UNIVERSITY OF Southampton
School of Physics and Astronomy

Theoretical implications of large θ_{13}

Steve King

Florence 25th June, 2012

What is υ?, INVISIBLES 12 and Alexei Smirnov Fest

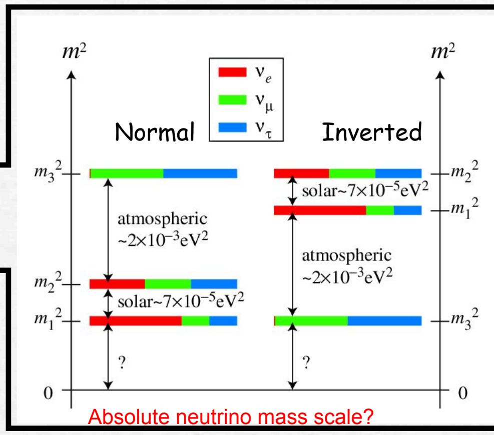
Three Neutrino Mixing

See talks by Brice and de Gouvea

Standard Model states

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L$$

Neutrino mass states



Pontecorvo
Maki
Nakagawa
Sakata

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

$$U_{PMNS} = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}e^{i\alpha_1/2} & 0 & 0\\ 0 & e^{i\alpha_2/2} & 0\\ 0 & 0 & 1\end{pmatrix}$$

$s_{ij} = \sin\theta_{ij}$

$c_{ij} = \cos\theta_{ij}$

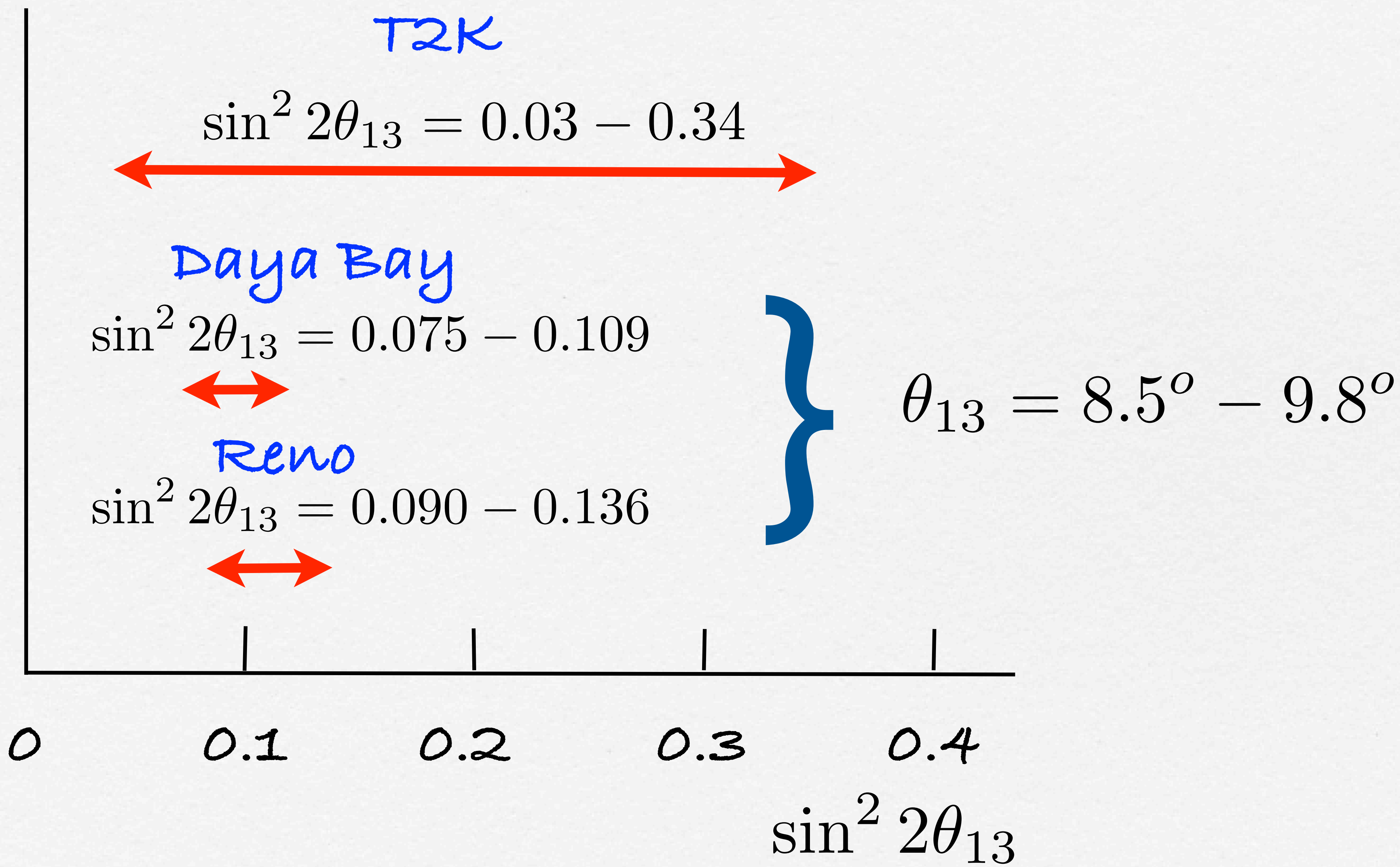
Atmospheric | Reactor | Solar | Majorana

