Entanglement Spectrum and Boundary Theories (in quantum magnets) using PEPS

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OUTLINE

- Entanglement concepts & tools for studying many-body systems
- Some motivation (and warm-up): entanglement spectra of Heisenberg ladder [DP, PRL **105**, 077202 (2010)]
- "Holographic mapping": Boundary Hamiltonians from PEPS wavefunctions
- Application to 2d AKLT wavefunctions: connection between bulk and boundary
- Application to 2d topological states: dimer and SU(2)-RVB wavefunctions

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- Phys. Rev. B 83, 245134 (2011) [1,2,3]
- arXiv:1202.0947 [1,2,4]
- arXiv:1203.4816 [1,2,4]

Exotic states of matter

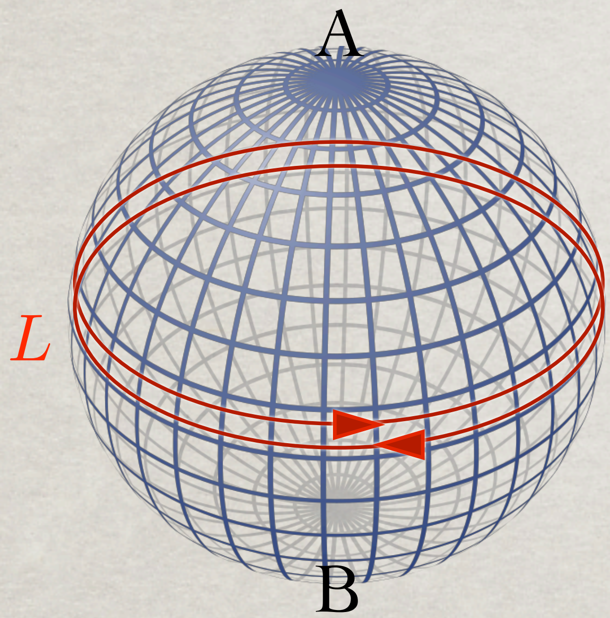
* no broken symmetry
* no local order
* GS degeneracy depends on **topology** of space

→ Topological order X. G. Wen

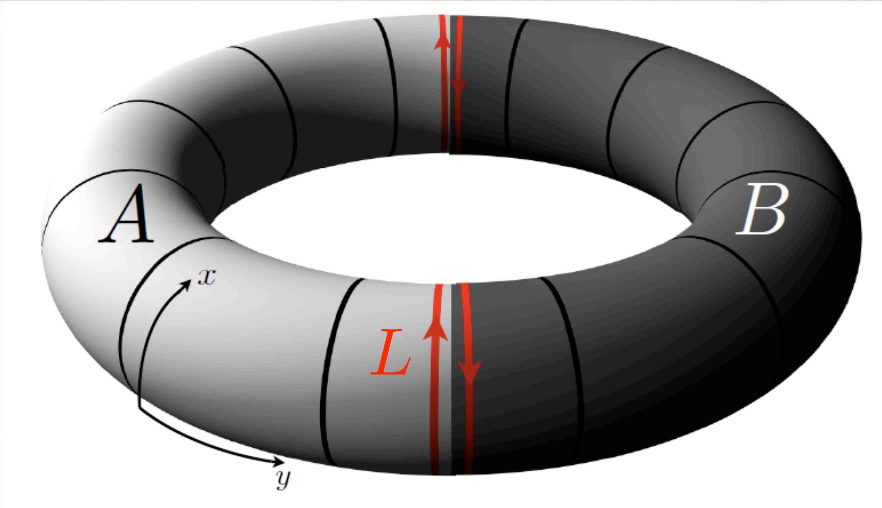
Exemple: (topological) spin liquid

Beyond the "order parameter paradigm": correlations "missed" by two-point correlation functions can be detected by entanglement measures

Edge states in (topological) FQH systems



Li & Haldane
PRL 2008



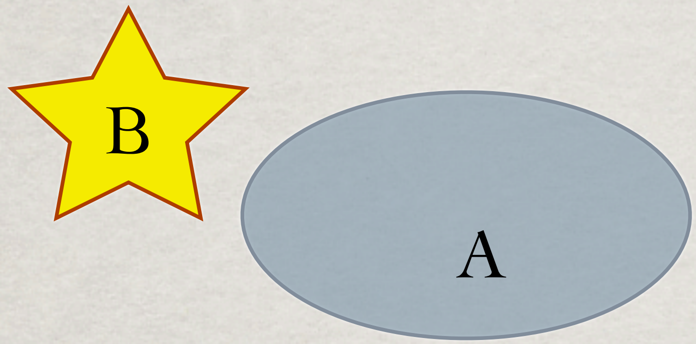
Lauchli et al., 2009

→ crutial role of edges !

Also topological insulators, etc...

Reduced density matrix

P. Dirac (1930)



$|\Psi\rangle \in \mathcal{E}_A \otimes \mathcal{E}_B$ $\qquad \rho = |\Psi\rangle\langle\Psi|$ projector

Definition: $\rho_A = \sum_j \langle j|_B \left(|\Psi\rangle\langle\Psi|\right)|j\rangle_B = \mathrm{Tr}_B\,\rho$

SINGLET: $\frac{1}{\sqrt{2}}(|\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B)$

$$\rho_A = (1/2)(|\uparrow\rangle_A\langle\uparrow|_A + |\downarrow\rangle_A\langle\downarrow|_A)$$

In general: $\rho_A = \sum_i \lambda_i^2\,|i\rangle_A\langle i|_A$ "mixed" ensemble

Entanglement Entropy

Kitaev & Preskill, 2006
Levin & Wen, 2006

A quantitative **measure:**

Reduced density matrix $\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$

$$S_{\mathrm{entanglement}} = -\mathrm{Tr}\{\rho_A \ln \rho_A\}$$ (Von Neumann)

$$S_{\mathrm{entanglement}} \propto \xi L^{d-1}$$ "area" law

d=2: $\propto L$ (perimeter)

More complex if critical or d=1 ...

Special Issue: Entanglement Entropy in Extended Quantum Systems, J. Phys. A **42**, N° 50, 500301-504012 (2009); Guest Editors: P. Calabrese, J. Cardy and B. Doyon.

Rewrite ρ_A as thermal density matrix

$$\rho(T) = \frac{1}{Z}\exp(-\beta H) = \sum_\alpha \exp(-\beta e_\alpha)|\alpha\rangle\langle\alpha|$$

