Paradoxes of neutrino oscillations

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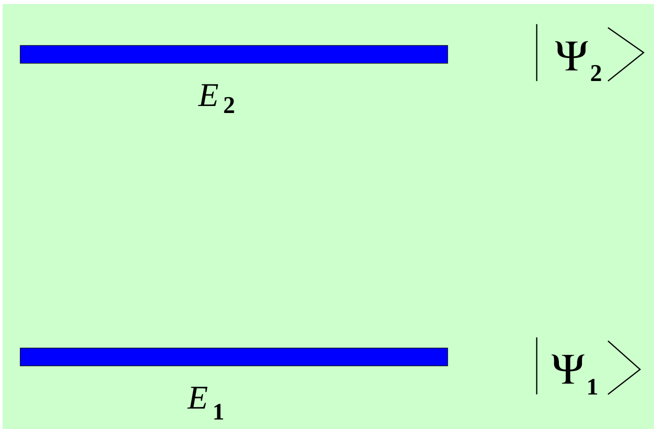
Oscillations: a well known QM phenomenon



$$\Psi_1(t) = e^{-i\,E_1\,t}\,\Psi_1(0)$$

$$\Psi_2(t) = e^{-i\,E_2\,t}\,\Psi_2(0)$$

$$\Psi(0) = a\,\Psi_1(0) + b\,\Psi_2(0) \quad (|a|^2 + |b|^2 = 1)\ ; \qquad \Rightarrow$$

$$\Psi(t) = a\,e^{-i\,E_1\,t}\,\Psi_1(0) + b\,e^{-i\,E_2\,t}\,\Psi_2(0)$$

Probability to remain in the same state $|\Psi(0)\rangle$ after time t:

$\diamondsuit$ $$P_{\rm surv} = |\langle\Psi(0)|\Psi(t)\rangle|^2 = \Big|\,|a|^2\,e^{-i\,E_1\,t} + |b|^2\,e^{-i\,E_2\,t}\Big|^2$$

$$= 1 - 4|a|^2|b|^2\sin^2[(E_2 - E_1)\,t/2]$$

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Appear to be a simple QM phenomenon

But: A closer look reveals a number of subtle and even paradoxical issues

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- What is the role of QM uncertainty relations?
- How can one make sure that the oscillation probability is Lorentz invariant?

Neutrino oscillations questions – contd.

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- When are the oscillations described by a universal probability?
- When is the emission of neutrinos from different space-tme points in extended sources coherent?
- Is the standard oscillation formula correct? If yes, what is the domain of its applicability?

Debating the basics of neutrino oscillations ...

Lipkin arXiv:0801.1465, arXiv:0905.1216, arXiv:0910.5049, Ivanov & Kienle arXiv:0909.1287, Merle arXiv:0907.3554, Peshkin arXiv:0804.4891, Faber arXiv:0801.3262, Gal arXiv:0809.1213, Giunti arXiv:0805.0431, Flambaum arXiv:0908.2039, Kienert, Kopp, Lindner & Merle arXiv:0808.2389, Walker Nature 453 (2008) 864, Giunti arXiv:0807.3818, Kleinert & Kienle ("Neutrino-pulsating vacuum") arXiv:0803.2938, Lambiase, Papini & Scarpeta arXiv:0811.2302, Burkhardt, Lowe, Stephenson, Goldman & McKellar, arXiv:0804.1099
Bilenky, v. Feilitzsch & Potzel arXiv:0804.3409, arXiv:0803.0527, J. Phys. G36 (2009) 078002, EA, Kopp & Lindner arXiv:0802.2513, arXiv:0803.1424,

Cohen, Glashow & Ligety arXiv:0810.4602, Visinelli & Gondolo arXiv:0810.4132, Keister & Polizou arXiv:0908.1404, Nishi & Guzzo arXiv:0803.1422, Lychkovskiy arXiv:0901.1198, Adhikari & Pal arXiv:0912.5266, Giunti arXiv:1001.0760, Ahluwalia & Schritt arXiv:0911.2965, Schmidt-Parzefall arXiv:0912.3620, Robertson arXiv:1004.1847 and many others.

Clarification of some of these issues and some apparent paradoxes of neutrino oscillations in:

EA, arXiv:0706.1216; EA & Smirnov, arXiv:0905.1903; arXiv:1008.2077; EA, Hernandez & Smirnov. arXiv:1201.4128; EA & Kopp arXiv:1001.4815

Kinematic constraints

Same momentum and same energy assumptions: contradict kinematics!

Pion decay at rest $(\pi^+ \to \mu^+ + \nu_\mu, \quad \pi^- \to \mu^- + \bar{\nu}_\mu)$:
For decay with emission of a massive neutrino of mass m_i:

$$E_i^2 = \frac{m_\pi^2}{4}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 + \frac{m_i^2}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_i^4}{4m_\pi^2}$$

$$p_i^2 = \frac{m_\pi^2}{4}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 - \frac{m_i^2}{2}\left(1 + \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_i^4}{4m_\pi^2}$$

For massless neutrinos: $E_i = p_i = E \equiv \frac{m_\pi}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 30 \;\; \mathrm{MeV}$

To first order in m_i^2:

$$E_i \simeq E + \xi\frac{m_i^2}{2E}\,, \qquad p_i \simeq E - (1-\xi)\frac{m_i^2}{2E}\,, \qquad \xi = \frac{1}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \approx 0.2$$

Oscillation phase

Oscillations are due to phase differences of different mass eigenstates:

$$\Delta\phi = \Delta E \cdot t - \Delta p \cdot x \qquad \left(E_i = \sqrt{p_i^2 + m_i^2}\right)$$

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Consider the case $\Delta E \ll E$ (relativistic or quasi-degenerate neutrinos) $\Rightarrow$

$$\Delta E \; = \; \frac{\partial E}{\partial p}\Delta p + \frac{\partial E}{\partial m^2}\Delta m^2 \; = \; v\Delta p \; + \; \frac{1}{2E}\,\Delta m^2$$

$$\Delta\phi \; = \; (v\Delta p + \frac{1}{2E}\Delta m^2)\,t \; - \; \Delta p \cdot x$$

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$$\Delta\phi = (v\Delta p + \frac{1}{2E}\Delta m^2)\,t - \Delta p\cdot x$$

$$= -(x - vt)\Delta p + \frac{\Delta m^2}{2E}t$$

In the center of wave packet $(x - vt) = 0$. In general, $|x - vt| \lesssim \sigma_x$;
if $\sigma_x\Delta p \ll 1$ (i,e, $\Delta p \ll \sigma_p$), $|x - vt|\Delta p \ll 1 \quad \Rightarrow$

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The reasons why wrong assumptions give the correct result:

- Neutrinos are relativistic or quasi-degenerate with $\Delta E \ll E$
- Neutrino energy uncertainty $\sigma_E \gg \Delta E$ (typically this means $\sigma_x \ll l_{\rm osc}$)

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The dichotomy led to a significant confusion in the literature.
How can it be resolved?

Possible solution: entanglement

Consider e.g. $\pi \to \mu + \nu$ decay.
Suppose that the 4-momentum of the pion p_π is well defined but the muon 4-momentum is correlated with that of the emitted ν_i:

$$p_{\nu i} + p_{\mu i} = p_\pi\,, \qquad i = 1, 2, 3$$

State produced in the pion decay: a coherent superposition of different neutrino mass eigenstates accompanied by the muon states with correlated 4-momenta (entangled state):

$$|\mu\,\nu\rangle = \sum_i U_{\mu i}^* |\mu(p_{\mu i})\rangle |\nu_i(p_{\nu i})\rangle.$$

If muon 4-momentum is measured very accurately (e.g. $p_\mu = p_{\mu 1}$) $\Rightarrow$ neutrino detector should observe only ν_1 with 4-momentum $p_{\nu 1}$.
A realization of the Einstein-Podolsky-Rosen correlation.

