Status and Prospects of Composite Higgs Models

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Firenze, 11 November

OUTLINE

- Review of Composite Higgs Models
 - Successes & Challenges
 - Phenomenology at LHC

- Beyond Minimal Models
 - Solving the Flavor Problem
 - Composite right-handed quarks

Soon we will know if the Higgs is fact or fiction.

Two paradigms:

• Weak Coupling: SM, Supersymmetry

 Strong Coupling: Technicolor, Composite Higgs, Higgsless, Extra-dimensions ... Soon we will know if the Higgs is fact or fiction.

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• Weak Coupling: SM, Supersymmetry

3+8 talks

 Strong Coupling: Technicolor, Composite Higgs, Higgsless, Extra-dimensions ...

2 talks

WHAT WE KNOW

SM is a gauge theory based on $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

$$\mathcal{L}_{Kinetic} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} W^b_{\mu\nu} W^{b\mu\nu} + i \sum_{j=1}^3 \left(\bar{\Psi}^j_L \ /\!\!\!D\Psi^j_L + \bar{\Psi}^j_R \ /\!\!\!D\Psi^j_R \right)$$

 $\Psi_{L,R} = (3,2)_{\frac{1}{6}} \oplus (3,1)_{\frac{2}{3}} \oplus (3,1)_{-\frac{1}{3}} \oplus (1,2)_{-\frac{1}{2}} \oplus (1,1)_{-1} \qquad (3 \text{ couplings})$

Unbroken gauge symmetry forbids mass terms: vacuum must respect a smaller symmetry

 $SU(3)_c \otimes U(1)_Q$

Mass terms can be written,

$$\mathcal{L}_{mass} = \sum_{i,j=1}^{3} \left[\bar{u}_{L}^{i} M_{i,j}^{u} u_{R} + \bar{d}_{L}^{i} M_{i,j}^{d} d_{R} + \bar{e}_{L}^{i} M_{i,j}^{e} e_{R} \right] + h.c.$$
$$+ m_{W}^{2} W^{2} + \frac{1}{2} m_{Z}^{2} Z^{2} \qquad \qquad O(20) \text{ parameters}$$

Mass for gauge bosons implies new degrees of freedom



The extra degrees of freedom are Goldstone Bosons

 $SU(2)_L \otimes U(1)_Y \to U(1)_Q$

which become longitudinal polarizations of W & Z

Important hint:

Custodial Symmetry $SU(2)_c$

In principle the Higgs scalar is not necessary for EWSB



$$A(W_L^+ W_L^- \to W_L^+ W_L^-) = \frac{1}{v^2} (s+t)$$

Interactions become strongly coupled around TeV. Perturbativity is violated at

 $\Lambda\sim 3\,{\rm TeV}$

In technicolor new strong interaction break EWS,

 $\langle \bar{\Psi}_{L}^{i} \Psi_{R}^{j} \rangle \sim v^{3} \delta^{ij} \longrightarrow \frac{SU(2)_{L} \otimes SU(2)_{R}}{SU(2)_{L+R}}$

Longitudinal polarizations of W&Z are composite states.

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Longitudinal polarizations of W&Z are composite states.

No Higgs scalar but techni-resonances (spin 0, 1/2, 1 etc.).

 $m_{\rho} < 3 \,\mathrm{TeV}$

 $\begin{array}{cc} & m_W = 80 \, \text{GeV} \\ \hline & 0 \end{array}$

Sadly phenomenologically highly problematic: precision electro-weak measurements and flavor

In the SM:



$$A(W_L^+ W_L^- \to W_L^+ W_L^-) \simeq \frac{1}{v^2} \left[s - \frac{s^2}{s - m_h^2} + (s \to t) \right]$$

Amplitude does not grow so SM can be valid up to the Planck scale.