Oscillation phase δ

Majorana phases α_1, α_2

3 masses + 3 angles + 1 (or 3) phase(s)
= 7 (or 9) new parameters for SM

Theta13 in 2011/12



Global Fits 2012

Schwetz talk

parameter	best fit $\pm 1\sigma$ (Forero, Tortola, Valle, Vanegas '12)	best fit $\pm 1\sigma$ (Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno '12)
Δm^2_{21} $[10^{-5}\mathrm{eV}^2]$	7.62 ± 0.19	$7.54^{+0.26}_{-0.22}$
Δm^2_{31} $[10^{-3}\mathrm{eV}^2]$	$2.53^{+0.08}_{-0.10}$ $-(2.40^{+0.10}_{-0.07})$	$2.43^{+0.07}_{-0.09}$ $-(2.42^{+0.07}_{-0.10})$
$\sin^2\theta_{12}$	$0.320^{+0.015}_{-0.017}$	$0.307^{+0.018}_{-0.016}$
$\sin^2\theta_{23}$	$0.49^{+0.08}_{-0.05}$ $0.53^{+0.05}_{-0.07}$	$0.398^{+0.030}_{-0.026}$ $0.408^{+0.035}_{-0.030}$
$\sin^2\theta_{13}$	$0.026^{+0.003}_{-0.004}$ $0.027^{+0.003}_{-0.004}$	$0.0245^{+0.0034}_{-0.0031}$ $0.0246^{+0.0034}_{-0.0031}$
δ	$\left(0.83^{+0.54}_{-0.64}\right)\pi$ 0.07π [a]	$(0.89^{+0.29}_{-0.44})\pi$ $(0.90^{+0.32}_{-0.43})\pi$