<u>But:</u> in this case no oscillations would occur!

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$\diamondsuit$ Kinematic entanglement is irrelevant to neutrino oscillations!

Wave packets

♢ Wave packets are necessary for describing localization of neutrino production and detection processes $\Rightarrow$ of neutrinos themselves!

WPs necessary for a proper definition of S-matrix

Neutrino energy and momentum have some uncertainties, σ_E and σ_p.

This does not mean that energy-momentum conservation is violated!
E-p conserv. is exact for closed systems. Satisfied exactly when applied to all particles in the system (including those that localize particles participating in ν production and detection in given space-time regions).

Energy and momentum uncertainties do not contradict E-p conservation!

At the technical level:

$$\mathcal{A}_i = \prod_j \int \frac{d\vec{p}_j}{(2\pi)^3} \tilde{f}_j(\vec{p}_j, \vec{\bar{p}}_j; T_S, \vec{X}_S) \prod_l \int \frac{d\vec{p}_l}{(2\pi)^3} \tilde{f}_l(\vec{p}_l, \vec{\bar{p}}_l; T_D, \vec{X}_D)\, \mathcal{A}_i^{pw}(\{p_j\}, \{p_l\})$$

$$\mathcal{A}_i^{pw}(\{p_j\}, \{p_l\}) \propto \delta^{(4)}\Big(\sum_f p_f - \sum_i p_i\Big)\,.$$

Oscillations and QM uncertainty relations

Neutrino oscillations – a QM interference phenomenon, owe their existence to QM uncertainty relations

Neutrino energy and momentum are characterized by uncertainties σ_E and σ_p related to the spatial localization and time scale of the production and detection processes. These uncertainties

- allow the emitted/absorbed neutrino state to be a coherent superposition of different mass eigenstates (Kayser, 1981)
- determine the size of the neutrino wave packets $\Rightarrow$ govern decoherence due to wave packet separation (Nussinov, 1976)

σ_E – the effective energy uncertainty, dominated by the smaller one between the energy uncertainties at production and detection. Similarly for σ_p.

The paradox of σ_E and σ_p

QM uncertainty relations: σ_p is related to the spatial localization of the production (detection) process, while σ_E to its time scale $\Rightarrow$ independent quantities.

On the other hand: Neutrinos propagating macroscopic distances are on the mass shell. For on-shell mass eigenstates $E^2 = p^2 + m_i^2$ means

$$E\sigma_E = p\sigma_p$$

How can this be understood?

The solution: At production, neutrinos are *not* on the mass shell. They go on shell only after they propagate $x \sim$ (a few)$\times$ De Broglie wavelengths. After that their energy and momentum get related by $E^2 = p^2 + m_i^2 \quad \Rightarrow$ the larger uncertainty shrinks towards the smaller one to satisfy $E\sigma_E = p\sigma_p$.

On-shell relation between E and p allows to determine the less certain of the two through the more certain one, reducing the error of the former.

What determines the length of ν w. packets?

The length of ν w. packets: $\sigma_x \sim 1/\sigma_p$. For propagating on-shell neutrinos:

$$\sigma_p \simeq \min\{\sigma_p^{\rm prod},\, (E/p)\sigma_E^{\rm prod}\} = \min\{\sigma_p^{\rm prod},\, (1/v_g)\sigma_E^{\rm prod}\}$$

Which uncertainty is smaller at production, $\sigma_p^{\rm prod}$ or $\sigma_E^{\rm prod}$?

Consider neutrino production in decays of an unstable particle localized in a box of size L_S. Time between two collisions with the walls of the box: T_S.

- If $T_S < \tau$ (τ – lifetime of the parent unstable particle) $\Rightarrow$ $\sigma_E \simeq T_S^{-1}$ (collisional broadening). Mom. uncertainty: $\sigma_p \simeq L_S^{-1}$.
  But: $L_S = v_S T_S \;\Rightarrow\; \sigma_E < \sigma_p$ (a consequence of $v_S < 1$)
- If $T_S > \tau$ (quasi-free parent particle) $\Rightarrow$ $\sigma_E \simeq \tau^{-1} = \Gamma$.
  $\sigma_p \simeq [(p/E)\tau]^{-1} \simeq [(p/E)\sigma_E]^{-1}$, i.e. $\sigma_E \simeq (p/E)\sigma_p < \sigma_p$.

The length of ν w. packets – contd.

In both cases $\boxed{\sigma_E^{\rm prod} < \sigma_p^{\rm prod}}$ $\Leftarrow$ also when $\nu' s$ are produced in collisions.

$$\Longrightarrow \qquad \sigma_{p\ \rm eff} \simeq \frac{\sigma_E}{v_g}\,, \qquad\qquad \boxed{\sigma_x \simeq \frac{v_g}{\sigma_E}}$$

In the stationary limit $(\sigma_E \to 0)$ one has $\sigma_{p\ \rm eff} \to 0$ even though σ_p is finite! Therefore $\sigma_x \to \infty$ and so the coherence length $l_{\rm coh} \to \infty$
– a well known result.

Lorentz invariance of oscillation probability

1. “Paradox” of neutrino w. packet length

For neutrino production in decays of unstable particles at rest (e.g. $\pi \to \mu\nu_\mu$):

$$\sigma_E \simeq \tau^{-1} = \Gamma_\pi\,, \qquad\qquad \sigma_x \simeq \frac{v_g}{\sigma_E} \simeq \frac{v_g}{\Gamma_\pi} \;(= v_g\tau)$$

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For decay in flight: $\Gamma_\pi' = (m_\pi/E_\pi)\Gamma_\pi$. One might expect

$$\sigma_x' \simeq \frac{E_\pi}{m_\pi}\sigma_x > \sigma_x\,.$$

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$$\sigma'_x \simeq \frac{E_\pi}{m_\pi}\sigma_x > \sigma_x \,.$$

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<u>The solution:</u> pion decay takes finite time. During the decay time the pion moves over distance $l = u\tau'$ ("chases" the neutrino if $u > 0$).

$$\sigma'_x \simeq v'_g/\Gamma' - l = v'_g\tau' - u\tau' = (v'_g - u)\gamma_u\tau = \frac{v_g\tau}{\gamma_u(1 + v_g u)} \,,$$

[the relativ. law of addition of velocities: $v'_g = (v_g + u)/(1 + v_g u)$].

Lorentz invariance issues – contd.

That is

$$\sigma'_x = \frac{\sigma_x}{\gamma_u(1+v_g u)}$$

For relativistic neutrinos $v_g \approx v'_g \approx 1 \;\; \Rightarrow$

$$\sigma'_x = \sigma_x \sqrt{\frac{1-u}{1+u}}$$

$\Rightarrow$ when the pion is boosted in the direction of neutrino emission $(u > 0)$ the neutrino wave packet gets contracted; when it is boosted in the opposite direction $(u < 0)$ – the wave packet gets dilated.

Lorentz invariance issues – contd.

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How can we see Lorentz invariance of the standard formula for the oscillation probability? P_{ab} depends on L/p (contains factors $\exp[-i\frac{\Delta m_{ik}^2}{2p}L]$). Is L/p Lorentz invariant?

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$$L' = \gamma_u(L + ut)\,, \qquad t' = \gamma_u(t + uL)\,,$$

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The stand. osc. formula results when (i) production and detection and (ii) propagation are coherent; for neutrinos from conventional sources (i) implies $\sigma_x \ll l_{\rm osc}$. $\Rightarrow$ one can consider neutrinos pointlike and set $L = v_g t$. $\Rightarrow$ $L' = \gamma_u L(1 + u/v_g)$.

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$\Rightarrow$ $p' = \gamma_u p(1+u/v_g)$.