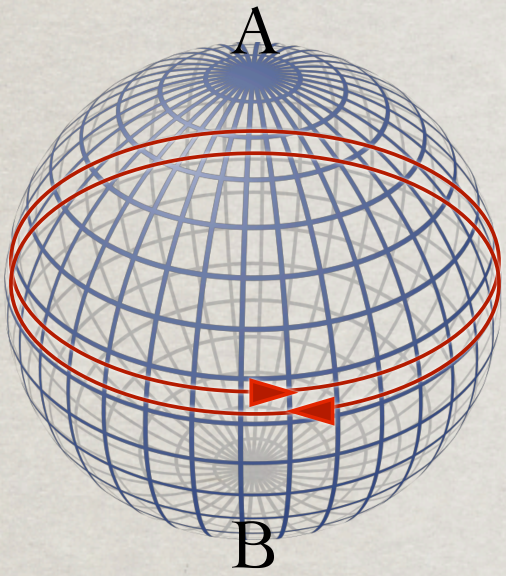
$\beta = 1/T$ inverse temperature

$$\rho_A = \sum_i \lambda_i^2 \, |i\rangle_A \langle i|_A$$

rewrite the weights as: $\lambda_i = \exp(-\xi_i/2)$

Entanglement spectrum : $\{\xi_i\}$

$$\rho_A = \exp(-\hat{\xi})$$



Li & Haldane, 2008

Regnault, Bernevig & Haldane, 2009

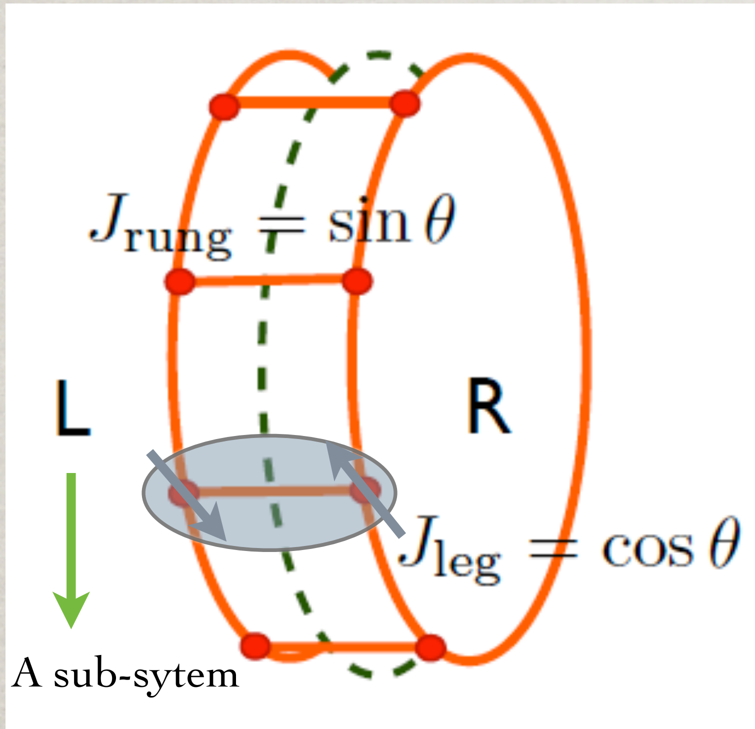
“Haldane” Conjecture:

Precise correspondence between the entanglement spectrum of a FQH system partitioned into two sub-systems linked by some “edge” and the true sub-system spectrum

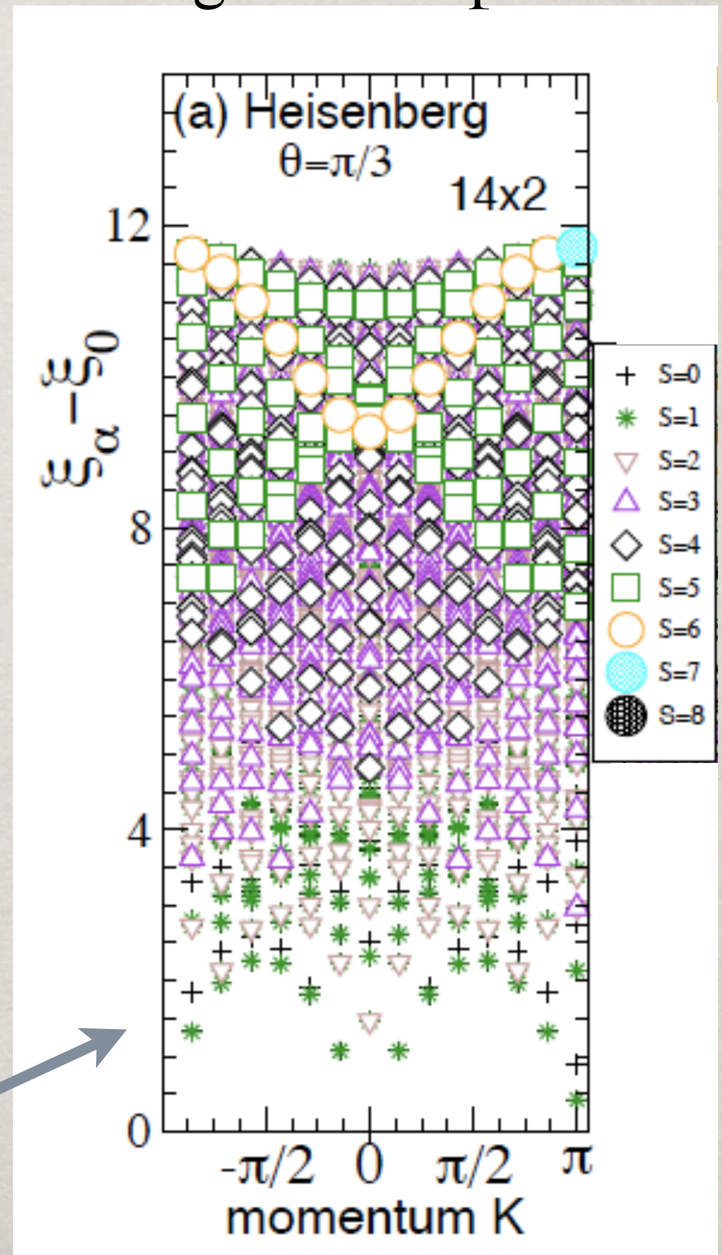
Questions:

- More generally, can the ES always be connected to the true edge spetrum ?
- Is there any edge property that reflect bulk properties ?

A simple example:
the 2-leg antiferromagnetic
spin "ladder"



Entanglement spectrum



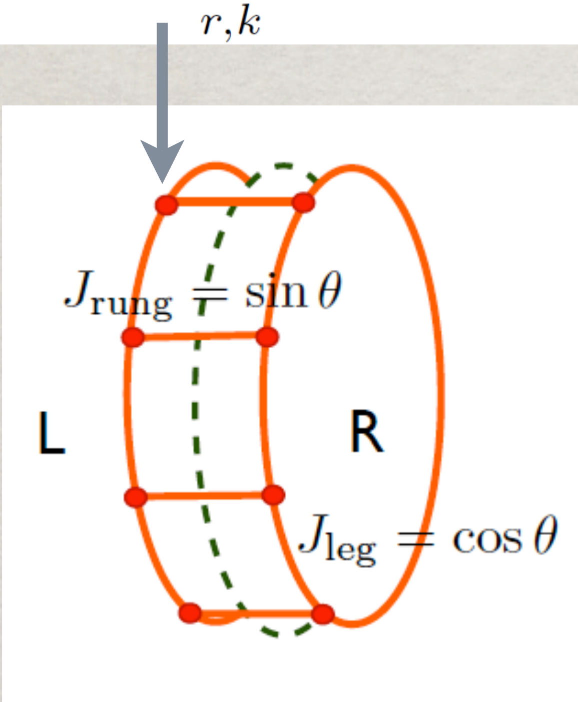
c=1 CFT

D.P., PRL **105**, 077202 (2010)

A precise characterization of the "boundary hamiltonien" is in fact possible !

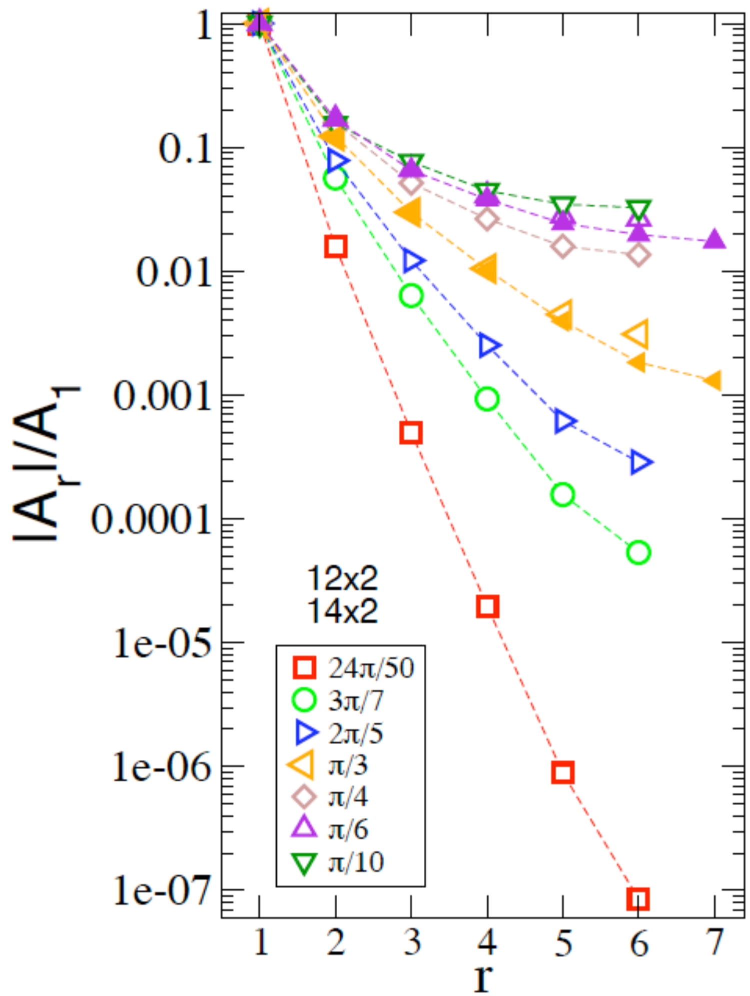
$$\rho_A = \exp\left(-H_b\right)$$

$$H_b = A_0 N_v + \sum_{r,k} A_r \, \mathbf{S}_k \cdot \mathbf{S}_{k+r} + R\hat{X}$$



