Neutrino masses : beyond d=5 tree-level operators

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based on arXiv:0907.3143, JHEP 10 (2009) 076 and arXiv:1205.5140 to appear in JHEP

In collaboration with Daniel Hernandez, Martin Hirsch, Toshi Ota and Walter Winter

What's ν? Invisibles12, Firenze, July 2012

Seesaw Mechanism

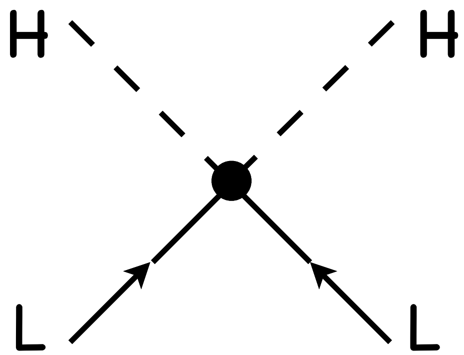
- Standard Model (SM) does not explain ν masses

Call for New Physics (NP) > EW

- Model independent approach : effective theories

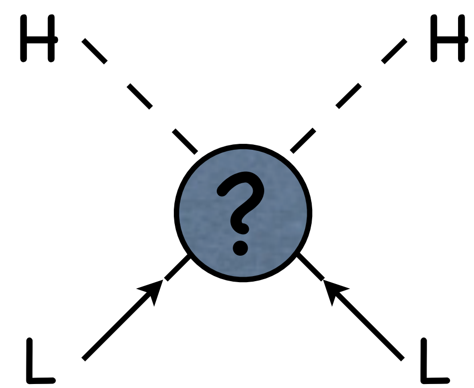
$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \delta\mathcal{L}^{d=5} + \delta\mathcal{L}^{d=6} + \ldots$$

- Lowest order: unique d=5 operator
  - Weinberg operator
  - Neutrino masses

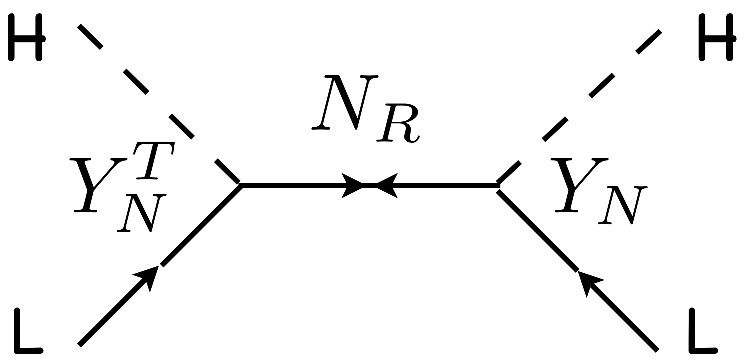


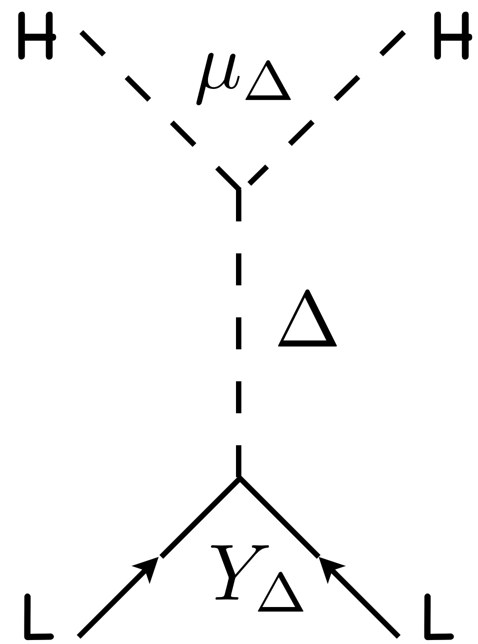
Recent review
A. Abada et al. '07

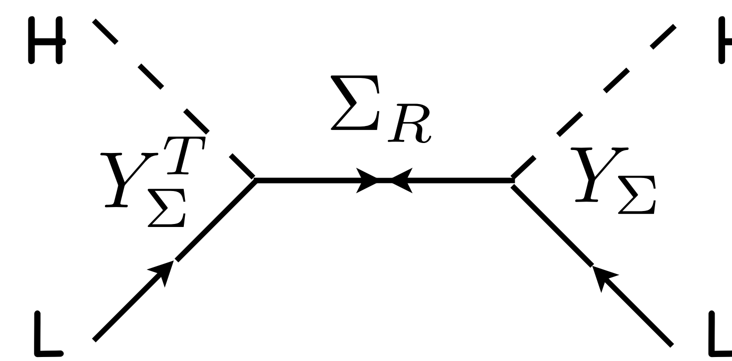
Seesaw Mechanism



Seesaw Mechanism







Type I

Minkowski 1977
Yanagida 1979
Gell-Mann et al. 1979
Mohapatra, Senjanovic 1980

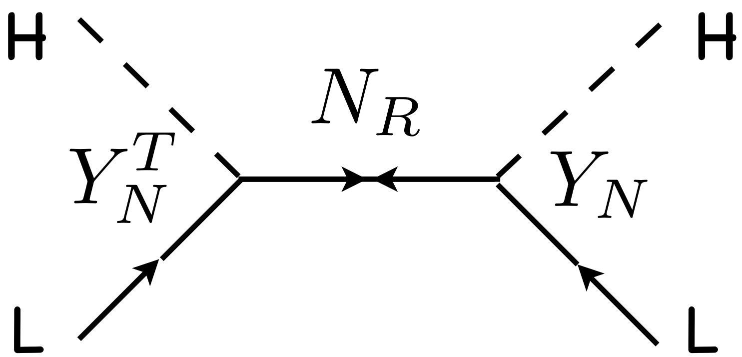
Type II

Magg, Wetterich 1980,
Schechter, Valle 1980,
Wetterich 1980,
Cheng, Li 1980,
Lazarides, Shafi, Wetterich 1981
Mohapatra, Senjanovic 1981,

Type III

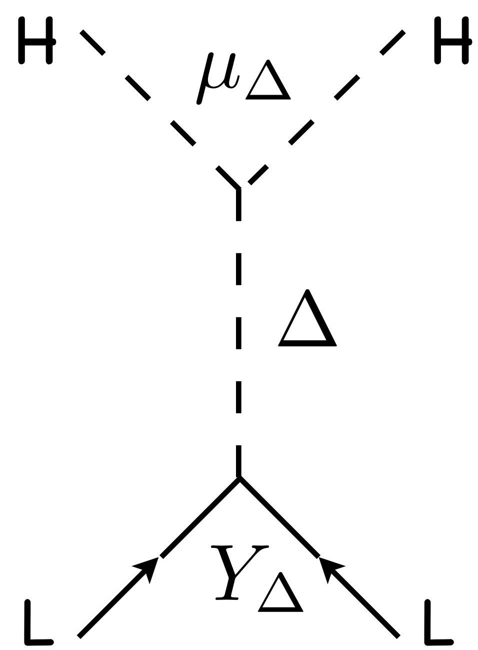
Foot, Lew, He and Joshi 1989

Seesaw Mechanism



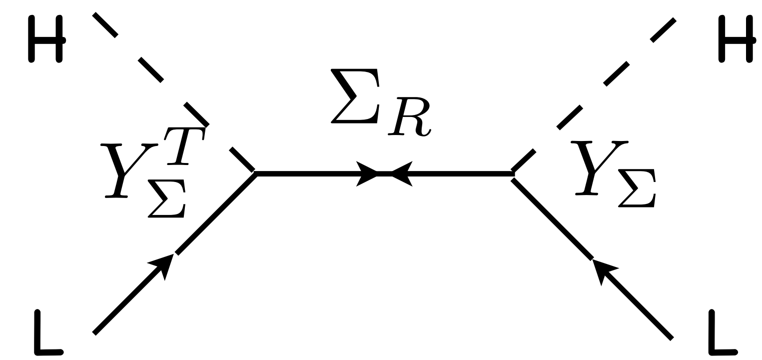
Type I

$$m_\nu \propto Y_N^T \frac{v^2}{M_N} Y_N$$



Type II

$$m_\nu \propto Y_\Delta \mu_\Delta \frac{v^2}{M_\Delta^2}$$



Type III

$$m_\nu \propto Y_\Sigma^T \frac{v^2}{M_\Sigma} Y_\Sigma$$

Problem :

$$m_\nu < \mathrm{eV} \Rightarrow \begin{cases} Y \sim \mathcal{O}(1),\ M \sim \mathrm{GUT} \\ Y \sim 10^{-5},\ M \sim \mathrm{TeV} \end{cases}$$

No LHC access
small couplings

Way out

Goals :
- New Physics @ TeV
- large couplings (LFV)

Means : need of additional source of suppression

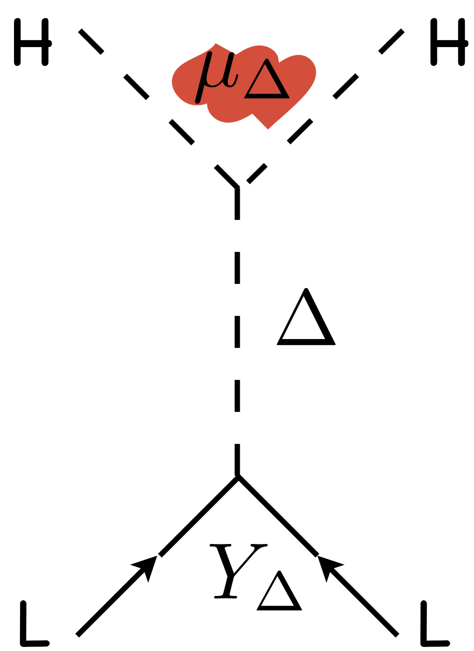
- Radiative generation of neutrino masses
- d>5 operator
- Small lepton number violating contributions

$$m_\nu \propto \frac{v^2}{\Lambda} \times \left(\frac{1}{16\pi^2}\right)^n \times \epsilon_{\mathrm{LNV}} \times \left(\frac{v}{\Lambda}\right)^{d-5}$$

Small lepton number violation contributions

Inverse/Linear Seesaw

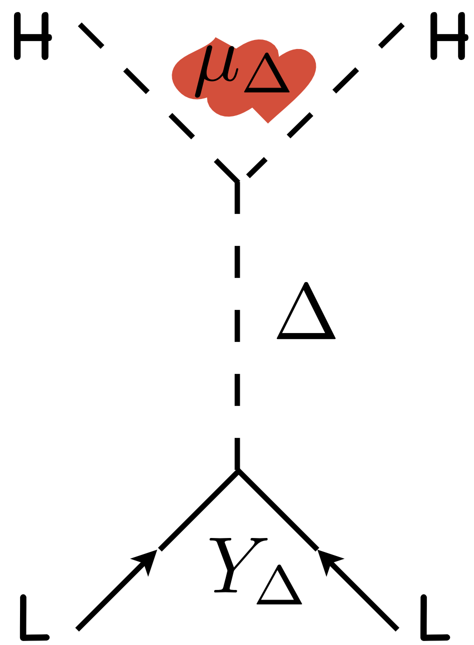
Type II : natural



m_ν	$Y_\Delta \mu_\Delta \frac{v^2}{M_\Delta^2}$
LFV	$Y_\Delta^\dagger Y_\Delta$

Inverse/Linear Seesaw

Type II : natural

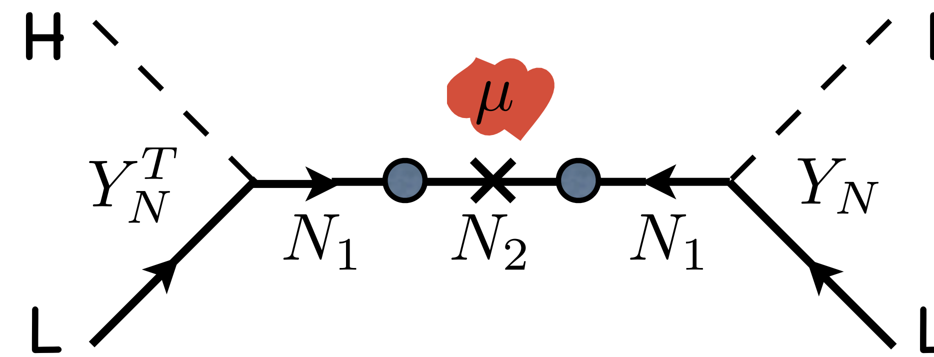


⚠ Inverse Seesaw

Type I/III : extra fermion

Mohapatra, Valle 1986

$$
\begin{array}{cc}
 & \begin{array}{ccc} \nu & N_1 & N_2 \end{array} \\
\begin{array}{c} \nu \\ N_1 \\ N_2 \end{array} & \begin{pmatrix} 0 & Y_N & 0 \\ Y_N^T & 0 & \Lambda \\ 0 & \Lambda & \mu \end{pmatrix}
\end{array}
$$

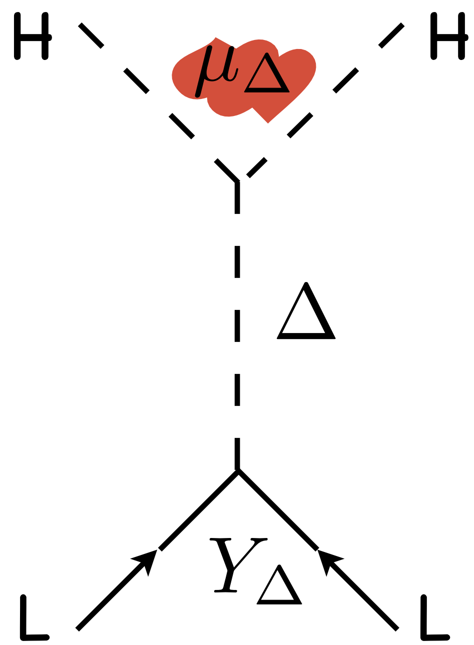


m_ν	$Y_\Delta \mu_\Delta \dfrac{v^2}{M_\Delta^2}$
LFV	$Y_\Delta^\dagger Y_\Delta$

m_ν	$-Y_N^T \dfrac{\mu}{\Lambda^2} Y_N v^2$
LFV	$Y_N^\dagger Y_N$

Inverse/Linear Seesaw

Type II : natural

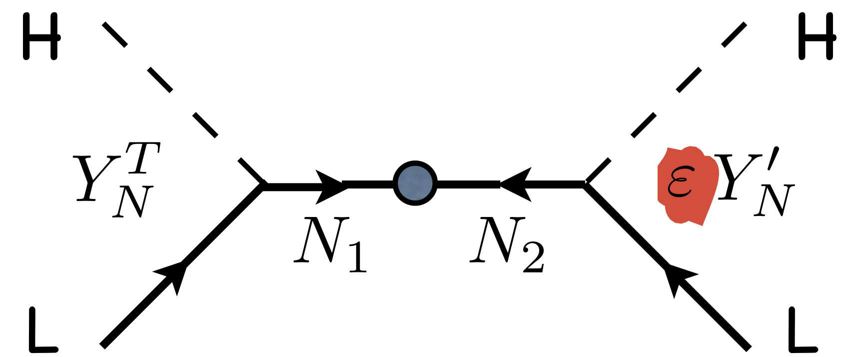


Linear Seesaw

Type I/III : extra fermion

Akhmedov et al. 1995

$$
\begin{array}{c} \\ \nu \\ N_1 \\ N_2 \end{array}
\begin{array}{c}
\begin{array}{ccc} \nu & N_1 & N_2 \end{array} \\
\begin{pmatrix} 0 & Y_N & \varepsilon Y'_N \\ Y_N^T & 0 & \Lambda \\ \varepsilon {Y'_N}^T & \Lambda & 0 \end{pmatrix}
\end{array}
$$



m_ν	$Y_\Delta \mu_\Delta \frac{v^2}{M_\Delta^2}$
LFV	$Y_\Delta^\dagger Y_\Delta$

m_ν	$\varepsilon \left({Y'_N}^T \frac{v^2}{\Lambda} Y_N + Y_N^T \frac{v^2}{\Lambda} Y'_N \right)$
LFV	$Y_N^\dagger Y_N$

d>5 operators

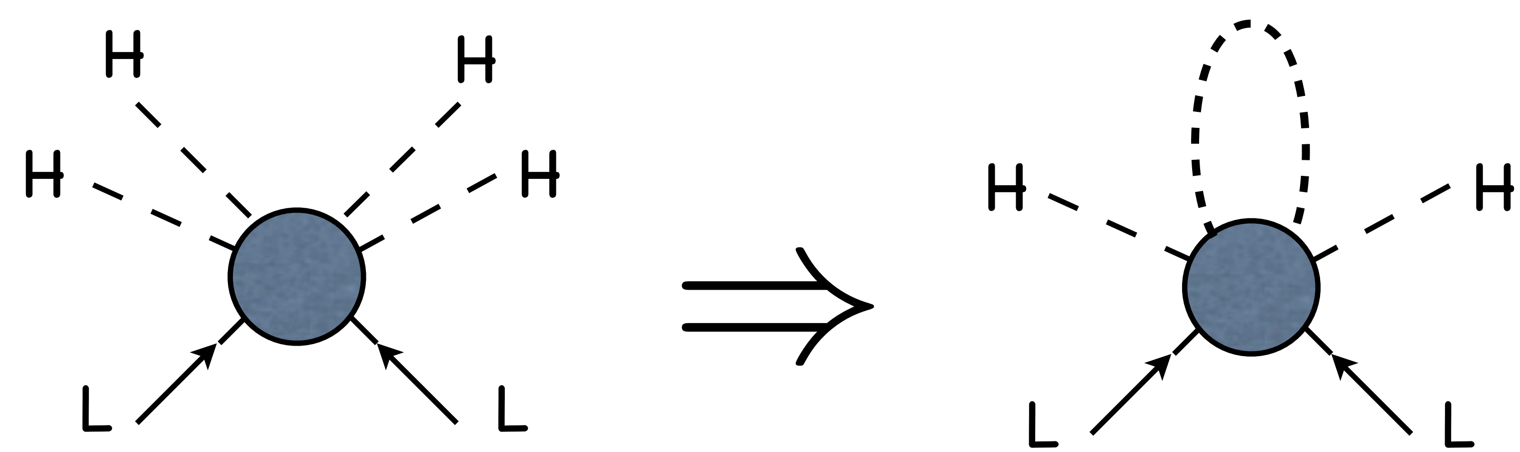
d>5 operator

arXiv:0907.3143, JHEP 10 (2009) 076

concept :

$$\mathcal{O}^{d=5} = LLHH$$
$$\mathcal{O}^{d=7} = (LLHH)(H^\dagger H)$$
$$\mathcal{O}^{d=9} = (LLHH)(H^\dagger H)^2$$
$$\vdots$$

problem :



$$\propto \frac{1}{\Lambda_{NP}^3}(LLHH)(H^\dagger H) \quad < \quad \propto \frac{1}{16\pi^2}\frac{1}{\Lambda_{NP}}(LLHH)$$

if

$\Lambda_{NP} > 3\,\mathrm{TeV}$

d>5 operator

concept :

$$
\begin{aligned}
\mathcal{O}^{d=5} &= LLHH \\
\mathcal{O}^{d=7} &= (LLHH)(H^\dagger H) \\
\mathcal{O}^{d=9} &= (LLHH)(H^\dagger H)^2 \\
&\vdots
\end{aligned}
$$

solution :

- genuine d=D operator as LO with all d<D forbidden
- new U(1) or discrete symmetry
- Pb : $H^\dagger H$ singlet -> need new fields

$$\mathcal{O}^{n+5} \sim (LLHH)S^n$$

Chen, de Gouvea, Dobrescu 2006
Gogoladze, Okada, Shafi, 2008

d>5 operator

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$$\mathcal{O}^{n+5} \sim (LLHH)S^n$$

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Gogoladze, Okada, Shafi, 2008

$$\boxed{\mathcal{O}^{2n+5} \sim (LLH_uH_u)(H_uH_d)^n}$$

simplest possibility : d=7 $(LLH_uH_u)(H_uH_d)$ with $\mathbb{Z}_5$

d>5 operator

decomposition : finding all possible heavy fields (mediators) for tree-level realizations of $(LLH_uH_u)(H_uH_d)$

SU(2) ⟵ X

L ⟶ Lorentz: S (scalar), V (vector), R/L (fermion)

Y ⟶ Hypercharge

d>5 operator

Type I (fermion singlet)

$$\mathbf{1}_0^{R/L}$$

Type II (scalar triplet)

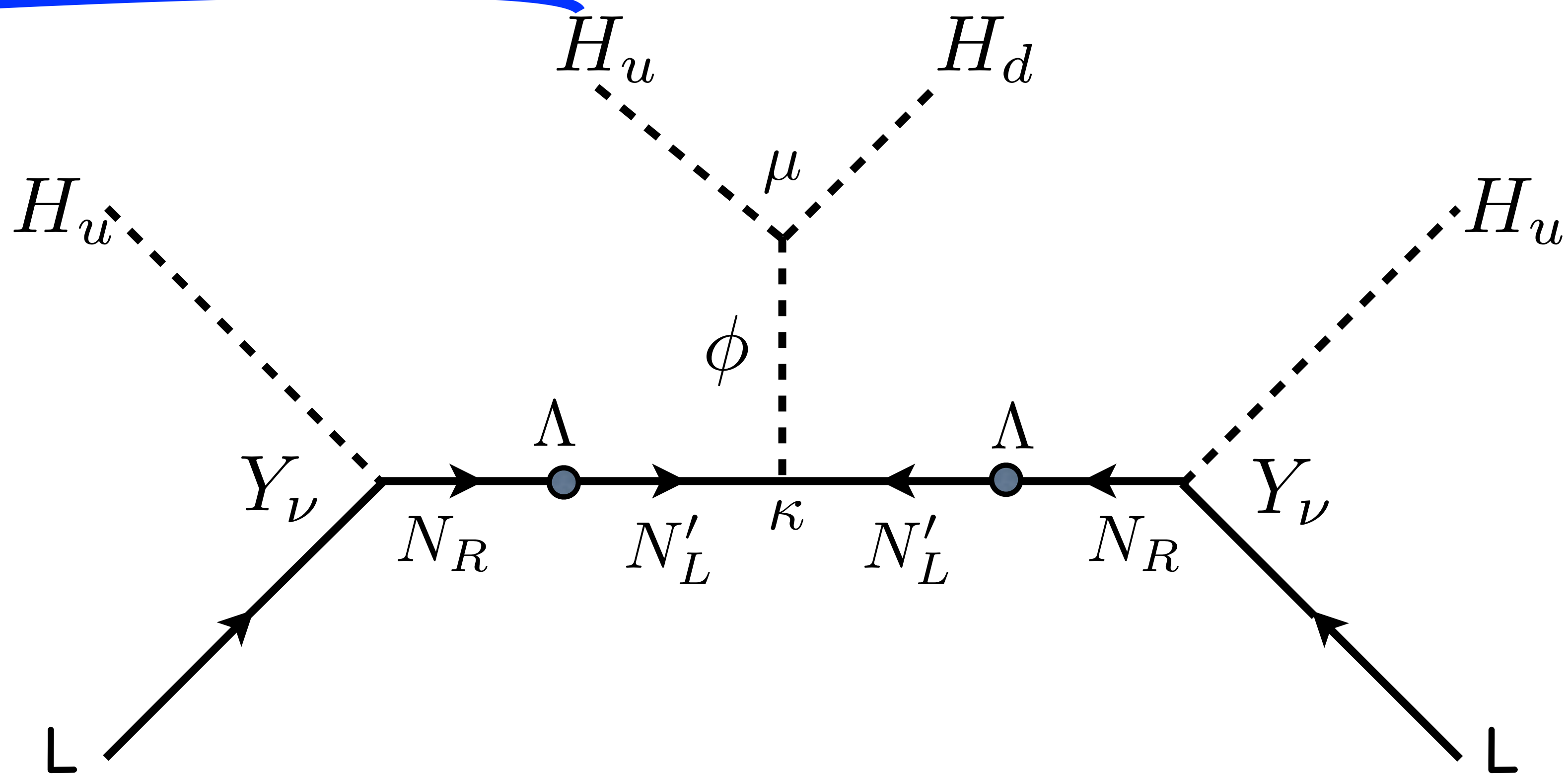
$$\mathbf{3}_{-1}^{S}$$

Type III (fermion triplet)