COMPOSITE HIGGS

Georgi, Kaplan '80s

A logical possibility is that Higgs doublet is a light remnant of strong dynamics.



spin I spin 1/2 spin 0.... 2₁

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Two parameters:



Relieves hierarchy problem:

$$\delta m_h^2 \sim \frac{3\,\lambda_t^2}{4\pi^2} m_\rho^2$$

Particularly compelling if the Higgs is a Goldstone Boson: Massless at leading order:

	SO(5)	CB = (2, 2)
EX:	$\overline{SU(2)_L \otimes SU(2)_R}$	GD = (2, 2)

Agashe , Contino, Pomarol, '04

Low energy lagrangian:

 $\mathcal{L} = f^2 D_{\mu} \Sigma^i D^{\mu} \Sigma^i + \dots \xrightarrow{SU(2)_L \otimes SU(2)_R} \rho \approx 1$

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Extended Higgs sectors:

 $\frac{SO(6)}{SO(5)} \qquad \frac{SO(6)}{SO(4) \otimes U(1)} \qquad \frac{SU(5)}{SU(4) \otimes U(1)} \qquad + \dots$

Gripaios, Pomarol, Serra '09

Mrazek, Pomarol, Rattazzi, MR, Serra, Wulzer 'I I

Ex:

Main difference from techni-color is that f is not linked to v. Increasing f CH approximates SM.

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Spectrum:

 $m_{\rho} \sim 3 \,\mathrm{TeV}$

 $m_{\rho} = g_{\rho} f$

 $m_h = ?$ $m_W = 80 \,\text{GeV}$ 0

Reasonable phenomenology can be obtained for $m_{\rho} \sim 3 \,\mathrm{TeV}$

Two sectors:

Strong sector: Higgs + (top) m_{ρ} g_{ρ} Contino, Kramer, Son, Sundrum '06 Giudice, Grojean, Pomarol, Rattazzi'07 Panico, Wulzer '11 de Curtis, MR, Tesi '11

Elementary: SM Fermions + Gauge Fields

Two sectors:





Contino, Kramer, Son, Sundrum '06 Giudice, Grojean, Pomarol, Rattazzi'07 Panico, Wulzer '11 de Curtis, MR, Tesi '11

Elementary: SM Fermions + Gauge Fields

They talk through linear couplings:

$$\mathcal{L}_{gauge} = g \, A_{\mu} J^{\mu}$$

$$\mathcal{L}_{mixing} = \lambda_L \bar{f}_L O_R + \lambda_R \bar{f}_R O_R$$

$$\frac{\tan\varphi \sim \frac{\lambda}{g_{\rho}}}{\longrightarrow} \qquad y$$

$$y \sim \frac{\lambda_L \lambda_R}{g_\rho}$$

Potential generated at 1-loop:

$$V(H) \propto \frac{m_{\rho}^4}{g_{\rho}^2} \frac{\lambda_{L,R}^2}{16\pi^2} \hat{V}\left(\frac{H}{f}\right)$$

Composite sector has $SU(2)_L \otimes SU(2)_R \otimes U(1)_X$ symmetry $(Y = T_{3R} + U(1)_X)$



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To generate Yukawa for the down sector



Corrections to SM couplings of down quarks are small and zero for right quarks.

Agashe , Contino, da Rold, Pomarol, '04

Recent progress started with Randall-Sundrum constructions.



Relevant physics largely independent from 5D constructions. Only first resonance accessible to LHC.

Potential can be computed with one resonance.



de Curtis, MR, Tesi '11

If light Higgs is found, nearby fermionic partners expected. If lucky visible at LHC7.

