GGI neutrino workshop

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Neutrino mass models and sizable θ_{13}

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Prologue

- remarkable results from neutrino oscillation experiments
- tri-bimaximal lepton mixing (until recently)
- family symmetries like A_4 and S_4
- origin of the Klein symmetry in the neutrino sector
- strategies of implementing a sizable reactor angle θ_{13} (post T2K)
  - tri-bimaximal mixing plus corrections (from extra ingredient)
  - new family symmetries
  - non-standard vacuum configurations

A brief history of neutrino mixing

- atmospheric neutrinos
  - ν_μ / $\overline{\nu}_\mu$ disappear – Super-Kamiokande (1998)
- accelerator neutrinos
  - ν_μ disappear – K2K (2002), MINOS (2006)
  - ν_μ converted to ν_τ – OPERA (2010 & 2012)
  - ν_μ converted to ν_e – T2K, MINOS (2011)
- solar neutrinos
  - ν_e disappear – Chlorine (1998), Gallium (1999 - 2009), Super-Kamiokande (2002), Borexino (2008)
  - ν_e converted to $(\nu_\mu + \nu_\tau)$ – SNO (2002)
- reactor neutrinos
  - $\overline{\nu}_e$ disappear – Double Chooz (2011), Daya Bay, RENO (2012)
  - $\overline{\nu}_e$ disappear – KamLAND (2002)

2011/2012 story of non-zero θ_{13}

T2K [arXiv:1106.2822]

- $\theta_{13} \neq 0$ disfavored at $\sim 2.5\sigma$

MINOS [arXiv:1108.0015]

- $\theta_{13} \neq 0$ disfavored at $\sim 1.6\sigma$

Double Chooz [arXiv:1112.6353]

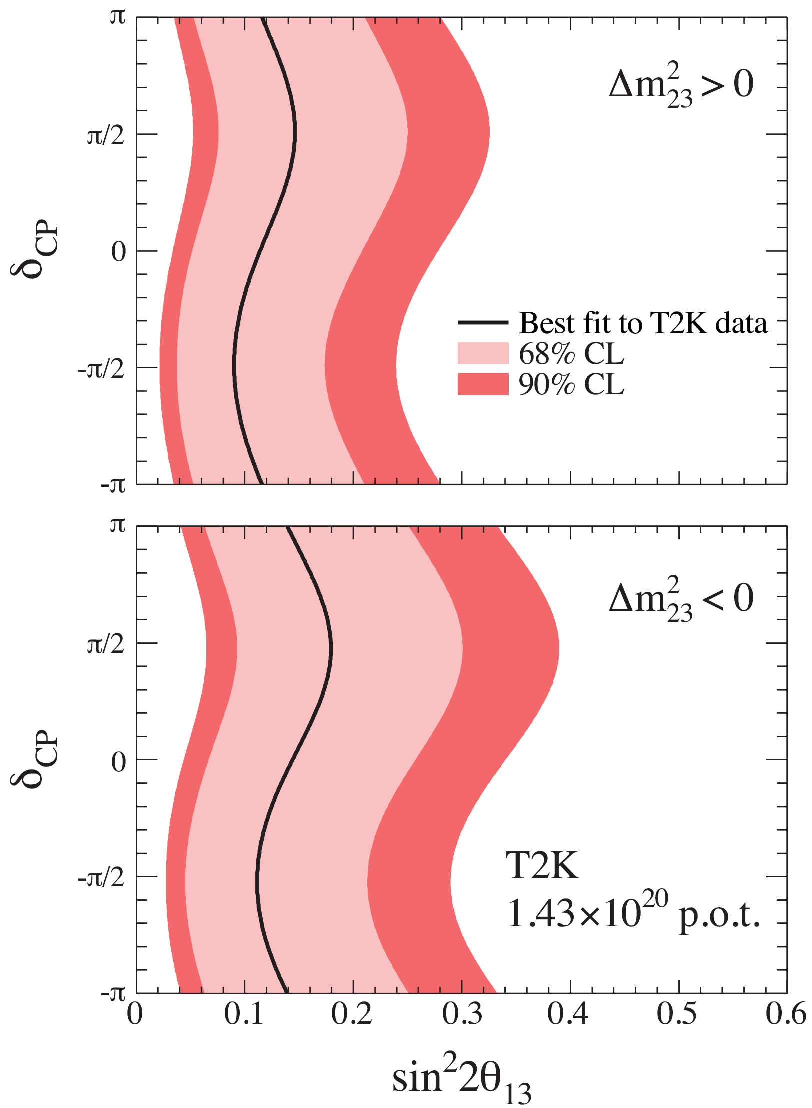
- $\theta_{13} \neq 0$ disfavored at $\sim 2\sigma$