$$\mathbf{3}_0^{R/L}$$

				Phenom.		
#	Operator	Top.	Mediators	NU	δg_L	4ℓ
1	$(H_u i\tau^2\overline{L^c})(H_u i\tau^2 L)(H_d i\tau^2 H_u)$	2	$\mathbf{1}_0^R, \mathbf{1}_0^L, \mathbf{1}_0^s$	✓		
2	$(H_u i\tau^2\vec{\tau}\overline{L^c})(H_u i\tau^2 L)(H_d i\tau^2\vec{\tau}H_u)$	2	$\mathbf{3}_0^R, \mathbf{3}_0^L\ \mathbf{1}_0^R, \mathbf{1}_0^L, \mathbf{3}_0^s$	✓	✓	
3	$(H_u i\tau^2\vec{\tau}\overline{L^c})(H_u i\tau^2\vec{\tau}L)(H_d i\tau^2 H_u)$	2	$\mathbf{3}_0^R, \mathbf{3}_0^L, \mathbf{1}_0^s$	✓	✓	
4	$(-i\epsilon^{abc})(H_u i\tau^2\tau^a\overline{L^c})(H_u i\tau^2\tau^b L)(H_d i\tau^2\tau^c H_u)$	2	$\mathbf{3}_0^R, \mathbf{3}_0^L, \mathbf{3}_0^s$	✓	✓	
5	$(\overline{L^c}i\tau^2\vec{\tau}L)(H_d i\tau^2 H_u)(H_u i\tau^2\vec{\tau}H_u)$	2/3	$\mathbf{3}_{-1}^s, \mathbf{3}_{-1}^s/\mathbf{1}_0^s$			✓
6	$(-i\epsilon^{abc})(\overline{L^c}i\tau^2\tau^a L)(H_d i\tau^2\tau^b H_u)(H_u i\tau^2\tau^c H_u)$	2/3	$\mathbf{3}_{-1}^s, \mathbf{3}_{-1}^s/\mathbf{3}_0^s$			✓
7	$(H_u i\tau^2\overline{L^c})(L i\tau^2\vec{\tau}H_d)(H_u i\tau^2\vec{\tau}H_u)$	2	$\mathbf{1}_0^R, \mathbf{1}_0^L, \mathbf{3}_0^R, \mathbf{3}_0^L, \mathbf{3}_{-1}^s$	✓	✓	
8	$(-i\epsilon^{abc})(H_u i\tau^2\tau^a\overline{L^c})(L i\tau^2\tau^b H_d)(H_u i\tau^2\tau^c H_u)$	2	$\mathbf{3}_0^R, \mathbf{3}_0^L, \mathbf{3}_0^R, \mathbf{3}_0^L, \mathbf{3}_{-1}^s$	✓	✓	
9	$(H_u i\tau^2\overline{L^c})(i\tau^2 H_u)(L)(H_d i\tau^2 H_u)$	1	$\mathbf{1}_0^R, \mathbf{1}_0^L, \mathbf{2}_{-1/2}^R, \mathbf{2}_{-1/2}^L, \mathbf{1}_0^s$	✓		
10	$(H_u i\tau^2\vec{\tau}\overline{L^c})(i\tau^2\vec{\tau}H_u)(L)(H_d i\tau^2 H_u)$	1	$\mathbf{3}_0^R, \mathbf{3}_0^L, \mathbf{2}_{-1/2}^R, \mathbf{2}_{-1/2}^L, \mathbf{1}_0^s$	✓	✓	
11	$(H_u i\tau^2\overline{L^c})(i\tau^2 H_u)(\vec{\tau}L)(H_d i\tau^2\vec{\tau}H_u)$	1	$\mathbf{1}_0^R, \mathbf{1}_0^L, \mathbf{2}_{-1/2}^R, \mathbf{2}_{-1/2}^L, \mathbf{3}_0^s$	✓		
12	$(H_u i\tau^2\tau^a\overline{L^c})(i\tau^2\tau^a H_u)(\tau^b L)(H_d i\tau^2\tau^b H_u)$	1	$\mathbf{3}_0^R, \mathbf{3}_0^L, \mathbf{2}_{-1/2}^R, \mathbf{2}_{-1/2}^L, \mathbf{3}_0^s$	✓	✓	
13	$(H_u i\tau^2\overline{L^c})(L)(i\tau^2 H_u)(H_d i\tau^2 H_u)$	1/4	$\mathbf{1}_0^R, \mathbf{1}_0^L, \mathbf{2}_{-1/2}^s, (\mathbf{1}_0^s)$	✓		
14	$(H_u i\tau^2\vec{\tau}\overline{L^c})(\vec{\tau}L)(i\tau^2 H_u)(H_d i\tau^2 H_u)$	1/4	$\mathbf{3}_0^R, \mathbf{3}_0^L, \mathbf{2}_{-1/2}^s, (\mathbf{1}_0^s)$	✓	✓	
15	$(H_u i\tau^2\overline{L^c})(L)(i\tau^2\vec{\tau}H_u)(H_d i\tau^2\vec{\tau}H_u)$	1/4	$\mathbf{1}_0^R, \mathbf{1}_0^L, \mathbf{2}_{-1/2}^s, (\mathbf{3}_0^s)$	✓		
16	$(H_u i\tau^2\tau^a\overline{L^c})(\tau^a L)(i\tau^2\tau^b H_u)(H_d i\tau^2\tau^b H_u)$	1/4	$\mathbf{3}_0^R, \mathbf{3}_0^L, \mathbf{2}_{-1/2}^s, (\mathbf{3}_0^s)$	✓	✓	
17	$(H_u i\tau^2\overline{L^c})(H_d)(i\tau^2 H_u)(H_u i\tau^2 L)$	1	$\mathbf{1}_0^R, \mathbf{1}_0^L, \mathbf{2}_{-1/2}^R, \mathbf{2}_{-1/2}^L$	✓		
18	$(H_u i\tau^2\vec{\tau}\overline{L^c})(\vec{\tau}H_d)(i\tau^2 H_u)(H_u i\tau^2 L)$	1	$\mathbf{3}_0^R, \mathbf{3}_0^L, \mathbf{2}_{-1/2}^R, \mathbf{2}_{-1/2}^L, \mathbf{1}_0^R, \mathbf{1}_0^L$	✓	✓	
19	$(H_u i\tau^2\overline{L^c})(H_d)(i\tau^2\vec{\tau}H_u)(H_u i\tau^2\vec{\tau}L)$	1	$\mathbf{1}_0^R, \mathbf{1}_0^L, \mathbf{2}_{-1/2}^R, \mathbf{2}_{-1/2}^L, \mathbf{3}_0^R, \mathbf{3}_0^L$	✓	✓	
20	$(H_u i\tau^2\tau^a\overline{L^c})(\tau^a H_d)(i\tau^2\tau^b H_u)(H_u i\tau^2\tau^b L)$	1	$\mathbf{3}_0^R, \mathbf{3}_0^L, \mathbf{2}_{-1/2}^R, \mathbf{2}_{-1/2}^L,$	✓	✓	
21	$(\overline{L^c}i\tau^2\tau^a L)(H_u i\tau^2\tau^a)(\tau^b H_d)(H_u i\tau^2\tau^b H_u)$	1/4	$\mathbf{3}_{-1}^s, \mathbf{2}_{+1/2}^s, (\mathbf{3}_{-1}^s)$			✓
22	$(\overline{L^c}i\tau^2\tau^a L)(H_d i\tau^2\tau^a)(\tau^b H_u)(H_u i\tau^2\tau^b H_u)$	1/4	$\mathbf{3}_{-1}^s, \mathbf{2}_{+3/2}^s, (\mathbf{3}_{-1}^s)$			✓
23	$(\overline{L^c}i\tau^2\vec{\tau}L)(H_u i\tau^2\vec{\tau})(H_u)(H_d i\tau^2 H_u)$	1/4	$\mathbf{3}_{-1}^s, \mathbf{2}_{+1/2}^s, (\mathbf{1}_0^s)$			✓
24	$(\overline{L^c}i\tau^2\tau^a L)(H_u i\tau^2\tau^a)(\tau^b H_u)(H_d i\tau^2\tau^b H_u)$	1/4	$\mathbf{3}_{-1}^s, \mathbf{2}_{+1/2}^s, (\mathbf{3}_0^s)$			✓
25	$(H_d i\tau^2 H_u)(\overline{L^c}i\tau^2)(\vec{\tau}L)(H_u i\tau^2\vec{\tau}H_u)$	1	$\mathbf{1}_0^s, \mathbf{2}_{+1/2}^L, \mathbf{2}_{+1/2}^R, \mathbf{3}_{-1}^s$			
26	$(H_d i\tau^2\tau^a H_u)(\overline{L^c}i\tau^2\tau^a)(\tau^b L)(H_u i\tau^2\tau^b H_u)$	1	$\mathbf{3}_0^s, \mathbf{2}_{+1/2}^L, \mathbf{2}_{+1/2}^R, \mathbf{3}_{-1}^s$			
27	$(H_u i\tau^2\overline{L^c})(i\tau^2 H_d)(\vec{\tau}L)(H_u i\tau^2\vec{\tau}H_u)$	1	$\mathbf{1}_0^R, \mathbf{1}_0^L, \mathbf{2}_{+1/2}^R, \mathbf{2}_{+1/2}^L, \mathbf{3}_{-1}^s$	✓		
28	$(H_u i\tau^2\tau^a\overline{L^c})(i\tau^2\tau^a H_d)(\tau^b L)(H_u i\tau^2\tau^b H_u)$	1	$\mathbf{3}_0^R, \mathbf{3}_0^L, \mathbf{2}_{+1/2}^R, \mathbf{2}_{+1/2}^L, \mathbf{3}_{-1}^s$	✓	✓	
29	$(H_u i\tau^2\overline{L^c})(L)(i\tau^2\vec{\tau}H_d)(H_u i\tau^2\vec{\tau}H_u)$	1/4	$\mathbf{1}_0^R, \mathbf{1}_0^L, \mathbf{2}_{+1/2}^s, (\mathbf{3}_{-1}^s)$	✓		
30	$(H_u i\tau^2\tau^a\overline{L^c})(\tau^a L)(i\tau^2\tau^b H_d)(H_u i\tau^2\tau^b H_u)$	1/4	$\mathbf{3}_0^R, \mathbf{3}_0^L, \mathbf{2}_{+1/2}^s, (\mathbf{3}_{-1}^s)$	✓	✓	
31	$(\overline{L^c}i\tau^2\tau^a H_d)(i\tau^2\tau^a H_u)(\tau^b L)(H_u i\tau^2\tau^b H_u)$	1	$\mathbf{3}_{+1}^L, \mathbf{3}_{+1}^R, \mathbf{2}_{+1/2}^L, \mathbf{2}_{+1/2}^R, \mathbf{3}_{-1}^s$	✓	✓	
32	$(\overline{L^c}i\tau^2\tau^a H_d)(\tau^a L)(i\tau^2\tau^b H_u)(H_u i\tau^2\tau^b H_u)$	1/4	$\mathbf{3}_{+1}^L, \mathbf{3}_{+1}^R, \mathbf{2}_{-3/2}^s, (\mathbf{3}_{-1}^s)$	✓	✓	
33	$(\overline{L^c}i\tau^2\vec{\tau}H_d)(i\tau^2\vec{\tau}H_u)(H_u)(H_u i\tau^2 L)$	1	$\mathbf{3}_{+1}^L, \mathbf{3}_{+1}^R, \mathbf{2}_{-3/2}^L, \mathbf{2}_{-3/2}^R, \mathbf{1}_0^L, \mathbf{1}_0^R$	✓	✓	
34	$(\overline{L^c}i\tau^2\tau^a H_d)(i\tau^2\tau^a H_u)(\tau^b H_u)(H_u i\tau^2\tau^b L)$	1	$\mathbf{3}_{+1}^L, \mathbf{3}_{+1}^R, \mathbf{2}_{-3/2}^L, \mathbf{2}_{-3/2}^R, \mathbf{3}_0^L, \mathbf{3}_0^R$	✓	✓	

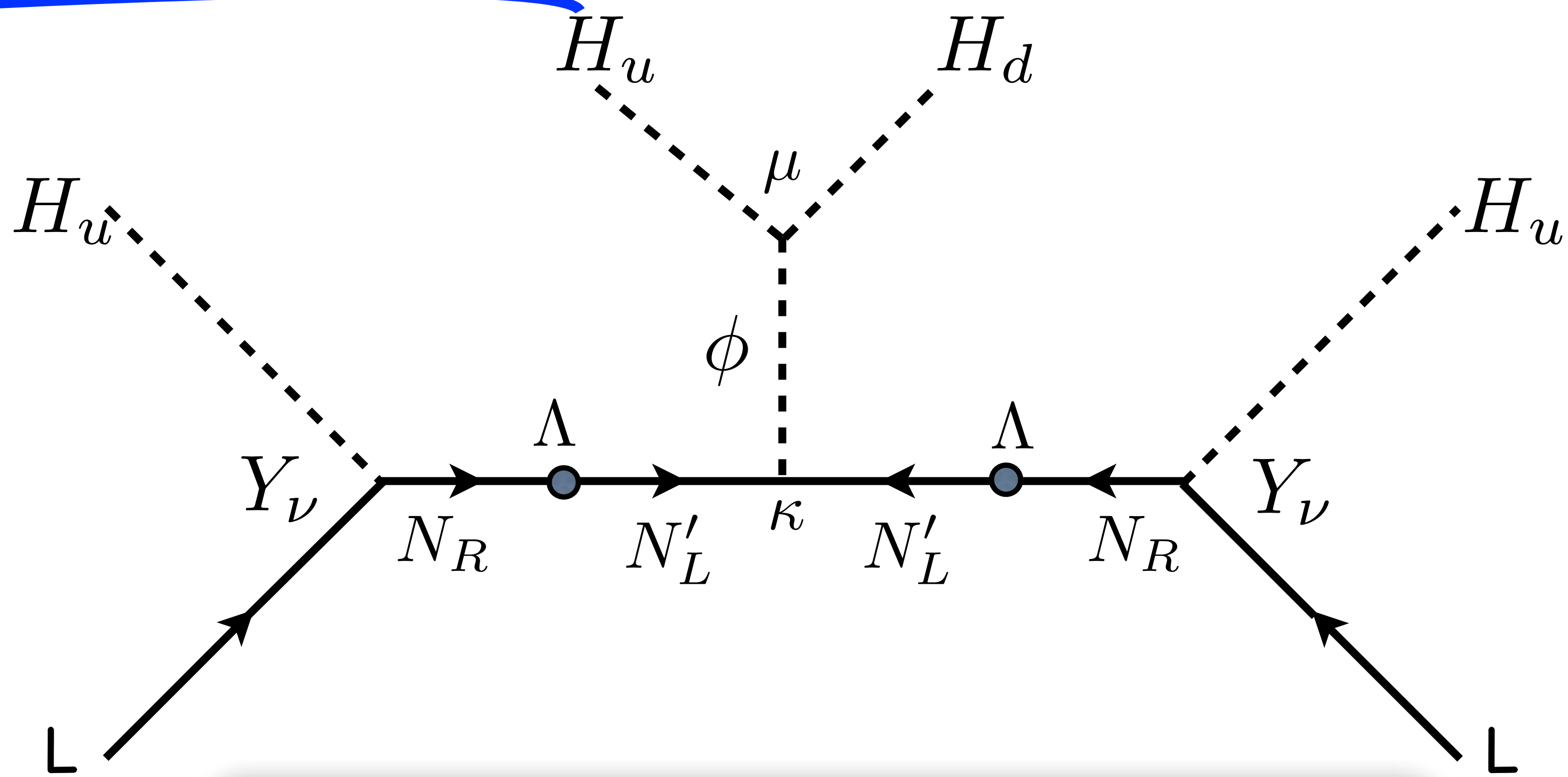
d>5 operator : first example



$$\phi \sim \mathbf{1}_0^S$$

$$N,\ N' \sim \mathbf{1}_0^F$$

d>5 operator : first example



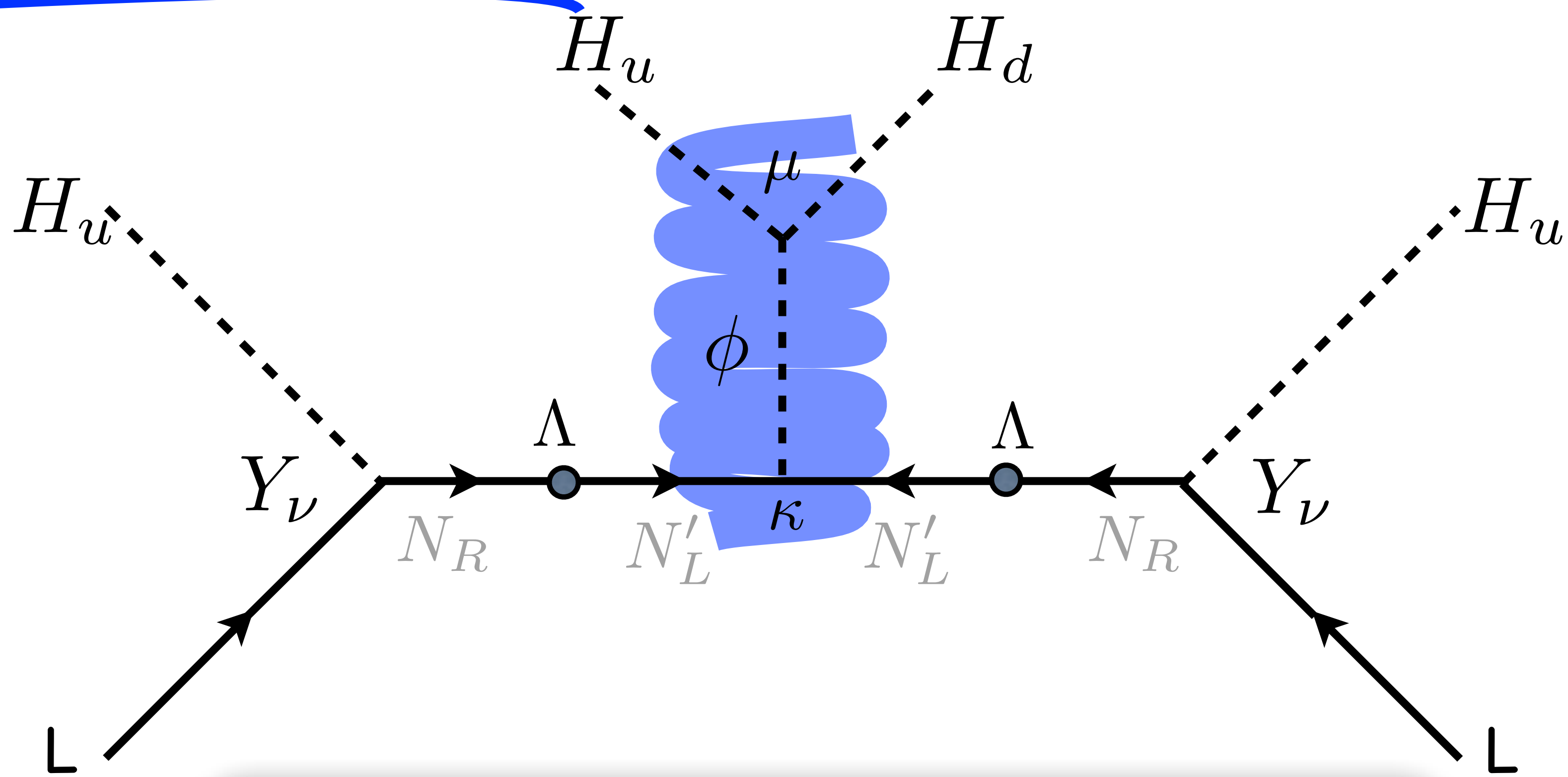
$$\phi \sim \mathbf{1}_0^S$$

$$N,\ N' \sim \mathbf{1}_0^F$$

$$\boxed{m_\nu = \frac{v_u^3 v_d}{2}\, Y_\nu^T\, (\Lambda^{-1})^T\, \frac{\kappa\mu}{M_\phi^2}\, \Lambda^{-1}\, Y_\nu}$$

Masses @TeV -> $Y_\nu \sim 10^{-4}$

d>5 operator : first example



$$\phi \sim \mathbf{1}_0^S$$

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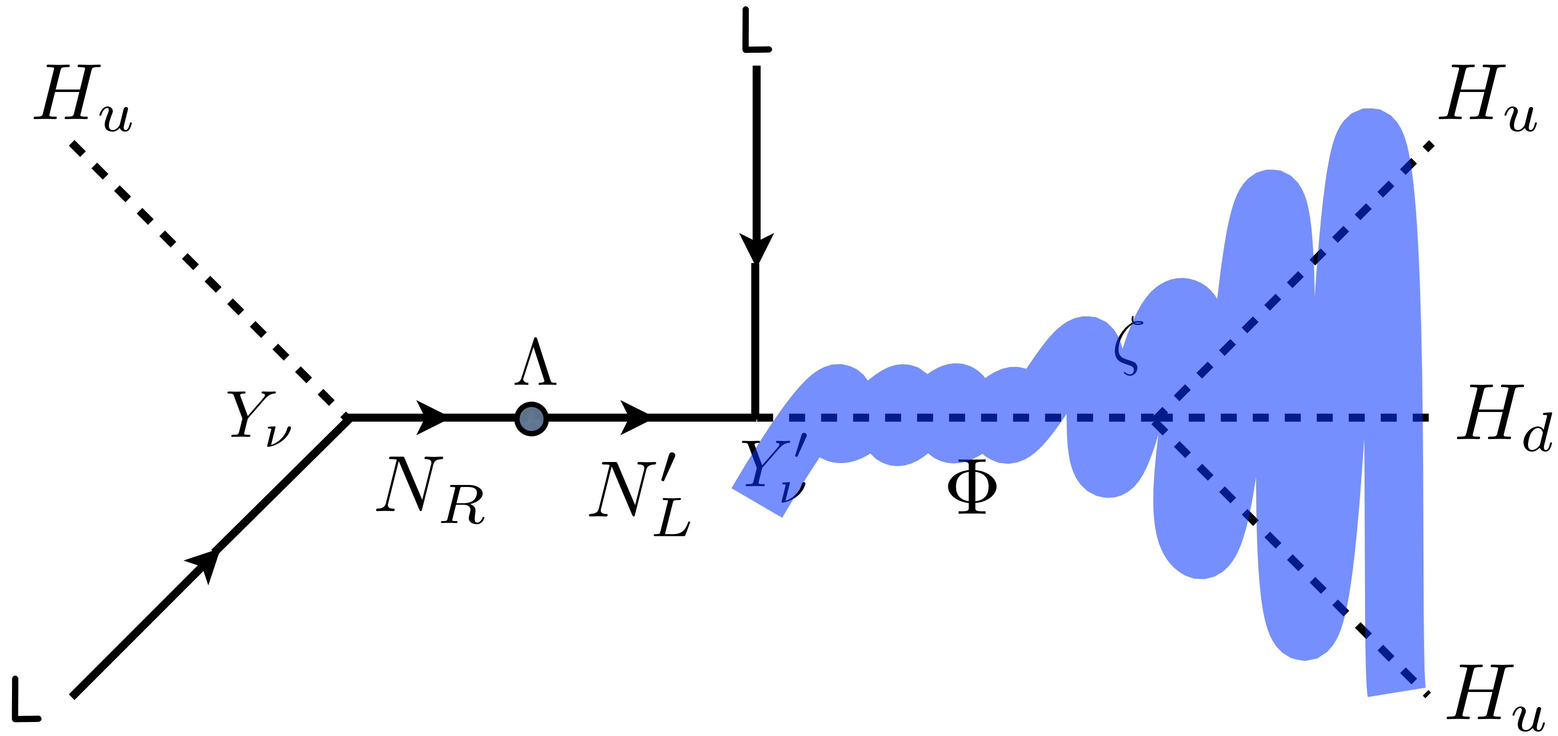
$$\boxed{m_\nu = \frac{v_u^3 v_d}{2}\, Y_\nu^T \,(\Lambda^{-1})^T \frac{\kappa\mu}{M_\phi^2} \Lambda^{-1}\, Y_\nu}$$

$$\begin{pmatrix} 0 & Y_\nu^T \langle H_u^0\rangle & 0 \\ Y_\nu \langle H_u^0\rangle & 0 & \Lambda \\ 0 & \Lambda & 2\kappa \frac{\mu}{M_\phi^2} \langle H_u^0 H_d^0\rangle \end{pmatrix} \xrightarrow[M_\phi \to \infty]{} \begin{pmatrix} 0 & Y_\nu^T \langle H_u^0\rangle & 0 \\ Y_\nu \langle H_u^0\rangle & 0 & \Lambda \\ 0 & \Lambda & \mu_{\rm LNV} \end{pmatrix}$$

d>5 operator : second example



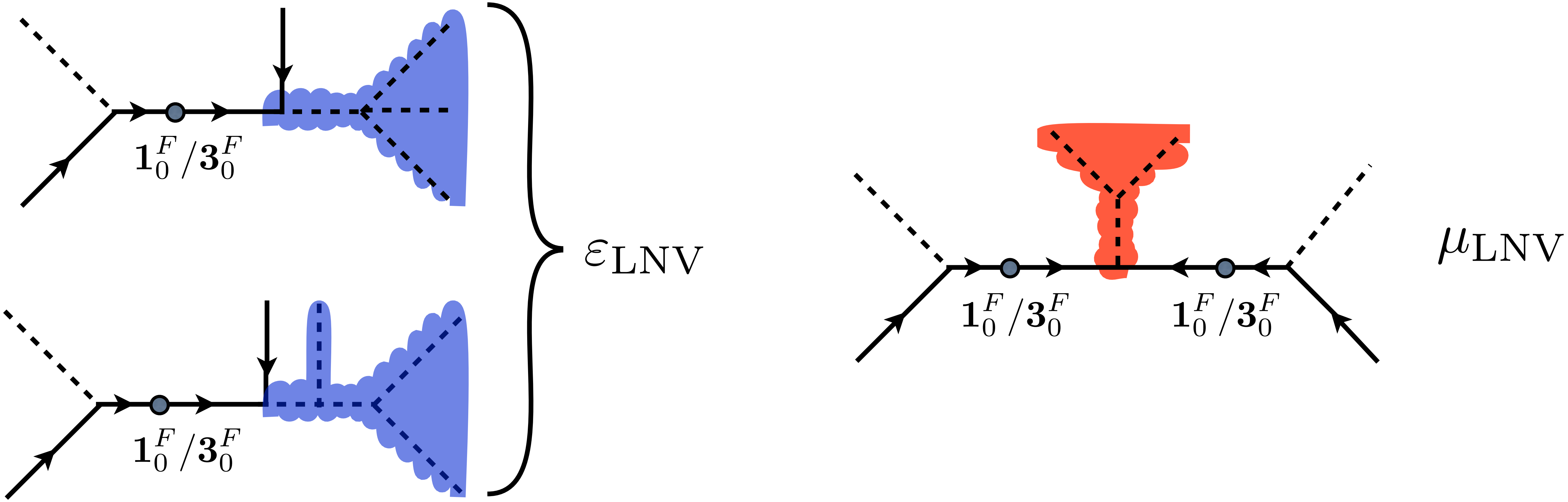
$$\Phi \sim \mathbf{2}^S_{+1/2}$$

$$N,\ N' \sim \mathbf{1}^F_0$$

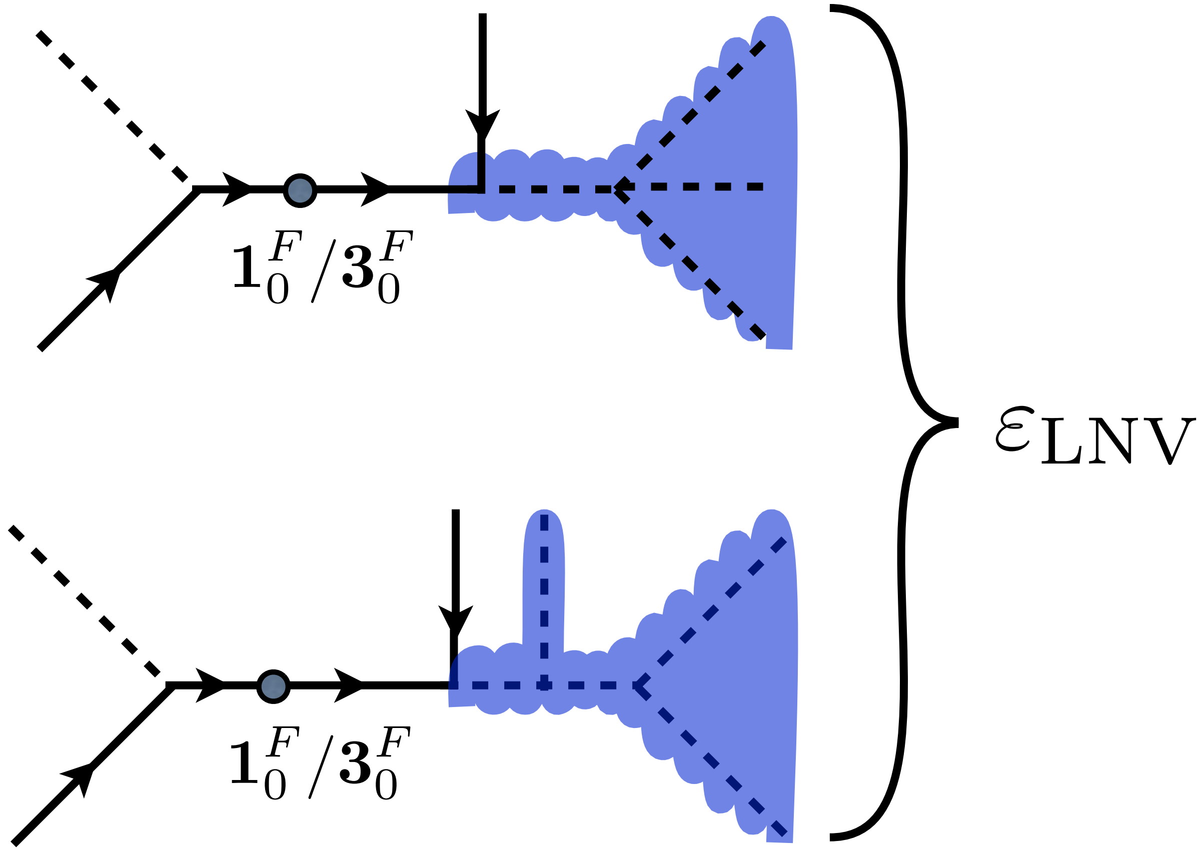
$$\boxed{m_\nu = \frac{\zeta v_u^3 v_d}{4 M_\Phi^4} \left(Y_\nu^T \Lambda^{-1} Y'_\nu + {Y'_\nu}^T \Lambda^{-1} Y_\nu \right)}$$

$$\begin{pmatrix} 0 & Y_\nu^T \langle H_u^0 \rangle & \frac{\zeta \langle H_d^0 \rangle \langle H_u^0 \rangle^2}{M_\Phi^2} {Y'_\nu}^T \\ Y_\nu \langle H_u^0 \rangle & 0 & \Lambda \\ \frac{\zeta \langle H_d^0 \rangle \langle H_u^0 \rangle^2}{M_\Phi^2} Y'_\nu & \Lambda & 0 \end{pmatrix} \xrightarrow[M_\Phi \to \infty]{} \begin{pmatrix} 0 & Y_\nu^T \langle H_u^0 \rangle & \varepsilon_{\rm LNV} {Y'_\nu}^T \\ Y_\nu \langle H_u^0 \rangle & 0 & \Lambda \\ \varepsilon_{\rm LNV} Y'_\nu & \Lambda & 0 \end{pmatrix}$$

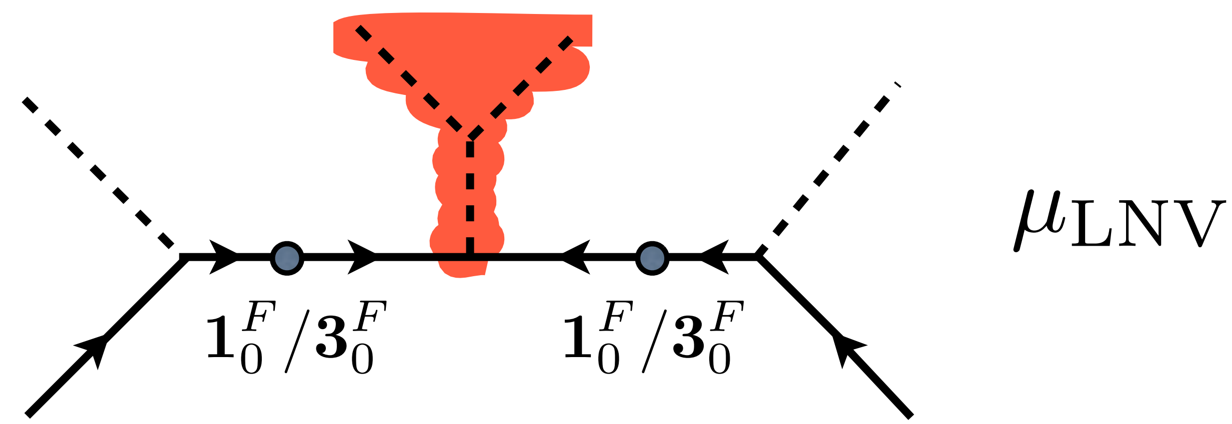
d>5 operator :


$\mathbf{1}_0^F/\mathbf{3}_0^F$
$\mathbf{1}_0^F/\mathbf{3}_0^F$
$\varepsilon_{\rm LNV}$
$\mathbf{1}_0^F/\mathbf{3}_0^F$
$\mathbf{1}_0^F/\mathbf{3}_0^F$
$\mu_{\rm LNV}$

