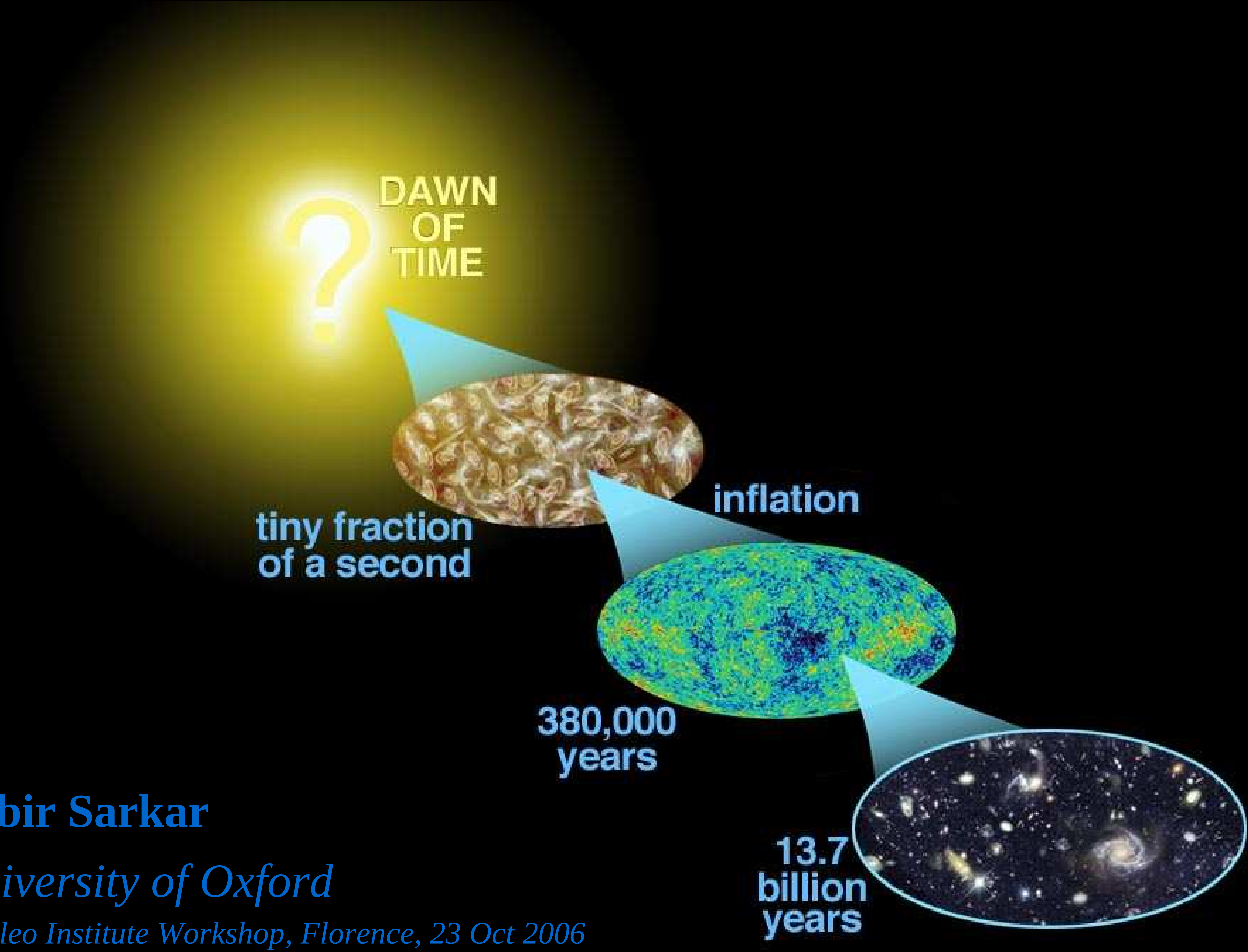
What have we learnt about inflation from *WMAP* ?



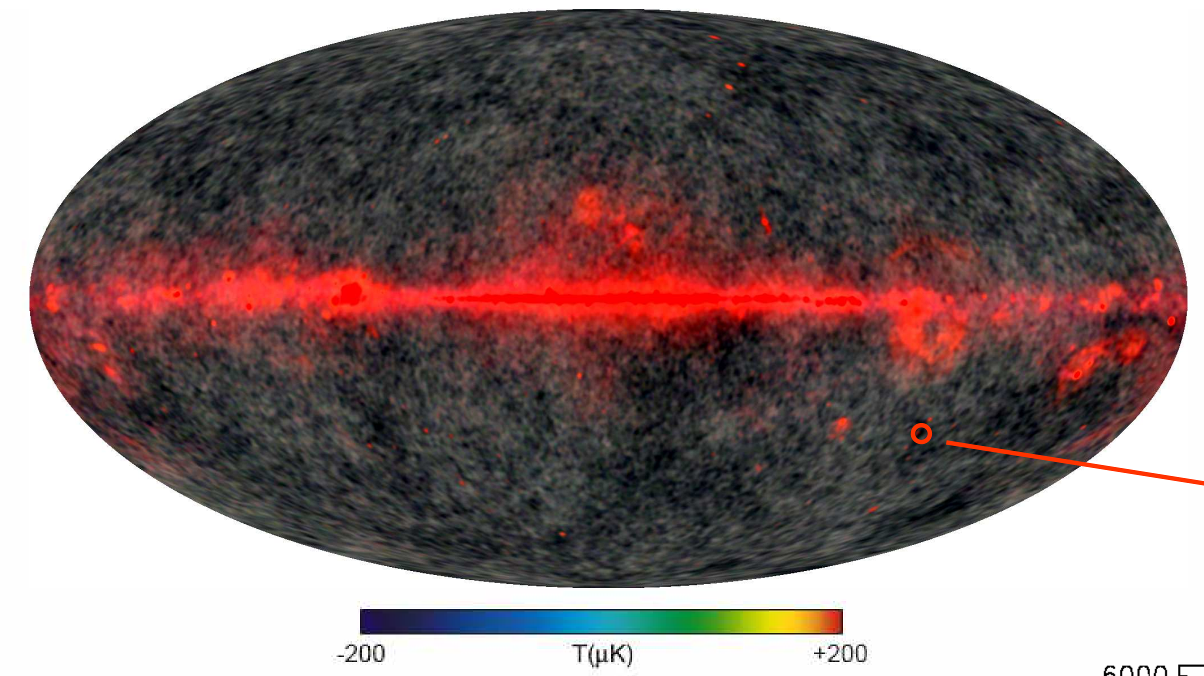
Courtesey: NASA/WMAP Science Team

Subir Sarkar

University of Oxford

Galileo Institute Workshop, Florence, 23 Oct 2006

'Internal Linear Combination' map (*circa* March 2006)



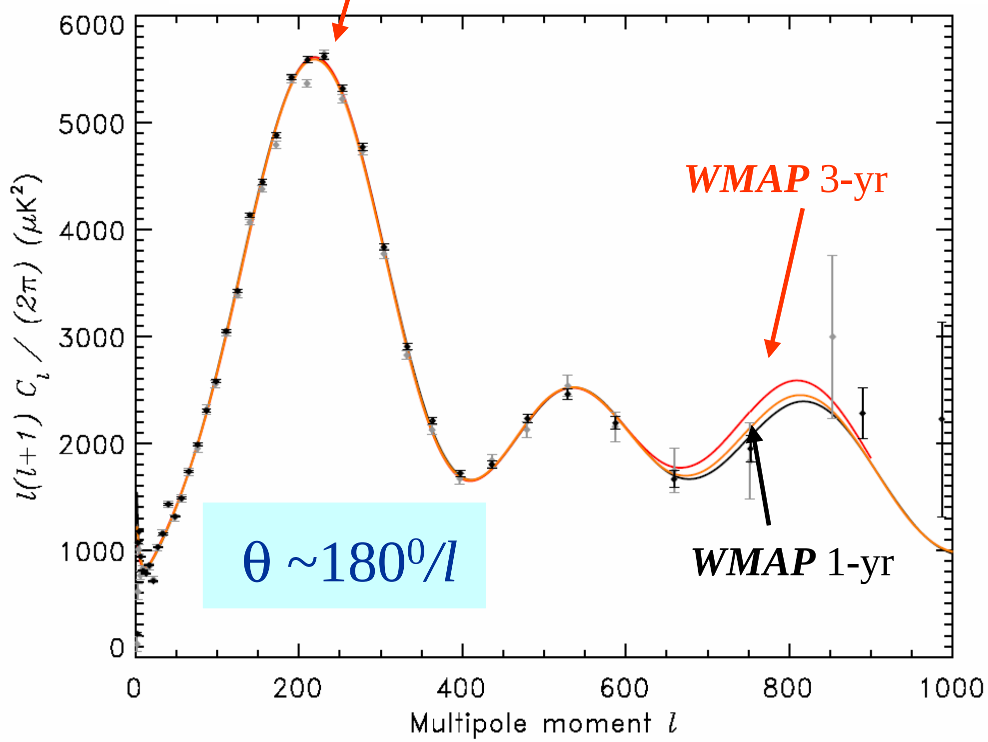
Coherent oscillations in photon-baryon plasma, excited by primordial density perturbations on *super*-horizon scales …

(Hubble radius at t_{rec})

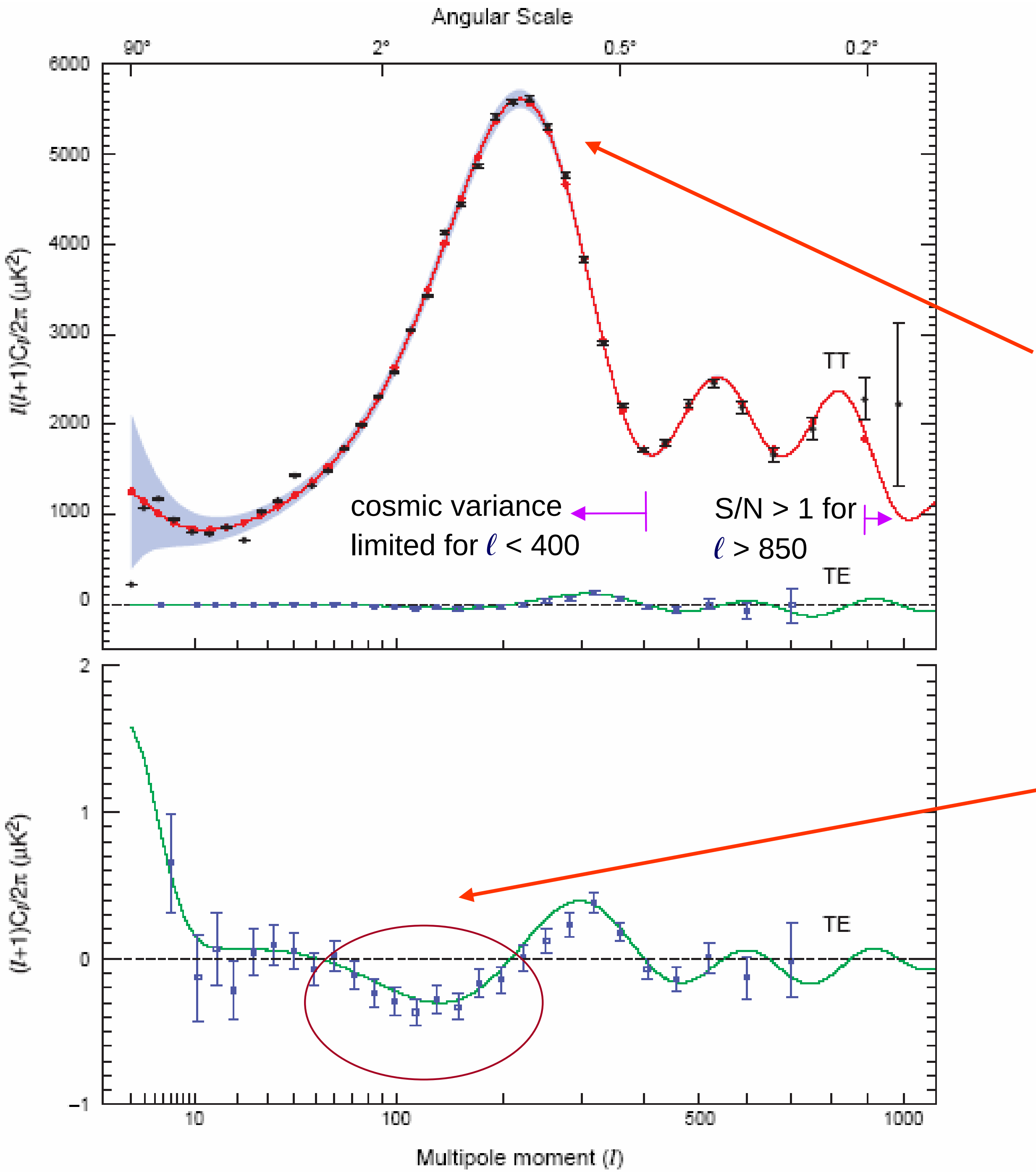
$$\Delta T(\mathbf{n}) = \sum a_{lm} Y_{lm}(\mathbf{n})$$

$$C_l \equiv \frac{1}{2l+1} \sum |a_{lm}|^2$$

C_l's mildly correlated since (due to Galactic foreground) only ~85% of sky can be used



WMAP does provide evidence for inflation …



The characteristic features of scalar density perturbations generated during a (quasi-) de Sitter phase of expansion:

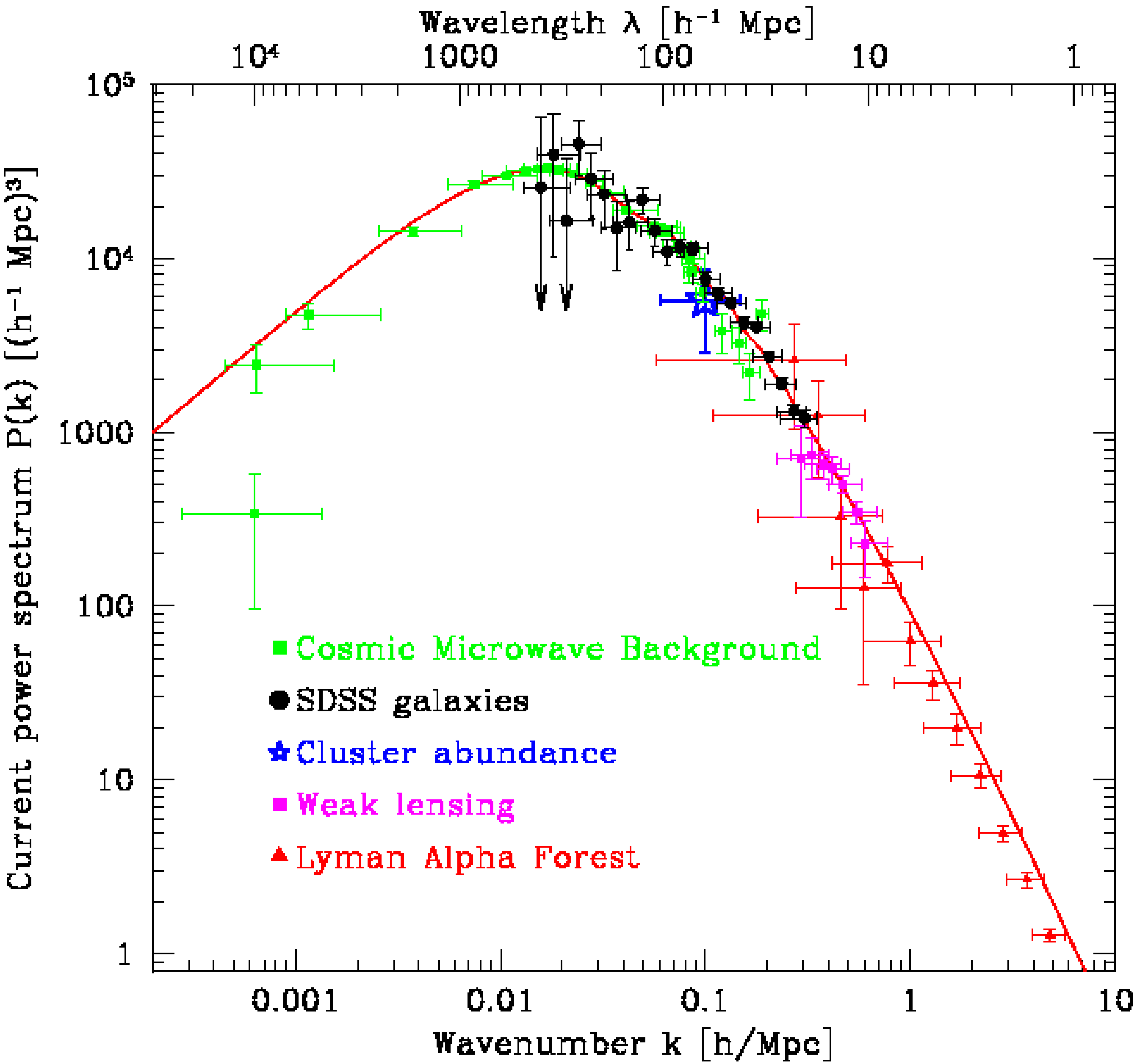
(a) ***Coherence*** of the Fourier modes → clean 'acoustic peak' structure on angular scales ($< 1^0$) which were *sub*-Hubble radius at last scattering ($z \sim 10^3$)

(b) Dipole out-of-phase with the monopole → ***negative cross-correlation*** between temperature and (electric) polarization on (*super*-Hubble radius) scales ~1-5^0

see Dodelson (2003)

Observations of large-scale structure are *consistent* with the ΛCDM model if the primordial fluctuations are ***adiabatic*** and ~***scale-invariant***

(as is apparently "expected in the simplest models of inflation")



Tegmark. (2004)

CMB (+ LSS) data indicates that inflation generated adiabatic, ~scale-invariant scalar density perturbations

But no ***tensor*** perturbations have yet been detected (through the expected *B*-mode polarization at low l) … can only set crude limit: $r \equiv T/S < 0.55$

$\Rightarrow$ Bound on inflationary energy scale: $V^{1/4} < 2 \times 10^{16}$ GeV

