A_4, θ_{13} and δ_{CP}

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Short History of A_4

In 1978, soon after the putative discovery of the third family of leptons and quarks, it was conjectured by Cabibbo and Wolfenstein independently that

$$U_{CW}^{l\nu} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$. This implies $\sin^2\theta_{12} = \sin^2\theta_{23} = 1/2$, $\sin^2\theta_{13} = 1/3$, $\delta_{CP} = \pm\pi/2$, i.e. bibitrimaximal mixing.

In 2001, Ma/Rajasekaran showed that U_{CW} occurs in A_4 which allows $m_{e,\mu,\tau}$ to be arbitrary, predicting also $\sin^2 2\theta_{atm} = 1,\ \theta_{e3} = 0$. In 2002, Babu/Ma/Valle showed how $\theta_{e3} \neq 0$ can be radiatively generated in A_4 with $\delta_{CP} = \pm\pi/2$, i.e. maximum CP violation.

In 2002, Harrison/Perkins/Scott, after abandoning their bimaximal and trimaximal hypotheses, proposed the tribimaximal mixing matrix, i.e.

$$U_{l\nu}^{HPS} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \sim (\eta_8, \eta_1, \pi^0)$$

This means $\sin^2 2\theta_{atm} = 1$, $\tan^2 \theta_{sol} = 1/2$, $\theta_{e3} = 0$. In 2004, I showed that this tribimaximal mixing may be obtained in A_4, with

$$U_{CW}^\dagger M_\nu U_{CW} = \begin{pmatrix} a+2b & 0 & 0 \\ 0 & a-b & d \\ 0 & d & a-b \end{pmatrix},$$

in the basis that M_l is diagonal. At that time SNO data gave $\tan^2 \theta_{sol} = 0.40 \pm 0.05$, but it was changed in early 2005 to 0.45 ± 0.05. Tribimaximal mixing and A_4 then became part of the lexicon of the neutrino theorist.

After the 2005 SNO revision, two A_4 models quickly appeared. (I) Altarelli/Feruglio:

$$U_{CW}^{\dagger} M_\nu U_{CW} = \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix},$$

i.e. $b = 0$, and (II) Babu/He:

$$U_{CW}^{\dagger} M_\nu U_{CW} = \begin{pmatrix} a' - d^2/a' & 0 & 0 \\ 0 & a' & d \\ 0 & d & a' \end{pmatrix},$$

i.e. $d^2 = 3b(b-a)$.

The challenge is to prove experimentally that A_4 exists. If A_4 is realized by a renormalizable theory at the electroweak scale, then the extra Higgs doublets required will bear this information. Specifically, A_4 breaks to the residual symmetry Z_3 in the charged-lepton sector, and all Higgs Yukawa interactions are determined in terms of lepton masses. This notion of lepton flavor triality [Ma(2010)] (exact if neutrino masses are zero) may be the key to such a proof, and these exotic Higgs doublets could be seen at the LHC: Cao/Khalili/Ma/Okada(2011); Cao/Damanik/Ma/Wegman(2011).

Nonzero θ_{13} in A_4

There is now very strong experimental evidence for nonzero θ_{13}.

Daya Bay: $\sin^2 2\theta_{13} = 0.089 \pm 0.010 \pm 0.005$,
RENO: $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$,
Double CHOOZ: $\sin^2 2\theta_{13} = 0.109 \pm 0.030 \pm 0.025$,

and also some evidence for nonmaximal θ_{23}:

MINOS: $\sin^2 2\theta_{23} = 0.96 \pm 0.04$.

In the A_4 basis, let

$$\mathcal{M}_\nu = \begin{pmatrix} a & f & e \\ f & a & d \\ e & d & a \end{pmatrix},$$

from 4 Higgs triplets $\sim \underline{1}, \underline{3}$ under A_4. The old idea was to enforce $e = f = 0$ to obtain tribimaximal mixing. Technically this was very difficult (but not impossible) to do. Suppose d, e, f are arbitrary (which is very easy to do), and let $b = (e+f)/\sqrt{2}$ and $c = (e-f)/\sqrt{2}$, then in the tribimaximal basis,

$$\mathcal{M}_\nu^{(1,2,3)} = \begin{pmatrix} a+d & b & 0 \\ b & a & c \\ 0 & c & a-d \end{pmatrix},$$

Note that the (1,3) and (3,1) entries are automatically zero. If a, b, c, d are all real, then

$$\sin^2 2\theta_{23} \simeq 1 - 2\sin^2 2\theta_{13}.$$

Since $\sin^2 2\theta_{23} > 0.92$, it would predict $\sin^2 2\theta_{13} < 0.04$, which is of course excluded by recent data. This looks like bad news, but it is actually good news.

Large δ_{CP} in A_4

In general, a, b, c, d are not real, although a may be chosen real by convention. What the A_4 structure tells us is that there are relationships among the three masses, the three angles and the three phases.

To see how this works, let $b = 0$ (which may be maintained by an interchange symmetry), then $\mathcal{M}_\nu^{(1,2,3)}$ can be diagonalized exactly by U_ϵ with an angle θ and a phase ϕ.

