Conformal Field Theory of Composite Fermions in the QHE

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Outline

- Introduction: wave functions and CFT
- wave functions looking for a theory: Jain wf & composite fermion
- CFT looking for wave functions: W-infinity minimal models
- they finally meet
- non-Abelian statistics of quasi-holes
- .....in progress........

Laughlin's wave function

$$\Psi_{gs}(z_1, z_2, \ldots, z_N) = \prod_{i<j} (z_i - z_j)^{2k+1} \, e^{-\sum |z_i|^2/2}$$

$$\nu = \frac{1}{2k+1} = 1, \frac{1}{3}, \frac{1}{5}, \ldots$$

- $\nu = 1$ filled Landau level: obvious gap $\omega = \frac{eB}{mc} \gg kT$
- $\nu = \frac{1}{3}$ non-perturbative gap due to Coulomb interaction

→ effective theories as e.g. CFTs

- one quasi-hole wf

$$\Psi_\eta = \prod_i (\eta - z_i) \, \Psi_{gs}$$

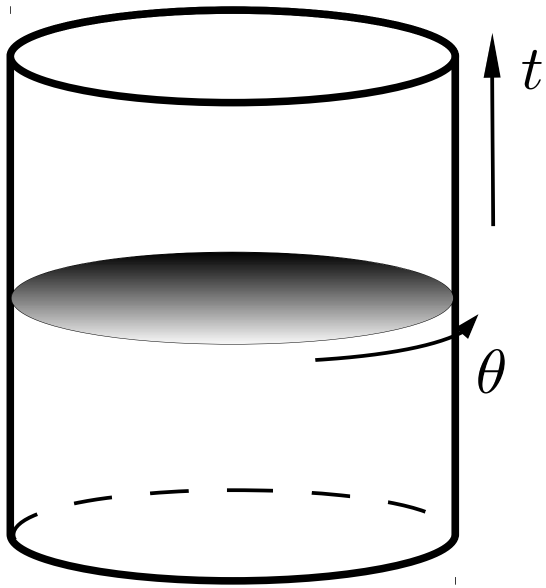
- two quasi-hole wf

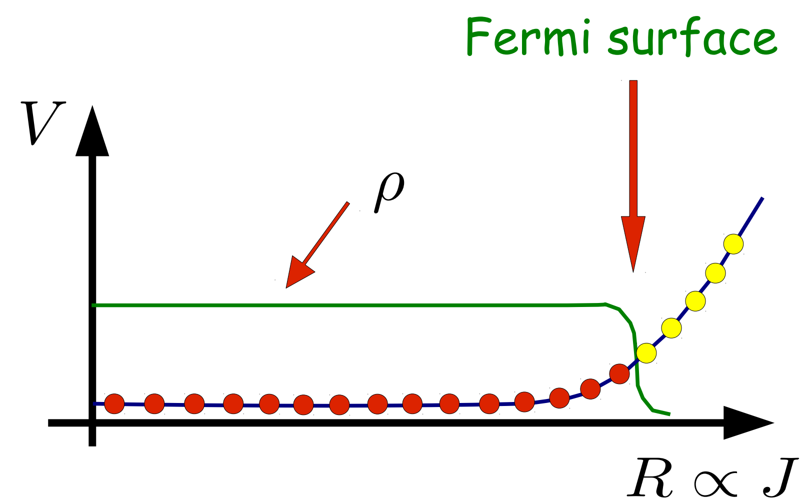
$$\Psi_{\eta_1,\eta_2} = (\eta_1 - \eta_2)^{\frac{1}{2k+1}} \prod_i (\eta_1 - z_i)(\eta_2 - z_i) \, \Psi_{gs}$$

→ fractional charge $Q = \frac{e}{2k+1}$ & statistics $\frac{\theta}{\pi} = \frac{1}{2k+1}$

