

On Minimal Models with Light Sterile Neutrinos

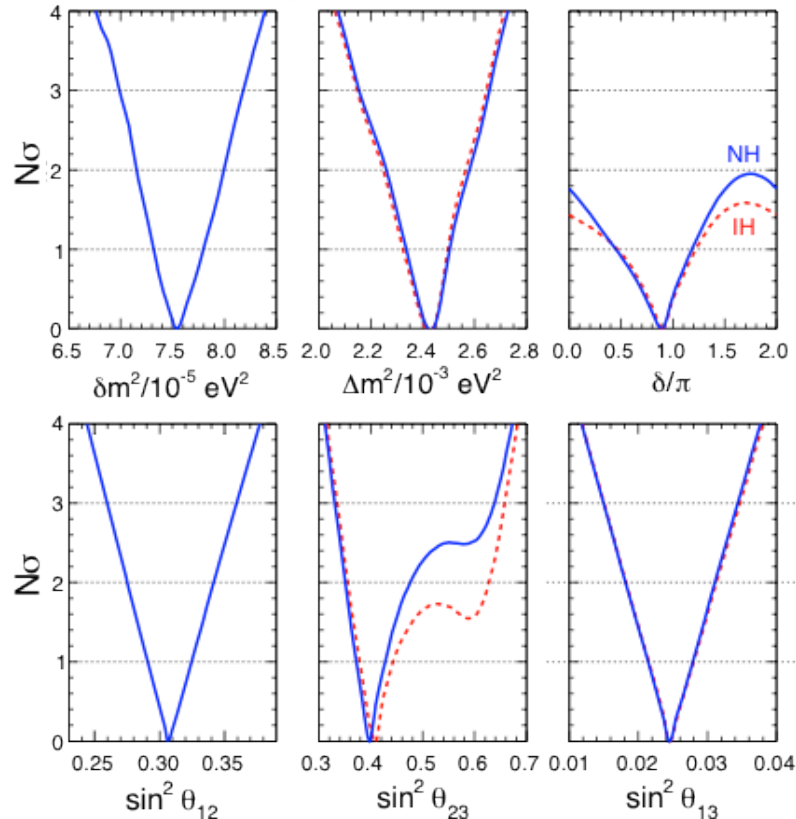
Pilar Hernández
University of Valencia/IFIC

Donini, López-Pavón, PH, Maltoni [arXiv:1106.0064](https://arxiv.org/abs/1106.0064)

Donini, López-Pavón, PH, Maltoni, Schwetz [arXiv:1205.5230](https://arxiv.org/abs/1205.5230)

SM + massive ν_s

Synopsis of global 3 ν oscillation analysis



Fogli et al 2012

(after T2K,

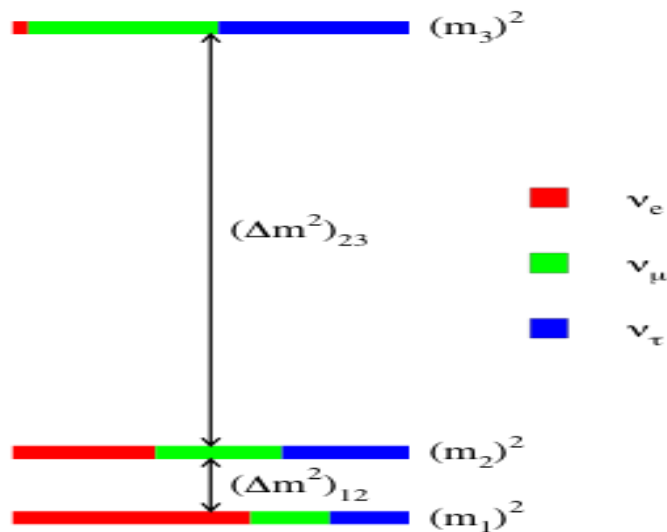
Double-CHOOZ, Daya Bay, RENO)

3 ν mixing:

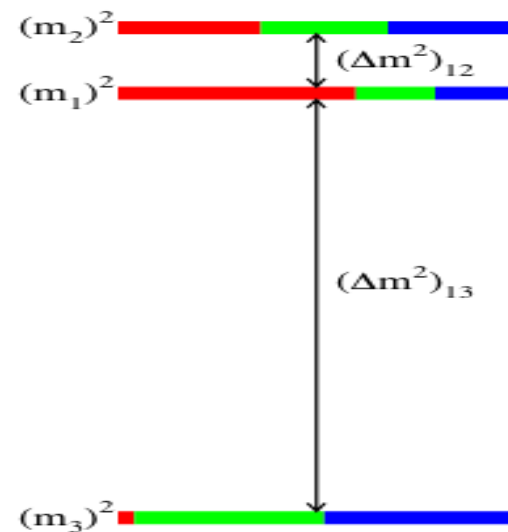
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS}(\theta_{12}, \theta_{23}, \theta_{13}, \delta, \dots) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Standard 3ν scenario

normal hierarchy



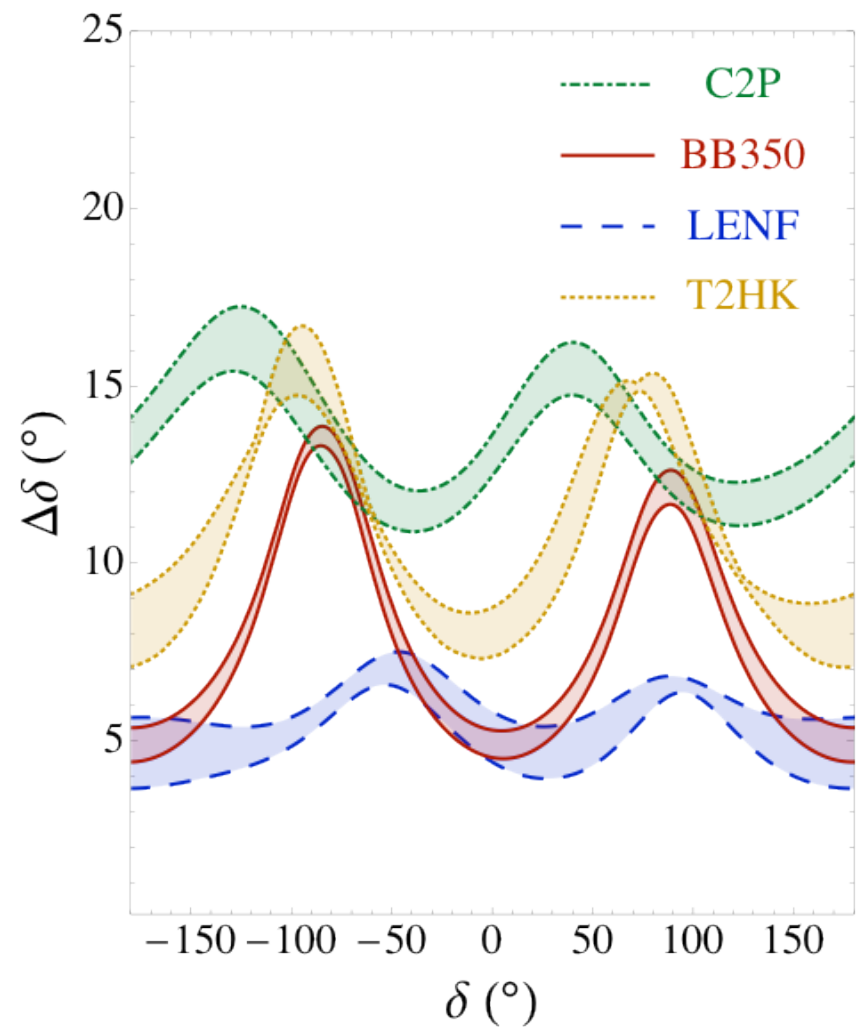
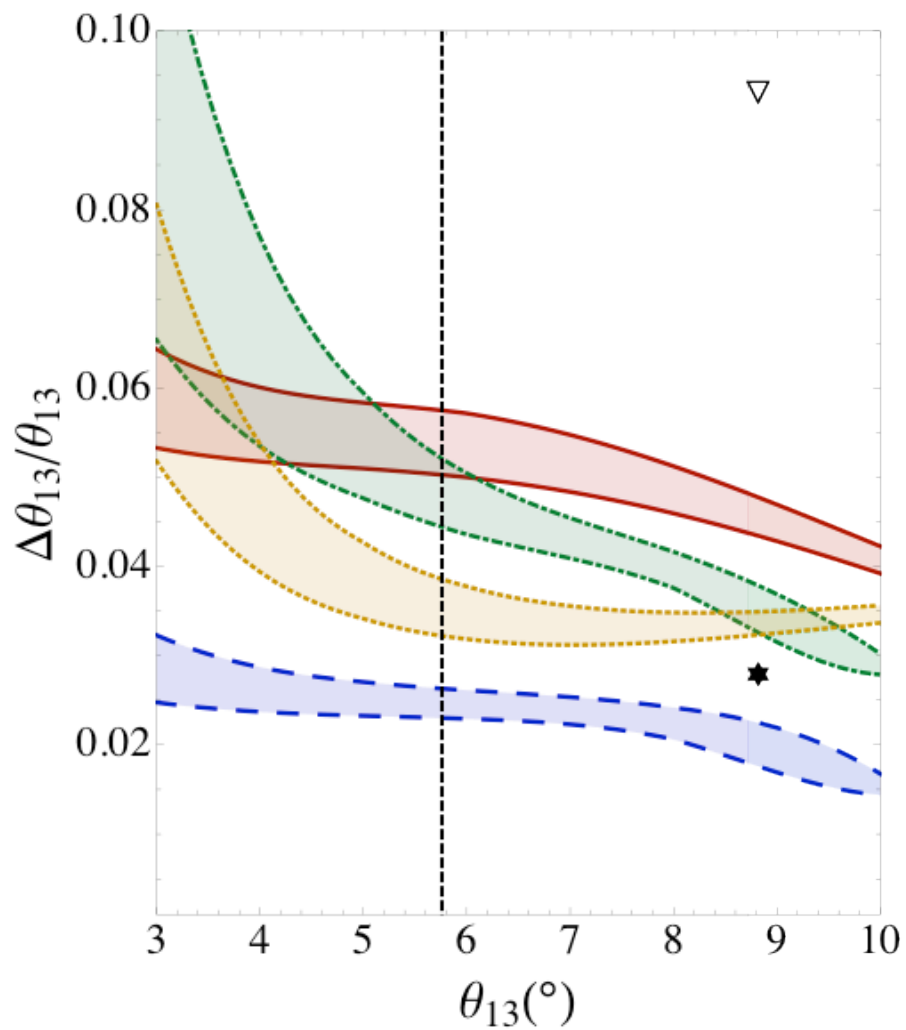
inverted hierarchy



The flavour observables:

Masses	Angles	CP-phases
$m_1^2 < m_2^2, m_3^2$	$\theta_{12}, \theta_{23}, \theta_{13}$	$\delta, \alpha_1, \alpha_2$

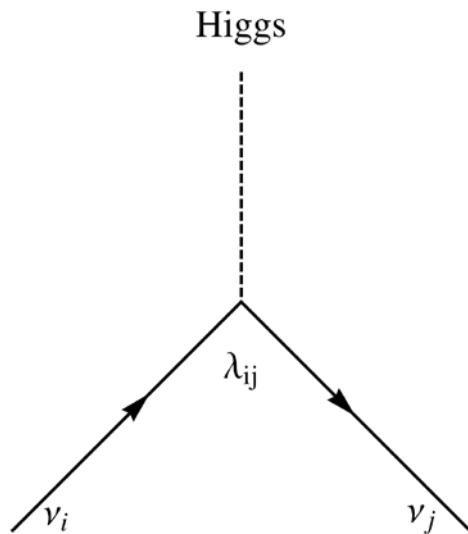
Good prospects for CP violation



New dofs needed !