2-leg ladder

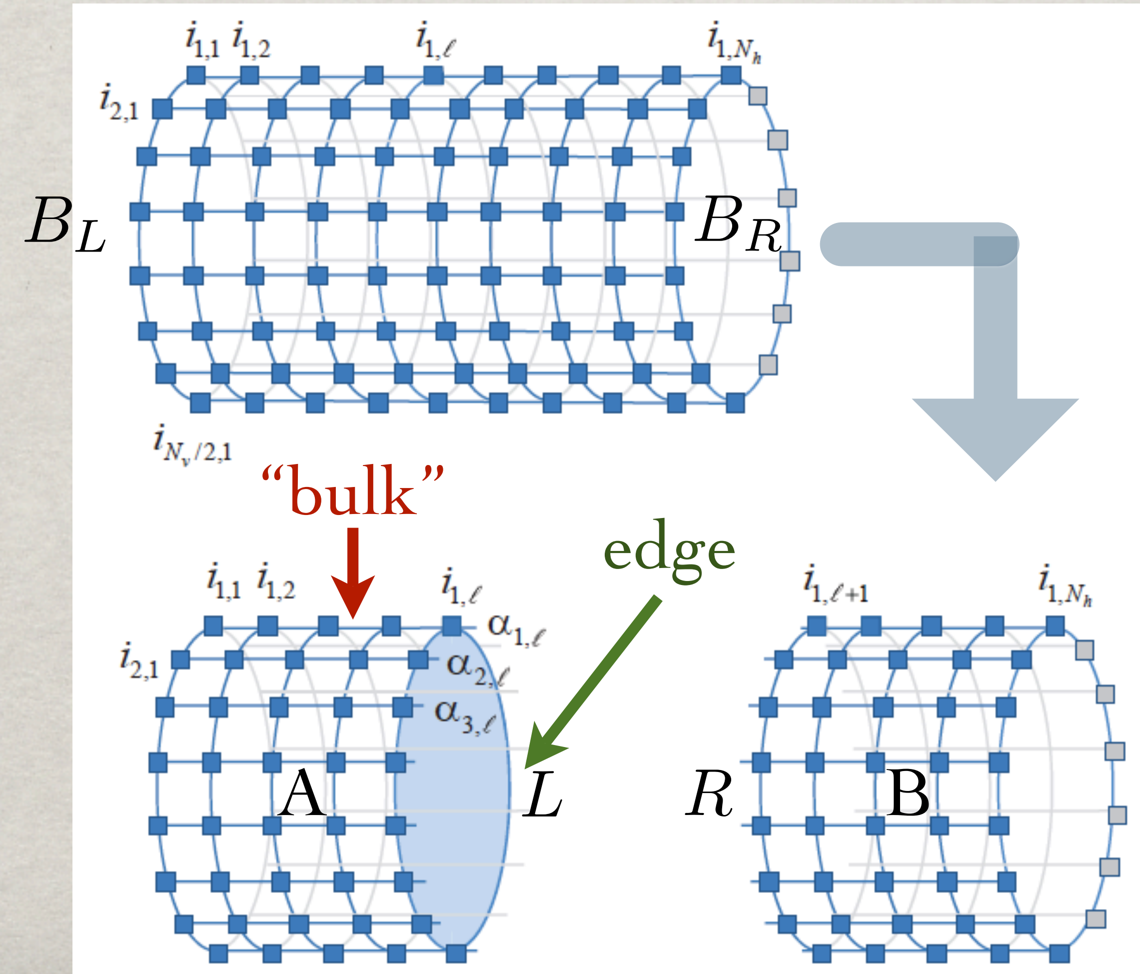
Exponential decay !



- Extend to long cylinders with Nh legs ?

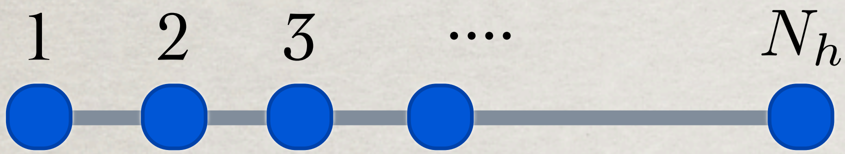
$N_h \to \infty$?

- Get a simple physical description of the degrees of freedom of H_b



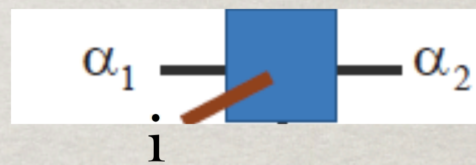
Tensor Network approaches

I. Cirac
F. Verstraete
G. Vidal



$$|\Psi\rangle = \sum_I c_I |i_1, i_2, ..., i_{N_h}\rangle \qquad i_k = -S, -S+1, ... , S-1, S$$

Matrix Product States (1D) : $M^i_{\alpha_1,\alpha_2}$ $D \times D$ matrix



Totsuka and M. Suzuki.,
J. Phys.: Condens.Matter,7(1639), 1995.
(see G. Sierra's talk)

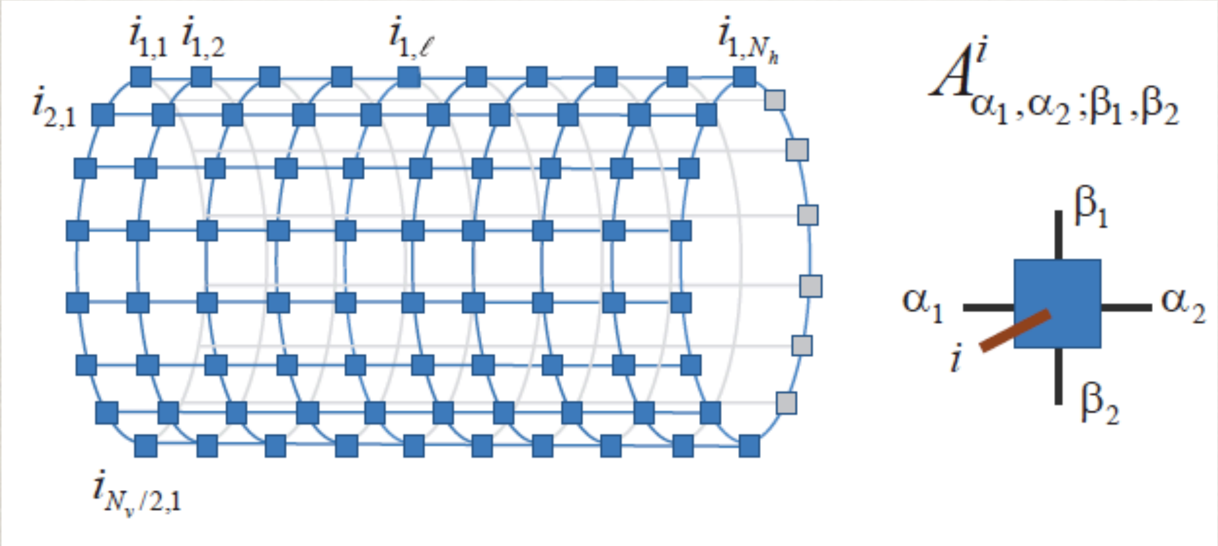
$$c_I = \sum_\alpha L^{i_1}_{\alpha_1} M^{i_2}_{\alpha_1 \alpha_2} ... M^{i_{N_h-1}}_{\alpha_{N_h-2}\alpha_{N_h-1}} R^{i_{N_h}}_{\alpha_{N_h-1}}$$

$$= \left(L^{i_1} M^{i_2} ... M^{i_{N_h-1}} R^{i_{N_h}} \right)$$

Equivalent to DMRG !! Romer and Ostlund (PRL, 1995)