Contino, Kramer, Son, Sundrum '06

Simplified 2 site picture: Each SM chirality has a Dirac fermionic partner

$$\mathcal{L}_{composite} = \bar{Q}(i \ D - m_Q)Q + \bar{T}(i \ D - m_T)T + Y_T \bar{Q}\tilde{H}T$$

$$\mathcal{L}_{mixing} = \frac{m_{\rho}}{g_{\rho}} \left[\lambda_L \bar{q}_L Q_R + \lambda_R t_R \bar{T}_L + \text{h.c.} \right]$$

Mass basis:

$$\begin{pmatrix} q_L \\ Q_L \end{pmatrix} = \begin{pmatrix} \cos \varphi_{q_L} & -\sin \varphi_{q_L} \\ \sin \varphi_{q_L} & \cos \varphi_{q_L} \end{pmatrix} \begin{pmatrix} q_L^{el} \\ Q_L^{co} \end{pmatrix}$$

$$m_H = \frac{m_Q}{\cos \varphi_{q_L}}$$

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Gauge fields:

$$\begin{pmatrix} A_{\mu} \\ \rho_{\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} A_{\mu}^{el} \\ \rho_{\mu}^{co} \end{pmatrix}$$

Yukawas:

$$y^{u} = \frac{\lambda_{Lu}}{g_{\rho}} \cdot Y^{U} \cdot \frac{\lambda_{Ru}^{\dagger}}{g_{\rho}} \qquad \qquad y^{d} = \frac{\lambda_{Ld}}{g_{\rho}} \cdot Y^{D} \cdot \frac{\lambda_{Rd}^{\dagger}}{g_{\rho}}$$

In the standard scenario $Y^{U,D}$ are anarchic. Small couplings are obtained from small mixings:

- Light generations mostly elementary
- Top strongly composite

Flavor hierarchies can be dynamically generated if the composite sector is conformal.

FIAV()R

Resonance exchange generates flavor violating 4-Fermi operators



FCNC of the light generation are suppressed by the mixings,



 $C_4^K \, \bar{d}_R^\alpha s_L^\alpha \bar{d}_L^\beta s_R^\beta$

 $C_4^K \sim \frac{g_\rho^2}{m_\rho^2} \frac{m_d \, m_s}{v^2}$

Flavor superior to TC theories.



Heavy resonances mostly coupled to 3rd generation + Higgs

SIGNATURES

Heavy resonances mostly coupled to 3rd generation + Higgs Spin-I: gluon, electro-weak



 $g_{\rho\bar{q}q} = g(\sin^2\varphi\cot\theta - \cos^2\varphi\tan\theta)$

First term negligible.

Decay into 3rd generation or heavy fermions.





Bini, Contino, Vignaroli '11 Barcelo', Carmona, Chala, Masip, Santiago '11 Bound:

 $m_{
ho} > 1000 \,\mathrm{GeV}$

CMS-pas-exo-11-006 866/pb Bound:

 $m_{\rho} > 1000 \,\mathrm{GeV}$

CMS-pas-exo-11-006 866/pb

Spin-1/2 : Top partners could be lighter + exotic charges

$$\left(\begin{array}{cc} T & T_{\frac{5}{3}} \\ B & T_{\frac{2}{3}} \end{array}\right) \qquad \qquad \tilde{T}$$

Production:



Double

Contino, Servant. '08



Single

Mrazek, Wulzer '09 Aguilar-Saveedra '09



Most sensitive experimental searches (1-slide snapshot)

Looking for pair production

[CMS L=1.14 fb ⁻¹] PAS-EXO-11-050	$T\bar{T} \to W b W \bar{b} \to b \bar{b} l^+ l^- \not\!$	$m_T > 422 \mathrm{GeV}$
[CMS L=0.80 fb ⁻¹] PAS-EXO-11-051	$T\bar{T} \rightarrow WbW\bar{b} \rightarrow b3jl^{\pm}E_{T}$	$m_T > 450 \mathrm{GeV}$
[CMS L=191 pb ⁻¹] PAS-EXO-11-005	$T\bar{T} \rightarrow tZ\bar{t}Z \rightarrow (l^+l^-)l^\pm jj$	$m_T > 417 \mathrm{GeV}$
[CMS L=1.14 fb ⁻¹] PAS-EXO-11-036	$\begin{split} B\bar{B} \to Wt W\bar{t} \to l^{\pm} l^{\pm} b 3j \not\!\!\!E_T \\ \to lll b 1j \not\!\!\!E_T \end{split}$	$m_B > 495{ m GeV}$