… thus no specific clue to the physics driving inflation

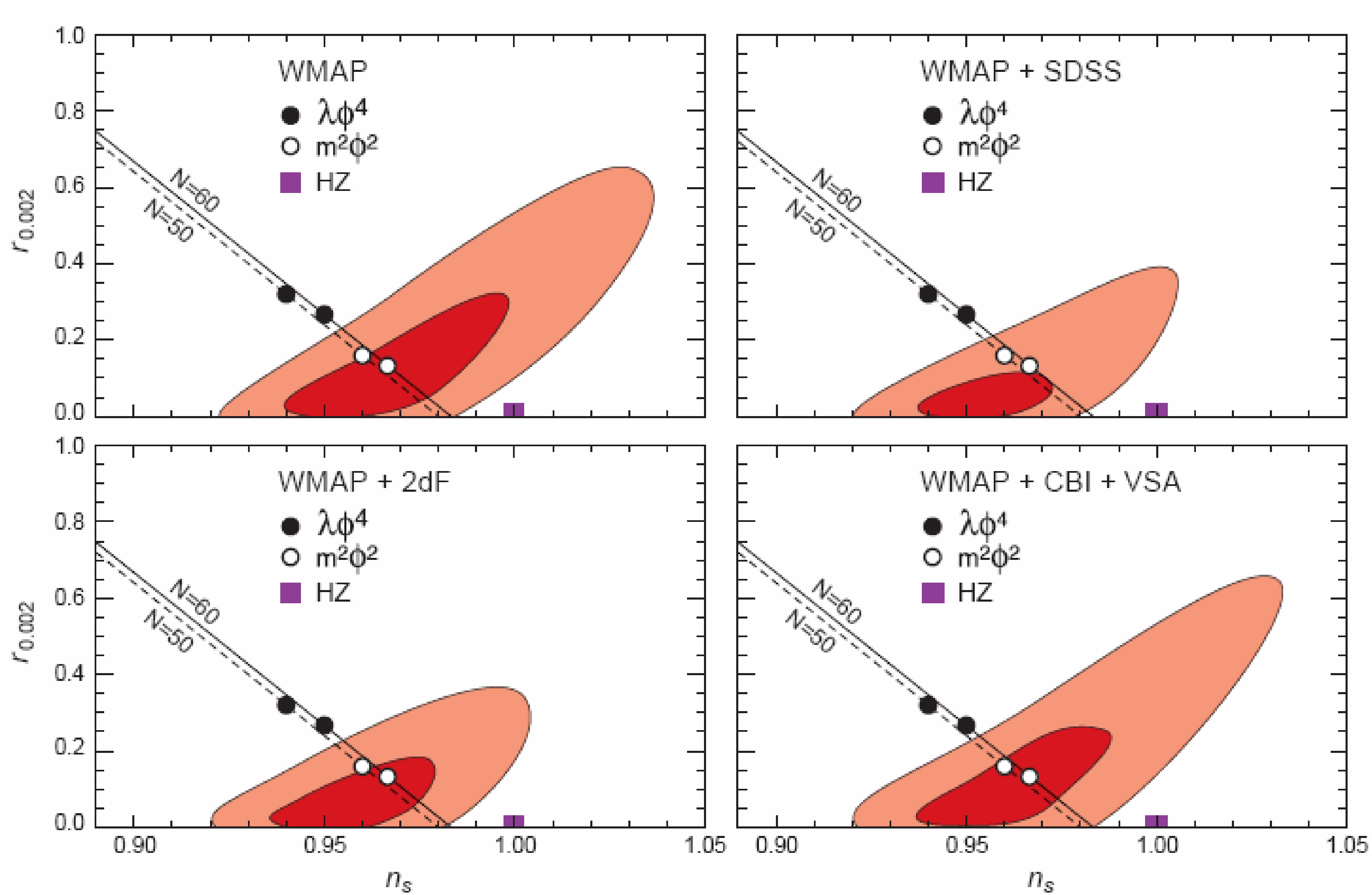
(GUT-scale? Hidden-sector scale? Electroweak scale?)

Can at best attempt to rule out 'toy models' (e.g. $V = \lambda\phi^4$) where inflation occurs at $\phi > M_P$ hence a large tensor signal is predicted …

Is there any signature in the data of the *physics* responsible for inflation?

… can discuss this sensibly only in the context of an *effective* field theory i.e. with $\phi \ll M_P$ …

WMAP-3 prefers $V = m^2\phi^2$ over $V = \lambda\phi^4$



But neither model has a ***physical*** basis ($\phi > M_P$!) and *both* are fine-tuned:
$\lambda \sim 10^{-12}$ or $m/M_P \sim 10^{-6}$ to generate the density perturbation $\delta_H \sim 10^{-5}$ ...

What we measure is the density perturbation, *not* the inflaton potential … so expand this around the field value $\phi^*_{\rm I}$ when the perturbation just entering our present Hubble radius ($H_0^{-1} \sim 3000\, h^{-1}$ Mpc) was generated

$$V(\phi) = V(0) + V'(0)\phi + \frac{1}{2}V''(0)\phi^2 + \ldots \qquad \boxed{\phi \equiv \phi^* - \phi^*_{\rm I}}$$

Then:
$$\delta^2_{\rm H}(k) = \frac{1}{75\pi^2}\frac{V(\phi^* = \phi^*_{\rm H})^3}{V'(\phi^* = \phi^*_{\rm H})^2 M^6}$$

on the scale k which exits the horizon when $\phi^* = \phi^*_{\rm H}$:

$$\boxed{k = aH,\ H \equiv \dot{a}/a \simeq (V/3M^2)^{1/2},\ M \equiv\ M_{\rm P}/\sqrt{8\pi} \simeq 2.44 \times 10^{18}\ {\rm GeV}}$$

If the *linear* term in the expansion of $V(\phi)$ dominates, then

$$V'(\phi^* = \phi^*_{\rm H}) = V'(0) + V''(0)\phi_{\rm H} + \ldots, \qquad V'(0) = cV(0)/M$$

So the energy scale required to generate $\delta_{\rm H} \sim 10^{-5}$ is indeed $\sim M_{\rm GUT}$:

$$V^{1/4}(\phi = 0) \simeq (75\pi^2\delta^2_{\rm H})^{1/4}c^{1/2}M \sim 2 \times 10^{-2}\sqrt{c}M$$

Question: What sort of models exhibit "linear inflation"?

Answer: *All* "chaotic" (large-field) models with $\boxed{V \propto \phi^{*n}}$

because then: $V^{p+1}(\phi = 0)\phi^p / V'(\phi = 0) \simeq (\phi/\phi_\mathrm{I}^*)^p \ll 1$

so $V = m^2\phi^2, \lambda\phi^4$ are *both equivalent* to: $V \approx V(0) + \alpha\phi$

But if ϕ transforms under a symmetry then *no* linear term

$\rightarrow$ "new inflation" with $V''(0) = \tilde{c}V(0)/M^2$

$$\Rightarrow \qquad \delta_\mathrm{H}^2 \simeq \frac{V(0)^3}{75\pi^2\tilde{c}^2 V(0)^2 \phi_\mathrm{H}^2 M^2}$$

So the energy scale of inflation gets *smaller* as $\phi_\mathrm{H} \rightarrow 0$:

$$V(\phi = 0)^{1/4} \simeq 2 \times 10^{-2} \sqrt{\tilde{c}}\phi_\mathrm{H}^{1/2} M^{1/2}$$

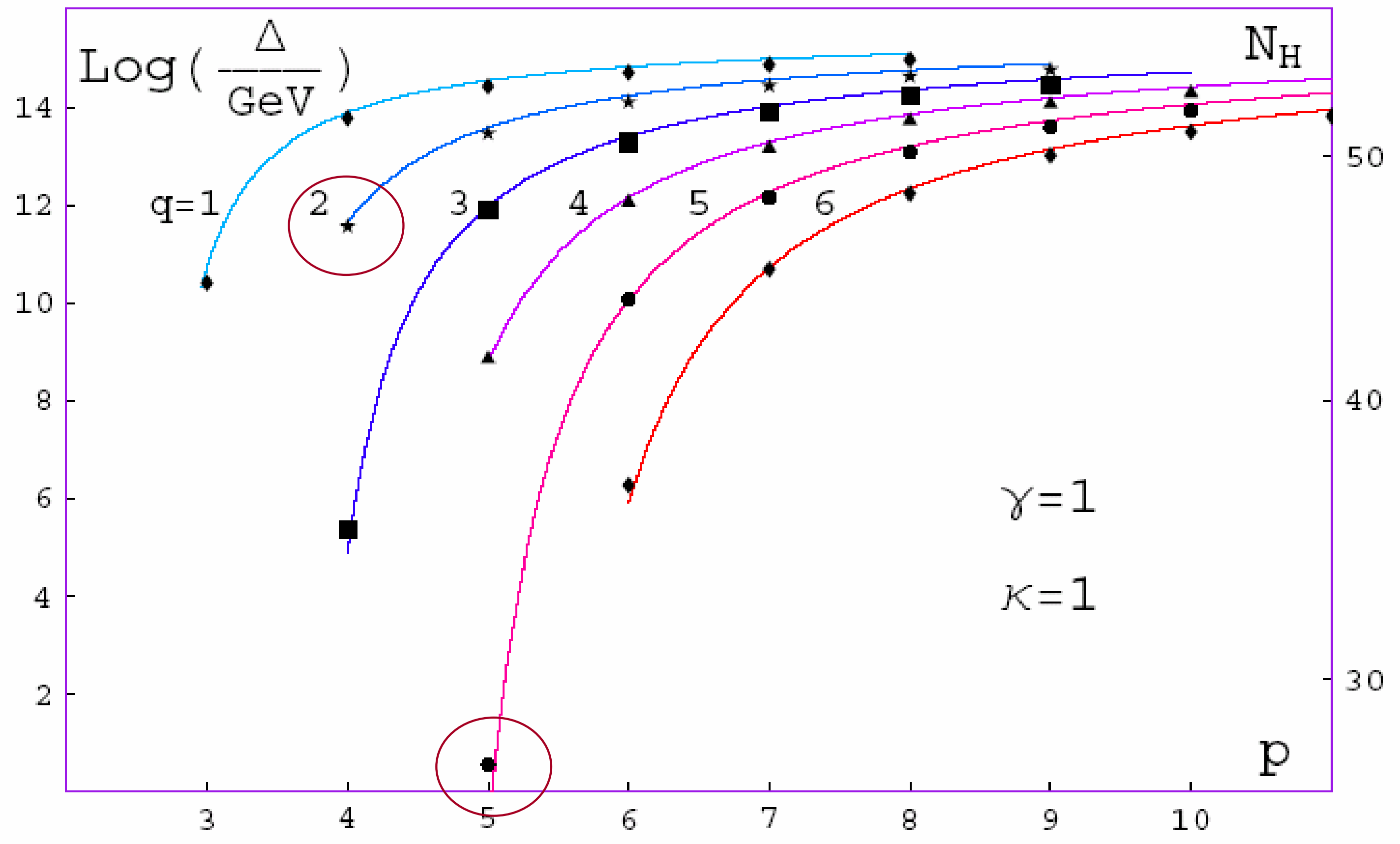
General 'new' inflaton potential: $\overline{V}(\phi) = \left(1 - \frac{\kappa}{\Delta^q}\phi^p\right)^2 + b\phi^2 + c$

Effective field theory: mass term + non-renormalizable operators

...can generate adequate inflation with correct δ_H at *any* energy scale

requires b < 1/20 (cf. 'natural' value: ~1 $\Rightarrow$ "η problem")

Energy Scale (GeV)



e-folds

German, Ross & Sarkar (2001)

The required NR operator can be realised in a *physical* theory

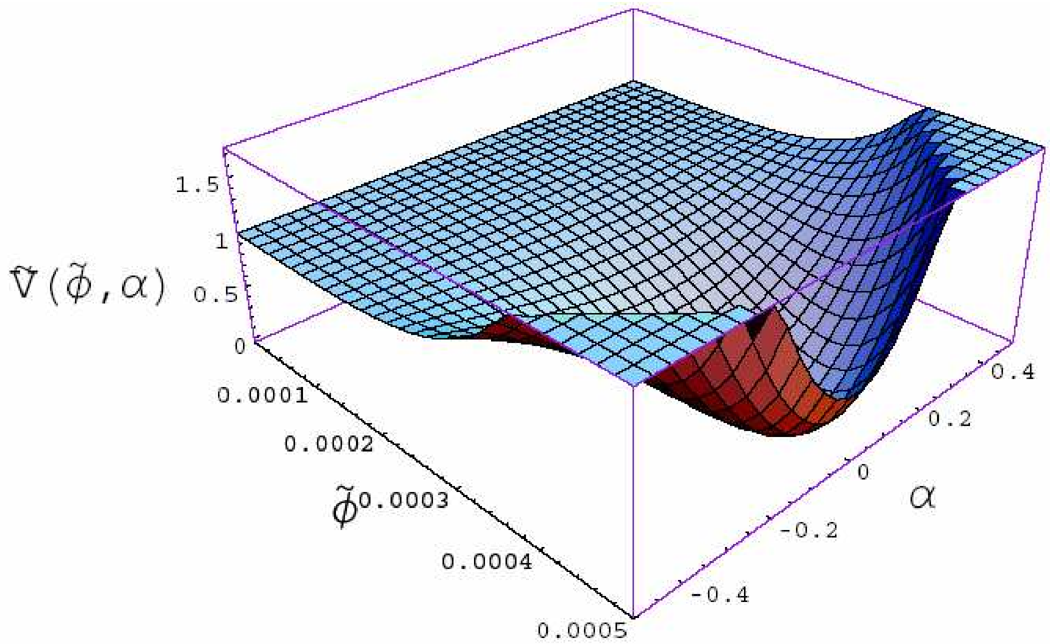


FIG. 5. The full supergravity potential (25) (in units of V_0/Δ^4) as a function of ϕ and its phase α for the case $(p, q, \kappa) = (4, 2, 1)$, corresponding to an inflationary scale of $\Delta \sim 5 \times 10^{11}$ GeV.

Inflation at SUGRA 'hidden-sector' scale

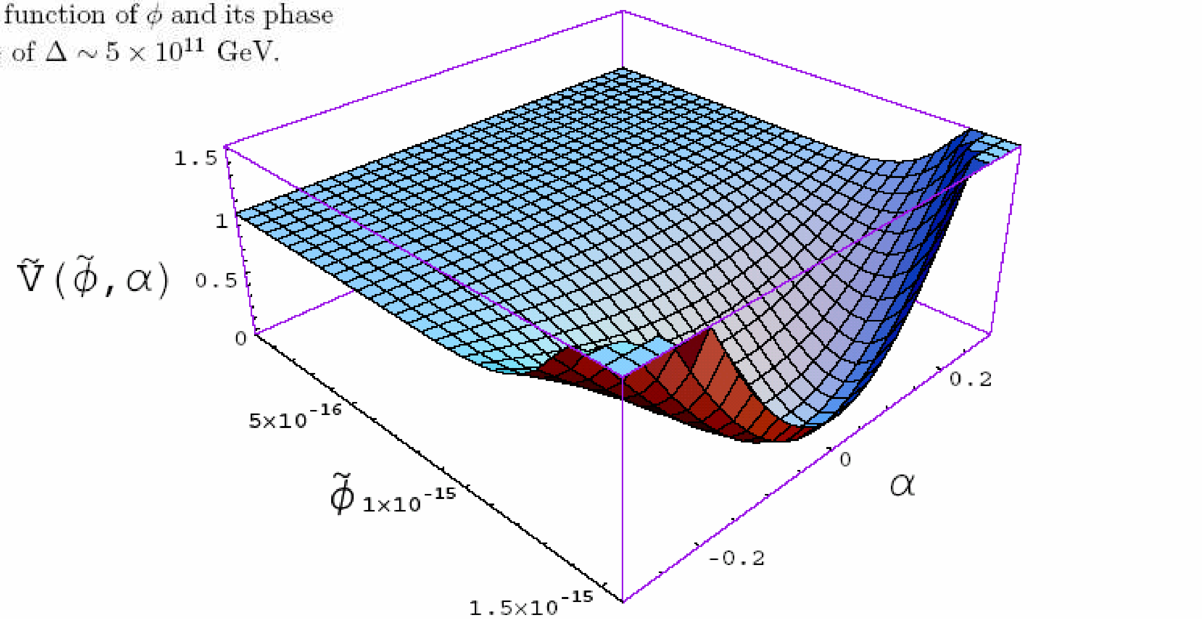
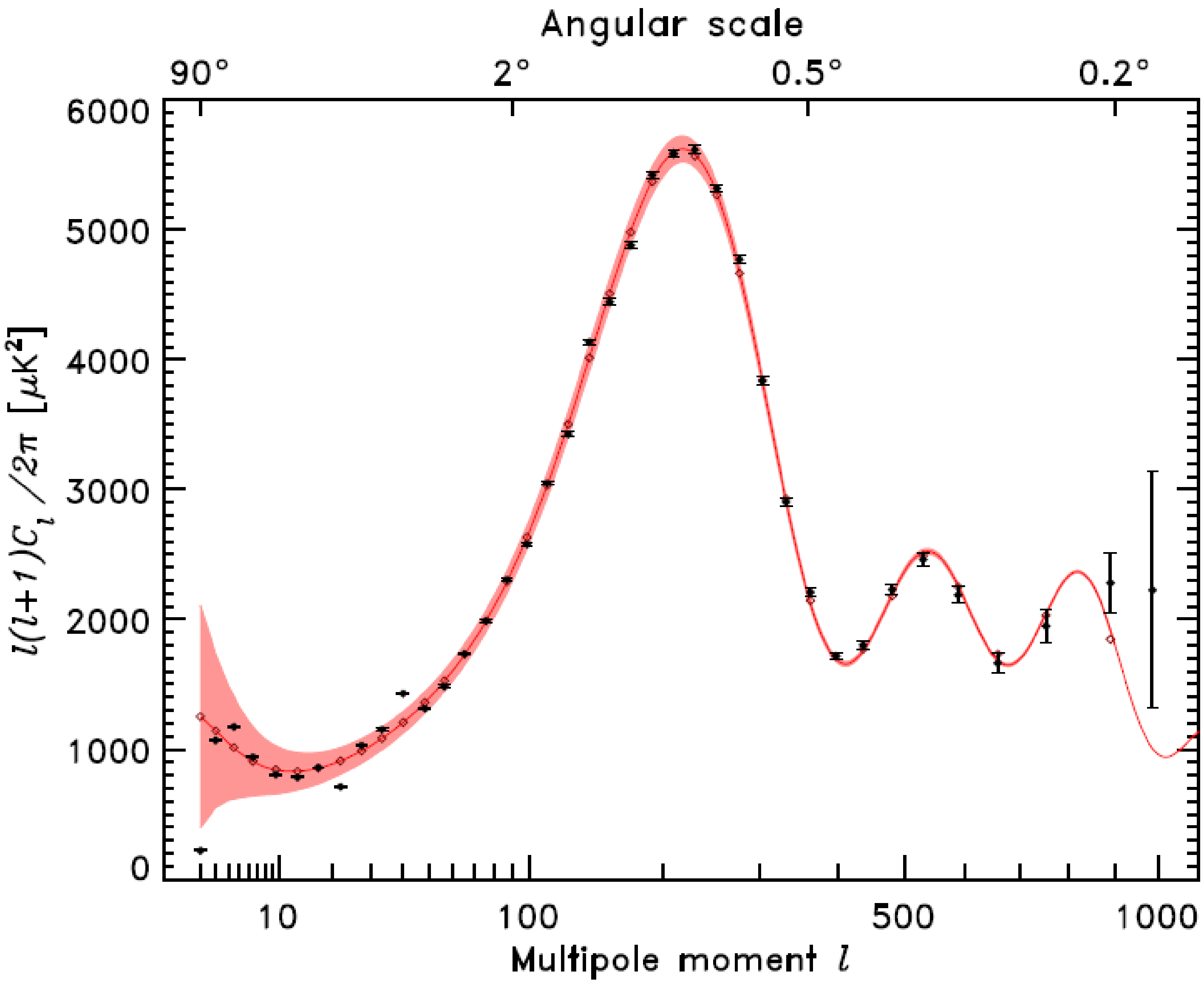


FIG. 6. Similar to Fig. 5, but for the case $(p, q, \kappa) = (5, 5, 1)$ corresponding to an inflationary scale of $\Delta \sim 1$ GeV.