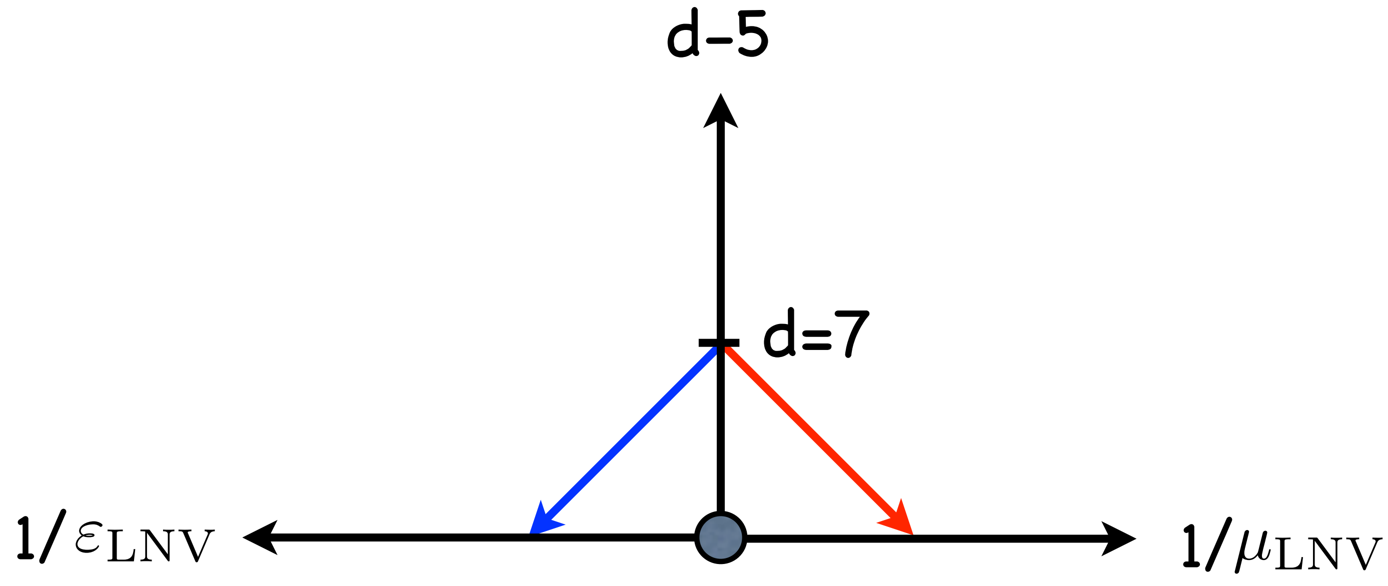
d>5 operator :



$\mathbf{1}_0^F/\mathbf{3}_0^F$
$\mathbf{1}_0^F/\mathbf{3}_0^F$
$\varepsilon_{\mathrm{LNV}}$

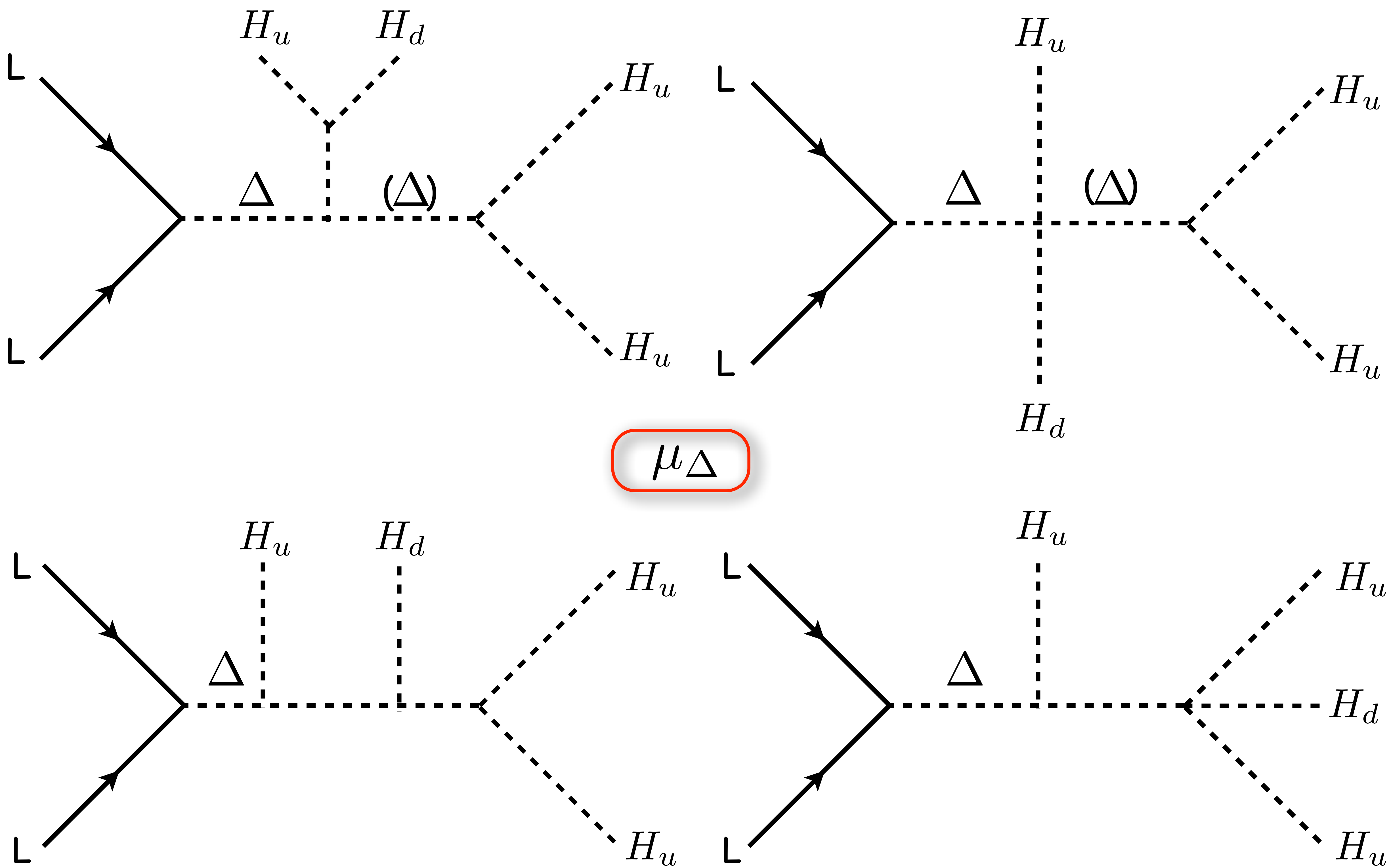


$\mathbf{1}_0^F/\mathbf{3}_0^F$
$\mathbf{1}_0^F/\mathbf{3}_0^F$
μ_{LNV}

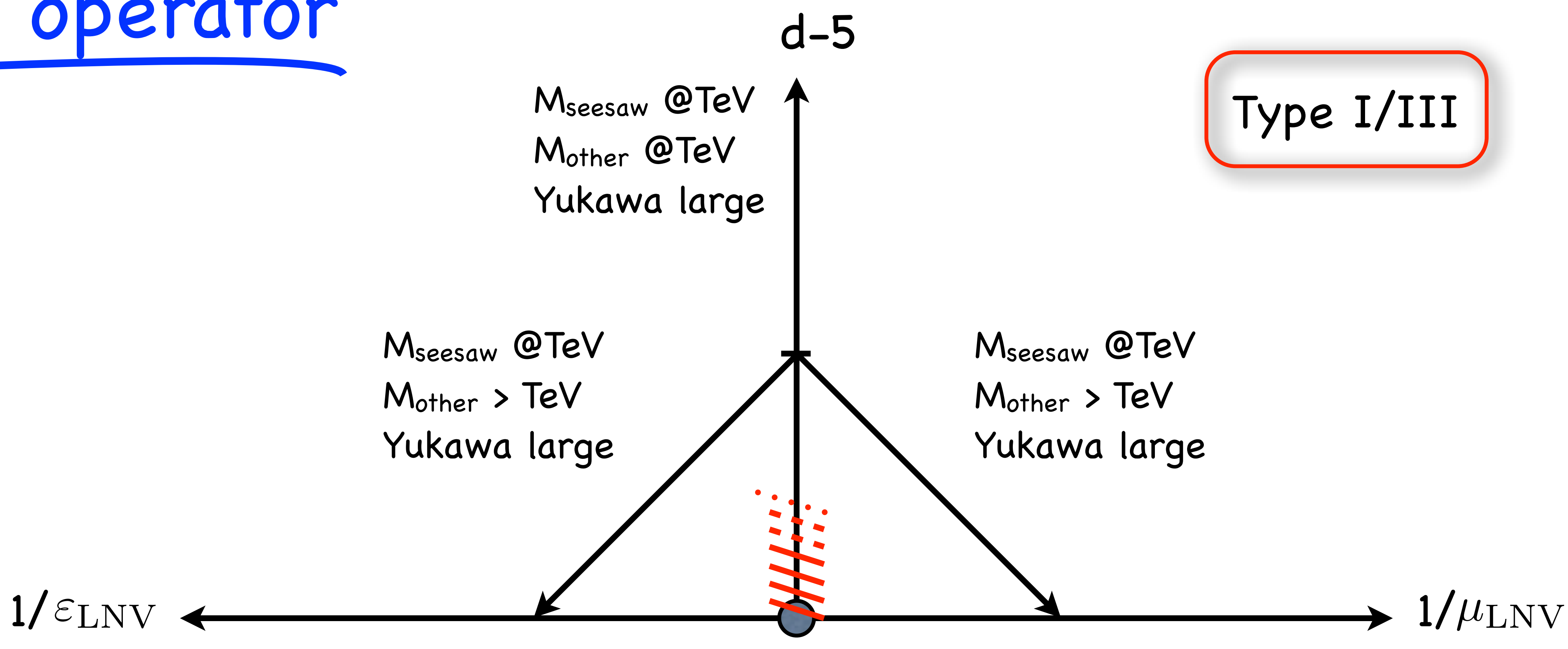


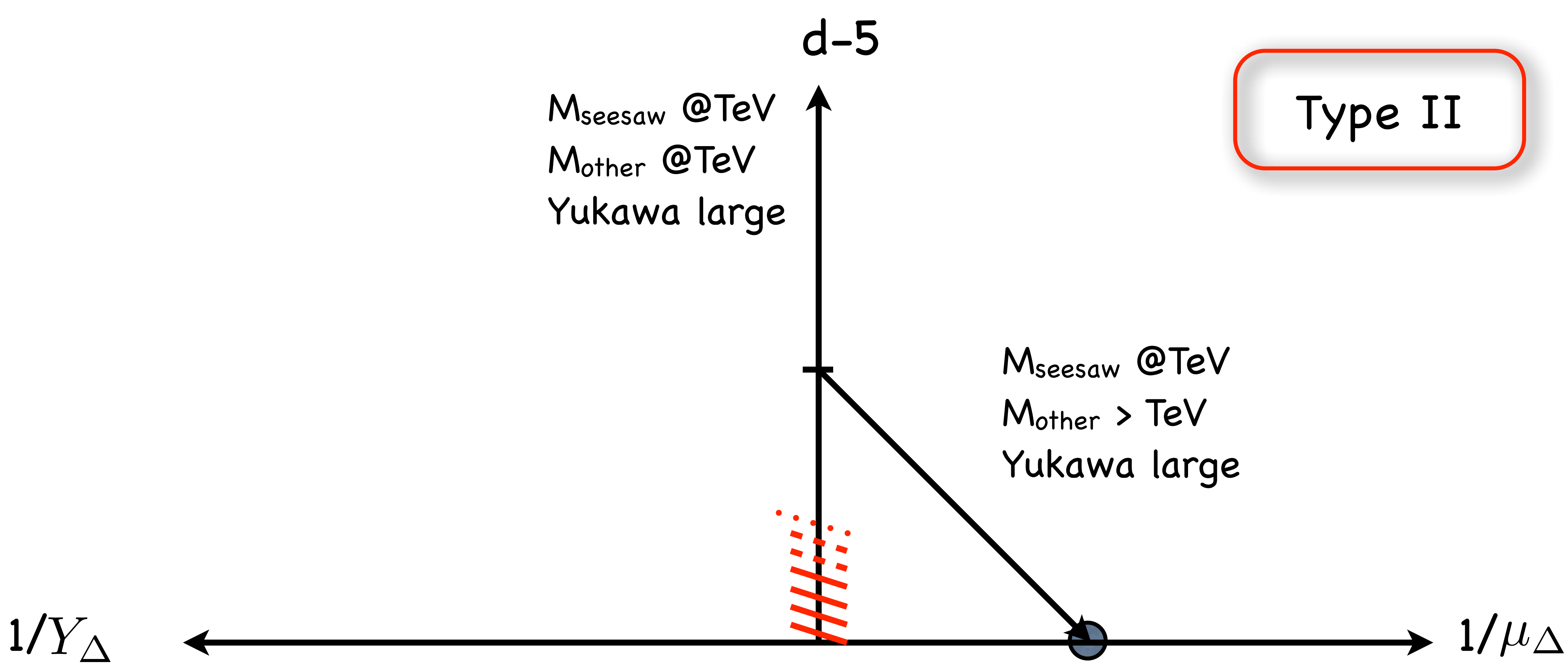
d-5
d=7
$1/\varepsilon_{\mathrm{LNV}}$
$1/\mu_{\mathrm{LNV}}$

d>5 operator : Type II



d>5 operator


d-5
Type I/III
M_{seesaw} @TeV
M_{other} @TeV
Yukawa large
M_{seesaw} @TeV
M_{other} > TeV
Yukawa large
M_{seesaw} @TeV
M_{other} > TeV
Yukawa large
$1/\varepsilon_{\rm LNV}$
$1/\mu_{\rm LNV}$

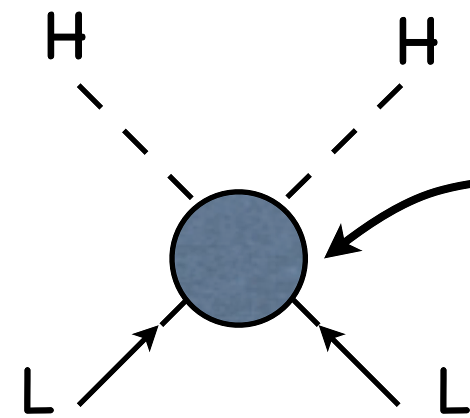

d-5
Type II
M_{seesaw} @TeV
M_{other} @TeV
Yukawa large
M_{seesaw} @TeV
M_{other} > TeV
Yukawa large
$1/Y_\Delta$
$1/\mu_\Delta$

Radiative neutrino masses

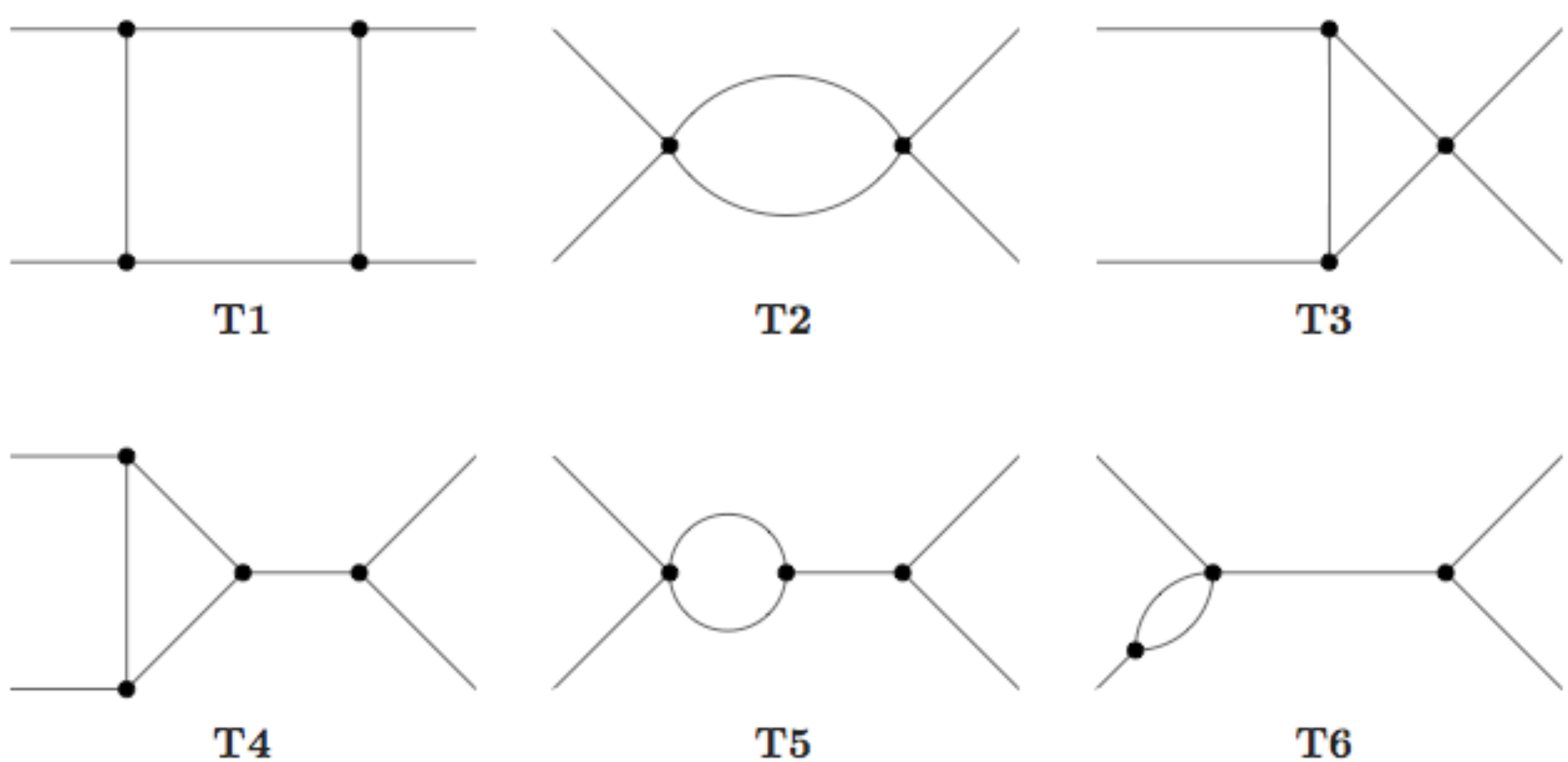
one-loop d=5

arXiv:1205.5140, to be published in JHEP

concept :



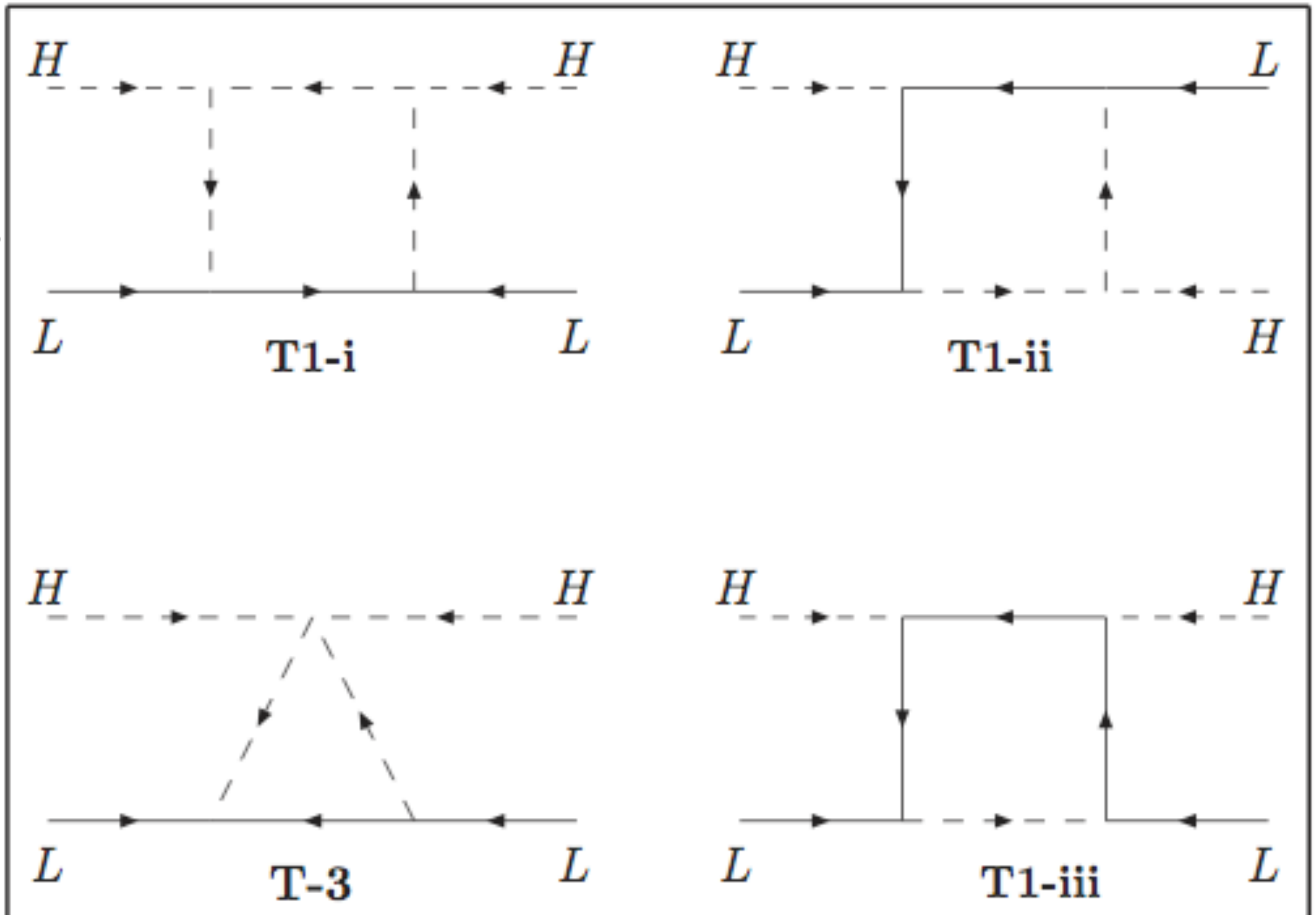
1 loop only, no self-energy



one-loop d=5

Other mechanism

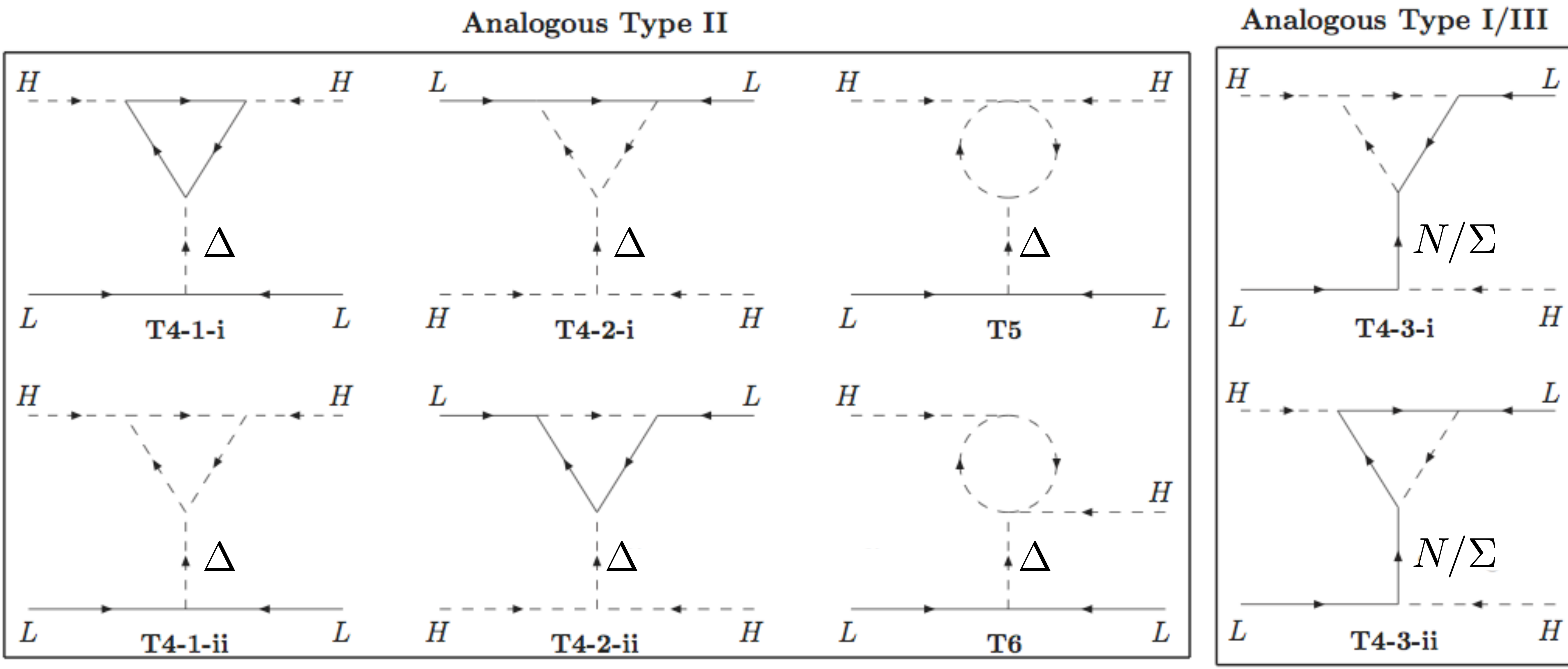
Include Dark doublet
Ma 2006
Kubo, Ma, Suematsu 2006



Include Zee Model
Zee 1980

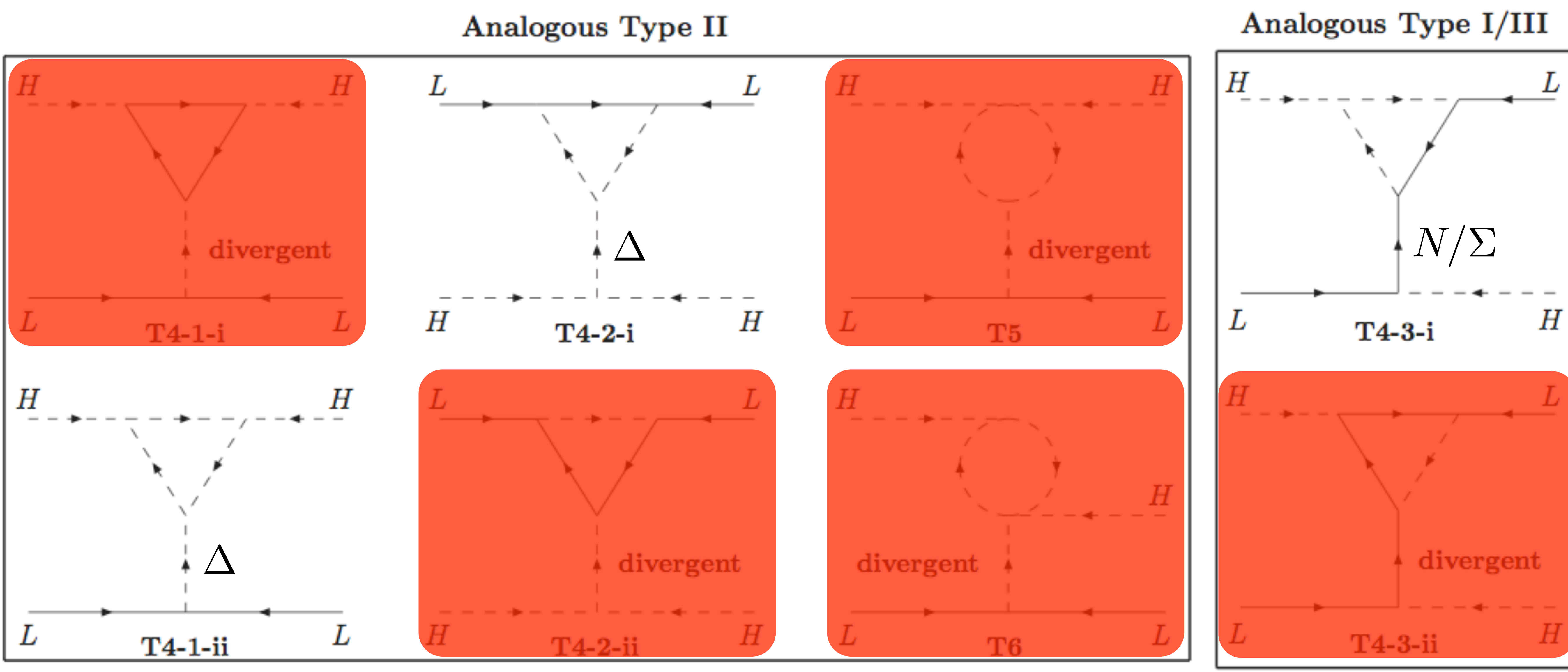
Partially Studied in Ma 1998

one-loop d=5



Analogous Type II
Analogous Type I/III
H
H
L
L
H
H
Δ
T4-1-i
T4-2-i
T5
T4-1-ii
T4-2-ii
T6
T4-3-i
T4-3-ii
N/Σ

one-loop d=5



Analogous Type II
Analogous Type I/III
H H L L T4-1-i divergent
L L H H T4-2-i Δ
H H L L T5 divergent
H L L H T4-3-i N/Σ
H H L L T4-1-ii Δ
L L H H T4-2-ii divergent
H H L L T6 divergent
H L L H T4-3-ii divergent

one-loop d=5

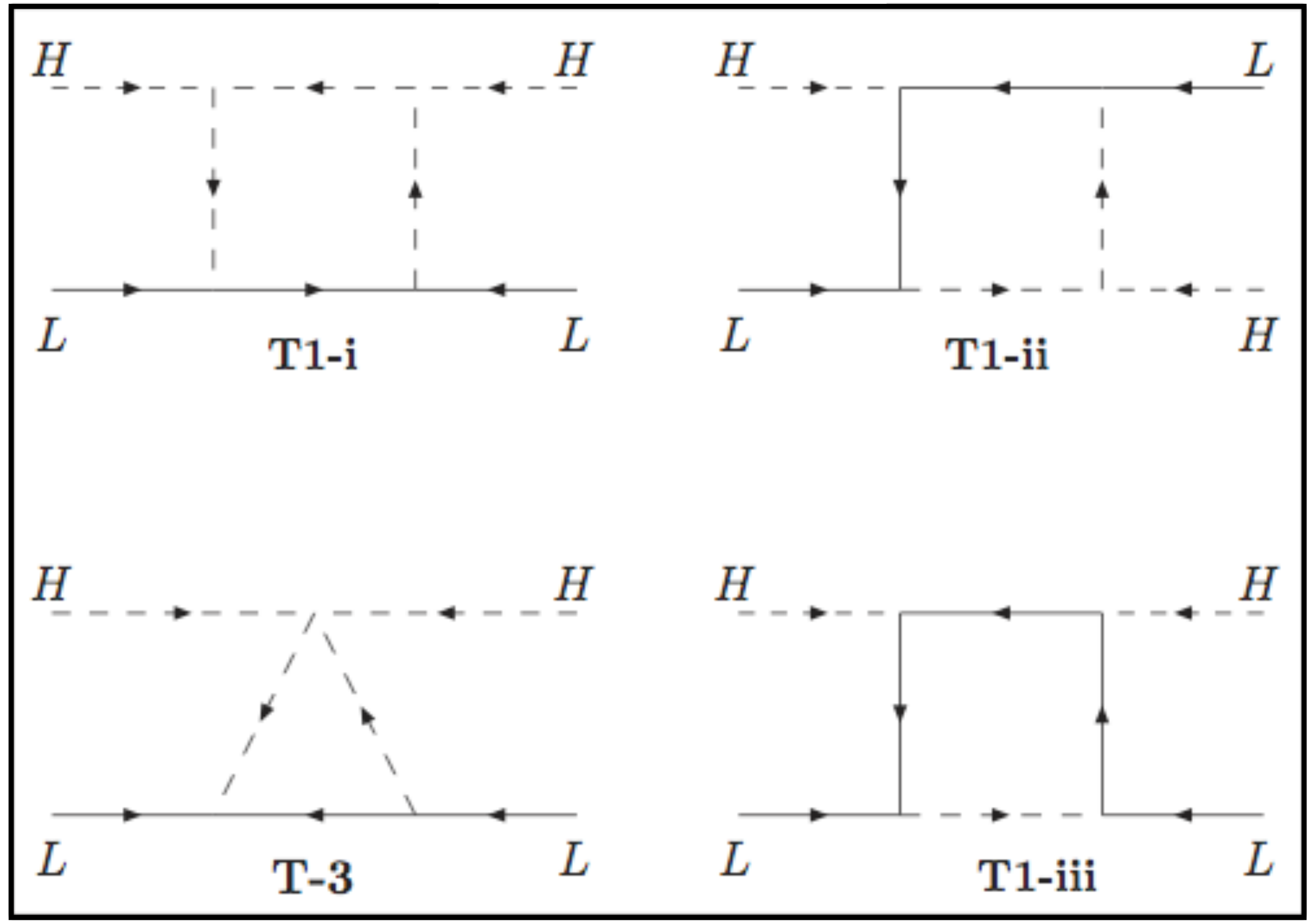
problem :