$\Rightarrow$ $$\boxed{L'/p' = L/p}$$

Lorentz invariance issues – contd.

A more general argument (applies also to Mössbauer neutrinos which are not pointlike): Consider the phase difference

$$\diamondsuit \qquad \Delta\phi \;=\; -\frac{1}{v_g}(L\;-\;v_g\,t)\Delta E\;+\;\frac{\Delta m^2}{2p}\,L$$

– a Lorentz invariant quantity, though the two terms are in not in general separately Lorentz invariant.

But: If the 1st term is negligible in all Lorentz frames, the second term is Lorentz invariant by itself $\Rightarrow$ L/p is Lorentz invariant.

The 1st term can be neglected when the production/detection coherence conditions are satisfied. In particular, it vanishes in the limit of pointlike neutrinos $L = v_g t$. N.B.:

$$L' - v_g' t' = \gamma_u\left[(L+ut) - \frac{v_g+u}{1+v_g u}(t+uL)\right] = \frac{L - v_g t}{\gamma_u(1+v_g u)}\,,$$

i.e. the condition $L = v_g t$ is Lorentz invariant. MB neutrinos: $\Delta E \simeq 0$.

Lorentz invariance issues – contd.

The oscillation probability must be Lorentz invariant even when the coherence conditions are not satisfied !
Lorentz invariance is enforced by the normalization condition.

$$P_{ab}(L) \;=\; \sum_{i,k} U_{ai} U_{bi}^* U_{ak}^* U_{bk}\, I_{ik}(L)\,, \qquad \text{where}$$

$$I_{ik}(L) \;\equiv\; \int dt\, \mathcal{A}_i(L,t) \mathcal{A}_k^*(L,t) e^{-i\Delta\phi_{ik}}$$

From the norm. cond. $\int dt\, |\mathcal{A}_i(L,t)|^2 \;=\; 1 \qquad \Rightarrow$

$$|\mathcal{A}_i|^2 dt \;=\; inv. \;\;\Rightarrow\;\; |\mathcal{A}_i||\mathcal{A}_k| dt \;=\; inv. \;\;\Rightarrow\;\; \mathcal{A}_i \mathcal{A}_k^* dt \;=\; inv.$$

The phase difference $\Delta\phi_{ik} = \Delta E_{ik} t - \Delta p_{ik} L$ is also Lorentz invariant $\Rightarrow$ so is $I_{ik}(L)$, and consequently $P_{ab}(L)$.

Do charged leptons oscillate?

What do we mean by charged leptons?

The usual $e^{\pm}$, $\mu^{\pm}$ and $\tau^{\pm}$ are mass eigenstates $\Rightarrow$ do not oscillate.

Is that the full answer?

Can we imagine a situation when one creates a coherent superposition of e, μ and τ and then also detects their coherent superposition (the same or different) rather than individual mass-eigenstate charged leptons?

Charged - current weak interactions look completely symmetric w.r.t. neutrinos and charged leptons!

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}}\left(\bar{e}_{aL}\gamma^{\mu}U_{ai}\nu_{iL}\right)W_{\mu}^{-} + h.c.\,, \qquad U = V_L^{\dagger}V_{\nu}$$

Why do we say that charged leptons are emitted and detected in mass eigenstates and neutrinos in flavour states (superpositions of mass eigenstates) and not vice versa? Or not both as some superpositions of mass eigenstates? E.g.

$|e_1\rangle = U_{1e}|e\rangle + U_{1\mu}|\mu\rangle + U_{1\tau}|\tau\rangle$ is emitted or detected together with ν_1,
$|e_2\rangle = U_{2e}|e\rangle + U_{2\mu}|\mu\rangle + U_{2\tau}|\tau\rangle$ is emitted or detected together with ν_2,
$|e_3\rangle = U_{3e}|e\rangle + U_{3\mu}|\mu\rangle + U_{3\tau}|\tau\rangle$ is emitted or detected together with ν_3.

Why do neutrinos oscillate?

Because they are emitted (and absorbed) alongside charged leptons of definite mass $e^\pm$, $\mu^\pm$ or $\tau^\pm$. (This “measures” the flavour of neutrinos). How do we know that charged leptons are in mass eigenstates?

(1) Beta decay: only electrons are emitted together with neutrinos. Emission of $\mu^\pm$ and $\tau^\pm$ is forbidden by energy conservation.

(2) Decays $\pi^\pm \to \mu^\pm\nu$, $\pi^\pm \to e^\pm\nu$ (or $K^\pm \to \mu^\pm\nu$, $K^\pm \to e^\pm\nu$). Here emission of both muons and electrons is allowed.

Assume a coherent superposition of e and μ is produced in pion decay (nearly) at rest. The energy uncertainty of the charged lepton:

$$\sigma_E \simeq \Gamma_\pi = 2.5 \cdot 10^{-8}\ \mathrm{eV}$$

Uncertainty in the mass determination ($\sqrt{(2E\sigma_E)^2 + (2p\sigma_p)^2]} \simeq 2\sqrt{2}E\sigma_E$):

$$\sigma_{m^2} \sim 2\sqrt{2}E\sigma_E \simeq 2\sqrt{2}\cdot(90\ \mathrm{MeV})\cdot(2.5\cdot 10^{-8}\ \mathrm{eV}) \simeq 6.4\ \mathrm{eV}^2$$

Do charged leptons oscillate?

This has to be compared with $m_\mu^2 - m_e^2 \simeq (106 \text{ MeV})^2 \quad \Rightarrow$

Different mass-eigenstate charged leptons are emitted incoherently!

This provides a “measurement” of the flavour of the emitted neutrino

For pion decay in flight: assume pion’s energy is E_0. The energies of the produced charged leptons are rescaled as $E \to E\,(E_0/m_\pi)$, but the pion decay width (and so σ_E) is rescaled as $\Gamma_\pi \to \Gamma_\pi(m_\pi/E_0) \quad \Rightarrow$ $[(2E\sigma_E)^2 + (2p\sigma_p)^2]^{1/2}$ remains the same (σ_{m^2} a Lorentz invariant quantity).

$$\Downarrow$$

- ♢ Charged leptons produced in $\pi^\pm \to l^\pm \nu$ and $K^\pm \to l^\pm \nu$ decays are always emitted as mass eigenstates and not as coherent superpositions of different mass eigenstates because of their very large Δm^2.

- ♢ Therefore even oscillations between e_1, e_1 and e_3 (or any other superpositions of e, μ and τ) are not possible.

Do charged leptons oscillate?

The masses and decay widths of $\pi^\pm$, $K^\pm$ are rather small $\Rightarrow$ σ_{m^2} small.

How about decays of $W^\pm$? For $W^\pm \to l^\pm \nu$ decays at rest:

$$\Gamma^0_{W\to l_a \nu} \simeq \frac{G_F m_W^3}{6\sqrt{2}\pi} \simeq 230\ \mathrm{MeV}$$

$$\Rightarrow \qquad \sigma_{m^2} \sim 2\sqrt{2}\,E\sigma_E \simeq 2\sqrt{2}\cdot 40\ \mathrm{GeV}\cdot 230\ \mathrm{MeV} \simeq (5\ \mathrm{GeV})^2\,.$$

Thus

$$\sigma_{m^2} \gg m_\mu^2 - m_e^2\,, \qquad \sigma_{m^2} > m_\tau^2 - m_\mu^2 \simeq (1.77\ \mathrm{GeV})^2\,,$$

$\Rightarrow$ all three charged leptons are produced *coherently* in $W^\pm$ decays.

Can one then observe oscillations between their different coh. superpositions?