Inflation at QCD scale

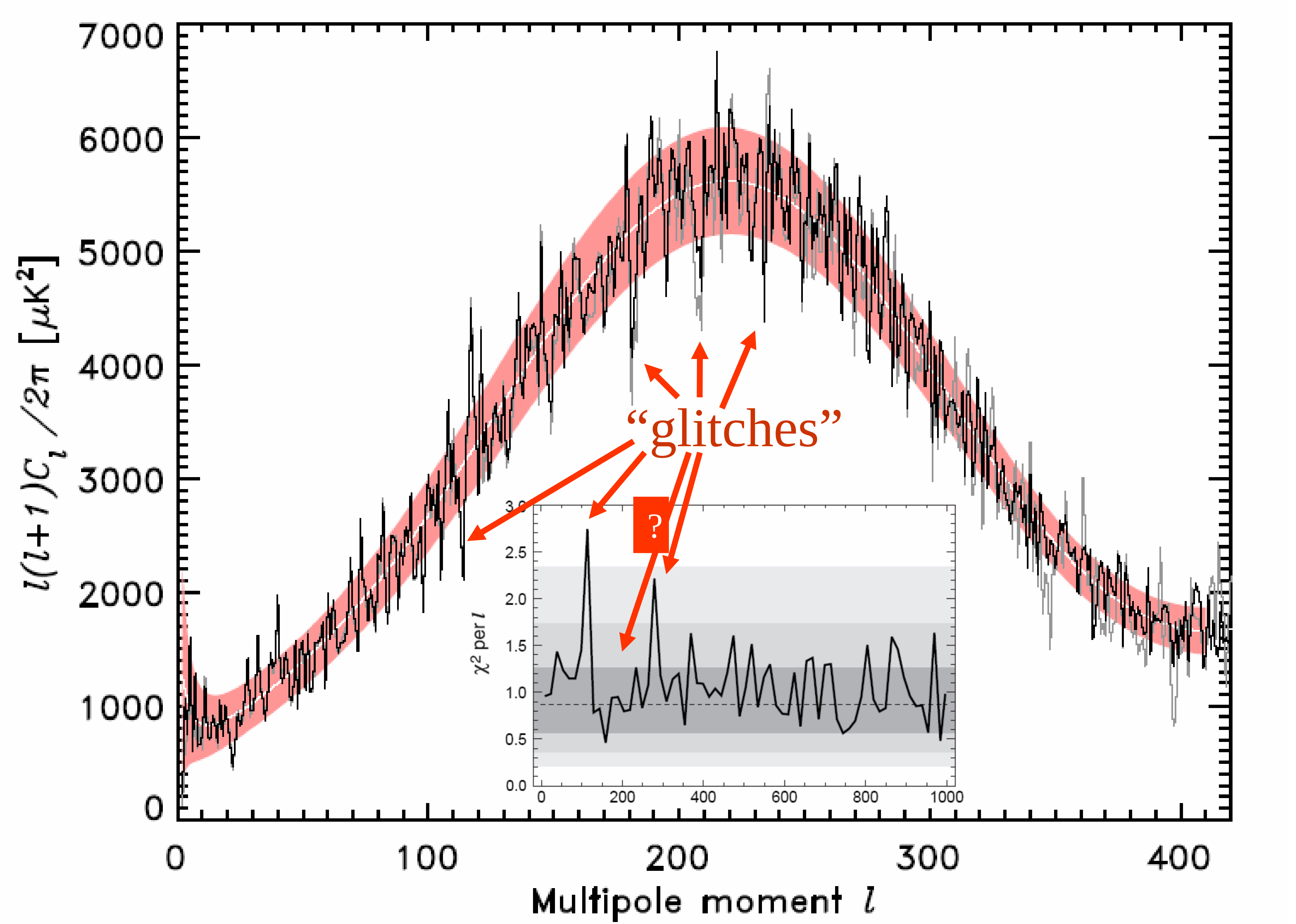
The 3-yr ***WMAP*** data is said to *confirm* the 'power-law ΛCDM model'

Best-fit: $\Omega_m h^2 = 0.13 \pm 0.01$, $\Omega_b h^2 = 0.022 \pm 0.001$, $h = 0.73 \pm 0.05$, $n = 0.95 \pm 0.02$

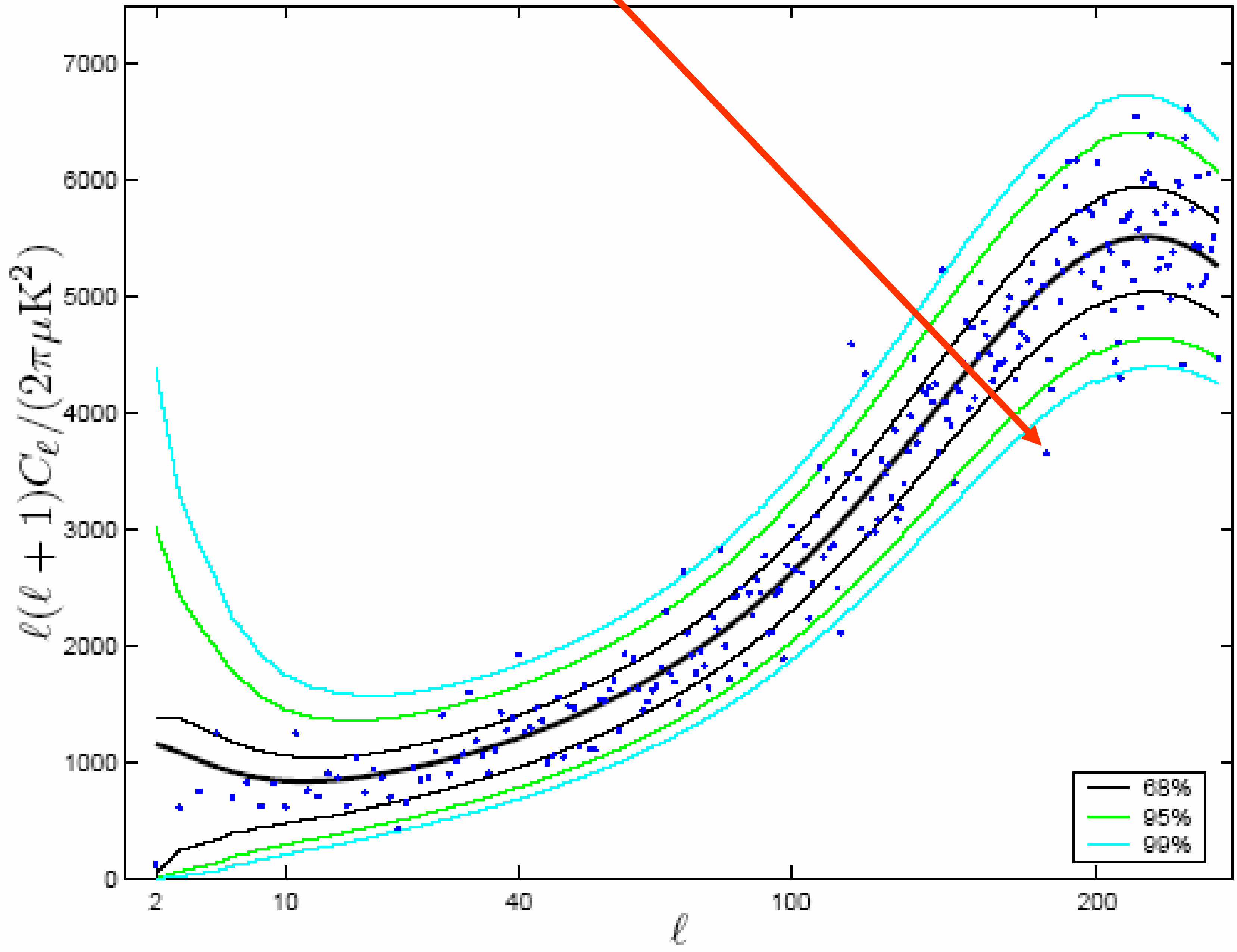


But the χ^2/dof = 1049/982 $\Rightarrow$ probability of only ~7% that this model is correct!

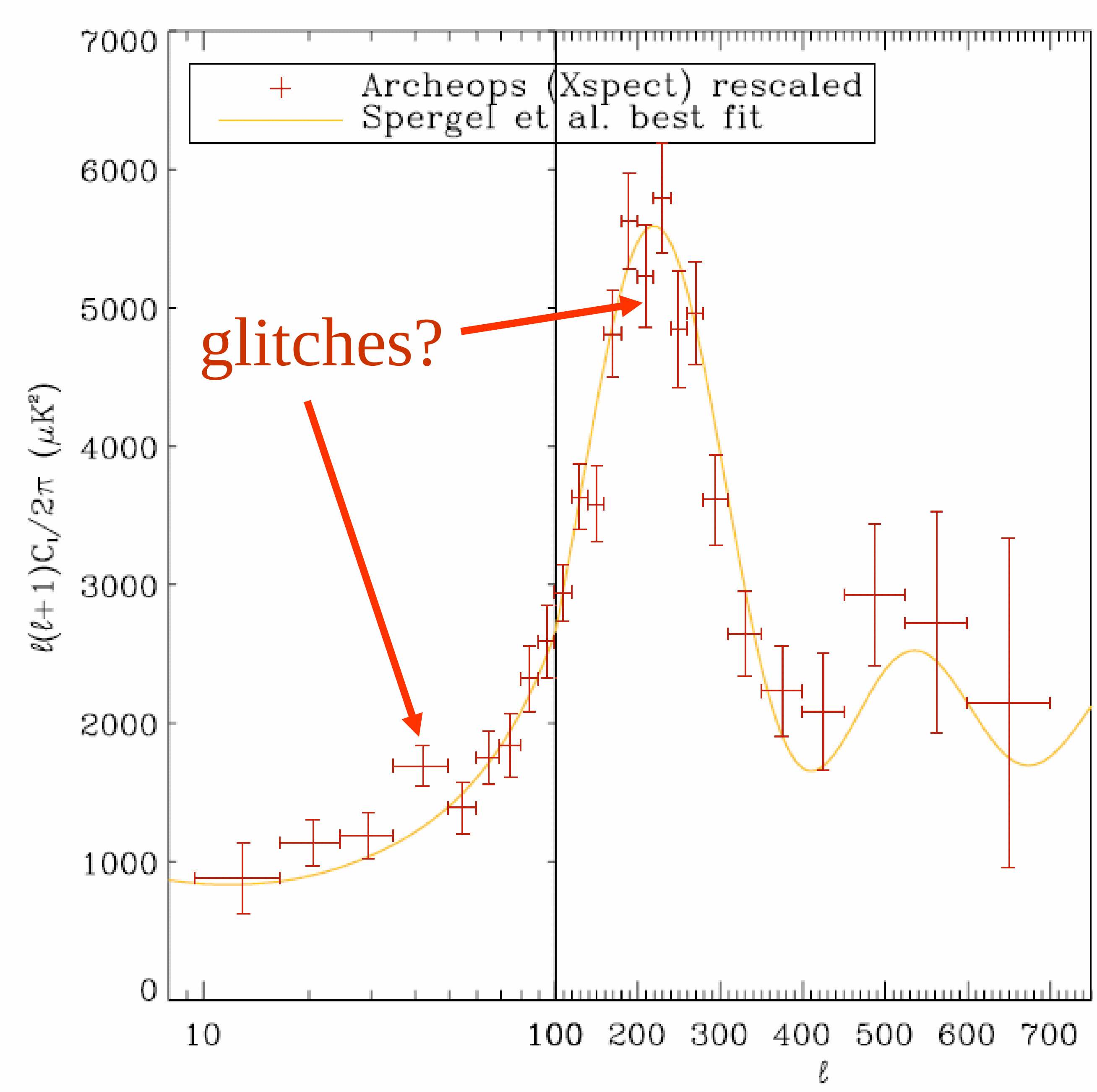
The excess χ^2 comes mostly from the *outliers* in the TT spectrum



***WMAP*-1**: Only 3 out of 16000 simulations would have a lower value of C_{181} than that observed (Lewis 2004)



Similar outliers have been seen by ***Archeops*** (although less significant)



Is the primordial density perturbation really **scale-free**?

"In the absence of an established theoretical framework in which to interpret these glitches … they will likely remain curiosities"

Spergel *et al* (2006)

Then why not also say:

"In the absence of an established theoretical framework in which to interpret *dark energy* … the *apparent acceleration of the universe* will likely remain a curiosity"

The formation of large-scale structure is akin to a scattering experiment

The **Beam:** **inflationary density perturbations**

No 'standard model' – usually *assumed* to be adiabatic and ~scale-invariant

The **Target:** **dark matter (+ baryonic matter)**

Identity unknown - usually taken to be cold (sub-dominant 'hot' component?)

The **Detector:** **the universe**

Modelled by a 'simple' FRW cosmology with parameters h, Ω_{CDM} , Ω_b , Ω_Λ , Ω_k …

The **Signal:** **CMB anisotropy, galaxy clustering …**

measured over scales ranging from ~ 1 – 10000 Mpc ($\Rightarrow$ ~8 e-folds of inflation)

We cannot simultaneously determine the properties of *both* the **beam** *and* the **target** with an unknown **detector**

… hence need to adopt suitable 'priors' on h, Ω_{CDM}, etc
in order to break inevitable parameter *degeneracies*

Astronomers have traditionally *assumed* a Harrison-Zeldovich spectrum:

$$P(k) \propto k^n, \;\; n = 1$$

But models of inflation generally predict departures from scale-invariance

In **single-field slow-roll models**: $n = 1 + 2V''/V - 3\,(V'/V)^2$

Since the potential $V(\phi)$ steepens towards the end of inflation, there will be a *scale-dependent spectral tilt* on cosmologically observable scales:

e.g. in model with *cubic* leading term: $V(\phi) \simeq V_o - \beta\phi^3 + \ldots \Rightarrow n \simeq 1 - 4/N_* \sim 0.94$

where $N_* \approx 60 + \ln\,(k^{-1}/3000h^{-1}\,\mathrm{Mpc})$ is the # of e-folds from the *end* of inflation

This agrees with the best-fit value power-law index inferred from the *WMAP* data

In **hybrid models**, inflation is ended by the 'waterfall' field, *not* due to the steepening of $V(\phi)$, so spectrum is generally closer to scale-invariant …