D ~ m parameter controling the DMRG truncation

Tensor Network for d=2 (and higher): Projected Entangled Paired States (PEPS)



"contract" product of tensors

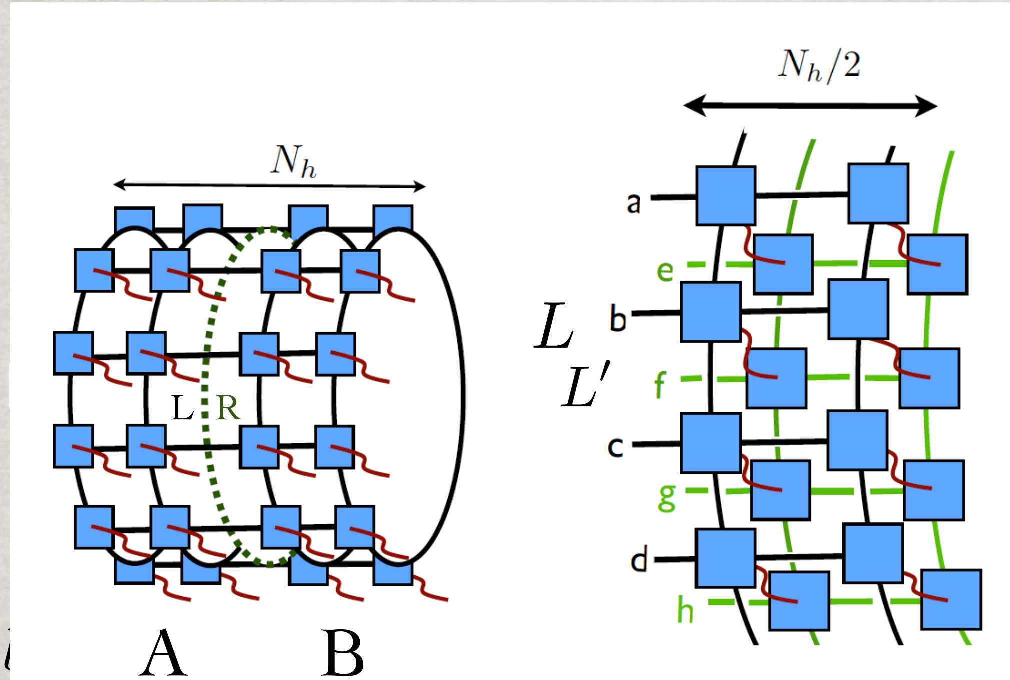
$$c_I = \sum_{\Lambda} L^{I_1}_{\Lambda_1} B^{I_2}_{\Lambda_1,\Lambda_2} \ldots B^{I_{N_h-1}}_{\Lambda_{N_h-2},\Lambda_{N_h-1}} R^{I_{N_h}}_{\Lambda_{N_h-1}}$$

$$B^{I_n}_{\Lambda_{n-1},\Lambda_n} = \mathrm{tr}\left[\prod_{k=1}^{N_v} \hat{A}^{i_{k,n}}_{\alpha_{k,n-1},\alpha_{k,n}}\right]$$

$$\Lambda_n = (\alpha_{1,n}, \alpha_{2,n}, \ldots, \alpha_{N_v,n})$$
$$I_n = (i_{1,n}, i_{2,n}, \ldots, i_{N_v,n})$$

Holographic framework

ρ_A



σ_b^2

"lives" on the boundary

Basic formula: $\rho_A = U\sigma_b^2 U^\dagger$

isometry: maps 2D onto 1D

$$\sigma_b^2 = \exp(-H_b)$$

Consequence: expect area law !

Boundary theories: main message

To what extend H_b is a local Hamiltonian ?

Can we describe TOPOLOGICAL systems ?

* gapped systems (AKLT):
  H_b is short-range

* approaching a critical point
  (deformed AKLT):
  H_b becomes long-range

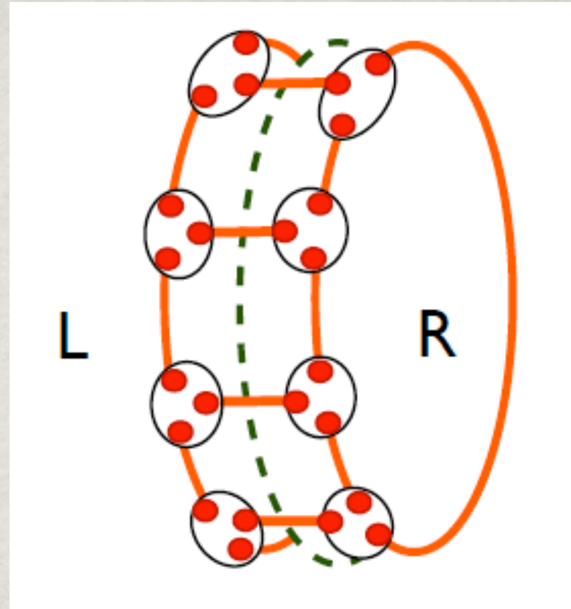
* for topological GS (Kitaev toric code, dimer wf, su(2)-RVB):
  => H_b non-local

Application to AKLT ladders

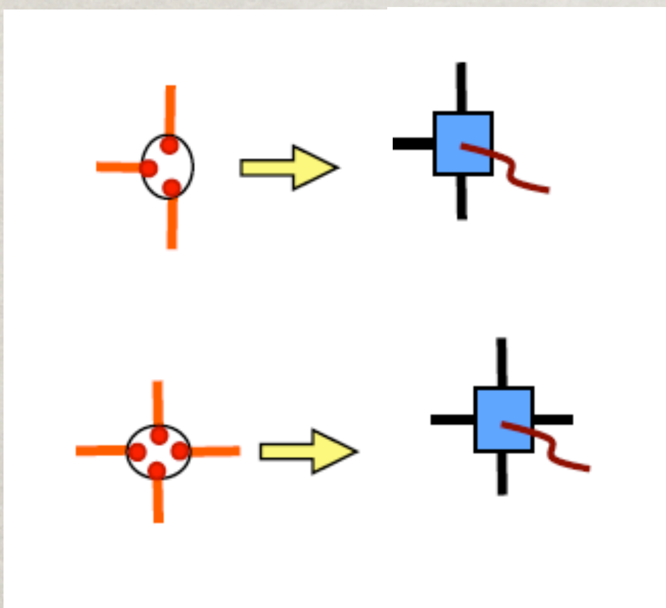
(Affleck-Lieb-Kennedy-Tasaki)