Neutrinos are massive -> **there must be new dofs in the SM**

$$-\mathcal{L}_{\text{Dirac}} = \bar{\nu}_L m_\nu \nu_R + h.c. \Leftrightarrow \bar{L} \tilde{\Phi} \lambda \nu_R + h.c.$$



$$m_\nu \sim \lambda v$$

New dofs needed

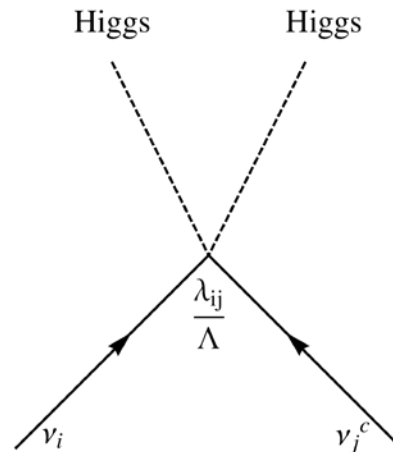
Neutrinos are massive \rightarrow there must be new dofs in the SM

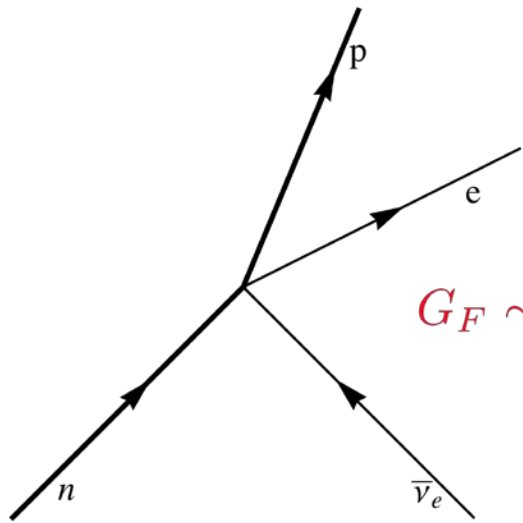
$$-\mathcal{L}_{\text{Majorana}} = \bar{\nu}_L m_\nu \nu_L^c + h.c. \leftrightarrow \bar{L} \tilde{\Phi} \alpha \tilde{\Phi} L^c + h.c.$$

Weinberg

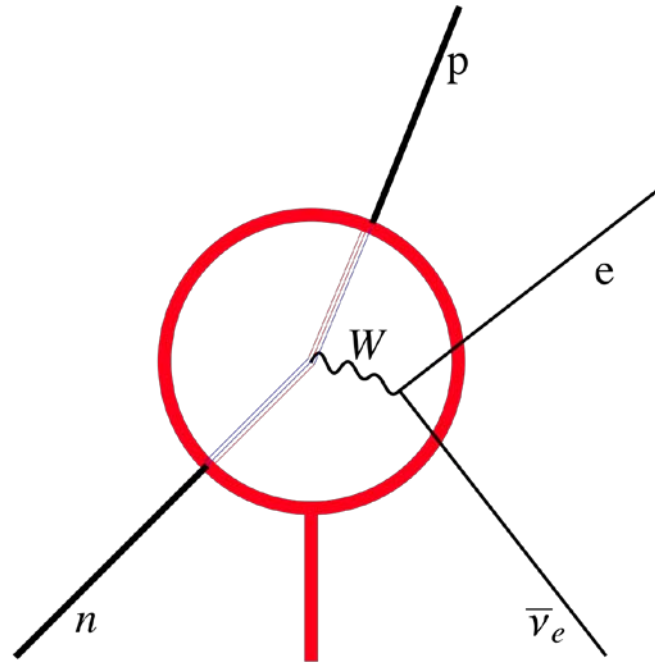
$$[\alpha] = -1$$

$$m_\nu \sim \lambda \frac{v^2}{\Lambda}$$

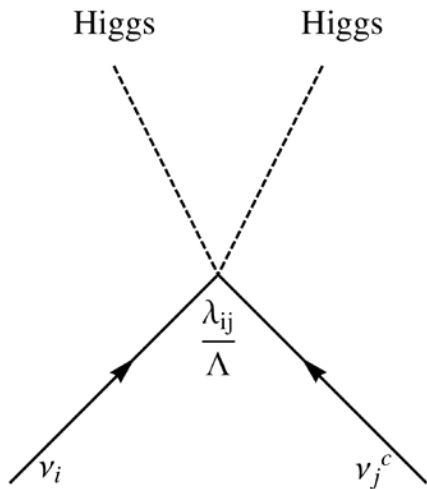




$$G_F \sim \frac{1}{M_W^2}$$



SM



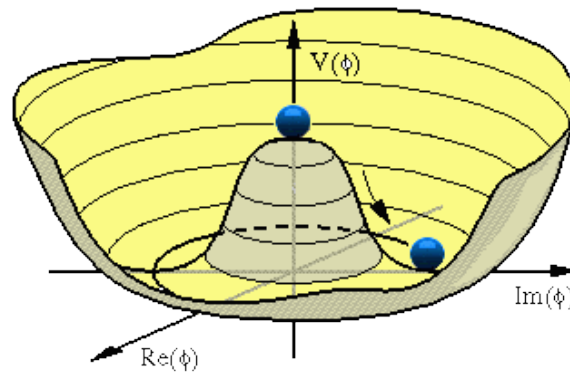
νSM ?

The good ν SM

- How does the ν scale relates to the EW scale ?

The good ν SM

- New scale versus EWSB ?

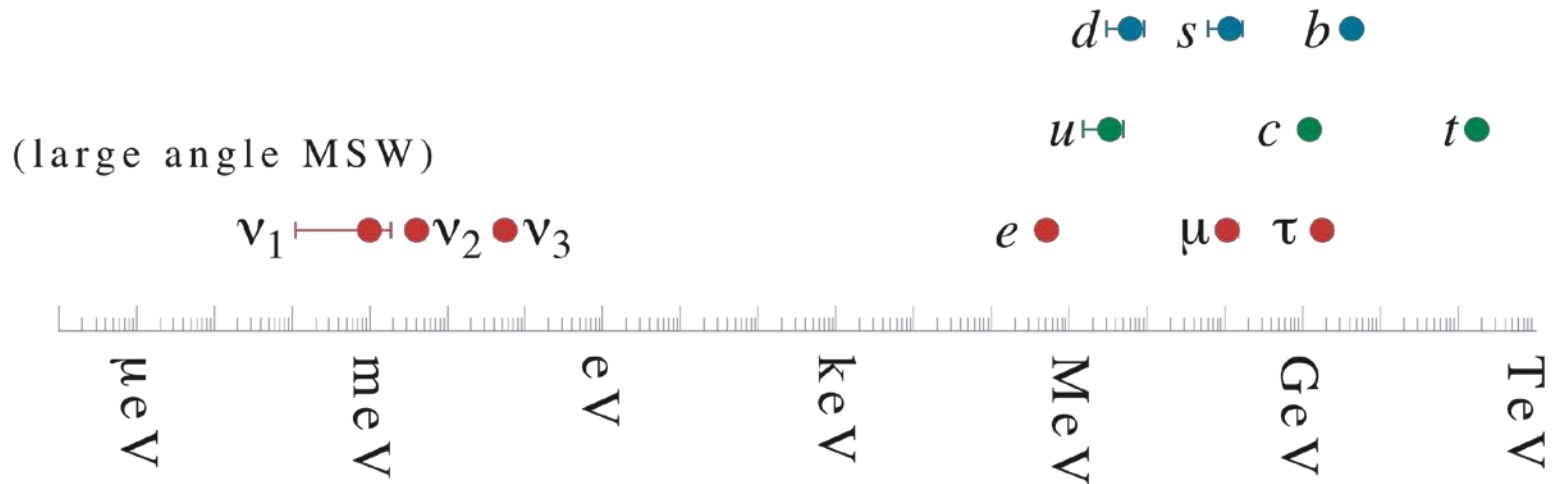


Besides the consistency of SM, now the Higgs...where does the new scale fit in this picture ?

The good ν SM

- New scale versus EWSB ?
- Explanation for the neutrino-charged lepton hierarchy

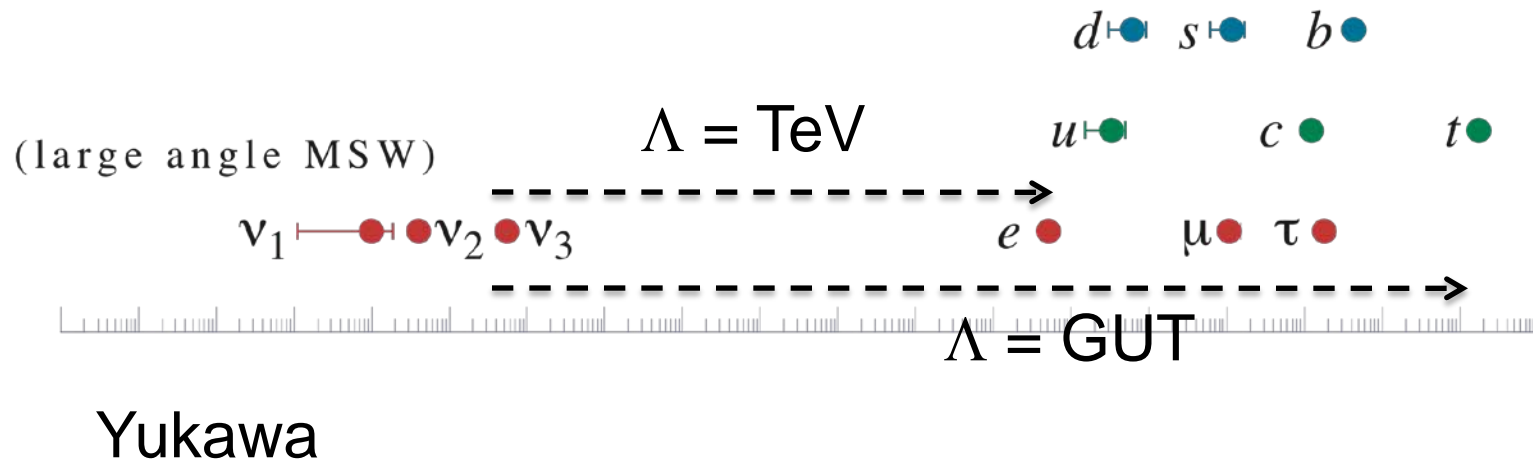
The ν flavour puzzle I



Why are neutrino masses so light ?

Seesaw of type I

Minkowski; Gell-Mann, Ramond
Slansky; Yanagida, Glashow...



The good ν SM

- New scale versus EWSB ?
- Explanation for the neutrino-charged lepton hierarchy
- Explain difference lepton/quark mixing

CKM

The ν flavour puzzle II

$V_{\text{CKM}} =$

$$\begin{pmatrix} 0.97383(24) & 0.2272(10) & 3.96(9) \times 10^{-3} \\ 0.2271(10) & 0.97296(24) & 42.21_{-0.80}^{+0.10} \times 10^{-3} \\ 8.14_{-0.64}^{+0.32} \times 10^{-3} & 41.61_{-0.78}^{+0.12} \times 10^{-3} & 0.999100_{-0.000004}^{+0.000034} \end{pmatrix}$$

PDG 2007

PMNS

$$|U|_{3\sigma} = \begin{pmatrix} 0.77 \rightarrow 0.86 & 0.50 \rightarrow 0.63 & 0.0 \rightarrow 0.22 \\ 0.22 \rightarrow 0.56 & 0.44 \rightarrow 0.73 & 0.57 \rightarrow 0.80 \\ 0.21 \rightarrow 0.55 & 0.40 \rightarrow 0.71 & 0.59 \rightarrow 0.82 \end{pmatrix}$$

Gonzalez-Garcia, Maltoni

Differences are striking!