Forbid tree-level d=5

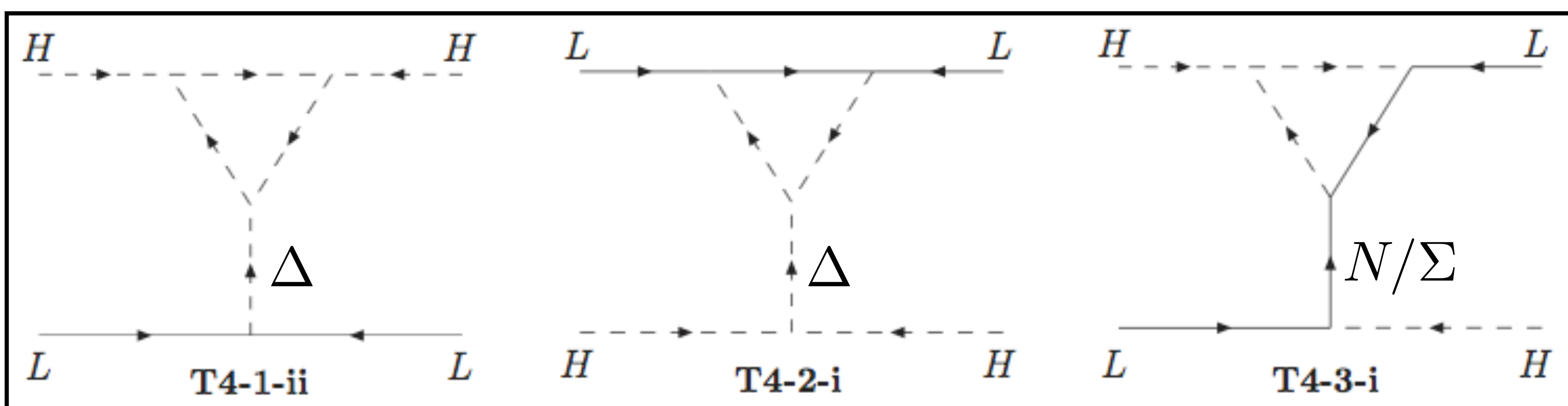
solution :

It depends ...

Other

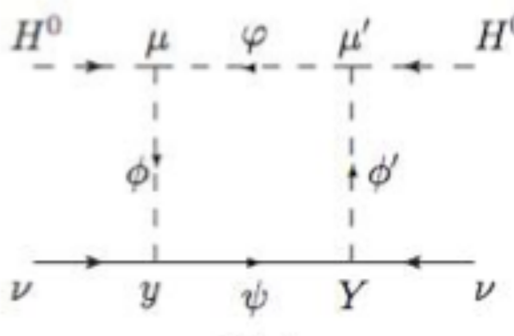


Loop Seesaw



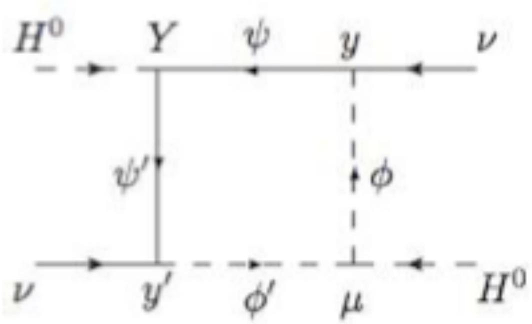
one-loop d=5

Diagram	Fields



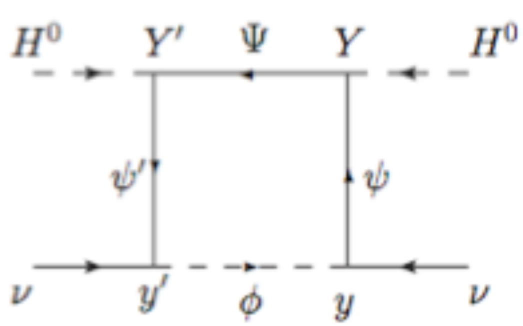
T1-1

φ	ϕ'	ψ	ϕ
1^S_α	$2^S_{\alpha-1}$	1^F_α	$2^S_{1+\alpha}$
1^S_α	$2^S_{\alpha-1}$	3^F_α	$2^S_{1+\alpha}$
2^S_α	$1^S_{\alpha-1}$	2^F_α	$1^S_{1+\alpha}$
2^S_α	$1^S_{\alpha-1}$	2^F_α	$3^S_{1+\alpha}$
2^S_α	$3^S_{\alpha-1}$	2^F_α	$1^S_{1+\alpha}$
2^S_α	$3^S_{\alpha-1}$	2^F_α	$3^S_{1+\alpha}$
3^S_α	$2^S_{\alpha-1}$	1^F_α	$2^S_{1+\alpha}$
3^S_α	$2^S_{\alpha-1}$	3^F_α	$2^S_{1+\alpha}$



T1-2

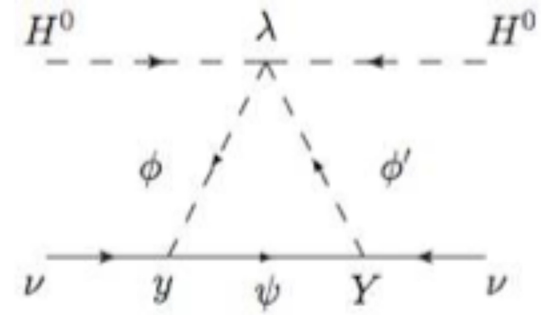
ψ	ϕ	ϕ'	ψ'
1^F_α	$2^S_{1+\alpha}$	1^S_α	$2^F_{1+\alpha}$
1^F_α	$2^S_{1+\alpha}$	3^S_α	$2^F_{1+\alpha}$
2^F_α	$1^S_{1+\alpha}$	2^S_α	$1^F_{1+\alpha}$
2^F_α	$1^S_{1+\alpha}$	2^S_α	$3^F_{1+\alpha}$
2^F_α	$3^S_{1+\alpha}$	2^S_α	$1^F_{1+\alpha}$
2^F_α	$3^S_{1+\alpha}$	2^S_α	$3^F_{1+\alpha}$
3^F_α	$2^S_{1+\alpha}$	1^S_α	$2^F_{1+\alpha}$
3^F_α	$2^S_{1+\alpha}$	3^S_α	$2^F_{1+\alpha}$



T1-3

Ψ	ψ'	ϕ	ψ
1^F_α	$2^F_{1+\alpha}$	1^S_α	$2^F_{\alpha-1}$
1^F_α	$2^F_{1+\alpha}$	3^S_α	$2^F_{\alpha-1}$
2^F_α	$1^F_{1+\alpha}$	2^S_α	$1^F_{\alpha-1}$
2^F_α	$1^F_{1+\alpha}$	2^S_α	$3^F_{\alpha-1}$
2^F_α	$3^F_{1+\alpha}$	2^S_α	$1^F_{\alpha-1}$
2^F_α	$3^F_{1+\alpha}$	2^S_α	$3^F_{\alpha-1}$
3^F_α	$2^F_{1+\alpha}$	1^S_α	$2^F_{\alpha-1}$
3^F_α	$2^F_{1+\alpha}$	3^S_α	$2^F_{\alpha-1}$

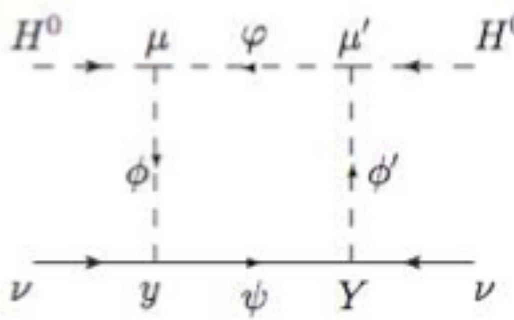
Diagram	Fields



T3

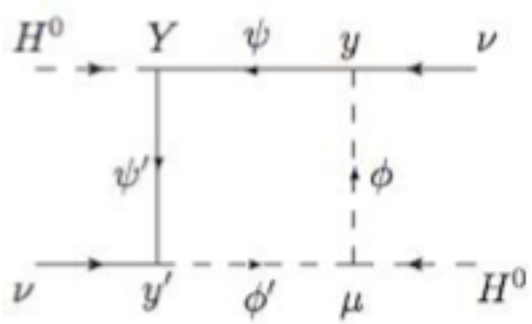
ϕ'	ϕ	ψ
1^F_α	$3^S_{2+\alpha}$	$2^S_{1+\alpha}$
2^F_α	$2^S_{2+\alpha}$	$1^S_{1+\alpha}$
2^F_α	$2^S_{2+\alpha}$	$3^S_{1+\alpha}$
3^F_α	$1^S_{2+\alpha}$	$2^S_{1+\alpha}$
3^F_α	$3^S_{2+\alpha}$	$2^S_{1+\alpha}$

one-loop d=5



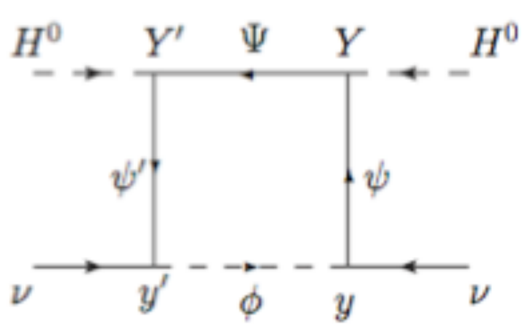
T1-1

φ	ϕ'	ψ	ϕ
1^S_α	$2^S_{\alpha-1}$	1^F_α	$2^S_{1+\alpha}$
1^S_α	$2^S_{\alpha-1}$	3^F_α	$2^S_{1+\alpha}$
2^S_α	$1^S_{\alpha-1}$	2^F_α	$1^S_{1+\alpha}$
2^S_α	$1^S_{\alpha-1}$	2^F_α	$3^S_{1+\alpha}$
2^S_α	$3^S_{\alpha-1}$	2^F_α	$1^S_{1+\alpha}$
2^S_α	$3^S_{\alpha-1}$	2^F_α	$3^S_{1+\alpha}$
3^S_α	$2^S_{\alpha-1}$	1^F_α	$2^S_{1+\alpha}$
3^S_α	$2^S_{\alpha-1}$	3^F_α	$2^S_{1+\alpha}$



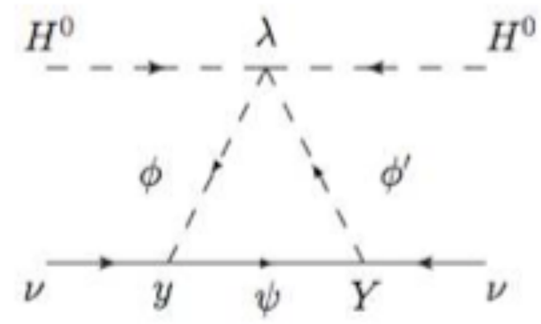
T1-2

ψ	ϕ	ϕ'	ψ'
1^F_α	$2^S_{1+\alpha}$	1^S_α	$2^F_{1+\alpha}$
1^F_α	$2^S_{1+\alpha}$	3^S_α	$2^F_{1+\alpha}$
2^F_α	$1^S_{1+\alpha}$	2^S_α	$1^F_{1+\alpha}$
2^F_α	$1^S_{1+\alpha}$	2^S_α	$3^F_{1+\alpha}$
2^F_α	$3^S_{1+\alpha}$	2^S_α	$1^F_{1+\alpha}$
2^F_α	$3^S_{1+\alpha}$	2^S_α	$3^F_{1+\alpha}$
3^F_α	$2^S_{1+\alpha}$	1^S_α	$2^F_{1+\alpha}$
3^F_α	$2^S_{1+\alpha}$	3^S_α	$2^F_{1+\alpha}$



T1-3

Ψ	ψ'	ϕ	ψ
1^F_α	$2^F_{1+\alpha}$	1^S_α	$2^F_{\alpha-1}$
1^F_α	$2^F_{1+\alpha}$	3^S_α	$2^F_{\alpha-1}$
2^F_α	$1^F_{1+\alpha}$	2^S_α	$1^F_{\alpha-1}$
2^F_α	$1^F_{1+\alpha}$	2^S_α	$3^F_{\alpha-1}$
2^F_α	$3^F_{1+\alpha}$	2^S_α	$1^F_{\alpha-1}$
2^F_α	$3^F_{1+\alpha}$	2^S_α	$3^F_{\alpha-1}$
3^F_α	$2^F_{1+\alpha}$	1^S_α	$2^F_{\alpha-1}$
3^F_α	$2^F_{1+\alpha}$	3^S_α	$2^F_{\alpha-1}$



T3

ϕ'	ϕ	ψ
1^F_α	$3^S_{2+\alpha}$	$2^S_{1+\alpha}$
2^F_α	$2^S_{2+\alpha}$	$1^S_{1+\alpha}$
2^F_α	$2^S_{2+\alpha}$	$3^S_{1+\alpha}$
3^F_α	$1^S_{2+\alpha}$	$2^S_{1+\alpha}$
3^F_α	$3^S_{2+\alpha}$	$2^S_{1+\alpha}$

one-loop d=5

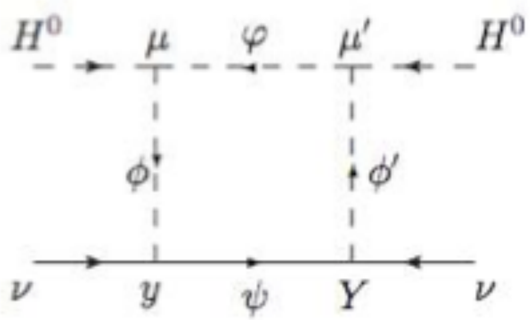
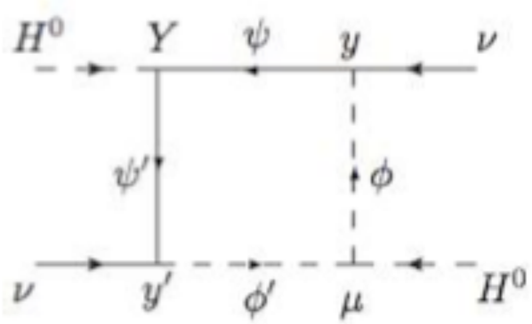
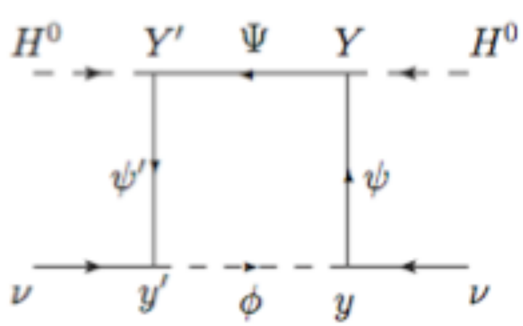
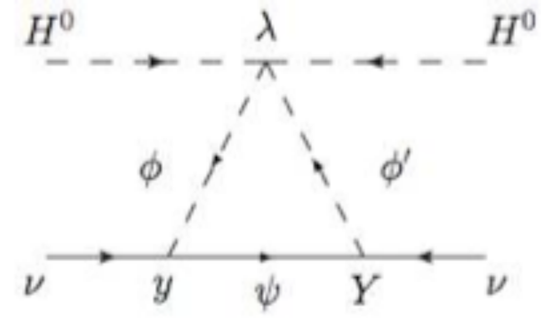
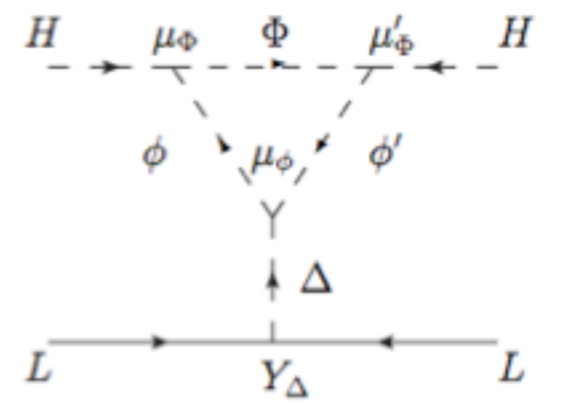
Diagram	Fields			
 T1-1	φ	ϕ'	ψ	ϕ
	1^S_α	$2^S_{\alpha-1}$	1^F_α	$2^S_{1+\alpha}$
	1^S_α	$2^S_{\alpha-1}$	3^F_α	$2^S_{1+\alpha}$
	2^S_α	$1^S_{\alpha-1}$	2^F_α	$1^S_{1+\alpha}$
	2^S_α	$1^S_{\alpha-1}$	2^F_α	$3^S_{1+\alpha}$
	2^S_α	$3^S_{\alpha-1}$	2^F_α	$1^S_{1+\alpha}$
	2^S_α	$3^S_{\alpha-1}$	2^F_α	$3^S_{1+\alpha}$
	3^S_α	$2^S_{\alpha-1}$	1^F_α	$2^S_{1+\alpha}$
	3^S_α	$2^S_{\alpha-1}$	3^F_α	$2^S_{1+\alpha}$
 T1-2	ψ	ϕ	ϕ'	ψ'
	1^F_α	$2^S_{1+\alpha}$	1^S_α	$2^F_{1+\alpha}$
	1^F_α	$2^S_{1+\alpha}$	3^S_α	$2^F_{1+\alpha}$
	2^F_α	$1^S_{1+\alpha}$	2^S_α	$1^F_{1+\alpha}$
	2^F_α	$1^S_{1+\alpha}$	2^S_α	$3^F_{1+\alpha}$
	2^F_α	$3^S_{1+\alpha}$	2^S_α	$1^F_{1+\alpha}$
	2^F_α	$3^S_{1+\alpha}$	2^S_α	$3^F_{1+\alpha}$
	3^F_α	$2^S_{1+\alpha}$	1^S_α	$2^F_{1+\alpha}$
	3^F_α	$2^S_{1+\alpha}$	3^S_α	$2^F_{1+\alpha}$
 T1-3	Ψ	ψ'	ϕ	ψ
	1^F_α	$2^F_{1+\alpha}$	1^S_α	$2^F_{\alpha-1}$
	1^F_α	$2^F_{1+\alpha}$	3^S_α	$2^F_{\alpha-1}$
	2^F_α	$1^F_{1+\alpha}$	2^S_α	$1^F_{\alpha-1}$
	2^F_α	$1^F_{1+\alpha}$	2^S_α	$3^F_{\alpha-1}$
	2^F_α	$3^F_{1+\alpha}$	2^S_α	$1^F_{\alpha-1}$
	2^F_α	$3^F_{1+\alpha}$	2^S_α	$3^F_{\alpha-1}$
	3^F_α	$2^F_{1+\alpha}$	1^S_α	$2^F_{\alpha-1}$
	3^F_α	$2^F_{1+\alpha}$	3^S_α	$2^F_{\alpha-1}$

Diagram	Fields		
 T3	ϕ'	ϕ	ψ
	1^F_α	$3^S_{2+\alpha}$	$2^S_{1+\alpha}$
	2^F_α	$2^S_{2+\alpha}$	$1^S_{1+\alpha}$
	2^F_α	$2^S_{2+\alpha}$	$3^S_{1+\alpha}$
	3^F_α	$1^S_{2+\alpha}$	$2^S_{1+\alpha}$
	3^F_α	$3^S_{2+\alpha}$	$2^S_{1+\alpha}$

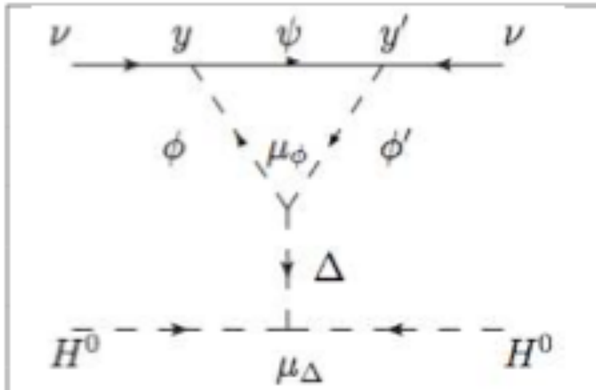
simple $\mathbb{Z}_2$ symmetry is enough

one-loop d=5



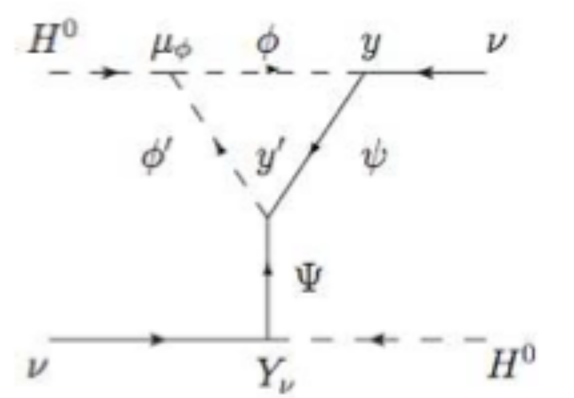
T4-1-ii

Δ	ϕ	ϕ'	Φ
3^S_{-2}	1^S_α	$3^S_{2+\alpha}$	$2^S_{1+\alpha}$
3^S_{-2}	2^S_α	$2^S_{2+\alpha}$	$1^S_{1+\alpha}$
3^S_{-2}	2^S_α	$2^S_{2+\alpha}$	$3^S_{1+\alpha}$
3^S_{-2}	3^S_α	$1^S_{2+\alpha}$	$2^S_{1+\alpha}$
3^S_{-2}	3^S_α	$3^S_{2+\alpha}$	$2^S_{1+\alpha}$



T4-2-i

Δ	ϕ'	ϕ	ψ
3^S_2	1^S_α	$3^S_{\alpha-2}$	$2^F_{\alpha-1}$
3^S_2	2^S_α	$2^S_{\alpha-2}$	$1^F_{\alpha-1}$
3^S_2	2^S_α	$2^S_{\alpha-2}$	$3^F_{\alpha-1}$
3^S_2	3^S_α	$1^S_{\alpha-2}$	$2^F_{\alpha-1}$
3^S_2	3^S_α	$3^S_{\alpha-2}$	$2^F_{\alpha-1}$

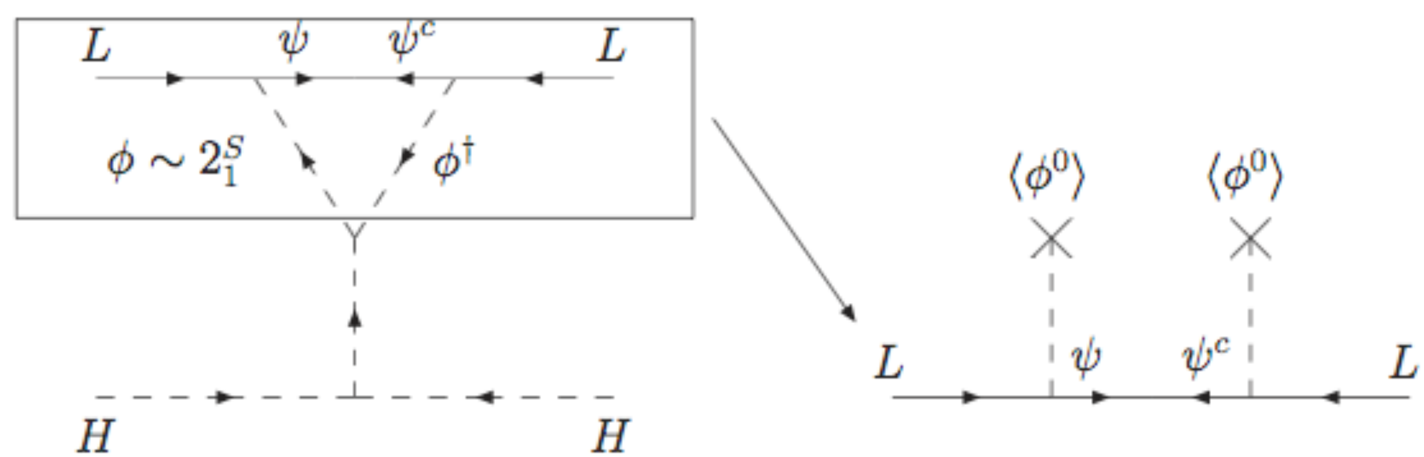


T4-3-i

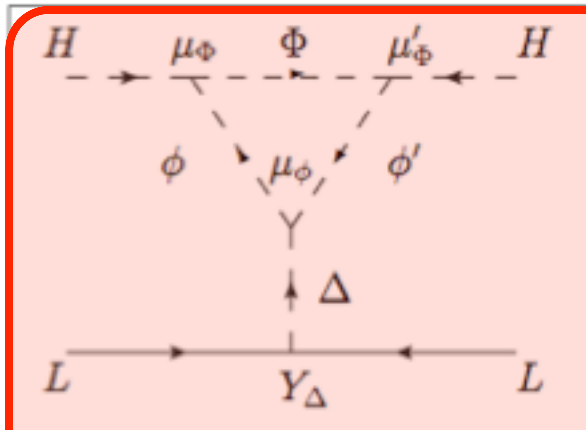
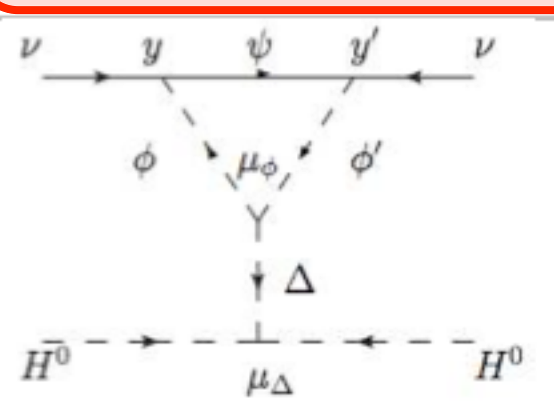
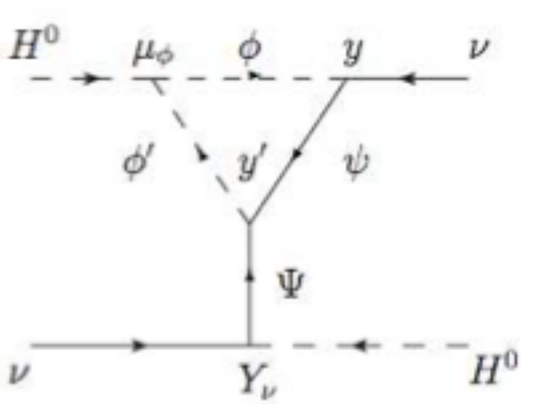
Ψ	ϕ	ϕ'	ψ
1^F_0	$1^S_{1+\alpha}$	1^S_α	2^F_α
1^F_0	$2^S_{1+\alpha}$	2^S_α	1^F_α
1^F_0	$2^S_{1+\alpha}$	2^S_α	3^F_α
1^F_0	$3^S_{1+\alpha}$	3^S_α	2^F_α
3^F_0	$1^S_{1+\alpha}$	3^S_α	2^F_α
3^F_0	$2^S_{1+\alpha}$	2^S_α	1^F_α
3^F_0	$2^S_{1+\alpha}$	2^S_α	3^F_α
3^F_0	$3^S_{1+\alpha}$	1^S_α	2^F_α
3^F_0	$3^S_{1+\alpha}$	3^S_α	2^F_α