$$S_i = z_i/2$$

$$H_{AKLT} = \sum_{<ij>} P_{S_i+S_j}$$



$N_h = 2$



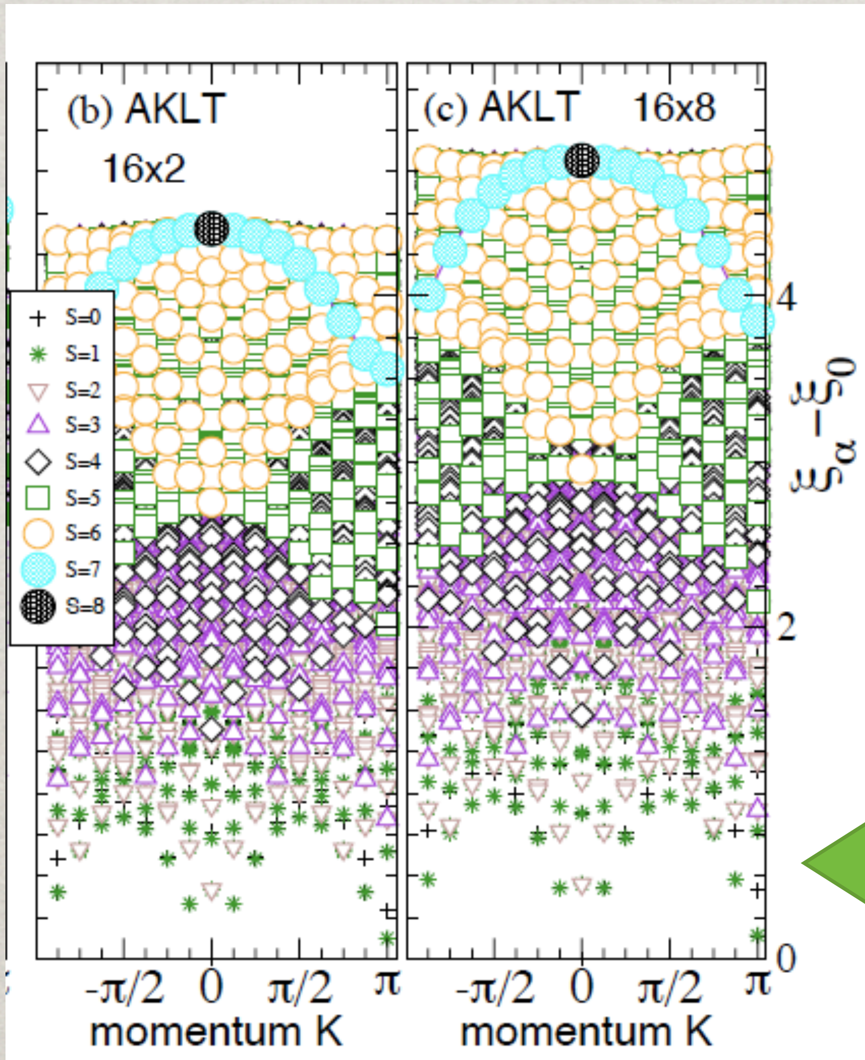
PEPS representation

D=2 !

# of legs N_h from 2 to ∞

Entanglement spectra of AKL ladders/cylinders

2 legs

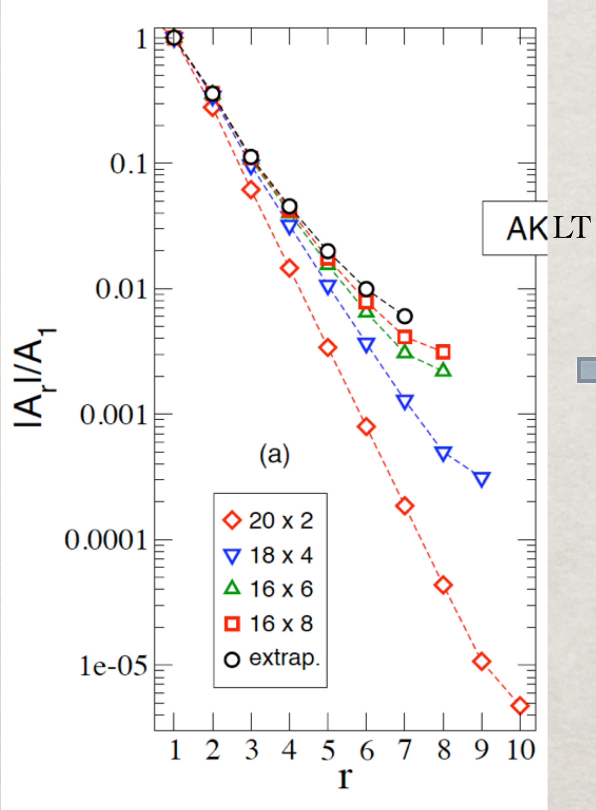


8 legs

same perimeter
= 16 sites

low energy
c=1 CFT

Extrapolate to infinite AKLT cylinders



$$A(r) \sim \exp\left(-r/\xi_b\right)$$

Short-range
boudary Hamiltonians

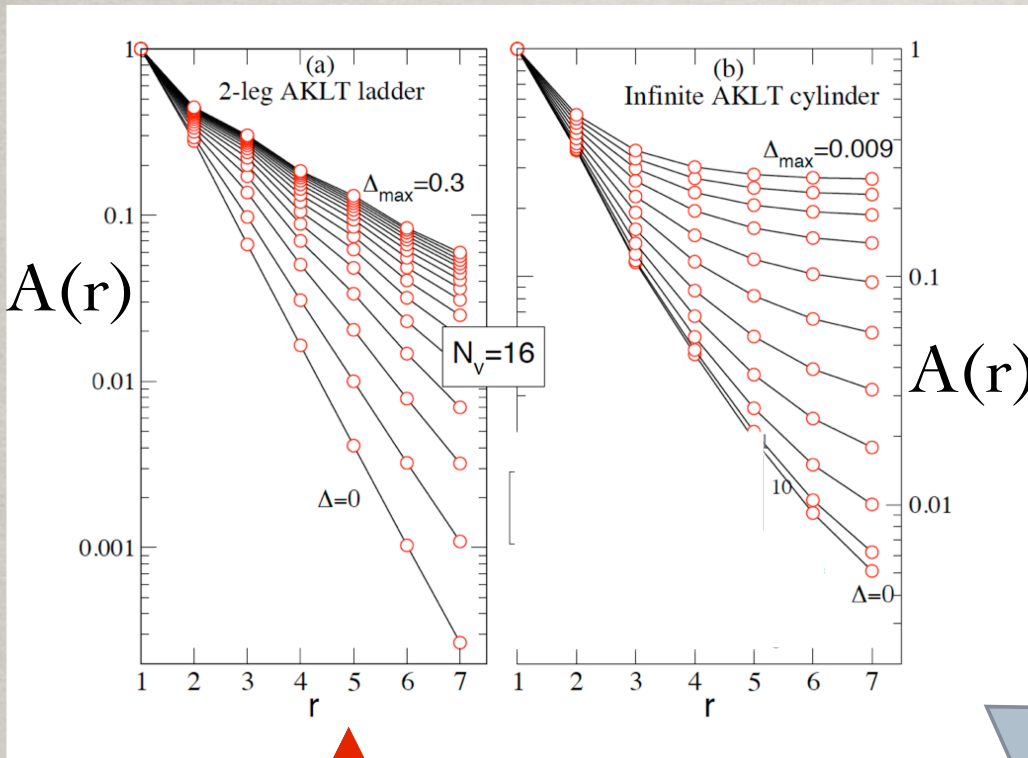
$$H_b = A_0 N_v + \sum_{r,k} A_r \, \mathbf{S}_k \cdot \mathbf{S}_{k+r} + R\hat{X}$$

Deformed AKLT model

$$A^m_{\alpha_1,\alpha_2,\alpha_3,\alpha_4} = \langle s_m | Q(\Delta) | \alpha_1,\alpha_2,\alpha_3,\alpha_4 \rangle$$

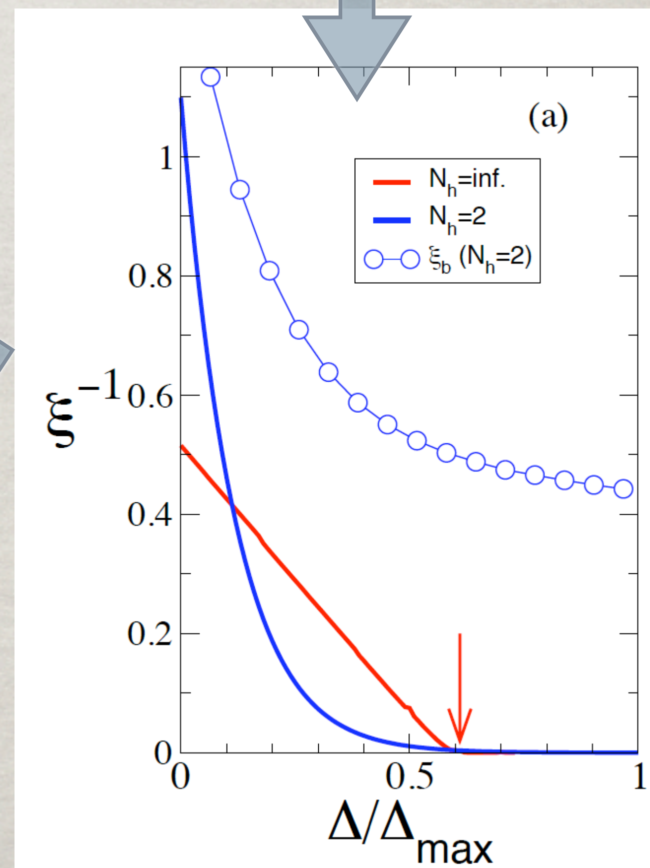
$$Q_n(\Delta) = e^{-8\Delta S^2_{z,n}}$$

breaks SU(2) down to U(1)