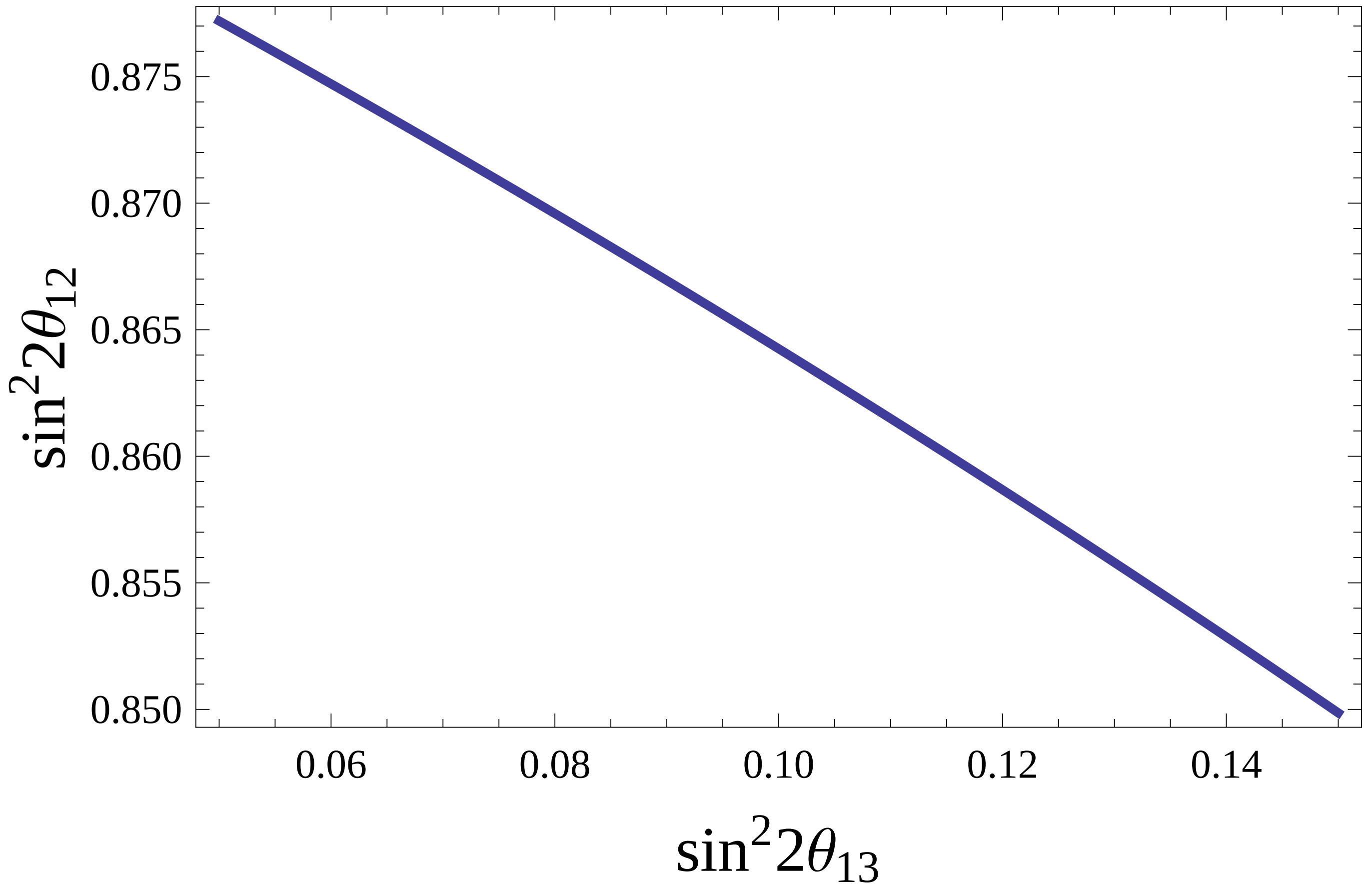
Let $U' = U_{TB} U_\epsilon^T$, then

$$U'_{e1} = \sqrt{\frac{2}{3}}, \quad U'_{e2} = \frac{\cos\theta}{\sqrt{3}}, \quad U'_{e3} = -\frac{\sin\theta}{\sqrt{3}} e^{-i\phi},$$

$$U'_{\mu 3} = -\frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{3}} e^{-i\phi}, \quad U'_{\tau 3} = \frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{3}} e^{-i\phi}.$$

The angles θ_{12}, θ_{23}, θ_{13}, and the phase δ_{CP} are extracted from $\tan^2\theta_{12} = |U'_{e2}/U'_{e1}|^2$, $\tan^2\theta_{23} = |U'_{\mu 3}/U'_{\tau 3}|^2$, and $\sin\theta_{13} e^{-i\delta_{CP}} = U'_{e3} e^{-i\alpha'_3/2}$, where α'_3 depends on the specific values of the mass matrix. As a result,

$$\tan^2\theta_{12} = \frac{1 - 3\sin^2\theta_{13}}{2},$$

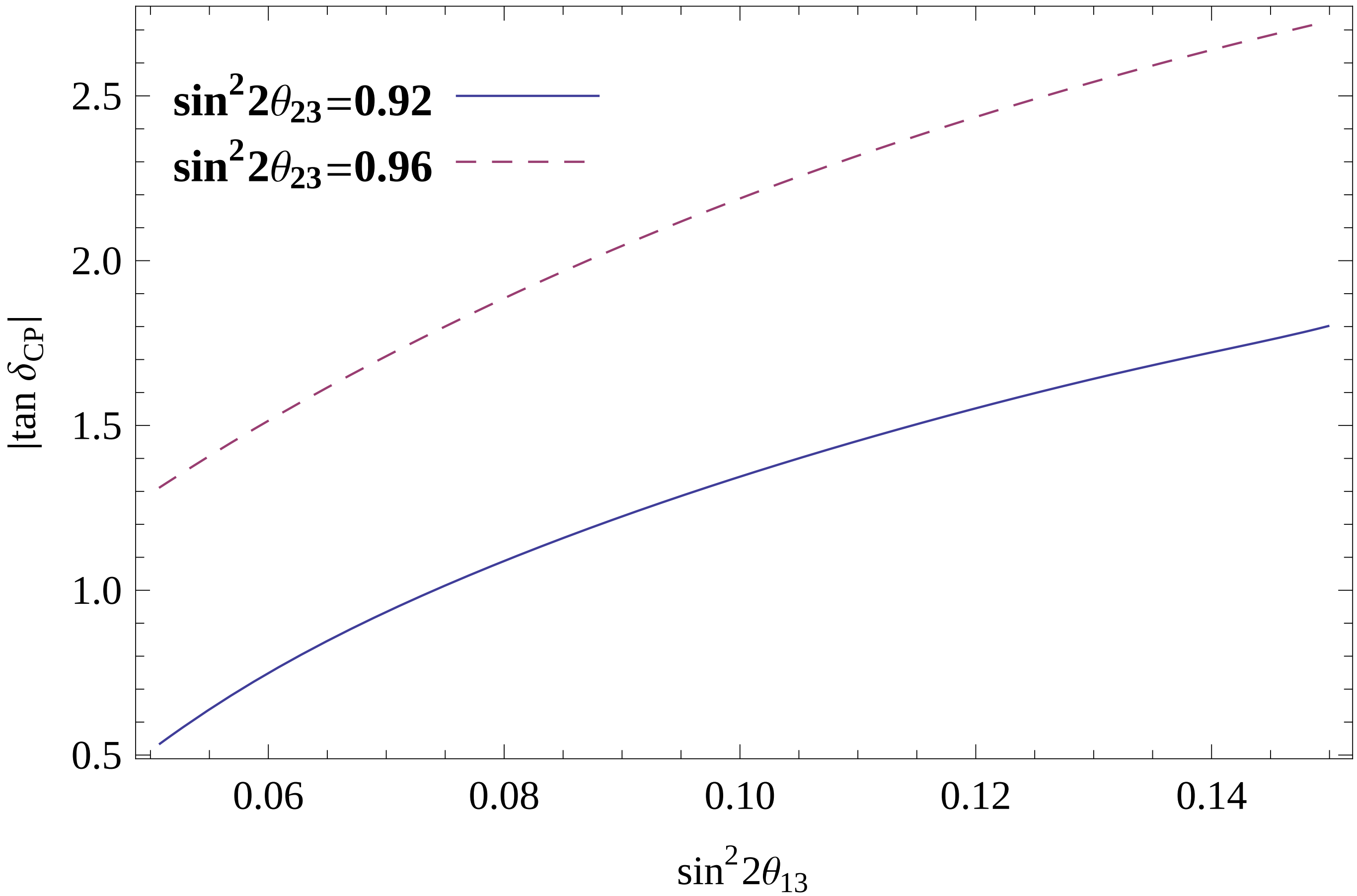


$\sin^2 2\theta_{12}$ versus $\sin^2 2\theta_{13}$.