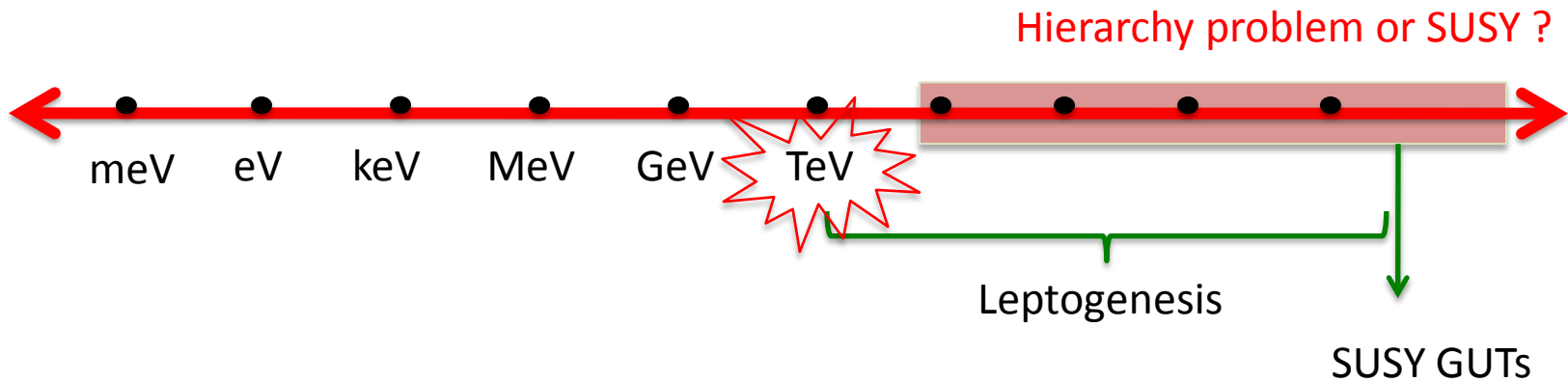
The good ν SM

- New scale versus EWSB ?
- Explanation for the neutrino-charged lepton hierarchy
- Explain the difference lepton/quark mixing (probably a very relevant question is how many ν dofs)
- Explain other open problems: DM, matter-antimatter, oscillation anomalies, cosmology anomalies...

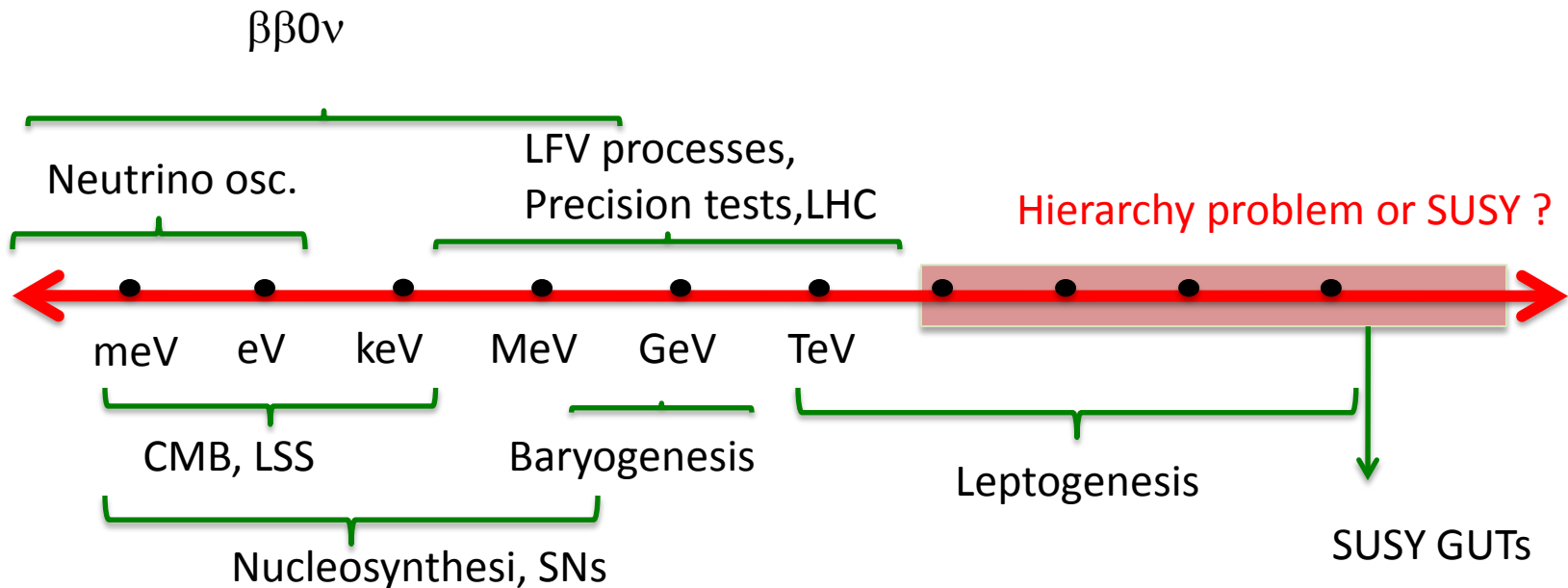
The good ν SM

- New scale versus EWSB ?
- Explanation for the neutrino-charged lepton hierarchy
- Explain the difference lepton/quark mixing
- Explain other open problems: DM, matter-antimatter, oscillation anomalies, cosmology anomalies...
- Do so, in a predictable and testable way !

Pinning down the New physics scale

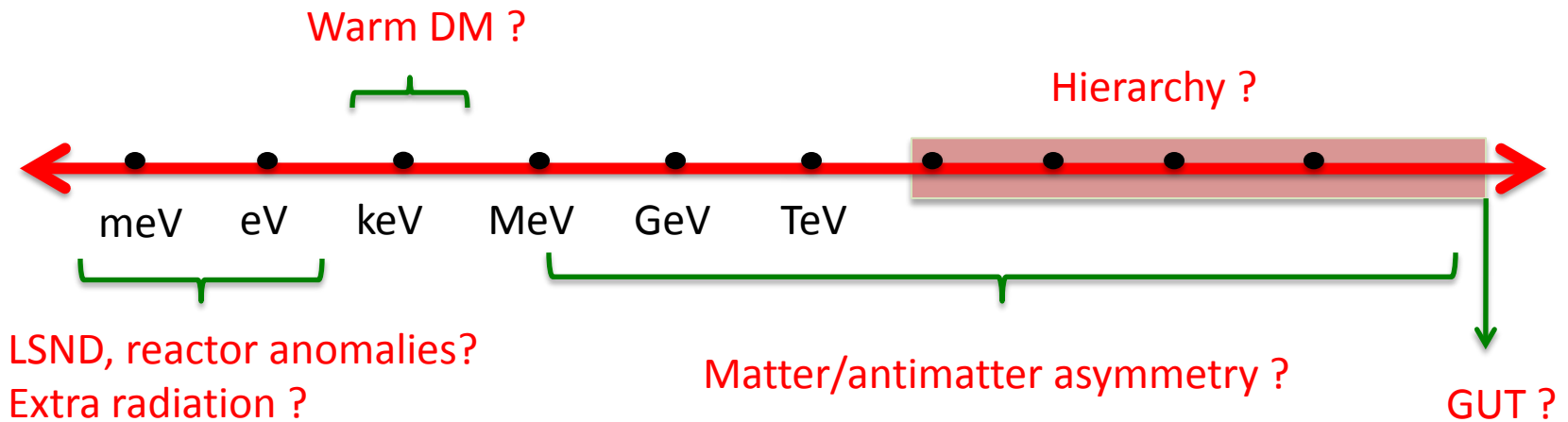


Pinning down the New physics scale



Light Sterile Neutrinos White Paper, Abazajian et al arXiv: 1204.5379 and refs. therein

Other uses of the New physics scale(s)



New dofs might help resolve open problems...or be excluded by observations

Outlier I: LSND anomaly

LSND vs KARMEN

$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

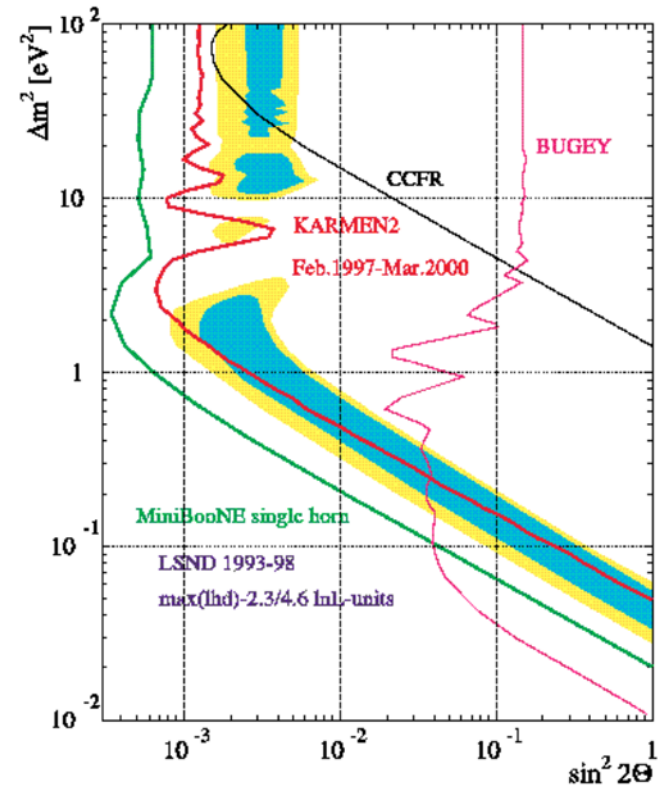
$$\nu_\mu \rightarrow \nu_e \text{ DIF } (28 \pm 6 / 10 \pm 2)$$

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e \text{ DAR } (64 \pm 18 / 12 \pm 3)$$

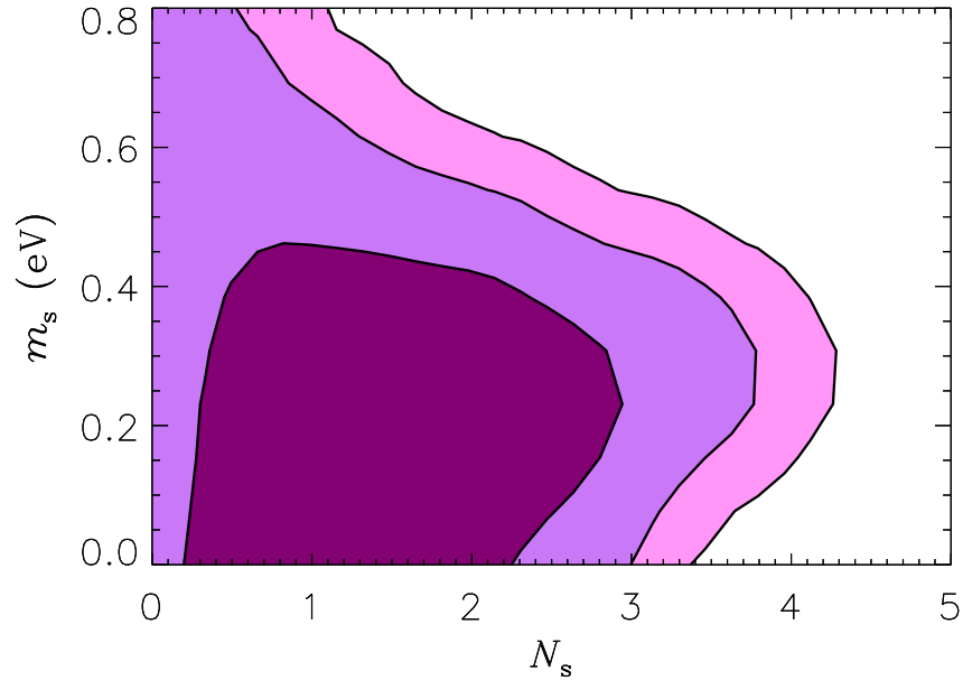
Appearance signal with very different

$$|\Delta m^2| \gg |\Delta m_{atm}^2|$$



Not yet disproved at an acceptable level of confidence...