Daya Bay [arXiv:1203.1669]

- $\theta_{13} \neq 0$ disfavored at $\sim 5.2\sigma$
- $7.9^\circ \lesssim \theta_{13} \lesssim 9.6^\circ$

RENO [arXiv:1204.0626]

- $\theta_{13} \neq 0$ disfavored at $\sim 4.9\sigma$
- $8.7^\circ \lesssim \theta_{13} \lesssim 10.8^\circ$

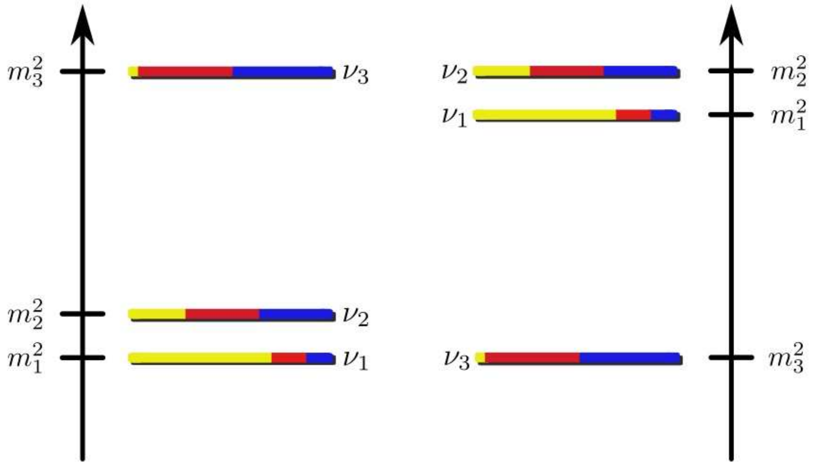


Three neutrino flavor mixing

(in diagonal charged lepton basis)

flavor PMNS mixing mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



atmospheric reactor + Dirac solar Majorana

$$U_{\mathrm{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & e^{\frac{\alpha_3}{2}} \end{pmatrix}$$

Tri-bimaximal lepton mixing vs. global neutrino fits

$$U_{\rm PMNS} \approx U_{\rm TB} \equiv \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