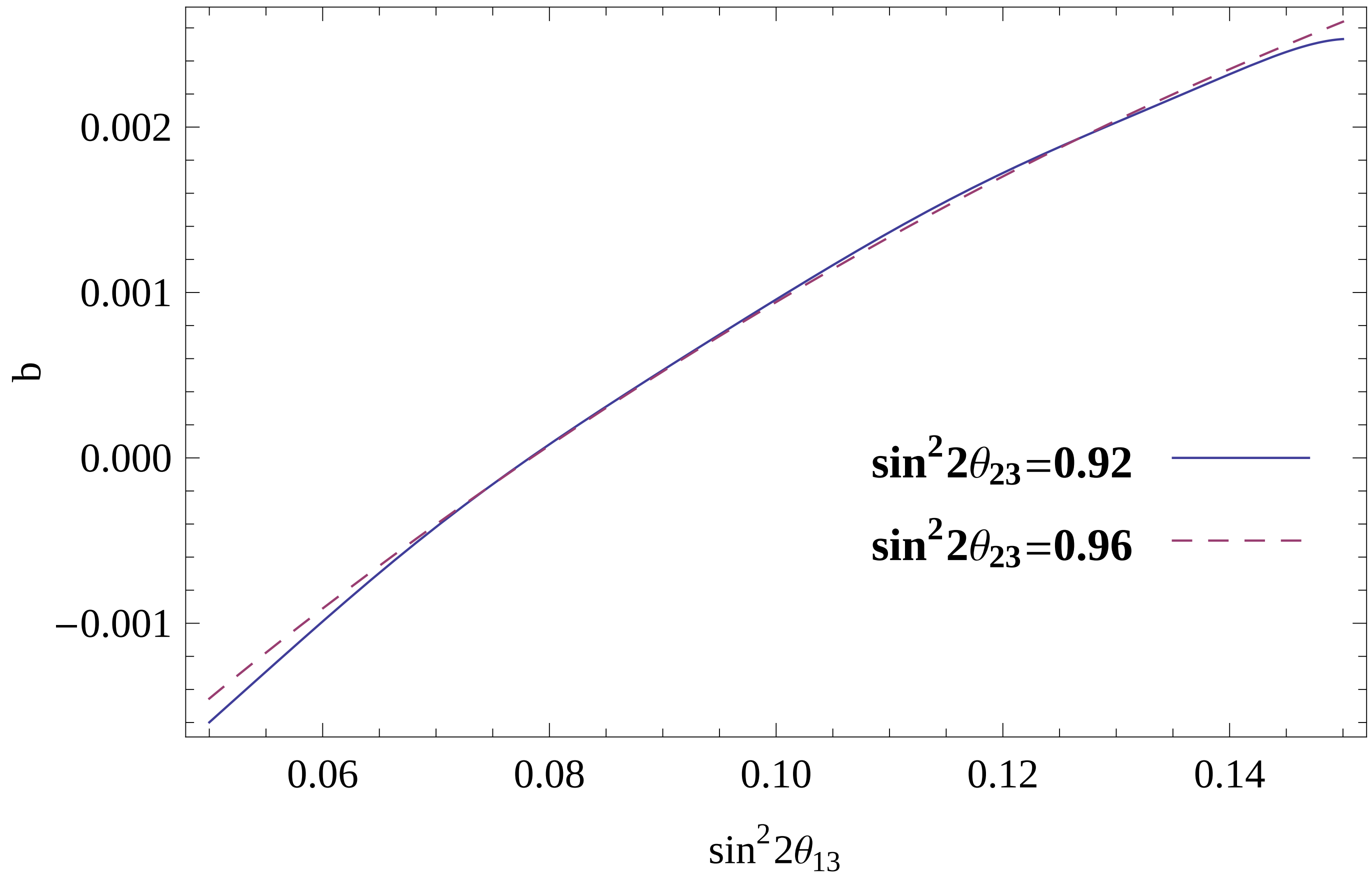
$$\tan^2\theta_{23} = \frac{\left(1 - \frac{\sqrt{2}\sin\theta_{13}\cos\phi}{\sqrt{1-3\sin^2\theta_{13}}}\right)^2 + \frac{2\sin^2\theta_{13}\sin^2\phi}{1-3\sin^2\theta_{13}}}{\left(1 + \frac{\sqrt{2}\sin\theta_{13}\cos\phi}{\sqrt{1-3\sin^2\theta_{13}}}\right)^2 + \frac{2\sin^2\theta_{13}\sin^2\phi}{1-3\sin^2\theta_{13}}}.$$

Let $\sin^2\theta_{13} = 0.16$ (i.e. $\sin^2 2\theta_{13} = 0.10$) and $Im(c) = 0$, then $\phi = 0$, and $\sin^2 2\theta_{23} = 0.80$, which is ruled out. Thus $\sin^2 2\theta_{23} > 0.92$ implies $|\tan\phi| > 1.2$.