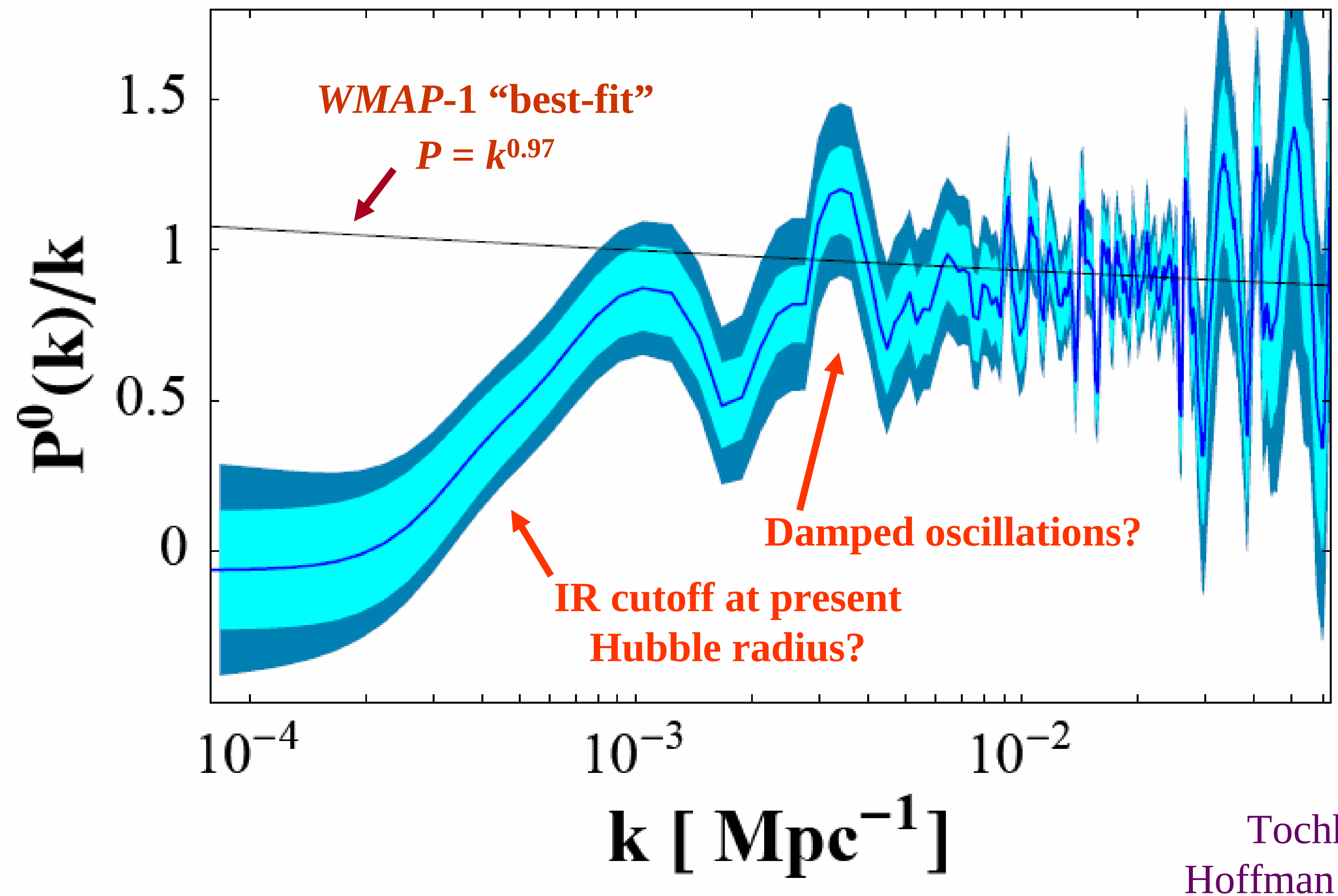
In general there would be *many* other fields present, whose own dynamics may *interrupt* the inflaton's slow-roll evolution (rather than terminate it altogether)

→ can generate features in the spectrum ('steps', 'oscillations', 'bumps' …)

Many attempts made to reconstruct the primordial spectrum (*assuming* ΛCDM)

Bridle, Lewis, Weller & Efstathiou 2003; Cline, Crotty & Lesgourgues 2003, Mukherjee & Wang 2003; Hannestad 2004; Kogo, Sasaki & Yokoyama 2004; Tocchini-Valentini, Douspis & Silk 2004, …

… Essential to use *non*-parametric methods (Shafieloo & Souradeep 2004)

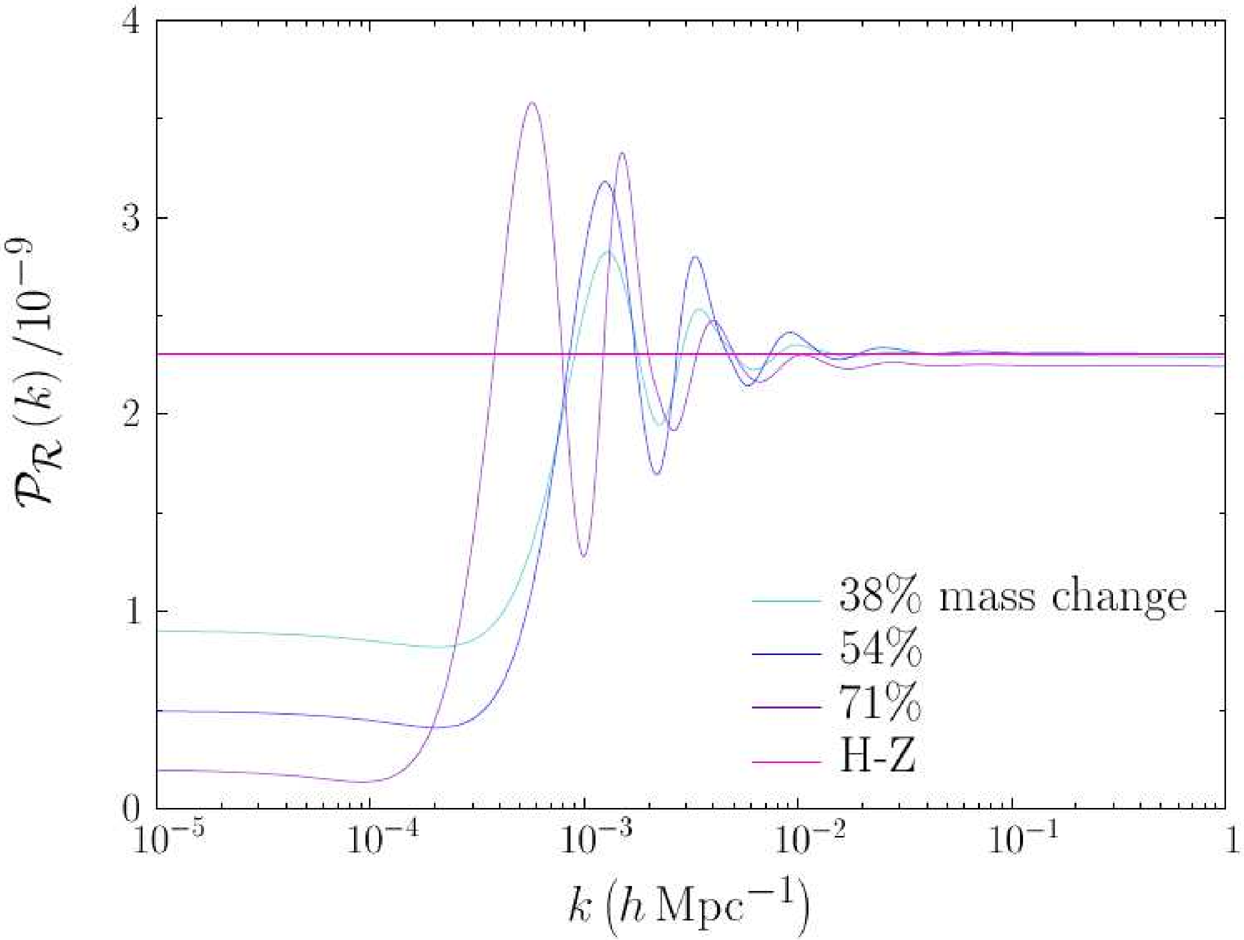


Tochhini-Valentini, Hoffman & Silk (2005)

Such spectra arise *naturally* if the inflaton mass changes suddenly, e.g. due to its coupling (through gravity) to a field which undergoes a fast symmetry-breaking phase transition in the rapidly cooling universe

(Adams, Ross & Sarkar 1997)

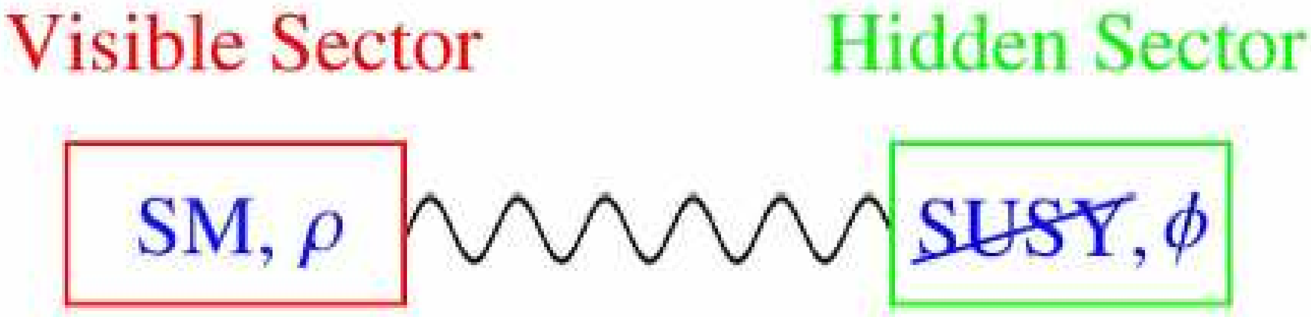
This must happen as cosmologically interesting scales 'exit the horizon' ... likely if (last phase of) inflation did not last longer than ~50-60 e-folds



Hunt & Sarkar (2005)

Consider inflation in context of *effective* field theory: $\mathcal{N}=1$ SUGRA

(successful description of gauge coupling unification, EW symmetry breaking, …)



The visible sector could be important during inflation if gauge symmetry breaking occurs

Supersymmetric theories contain 'flat directions' in field space where the potential vanishes in the limit of unbroken SUSY

This is due to various symmetries and non-renormalisation theorems

Flat directions are lifted by

- ~~SUSY~~ .
- Higher dimensional operators $\rho^n / M_{\mathrm{P}}^{n-4}$ which appear after integrating out heavy degrees of freedom

These fields get a large mass ($m^2 \sim \pm H^2$) *during* inflation, thus perturbing the inflaton

For canonically normalised fields with

$$K = \sum_{\alpha} |\phi_\alpha|^2$$

the SUGRA scalar potential is

$$\begin{aligned}
V_{\rm SUGRA} &= \exp\left(\frac{K}{M_{\rm P}^2}\right)\left[\sum_{\alpha,\beta}\left(\frac{\partial^2 K}{\partial\overline{\phi}_\alpha \partial\phi_\beta}\right)^{-1}\left(\frac{\partial W}{\partial\phi_\alpha} + \frac{W}{M_{\rm P}^2}\frac{\partial K}{\partial\phi_\alpha}\right)\right. \\
&\quad \left.\times\left(\frac{\partial\overline{W}}{\partial\overline{\phi}_\beta} + \frac{\overline{W}}{M_{\rm P}^2}\frac{\partial K}{\partial\overline{\phi}_\beta}\right) - \frac{3}{M_{\rm P}^2}|W|^2\right] + \text{D-terms} \\
&= \exp\left(\frac{1}{M_{\rm P}^2}\sum_{\gamma}|\phi_\gamma|^2 + \ldots\right)\left\{\sum_{\alpha,\beta}(\delta_{\alpha\beta} + \ldots)\right. \\
&\quad \times\left[\frac{\partial W}{\partial\phi_\alpha} + \frac{W}{M_{\rm P}^2}(\overline{\phi}_\alpha + \ldots)\right]\left[\frac{\partial\overline{W}}{\partial\overline{\phi}_\beta} + \frac{\overline{W}}{M_{\rm P}^2}(\phi_\beta + \ldots)\right] \\
&\quad \left. - \frac{3}{M_{\rm P}^2}|W|^2\right\} \\
&\simeq V_{\rm global} \pm \frac{V_{\rm global}}{M_{\rm P}^2}\sum_{\alpha}|\phi_\alpha|^2, \quad V_{\rm global} = \sum_{\alpha}\left|\frac{\partial W}{\partial\phi_\alpha}\right|^2
\end{aligned}$$

i.e. there is a contribution of $\pm H^2$ to the mass-squared of all scalar fields

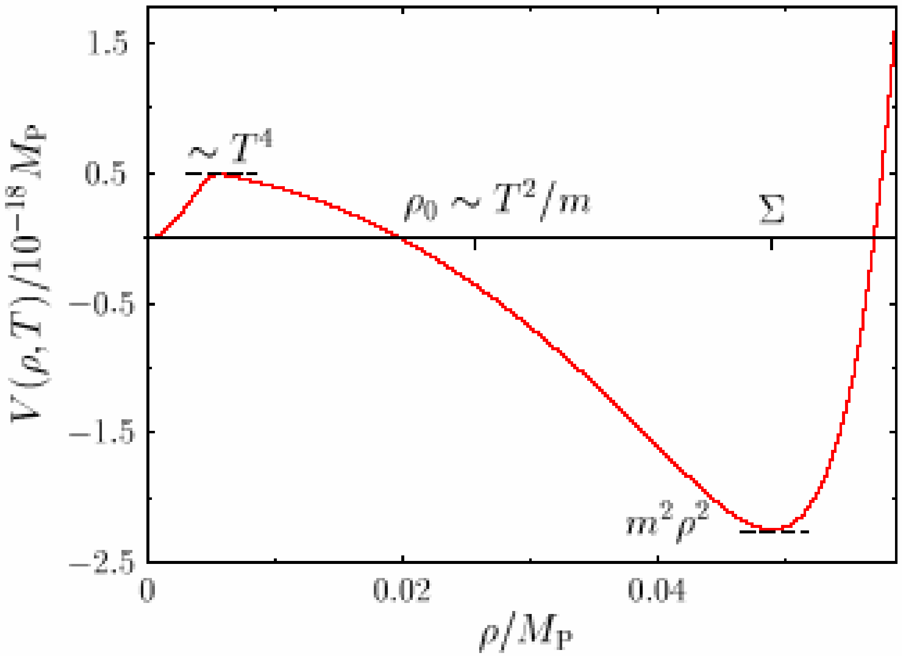
If m^2 is negative, ρ is stabilised at $\Sigma \sim \mathcal{O}\left(M_{\rm P}^2 \left|m^2\right|\right)^{1/(n-4)}$, by $\rho^n/M_{\rm P}^{n-4}$ terms

Assume that in the era preceding observable inflation, all fields (with gauge and/or Yukawa couplings) are in thermal equilibrium

Including the one-loop finite temperature correction

$$V(\rho, T) \quad \simeq \quad \begin{cases} C_1 T^2 \rho^2, & \text{for} \quad \rho \ll T \\ -m^2\rho^2 + \frac{1}{90}\pi^2 N_{\rm h}(T) T^4 + \frac{\gamma \rho^n}{M_{\rm P}^{n-4}}, & \text{for} \quad T \ll \rho < \Sigma \end{cases}$$

Here $N_{\rm h}(T)$ is the number of helicity states with mass much less than the temperature

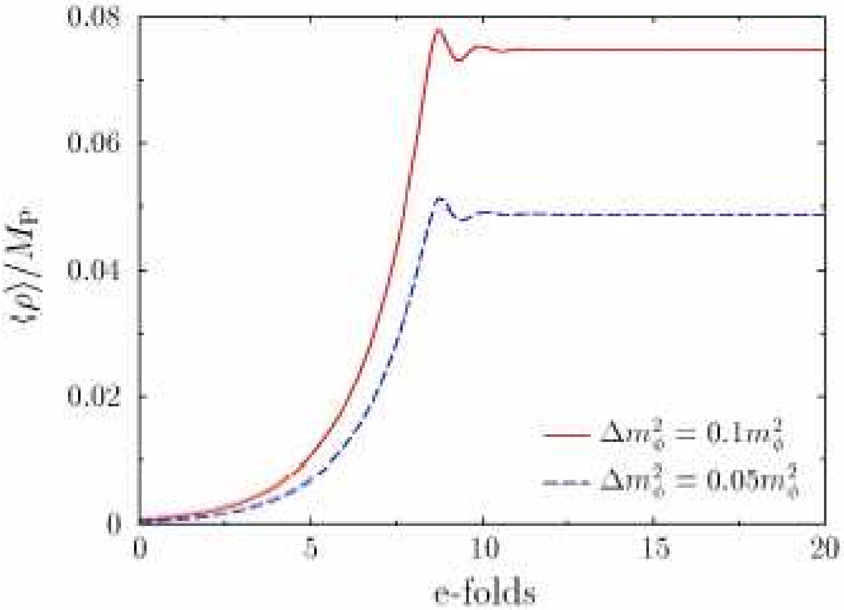


The tunneling rate through the thermal barrier between $\rho = 0$ and $\rho \sim T^2/m$ is negligible, so $\rho = 0$ until $T \sim m$ when the barrier disappears (Yamamoto 1985)

ρ evolves to the global minimum at Σ as

$$\ddot{\rho} + 3H\dot{\rho} = -\frac{dV}{d\rho}$$

$$\Rightarrow \rho \quad \simeq \quad \begin{cases} \rho_0 \exp\left[\frac{3Ht}{2}\left(\sqrt{1+\frac{8m^2}{9H^2}}\right) - 1\right], & \langle\rho\rangle \ll \Sigma \\ \Sigma + K_1 \exp\left(-\frac{3Ht}{2}\right) \sin\left[\frac{3Ht}{2}\sqrt{(n-2)\frac{8m^2}{9H^2} - 1} + K_2\right], & \langle\rho\rangle \sim \Sigma \end{cases}$$



After the phase transition slow-roll inflation continues but at a reduced scale

$$V(\phi) \quad \rightarrow \quad \left[1 - (\Sigma/M_{\mathrm{P}})^2\right] V(\phi)$$

For $\Sigma \ll M_{\mathrm{P}}$ the change is negligible and so H can be taken to be sensibly constant

However ρ and ϕ are coupled by gravity.

Then with $K \subset \kappa\phi\phi^\dagger\rho^2/M_{\rm P}^2$ for example

$$V(\phi,\rho) = V_0 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}\mu^2\rho^2 + \frac{1}{2}\lambda\phi^2\rho^2 + \frac{\gamma}{M_{\rm P}^{n-4}}\rho^n + \ldots, \qquad \lambda = \frac{\kappa H^2}{M_{\rm P}^2},$$

$\Rightarrow$ change in inflaton effective mass-squared $m_\phi^2 \equiv {\rm d}^2 V/{\rm d}\phi^2$

$$m_\phi^2 = -m^2 \quad \rightarrow \quad m_\phi^2 = -m^2 + \lambda\Sigma^2, \quad \Sigma \simeq \left(\frac{2m^2 M_{\rm P}^{n-4}}{n\gamma}\right)^{1/(n-2)} .$$

Phase transition must occur as cosmological scales are leaving the horizon for its effects to be observable (eg in LSS or CMB).

But we expect many flat directions which each cause a phase transition at a different temperature

$\Rightarrow$ increased likelihood that one will be observed.

All this happens if the **initial conditions are thermal** (i.e. ρ starts at origin) and this (last) phase of **inflation lasts just long enough to create present Hubble volume**

may seem fine-tuned but the data *does* indicate an **IR cutoff** at the present Hubble radius!