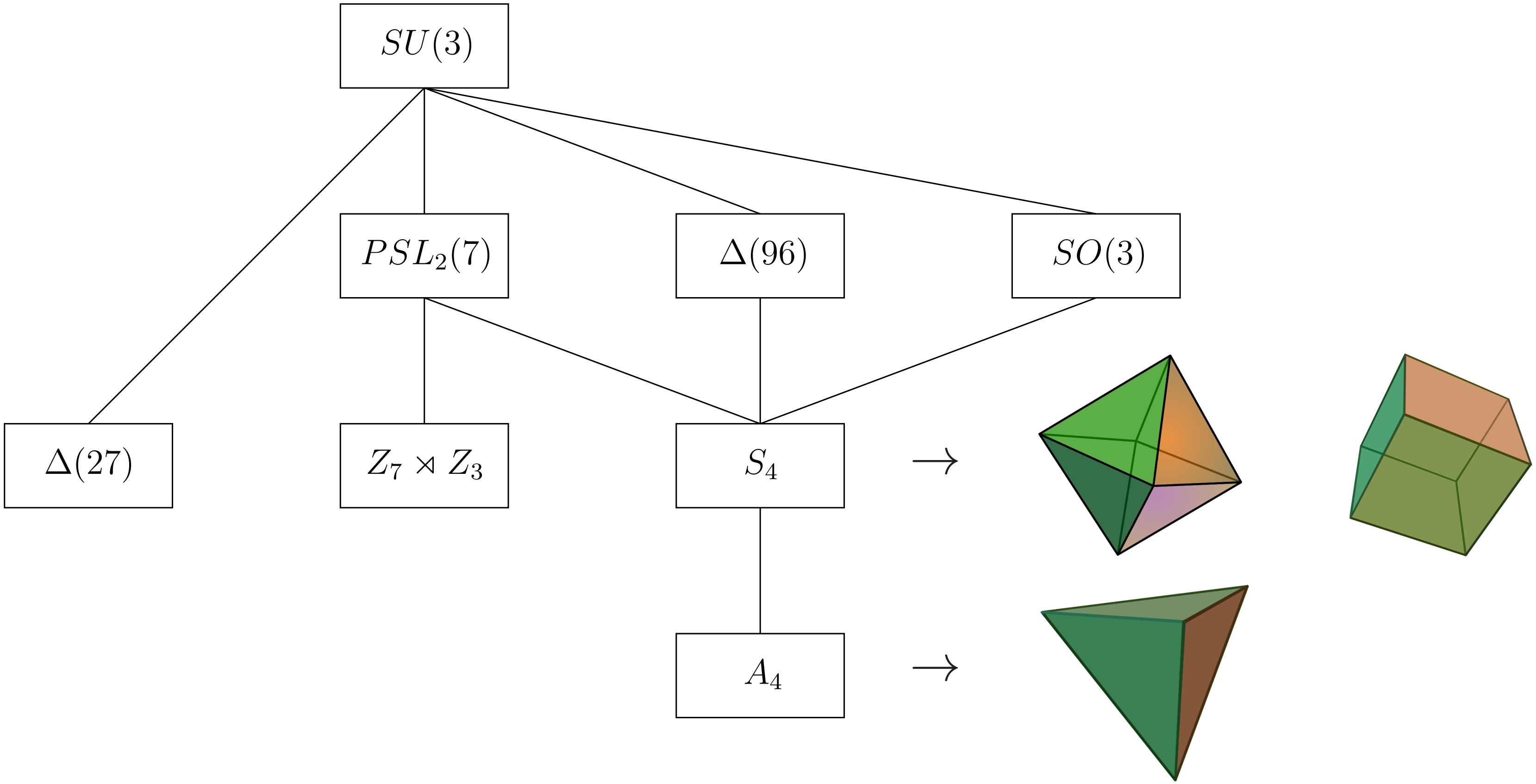
$\Rightarrow$

PMNS-angles	tri-bimax.	1σ exp.	1σ exp.
$\sin^2\theta_{12}$:	$\frac{1}{3}$	$0.303 - 0.335$	$0.291 - 0.325$
$\sin^2\theta_{23}$:	$\frac{1}{2}$	$0.44 - 0.58$	$0.37 - 0.44$
$\sin^2\theta_{13}$:	0	$0.022 - 0.030$	$0.021 - 0.028$
		Forero et al. (2012)	Fogli et al. (2012)

- TB mixing fits relatively well $\rightarrow$ family symmetry, e.g. A_4, S_4
- how to accommodate sizable $\theta_{13} \sim 8° - 10°$?

