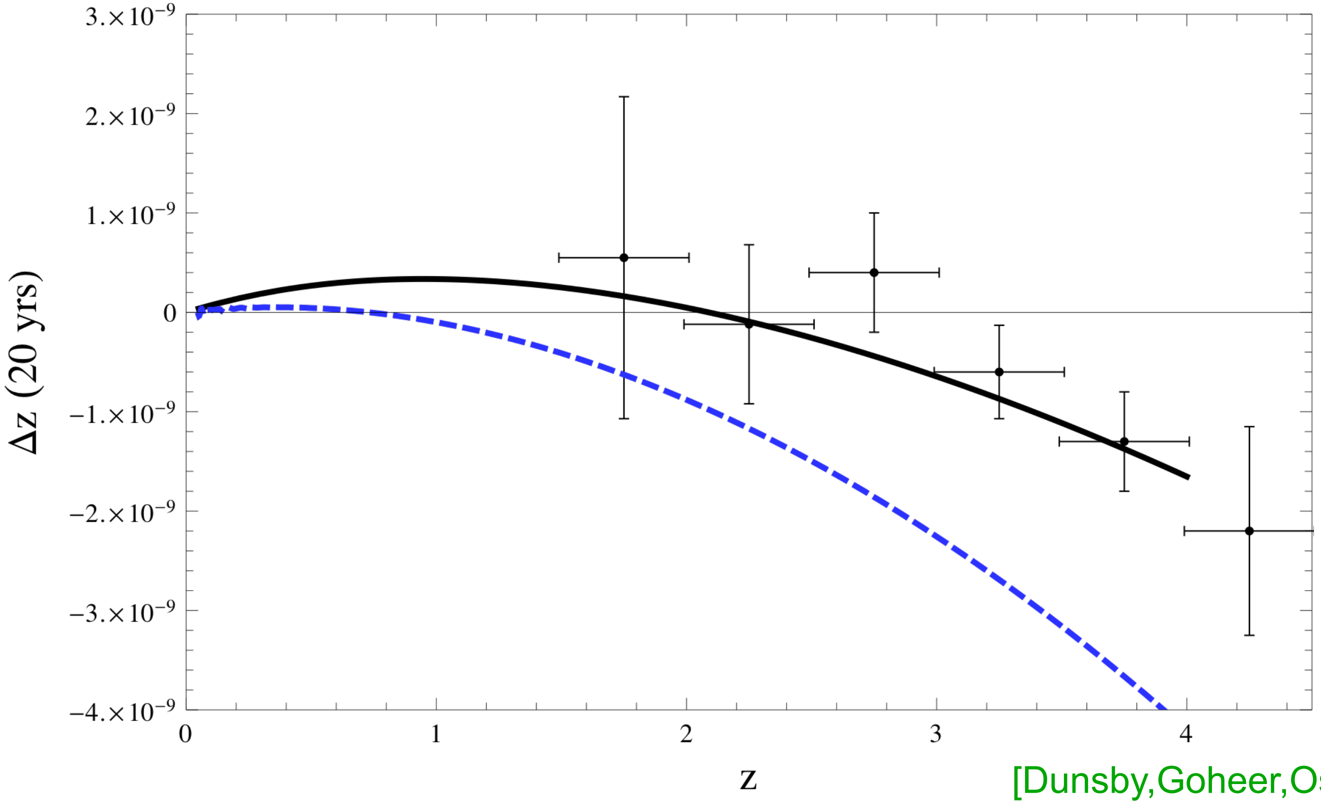
Importance of the Copernican Principle

Construct a LTB model such thatthat



$$8\pi G\rho(z) = 8\pi G\rho_{FL}(z) = 3\Omega_{\rm m0}H_0^2(1+z)^3$$

i.e. same $D_L(z)$ & same matter profile
BUT NO cosmological constant



[Dunsby,Goheer,Osano,JPU, 1002.2397]

Comparison to FL

Copernican principle:

- Geometry reduces to a(t).
- Reconstruction requires H(a) or equivalently H(z) since 1+z~1/a.
- One needs only 1 observable [$D_L(z)$ or $D_A(z)$].
- Data on the light cone are sufficient to reconstruct the full spacetime.
- δz is then predicted.

Lemaître-Tolman-Bondi solutions:

- Spherical symmetry
- Geometry depends on 2 arbitray functions of r.
- Bacground data [$D_L(z)$ or $D_A(z)$] are not enough.