Coherence length $l_{\rm coh} \simeq \sigma_x/\Delta v_g$:

$$(l_{\rm coh})_{\rm max} \simeq [\Gamma^0_{W\to l_a\nu}(\Delta v_g)_{\rm min}]^{-1} \simeq \frac{3\sqrt{2}\pi}{G_F m_W (m_\mu^2 - m_e^2)} \simeq 2.5\times 10^{-8}\ \mathrm{cm}\,.$$

$\Rightarrow$ $l^\pm$ loose their coherence almost immediately after their production

When are neutrino oscillations observable?

Keyword: *Coherence*

Neutrino flavour eigenstates ν_e, ν_μ and ν_τ are coherent superpositions of mass eigenstates ν_1, ν_2 and ν_3 $\Rightarrow$ oscillations are only observable if

- neutrino production and detection are coherent
- coherence is not (irreversibly) lost during neutrino propagation.

Possible decoherence at production (detection): If by accurate E and p measurements one can tell (through $E = \sqrt{p^2 + m^2}$) which mass eigenstate is emitted, the coherence is lost and oscillations disappear!

Production and detection coherence $\Leftrightarrow$ localization cond.:

$$l_{\rm prod} \ll l_{\rm osc}\,, \qquad\qquad l_{\rm det} \ll l_{\rm osc}$$

Usually satisfied with large margins.

Propagation coherence: $\qquad L < l_{\rm coh} \simeq \frac{v}{\Delta v}\sigma_x = \frac{2E^2}{\Delta m^2}\,v\sigma_x$

A manifestation of neutrino coherence

Even non-observation of neutrino oscillations at distances $L \ll l_{\rm osc}$ is a consequence of and an evidence for coherence of neutrino emission and detection! Two-flavour example (e.g. for ν_e emission and detection):

$$A_{\rm prod/det}(\nu_1) \sim U_{e1} = \cos\theta\,, \qquad A_{\rm prod/det}(\nu_2) \sim U_{e2} = \sin\theta \qquad \Rightarrow$$

$$A(\nu_e \to \nu_e) = \sum_{i=1,2} A_{\rm prod}(\nu_i) A_{\rm det}(\nu_i) = \cos^2\theta + e^{-i\Delta\phi}\sin^2\theta$$

Phase difference $\Delta\phi$ vanishes at short L $\Rightarrow$

$$P(\nu_e \to \nu_e) = (\cos^2\theta + \sin^2\theta)^2 = 1$$

If ν_1 and ν_2 were emitted and absorbed incoherently) $\Rightarrow$ one would have to sum probabilities rather than amplitudes:

$$P(\nu_e \to \nu_e) \sim \sum_{i=1,2} |A_{\rm prod}(\nu_i) A_{\rm det}(\nu_i)|^2 \sim \cos^4\theta + \sin^4\theta \;<\; 1$$

A universal oscillation probability?

Q.: When are the oscillations described by a universal (production and detection independent) oscillation probability?

A.: When neutrinos are relativistic or quasi-degenerate in mass and the conditions of coherent neutrino emission and detection

$$\Delta E \ll \sigma_E\,, \qquad\qquad \Delta p \ll \sigma_p$$

are satisfied.

Under these conditions the rate of the overall neutrino production-propagation-detection process can be factorized into the production rate $d\Gamma_\alpha^{\rm prod}(E)/dE$, propagation (oscillation) probability $P_{\alpha\beta}(E,L)$ and detection cross section $\sigma_\beta(E) \quad \Rightarrow \quad P_{\alpha\beta}(E,L)$ can be extracted.

Coherence of ν production in different points

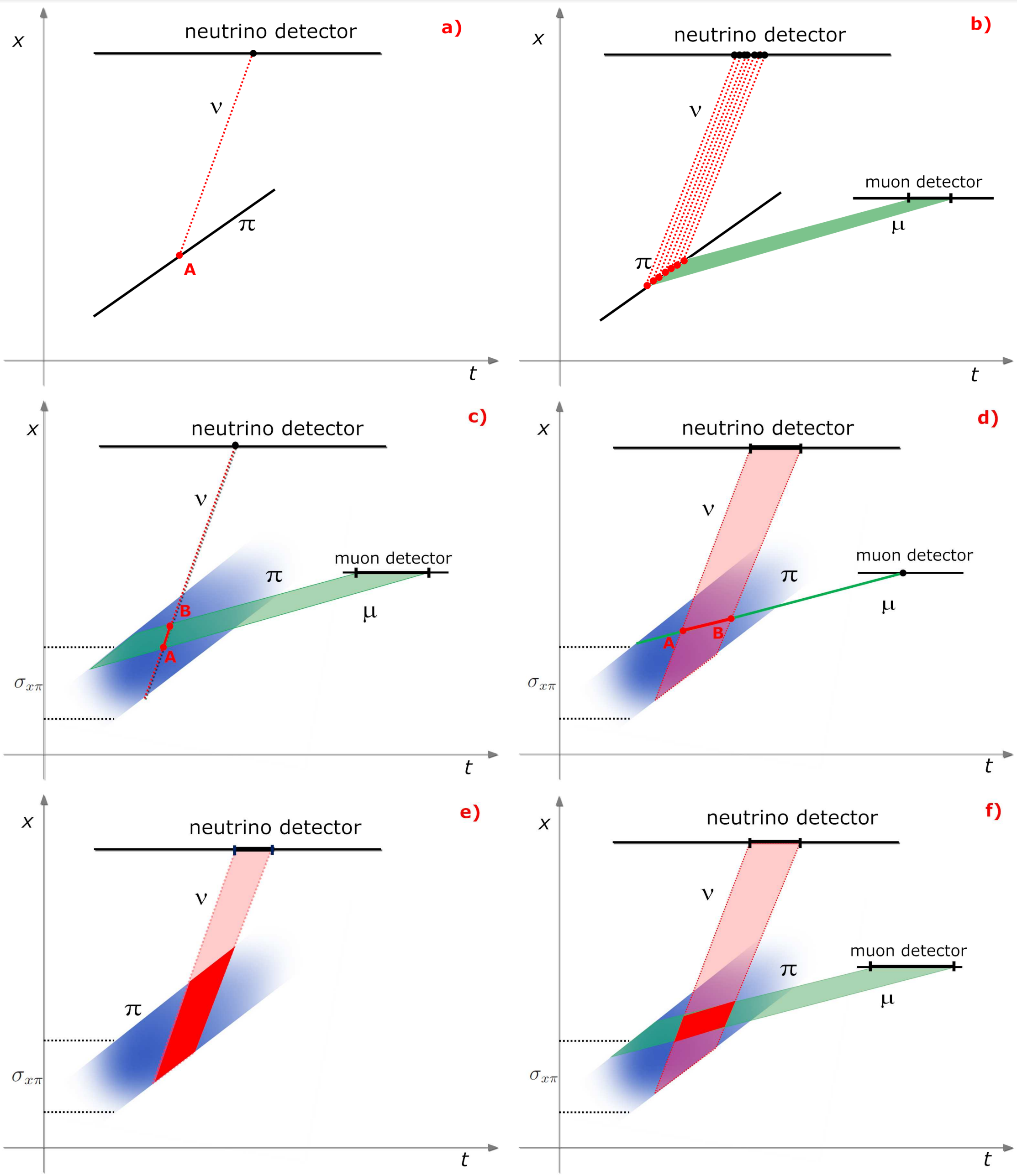
Neutrino production in extended sources: Amplitudes of neutrino emission in different points must be summed – a consistent QM procedure.

The standard approach: calculate the probability that neutrino produced at a fixed point x oscillates, and then integrate over all x in the source (probability summation procedure – classical in nature).