Non-Abelian family symmetries

- unify three families in multiplets of family symmetry
- group should have three-dimensional representations



Symmetries of the mass matrices

charged leptons $M_\ell \;=\; \mathrm{diag}\,(m_e, m_\mu, m_\tau)$

symmetric under diagonal phase transformation h

$\boxed{M_\ell \;=\; h^T M_\ell\, h^*}$ e.g. $h = \mathrm{diag}\,(1, e^{\frac{4\pi i}{3}}, e^{\frac{2\pi i}{3}})$

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neutrinos $M_\nu \;=\; U_{\mathrm{PMNS}}\,\mathrm{diag}\,(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\; U_{\mathrm{PMNS}}^T$

symmetry of M_ν depends on U_{PMNS}

$\boxed{M_\nu \;=\; k^T M_\nu\, k}$ $k = U_{\mathrm{PMNS}}^*\,\mathrm{diag}\,(\pm 1, \pm 1, \pm 1)\; U_{\mathrm{PMNS}}^T$

require $\det k = 1$

four different k $\rightarrow$ generate $Z_2 \times Z_2$ symmetry group

Klein symmetry $\mathcal{K} = \{1, k_1, k_2, k_3\}$

for $U_{\mathrm{PMNS}} = U_{\mathrm{TB}}$:

$$k_1 = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad k_2 = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad k_3 = k_1 k_2$$

Origin of the Klein symmetry

- "direct" models
  - Klein symmetry $\mathcal{K} \;\subset\;$ family symmetry $\mathcal{G}$
  - flavon fields ϕ break $\mathcal{G}$ down to $\mathcal{K}$ in neutrino sector
  - for TB mixing (k_1, k_2, h) generate S_4
- "indirect" models
  - Klein symmetry $\mathcal{K}$ not necessarily $\;\subset\;$ family symmetry $\mathcal{G}$
  - $\mathcal{G}$ responsible for generating particular flavon VEV configurations
  - for TB mixing – from e.g. $\Delta(27)$, $Z_7 \rtimes Z_3$

$$\langle\phi_1\rangle \propto \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_2\rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_3\rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\Rightarrow \quad \mathcal{L}_\nu \quad \sim \quad \nu\,(\phi_1\phi_1^T + \phi_2\phi_2^T + \phi_3\phi_3^T)\,\nu\;H\,H$$

Typical model setup

M_{Pl}

M_{GUT}

family symmetry broken

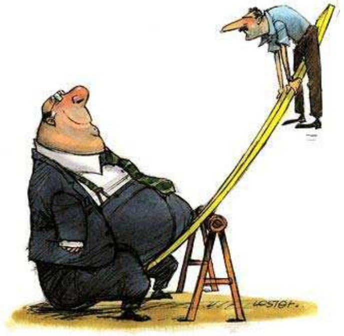
M_{seesaw} $\rightarrow$ M_R / Yukawas generated

M_{EW} electroweak symmetry broken

$\rightarrow$ light fermion masses generated

<u>ingredients:</u>

Majorana ν_L



seesaw

Implementing sizable θ_{13}

direct models	indirect models
TB plus corrections	TB plus corrections
other family symmetries with non-standard $\mathcal{K}$	
	non-standard flavon VEV configurations

TB plus non-diagonal charged leptons

Charged lepton corrections to TB mixing

- charged lepton mass matrix might not be diagonal (GUTs)
- $U_{\mathrm{PMNS}} = V_{\ell_L} V_{\nu_L}^\dagger$ and $V_{\nu_L}^\dagger = U_{\mathrm{TB}}$

$$U_{\mathrm{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & \hat{s}_{23} \\ 0 & -\hat{s}_{23}^* & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & \hat{s}_{13} \\ 0 & 1 & 0 \\ -\hat{s}_{13}^* & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & \hat{s}_{12} & 0 \\ -\hat{s}_{12}^* & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{\begin{aligned} c_{ij} &= \cos\theta_{ij} \\ \hat{s}_{ij} &= \sin\theta_{ij}\, e^{-i\delta_{ij}} \end{aligned}}$$

$$\begin{aligned}
s_{12}\, e^{i\delta_{12}} &\approx \frac{1}{\sqrt{3}} \left(e^{i\delta_{12}^\nu} - \theta_{12}^\ell\, e^{i\delta_{12}^\ell} + \theta_{13}^\ell\, e^{i(\delta_{13}^\ell - \delta_{23}^\nu)} \right) \\
s_{23}\, e^{i\delta_{23}} &\approx \frac{1}{\sqrt{2}} \left(e^{i\delta_{23}^\nu} - \theta_{23}^\ell\, e^{i\delta_{23}^\ell} \right) \\
s_{13}\, e^{i\delta_{13}} &\approx \frac{1}{\sqrt{2}} \left(-\theta_{12}^\ell\, e^{i(\delta_{12}^\ell + \delta_{23}^\nu)} - \theta_{13}^\ell\, e^{i\delta_{13}^\ell} \right)
\end{aligned}$$