Rontino

Looking for single production

 $\begin{bmatrix} D0 & L=5.4 \text{ fb}^{-1} \end{bmatrix} \qquad Q\bar{q} \rightarrow Wq\bar{q} \rightarrow l^{\pm}jj \not E_T \\ \text{arXiv:1010.1466} \qquad \rightarrow Zq\bar{q} \rightarrow (l^+l^-)jj$

Notice: All analyses assume 100% BR to the considered channel

Modified couplings:



a, b, c receive corrections of order (v/f)^2. Modified Higgs cross-sections hard to see at LHC7.

WW scattering is not exactly unitarized. Unfortunately beyond LHC reach unless v/f~I.

Contino et al.'10

Within "anarchic" hypothesis only third generation and Higgs affected by compositeness: hard to distinguish from SM at LHC7.



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Models are however imperfect. Some tension exists particularly with flavor

- Kaon: $m_{
 ho} > 10 \, \frac{g_{
 ho}}{Y^D} \, {
 m TeV}$
- EDMs: $m_{\rho} > 2 Y^D \text{ TeV}$

Also $b \rightarrow s\gamma$ and ϵ'/ϵ , leptons...

Abandon the anarchic hypothesis?

FLAVOR SYMMETRIES

with Andreas Weiler arxiv:1106.6357[hep-ph] + work in progress

> See also: Weiler et al. '07 Barbieri, Isidori, Pappadopulo '08 Delaunay, Gedalia, Lee, Perez, Ponton '11

Adding a flavor symmetry:



 $egin{array}{ccc} \lambda_{Lu} & \lambda_{Ru} \ \lambda_{Ld} & \lambda_{Rd} \end{array}$

All flavor violation comes from the mixings.

 $y_u \propto \lambda_{Lu} \lambda_{Ru}$

 $y_d \propto \lambda_{Ld} \lambda_{Rd}$

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Simple realizations of Minimal Flavor Violation:

mixings ~ SM Yukawas

• Left-handed compositeness:

$\lambda_{Lu} \propto Id$,	$\lambda_{Ld} \propto Id$	\bot $SU(3)_F$
$\lambda_{Ru} \propto y_u ,$	$\lambda_{Rd} \propto y_d$	$\top \qquad \qquad L_U, L_D, U, D \in \mathcal{B}_F$

• Right-handed compositeness:



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• Right-handed compositeness:



 $SU(3)_U \otimes SU(3)_D$ $L_U, U \in (3, 1) \quad L_D, D \in (1, 3)$



Mixing of one chirality of light quarks is large.

Flavor bounds are automatically satisfied. No EDMs are generated to leading order. Flavor bounds are automatically satisfied. No EDMs are generated to leading order.

LEP bounds,

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to q\bar{q})} = .21629 \pm .00066$$

$$R_h = \frac{\Gamma(Z \to q\bar{q})}{\Gamma(Z \to \mu\bar{\mu})} = 20.767 \pm .025$$

Modified couplings strongly constrained



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Modified couplings strongly constrained



Similar bound is found from unitarity of CKM

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \approx 1 - .7 \,\frac{\delta g_{Lu}}{g_{Lu}}$$

 $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = .9999 \pm .0012$

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LH COMPOSITENESS:



Strongly constrained and only possible if tR is composite.

RH COMPOSITENESS:

No bounds from LEP!!

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No bounds from LEP!!