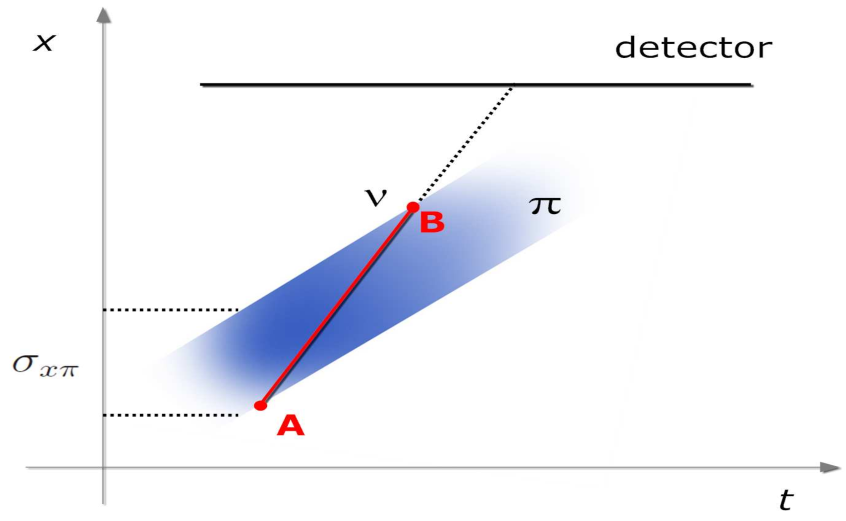
Both procedures give identical answers under realistic conditions!

The two approaches lead to different results whenever the localization properties of the parent particles at neutrino production and of the detection process are such that they prevent the precise localization of the point of neutrino emission – difficult to realize in practice.

Graphical interpretation



Finite-width pion WP



Additional phase for the segment AB:

$$\Delta\phi = -[E_j(P_j) - E_k(P_k)]\Delta t + (P_j - P_k)\Delta x\,.$$

Δt and Δx: projections of AB on the t and x axes. $\Rightarrow$

$$\Delta t = \frac{\sigma_{x\pi}}{v_g - v_\pi}\,, \qquad \Delta x = \sigma_{x\pi}\frac{v_g}{v_g - v_\pi}\,.$$

$$\Delta\phi \simeq -\frac{v_g}{v_g - v_\pi}\cdot\frac{\Delta m_{jk}^2}{2P}\sigma_{x\pi}$$

Finite-width pion WP – contd.

Are deviations between the results of the coherent amplitude summation and incoherent probability summation approaches experimentally observable? Requires extremely high energies of the parent pion:

$$2\,(E_\pi \sigma_{x\pi})\,\frac{\Delta m^2}{m_\pi^2} \gtrsim 1\,.$$

E.g. for $\sigma_{x\pi} \sim 10^{-4}$ cm and $\Delta m^2 \sim 1$ eV^2 $\Delta\phi$ would be ~ 1 for pion energies $E_\pi \gtrsim 10^3$ TeV – not feasible,

Another possibility: increase significantly the spatial width of w. packets of ancestor protons, which would increase the values of $\sigma_{x\pi}$. But: not clear how this could be achieved.

Other possibilities...

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<u>But:</u> Conditions for partial decoherence are difficult to realize

A long-lasting collaboration...

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THANKS !

A tough guy...

Backup slides

Coherence production conditions

Coherence production conditions:

$$|\Delta E| \ll \sigma_E\,, \qquad\qquad |\Delta p| \ll \sigma_p\,.$$

On the other hand:

$$\Delta E \simeq v_g \Delta p + \frac{\Delta m^2}{2E}.$$

Constraint $|\Delta E| \ll \sigma_E \quad \Rightarrow$

$$\left|\frac{v_g \Delta p}{\sigma_E} + \frac{\Delta m^2}{2E\sigma_E}\right| \ll 1. \qquad\qquad (*)$$

(a) The two terms in ΔE do not approximately cancel each other. $\Rightarrow$ $v_g|\Delta p| \ll \sigma_E \le \sigma_p$, i.e. for relativistic neutrinos $|\Delta p| \ll \sigma_p$ follows from $|\Delta E| \ll \sigma_E$.

(b1) There is a strong cancellation, but both terms on the l.h.s. of (*) are small – see case (a).

(b2) Strong cancellation, but both terms on the l.h.s. of (*) are $\gtrsim 1$: momentum condition is independent. But: the only known case – Mössbauer neutrinos.

Do charged leptons oscillate?

What do we mean by charged leptons?

The usual $e^{\pm}$, $\mu^{\pm}$ and $\tau^{\pm}$ are mass eigenstates $\Rightarrow$ do not oscillate.

[Also: unlike neutrinos, they participate also in EM interactions (and are normally detected via these interactions) which are flavour-blind.]

Assume we create a muon at $t_0 = 0$ and $\vec{x}_0 = 0$. Neglecting muon decay, we have

$$|\Psi(0)\rangle = |\mu\rangle\,; \quad |\Psi(\vec{x},t)\rangle = e^{-ip_\mu x}|\mu\rangle \quad \Rightarrow \quad P_{\mu\mu} = |\langle\mu|\Psi(\vec{x}\,,t)\rangle|^2 = 1$$

Assume now we manage to create a coherent superposition of μ and e:

$$|\Psi(0)\rangle = \cos\theta|\mu\rangle + \sin\theta|e\rangle$$

The weights of μ and e in the initial state: $\cos^2\theta$ and $\sin^2\theta$.

Do charged leptons oscillate?

Evolved state:

$$|\Psi(\vec{x},\, t)\rangle \;=\; e^{-ip_\mu x}\cos\theta|\mu\rangle \;+\; e^{-ip_e x}\sin\theta|e\rangle$$

The probabilities of finding μ and e:

$$P_\mu \;=\; |\langle\mu|\Psi(\vec{x}\,,t)\rangle|^2 \;=\; |e^{-ip_\mu x}\cos\theta|^2 \;=\; \cos^2\theta$$
$$P_e \;=\; |\langle e|\Psi(\vec{x}\,,t)\rangle|^2 \;=\; |e^{-ip_e x}\sin\theta|^2 \;=\; \sin^2\theta$$

– are the same! $\Rightarrow$ There are no oscillations between mass eigenstates, no matter if the initial state is pure or (coherently) mixed

$$\Downarrow$$

$\boxed{\text{There are no oscillations between } e\,,\ \mu \text{ and } \tau\,!}$

[NB: The same for neutrinos – initially produced ν_e can with some probability oscillate into ν_μ or ν_τ, but the weights of ν_1, ν_2 and ν_3 that were in the initial state will remain the same!]

Do charged leptons oscillate?

What about $W^{\pm} \to l^{\pm}\nu$ decays in flight? Let γ be the Lorentz factor of $W^{\pm}$. $(\Delta v_g)_{\rm min} \simeq \Delta m_{\mu e}^2/2E^2 \equiv (m_\mu^2 - m_e^2)/2E^2$ and the partial decay width of $W^{\pm}$ scale with γ as

$$(\Delta v_g)_{\rm min} \to \gamma^{-2}(\Delta v_g)_{\rm min}\,, \qquad \Gamma^0_{W\to l_a\nu} \to \gamma^{-1}\Gamma^0_{W\to l_a\nu}\,.$$

Therefore the maximum coherence length $(l_{\rm coh})_{\rm max} \simeq \sigma_x/(\Delta v_g)_{\rm min} \simeq 1/[\Gamma^0_{W\to l_a\nu}(\Delta v_g)_{\rm min}]$ scales as

$$(l_{\rm coh})_{\rm max} \to \gamma^3 (l_{\rm coh})_{\rm max}\,.$$

In order for $(l_{\rm coh})_{\rm max}$ to be larger than e.g. 1 m, one would need $\gamma \gtrsim 1600$, or $E_W \gtrsim 130$ TeV – far above presently feasible energies.

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N.B.: Even if coherence was satisfied for charged leptons, to fix the composition of the mixed $l^\pm$ state in terms of e, μ and τ one would have to detect the accompanying neutrino as a state different from $\nu_{\rm fl}$ – e.g. as a mass eigenstate. Not possible within the standard model!