- $\theta_{12}^\ell \sim \theta_C \sim 0.22 \quad \to \quad \theta_{13} \sim 9^\circ$
- not (easily) compatible with Georgi-Jarlskog relations

TB plus new TB breaking flavon

An S_4 model of leptons

King, Luhn (2011)

matter	L	τ^c	μ^c	e^c	N^c	H_u	H_d
S_4	$\mathbf{3}$	$\mathbf{1'}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$
Z_3^ν	1	2	2	2	2	0	0
Z_3^ℓ	0	2	1	0	0	0	0

flavons	φ_ℓ	η_μ	η_e	φ_ν	η_ν	ξ_ν	ζ_ν
S_4	$\mathbf{3'}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3'}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1'}$
Z_3^ν	0	0	0	2	2	2	0
Z_3^ℓ	1	1	2	0	0	0	0

$$\langle\varphi_\ell\rangle = \begin{pmatrix} 0 \\ v_\ell \\ 0 \end{pmatrix} \qquad \langle\eta_\mu\rangle = \begin{pmatrix} 0 \\ w_\mu \end{pmatrix} \qquad \langle\eta_e\rangle = \begin{pmatrix} w_e \\ 0 \end{pmatrix}$$

$$\langle\varphi_\nu\rangle = v_\nu \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\eta_\nu\rangle = w_\nu \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \langle\xi_\nu\rangle = u_\nu \qquad \langle\zeta_\nu\rangle = z_\nu$$

Charged lepton sector

$$\boxed{W_\ell \;\sim\; \left[\tfrac{1}{M}(L\varphi_\ell)_{1'}\,\tau^c + \tfrac{1}{M^2}(L\varphi_\ell)_2\,\eta_\mu\,\mu^c + \tfrac{1}{M^2}(L\varphi_\ell)_2\,\eta_e\,e^c\right]H_d}$$

- Z_3^ℓ controls pairing of flavons with right-handed charged fermions
- different S_4 contractions of $(L\varphi_\ell)$ pick out different L_i components

$$
\begin{aligned}
(L\varphi_\ell)_{1'} &= \quad L_1\varphi_{\ell 1} + L_2\varphi_{\ell 3} + L_3\varphi_{\ell 2} \quad \rightarrow \quad L_3 \\
(L\varphi_\ell)_2 &= \begin{pmatrix} L_1\varphi_{\ell 3} + L_2\varphi_{\ell 2} + L_3\varphi_{\ell 1} \\ L_1\varphi_{\ell 2} + L_2\varphi_{\ell 1} + L_3\varphi_{\ell 3} \end{pmatrix} \rightarrow \begin{pmatrix} L_2 \\ L_1 \end{pmatrix}
\end{aligned}
$$

- mass matrix diagonal by construction
- m_τ heavier than m_μ and m_e
- hierarchy between m_μ and m_e due to hierarchy of VEVs w_μ and w_e
- just a toy model of charged lepton sector (with GUTs off-diagonals)

Neutrino sector

$$\boxed{W_\nu \;\sim\; LN^cH_u + (\varphi_\nu + \eta_\nu + \xi_\nu)N^cN^c + \frac{1}{M}\zeta_\nu\eta_\nu N^cN^c}$$

- trivial Dirac neutrino Yukawa ✓
- neutrino mixing governed by heavy right-handed neutrinos
- S_4 multiplication rule ($N^c \sim \mathbf{3}$)

$$\mathbf{3} \otimes \mathbf{3} = (\mathbf{3}' + \mathbf{2} + \mathbf{1})_s$$

- three TB conserving flavons $\varphi_\nu \quad \eta_\nu \quad \xi_\nu$
- ζ_ν flavon is neutral except for S_4 ($\zeta_\nu \sim \mathbf{1}'$)

$$\mathbf{1}' \otimes (\mathbf{3} \otimes \mathbf{3}) = (\mathbf{3} + \mathbf{2} + \mathbf{1}')_s$$

- only one extra term involving ζ_ν
- this breaks TB structure (at higher order) ...