$\mathbb{Z}_n$:
- loop = singlet
  - loop $\Rightarrow$ tree

solution :
- No LNV couplings
- Fermion in loop : Majorana
- $\mathbb{Z}_2$ to prevent scalar vev



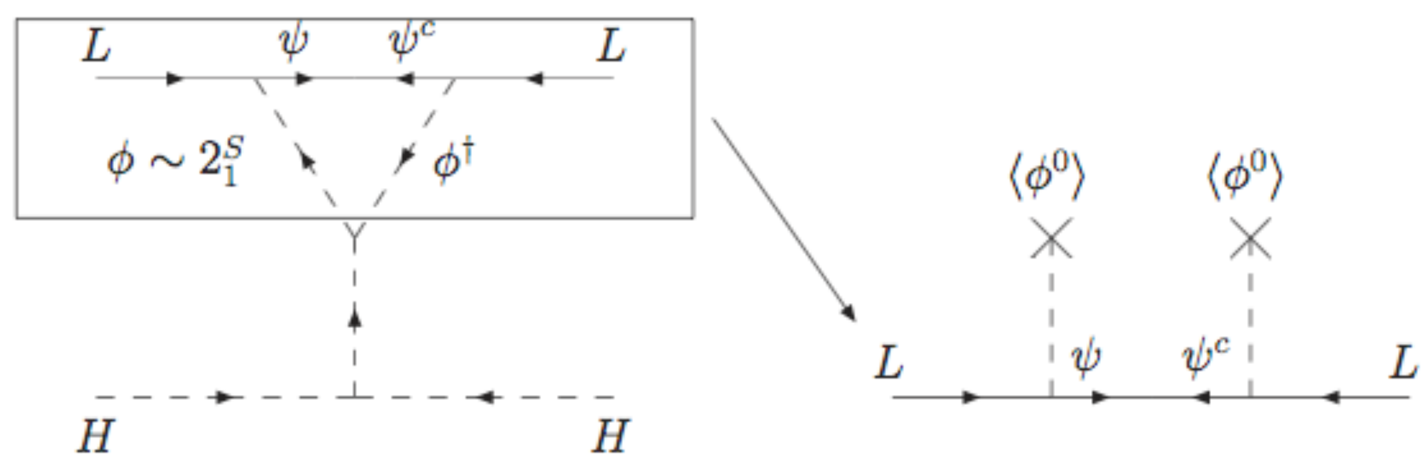
one-loop d=5

Diagram	Fields			
 T4-1-ii	Δ	ϕ	ϕ'	Φ
	3^S_{-2}	1^S_α	$3^S_{2+\alpha}$	$2^S_{1+\alpha}$
	3^S_{-2}	2^S_α	$2^S_{2+\alpha}$	$1^S_{1+\alpha}$
	3^S_{-2}	2^S_α	$2^S_{2+\alpha}$	$3^S_{1+\alpha}$
	3^S_{-2}	3^S_α	$1^S_{2+\alpha}$	$2^S_{1+\alpha}$
	3^S_{-2}	3^S_α	$3^S_{2+\alpha}$	$2^S_{1+\alpha}$
 T4-2-i	Δ	ϕ'	ϕ	ψ
	3^S_2	1^S_α	$3^S_{\alpha-2}$	$2^F_{\alpha-1}$
	3^S_2	2^S_α	$2^S_{\alpha-2}$	$1^F_{\alpha-1}$
	3^S_2	2^S_α	$2^S_{\alpha-2}$	$3^F_{\alpha-1}$
	3^S_2	3^S_α	$1^S_{\alpha-2}$	$2^F_{\alpha-1}$
	3^S_2	3^S_α	$3^S_{\alpha-2}$	$2^F_{\alpha-1}$
 T4-3-i	Ψ	ϕ	ϕ'	ψ
	1^F_0	$1^S_{1+\alpha}$	1^S_α	2^F_α
	1^F_0	$2^S_{1+\alpha}$	2^S_α	1^F_α
	1^F_0	$2^S_{1+\alpha}$	2^S_α	3^F_α
	1^F_0	$3^S_{1+\alpha}$	3^S_α	2^F_α
	3^F_0	$1^S_{1+\alpha}$	3^S_α	2^F_α
	3^F_0	$2^S_{1+\alpha}$	2^S_α	1^F_α
	3^F_0	$2^S_{1+\alpha}$	2^S_α	3^F_α
	3^F_0	$3^S_{1+\alpha}$	1^S_α	2^F_α
	3^F_0	$3^S_{1+\alpha}$	3^S_α	2^F_α

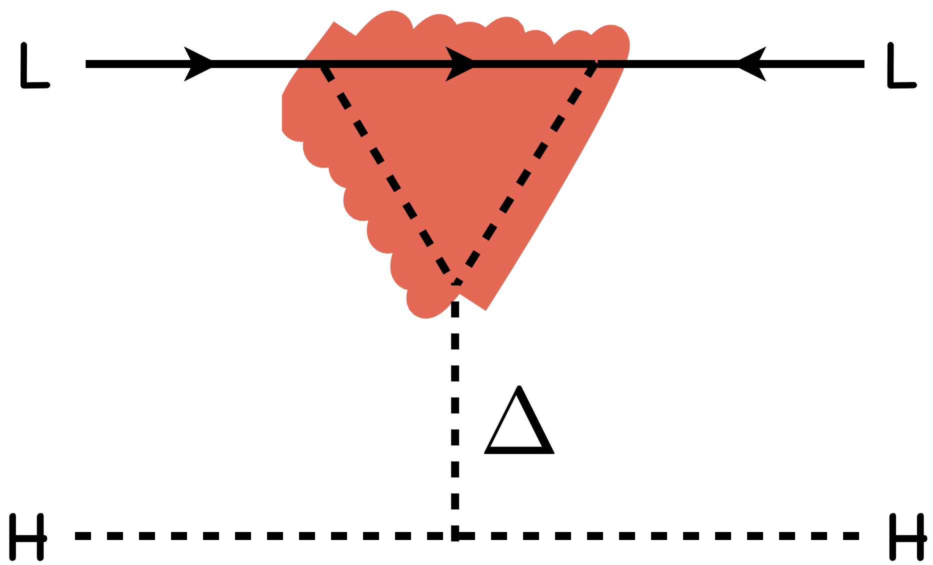
$\mathbb{Z}_n$:
- loop = singlet

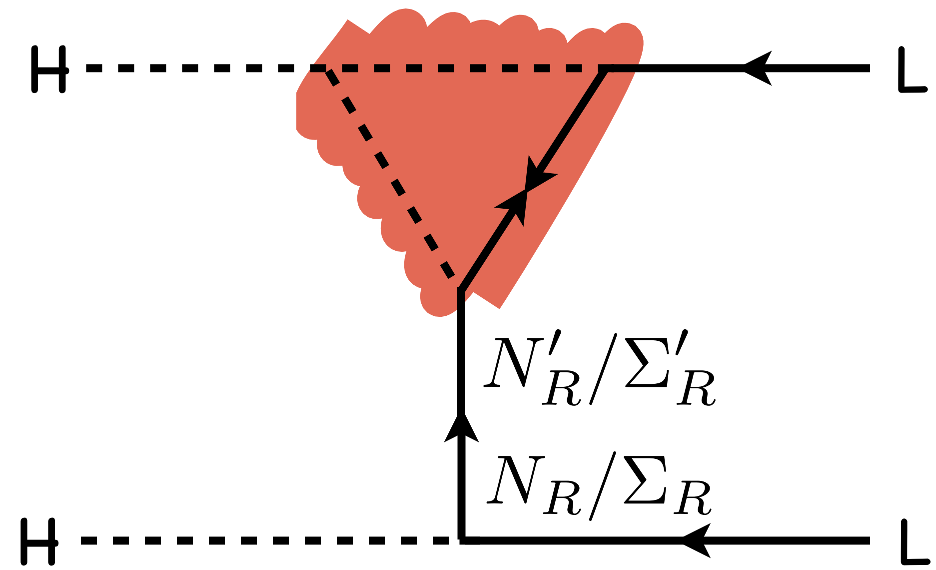
  loop $\Rightarrow$ tree

solution :
- No LNV couplings
- Fermion in loop : Majorana
- $\mathbb{Z}_2$ to prevent scalar vev



one-loop d=5





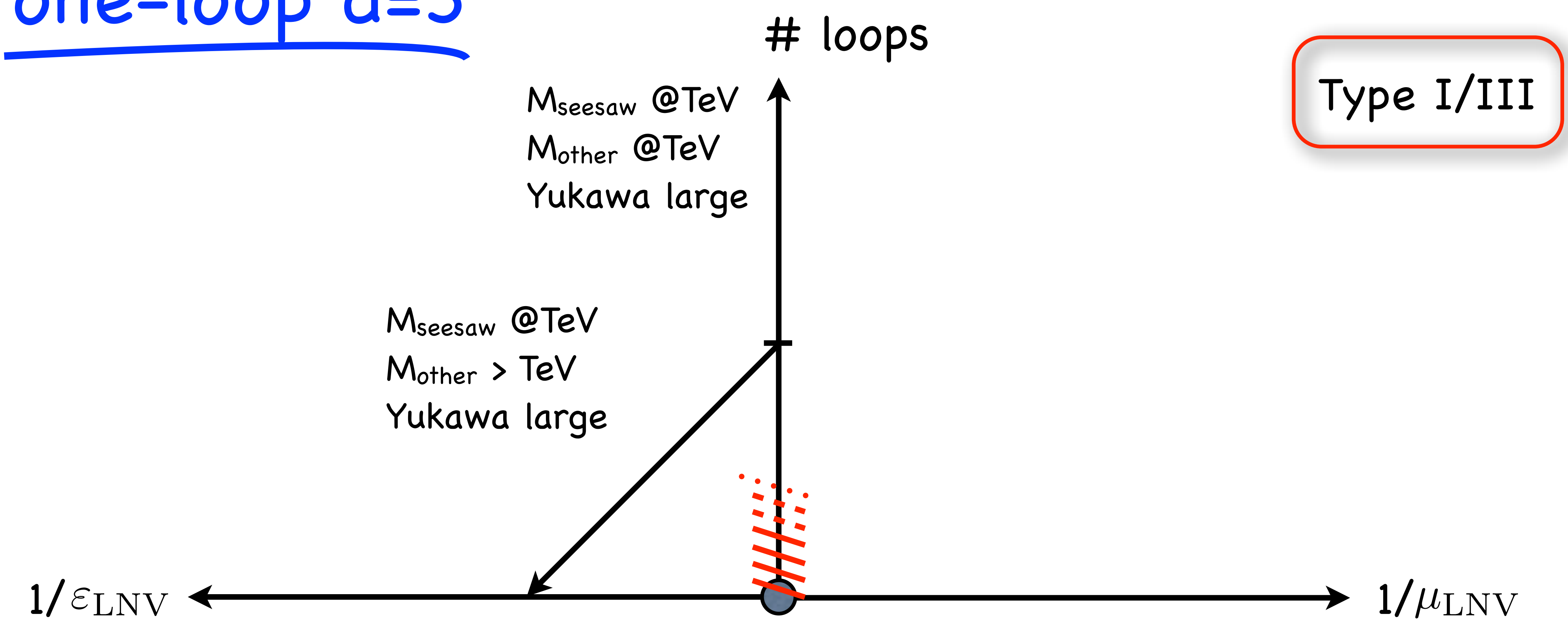
$$Y_{\Delta_{\rm loop}} \sim \varepsilon Y'_\Delta$$

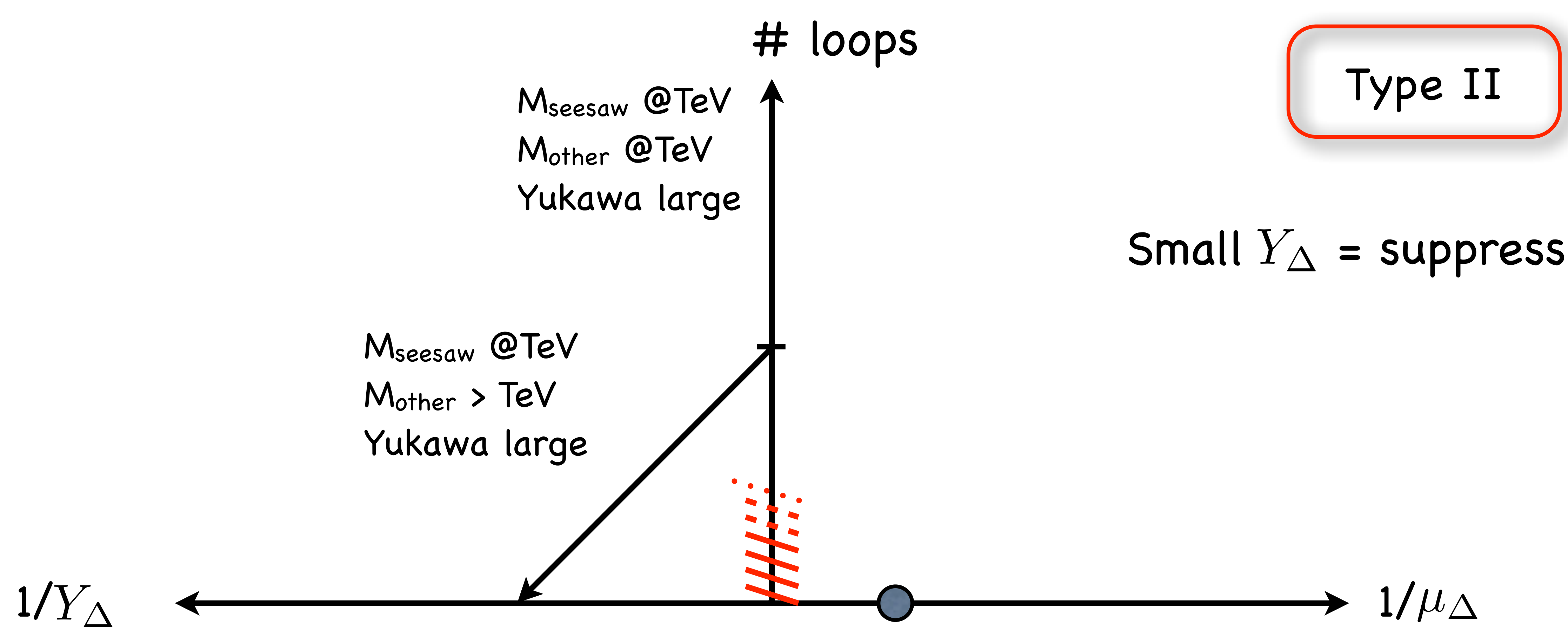
$$\begin{pmatrix} 0 & Y_\nu & \varepsilon Y'_\nu \\ Y_\nu^T & 0 & \Lambda \\ \varepsilon Y'^T_\nu & \Lambda & 0 \end{pmatrix}$$

Type II

Type I/III

one-loop d=5

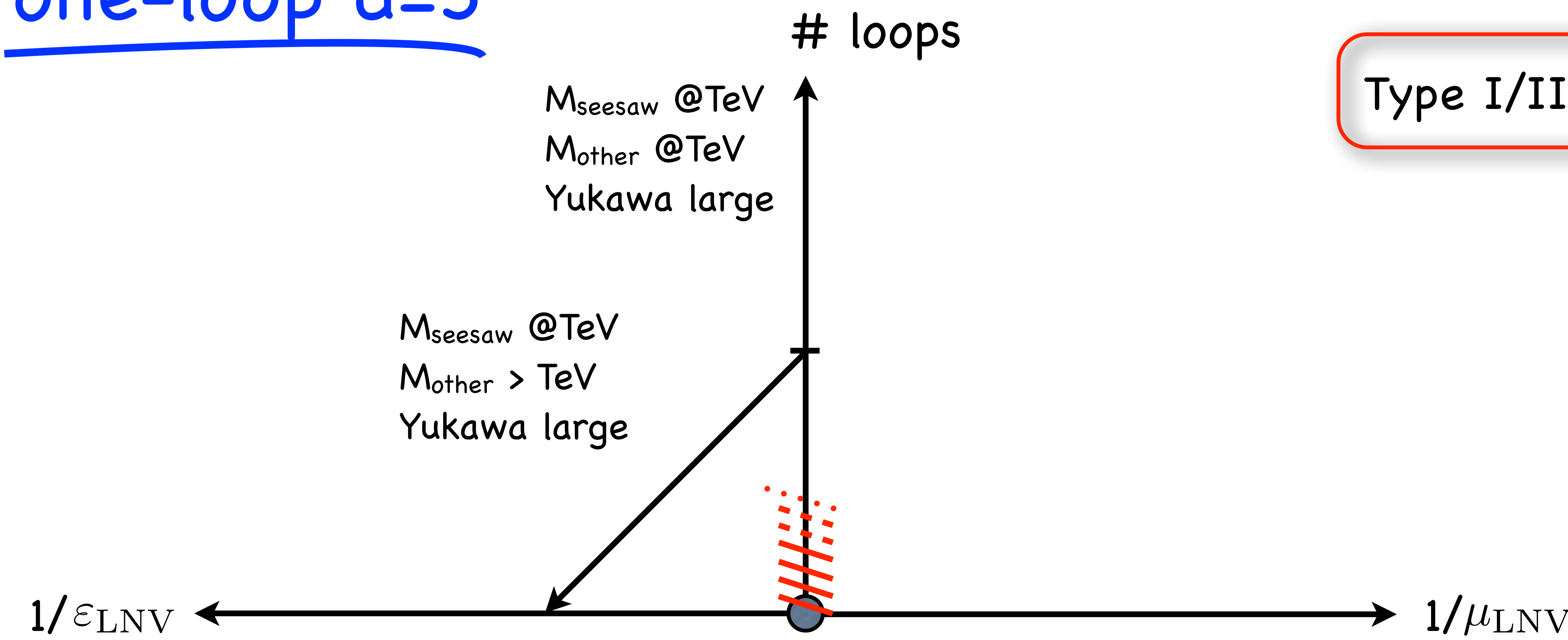




Small Y_Δ = suppressed LFV

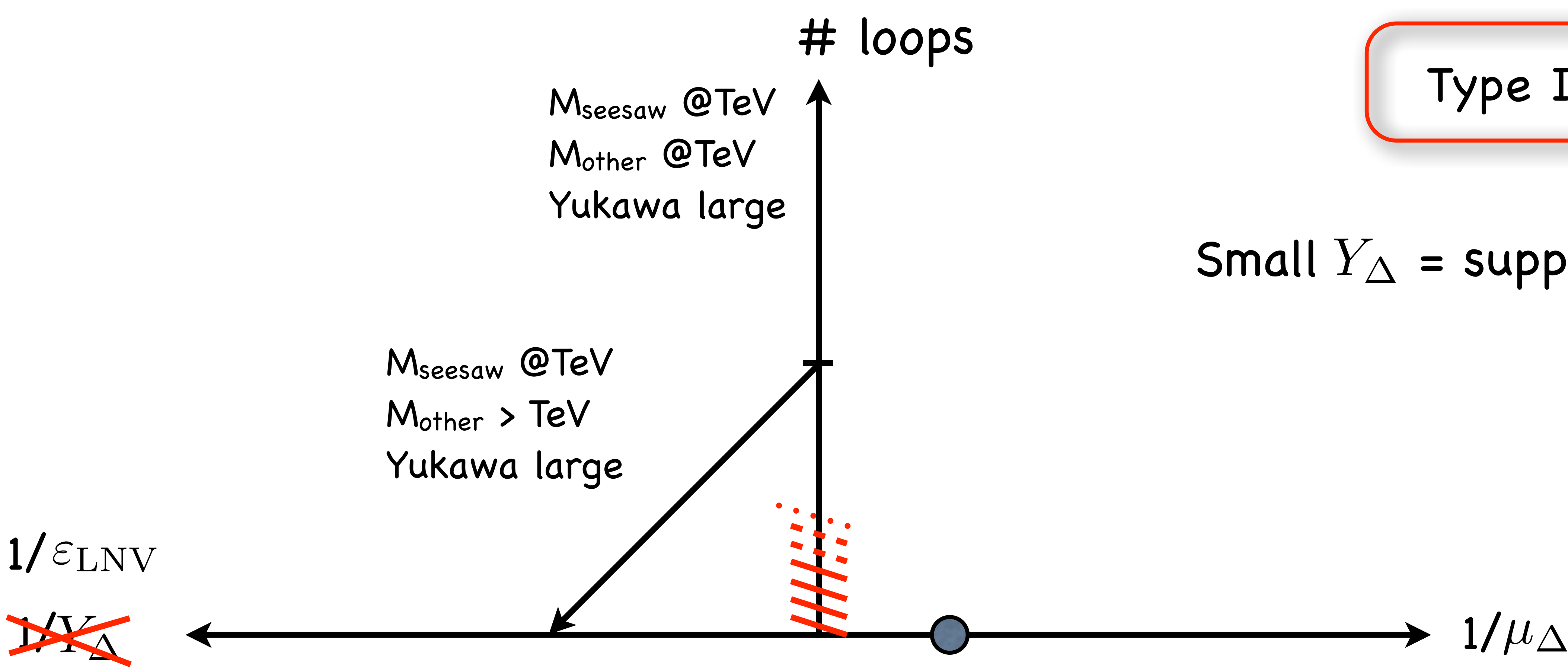
one-loop d=5

Type I/III



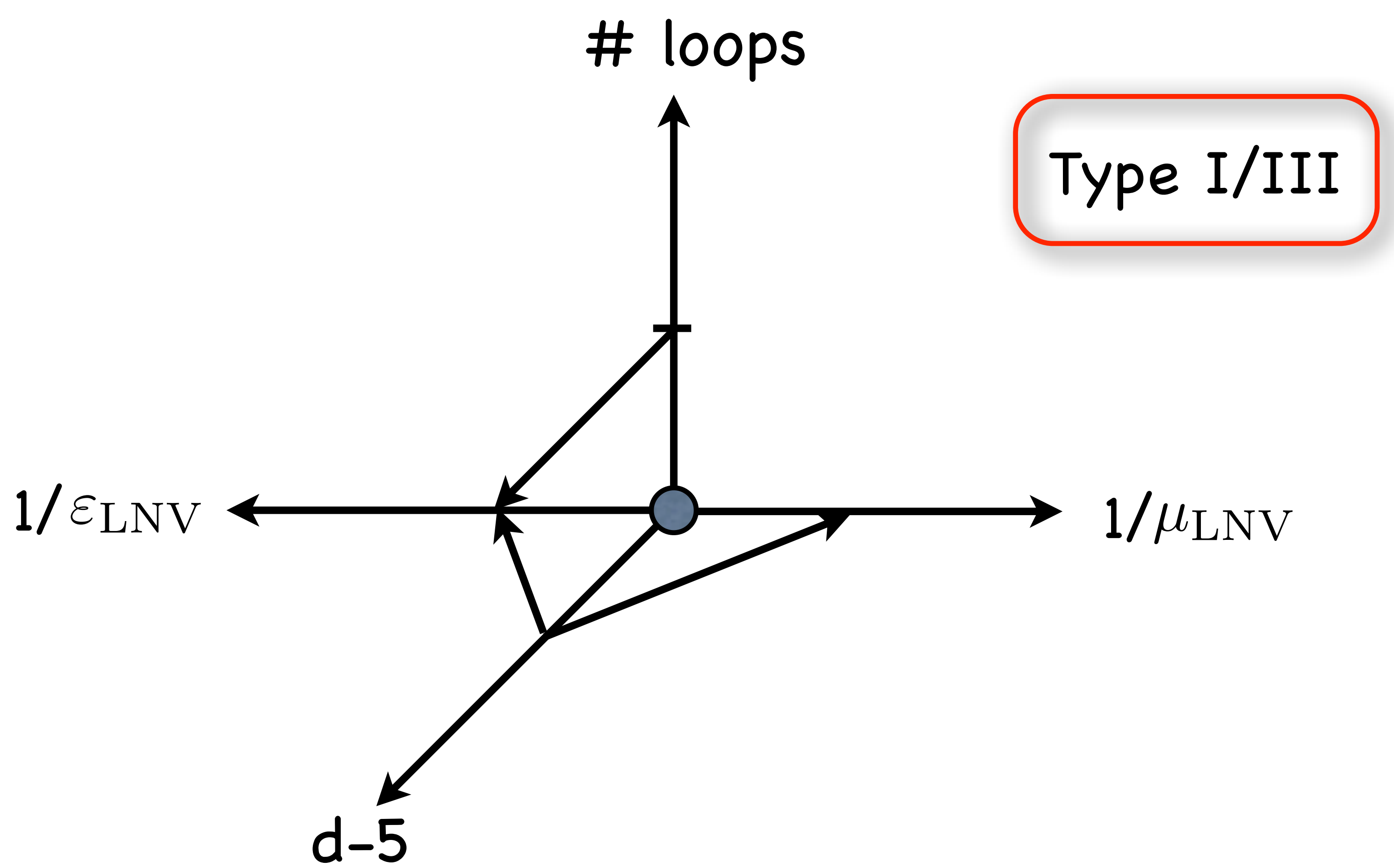
Type II

Small Y_Δ = suppressed LFV

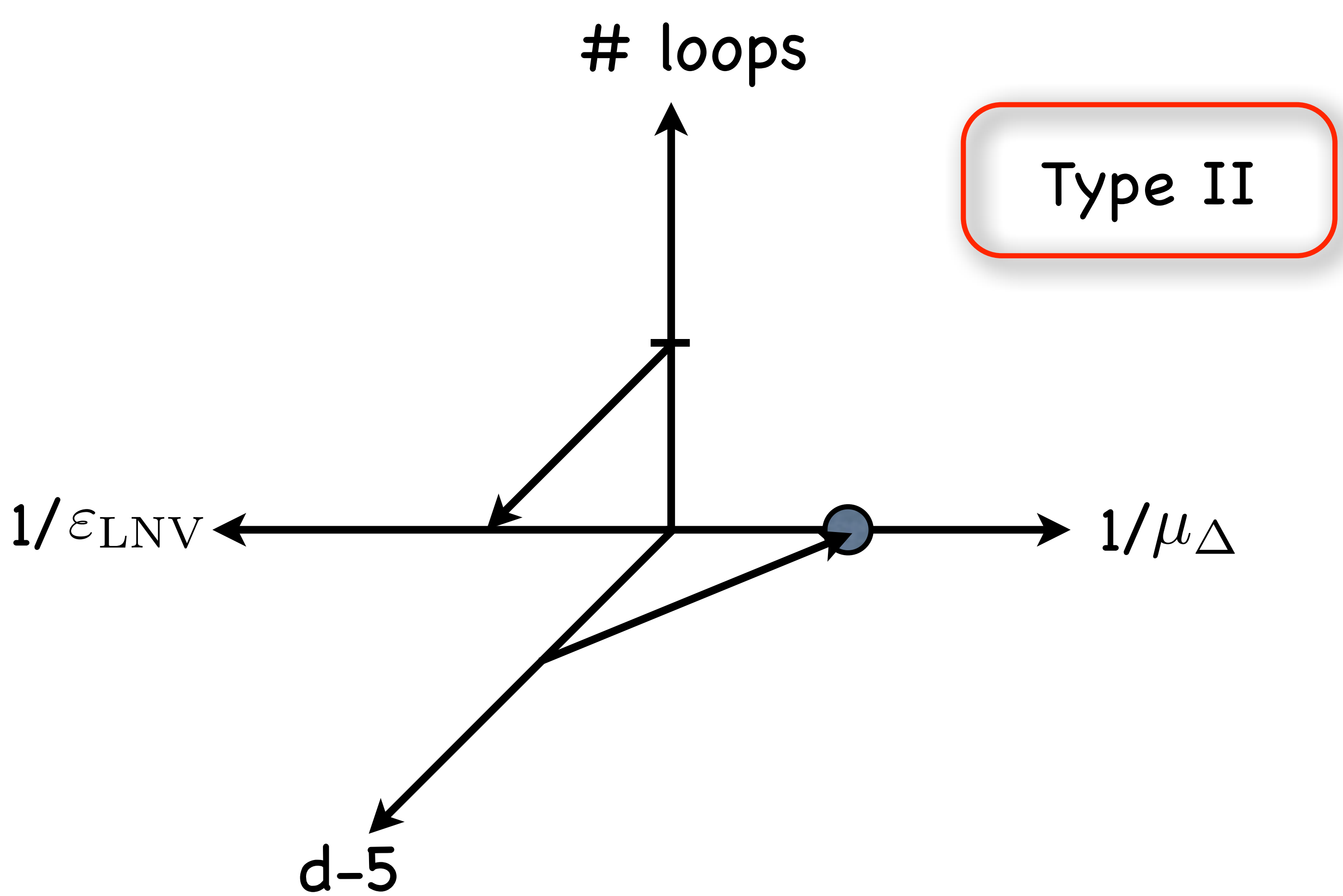


Is there a pattern here?