Extensions of the standard model?

Consider the SM amended by three heavy RH neutrinos N_i (seesaw model) plus an extra Higgs doublet. In this model N_i can decay into a charged lepton and charged Higgs boson:

$$N_i \to e_i^- + \Phi^+ \,.$$

Decays are caused by the Yukawa coupling Lagrangian

$$\mathcal{L}_Y = Y_{ai}\bar{L}_a N_{Ri}\Phi \;+\; h.c.\,,$$

In the basis where the mass matrices of N_i and $l^\pm$ have been diagonalized, the Yukawa coupling matrix Y_{ai} is in general not diagonal $\Rightarrow$ in the decay of a mass-eigenstate sterile neutrino N_i any of the three charged leptons $e_a = e,\, \mu,\, \tau$ can be produced.
What are the conditions for the produced charged lepton state e_i to be a coherent superposition of the mass eigenstates e_a:

$$|e_i\rangle = [(Y^\dagger Y)_{ii}]^{-1/2}\sum_a Y^\dagger_{ia}\,|e_a\rangle\,,$$

and how long this state can maintain its coherence?

Extensions of the standard model?

Neglecting the masses of $\Phi^\pm$ and $l^\pm$ compared to the mass M_i of the sterile neutrino:

$$\Gamma_i^0 \simeq \alpha_i M_i\,, \qquad \text{where} \qquad \alpha_i \equiv \frac{(Y^\dagger Y)_{ii}}{16\pi}\,.$$

Coherent production condition:

$$2\sqrt{2}\,E\,\Gamma_i^0 \simeq 2\sqrt{2}\,(M_i/2)\,\alpha_i M_i > \max\{m_\mu^2 - m_e^2,\ m_\tau^2 - m_\mu^2\}\,,$$

or

$$\alpha_i > 2.2\,(\mathrm{GeV}/M_i)^2\,.$$

From $l_{\rm coh} = \sigma_x v_g/\Delta v_g$ the coherence length for the emitted charged lepton state:

$$l_{\rm coh} \simeq \frac{M_i^2}{2\Gamma_i^0(m_\tau^2 - m_\mu^2)} \simeq 3.1\times 10^{-15}\,\alpha_i^{-1}\frac{M_i}{\mathrm{GeV}}\ \mathrm{cm}\,.$$

$\Rightarrow$

Extensions of the standard model?

$$l_{\mathrm{coh}} < 1.4 \times 10^{-15}\ \mathrm{cm}\ (M_i/\mathrm{GeV})^3\,.$$

For N_i decays in flight the r.h.s. has to be multiplied by γ^3 $\Rightarrow$ $(M_i/\mathrm{GeV})^3$ has to be replaced by $(E_i/\mathrm{GeV})^3$.
The charged lepton state will maintain its coherence over the distance $\sim 1\ m$ if

$$E_i \gtrsim 400\ \mathrm{TeV} \quad \Rightarrow \quad (Y^\dagger Y)_{ii} \gtrsim 1.3 \times 10^{-11}\,.$$

If only e and μ are to be produced coherently, a milder lower limit on E_i results:

$$E_i \gtrsim 10\ \mathrm{TeV}\,, \qquad (Y^\dagger Y)_{ii} \gtrsim 8.5 \times 10^{-11}\,.$$

Extensions of the standard model?

If the condition for coherent creation of the charged lepton state is satisfied and this state is detected through the inverse decay process before it loses its coherence, it may exhibit oscillations: a mass eigenstate sterile neutrino N_j different from N_i can be produced in the detection process $\Rightarrow$ the state e_i has oscillated into e_j.

Charged leptons would be able to oscillate, leading to a non-zero probability of the emission or absorption of a different sterile neutrino mass eigenstate N_j in the processes $e_j^{\pm} + \Phi^{\mp} \to N_j$ or $e_j^{\pm} + N_j \to \Phi^{\pm}$.

$\Rightarrow$ The roles of neutrinos and charged leptons reversed compared to the usual situation because of sterile neutrinos being much heavier than the charged leptons.

Finite-width pion WP

Two models of finite-size pion WP, Gaussian and box-type. For $\Gamma l_p/v_\pi \gg 1$:

$$\diamondsuit \quad P^{\rm eff}_{\mu\mu} = c^4 + s^4 + \frac{2c^2 s^2}{\xi^2+1}\left[(\cos\phi + \xi\sin\phi) - A_\pi \xi(\xi\cos\phi - \sin\phi)\right]$$

The parameter A_π:

$$A_{\pi\,\rm box} = \frac{v_g}{v_\pi}\frac{\Gamma}{v_g - v_\pi}\sigma_{x\pi}\,, \qquad A_{\pi\,\rm Gauss} = \frac{2}{\sqrt{2\pi}}\frac{v_g}{v_\pi}\frac{\Gamma}{v_g - v_\pi}\sigma_{x\pi}\,.$$

i.e. $A_\pi \sim (v_g/v_\pi)\sigma_{x\pi}/\sigma_{x\nu}$. The correction is of order

$$A_\pi \xi \sim \left[\frac{\Delta m^2}{2P}\sigma_{x\pi}\right]\cdot\frac{v_g}{v_g - v_\pi} = 2\pi\frac{\sigma_{x\pi}}{l_{\rm osc}}\cdot\frac{v_g}{v_g - v_\pi}$$

– small since $\sigma_{x\pi} \lll l_{\rm osc}$ (unless $v_\pi \simeq v_g$ to a very high accuracy).

An interesting point: summation at the probabilities level for finite-thickness $(= d)$ proton target and point-like neutrino production gives similar expression, but with $A_\pi \xi = (\Delta m^2/2P)d$ (no factor $[v_g/(v_g - v_\pi)]$).

Effects of muon detection (for pointlike pion)

If muons is detected: plane wave $\rightarrow$ wave packet

$$\psi_\mu(x,t) = e^{iKx - iE_\mu(K)t}\, g_\mu[(x-x_S) - v_\mu(t-t_S)]\,.$$

Shape factor $g_\mu[(x-x_S) - v_\mu(t-t_S)]$ determined by the muon detection process. The argument of g_μ: initial condition that at time $t = t_S$ the peak of the w. packet is at $x = x_S$. Choose x_S as the coordinate of the center of the muon w. packet at the neutrino production time. For pointlike pions x_S should lie on the pion's trajectory $\Rightarrow$ $x_S = v_\pi t_S$.

$$I_{jk}(L) = C_0 \int_0^{l_p} dx \left| g_\mu\Big((x-x_S)\frac{v_\pi - v_\mu}{v_\pi}\Big)\right|^2 e^{-i\frac{\Delta m_{jk}^2}{2P}(L-x) - \Gamma\frac{x}{v_\pi}}\,.$$

When the muon is undetected: $g_\mu \rightarrow 1$. Eff. width of the muon w. packet:

$$\tilde{\sigma}_{x\mu} \equiv \sigma_{x\mu}\frac{v_\pi}{v_\pi - v_\mu}\,.$$

The results of amplitude summation and probability summation approaches again coincide.

Limiting cases

(1) $\tilde{\sigma}_{x\mu} \to \infty$: plane wave limit. $g_\mu \to const$ – previous results recovered.

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$$I_{jk}(L) = const.\, e^{-\Gamma \frac{x_S}{v_\pi}}\, e^{-i\frac{\Delta m_{jk}^2}{2P}(L-x_S)} \qquad \Rightarrow \qquad P_{\alpha\beta}^{\rm stand}(L - x_S).$$

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$$P_{\mu\mu} = c^4 + s^4 + 2s^2c^2\, e^{-\frac{1}{2}\left(\frac{\Delta m^2}{2P}\right)^2 \tilde{\sigma}_{x\mu}^2}\, \cos\left(\frac{\Delta m^2}{2P}(L - x_S)\right).$$

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$\Rightarrow$ the decoherence parameter is $\frac{\Delta m^2}{2P}\tilde{\sigma}_{x\mu}$. For $\tilde{\sigma}_{x\mu} \ll l_{\rm osc}/2\pi$ the stand. probability is recovered.