Outlier II: Cosmology



Hamann et al, ArXiv: 1006.5276

Sterile species favoured by [LSS](#) and [CMB](#)

Nucleosynthesis:

$$N_s = 0.68^{+0.80}_{-0.70}$$

Izotov, Thuan

LSND anomaly

In order to accommodate a new $|\Delta m_{LSND}^2| \simeq \mathcal{O}(1eV)$

- Need at least four ($n_s \geq 1$) distinct eigenstates
- Apparently CP violating effect needed
(signal LSND/MB anti- ν not MB ν) $n_s \geq 2$
(seems not the case with new MiniBOONE data...)
- Tension appearance (signal) and disappearance (no signal) ?
- Tension with cosmology ?

Oscillation terms associated with the larger mass splittings...

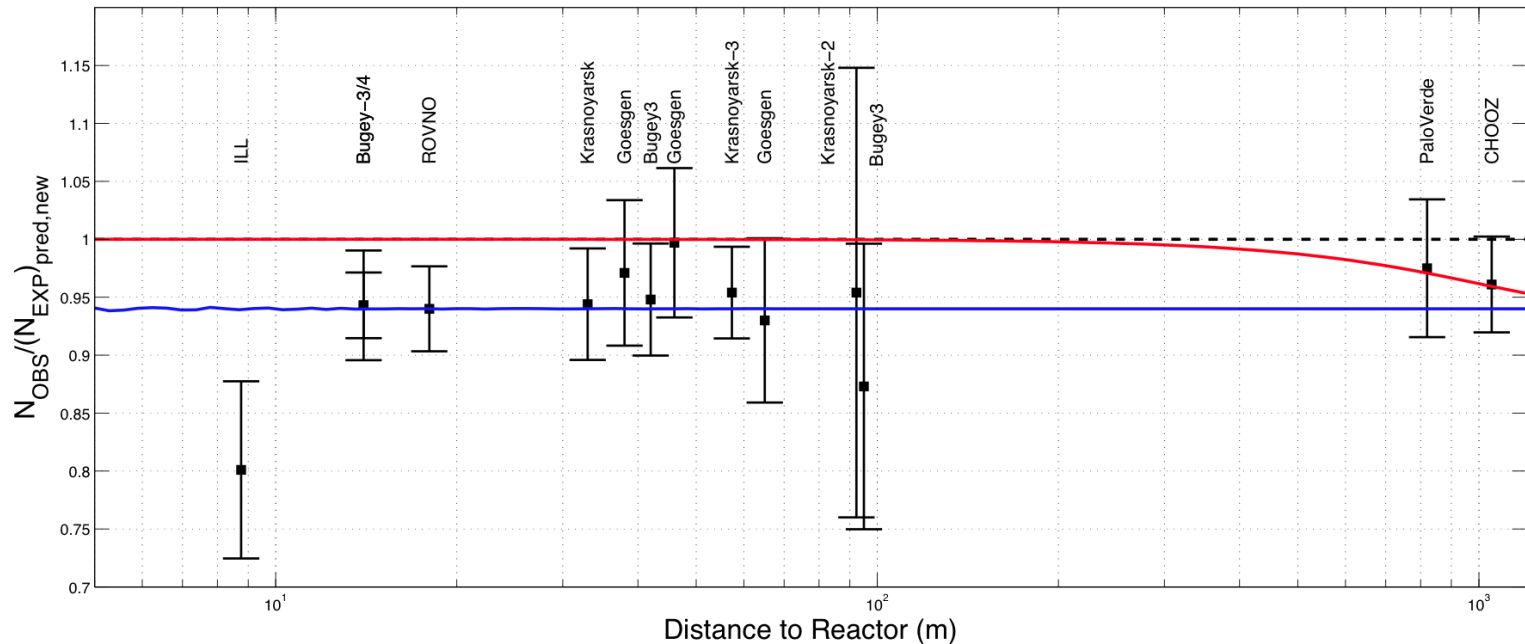
$$P(\nu_e \rightarrow \nu_\mu) = O(|U_{ei}|^2 |U_{\mu i}|^2)$$

$$P(\nu_\mu \rightarrow \nu_\mu) = O(|U_{\mu i}|^2)$$

$$P(\nu_e \rightarrow \nu_e) = O(|U_{ei}|^2)$$

A convincing signal would be to find it in all the three...

Outlier III: reactor anomaly



Re-calculation of reactor fluxes: old fluxes underestimated by 3%:

Mueller et al, ArXiv: 1101.2663

Still to be confirmed by the new reactor experiments !

3+2 neutrino mixing model

Parametrized in terms of a general unitary 5x5 mixing matrix
(9 angles, 5 phases physical)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \\ \nu'_s \end{pmatrix} = U_{5 \times 5} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \nu_5 \end{pmatrix}$$

	Δm_{41}^2	$ U_{e4} $	$ U_{\mu 4} $	Δm_{51}^2	$ U_{e5} $	$ U_{\mu 5} $	δ/π	χ^2/dof
3+2	0.47	0.128	0.165	0.87	0.138	0.148	1.64	110.1/130
1+3+1	0.47	0.129	0.154	0.87	0.142	0.163	0.35	106.1/130

	3+1	3+2
χ^2_{\min}	100.2	91.6
NDF	104	100
GoF	59%	71%
Δm_{41}^2 [eV ²]	0.89	0.90
$ U_{e4} ^2$	0.025	0.017
$ U_{\mu 4} ^2$	0.023	0.018
Δm_{51}^2 [eV ²]		1.60
$ U_{e5} ^2$		0.017
$ U_{\mu 5} ^2$		0.0064
η		1.52π
$\Delta\chi^2_{\text{PG}}$	24.1	22.2
NDF _{PG}	2	5
PGoF	6×10^{-6}	5×10^{-4}

Kopp, Maltoni, Schwetz (KMS) arXiv:1103.4570

Giunti, Laveder, (GL) arXiv:1107.1452

Significant improvement over 3ν scenario, but **tension appearance/disappearance remains**

What is this Pheno $3+n_s$ mixing model ?

Assumes a general mass matrix for $3+n_s$ neutrinos:

$$\left(\begin{array}{cc} \overbrace{M_{LL}}^{3 \times 3} & \overbrace{M_{LR}}^{3 \times n_s} \\ M_{LR}^T & M_{RR} \end{array} \right)$$

What is this Pheno $3+n_s$ mixing models ?

Assumes a general mass matrix for $3+n_s$ neutrinos:

Gauge invariance

$$\begin{pmatrix} \cancel{M_{LL}} & M_{LR} \\ M_{LR}^T & M_{RR} \end{pmatrix}$$



Effective theory: M_{LL} parametrizes our ignorance about the underlying dynamics

UV extension 1: a model with $n_R \geq 3+n_s$, where 3 heavier states are integrated out

UV extension 2: a model with $n_R = 3 + 2 n_s$ and an exact lepton number....

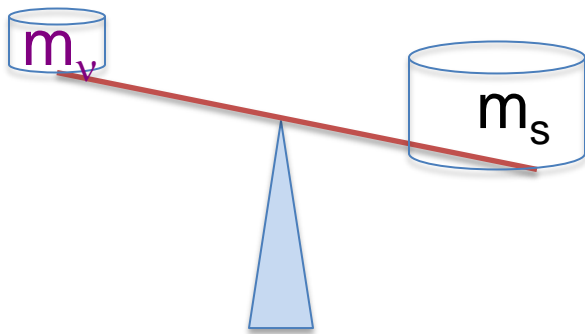
Type I Seesaw

Most general (renormalizable) Lagrangian compatible with SM gauge symmetries:

$$\mathcal{L} = \mathcal{L}_{SM} - \sum_{i=1}^{n_R} \bar{l}_L^\alpha Y^{\alpha i} \tilde{\Phi} \nu_R^i - \sum_{i,j=1}^{n_R} \frac{1}{2} \bar{\nu}_R^{ic} M_N^{ij} \nu_R^j + h.c.$$

$Y: 3 \times n_R$ $M_N: n_R \times n_R$

$$M_\nu = \begin{pmatrix} 0 & Yv \\ Yv & M_N \end{pmatrix}$$

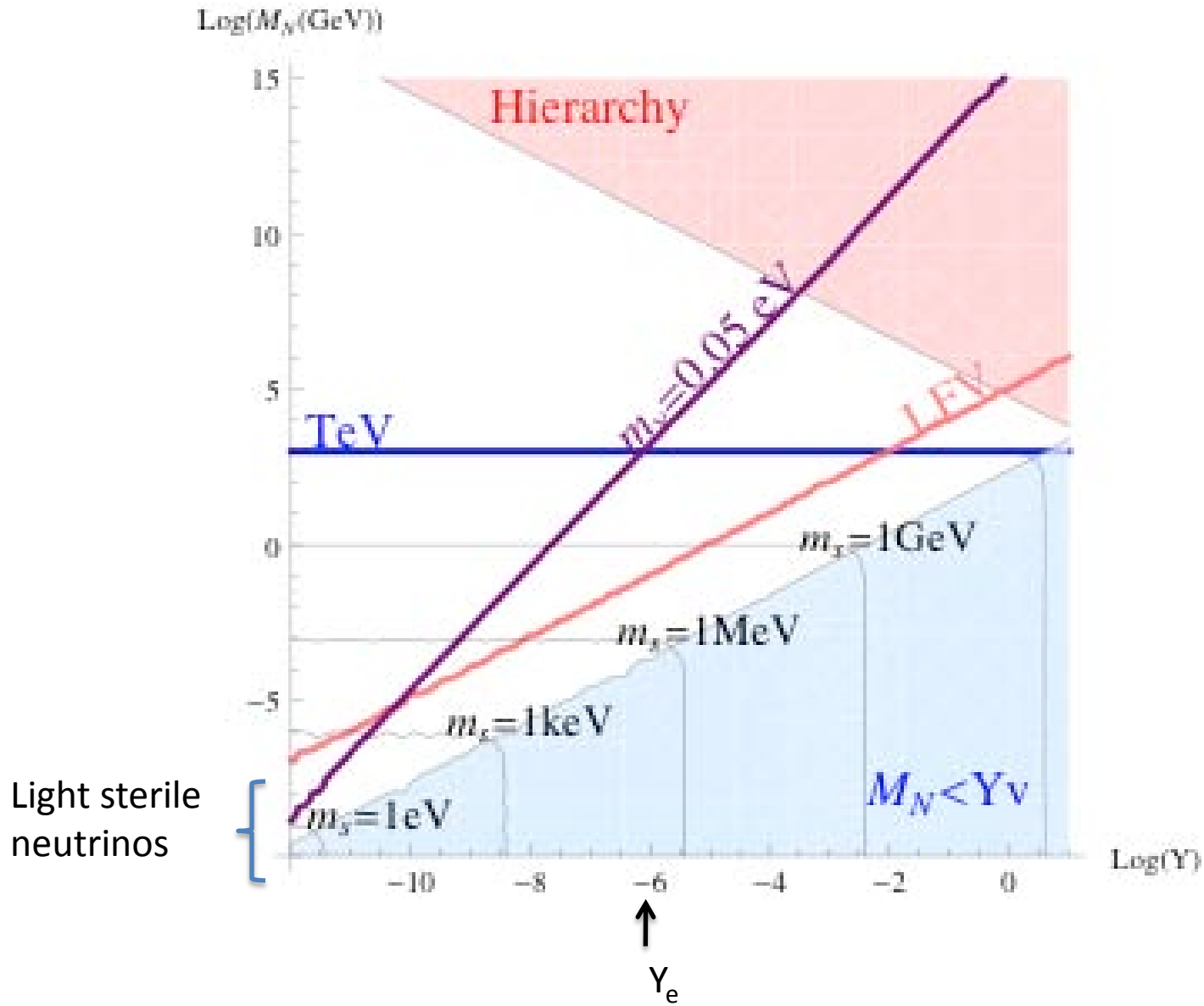


$$m_\nu = Y^T \frac{v^2}{M_N} Y$$

$$m_s \sim M_N$$

$$LFV \sim Y^\dagger \frac{v^2}{M_N^2} Y$$

One scale see-saw models



3+n_R Minimal Models

$$\mathcal{L} = \mathcal{L}_{SM} - \sum_{i=1}^{n_R} \bar{l}_L^\alpha Y^{\alpha i} \tilde{\Phi} \nu_R^i - \sum_{i,j=1}^{n_R} \frac{1}{2} \bar{\nu}_R^{ic} M_N^{ij} \nu_R^j + h.c.$$