Breaking of the TB Klein symmetry $\mathcal{K}$

Dirac term LN^cH_u respects $\mathcal{K} \subset S_4$

Majorana terms $\left(\varphi_\nu + \eta_\nu + \xi_\nu + \frac{1}{M}\zeta_\nu\eta_\nu\right) N^cN^c$ respect k_1 but break k_2

S_4 irrep	k_1	k_2	alignment
$\mathbf{3'}$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	$\langle\varphi_\nu\rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
$\mathbf{2}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\langle\eta_\nu\rangle \propto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
$\mathbf{1}$	1	1	$\langle\xi_\nu\rangle \propto 1$
$\mathbf{1'}$	1	-1	$\langle\zeta_\nu\rangle \propto 1$

Resulting mixing

$$M_R = \frac{M_1+M_3}{6}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \frac{2M_2+M_3-M_1}{6}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + \frac{M_1+M_2-M_3}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$+ \Delta \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \quad \leftarrow \quad \text{small TB breaking term}$$

$$\Longrightarrow \quad \boxed{U_{\text{PMNS}} \approx \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}}\alpha^* \\ \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{2}}\alpha & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha^* \\ \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}}\alpha & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha^* \end{pmatrix}}$$

$$\text{Re}\ \alpha \approx -\sqrt{3}\cdot\left[\text{Re}\left(\frac{\Delta}{M_1 - M_3}\right) + \text{Im}\left(\frac{\Delta}{M_1 - M_3}\right)\frac{\text{Im}\left(\frac{M_1+M_3}{M_1-M_3}\right)}{\text{Re}\left(\frac{M_1+M_3}{M_1-M_3}\right)}\right]$$

$$\text{Im}\ \alpha \approx \sqrt{3}\cdot\frac{\text{Im}\left(\frac{\Delta}{M_1-M_3}\right)}{\text{Re}\left(\frac{M_1+M_3}{M_1-M_3}\right)}$$

Trimaximal neutrino mixing

- second column of $U_{\rm PMNS} \propto (1,1,1)^T$
- one could have guessed this special structure

  (i) $(1,1,1)^T$ is an eigenvector of M_R

  (ii) k_1 generator of TB Klein symmetry $\mathcal{K}$ unbroken
- such a TB breaking affects θ_{13} and θ_{23} – but not θ_{12}
- get correlations between deviation parameters r, a, s

$$\sin\theta_{13} = \frac{1}{\sqrt{2}}\, r \qquad \sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a) \qquad \sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s)$$

$$r\cos\delta \approx -\frac{2}{\sqrt{3}}\,\mathrm{Re}\,\alpha \qquad a \approx \frac{1}{\sqrt{3}}\,\mathrm{Re}\,\alpha \qquad \delta \approx \pi + \arg\,\alpha$$

$\rightarrow$ testable sum rules $\boxed{a \approx -\frac{1}{2}\, r\cos\delta \qquad s \approx 0}$

Revisiting a GUT model with TB neutrino mixing

An $S_4 \times SU(5)$ model

matter	T_3	T	F	N^c	H_5	$H_{\overline{5}}$	$H_{\overline{45}}$
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{45}}$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)$	0	5	4	-4	0	0	1