The case of muon interacting with medium

The case when the muon interacts with the medium but there are no muon detectors (the muon's position not measured). Neutrinos are not tagged $\Rightarrow$ one has to integrate

$$I_{jk}(L) = C_0 \int_0^{l_p} dx \left| g_\mu \Big((x - x_S) \frac{v_\pi - v_\mu}{v_\pi} \Big) \right|^2 e^{-i \frac{\Delta m_{jk}^2}{2P}(L-x) - \Gamma \frac{x}{v_\pi}} .$$

over x_S.

Integration of $|g_\mu|^2$ gives the normalization constant of this function which does not influence the oscillation probabilities. The results obtained in the case when the muon is not detected are recovered.

Pion interactions

Interaction of the pions in the bunch btw themselves or with other particles may identify the individual pion whose decay produces a given neutrino. E.g. pion decay may lead to some recoil of the neighbouring particles which may be detected. $\Rightarrow$

Would localize the neutrino production point up to an uncertainty of order of the inter-pionic distance (or the distance between the pion and the other particles in the source) r_0 $\Rightarrow$ neutrino tagging.
Production decoherence parameter: $(\Delta m^2/2P)r_0$.

If the information about the interaction between the decaying pion and the surrounding particles is not recorded and not used for neutrino tagging, the oscillations occur in exactly the same way as if pions did not interact with each other or with other particles.

Production coherence for some experiments

Unless otherwise specified, $\Delta m^2 = 2$ eV^2. For β-beams $E_0 = 2$ MeV, $\tau_0 = 1$s, $\gamma = 100$.

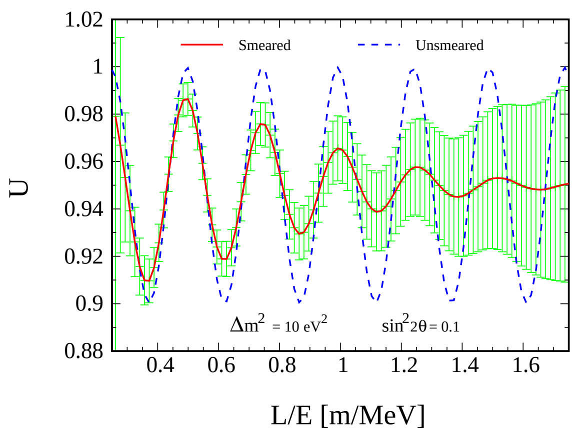
Experiment	$\langle E_\nu \rangle$(MeV)	L(m)	$l_p(m)$	$l_{\rm dec}$(m)	$l_{\rm osc}$(m)	ϕ	$\Gamma l_p/v_P$	ϕ_p	ξ
LSND	$\sim$40	30	0	0	50	3.8	-	0	0
KARMEN	$\sim$40	17.7	0	0	50	2.24	-	0	0
MiniBooNE	$\sim$800	541	50	89	992	3.43	0.56	0.32	0.56
NOMAD	$2.7 \cdot 10^3$	770	290	3009	33480	0.145	0.1	0.054	0.56
(20 eV^2)					3348	1.45	0.1	0.54	5.64
CCFR(10^2 eV^2)	$5 \cdot 10^4$	891	352	5570	1240	4.51	0.06	1.78	28.2
CDHS	3000	130	52	334	3720	0.22	0.155	0.088	0.56
(20 eV^2)					372	2.2	0.155	0.878	5.64
K2K	1500	300	200	167	1861	1.01	1.2	0.68	0.56
T2K	600	280	96	66.4	744	2.36	1.45	0.81	0.56
Minos	3300	1040	675	368	4092	1.6	1.84	1.04	0.56
NOνA	2000	1040	675	223	2480	2.64	3.03	1.71	0.56
β-beams	400	$1.3 \cdot 10^5$	2500	$3 \cdot 10^{10}$	496	1647	$8.3 \cdot 10^{-8}$	31.7	$3.8 \cdot 10^8$

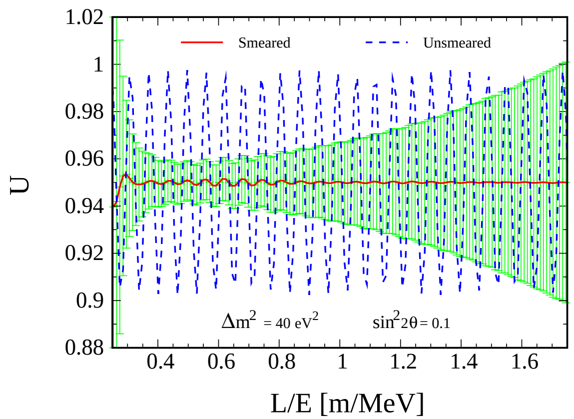
Noticeable effects for MiniBooNE, NOMAD (20 eV^2), CCFR (100 eV^2), CDHS (20 eV^2), K2K, T2K, MINOS, NOνA, very large effects for β-beams

Examples of prod. coherence violation

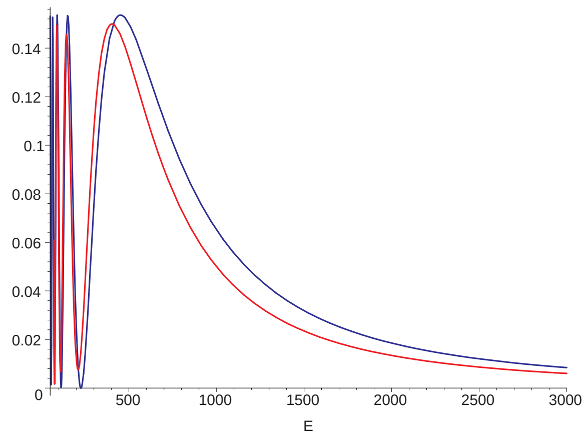
$\nu_e \to \nu_s$ oscillations in β-beam expts. (Agarwalla, Huber & Link, arXiv:0907.3145).

Ratio of oscillated and unoscillated fluxes ($\gamma = 30,\ l_p = 10$m, $L = 50$ m):