3+2 Minimal Model much more predictive than 3+2 Phenomenological Model

Model	# Δm^2	# Angles	# Phases
3 ν	2	3	1 -3
3+2 MM	4	4	3
3+2 PM	4	9	5

On parametrizations

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_N \end{pmatrix}$$

- Independent (physical) parameters only
- Convenient to impose existing constraints

$$\mathcal{M}_\nu = U^* \text{Diag}(0, m_2, m_3, M_1, M_2) U^T$$

$$U = \begin{pmatrix} U_{aa} & U_{as} \\ U_{sa} & U_{ss} \end{pmatrix},$$

Casas-Ibarra parametrization

For $m_i \ll M_j$

$$U_{aa} \simeq U_{\text{PMNS}} \quad U_{as} \simeq iU_{\text{PMNS}} \begin{pmatrix} 0 \\ m_l^{1/2} R^\dagger M_h^{-1/2} \end{pmatrix}$$

$$U_{\text{PMNS}}(\theta_{12}, \theta_{13}, \theta_{23}, \delta, \alpha) \quad \text{Unitary}$$

$$R(\theta_{45}, \gamma_{45}) = \begin{pmatrix} \cos(\theta_{45} + i\gamma_{45}) & \sin(\theta_{45} + i\gamma_{45}) \\ -\sin(\theta_{45} + i\gamma_{45}) & \cos(\theta_{45} + i\gamma_{45}) \end{pmatrix} \quad \text{Complex orthogonal}$$

If $M \leq \text{O}(\text{eV})$ corrections are important !

Beyond Casas-Ibarra

More generally (extended Casas-Ibarra)

$$U_{aa} = U_{PMNS} \begin{pmatrix} 1 & 0 \\ 0 & H \end{pmatrix}, \quad U_{as} = iU_{PMNS} \begin{pmatrix} 0 \\ H m_l^{1/2} R^\dagger M_h^{-1/2} \end{pmatrix}$$

$$H^{-2} = I + m_l^{1/2} R^\dagger M_h^{-1} R m_l^{1/2}$$

Donini, et al 1205.5230

Incorporates the expected non-unitarity effects in the light sector

Alternative parametrization same philosophy

Blennow, Fernandez-Martinez arXiv:1107.3992

Heavy-Light Mixings

Heavy-light mixings are **predicted** up to a complex angle z_{45} and two CP phases !

Normal Hierarchy

Ex:
$$U_{e4} \simeq i \left(\sqrt{\frac{m_3}{M_1}} e^{i(\alpha-\delta)} s_{13} \sin z_{45} + \sqrt{\frac{m_2}{M_1}} c_{13} s_{12} \cos z_{45} \right)$$

Suppressed in $\sqrt{\frac{m_i}{M_j}}$ and θ_{reactor} OR $\sqrt{\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2}}$

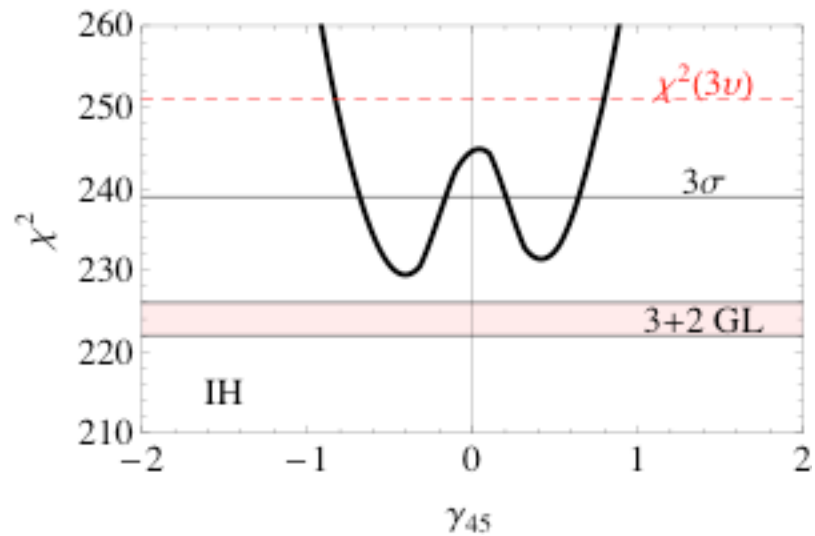
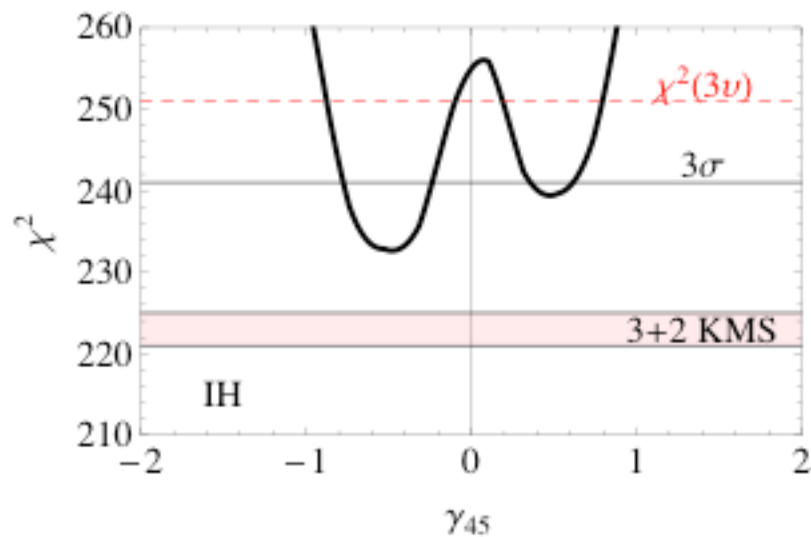
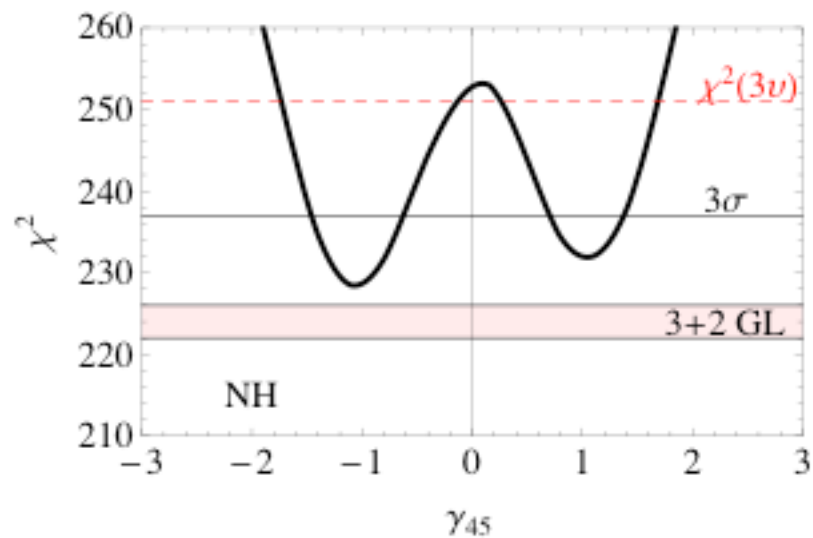
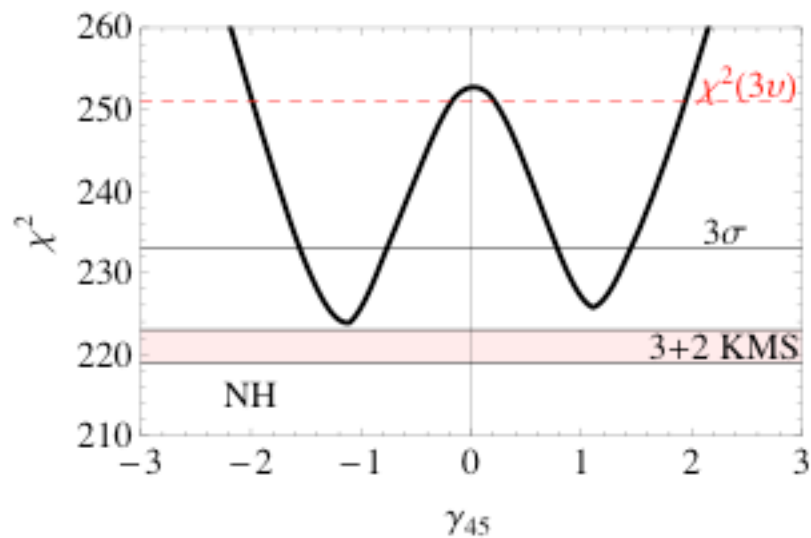
Inverse Hierarchy Suppressed only in $\sqrt{\frac{m_i}{M_j}}$ Right ballpark!