Global view

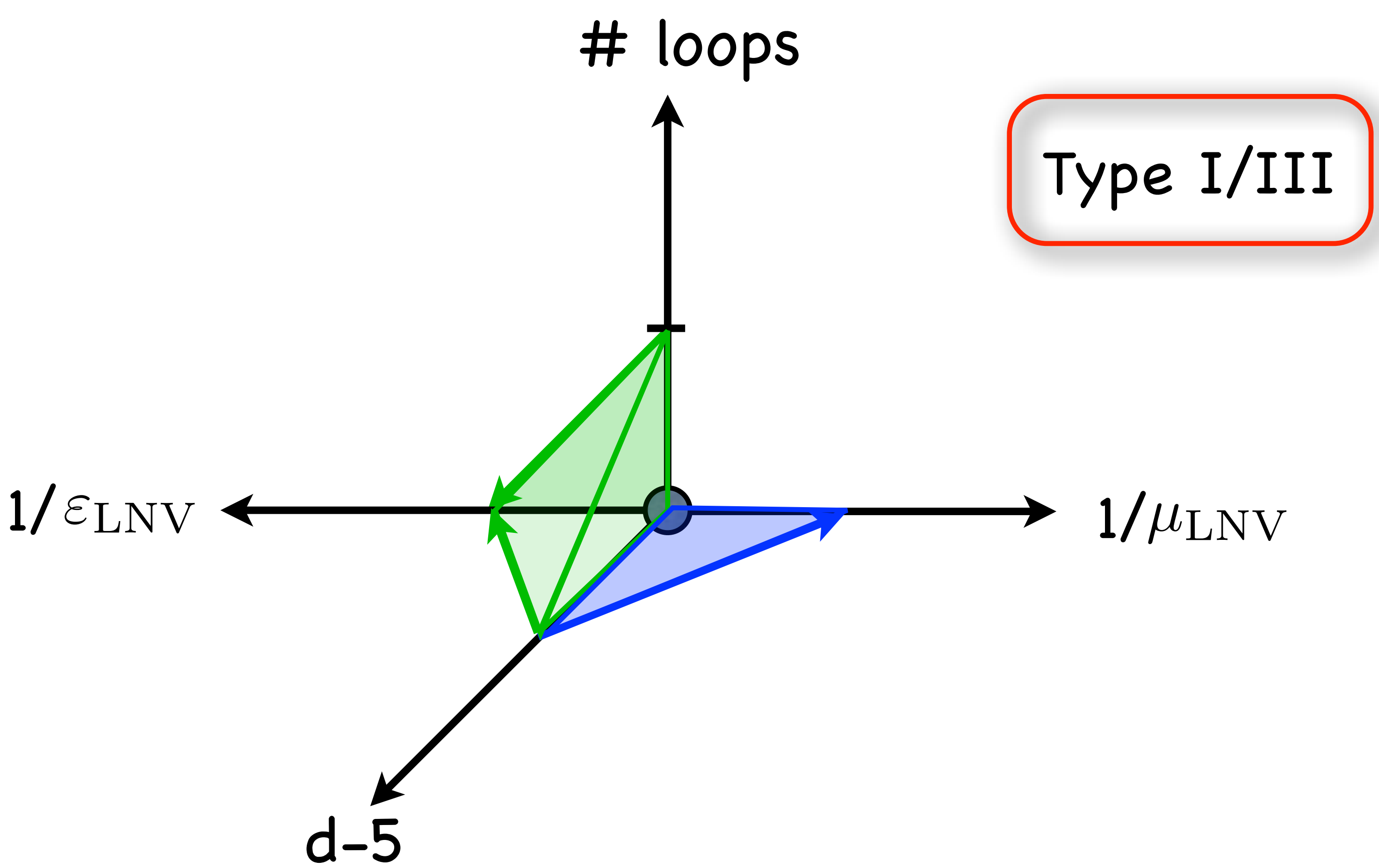


loops
Type I/III
$1/\varepsilon_{\rm LNV}$
$1/\mu_{\rm LNV}$
d-5

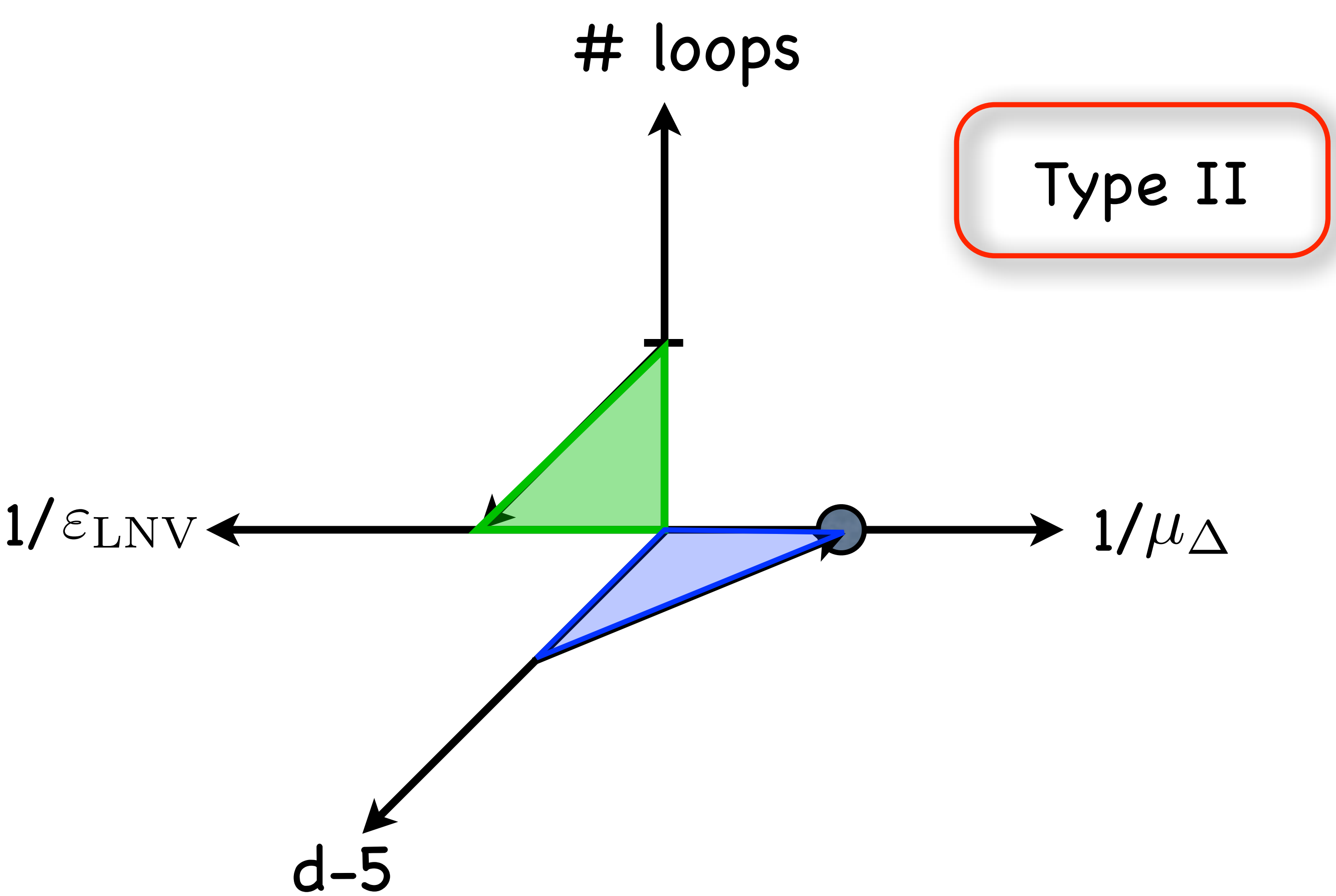


loops
Type II
$1/\varepsilon_{\rm LNV}$
$1/\mu_{\Delta}$
d-5

Global view

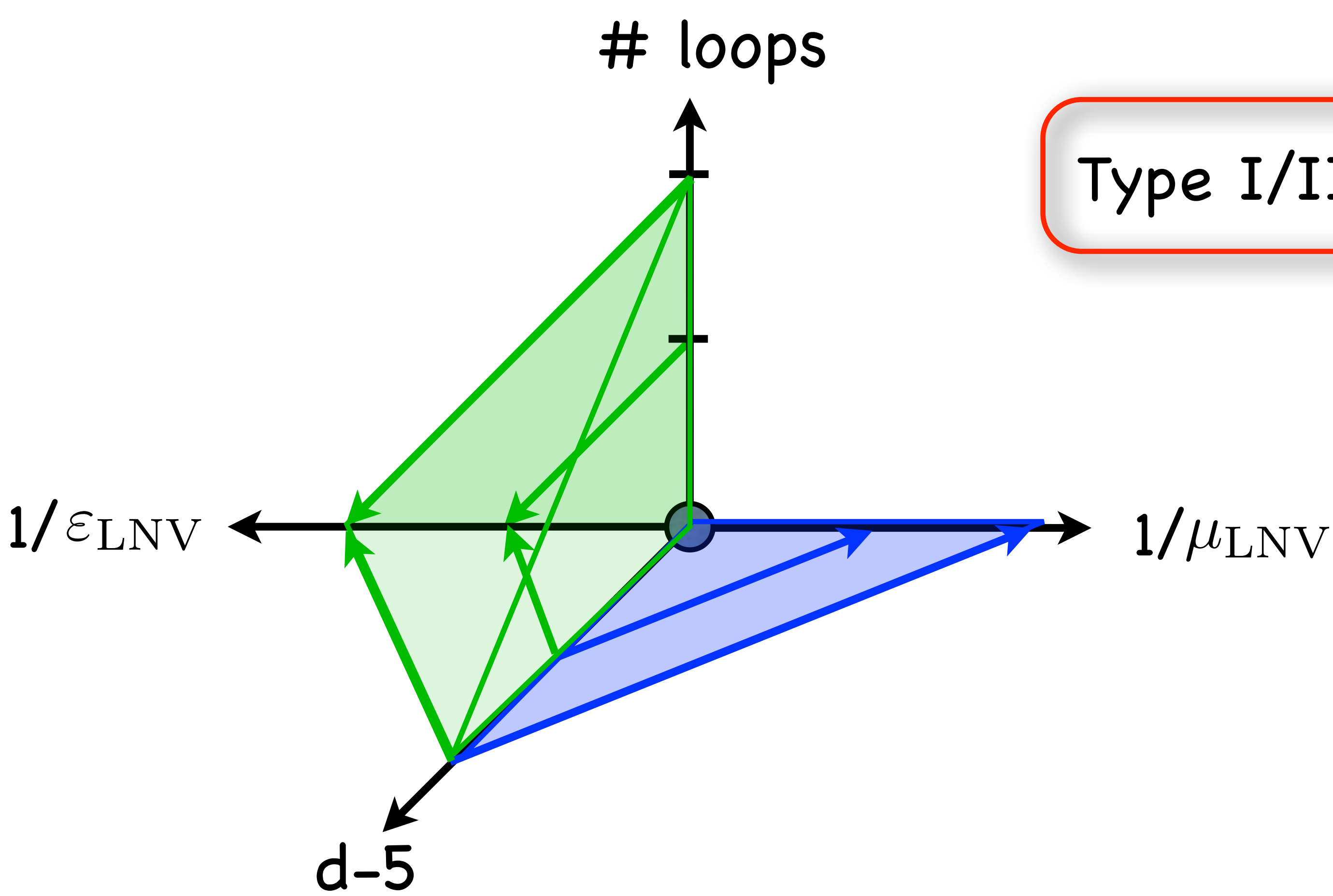


loops
Type I/III
$1/\varepsilon_{\rm LNV}$
$1/\mu_{\rm LNV}$
d-5



loops
Type II
$1/\varepsilon_{\rm LNV}$
$1/\mu_{\Delta}$
d-5

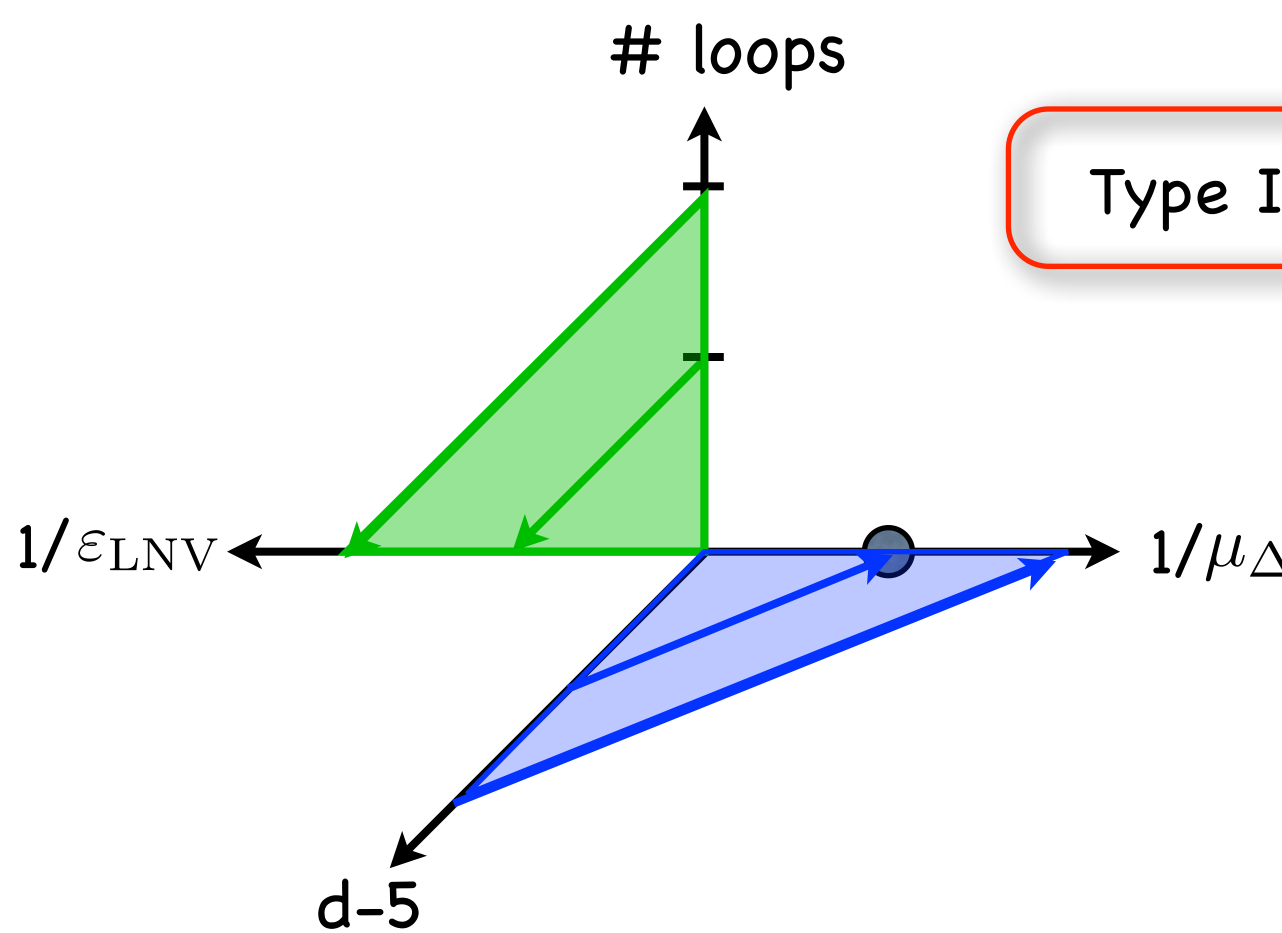
Global view



Type I/III

?

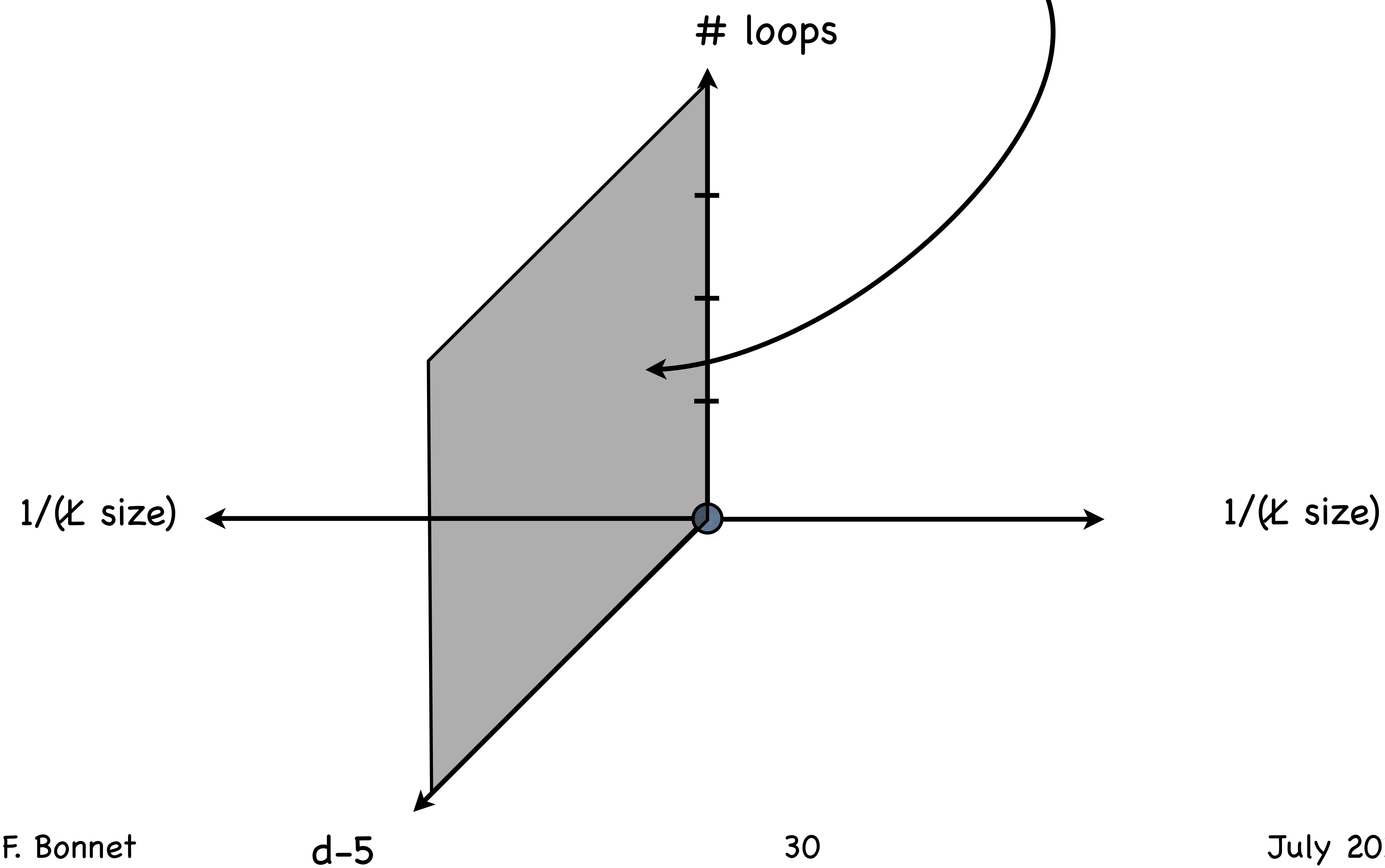
?



Type II

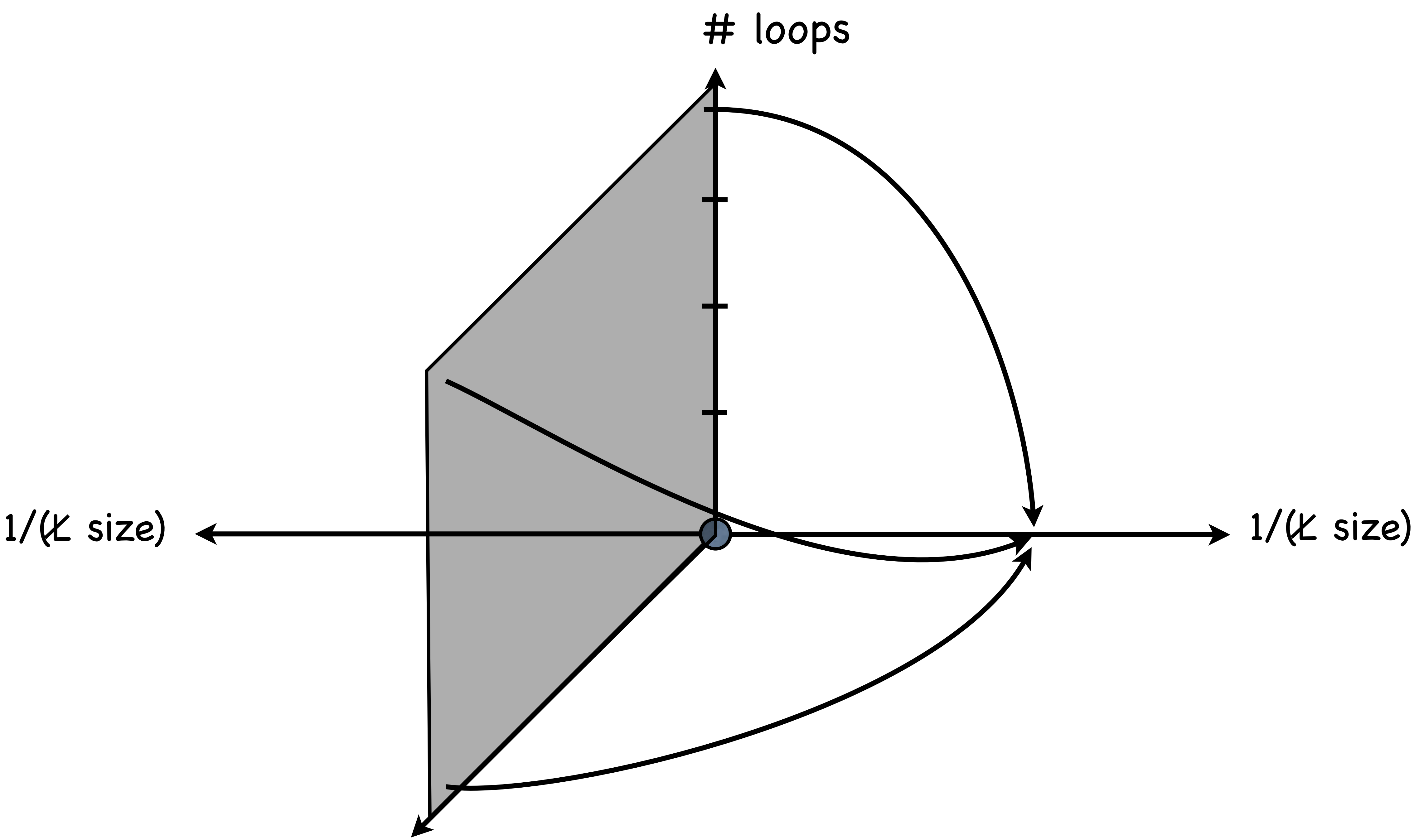
Global view

$$
\begin{aligned}
\mathcal{L}_{eff} \quad = \mathcal{L}_{SM} \quad & +\delta\mathcal{L}^{(0)}_{d=5} + \delta\mathcal{L}^{(1)}_{d=5} + \delta\mathcal{L}^{(2)}_{d=5} + \ldots \\
& +\delta\mathcal{L}^{(0)}_{d=7} + \delta\mathcal{L}^{(1)}_{d=7} + \delta\mathcal{L}^{(2)}_{d=7} + \ldots \\
& +\delta\mathcal{L}^{(0)}_{d=9} + \delta\mathcal{L}^{(1)}_{d=9} + \delta\mathcal{L}^{(2)}_{d=9} + \ldots \\
& + \ldots
\end{aligned}
$$



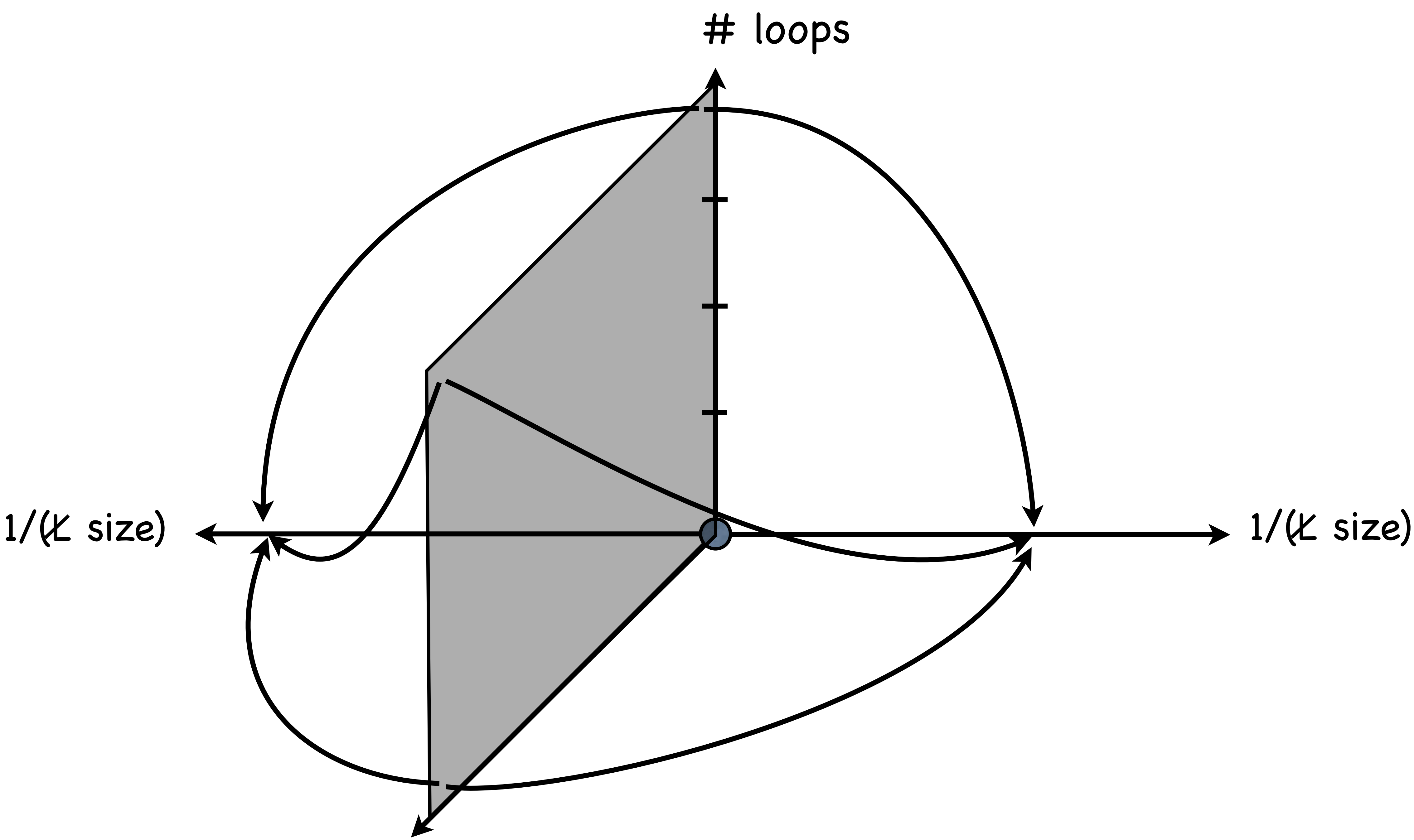
Introduction

$$
\begin{aligned}
\mathcal{L}_{eff} \quad = \mathcal{L}_{SM} \quad & +\delta\mathcal{L}^{(0)}_{d=5} + \delta\mathcal{L}^{(1)}_{d=5} + \delta\mathcal{L}^{(2)}_{d=5} + \ldots \\
& +\delta\mathcal{L}^{(0)}_{d=7} + \delta\mathcal{L}^{(1)}_{d=7} + \delta\mathcal{L}^{(2)}_{d=7} + \ldots \\
& +\delta\mathcal{L}^{(0)}_{d=9} + \delta\mathcal{L}^{(1)}_{d=9} + \delta\mathcal{L}^{(2)}_{d=9} + \ldots \\
& + \ldots
\end{aligned}
$$