The Spectrum

Metric describing scalar perturbations in a flat universe can be written as

$$\mathrm{d}s^2 = a^2 \left[(1 + 2A_\mathrm{s})\,\mathrm{d}\eta^2 - 2\partial_i B_\mathrm{s} \mathrm{d}\eta \mathrm{d}x^i - \{(1 - 2D_\mathrm{s})\,\delta_{ij} + 2\partial_i \partial_j E_\mathrm{s}\}\,\mathrm{d}x^i \mathrm{d}x^j \right].$$

Use Sasaki-Mukhanov variable

$$u = a\left(\delta\phi + H\frac{D_\mathrm{s}}{\dot{\phi}}\right) = -z\mathcal{R}, \quad z = \frac{a\dot{\phi}}{H}, \quad \mathcal{R} = D_\mathrm{s} + H\frac{\delta\phi}{\dot{\phi}}.$$

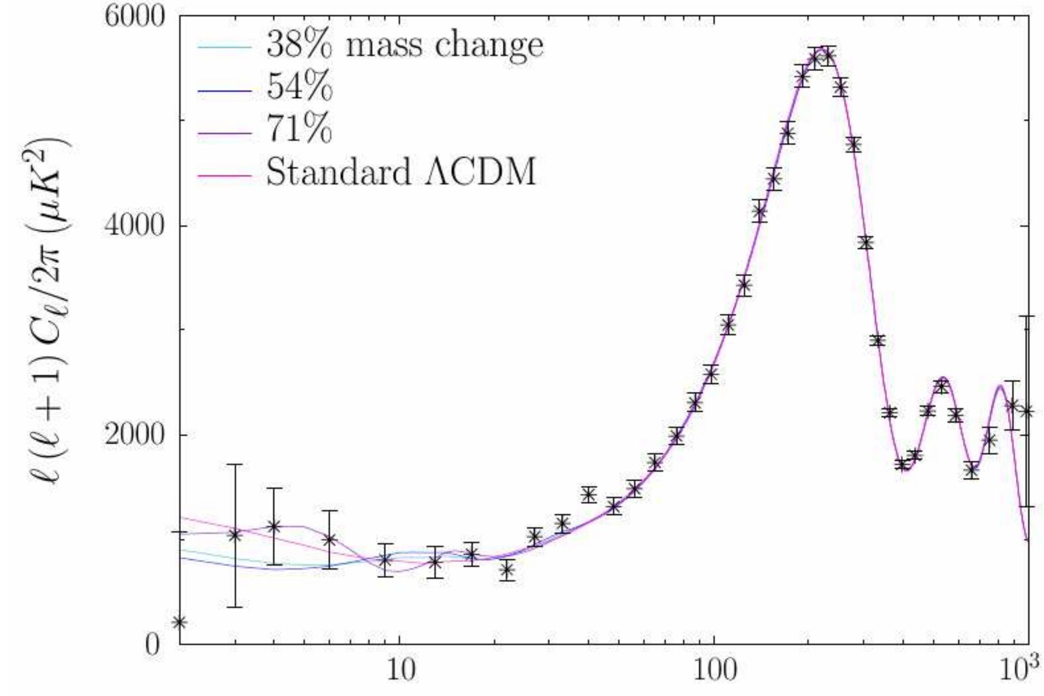
Fourier components of u satisfy

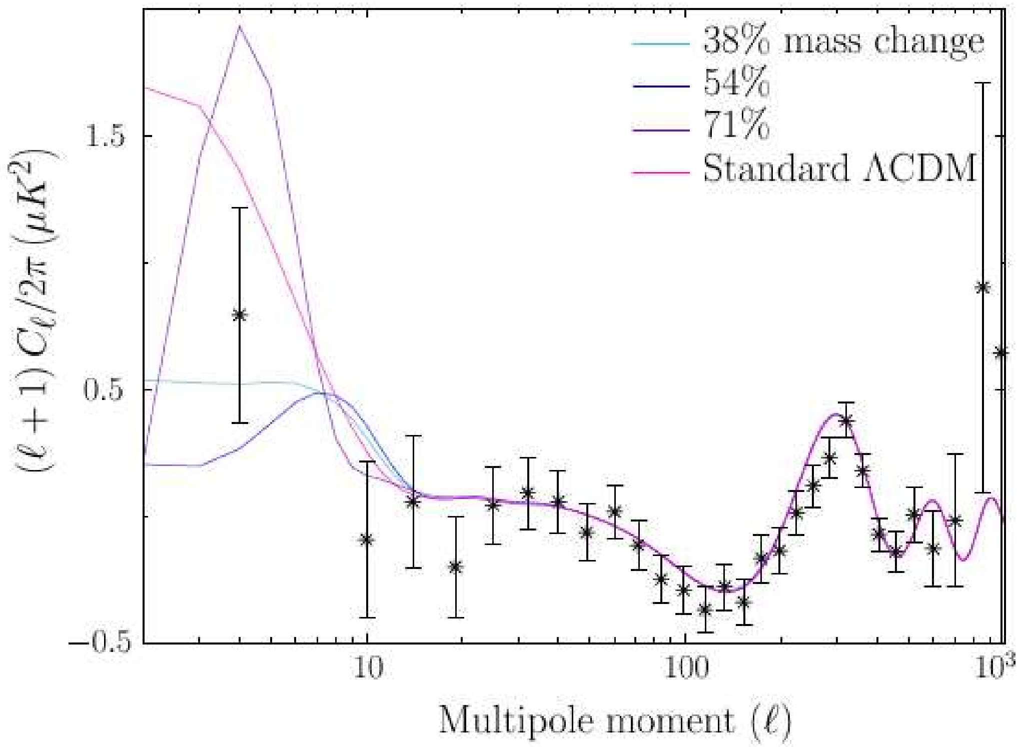
$$u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k = 0, \quad \frac{z''}{z} = a^2 \left(2H^2 + m^2 - \lambda\rho^2 - \frac{2\lambda\rho\dot{\rho}\phi}{\dot{\phi}}\right).$$

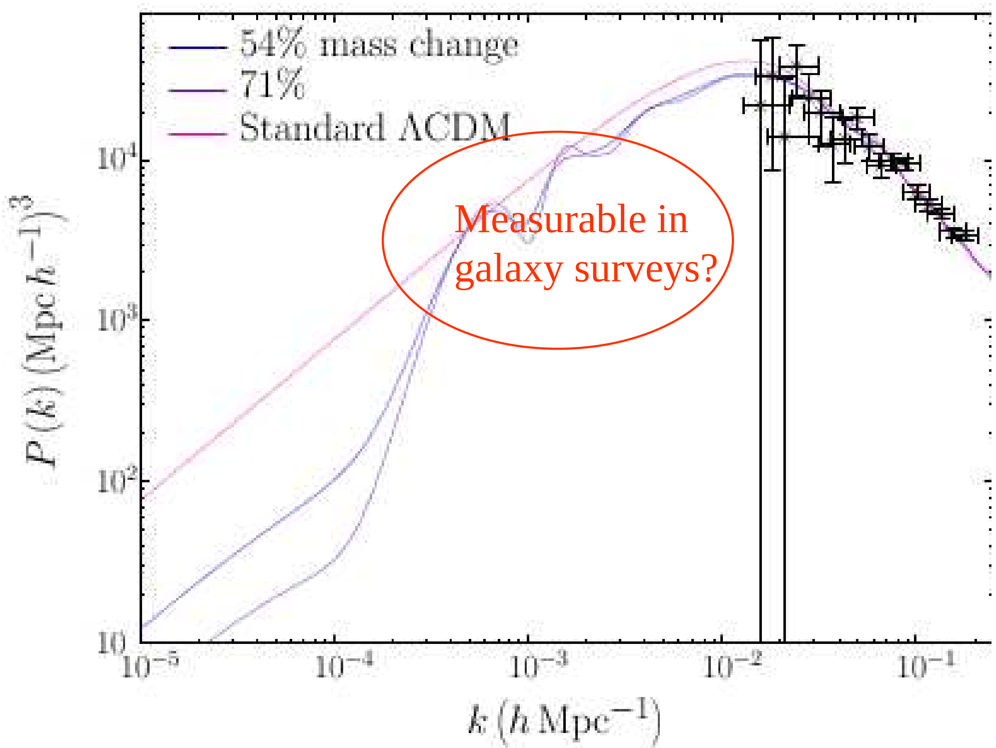
Spectrum is given by

$$\mathcal{P}_\mathcal{R}{}^{1/2} = \sqrt{\frac{k^3}{2\pi^2}}\,|\mathcal{R}_k| = \sqrt{\frac{k^3}{2\pi^2}}\left|\frac{u_k}{z}\right|.$$

Use WKB method (Martin & Schwarz 2003) to obtain P_R when slow-roll is violated …







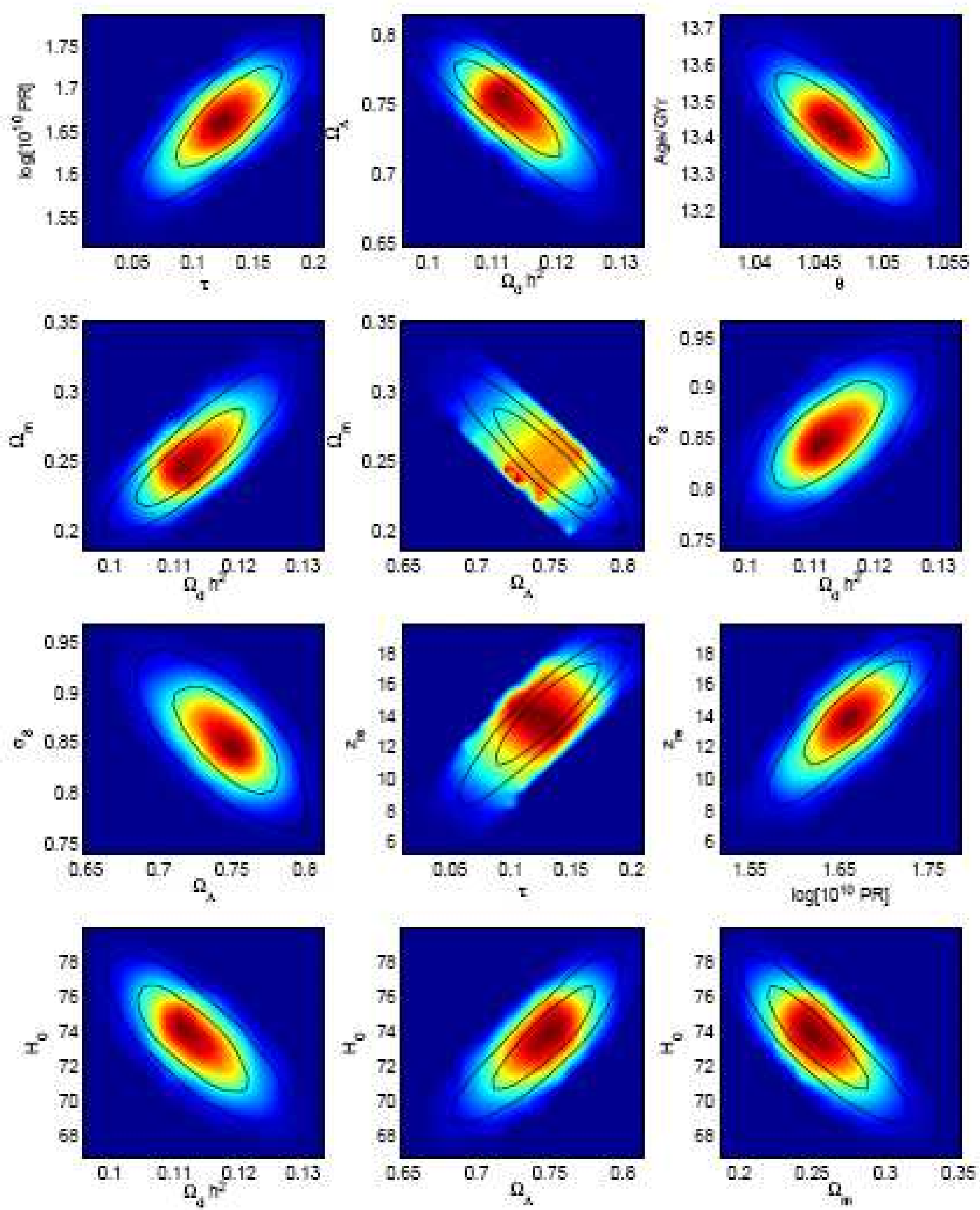
Fits are all acceptable but fit parameters change little except for large-scale amplitude

n	χ^2	$\frac{\Delta m_\phi^2}{m_\phi^2}$	$\Omega_b h^2$	$\Omega_c h^2$	H_0	τ	$10^4 k_0$	$10^{10} A_s$
15	5628.9	0.38	0.0237 $\pm$0.0010	0.0982 $\pm$0.0204	78.9 $\pm$8.1	0.150 $\pm$0.076	8.04 $\pm$5.84	9.54 $\pm$1.09
16	5629.4	0.54	0.0236 $\pm$0.0011	0.0992 $\pm$0.0217	78.8 $\pm$8.5	0.150 $\pm$0.075	7.89 $\pm$5.16	5.23 $\pm$0.49
17	5629.6	0.71	0.0238 $\pm$0.0011	0.1010 $\pm$0.0233	78.0 $\pm$9.2	0.131 $\pm$0.075	3.62 $\pm$4.74	2.21 0.20

***WMAP* does not require the primordial density perturbation to be scale-free!**

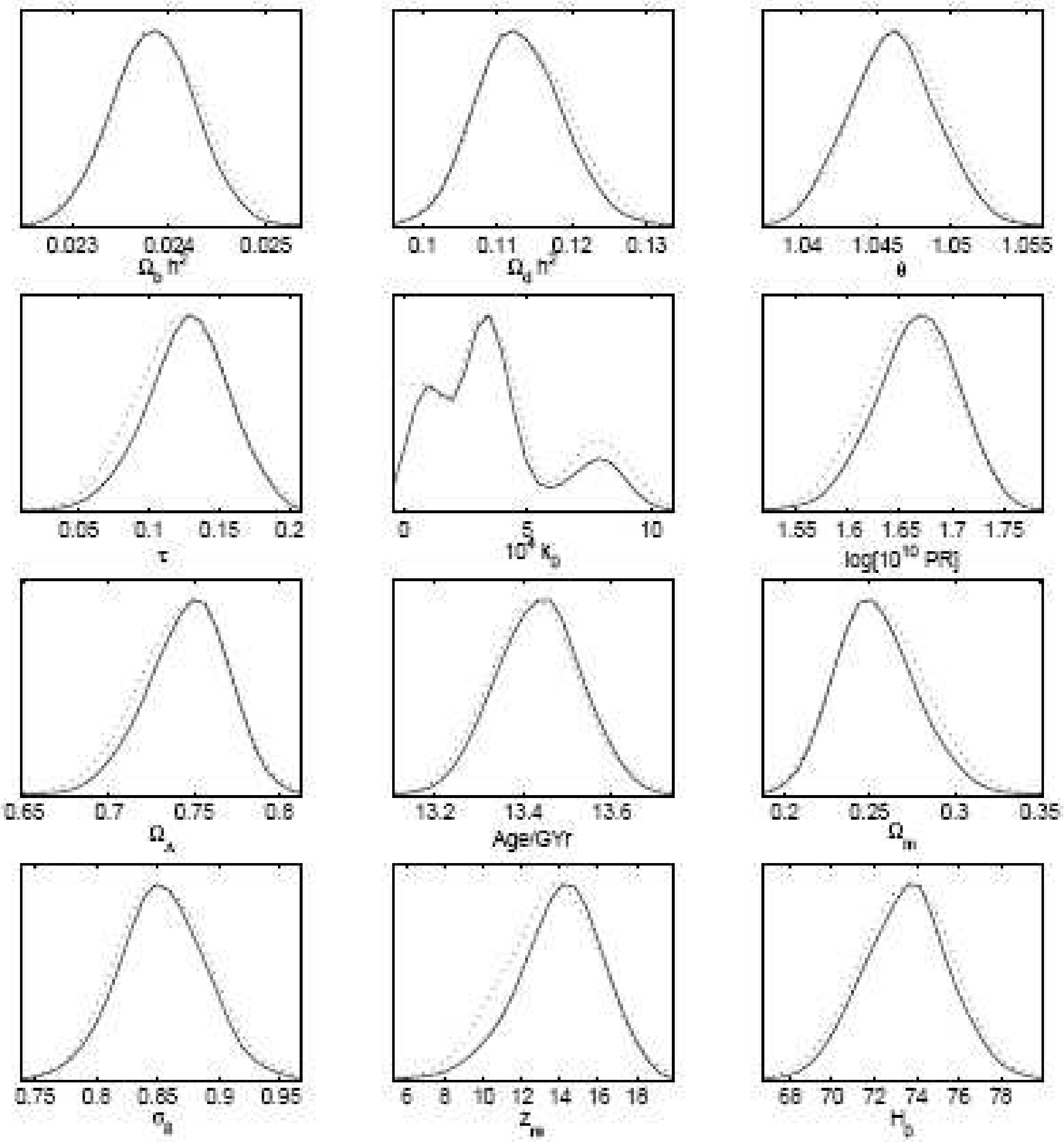
Hunt & Sarkar (2006)

Parameter degeneracies - ΛCDM universe (‘step’ spectrum)



Hunt & Sarkar
(to appear)