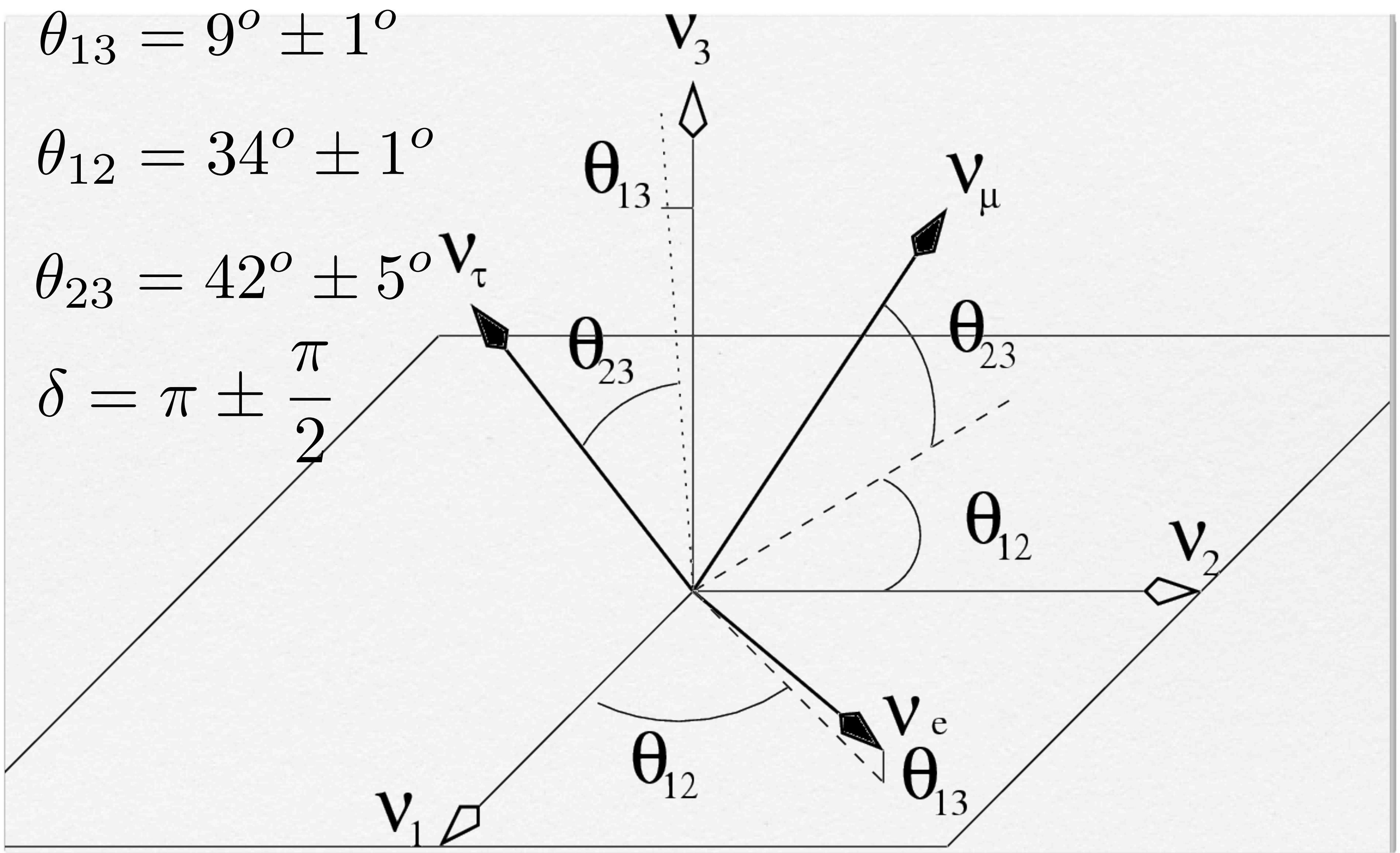
Neutrino Mixing

$$\theta_{13} = 9^o \pm 1^o$$

$$\theta_{12} = 34^o \pm 1^o$$

$$\theta_{23} = 42^o \pm 5^o$$

$$\delta = \pi \pm \frac{\pi}{2}$$



See-saw mechanism

P.Minkowski, PLB67(1977)421 ...

Possible type II contribution

Dirac matrix

$$\begin{pmatrix}\overline{\nu_L} & \overline{\nu_R^c}\end{pmatrix}\begin{pmatrix}0 & m_{LR}\\ m_{LR}^T & M_{RR}\end{pmatrix}\begin{pmatrix}\nu_L^c\\ \nu_R\end{pmatrix}$$