Donini, et al 1106.0064; de Gouvea, Huang 1110:6122; Fan, Langacker 1201:6662

Global fits

- Not possible to decouple LBL and SBL analyses: too large correlations in U_{aa} and U_{as}
- Need to include corrections to Casas-Ibarra-> more general parametrization
- Use M_1 , M_2 from KMS and GL fits

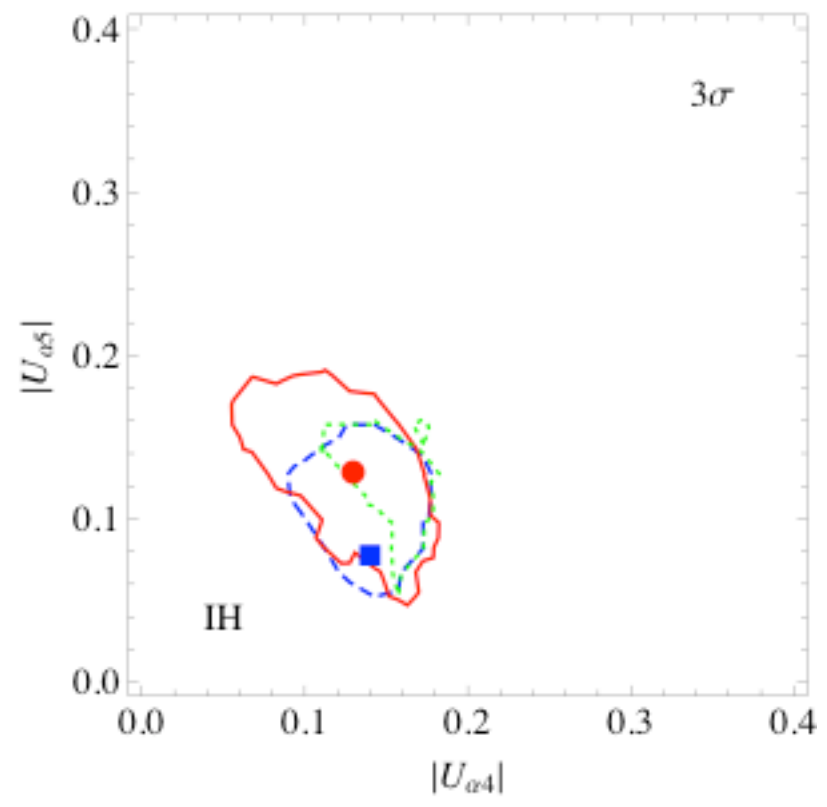
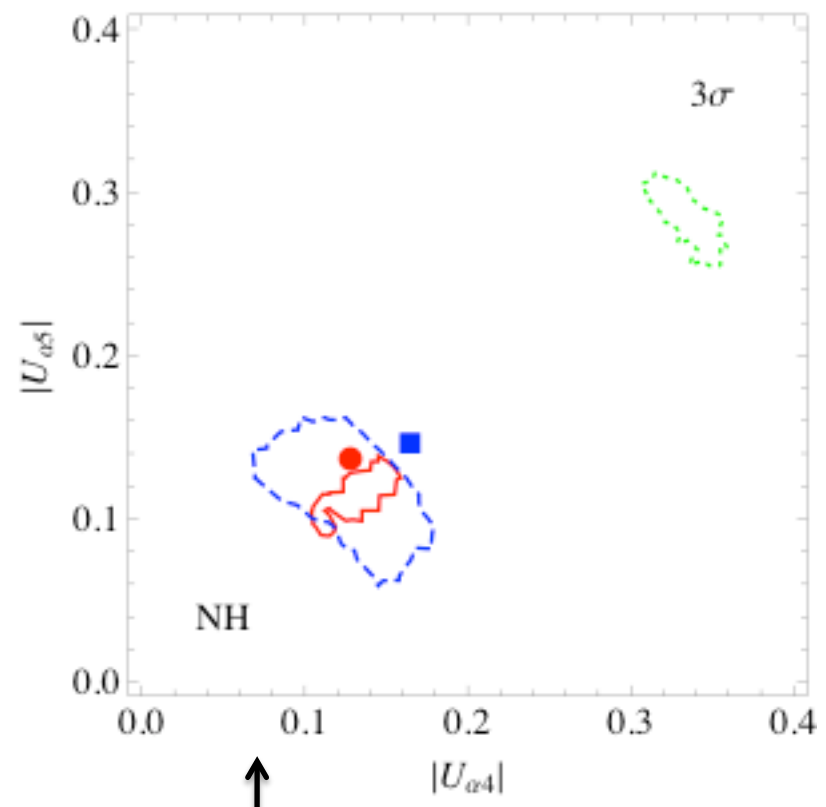
3+2 MM vs 3+2 PM vs 3 ν



$$z_{45} = \theta_{45} + i\gamma_{45}$$

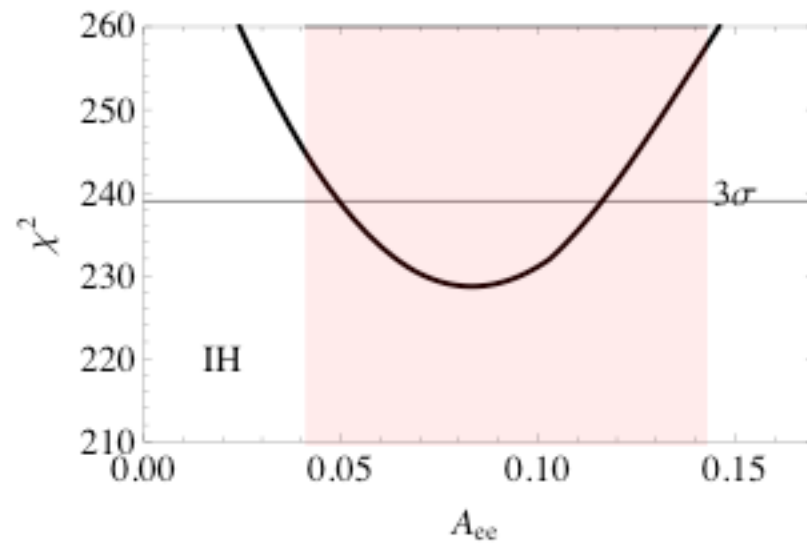
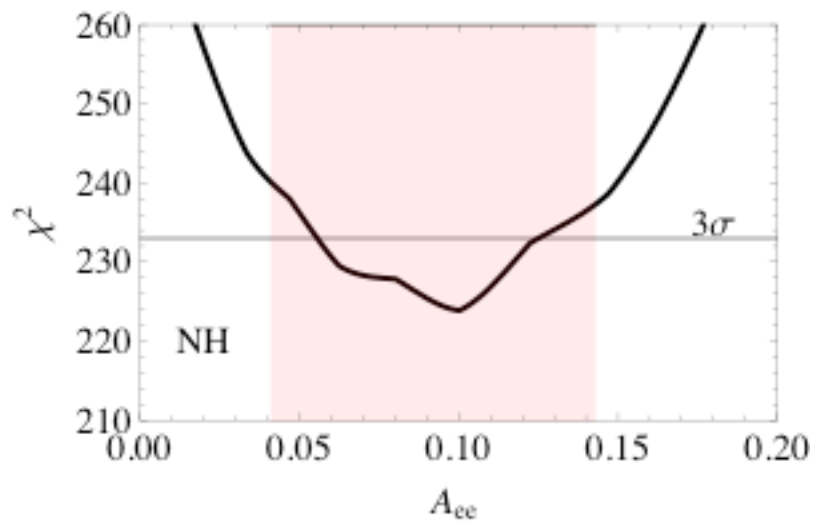
Heavy-Light Mixings

e μ τ

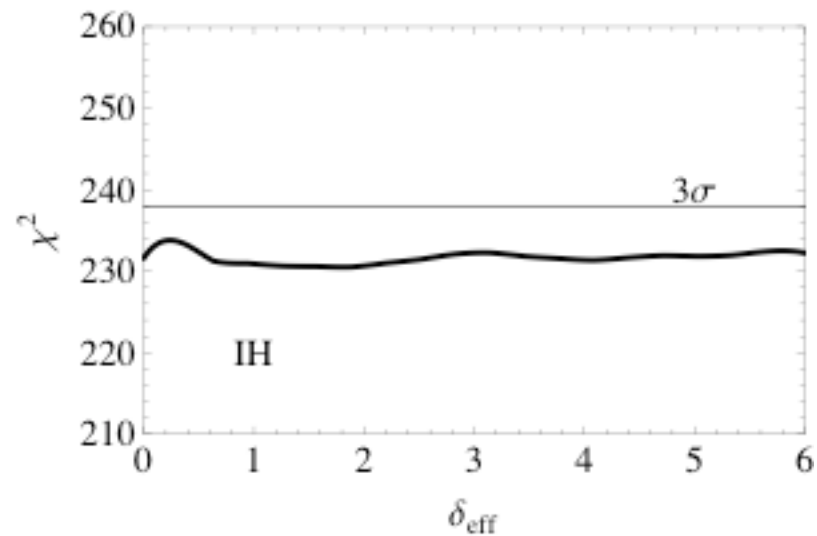
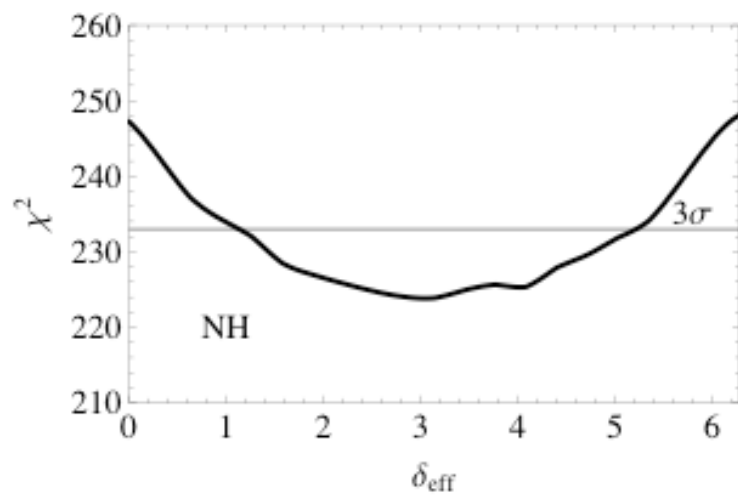


↑
Large tau mixings to heavy states

Constraint on $\sin^2 2\theta_{\text{reactor}}$



Constraint on δ_{eff}

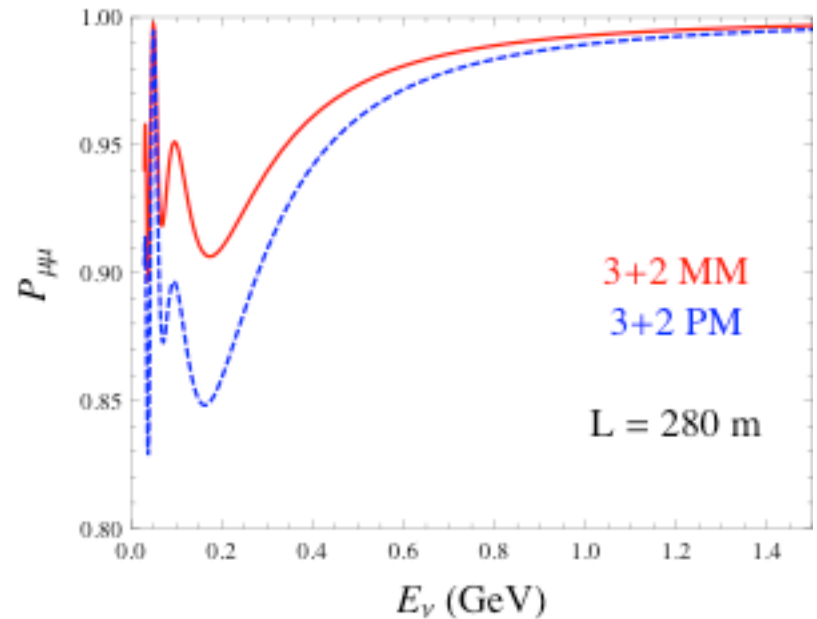
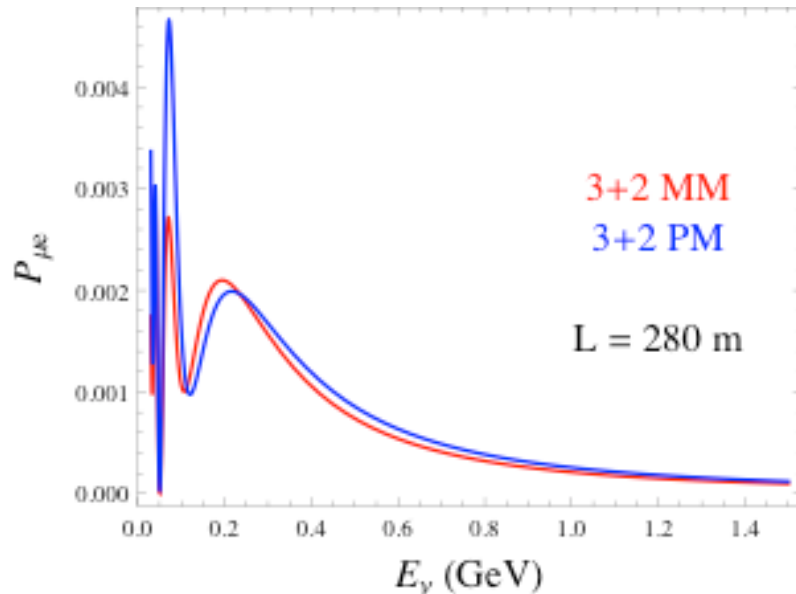


BUT....