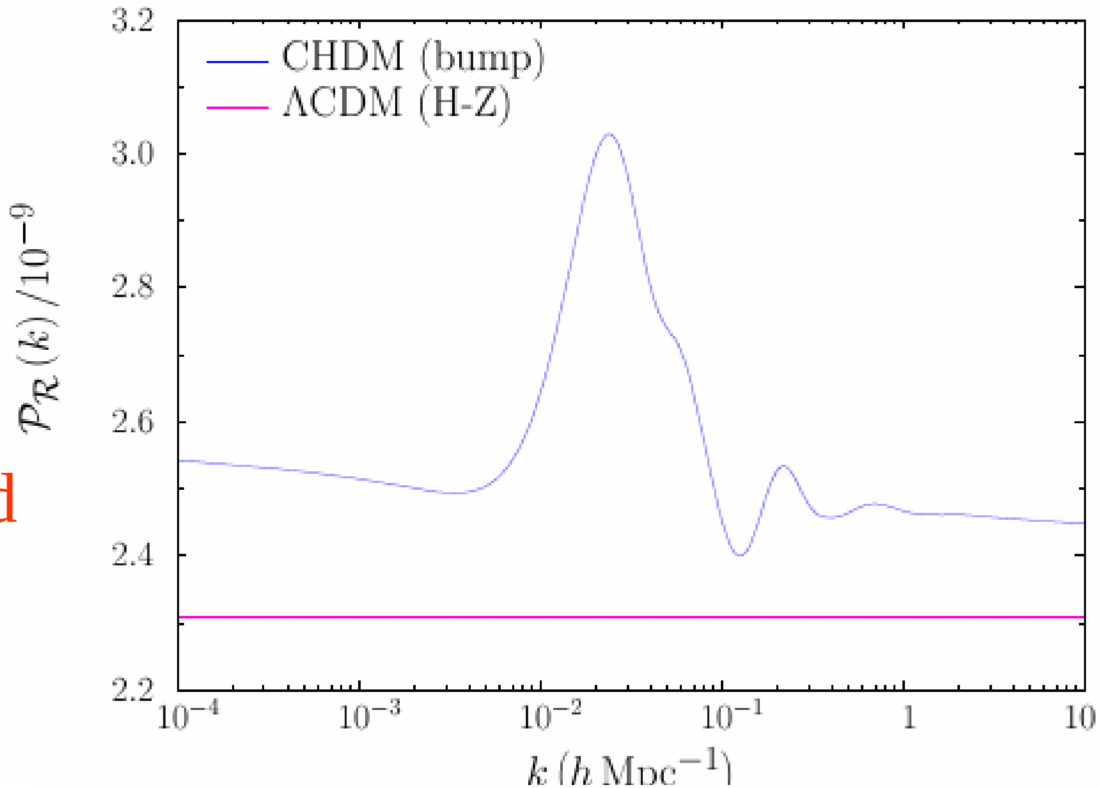
MCMC likelihood distributions for ΛCDM ('step' spectrum)



… not too different from 'power law ΛCDM'

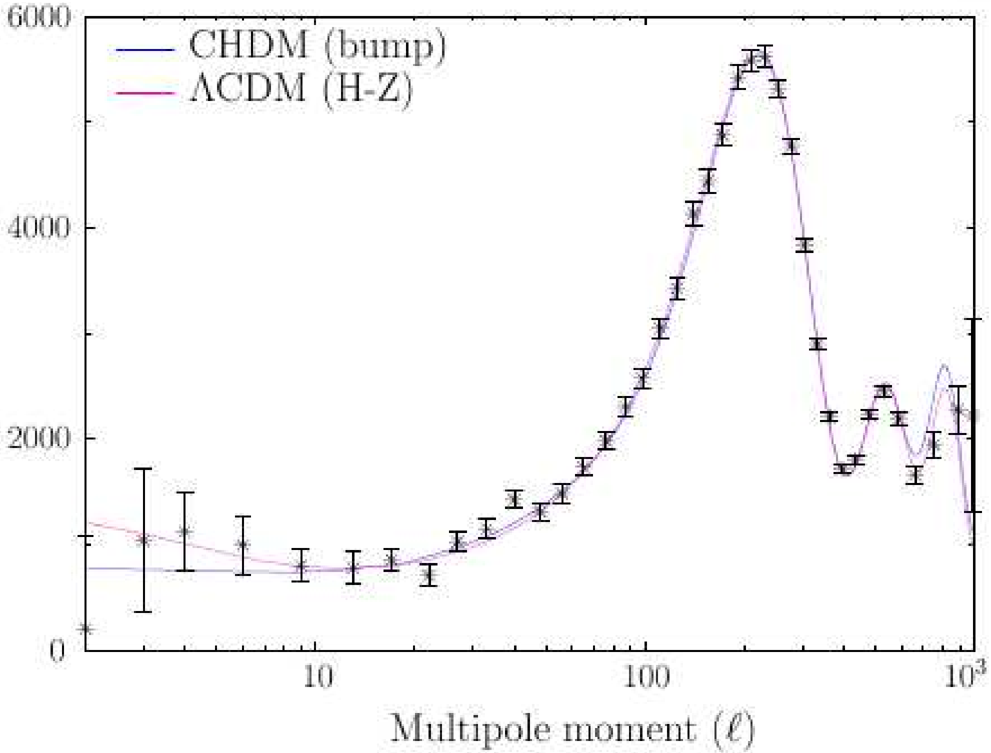
Hunt & Sarkar (to appear)

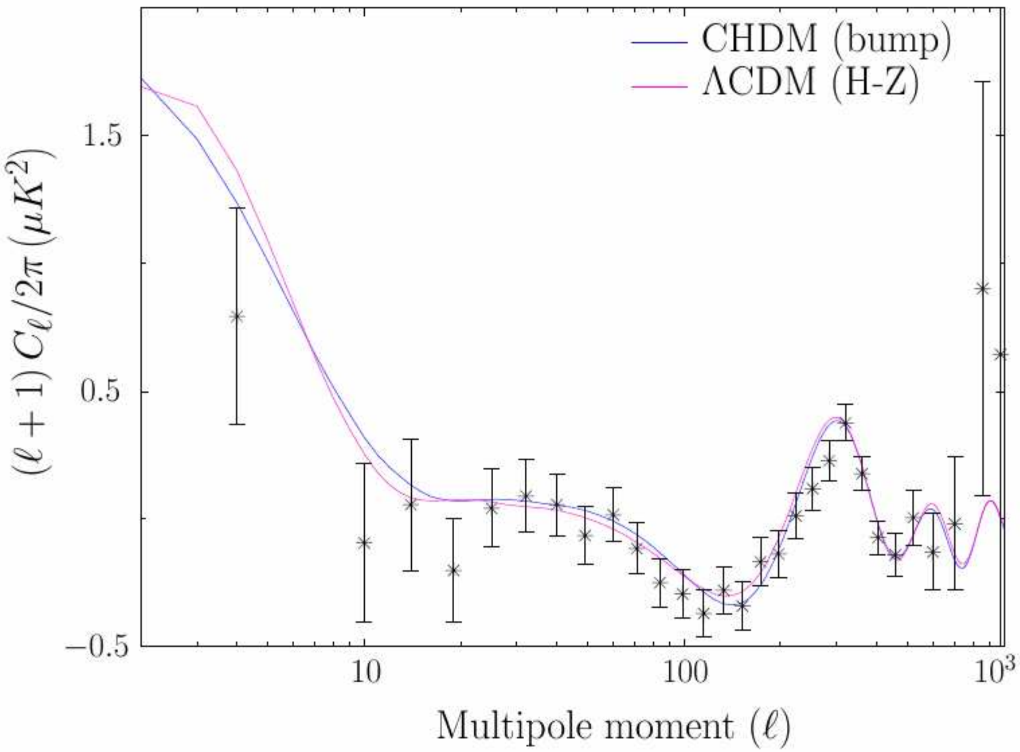
But if there are *many* flat direction fields, then two phase transitions may occur in quick succession, creating a 'bump' in the primordial spectrum on cosmologically relevant scales



The *WMAP* data can then be fitted just as well with ***no dark energy***

($\Omega_m = 1$, $\Omega_\Lambda = 0$, $h = 0.46$)

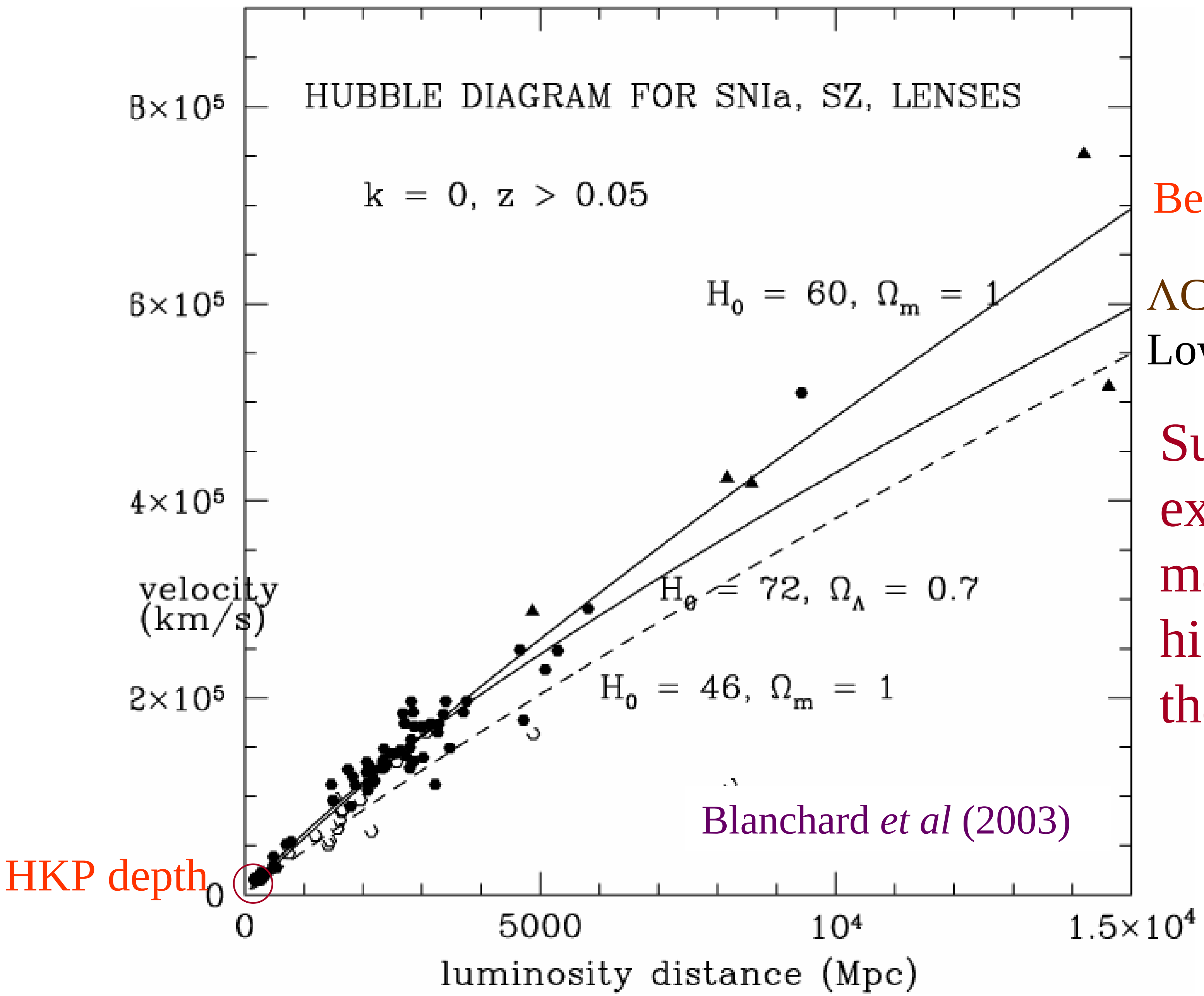




h = 0.46 is inconsistent with Hubble Key Project value (h = 0.72 ± 0.08)

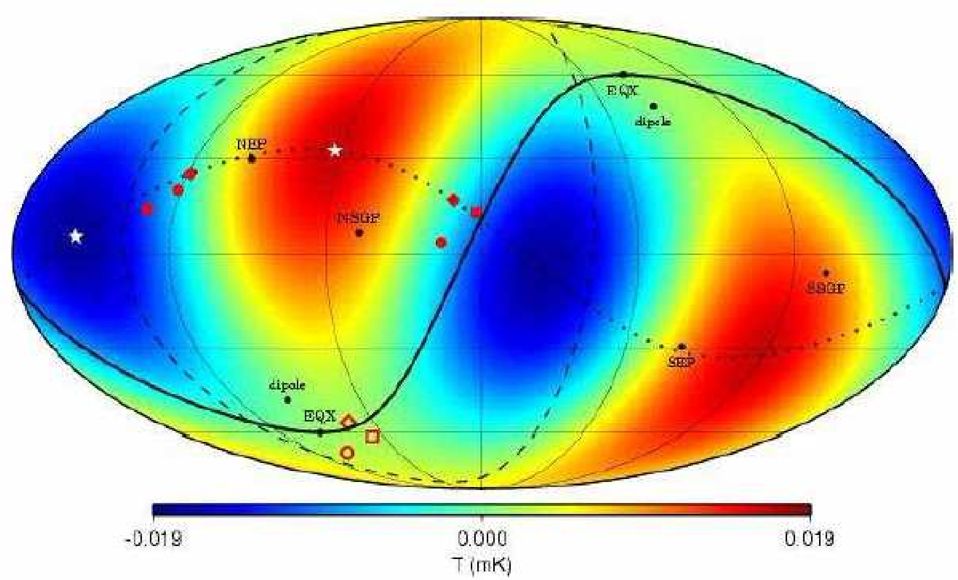
but is in fact *indicated* by direct (and much deeper) determinations

e.g. gravitational lens time delays (h = 0.48 ± 0.03)



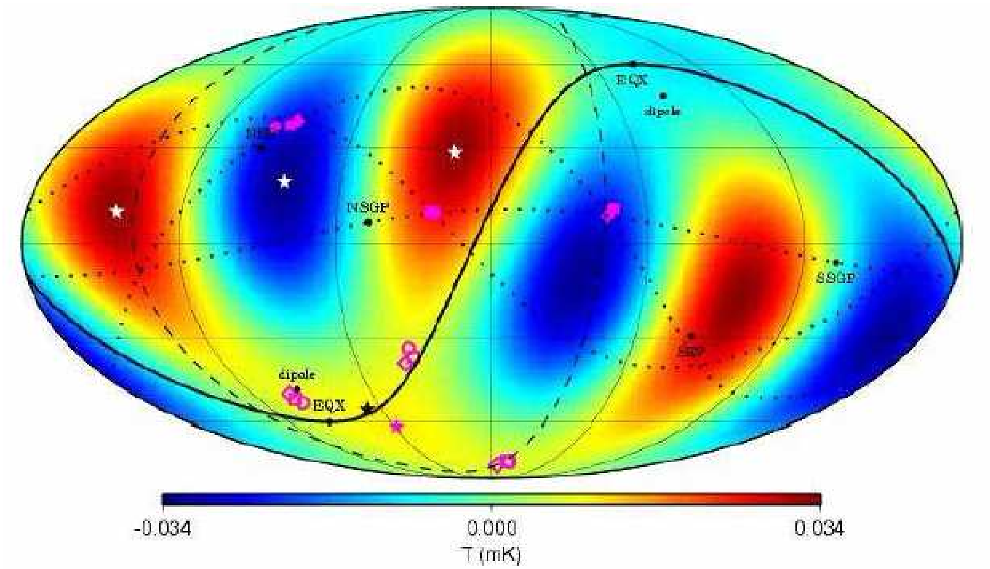
Suggests expansion rate may be 30% higher locally than globally!

Are we located in a ~500 Mpc void which is expanding faster than the average rate (inhomogeneous Lemaitré-Tolman-Bondi model)?

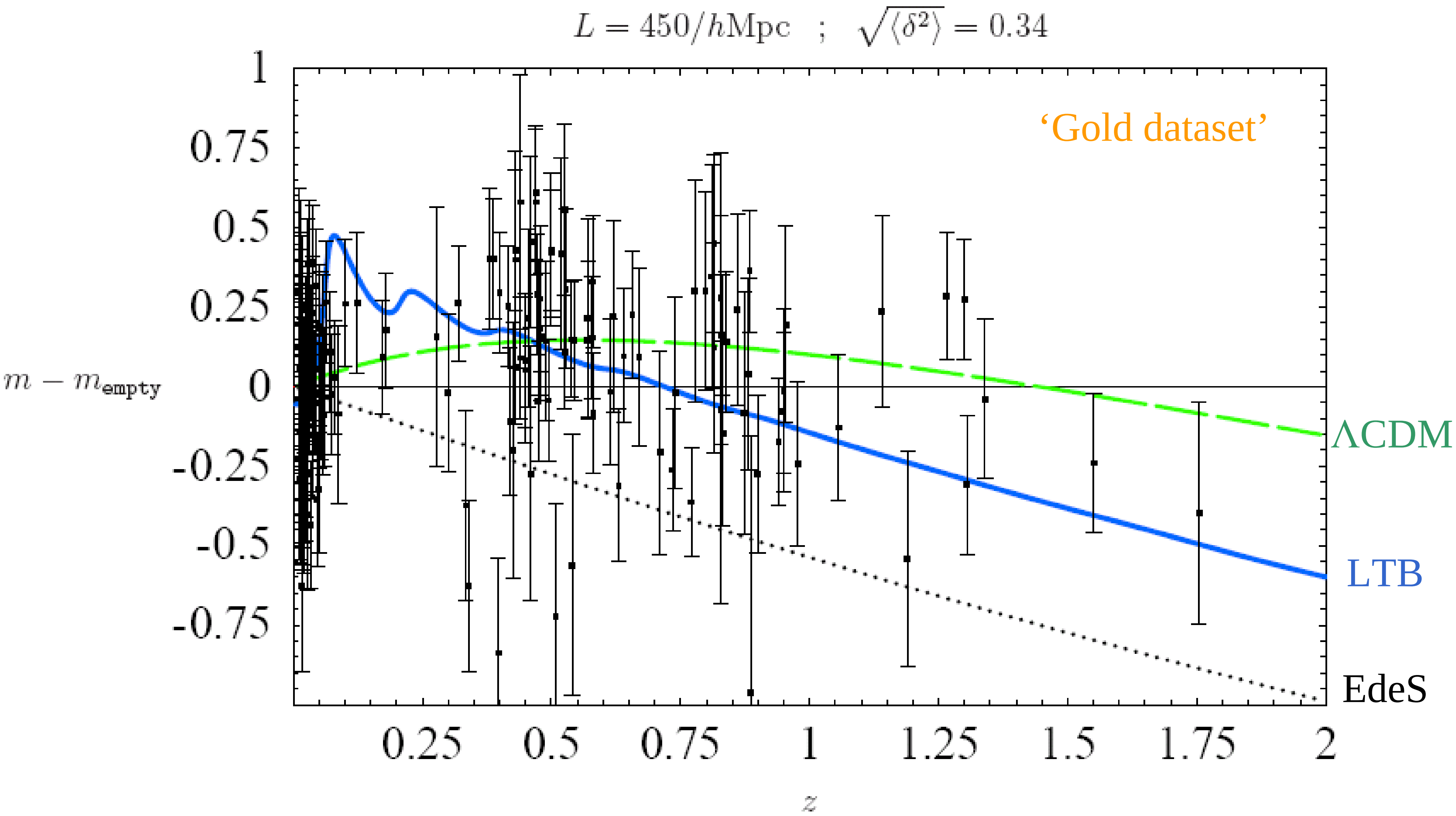


Can the 'Rees-Sciama effect' due to our *local* inhomogeneity then explain the mysterious alignment of the quadrupole and octupole?