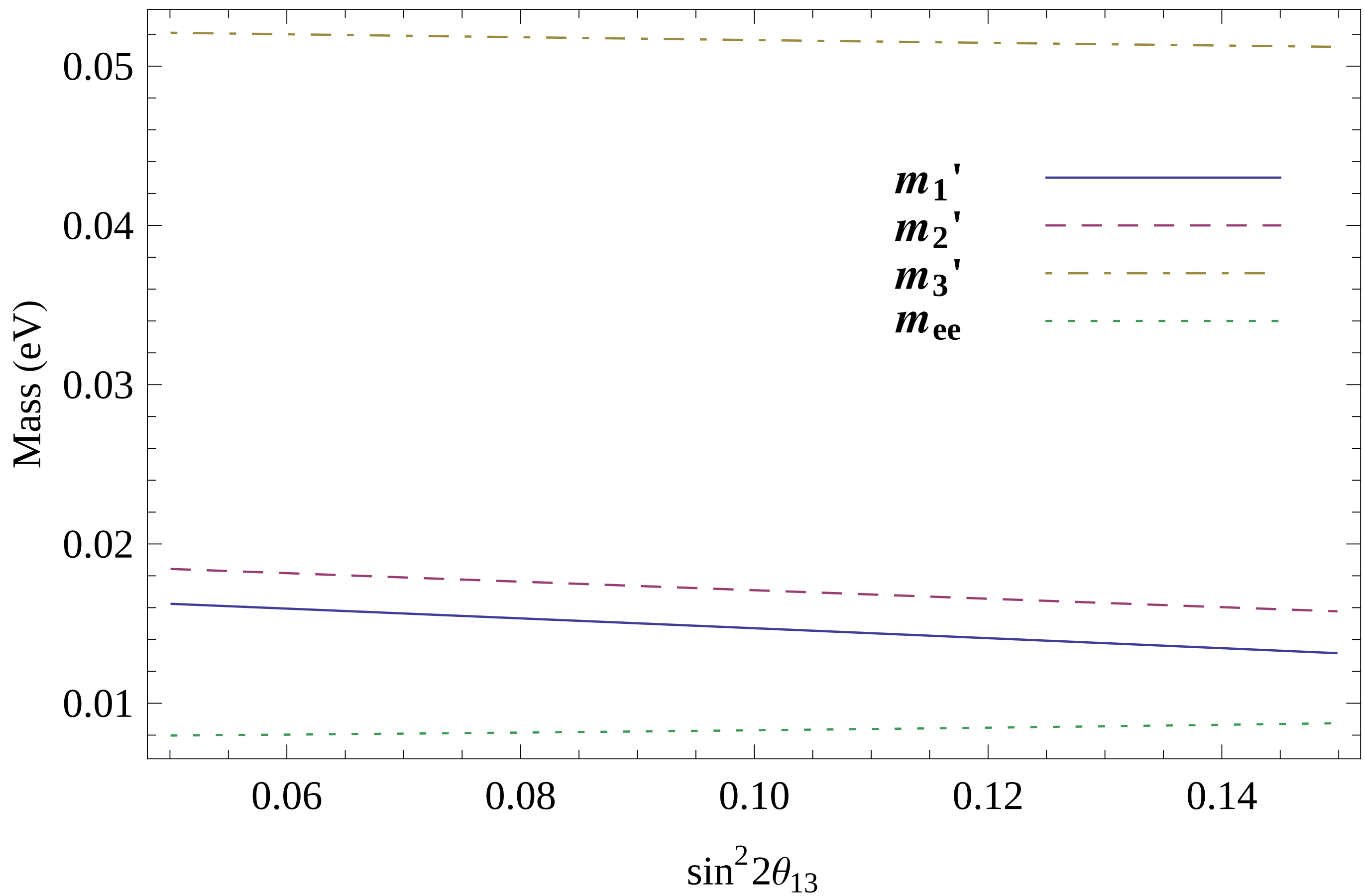
In a full numerical analysis with b, d real and c complex [Ishimori/Ma(2012)], $|\tan\delta_{CP}|$ is obtained as a function of $\sin^2 2\theta_{13}$ (for normal hierarchy only).



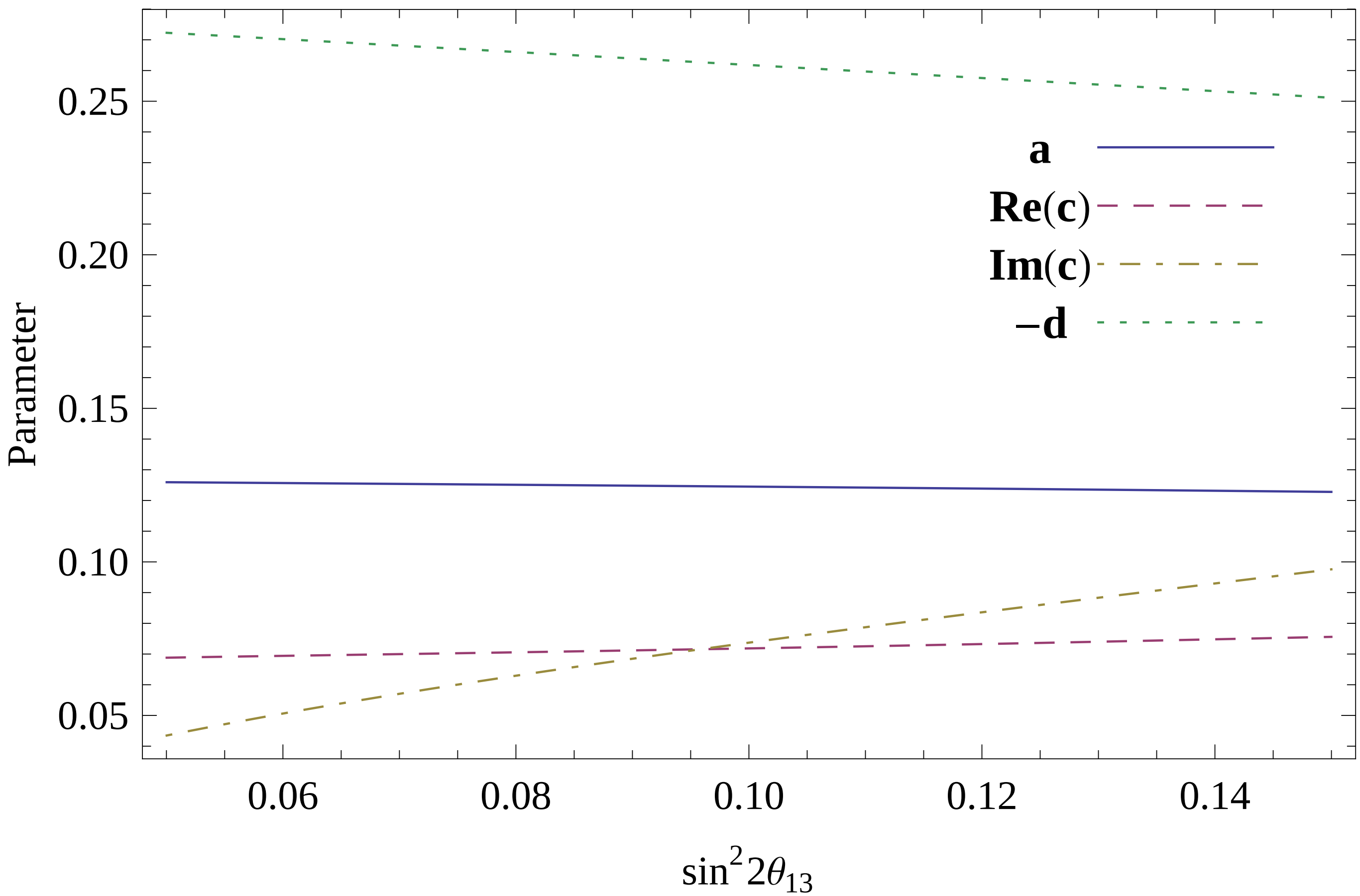
$|\tan \delta_{CP}|$ versus $\sin^2 2\theta_{13}$ for $\sin^2 2\theta_{23} = 0.92$ and 0.96.