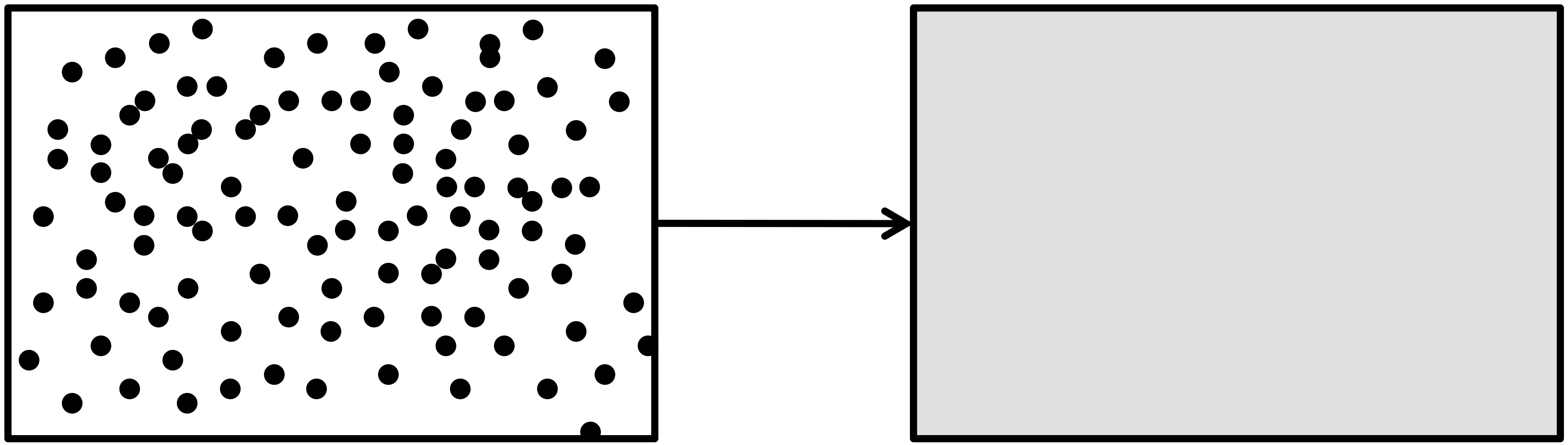
$$R[t_*(z), r_*(z)] = D_A(z)$$

- δz is an extra-piece of information that allows the reconstruction.
- This allows to get $H_\parallel$ and $H_\perp$ on the past light cone.

If FL is a good description, we must find that $H_\parallel = H_\perp$

Description of matter

Fluid approximation



Matter clustered / under dense regions

$$R_{\mu\nu} = 0$$
$$C_{\mu\nu\alpha\beta} \neq 0$$

Homegeneous density

$$R_{\mu\nu} \neq 0$$
$$C_{\mu\nu\alpha\beta} = 0$$

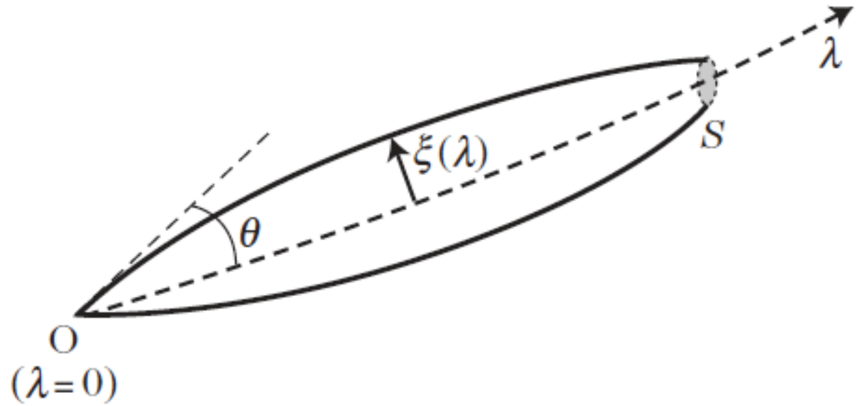
Two spacetimes are very different.
Can we understand why such a smoothing works.

Different from the backreaction problem.

Propagation of light

Geodesic deviation equation

$$k^\alpha k^\beta \nabla_\alpha \nabla_\beta \eta^\mu = R^\mu{}_{\nu\alpha\beta} k^\nu k^\alpha \eta^\beta.$$



Sachs equation

$$\frac{\mathrm{d}^2}{\mathrm{d}v^2}\eta_a = \mathcal{R}_{ab}\eta^b, \qquad \mathcal{R}_{ab} = \begin{pmatrix} \Phi_{00} & 0 \\ 0 & \Phi_{00} \end{pmatrix} + \begin{pmatrix} -\mathrm{Re}\,\Psi_0 & \mathrm{Im}\,\Psi_0 \\ \mathrm{Im}\,\Psi_0 & \mathrm{Re}\,\Psi_0 \end{pmatrix}$$

Ricci focusing Weyl focusing

$$\Phi_{00} = -\frac{1}{2} R_{\mu\nu} k^\mu k^\nu, \quad \Psi_0 = -\frac{1}{2} C_{\mu\nu\alpha\beta} m^\mu k^\nu m^\beta k^\beta,$$

For narrow beams, magnification and distorsion probe the small scale structure of spacetime.

Fluid approximation

Supernovae observation

beam is very thin: 1 AU @ z=1 corresponds to 10^{-7} arcsec
this is typically smaller than the distance between any massive object

beam propagates mostly in underdense regions

Zel'dovich, Dyer, Roeder

distribution of magnification

scatter of the m-z diagram allow to constrain the smoothness of the matter distribution.

systematic shift + scatter

On which scale are we allowed to use the fluid approximation?

Different from the backreaction approach.

See clarkson et al. arXiv:1109.2484

Conclusions

GR well tested in the Solar system but there is still place for modifications

Many extensions have been condered.

Field theory extentions are constrained

- Hamiltonian bounded from below (no ghost – tachyon)
- Cauchy problem
- need to go beyond a pure fit of the data

String inspired models

- generally leads to scalar-tensor theories [compactification] but usually induce variation of constants.
- brane models usually inlude massive gravitons.

Non-local models may avoid the general theorems.

Tests require combination of Solar system/strong-field/cosmology.

Cosmology does not reduce to large scale structure