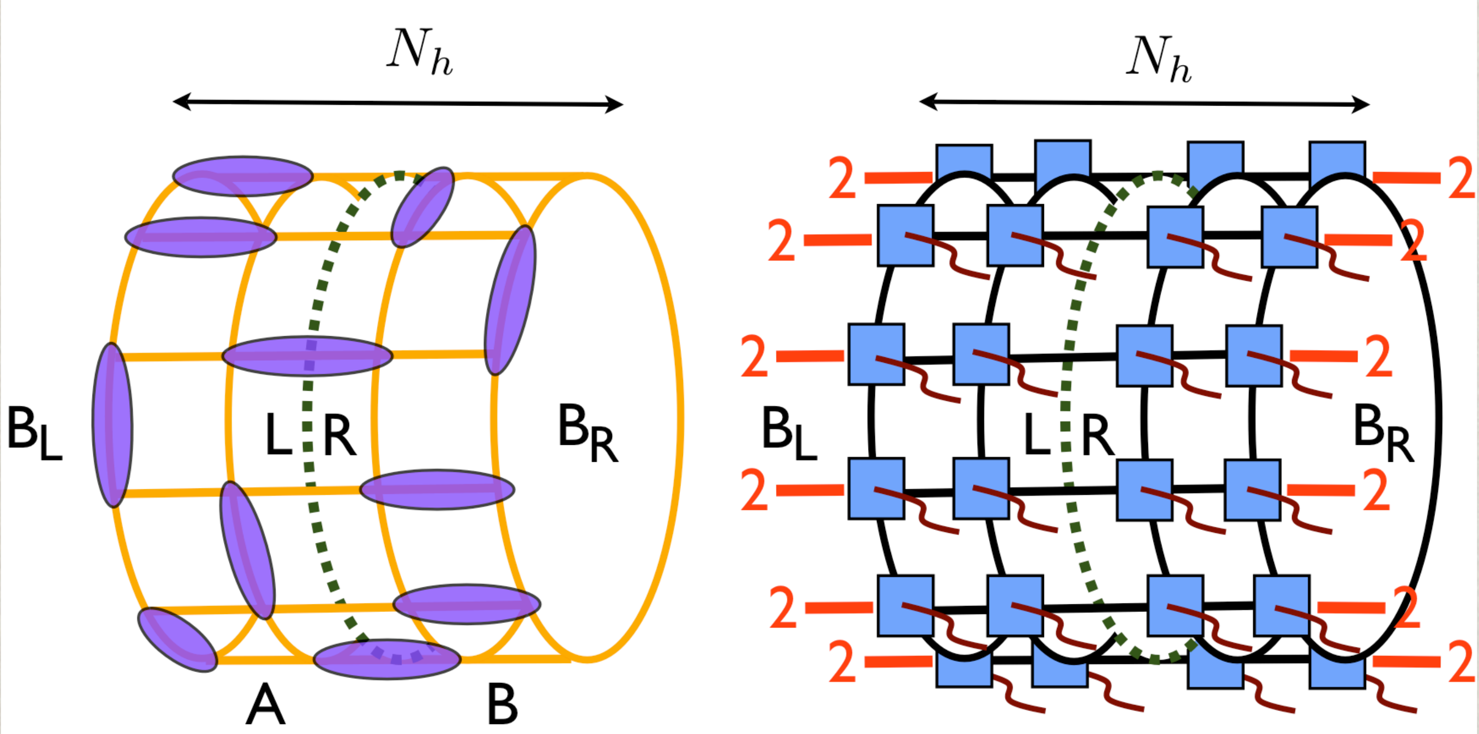


$A(r) \sim \exp(-r/\xi_b)$

Critical point:



Topological spin liquids



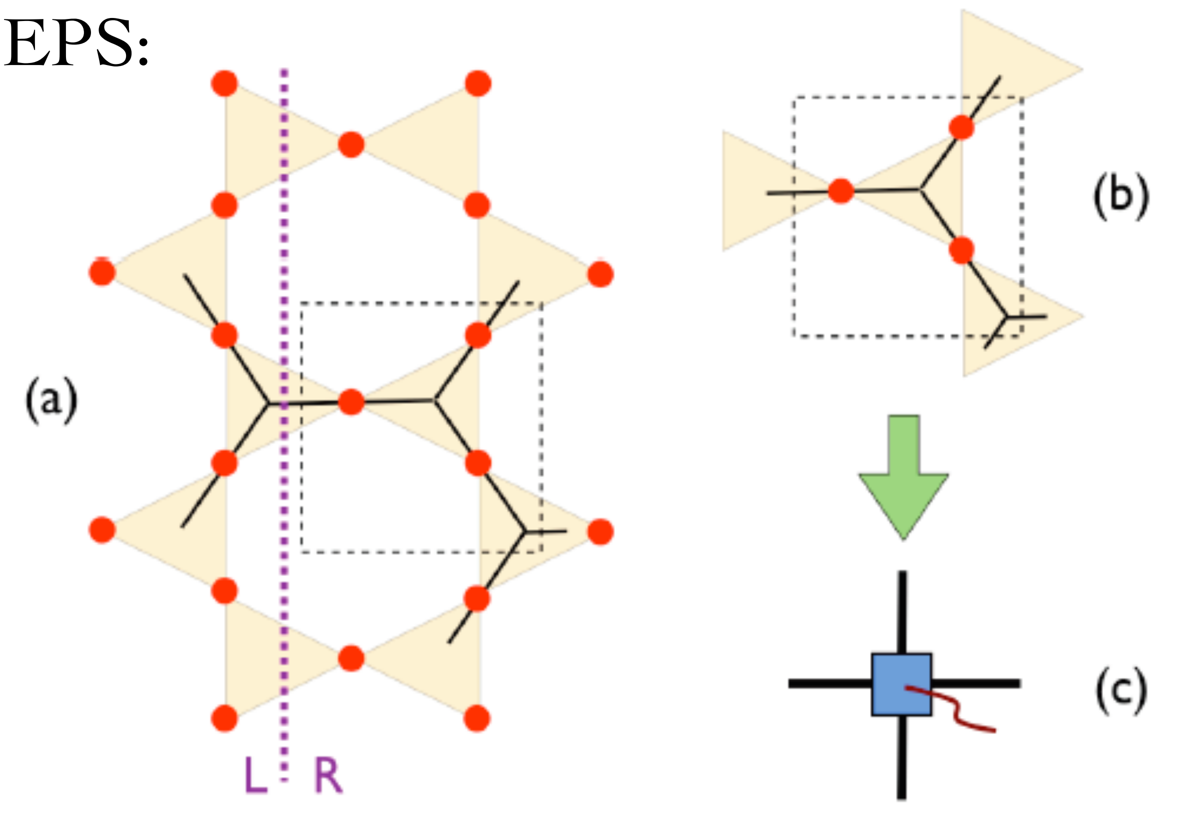
RVB = equal-weight superposition of NN singlet coverings → D=3 PEPS

Short-range spin-spin correlations

RVB on the kagome lattice

Evidence for Z2 liquid from recent numerics:
Yan, Huse & White, Science 2011

PEPS:



map on a square lattice
(but no reflection symmetry)