Parameter b versus $\sin^2 2\theta_{13}$ for $\sin^2 2\theta_{23} = 0.92$ and 0.96.



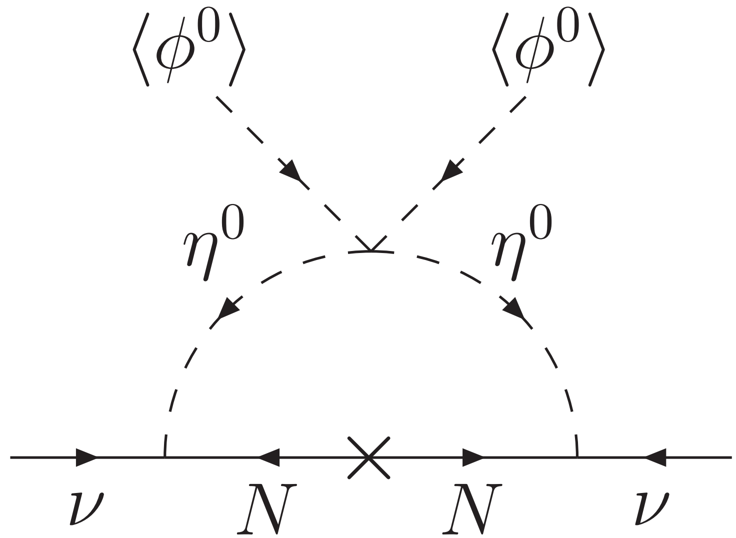
Physical neutrino masses and the effective neutrino mass m_{ee} in neutrinoless double beta decay for $\sin^2 2\theta_{23} = 0.96$.



A_4 parameters for $\sin^2 2\theta_{23} = 0.96$.

Scotogenic Majorana Neutrino Mass

Neutrino mass is linked to dark matter in a one-loop mechanism [Ma(2006)] by having a second scalar doublet (η^+, η^0) and three neutral fermion singlets N_i, all of which are odd under an exactly conserved Z_2 whereas all standard-model particles are even. This may be called 'scotogenic' from the Greek 'scotos' meaning darkness. The η doublet was proposed two months later by itself [Barbieri/Hall/Rychkov(2006)] and became known as 'inert', although it has both gauge and scalar interactions.



Scotogenic Majorana neutrino mass.

The one-loop diagram for scotogenic Majorana neutrino mass is exactly calculable from the exchange of $Re(\eta^0)$ and $Im(\eta^0)$ and is given by

$$\sum_k \frac{h_{ik}h_{jk}M_k}{16\pi^2}\left[\frac{m_R^2}{m_R^2-M_k^2}\ln\frac{m_R^2}{M_k^2}-\frac{m_I^2}{m_I^2-M_k^2}\ln\frac{m_I^2}{M_k^2}\right].$$

In the limit

$m_R^2 - m_I^2 = 2\lambda_5 v^2 << m_0^2 = (m_R^2 + m_I^2)/2 << M_k^2$,

this reduces to the so-called radiative seesaw:

$$\frac{\lambda_5 v^2}{8\pi^2}\sum_k \frac{h_{ik}h_{jk}}{M_k}\left[\ln\frac{M_k^2}{m_0^2}-1\right].$$

Scotogenic Nonzero θ_{13} and Large δ_{CP} in A_4

Let $(\nu_i, l_i) \sim \underline{3}$, $l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$ as before. Add $(\eta^+, \eta^0) \sim \underline{1}$, and $N_i \sim \underline{3}$, then ν_i is connected to N_i by the identity matrix. The structure of the $N_i N_j$ Majorana mass matrix is then communicated to ν_i through U_{CW} to l_j. Assume

$$\mathcal{M}_N = \begin{pmatrix} A & F & E \\ F & A & D \\ E & D & A \end{pmatrix},$$

with $F = -E$, which may be maintained by gauging $B - L$ with scalars $\sigma_0 \sim \underline{1}$ and $\sigma_i \sim \underline{3}$ under A_4.

The breaking of A_4 is accompanied by soft terms respecting the interchange symmetry $\sigma_1 \longrightarrow \sigma_1$, $\sigma_2 \longrightarrow -\sigma_3$, $\sigma_3 \longrightarrow -\sigma_2$. In the tribimaximal basis,

$$\mathcal{M}_N^{(1,2,3)} = \begin{pmatrix} A+D & 0 & 0 \\ 0 & A & C \\ 0 & C & A-D \end{pmatrix},$$

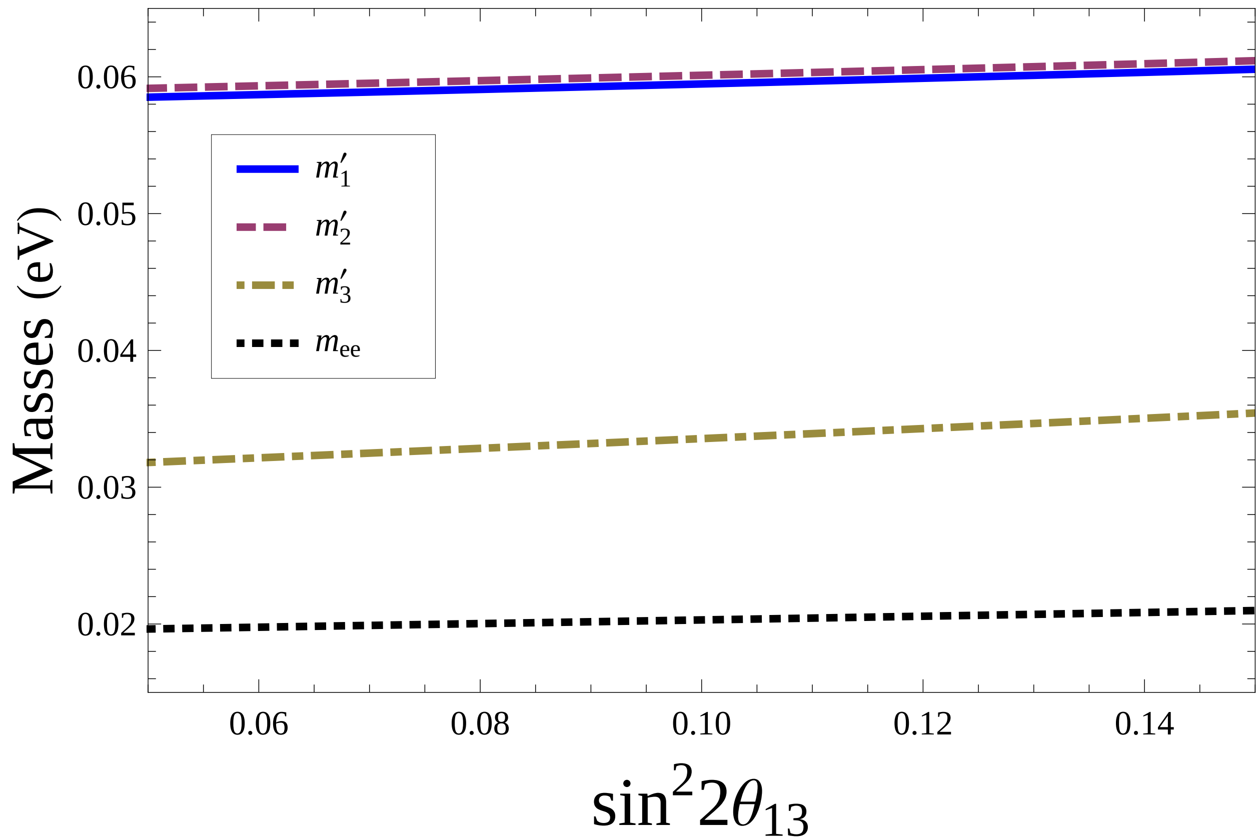
where $C = (E-F)/\sqrt{2} = \sqrt{2}E$. Rescale M_k so that