(e.g. Inoue & Silk 2006)



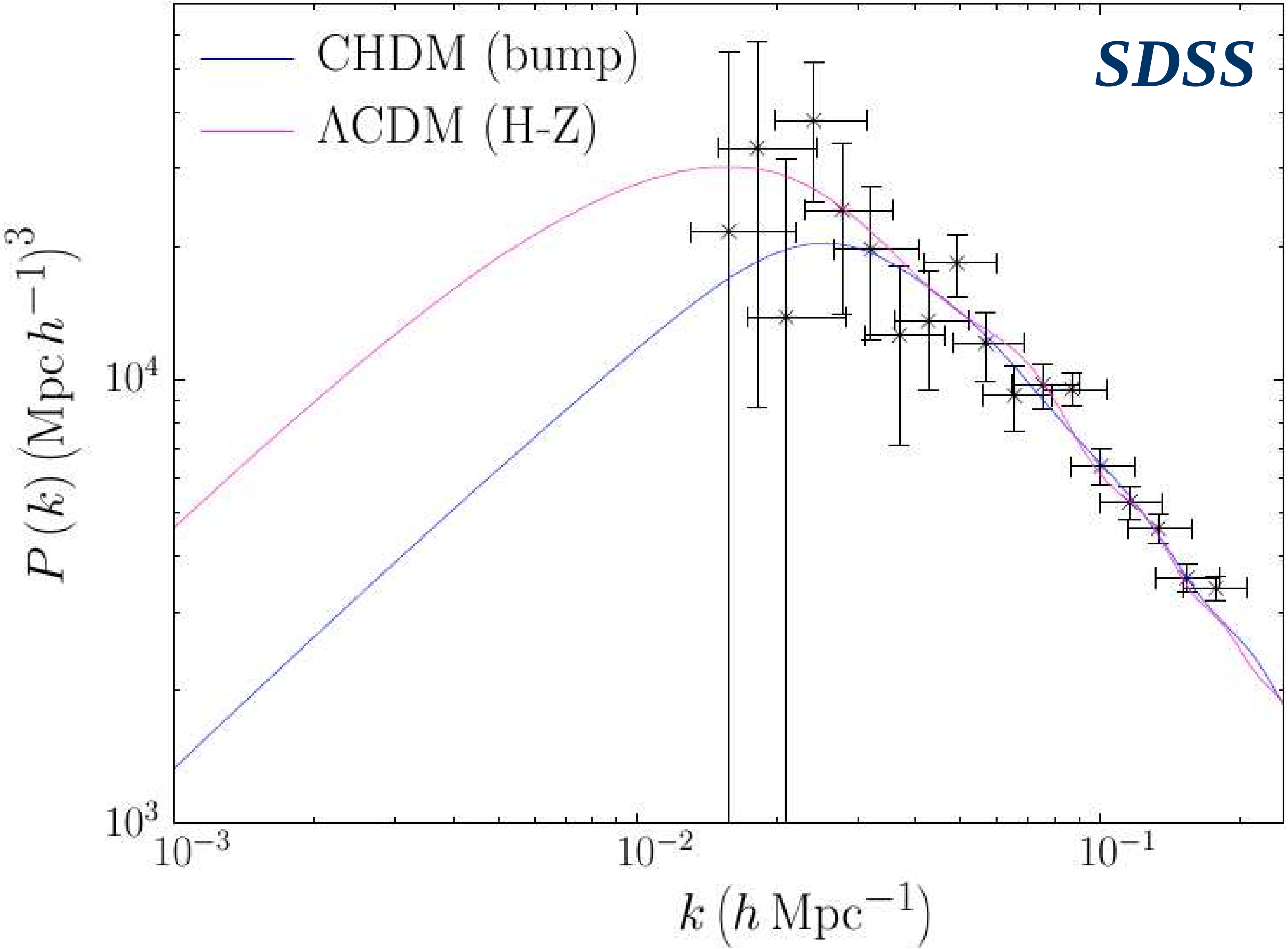
The Lemaitré-Tolman-Bondi model may even explain the SNIa Hubble diagram *without* acceleration!



Biswas, Mansouri & Notari (2006)

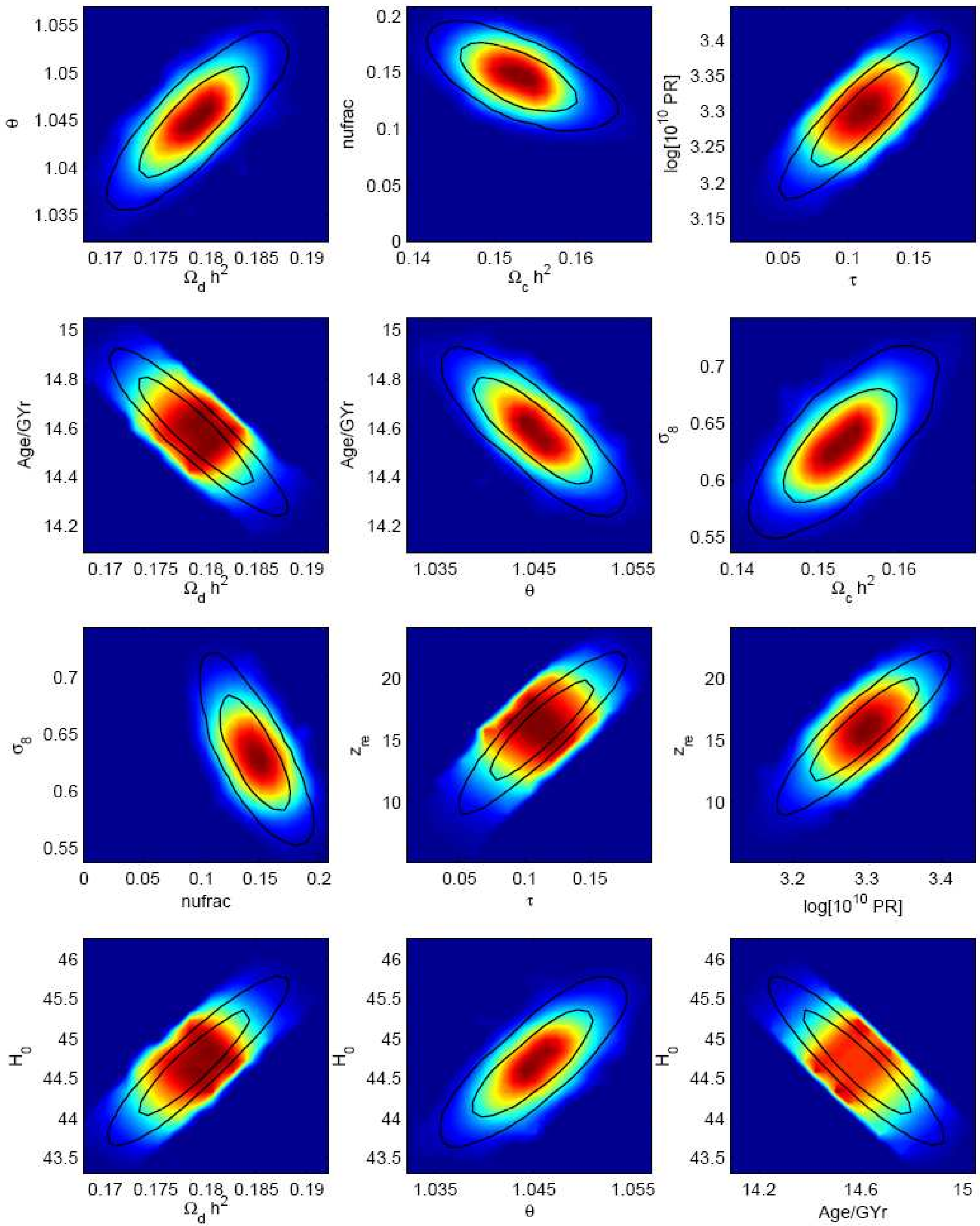
The small-scale power would be excessive unless damped by free-streaming ...

Adding 3 νs of mass 0.8 eV ($\Rightarrow \Omega_\nu \approx 0.14$) gives *good* match to large-scale structure
(note that $\Sigma\, m_\nu \approx 2.4$ eV – well above '*WMAP* bound'!)



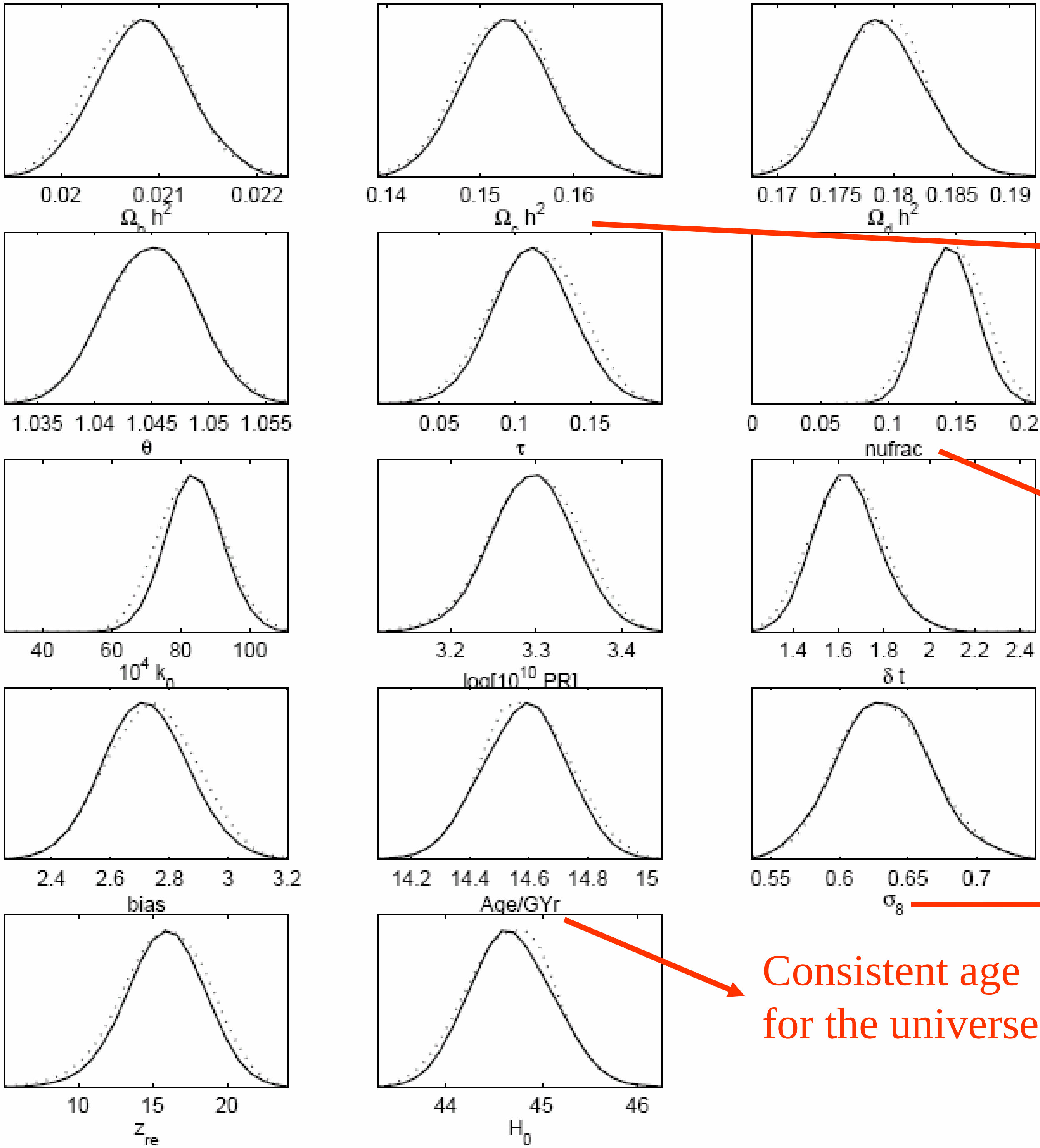
Fit gives $\Omega_b h^2 \approx 0.021 \rightarrow$ BBN √ $\Rightarrow$ baryon fraction in clusters predicted to be ~11% √

Parameter degeneracies - CHDM universe ('bump' spectrum)



Hunt & Sarkar
(to appear)

MCMC likelihoods - CHDM universe (‘bump’ spectrum)



This is ~50% higher than the ‘WMAP value’ used widely for CDM abundance

To fit the large-scale structure data *requires* ~eV mass neutrinos

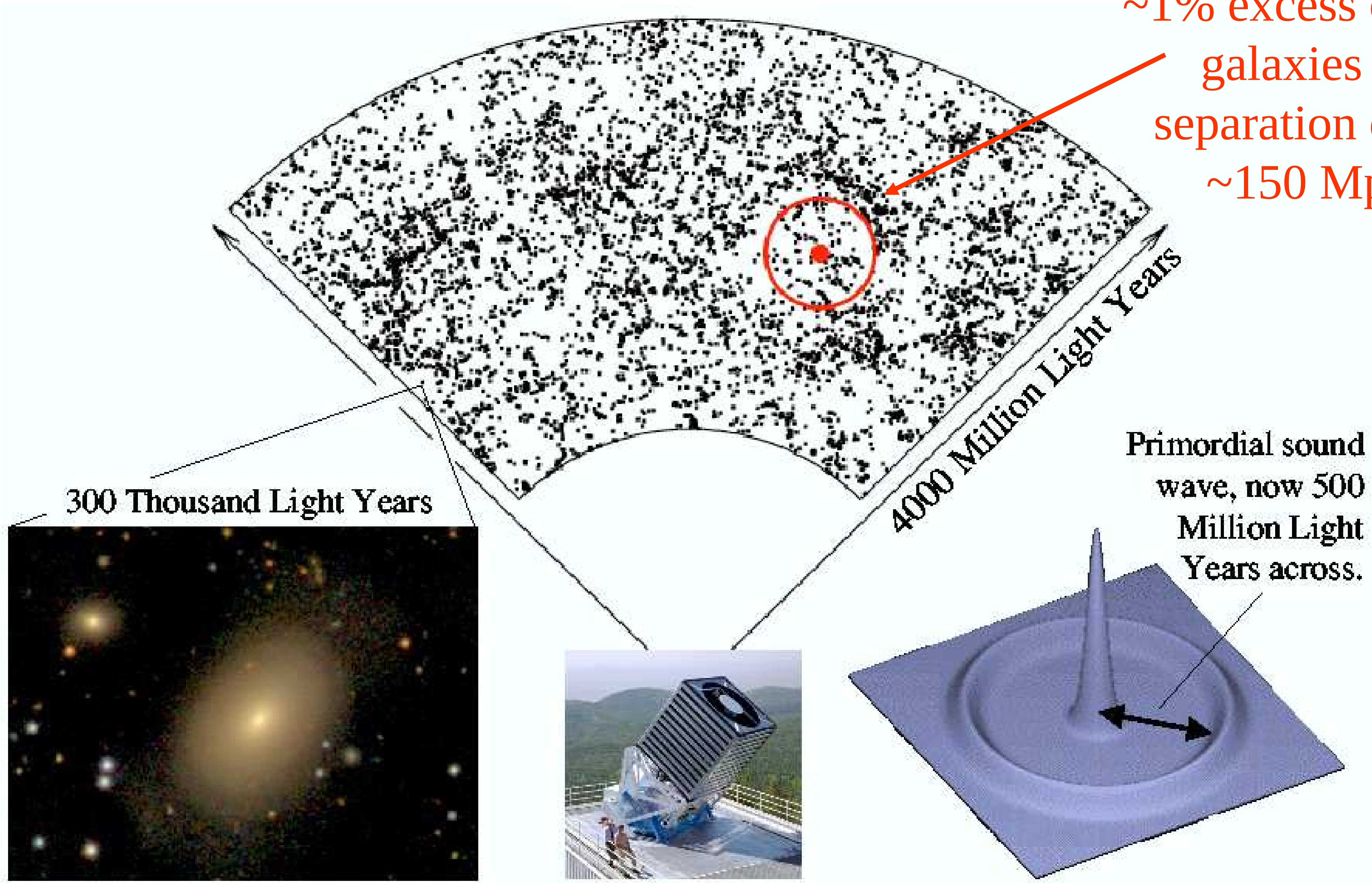
Consistent with data on clusters and weak lensing