Main constraint from recent di-jet searches,

 $\frac{g_{\rho}^2}{4\,m_{\rho}^2}\,\sin^4\varphi_{q_R}\,\left(\bar{q}_{R\alpha}^i\gamma^\mu q_{R\beta}^i\bar{q}_{R\beta}^j\gamma_\mu q_{R\alpha}^j\right)\qquad\frac{\text{COMPOSI}}{-}$

$$\xrightarrow{\text{TENESS}} \quad \sin^2 \varphi_{q_R} \le \frac{2}{g_{\rho}} \left(\frac{m_{\rho}}{3 \,\text{TeV}} \right)$$

Large or full compositeness is still allowed with $m_{\rho} = 3 \text{ TeV}$

If RH quarks are fully composite MFV follows from the flavor symmetry.

LHC PHENO

Exciting phenomenology with RH compositeness: proton could be half composite!

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Exciting phenomenology with RH compositeness: proton could be half composite!

Spin-I: gluon, electro-weak, flavor resonances



 $g_{\rho\bar{q}q} = g(\sin^2\varphi\cot\theta - \cos^2\varphi\tan\theta)$

First term easily dominates for RH compositeness.

Gluon resonances:



Cross-sections O(10) larger. Decay into light generation can be important.

$g_{ ho}$	$\sin \varphi_{u_R}$	$\sin \varphi_{d_R}$	$\sigma(\text{pb})$	$\Gamma(\text{GeV})$	$\operatorname{Br}(u\overline{u})$	$\operatorname{Br}(t_L \overline{t}_L)$	$\operatorname{Br}(t_R \overline{t}_R)$
3	AN	AN	0.31	190	0.03	0.004	0.84
5	AN	AN	0.1	480	0.004	0.0005	0.98
3	0.4	0.4	0.17	50	0.07	0.35	0.005
5	0.4	0.4	0.38	60	0.14	0.1	0.12
3	0.8	0.8	2.8	340	0.17	0.0003	0.16
5	0.5	0.5	1.1	140	0.17	0.01	0.16
3	1	0.25	5.2	490	0.33	0.001	0.32
5	1	0.25	15	1400	0.33	0.0002	0.33

LHC14

LHC7 bounds already relevant:



Expected signals in di-jet!

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Expected signals in di-jet.

Spin-1/2 :

Partners of first generation mostly produced.



Large enhancement from access to valence quarks.







Atre, Azuelos, Carena, Han, Ozcan, Santiago, Unel 'I I

LHC searches could already probe fermions up to 1 TeV. LHC 14 will either discover or exclude the model.

Right quark partners produced by resonance exchange.





$$g_{\rho} = 3, \sin \varphi_{u_R} = .7, \sin \varphi_{d_R} = 1/6$$



MR, Sanz, Weiler, in progress

3-4 jets final states.

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CONCLUSIONS

• As in the SM in CH the Higgs will be found at LHC7.

• Within standard assumptions it will be hard to distinguish SM and CH at LHC7.

• If flavor symmetries are at work light quarks are partially composite and exciting signals could be seen at LHC7.

Flavor vs. Composite Higgs at LHC

Contino, Kramer, Son, Sundrum '06

Simplified 2 site picture: Each SM chirality has a Dirac fermionic partner

$$\mathcal{L}_{composite} = \bar{Q}^{i}(i \not D - m_{Q}^{i})Q^{i} + \bar{U}^{i}(i \not D - m_{U}^{i})U^{i} + \bar{D}^{i}(i \not D - m_{D}^{i})D^{i} + Y_{ij}^{U}\bar{Q}^{i}\tilde{H}U^{j} + Y_{ij}^{D}\bar{Q}^{i}HD^{j}$$

$$\mathcal{L}_{mixing} = \frac{m_{\rho}}{g_{\rho}} \left[\lambda_q^{ij} \bar{q}_{Li} Q_{Rj} + \lambda_u^{ij} u_{Ri} \bar{U}_{Lj} + \lambda_d^{ij} d_{Ri} \bar{D}_{Lj} + \text{h.c.} \right]$$

Mass basis:

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