$$m'_k = \frac{1}{M_k}\left(\ln\frac{M_k^2}{m_0^2} - 1\right).$$

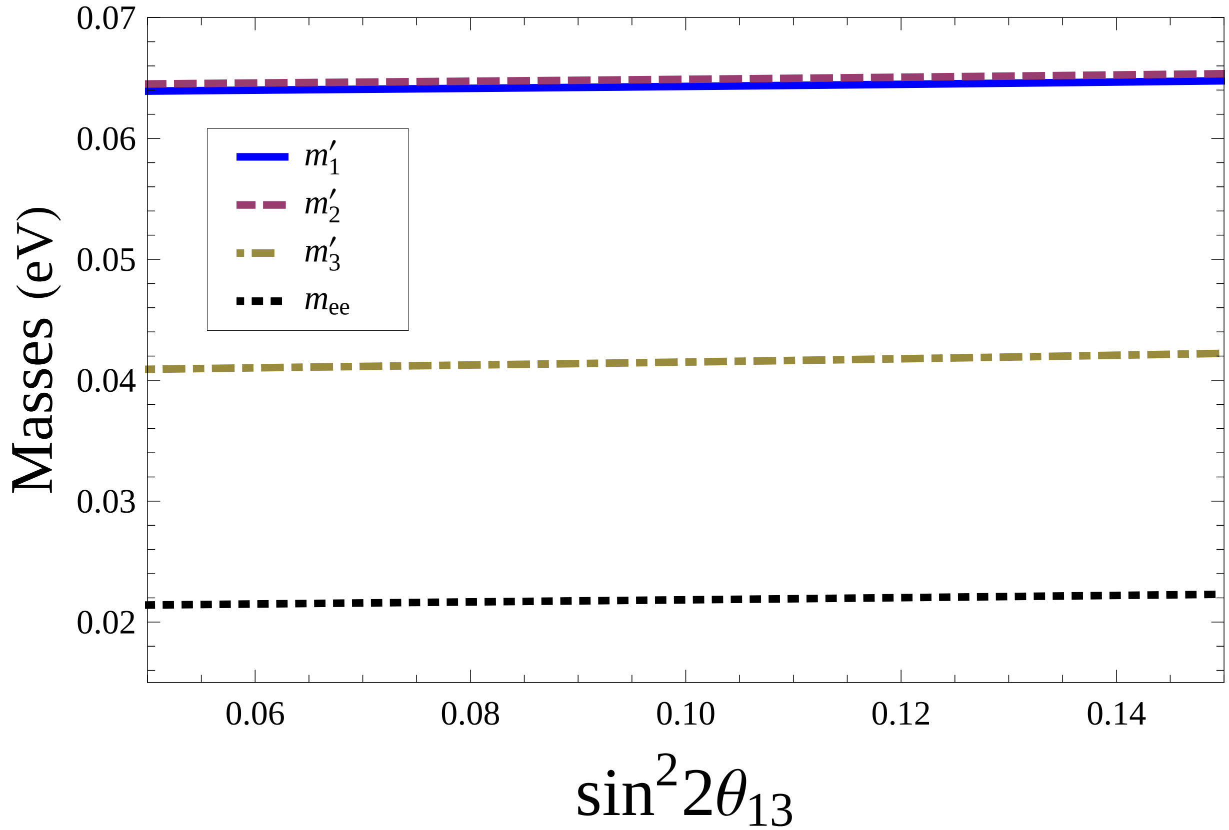
Inputs: $\Delta m_{21}^2 = 7.59 \times 10^{-5}\ \mathrm{eV}^2$,
$\Delta m_{32}^2 = 2.45 \times 10^{-3}\ \mathrm{eV}^2$.

Five representation solutions for $\sin^2 2\theta_{23} = 0.96$ and $\sin^2 2\theta_{13} = 0.10$. [Ma/Natale/Rashed(2012)]