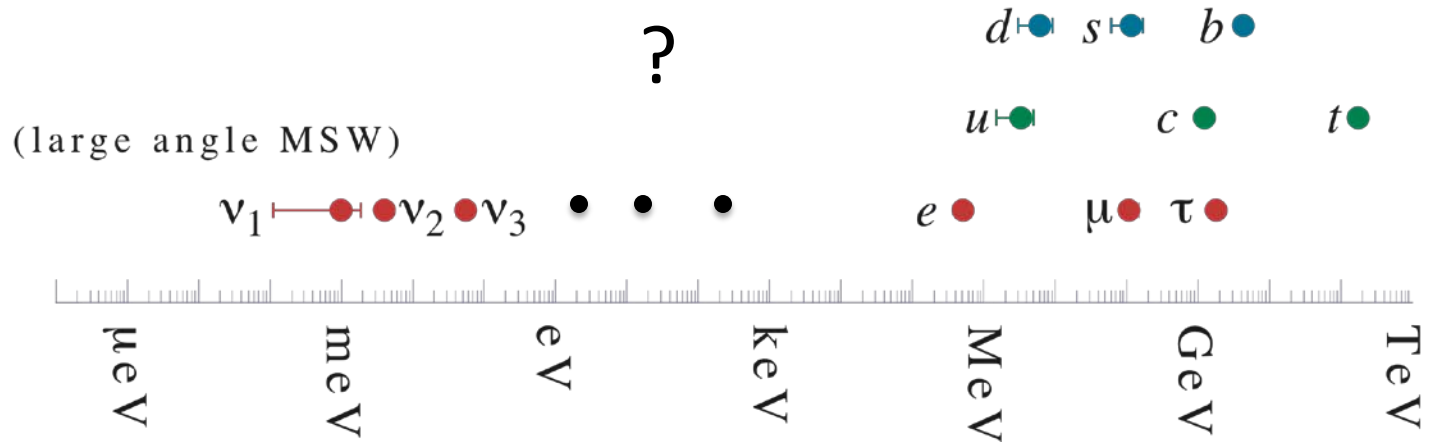
Significant improvement over 3ν scenario, but large tension appearance/disappearance in μ sector remains (MINOS CC high energy)

Tension with cosmology that favours extra but lighter states

Should be easy to clarify...



Even if LSND not correct we should clarify whether the picture does not look like ...



Hierarchy ?

Minimal models

Most general (renormalizable) Lagrangian compatible with SM gauge symmetries:

$$\mathcal{L} = \mathcal{L}_{SM} - \sum_{i=1}^{n_R} \bar{l}_L^\alpha Y^{\alpha i} \tilde{\Phi} \nu_R^i - \sum_{i,j=1}^{n_R} \frac{1}{2} \bar{\nu}_R^{ic} M_N^{ij} \nu_R^j + h.c.$$

$Y: 3 \times n_R$

$M_N: n_R \times n_R$

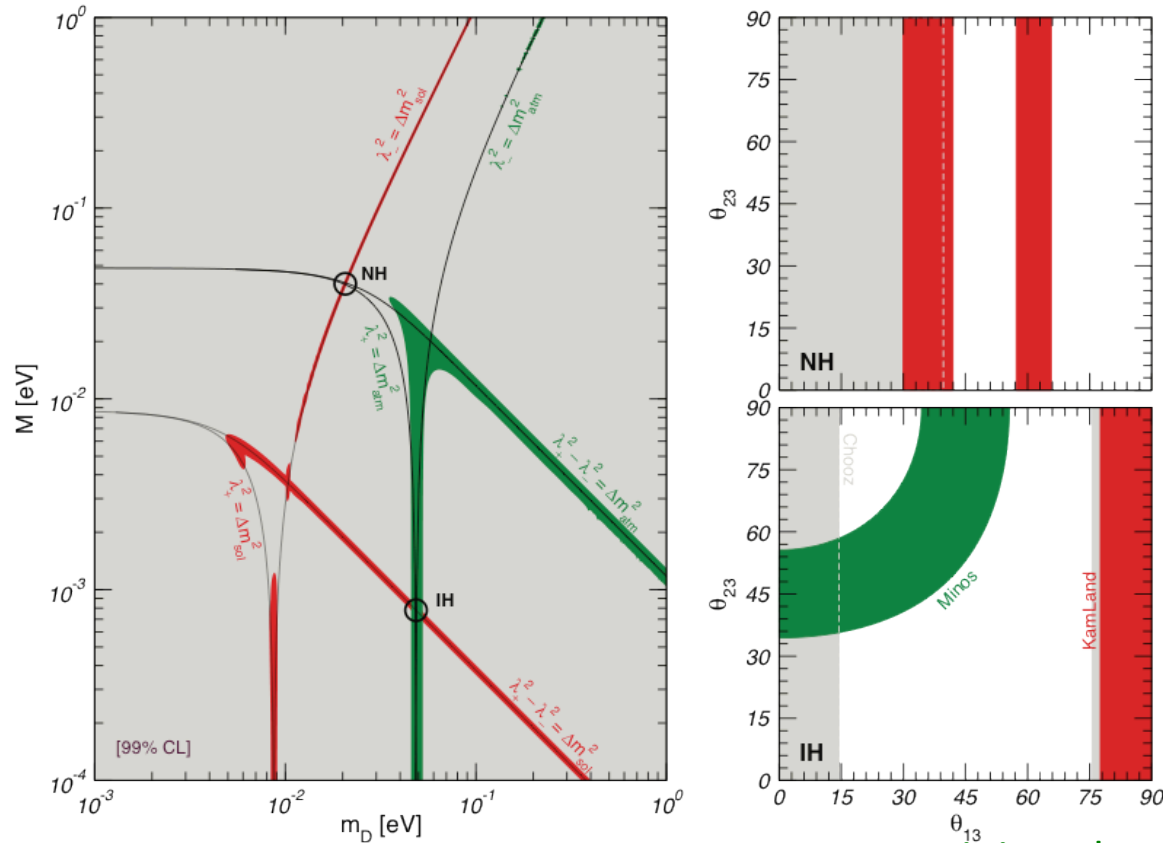
Number of Physical Parameters

n_R	L_i	# zero modes	# masses	# angles	# CP phases	
1	-	2	2	2	0	→ 3+1 minimal
	+1	2	1	2	0	→ 1 Dirac
2	-	1	4	4	3	→ 3+2 minimal
	(+1,+1)	1	2	3	1	→ 2 Dirac
	(+1,-1)	3	1	3	1	
3	-	0	6	6	6	
	(+1,+1,+1)	0	3	3	1	→ 3 Dirac
	(+1,-1,+1)	2	2	6	4	
	(+1,-1,-1)	4	1	4	1	

Complexity ↓
↑ predictivity

Minimal 3+1

Two massless +two massive eigenstates, only two physical angles, no CP violation

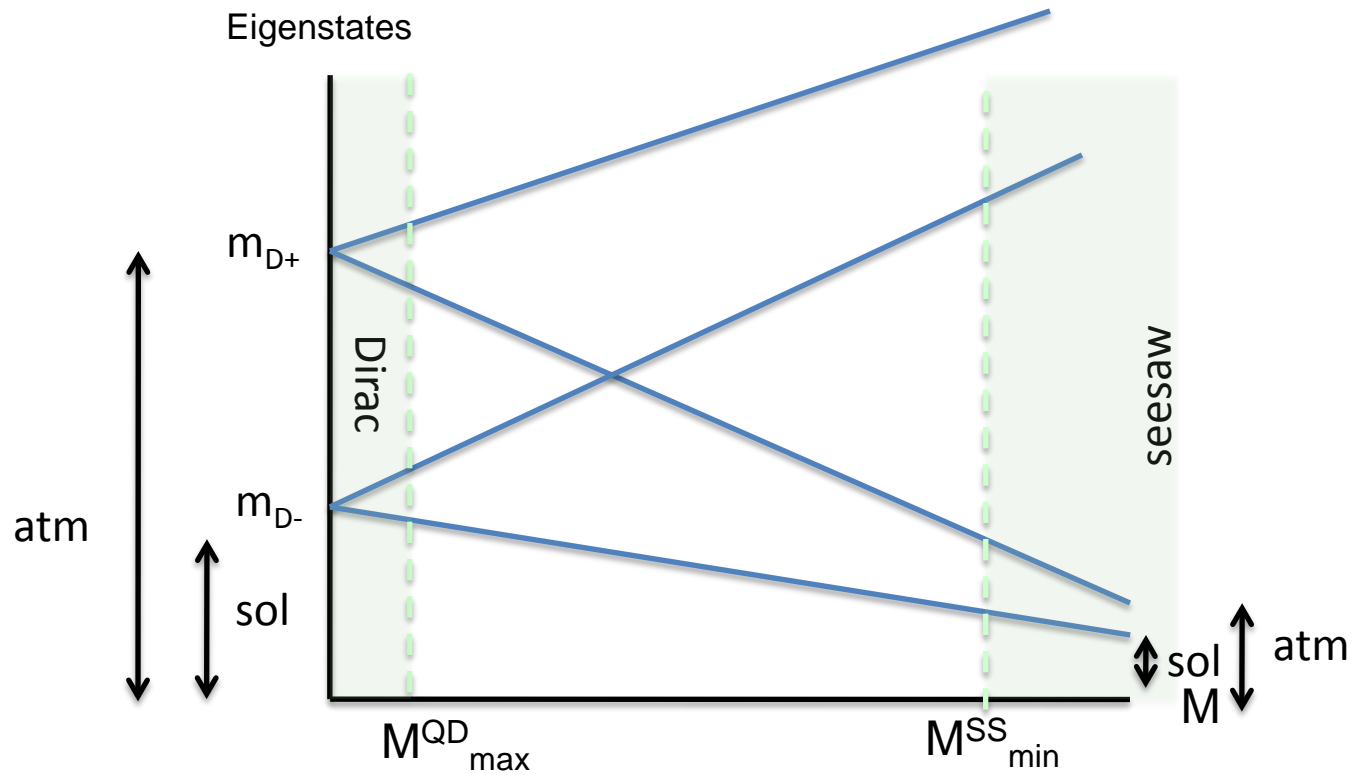


Donini et al 1106.0064

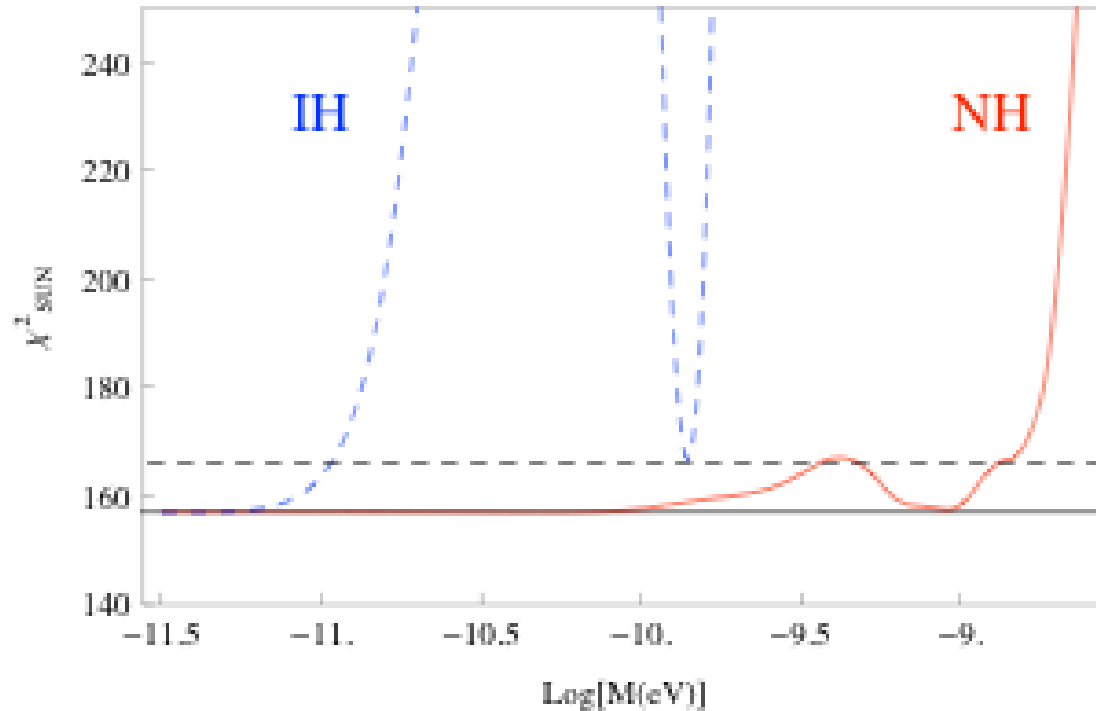
Strong incompatibility between Chooz+KamLAND vs Chooz+MINOS