Some properties of RVB wavefunctions

* Square lattice: algebraic dimer-dimer correlations

Albuquerque & Alet, PRB 2010

* Kagome lattice: short-range dimer-dimer correlations

→ $\mathbb{Z}_2$ liquid

Misguich et al., PRL 2002
Moessner & Sondhi, PRL 2001

Disconnected topological sectors
in the space of dimer lattice coverings

E.g. on a cylinder:
* square lattice : Nh+1 sectors
* kagome lattice: 2 sectors

Structure of the Boundary Hamiltonian

Topological sectors translate into
CONSERVATION LAWS of "transfer matrix"
[i.e. on (parity) of # of "2" on each row]

$$\sigma_b^2 = \exp\left(-\tilde{H}_b\right)$$

$$\tilde{H}_b = H_1 + \beta_\infty(\mathbf{1}^{\otimes N_v} - \mathcal{P})$$

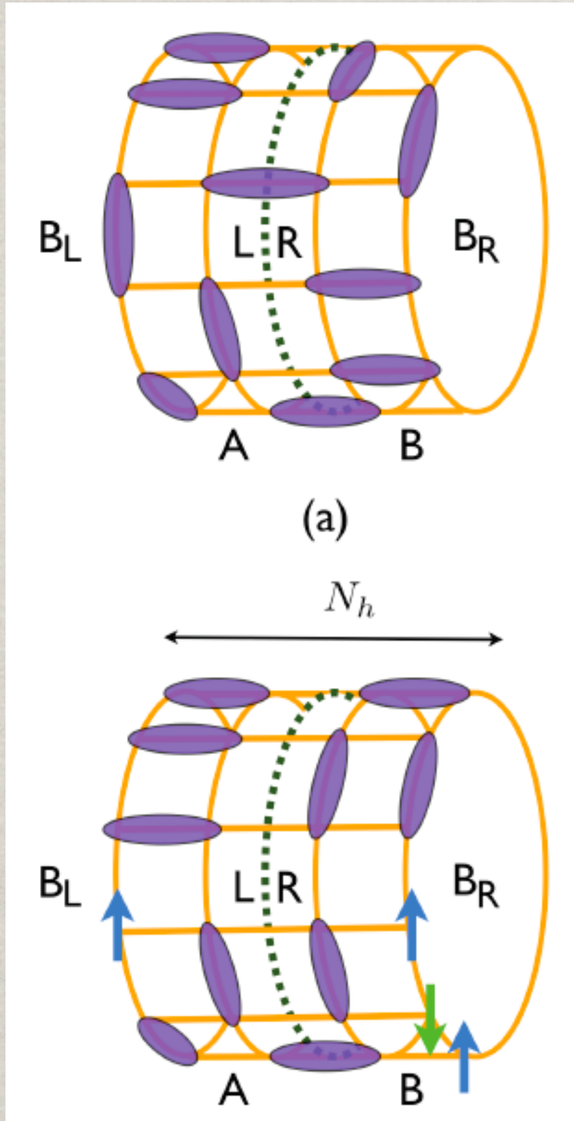
$$\beta_\infty \to \infty$$

supported by the non-zero eigenvalue sector of the RDM

$$H_1 = H_{\text{local}}\mathcal{P}$$

projector characterizing the sectors

Different sectors can be obtained
by choosing different boundary conditions



(kagome)

$$\mathcal{P} = \mathcal{P}_{\text{even}}$$

$$\mathcal{P} = \mathcal{P}_{\text{odd}}$$

NB: for the square lattice, N_v sectors/projectors

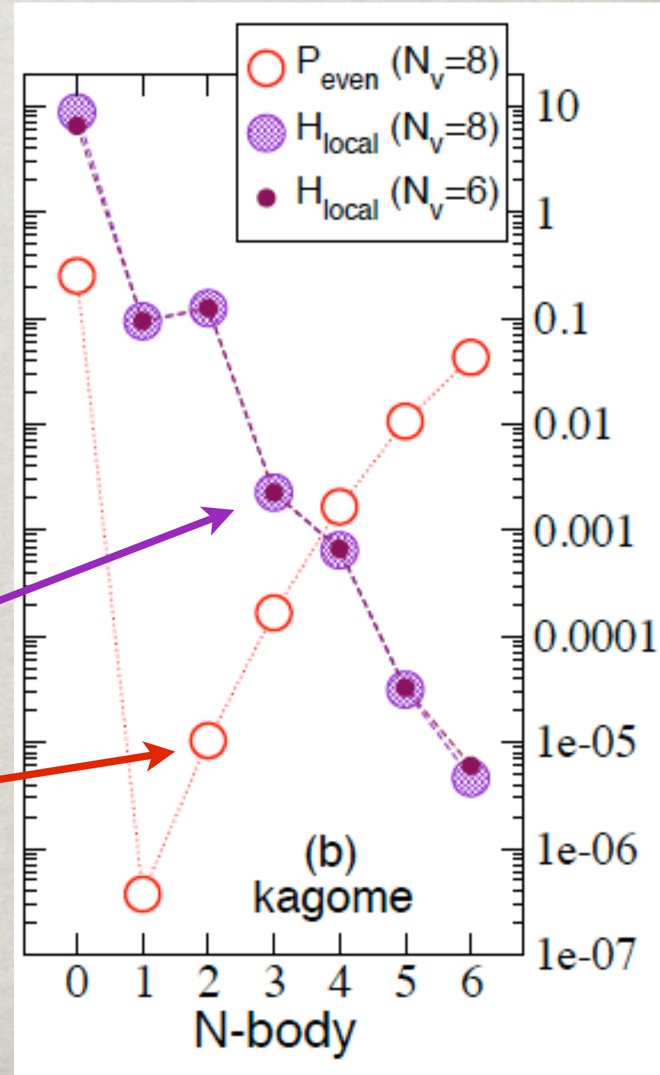
Non-locality of Boundary Hamiltonian (acting on the edge)

local operator (on the edge):
$D \times D$ matrix $\Rightarrow$ basis of D^2 operators

$$\mathcal{O}_{\rm edge} = c_0 N_v + \sum_{\lambda,i} c_\lambda \hat{x}^i_\lambda + \sum_{\lambda,\mu,r,i} d_{\lambda\mu}(r)\,\hat{x}^i_\lambda \hat{x}^{i+r}_\mu + \sum_{\lambda,\mu,\nu,r,r',i} e_{\lambda\mu\nu}(r,r')\,\hat{x}^i_\lambda \hat{x}^{i+r}_\mu \hat{x}^{i+r'}_\nu + \cdots ,$$

$$H_1 = H_{\rm local}\mathcal{P}$$

$H_{\rm local}$ is an extended t-J model !