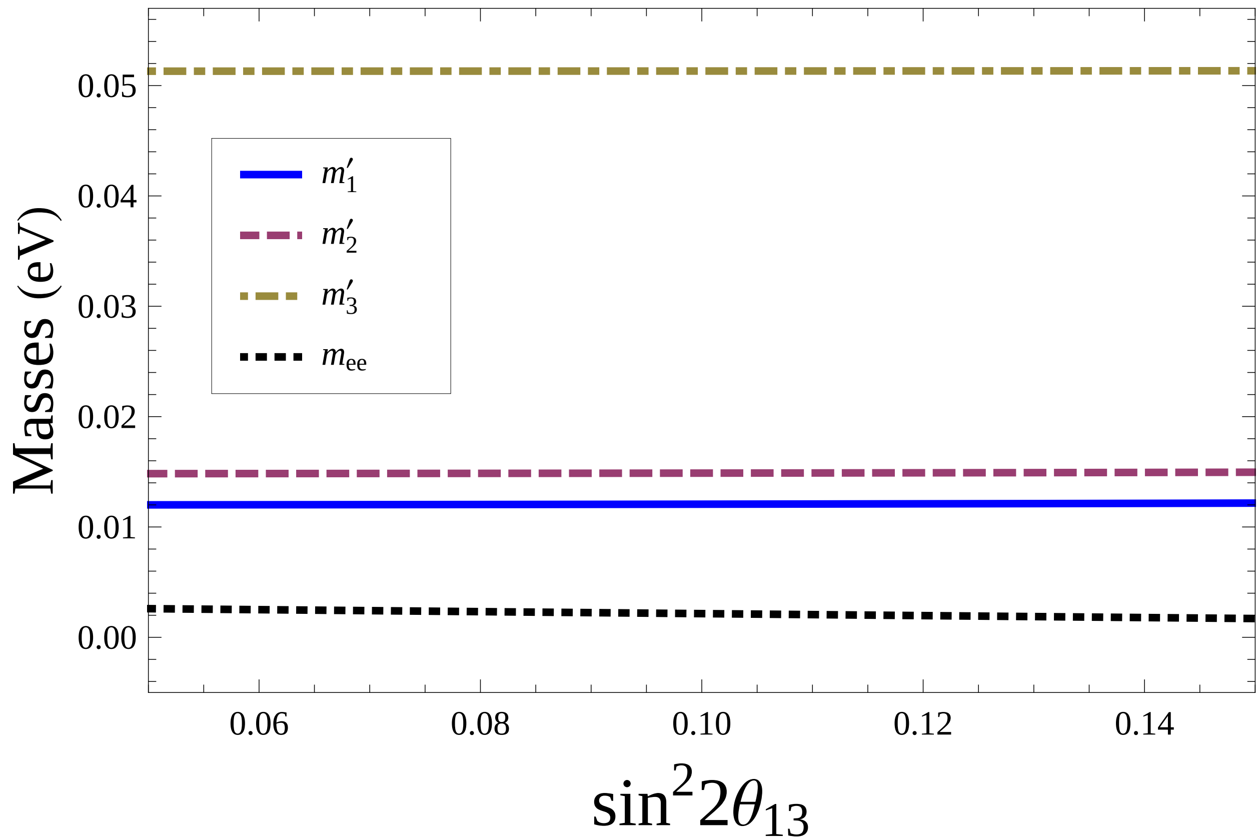
solution	Im(D)	class	$\lvert\tan\delta_{CP}\rvert$	m_{ee}
I	0	IH	2.05	0.020
II	Re(D)	IH	4.64	0.022
III	0	NH	3.59	0.002
IV	0	QD	2.20	0.046
V	Re(D)	QD	1.84	0.051



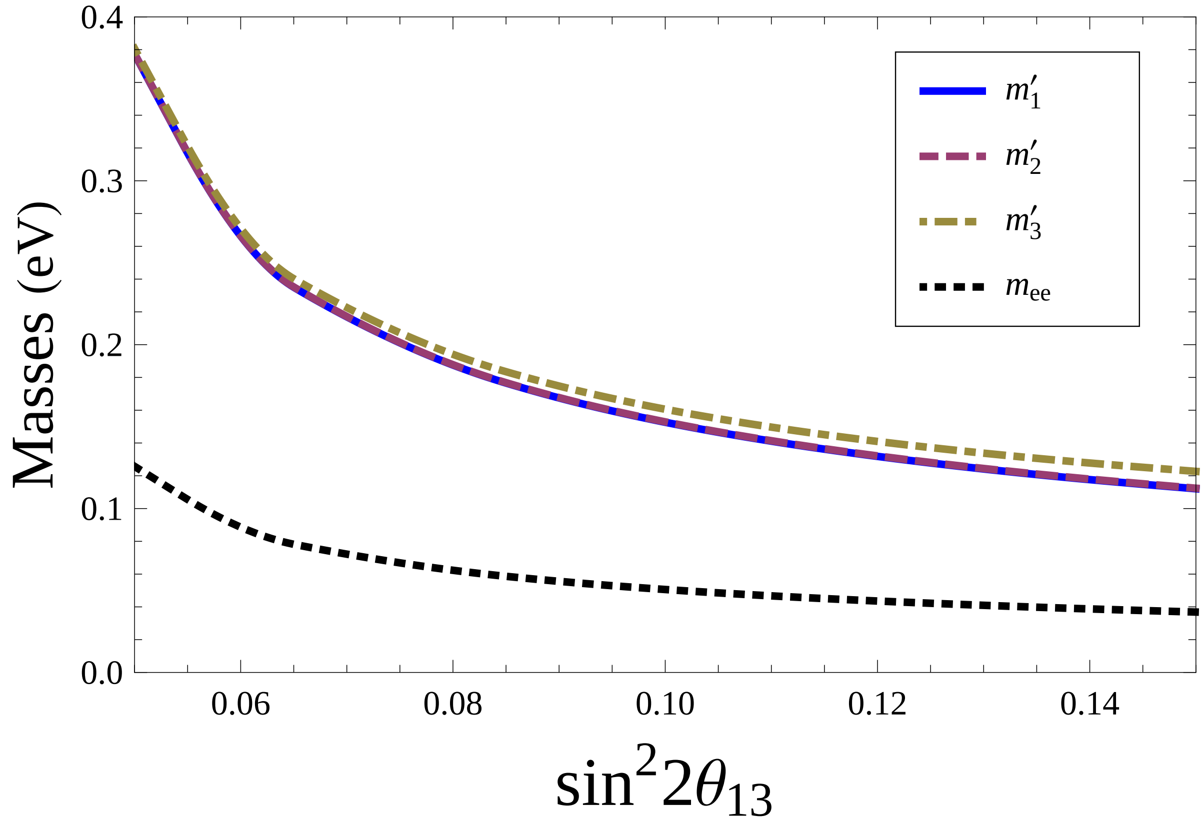
Neutrino masses and m_{ee} for inverted hierarchy
with Im(D)=0 and $\sin^2 2\theta_{23} = 0.96$.