Minimal 3+2

Degenerate case: $M_1 = M_2 = M$, 3 angles, no CP violation



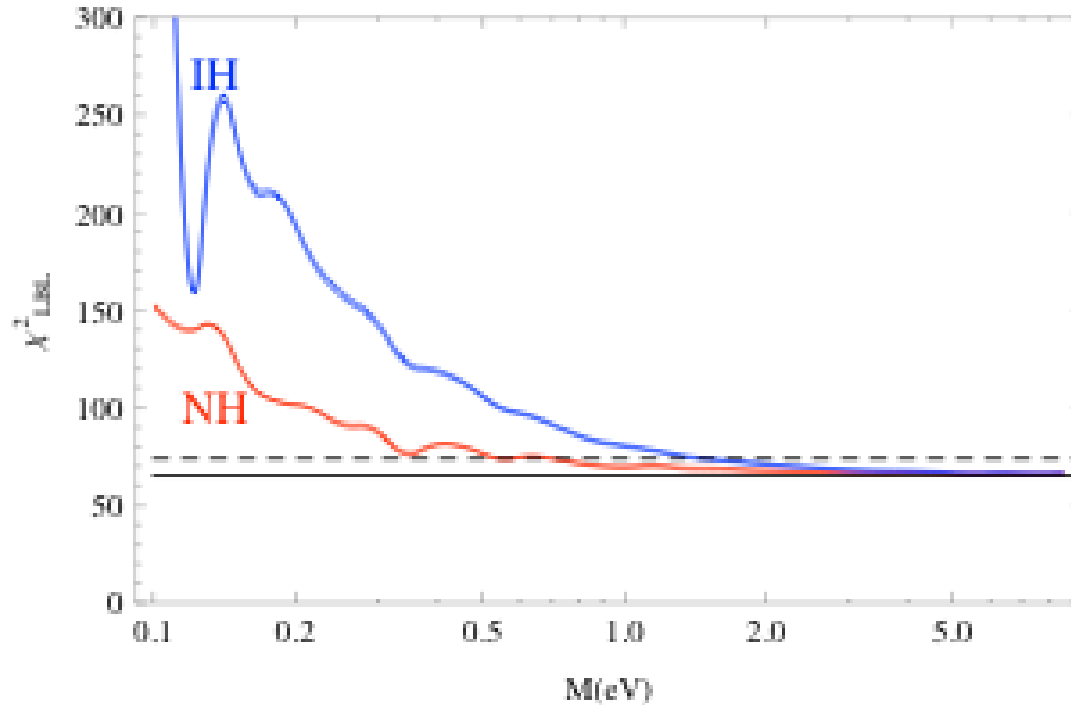
SOLAR data: $M_{\text{max}}^{\text{QD}}$



Impressive sensitivity of solar neutrinos to tiny departures from diracness!

See also [De Gouvea, Huang, Jenkins arXiv: 0906.1611](#)

LBL data: M_{\min}^{SS}



$M > 0.6$ eV (NH), 1.4 eV (IH) as good fits as 3v scenario

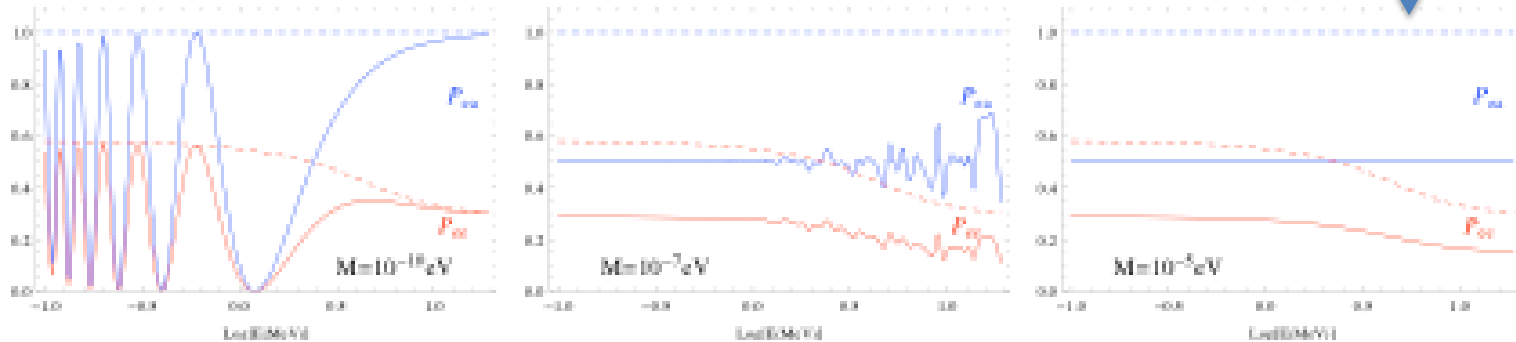
Conclusions

- Minimal models of neutrino masses are also models with extra sterile states but ones that are much more constrained/predictive than those used in phenomenological fits
- They have a rich phenomenology if their mass is below the EW scale
- This phenomenology has to be explored systematically
- Optimistically we might solve some other problem...or at least severely constraint the ν physics scale

SOLAR data: $M^{\text{QD}}_{\text{max}}$

IH

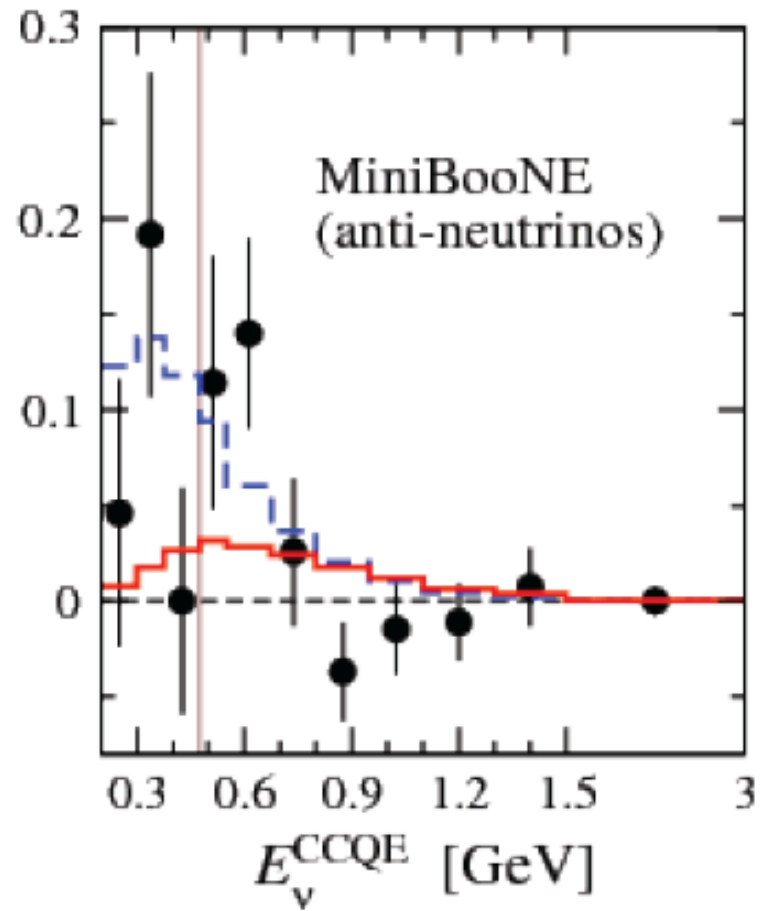
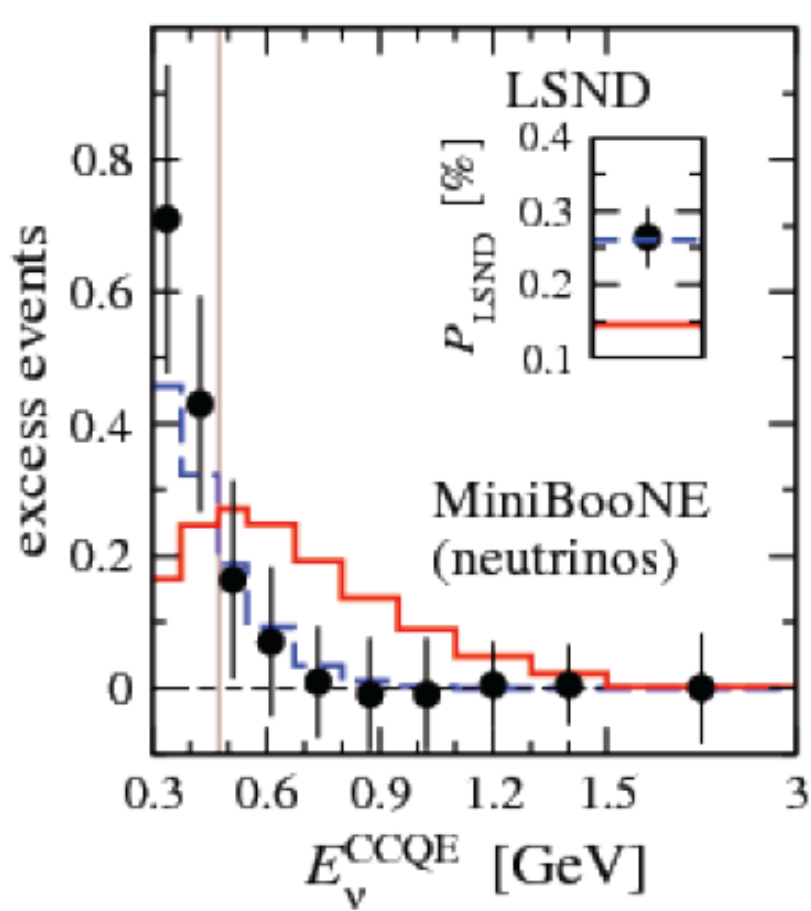
Adiabatic approx.



Adiabaticity limit:

$$M(\text{eV}) < \begin{cases} 10^{-7} \times E_\nu(\text{MeV}) & \text{NH,} \\ 2 \times 10^{-8} \times E_\nu(\text{MeV}) & \text{IH.} \end{cases}$$

Vacuum oscillations:
$$L_{osc} \sim \frac{E_\nu}{M m_{D-}}$$



3+2 PM, KMS fit