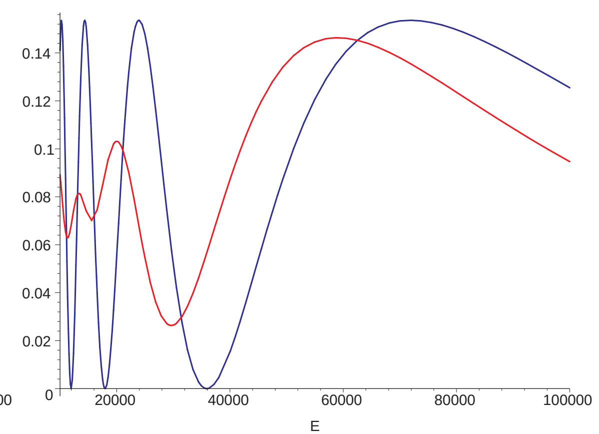




T2K



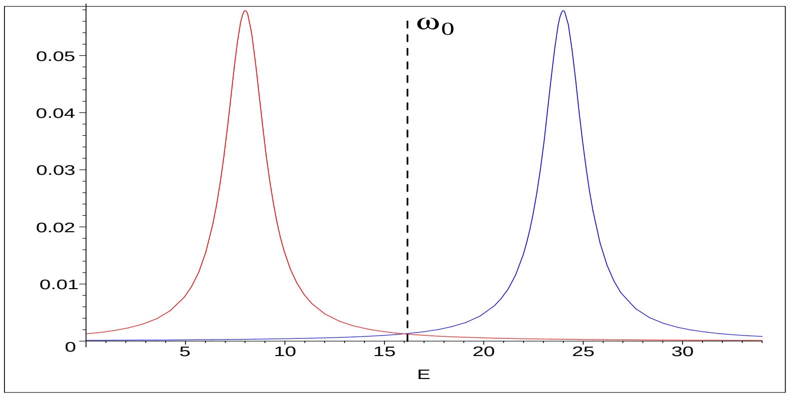
CCFR

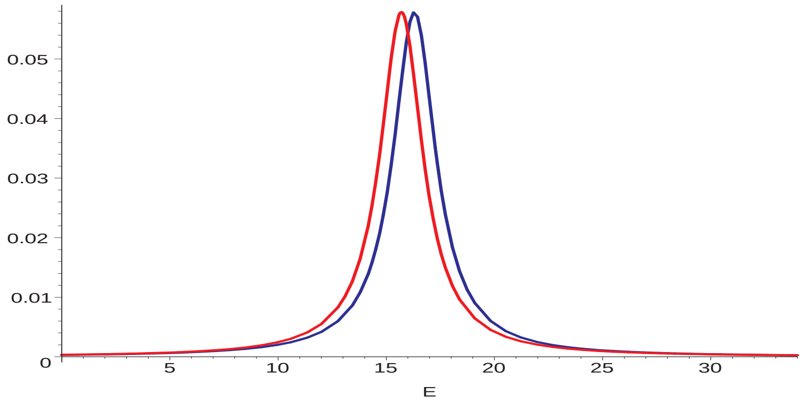


Mössbauer effect

Conventional Mössbauer effect – Res. absorption of γ quanta:

$$A^* \to A + \gamma; \qquad A + \gamma \to A^*$$





Nuclear exc. energy: ω_0.

Recoil energy: $R = \frac{\omega_0^2}{2M}$

$$E_e = \omega_0 - \frac{\omega_0^2}{2M}$$

$$E_a = \omega_0 + \frac{\omega_0^2}{2M}$$

Recoilless emission and absorption (Mössb. eff.):

$$E_e \simeq E_a \simeq \omega_0$$

Strong enhancement of absorption

Mössbauer effect with neutrinos?

Beta decay with 2 - body final state:

$$A(N,Z) \to A(N-1,Z+1) \ + e_B^- \ + \ \bar{\nu}_e$$

Inverse process:

$$\bar{\nu}_e \ + \ e_B^- \ + \ A(N-1,Z+1) \to A(N,Z)$$

If the neuclei are embedded in solid state lattice, recoilless emission and absorption in principle possible.

Possibility of Mössbauer effect with neutrinos:

Visscher, 1959; Kells & Schiffer, 1983; Raghavan, 2005, 2006

Relevant processes considered:

Bahcall, 1961 – bound state β decay;

Mikaelyan, Tsinoev & Borovoi, 1967 – inverse process (stimulated K-electron capture)

Mössbauer effect with neutrinos?

Mössbauer effect with neutrinos on ^{3}H – ^{3}He system:

$$^3\mathrm{H} \;\to\; (^3\mathrm{He} \,+\, e^-_B) + \bar{\nu}_e\,; \qquad \bar{\nu}_e \,+\, (^3\mathrm{He} \,+\, e^-_B) \;\to\; {}^3\mathrm{H}$$

Energy release: $Q = 18.6$ keV. Mean lifetime of ^{3}H is 17.8 yr $\Rightarrow$
Nat. linewidth $\Gamma_{^3\mathrm{H}} = 1.17 \times 10^{-24}$ eV – extremely small: $\Delta E/E \sim 10^{-28}$!

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Various (homogeneous and inhomogeneous) broadening effects exist. By suppressing them probably an effective linewidth $\Gamma_{\mathrm{eff}} \sim 10^{-11}\ \mathrm{eV}$ can be achieved (W. Potzel) $\Rightarrow$ $\Delta E/E \sim 10^{-15}$ – still very small.

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Number of ^{3}H atoms produced in the target can be counted by detecting their decay or using mass spectroscopy.

Very serious technical difficulties exist, but apparently realization of a Mössbauer experiment with neutrinos is not impossible (Raghavan, Potzel).
If realized: for $\Gamma \sim 10^{-11}$ eV, $\sigma \sim 10^{-33}\, cm^2$!

Mössbauer effect with neutrinos?

If a Mössbauer neutrino experiment is realized $\Rightarrow$ a unique source of extremely monochromatic low energy neutrinos. Would open up possibilities

- to detect for the first time keV neutrinos
- to detect neutrinos with g or 100 g scale (rather than t or kt scale) detectors
- to observe gravitational redshift of neutrinos
- to study neutrino oscillations at distances ~ 10 m rather than km or hundreds/thousands of km
- to search for the effects of yet unmeasured mixing angle θ_{13} and possibly measure it
- to discriminate between the normal and inverted neutrino mass hierarchies without using matter effects
- to study possible oscillations into sterile neutrino states

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Arguments in the literature (Bilenky et al.):

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Is that true?

Will Mössbauer neutrinos oscillate?

Neutrino oscillations require some intrinsic uncertainty of energy and momentum of the emitted and detected neutrino states !

If E and p were known precisely, from $E^2 = p^2 + m_i^2$ one would determine which mass eihenstate has been emitted $\Rightarrow$ neutrinos of different mass would not be emitted coherently.

For Mössbauer effect with neutrinos in ^{3}H – ^{3}He system:

$$\diamondsuit \quad \frac{\Delta m^2}{2E} = \frac{2.5 \times 10^{-3}\,\mathrm{eV}^2}{2 \cdot 18.6\,\mathrm{keV}} \simeq 6.7 \cdot 10^{-8}\,\mathrm{eV} \gg \Gamma \sim 10^{-11}\,\mathrm{eV}\,!$$

Can neutrinos of different mass be accommodated within such a small energy uncertainty?

Will neutrinos with such small energy uncertainty oscillate ?

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By knowing the neutrino energy and momentum one can determine its mass
But: Mass eigenstates do not oscillate !

Two “standard” approaches to ν oscillations

The oscillation phase: $\phi = p_\mu x^\mu = E \cdot t - p \cdot x \quad \Rightarrow$

$$\boxed{\Delta\phi \;=\; \Delta E\cdot t \,-\, \Delta p\cdot L}$$

I. Same momentum approach $(\Delta p \,=\, 0)$. The oscillation phase

$$\Delta\phi \;=\; \Delta E \cdot t \;-\; \Delta p \cdot L \;\;\Rightarrow\;\; \Delta E \cdot t$$

– evolution in time; needs to use $L \,\simeq\, t$.

II. Same energy approach $(\Delta E \,=\, 0)$:

$$\Delta\phi \;=\; -\,\Delta p \cdot L$$

– evolution in space.

Will Mössbauer neutrinos oscillate?

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Our point of view: in general, there is no reason to believe that ν_i have either same energy or same momentum. No need to perform Mössbauer ν experiment to decide which approach is correct – it is sufficent to carefully examine the validity of the approximations used.

How about Mössbauer neutrinos?

Very small effective linewidth Γ $\Rightarrow$ small energy uncertainty of the emitted neutrino state. Can different neutrino mass eigenstates be emitted coherently?

$$\sigma_{m^2} = \left[(2E\sigma_E)^2 + (2p\sigma_p)^2\right]^{1/2}$$

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$\Rightarrow \quad \sigma_p \sim 10\ \mathrm{keV}$, i.e. $\quad \sigma_m^2 \simeq 2p\sigma_p \sim 4\times 10^8\ \mathrm{eV}^2 \gg \Delta m^2$

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$\Rightarrow$ Oscillations must occur !