Consistent age for the universe

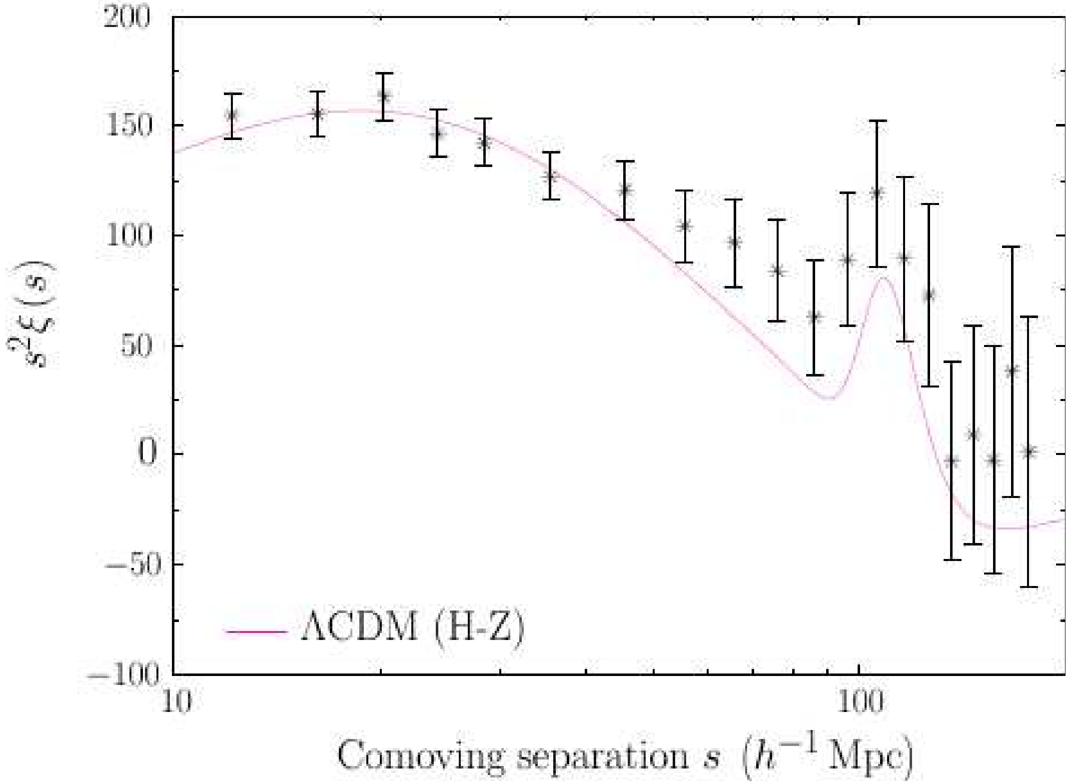
Hunt & Sarkar (to appear)

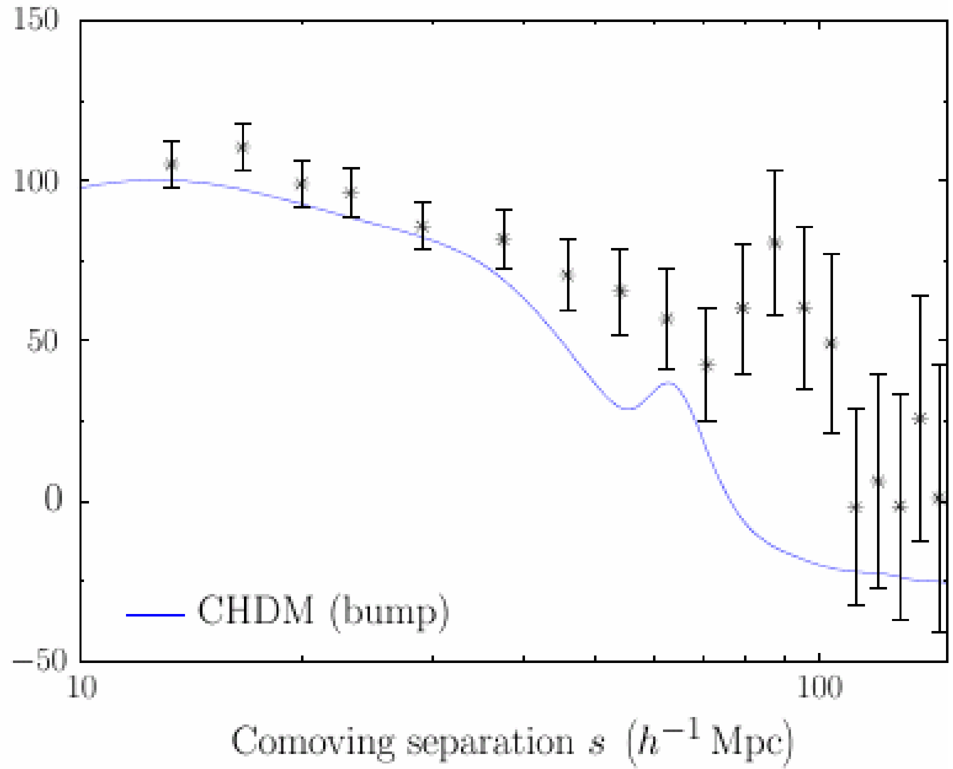
New Test: Baryon Acoustic Peak in the Large-Scale Correlation Function of *SDSS* Luminous Red Galaxies

~1% excess of galaxies at separation of ~150 Mpc



In EdeS model with **no dark energy**, the baryon bump is at the ~same *physical* scale, but at a different location in observed (redshift) space





We *can* match the angular size of the 1st acoustic peak at z ~ 1100 by taking h ~ 0.5, but we *cannot* then also match the angular size of the baryonic feature at z ~ 0.35

But for inhomogeneous LTB model (h ~ 0.7 for z < 0.08, then $h \rightarrow 0.5$) **angular diameter distance @ z = 0.35 is similar to ΛCDM!**

Biswas, Mansouri, Notari (2006)

Conclusions

WMAP is supposed to have confirmed the need for a dominant component of **dark energy** from precision observations of the CMB

➢ However we cannot *simultaneously* determine both the primordial spectrum *and* the cosmological parameters from CMB (and LSS) data

We do not know the physics behind inflation hence are not justified in *assuming* that the generated scalar density perturbation is scale-free (and then conclude that the data *confirm* the power-law ΛCDM model)

The data provides intriguing hints for features in the primordial spectrum … this has crucial implications for parameter extraction e.g. a 'bump' in the spectrum allows the data to be well-fitted *without* any dark energy!

➢ Given the *unacceptable* degree of fine-tuning required to accommodate dark energy, we should explore if the SNIa Hubble diagram, BAO etc can be equally well accounted for in the *inhomogeneous* **LTB model**

The FRW model may be *too* simple a description of the real universe!

MCMC method

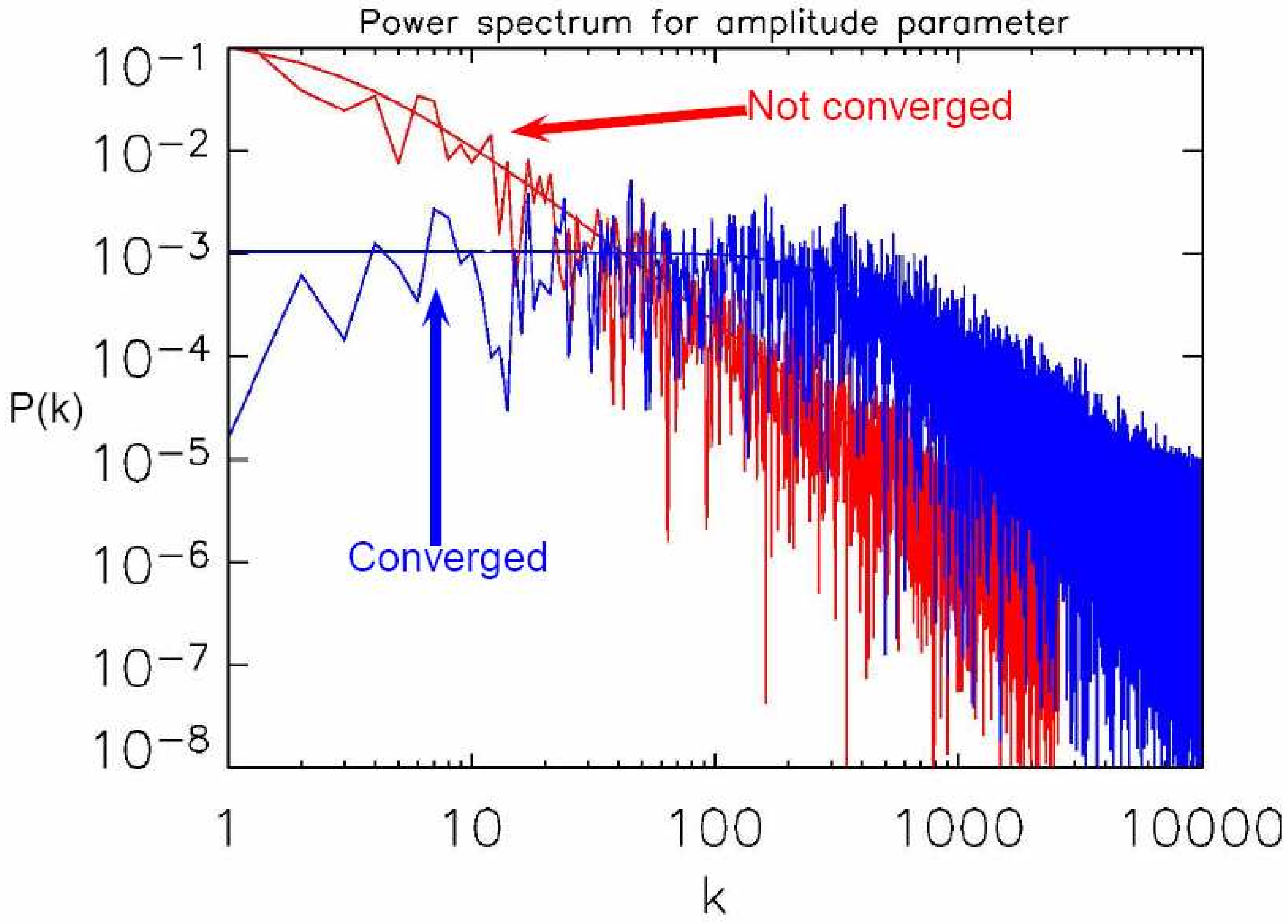
Used Metropolis algorithm with multivariate Gaussian proposal distribution.

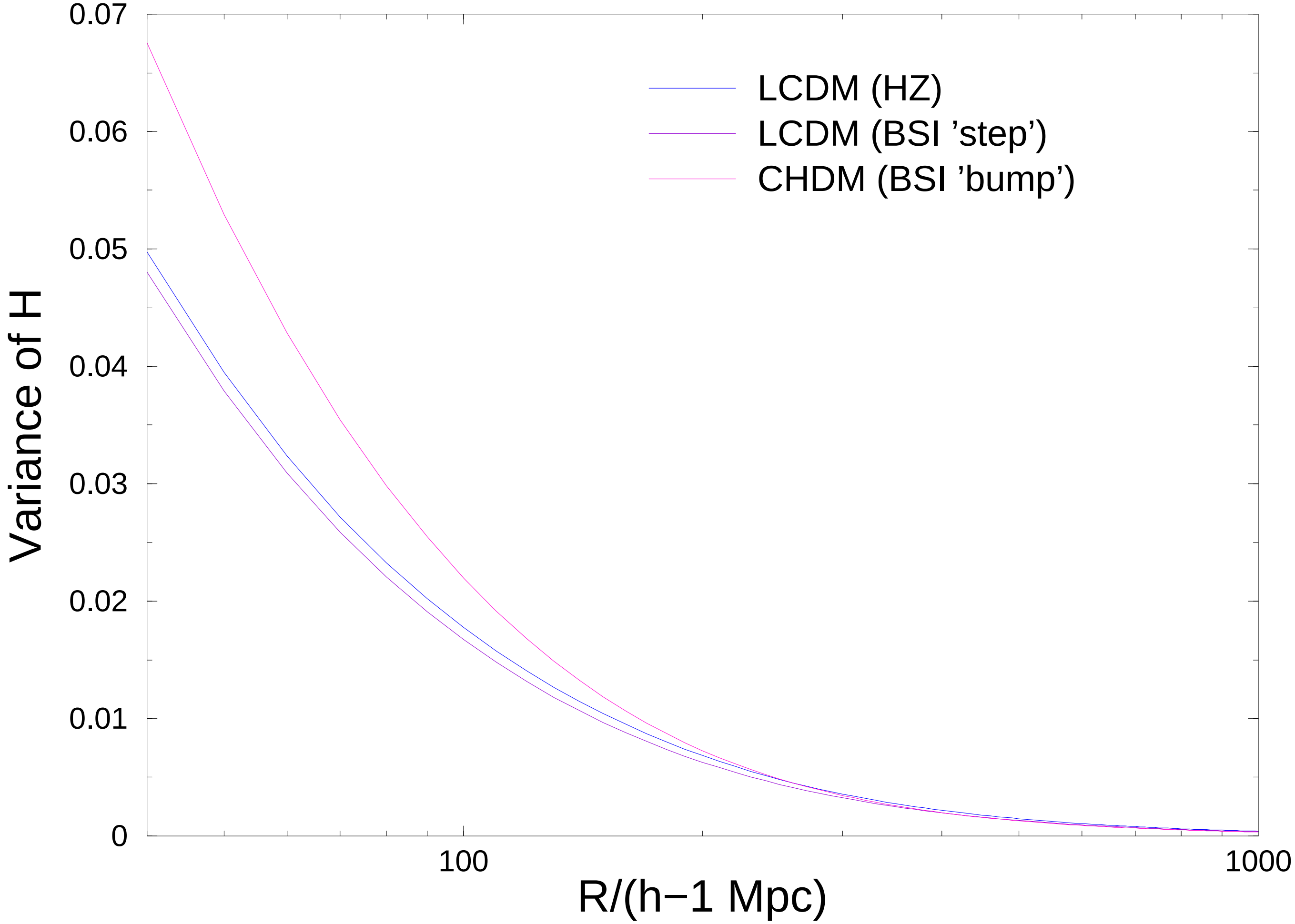
Optimised the covariance matrix of proposal distribution C_{T} using method of J. Dunkley et. al.:

1. Guess covariance matrix of underlying distribution C.
2. Set $C_{\mathrm{T}} = \left(2.4^2/D\right) C$ and run chain. Value of 2.4 found empirically to produce best results.
3. Use new chain to refine estimate of C.
4. Repeat steps 2. and 3. until chain converges.

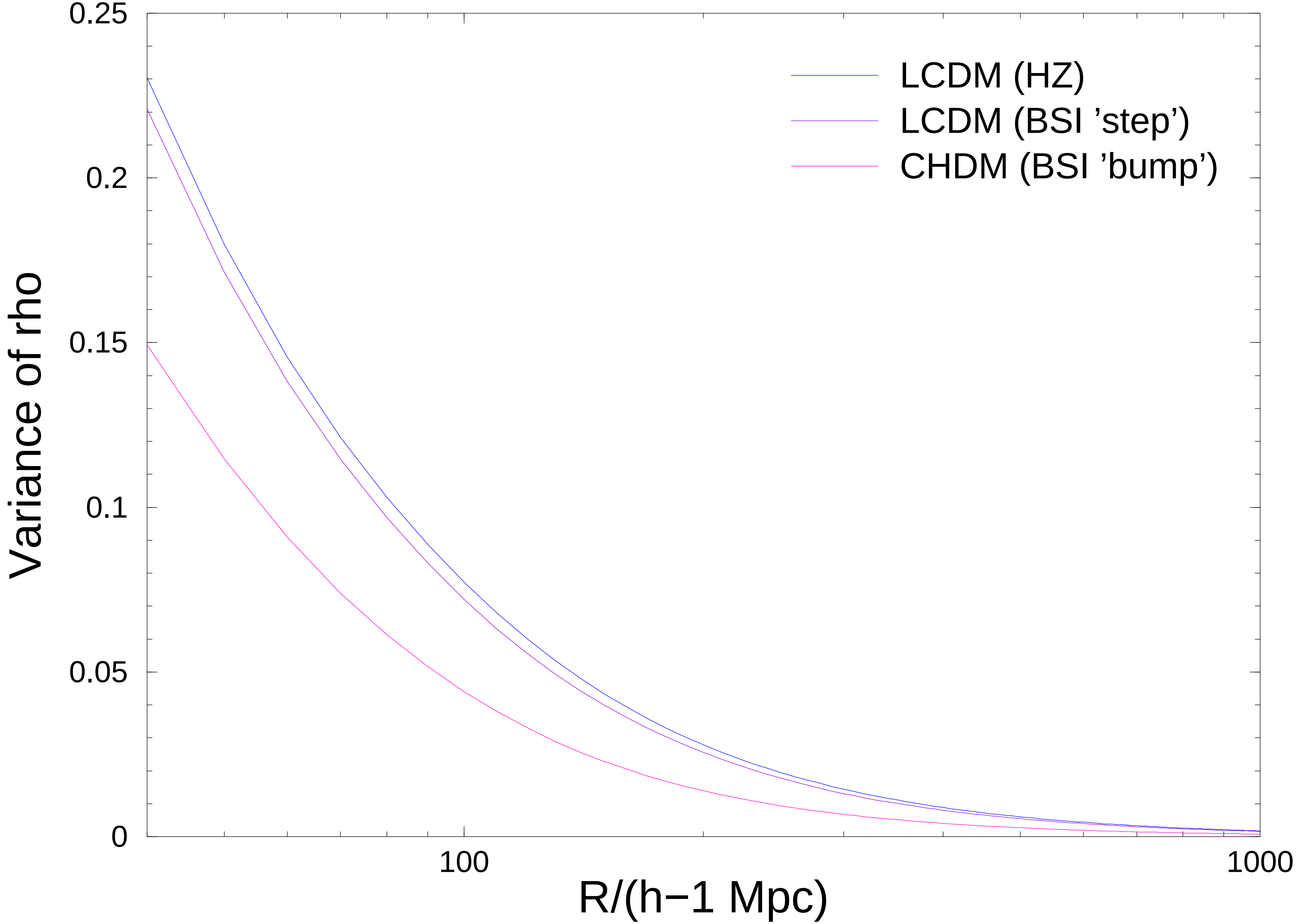
In practice 2 updates of C were necessary.

Tested convergence using spectral method of J. Dunkley *et. al.*


Power spectrum for amplitude parameter
Not converged
Converged
P(k)
k
10−1
10−2
10−3
10−4
10−5
10−6
10−7
10−8
1
10
100
1000
10000



0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Variance of H
100
1000
R/(h−1 Mpc)
LCDM (HZ)
LCDM (BSI 'step')
CHDM (BSI 'bump')


LCDM (HZ)
LCDM (BSI 'step')
CHDM (BSI 'bump')
Variance of rho
R/(h−1 Mpc)
0.25
0.2
0.15
0.1
0.05
0
100
1000