Conformal field theory of edge excitations

The edge of the droplet can fluctuate: edge waves are massless

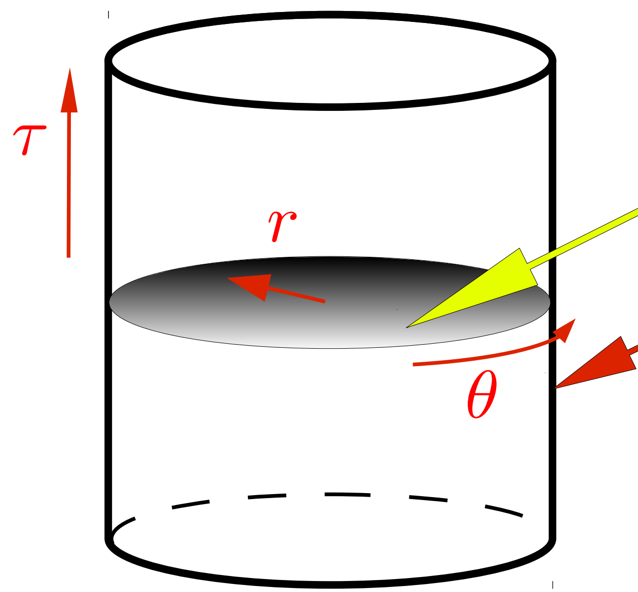




- edge ~ Fermi surface: linearize energy $\varepsilon(k) = \frac{v}{R}(k - k_F), \quad k = 0, 1, \ldots$
- relativistic field theory in 1+1 dimensions, massless, chiral

  ➡ chiral compactified c=1 U(1) CFT (chiral Luttinger liquid)
- more involved CFTs are possible: U(1) x neutral part

CFT descriptions of QHE



$z = re^{i\theta}$ plane (bulk excit)

$\zeta = e^{\tau} e^{i\theta}$ cylinder (edge excit)

$\langle \; \phi_1 \quad \phi_2 \; \rangle_{CFT}$ → $(z_1 - z_2)^{\alpha^2}$ anyon wavefunction

$\langle \; \phi_1 \quad \phi_2 \; \rangle_{CFT}$ → $(\zeta_1 - \zeta_2)^{\alpha^2}$ edge-excit. correlator

- equivalence of descriptions: analytic continuation from the circle,

  use map CFT ↔ Chern-Simons theory in 2+1 dim

- c=1 Luttinger CFT:
  - wavefunctions: spectrum of anyons and braiding
  - edge correlators: physics of conduction experiments

Non-Abelian fractional statistics

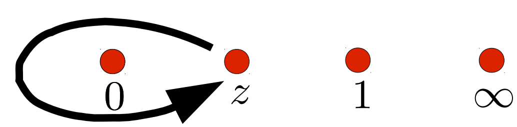
- $\nu = \frac{5}{2}$ described by Moore-Read "Pfaffian state" ~ Ising CFT x U(1)
- Ising fields: I identity, ψ Majorana = electron, σ spin field = q-hole
- fusion rules:
  - $\psi \cdot \psi = I$ electrons fuse into Bosonic bound state
  - $\sigma \cdot \sigma = I + \psi$ 2 channels of fusion = 2 conformal blocks

$\langle \sigma(0)\sigma(z)\sigma(1)\sigma(\infty)\rangle = a_1 F_1(z) + a_2 F_2(z)$ hypergeometric

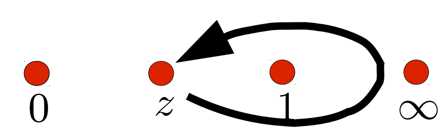
→ state of 4 anyons is two-fold degenerate (Moore, Read '91)

- statistics of anyons ~ analytic continuation → 2x2 unitary matrix

$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}(ze^{i2\pi}) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}(z)$$



$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}(1+(z-1)e^{i2\pi}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}(z)$$



- Topological Quantum Computation (Nayak et al. '07)

Jain composite fermion

$$\Psi_{\nu=\frac{1}{p+1}} = \prod_{i<j}(z_i - z_j)^p \prod_{i<j}(z_i - z_j) = \prod_{i<j}(z_i - z_j)^p \; \Psi_{\nu=1}, \qquad p \text{ even}$$

- Correspondence FQHE ⟷ IQHE

FQHE	IQHE
$\frac{N_\Phi}{N_e} = \frac{1}{\nu} = p + 1$	$\frac{1}{\nu^*} = 1$
B	$B^* = B - p\,\rho\,\Phi_o$

- generalize to n filled Landau levels

$\frac{1}{\nu} = p + \frac{1}{n}$	$\frac{1}{\nu^*} = \frac{1}{n}$

$$\Psi_{\nu=\frac{n}{np+1}} = \mathcal{P}_{LLL} \prod_{i<j}(z_i - z_j)^p \; \Psi_{\nu^*=n}$$

- composite fermion: quasiparticle feeling the reduced B^*
- many experimental confirmations — no definite theory
- $\Psi_{\nu=\frac{n}{np+1}}$ written directly in LLL using projection $\bar{z}_i \to \partial_{z_i}$ in $\Psi_{\nu^*=n}$

(Jain, Kamilla '97)