$$M^{\nu} = m_{LR}.\frac{1}{M_{RR}}.m_{LR}^T$$

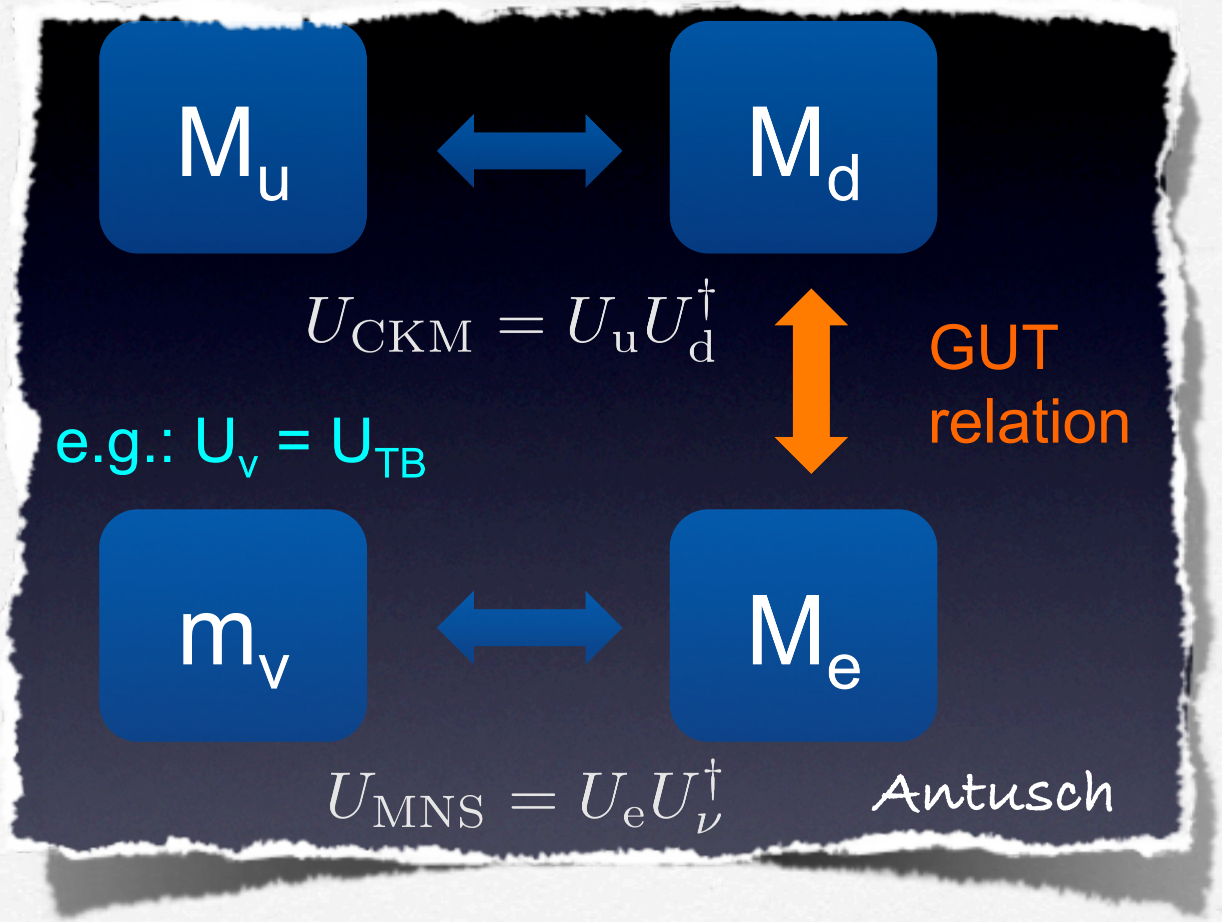
Light Majorana matrix

Heavy Majorana matrix

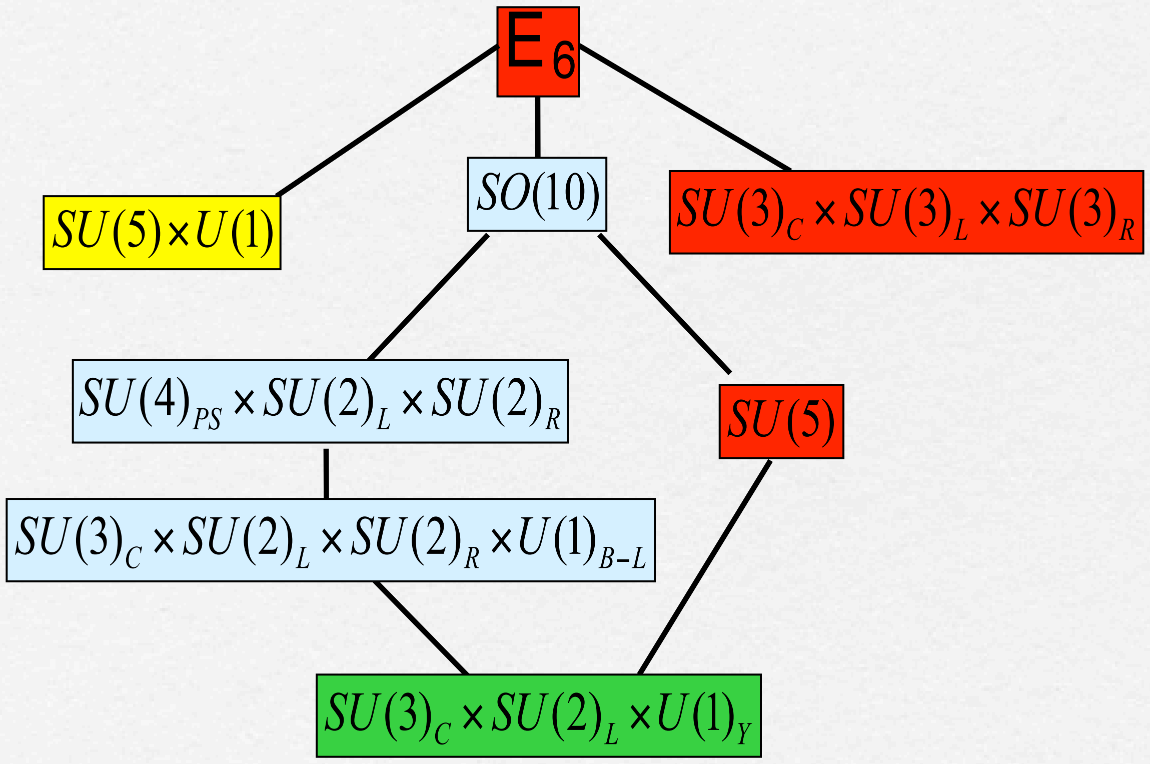
- Neutrinos are light because RH neutrinos are heavy
- Neutrino mass suggests connection with GUT scale physics