Hagedorn, King, Luhn (2012)

$$\langle \Phi_2^u \rangle \propto \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\langle \widetilde{\Phi}_2^u \rangle \propto \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

flavons	Φ_2^u	$\widetilde{\Phi}_2^u$	Φ_3^d	$\widetilde{\Phi}_3^d$	Φ_2^d	$\Phi_{3'}^\nu$	Φ_2^ν	Φ_1^ν	η
S_4	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{2}$	$\mathbf{3'}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)$	-10	0	-4	-11	1	8	8	8	7

$$\langle \Phi_3^d \rangle \propto \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \langle \widetilde{\Phi}_3^d \rangle \propto \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \quad \langle \Phi_2^d \rangle \propto \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \langle \Phi_{3'}^\nu \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \langle \Phi_2^\nu \rangle \propto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Charged fermions

up sector $\mathbf{10\ 10\ 5_H}$

$$M_u \sim \begin{pmatrix} \lambda^8 & 0 & 0 \\ 0 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix} v_u$$

· diagonal up quark matrix

· renormalizable top Yukawa

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down sector $\mathbf{\overline{5}\ 10\ \overline{5}_H}$ & $\mathbf{\overline{5}\ 10\ \overline{45}_H}$

$$M_d \sim \begin{pmatrix} 0 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^4 \\ 0 & 0 & \lambda^2 \end{pmatrix} v_d \qquad M_\ell \sim \begin{pmatrix} 0 & \lambda^5 & 0 \\ \lambda^5 & 3\,\lambda^4 & 0 \\ \lambda^5 & 3\,\lambda^4 & \lambda^2 \end{pmatrix} v_d$$

· Georgi-Jarlskog relations $m_b \sim m_\tau$, $m_s \sim \frac{1}{3} m_\mu$, $m_d \sim 3 m_e$

· Gatto-Sartori-Tonin relation $M^d_{12} = M^d_{21} \;\rightarrow\; \theta^d_{12} \sim \sqrt{\frac{m_d}{m_s}} \sim \lambda$

· charged lepton mixing $\theta^\ell_{12} \sim \frac{1}{3}\lambda \sim 4^\circ$

Neutrino sector and PMNS mixing

$$\boxed{W_\nu \;\sim\; F N^c H_5 + (\Phi^\nu_{3'} + \Phi^\nu_2 + \Phi^\nu_1) N^c N^c + \frac{1}{M}\eta \Phi^d_2 N^c N^c}$$

- new flavon η does not break TB symmetry
- effective doublet $\langle \eta \Phi^d_2 \rangle \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\underline{\underline{\text{respects } k_1}}$ but breaks k_2
- neutrino sector has trimaximal structure

- charged lepton corrections modify previous sum rules
- additional phases enter $\rightarrow$ sum rule bounds

$$|a| \;\lesssim\; \frac{1}{2}\left(r + \frac{\theta_C}{3}\right)|\cos\delta| \qquad\qquad |s| \;\lesssim\; \frac{\theta_C}{3}$$

Does this work in A_4 models too?

A_4 models with trimaximal neutrino mixing

- $A_4 \subset S_4$ but without the k_2 generator
- A_4 models with TB mixing: absence of flavons in the $\mathbf{1}'$ and $\mathbf{1}''$
  $\rightarrow$ k_2 symmetry arises accidentally
- add $\mathbf{1}'$ and $\mathbf{1}''$ flavons to break k_2 King, Luhn (2011)

$$\boxed{W_\nu \;\sim\; LN^cH_u + (\varphi_\nu + \xi_\nu + \xi'_\nu + \xi''_\nu)N^cN^c}$$

- k_1 symmetry is left unbroken
- trimaximal neutrino mixing
- effects of ξ'_ν and ξ''_ν not suppressed
- partial cancellation to a level of about 20% required
- a possible such $A_4 \times SU(5)$ GUT model exists Cooper, King, Luhn (2012)

Conclusion

- experimental measurement of $\theta_{13} \sim 8^\circ - 10^\circ$
- review role of family symmetries
- implementing non-zero θ_{13} via corrections to TB mixing
  - from non-diagonal charged leptons
  - S_4 model of leptons with TB breaking flavon
  - $S_4 \times SU(5)$ model
  - similarly possible for A_4
- sum rules/sum rule bounds for mixing angles

Thank you