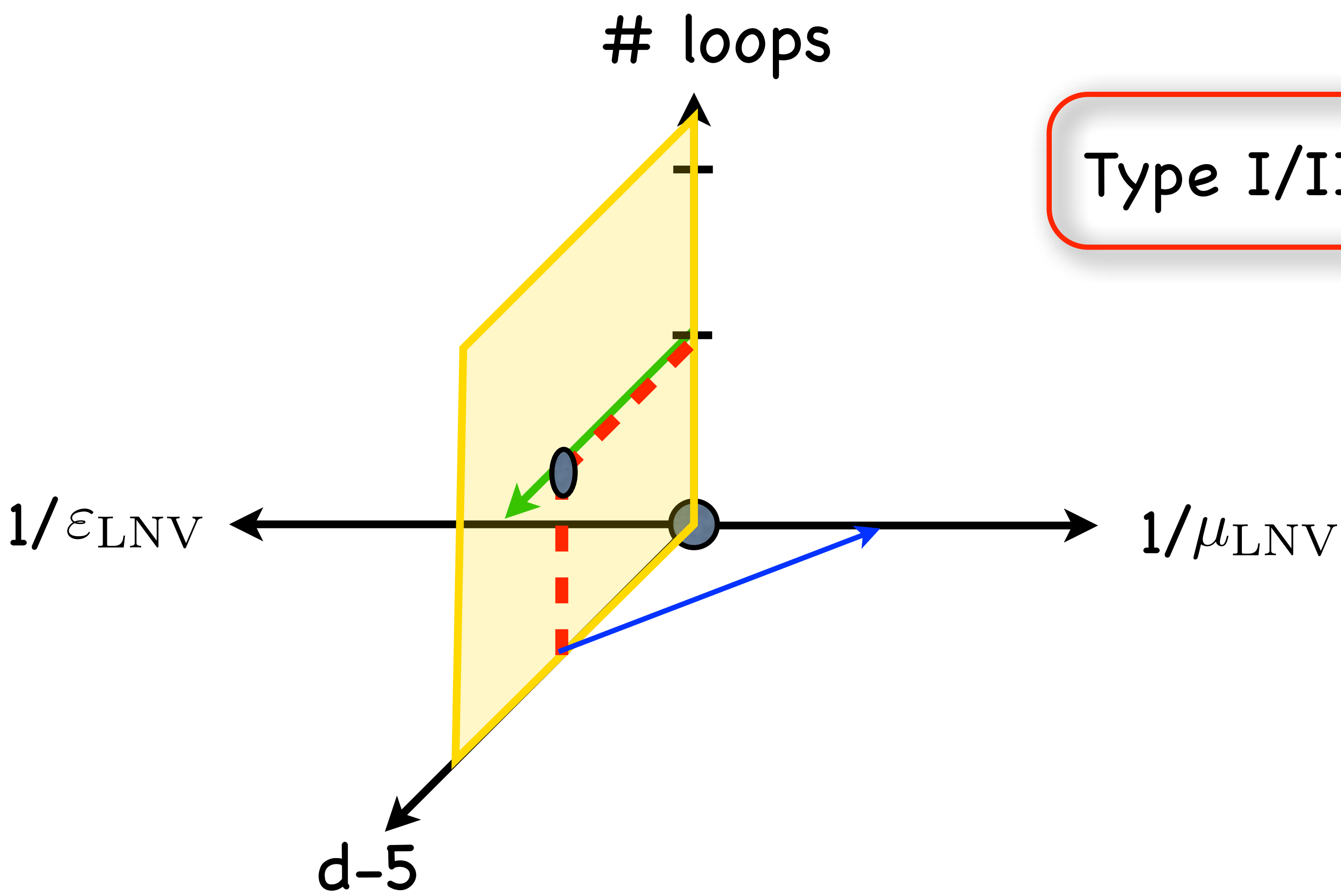


Introduction

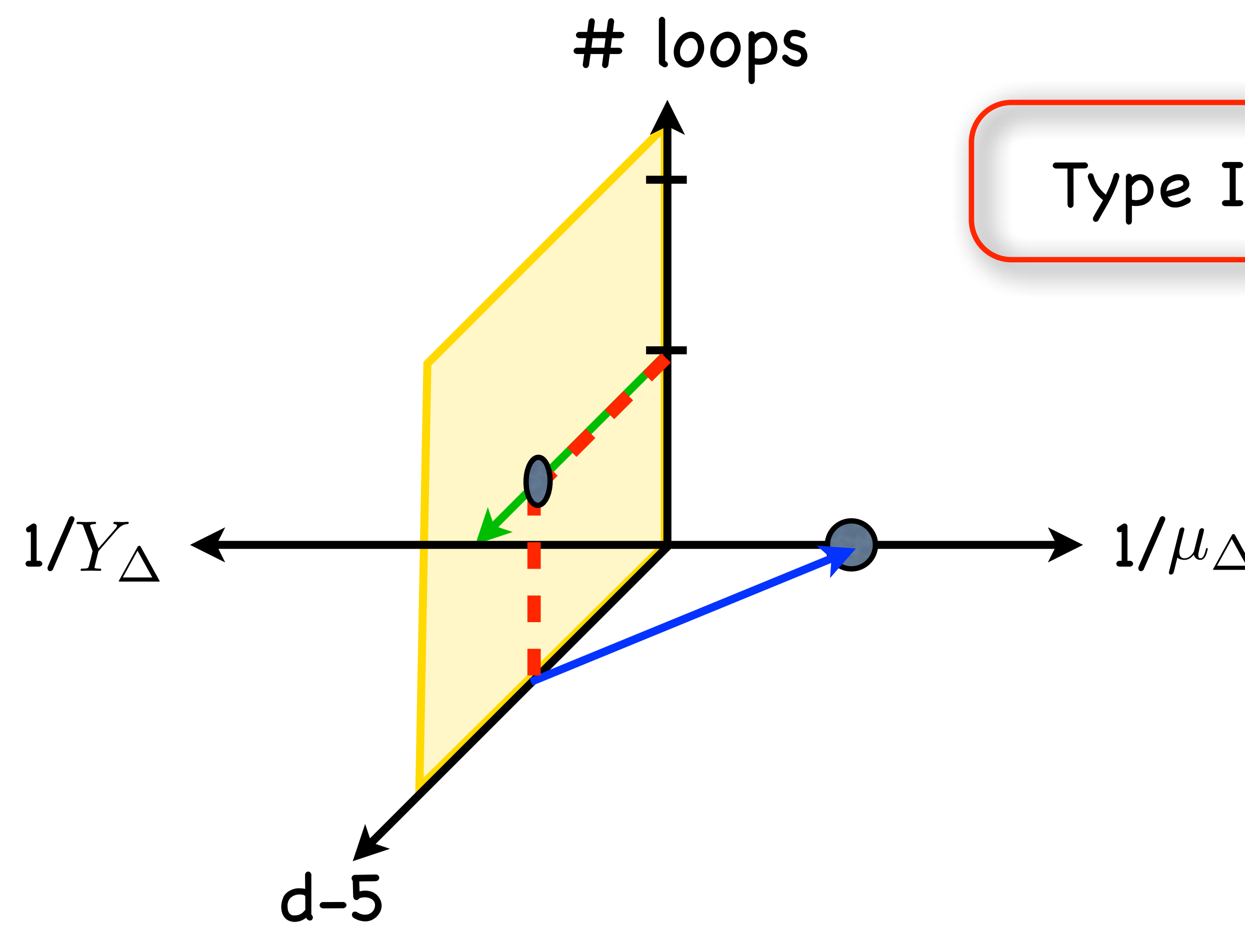
$$\begin{aligned}\mathcal{L}_{eff} \quad = \mathcal{L}_{SM} \quad & +\delta\mathcal{L}^{(0)}_{d=5} + \delta\mathcal{L}^{(1)}_{d=5} + \delta\mathcal{L}^{(2)}_{d=5} + \ldots \\ & +\delta\mathcal{L}^{(0)}_{d=7} + \delta\mathcal{L}^{(1)}_{d=7} + \delta\mathcal{L}^{(2)}_{d=7} + \ldots \\ & +\delta\mathcal{L}^{(0)}_{d=9} + \delta\mathcal{L}^{(1)}_{d=9} + \delta\mathcal{L}^{(2)}_{d=9} + \ldots \\ & + \ldots\end{aligned}$$



Global view

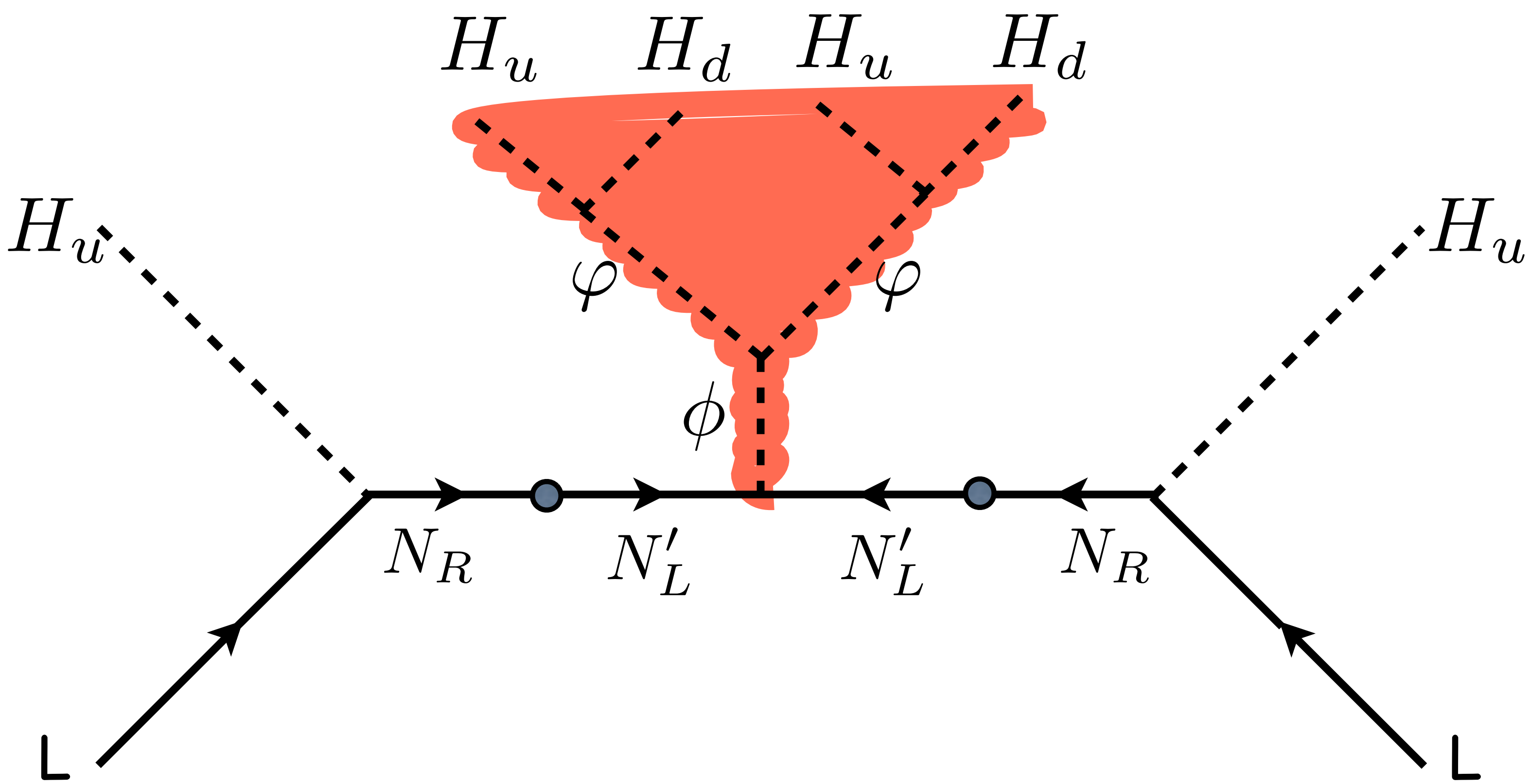


Type I/III



Type II

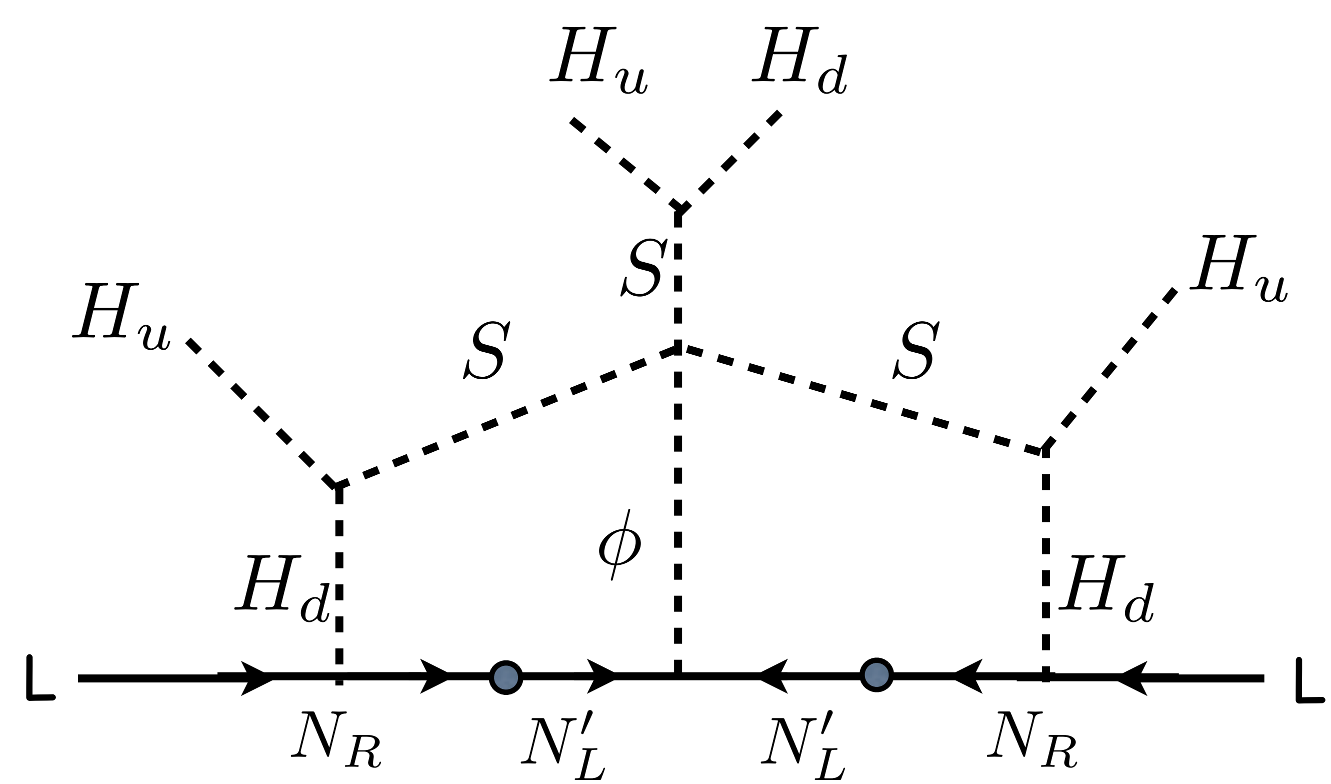
Global view



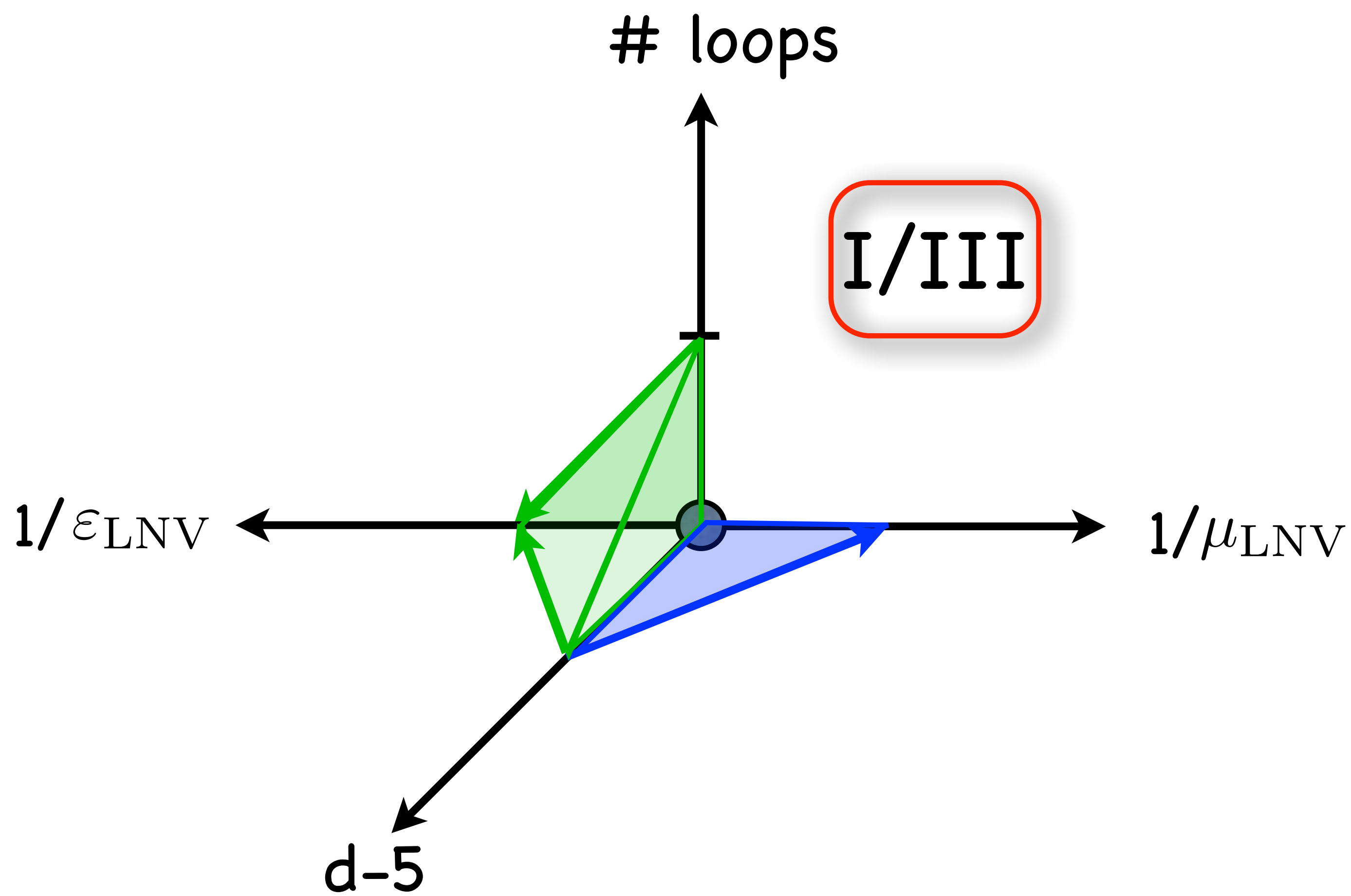
d=9, tree

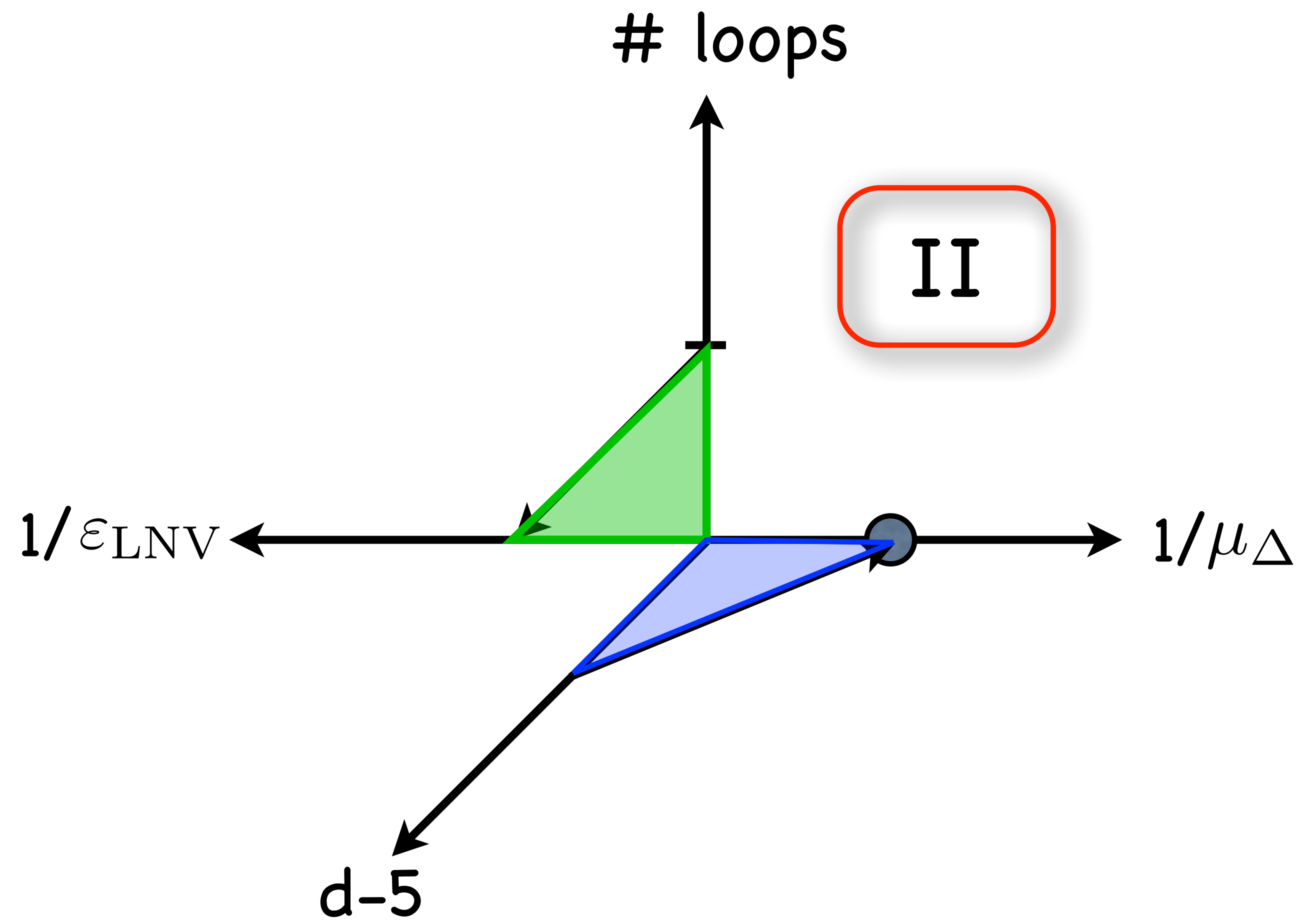
d=9, 2-loop

$$\begin{pmatrix} m_\nu^{(2-\text{loop})} & Y_N^T\langle H_d^0\rangle & (\varepsilon Y'_\nu)^{(1-\text{loop})\text{T}} \\ Y_N\langle H_d^0\rangle & \mu'^{(\text{tree})} & \Lambda \\ (\varepsilon Y'_\nu)^{(1-\text{loop})} & \Lambda^T & \mu^{\text{tree}} \end{pmatrix}$$



Conclusions

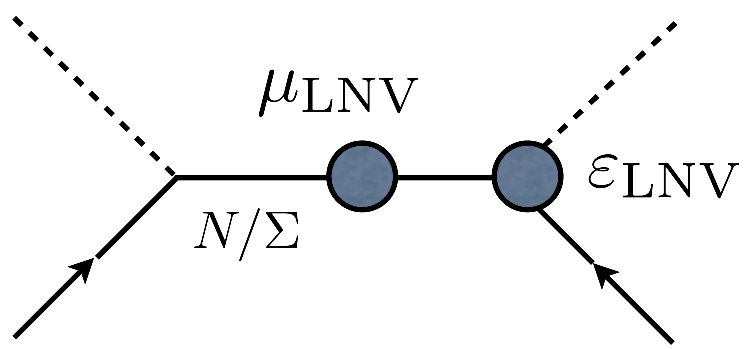


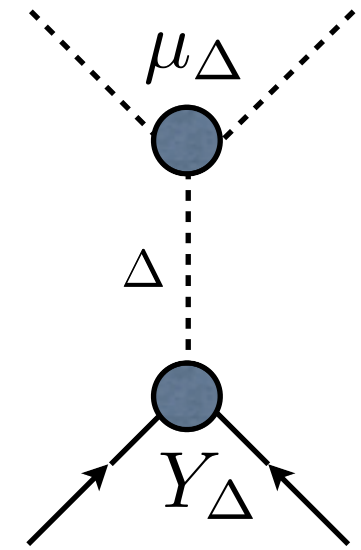


- Inverse/Linear Seesaw = Effective root for d=7/1-loop generalization of Type I, II, III Seesaws

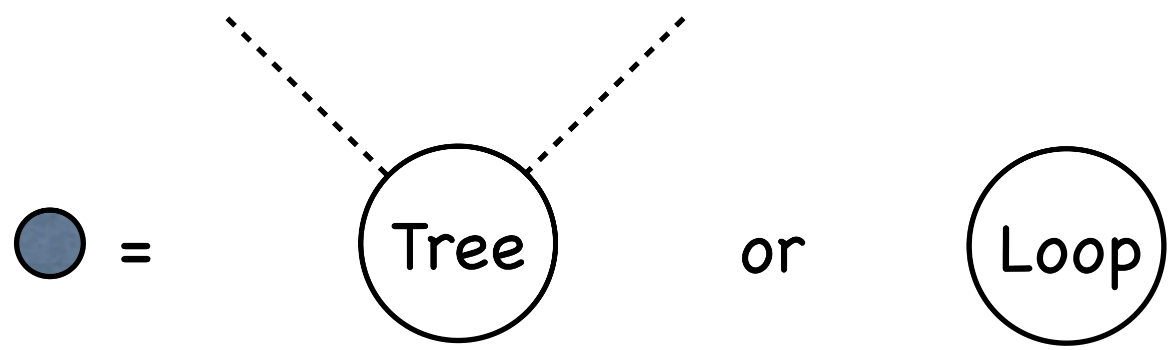
- $\varepsilon_{\rm LNV}$ and $\mu_{\rm LNV}$ can discriminate loop Vs tree level