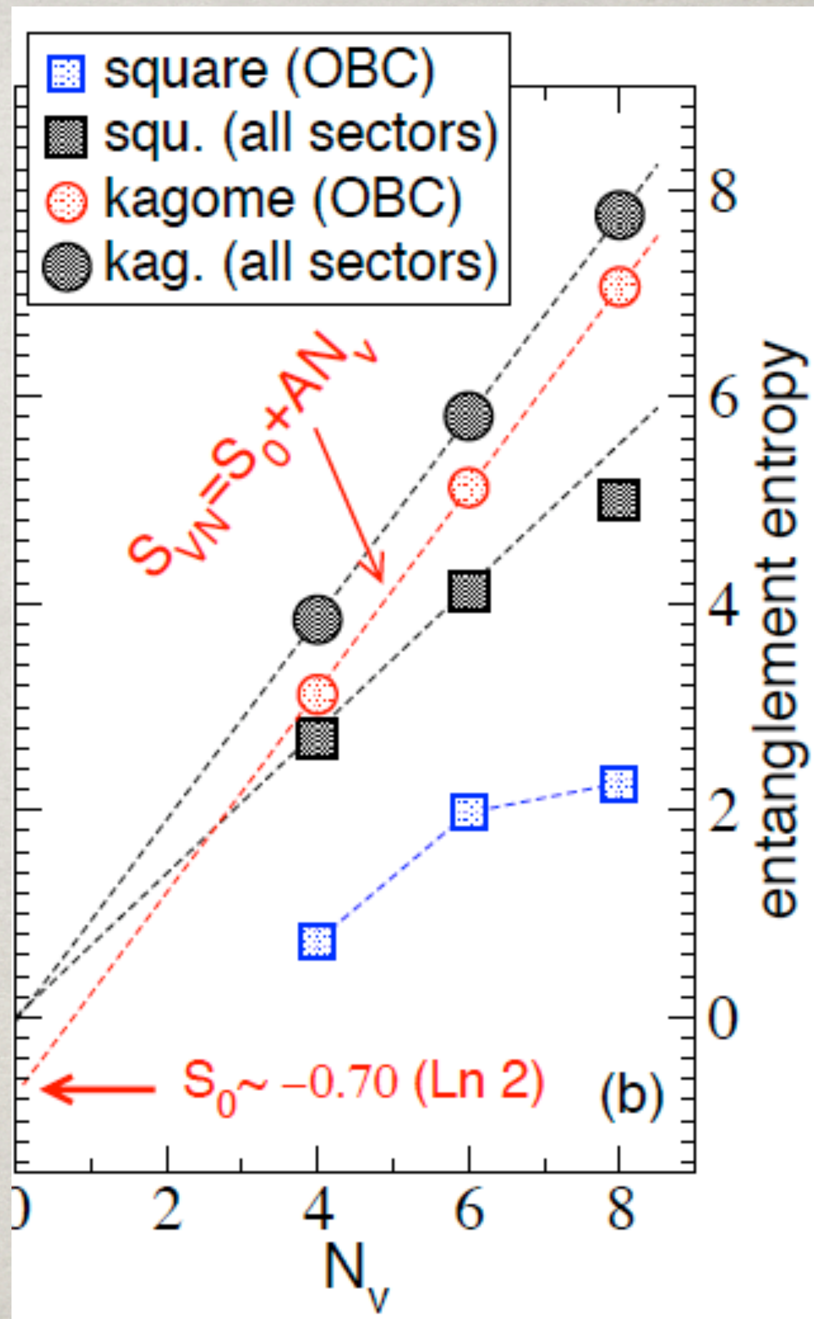
Topological entropy

# of states on the edge contributing to even sector:

$$\# = 3^{N_v}/2$$

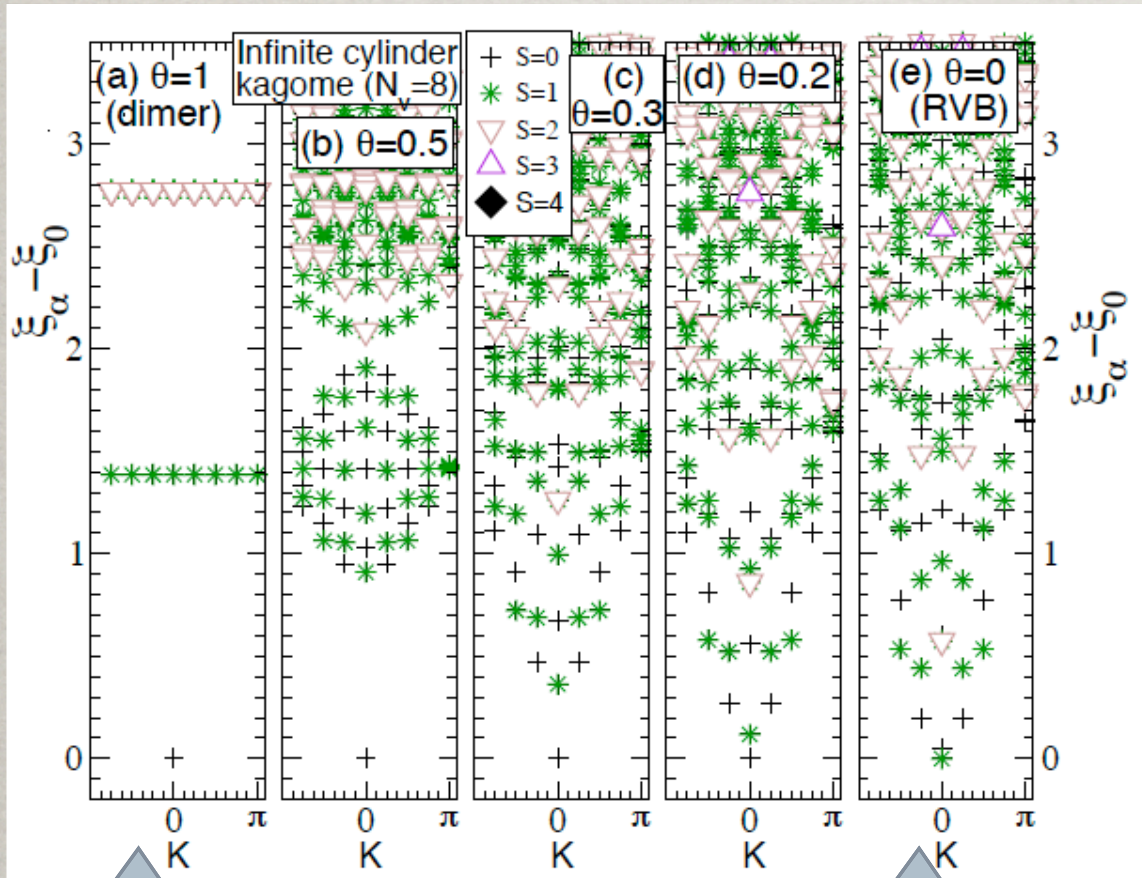
$$S_{\mathrm{VN}} \sim -\ln \#$$

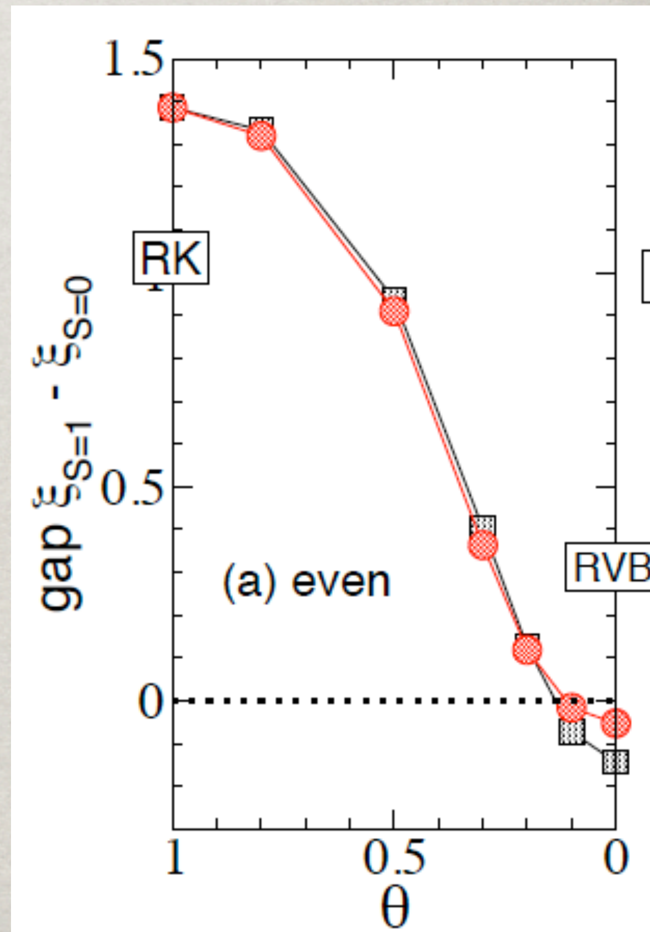
$$S_{\mathrm{VN}} = S_0 + AN_v$$



Interpolation between orthogonal dimer and RVB states

ES spectrum reflects the edge spectrum (conjecture)





can be mapped onto Kitaev's toric code

gapless ES !

Conclusion and outlook

* natural mapping between bulk and boundary
➔ properties of bulk reflected in the property of the boundary Hamiltonian
➔ property of the bulk can be “read off” the property of boundary Hamiltonian

* applied also to TOPOLOGICAL states
➔ tool to identify spin liquids in microscopic models

* extensions to chiral SL, fermions, (non-Abelian) anyons or gauge models ...