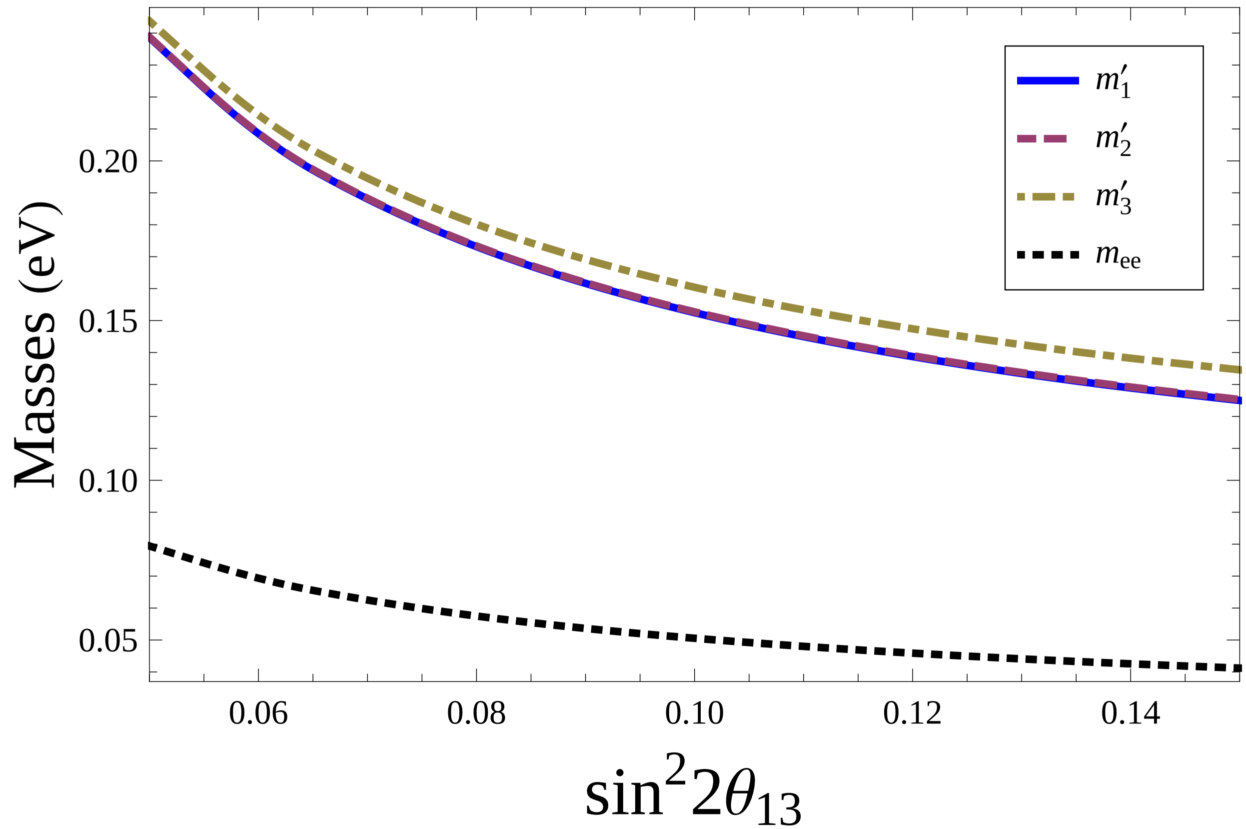
Neutrino masses and m_{ee} for inverted hierarchy
with Im(D)=Re(D) and $\sin^2 2\theta_{23} = 0.92$.



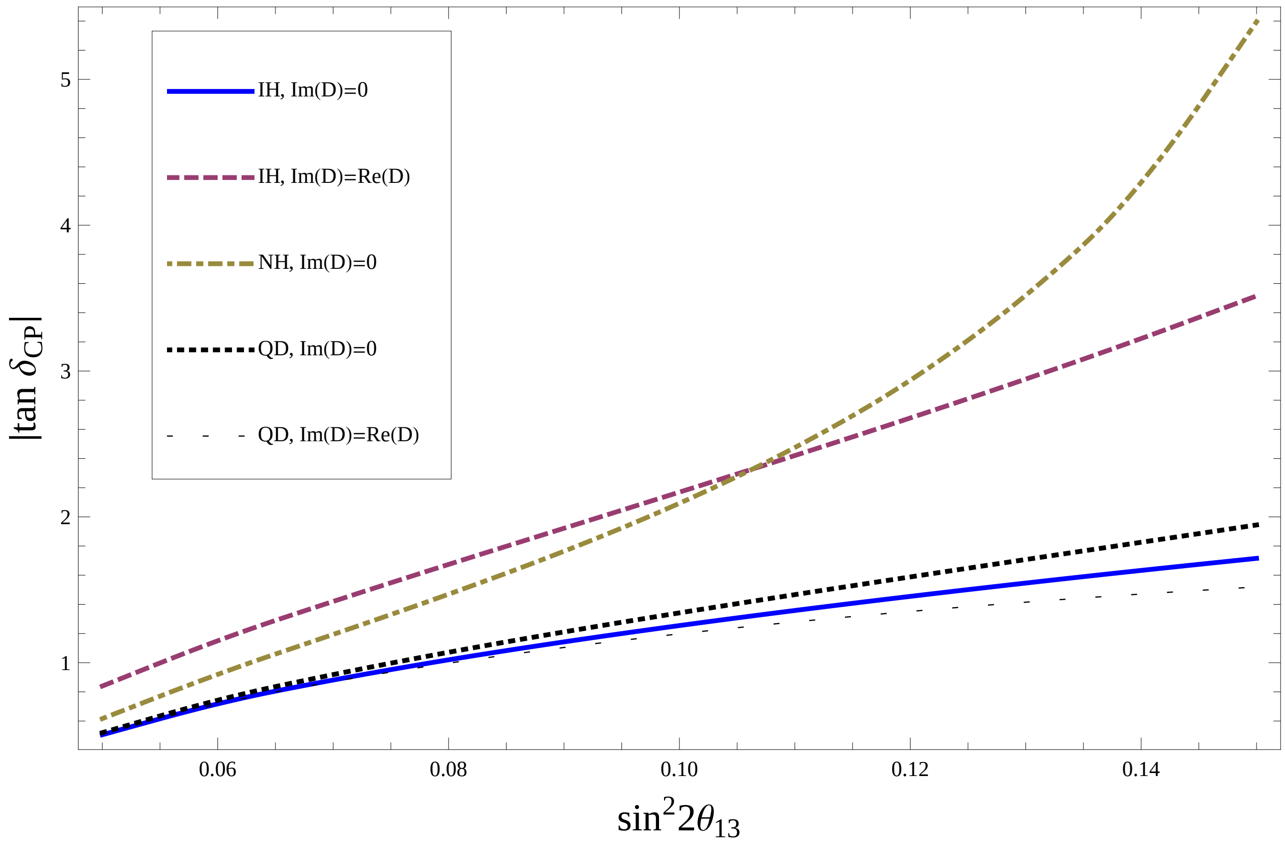
Neutrino masses and m_{ee} for normal hierarchy
with Im(D)=0 and $\sin^2 2\theta_{23} = 0.96$.



Neutrino masses and m_{ee} for quasi-degenerate normal ordering with Im(D)=0 and $\sin^2 2\theta_{23} = 0.96$.



Neutrino masses and m_{ee} for quasi-degenerate normal ordering with Im(D)=Re(D) and $\sin^2 2\theta_{23} = 0.96$.



$|\tan \delta_{CP}|$ versus $\sin^2 2\theta_{13}$ for $\sin^2 2\theta_{23} = 0.92$.

Conclusion

With the new precise measurements of $\sin^2 2\theta_{13}$, tribimaximal mixing is dead, but not A_4. In fact, the original A_4 model had two important parts: (A) diagonalizing the charged-lepton mass matrix with U_{CW} for arbitrary values of $m_{e,\mu,\tau}$, (B) allowing the neutrino mass matrix to be restricted. The special case of tribimaximal mixing requires a condition which is very difficult to enforce theoretically. Relaxing (B) and keeping (A) do very well with present data. Predictions for $|\tan \delta_{CP}|$ and m_{ee} are given in two models.