Hansson et al. ('07)

$$\Psi_{\nu=\frac{2}{2p+1}} = \mathcal{A}\left[\prod^{N/2} w_{ij}^{p+1}\, \partial_{z_1}\cdots\partial_{z_{N/2}} \prod^{N/2} z_{ij}^{p+1} \prod^{N/2} (z_i - w_j)^p\right]$$

$$\frac{1}{\nu} = p + \frac{1}{2}$$

$$w_{ij} = w_i - w_j$$

$$z_{ij} = z_i - z_j$$

- result based on non-trivial algebraic identites
- related to Abelian two-component edge theory,

$$K = \begin{pmatrix} p+1 & p \\ p & p+1 \end{pmatrix}$$

but not quite

(Wen, Zee; Read,...)

$$\Psi_{\nu=\frac{2}{2p+1}} = \mathcal{A}\left[\left\langle (\partial_{z_1} V_+)\cdots(\partial_{z_{N/2}} V_+)\, V_- \cdots V_- \right\rangle\right]$$

$$V_\pm = e^{i\sqrt{p+\frac{1}{2}}\,\varphi}\, e^{\pm i\frac{1}{\sqrt{2}}\phi}$$

$U(1)$ neutral

$\mathcal{A}$: two fermions V_+, V_- → one fermion;

descendant fields needed for non-vanishing result,

also yield correct "shift"

Properties of Jain wf

$$\Psi_{\nu=\frac{2}{2p+1}} = \mathcal{A}\left[\prod^{N/2} w_{ij}^{p+1}\ \partial_{z_1}\cdots\partial_{z_{N/2}} \prod^{N/2} z_{ij}^{p+1} \prod^{N/2}(z_i - w_j)^p\right], \qquad \frac{1}{\nu} = p + \frac{1}{2}$$

- reminds of Paffian state in Abelian version (A.C, Georgiev, Todorov '01)

$$\Psi_{\rm Pfaff} = \mathcal{A}\left[\prod^{N/2} w_{ij}^{M+2} \prod^{N/2} z_{ij}^{M+2} \prod^{N/2}(z_i - w_j)^M\right], \quad M \text{ odd}, \quad K = \begin{pmatrix} M+2 & M \\ M & M+2 \end{pmatrix}$$

- same vanishing behaviour:

$$\Psi \sim z_{12}^{p-1}\left(z_{13}^2\ z_{14}^2\cdots\right), \qquad \frac{1}{\nu} = p + \frac{1}{2}$$

$$\Psi \sim \left(z_{12}\, z_{13}\, z_{23}\right)^{p-1}\left(z_{14}^2\ z_{15}^2\cdots\right), \qquad \frac{1}{\nu} = p + \frac{1}{3}$$

⟹ $p = 1$ Jain is excited state of the $M = 0$ Pfaffian!

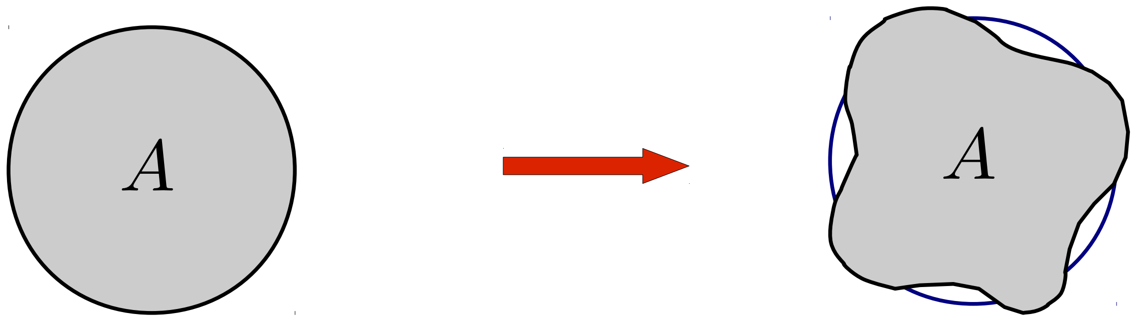
- same pairing?
- fractional statistics?

cf. Simon, Rezayi, Cooper '07;
Regnault, Bernevig, Haldane '09

W-infinity symmetry

Area-preserving diffeomorphisms of incompressible fluid

$$\int d^2x\ \rho(x) = N = \rho_o A$$ → A = constant



- W-infinity symmetry can be implemented in the edge CFT
- CFT with higher currents characteristic of 1d fermions

$$W^0 = J, \qquad W^1 =: J^2 :\sim H, \qquad W^2 =: J^3 :, \qquad \cdots$$

- representations completely known (V.Kac, A. Radul '92)
- $c = n$ and generically equivalent to $\widehat{U(1)}^n$ symmetry