Conjecture





With

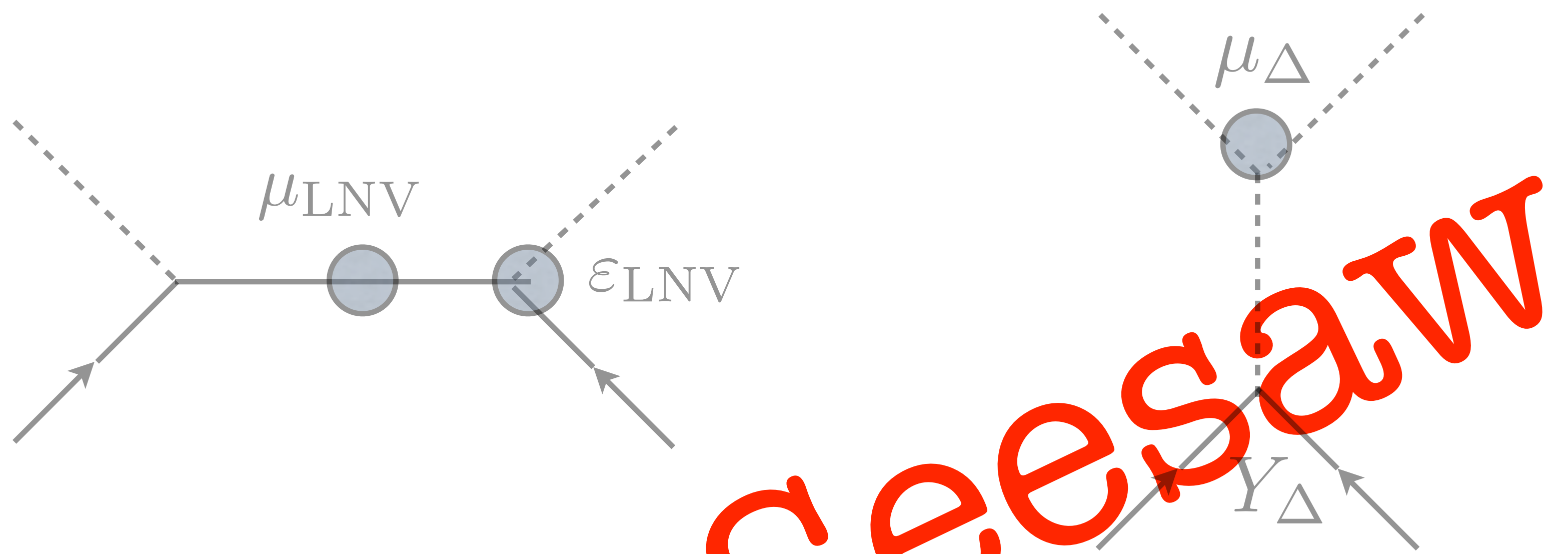


Inverse/Linear Seesaw

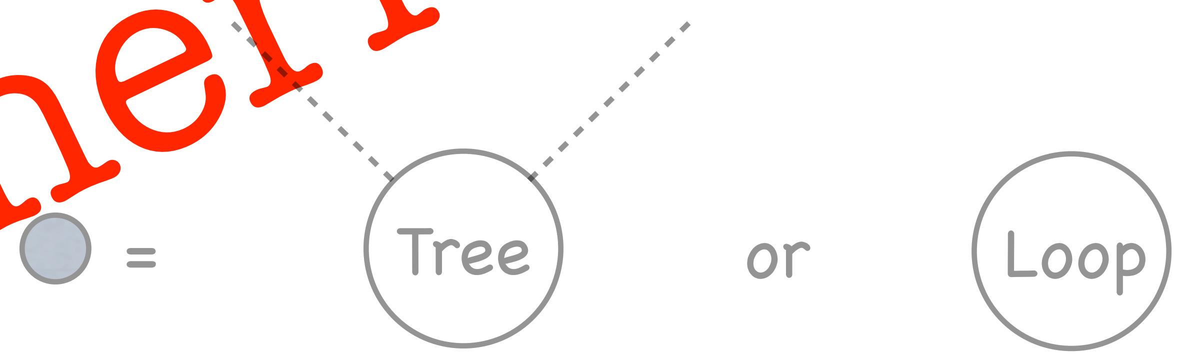
=

Effective theory of all Type I, II and III models

Global view



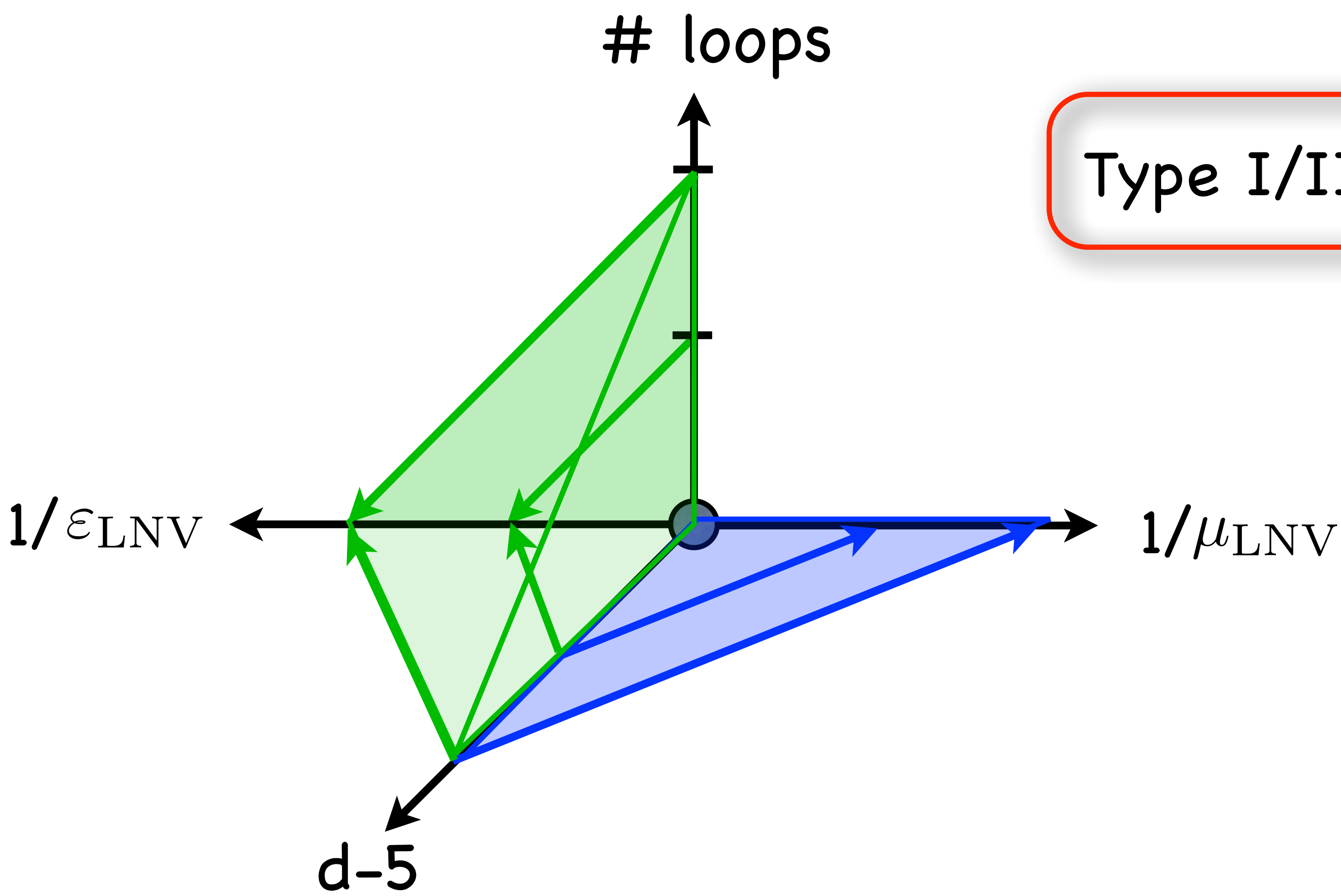
With



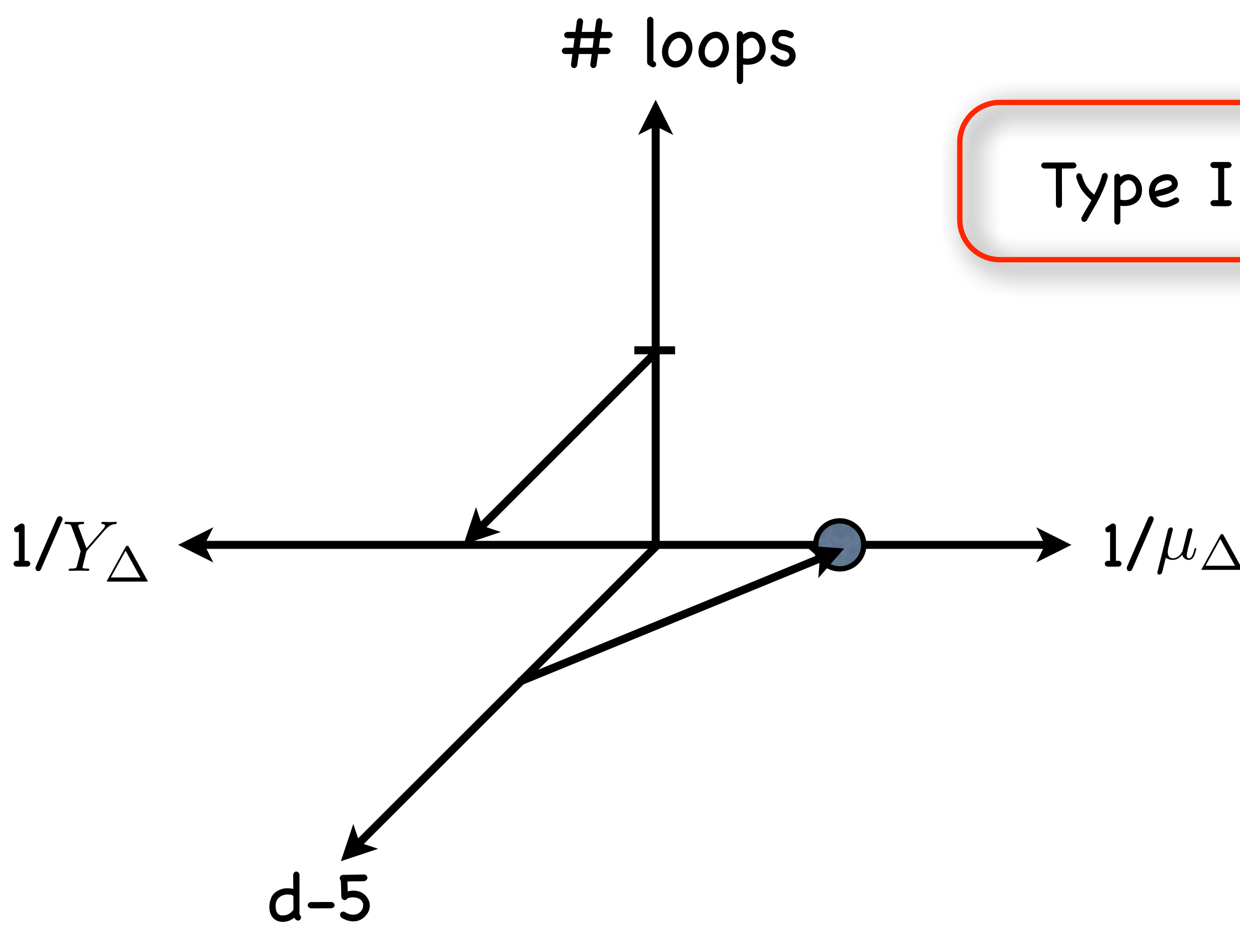
Generic Seesaw

Back up

Global view

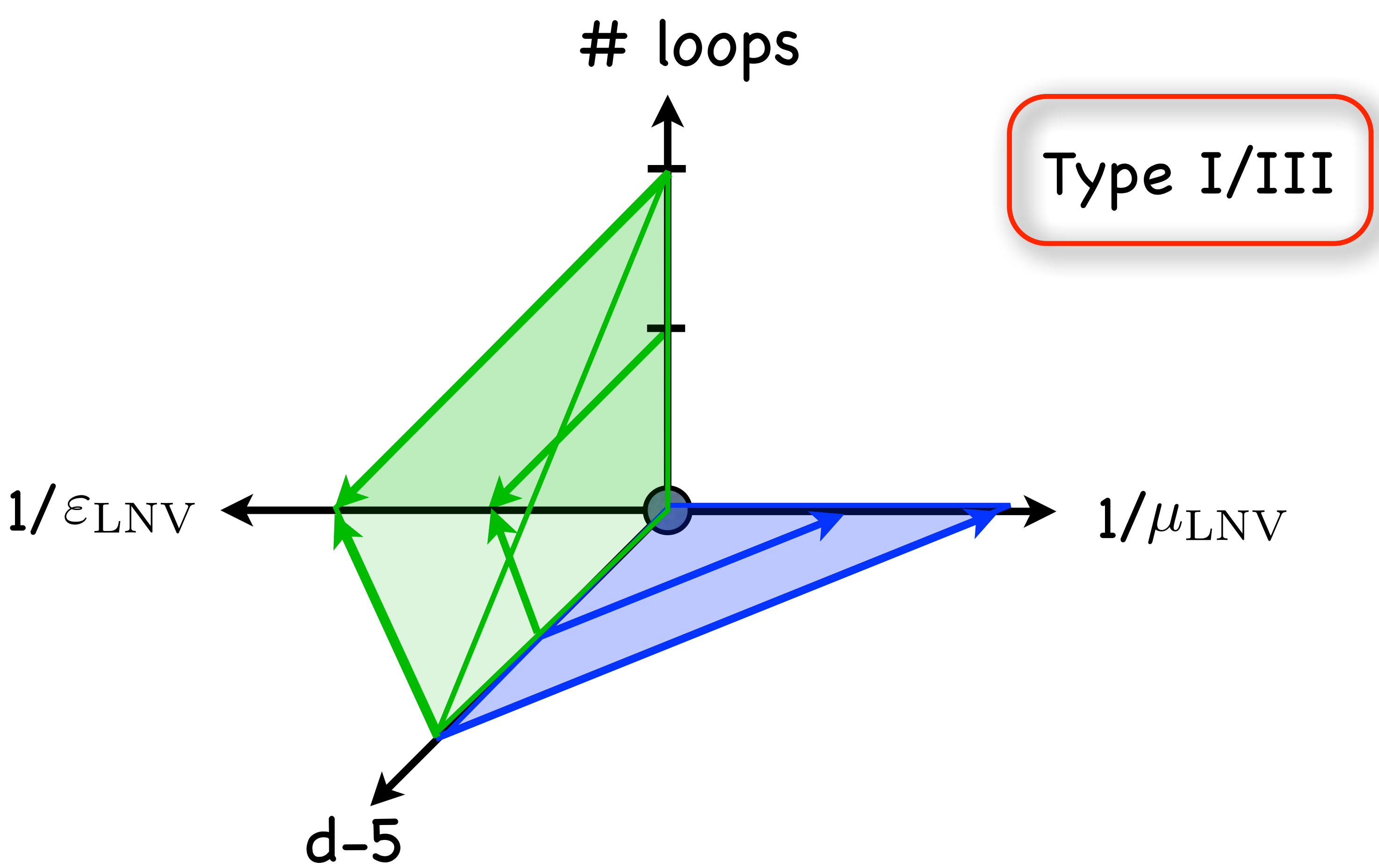


Type I/III

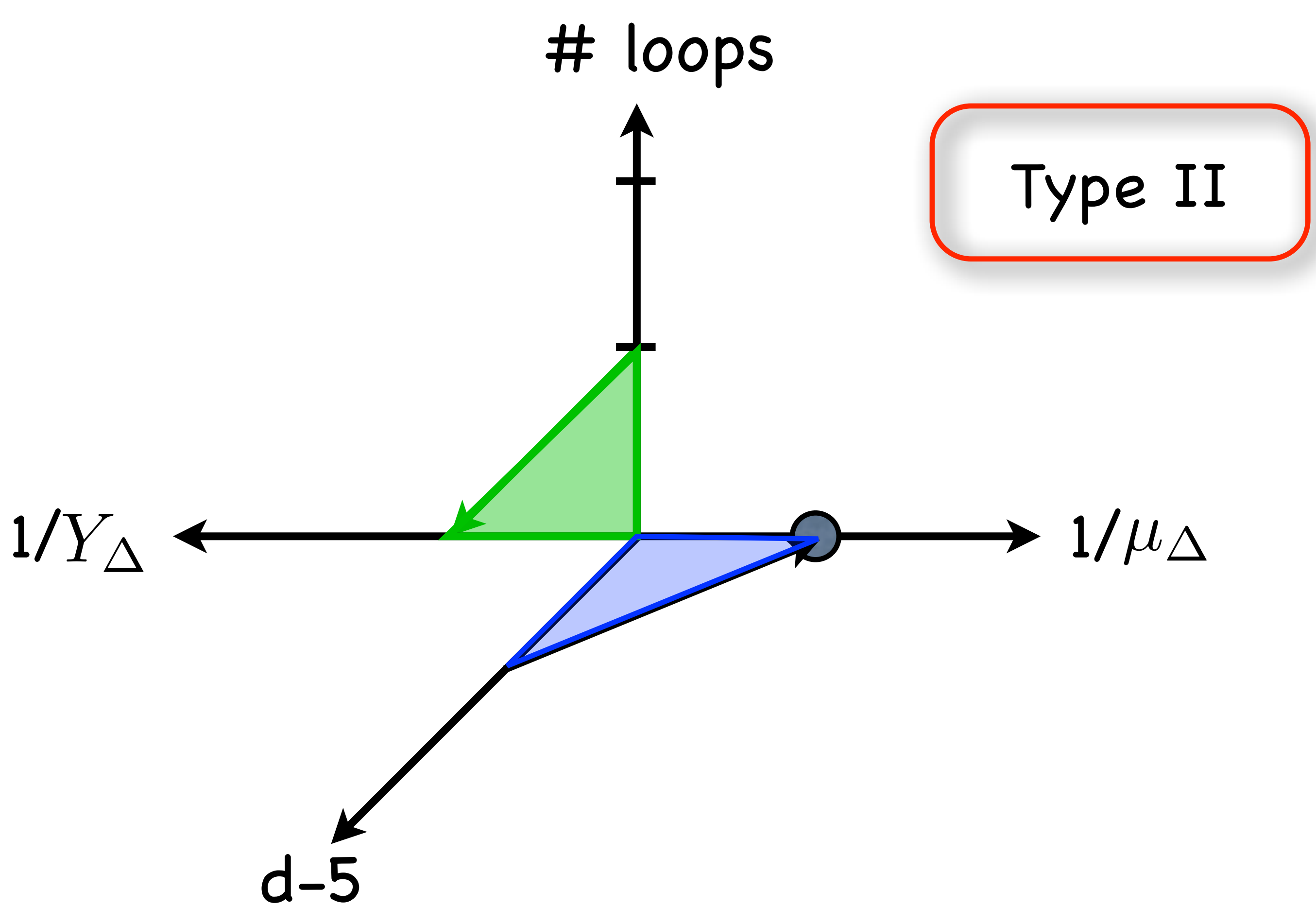


Type II

Global view

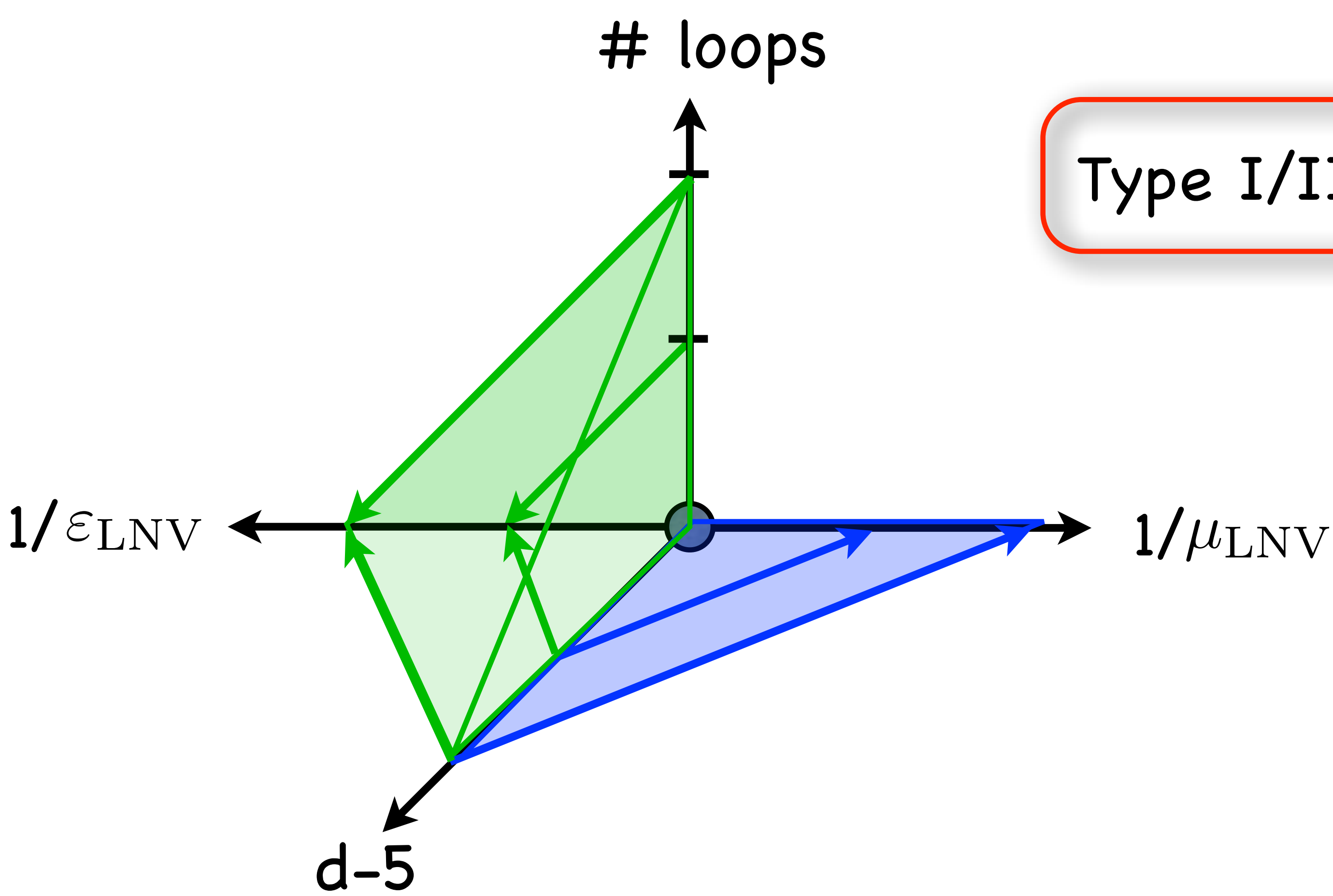


loops
Type I/III
$1/\varepsilon_{\rm LNV}$
$1/\mu_{\rm LNV}$
d-5

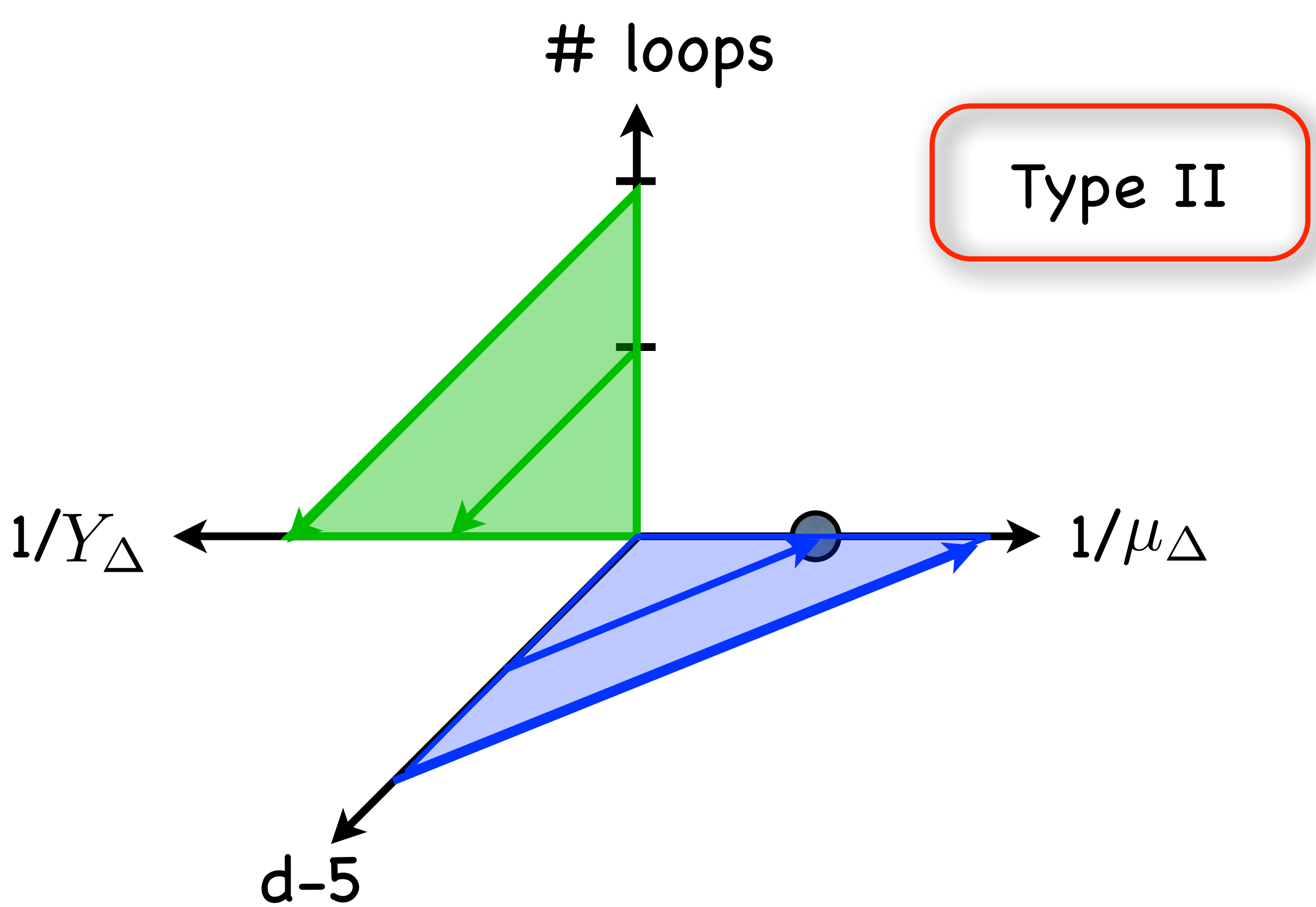


loops
Type II
$1/Y_\Delta$
$1/\mu_\Delta$
d-5

Global view

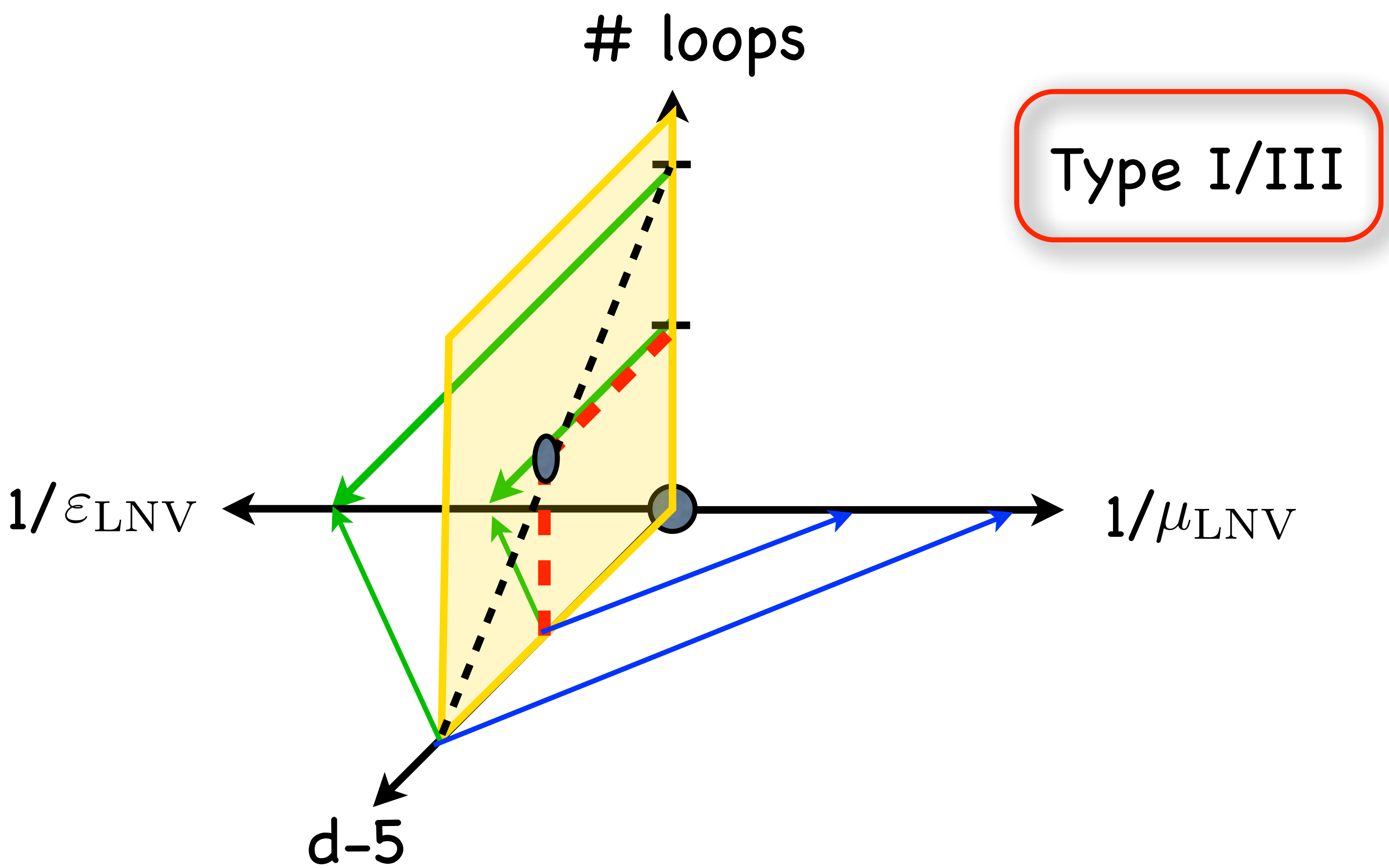


loops
Type I/III
$1/\varepsilon_{\rm LNV}$
$1/\mu_{\rm LNV}$
d-5

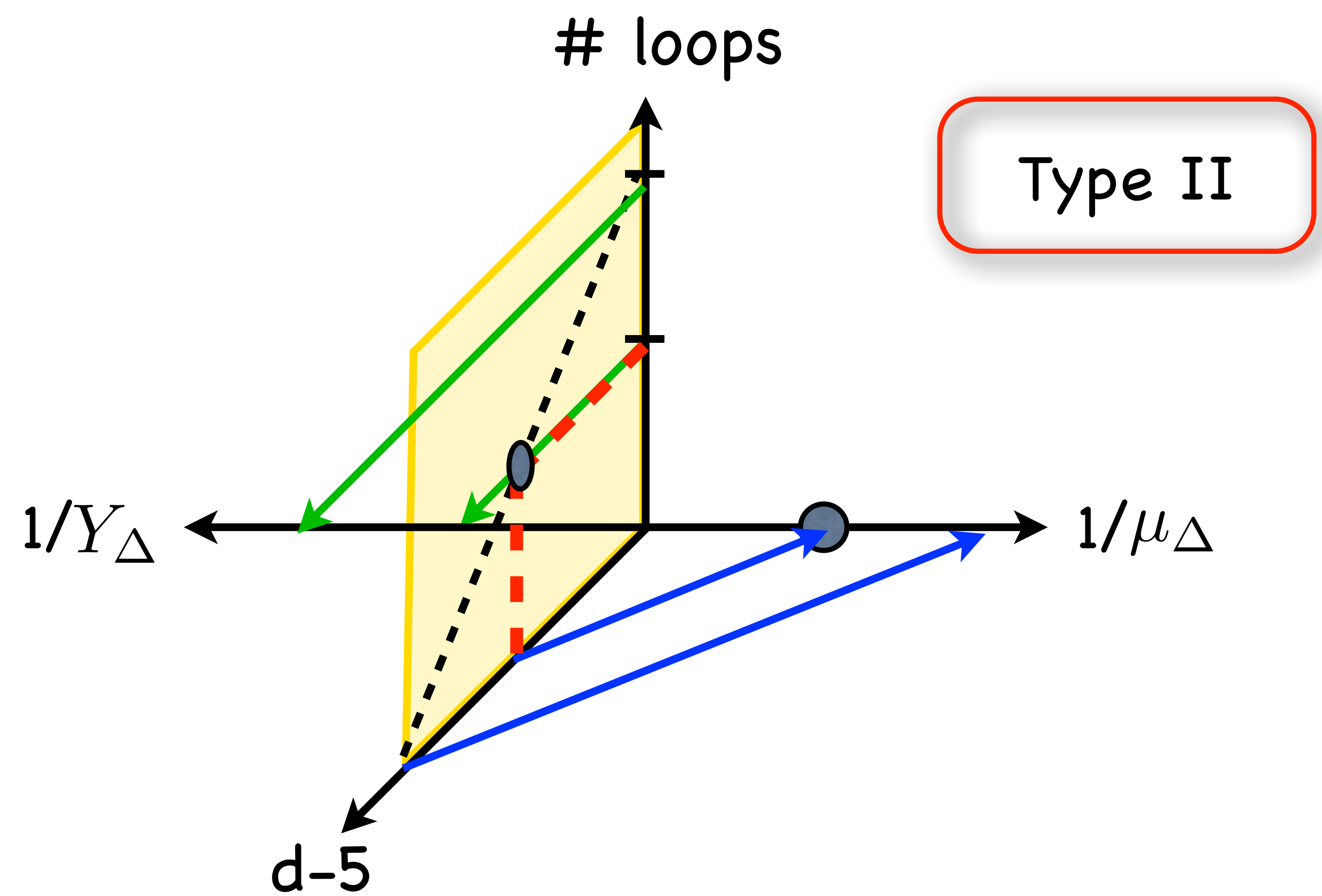


loops
Type II
$1/Y_\Delta$
$1/\mu_\Delta$
d-5

Global view



loops
Type I/III
$1/\varepsilon_{\rm LNV}$
$1/\mu_{\rm LNV}$
d-5



loops
Type II
$1/Y_\Delta$
$1/\mu_\Delta$
d-5