W-infinity minimal models

- repres. with extended symmetry are degenerate and allow for a projection: W_∞ minimal models (A.C., Trugenberger, Zemba '96)

$$\widehat{U(1)}^n \longrightarrow \widehat{U(1)} \times \widehat{SU(n)}_1 \longrightarrow \widehat{U(1)} \times \frac{\widehat{SU(n)}_1}{SU(n)}$$

- these edge theories reproduce Jain fillings and symmetries $\nu = \dfrac{n}{p\, n \pm 1}$
- properties:
  - single electron excitation
  - reduced multiplicities of edge states
  - non-Abelian statistics of quasi-particles & electron (????)
- ex: $c = 2, \quad \dfrac{1}{\nu} = p + \dfrac{1}{2}, \qquad \widehat{U(1)} \times \mathrm{Vir}$ Vir = SU(2) Casimir subalgebra
- neutral part $h = \frac{k^2}{4}$ with spin $s = \frac{k}{2}$, ~~s_z~~ ; electron & q-hole have $s = \frac{1}{2}$

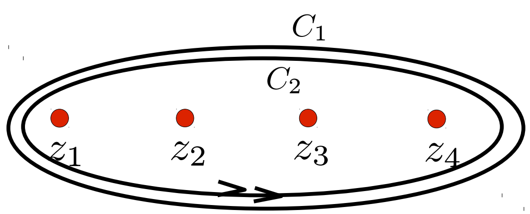
$$V_\pm = e^{\pm \frac{i}{\sqrt{2}} \phi}, \quad s_z = \pm \frac{1}{2}, \quad V_- \sim V_+ = Q_+ V_-, \quad Q_+ = J_0^+ = \oint du J^+(u)$$

Dotsenko-Fateev screening charge

Derivation of Jain-Hansson wf

- extend W-infinity minimal models from edge to bulk
- 4-el. wf has two channels, $\{\frac{1}{2}\} \times \{\frac{1}{2}\} = \{0\} + \{1\}$, given by choices of C_i

$$\Psi = \left\langle \oint_{C_1} J^+ \oint_{C_2} J^+ \, V_-(z_1) V_-(z_2) V_-(z_3) V_-(z_4) \right\rangle$$



- impose antisymm of electrons ➡ $\Psi = 0$
- consider descendant with same charge: $J_0^+ \;\rightarrow\; J_{-1}^+,$

$$J_{-1}^+ V_- \sim \partial_z V_+$$

$$\Psi' = \langle J_{-1}^+ V_-(z_1) \, J_{-1}^+ V_-(z_2) \, V_-(z_3) V_-(z_4) \rangle + \text{perm} = \Psi_{\text{Jain}}$$

OK filling fraction and shift

- equivalence extends to $c = 3:\;\; SU(3)$ descendants

➡ Underlying theory of Jain wf is W-infinity minimal model

+ Fermi statistics for electrons

Non-Abelian statistics of q-holes

- smallest q-hole, e.g. $Q=\frac{1}{5}$ at $\nu=\frac{2}{5}$, has neutral part $s=\frac{1}{2}$ again:
  → two components $s_z=\pm\frac{1}{2}$ identified by the projection

  $H_+ \sim J_0^+ \, H_-$ q-holes in "upper/lower effective LL" are identical

- fusion $\begin{cases} H_\pm H_\pm \sim H_{s=1} \\ H_+ H_- \sim H_{s=0} = I \end{cases}$ two channels → non-Abelian statistics

- first non-trivial case is 4 q-holes: two independent states

$$\Psi_{(12,34)} = \left\langle H_+(\eta_1) H_+(\eta_2) H_-(\eta_3) H_-(\eta_4) \prod V_e(z_i) \right\rangle$$

$$\Psi_{(13,24)} = \left\langle H_+(\eta_1) H_-(\eta_2) H_+(\eta_3) H_-(\eta_4) \prod V_e(z_i) \right\rangle$$

- detailed properties of $2k$ q-hole states yet to be worked out;

  quantum dimension $d_k \sim 2^k$ (but not Rational CFT)

Conclusions

- CFT description of Jain states supports their universality and suggests non-Abelian q-hole excitations
- open problems:
  - which model Hamiltonian?
  - details of q-hole states and NA statistics
- experimental tests:
  - thermopower, if measures can be extended to higher B
  - puzzles in the $\nu = \frac{2}{5}$ resonant tunneling experiment?