GUTs

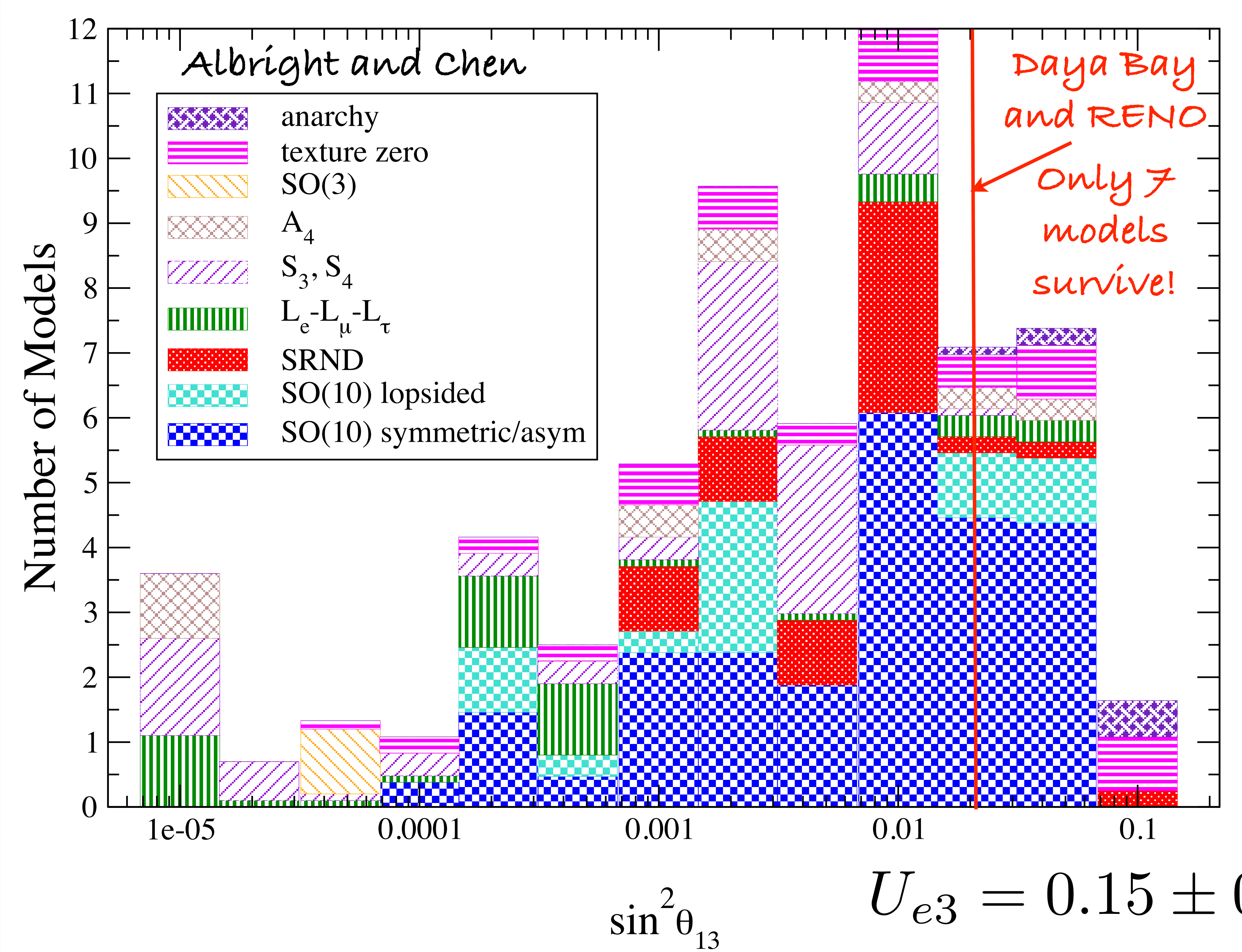
Talks by Antusch, Mohapatra



Possible new combinations of Clebsches leading to large Theta13: Antusch, Maurer ('11)
Mazocca, Petcov, Romanino, Spinrath ('11)
Meroni, Petcov, Spinrath ('12)



Models Survey c.2006



$$U_{e3} = 0.15 \pm 0.02$$

Theory Road Map

Daya Bay/RENO

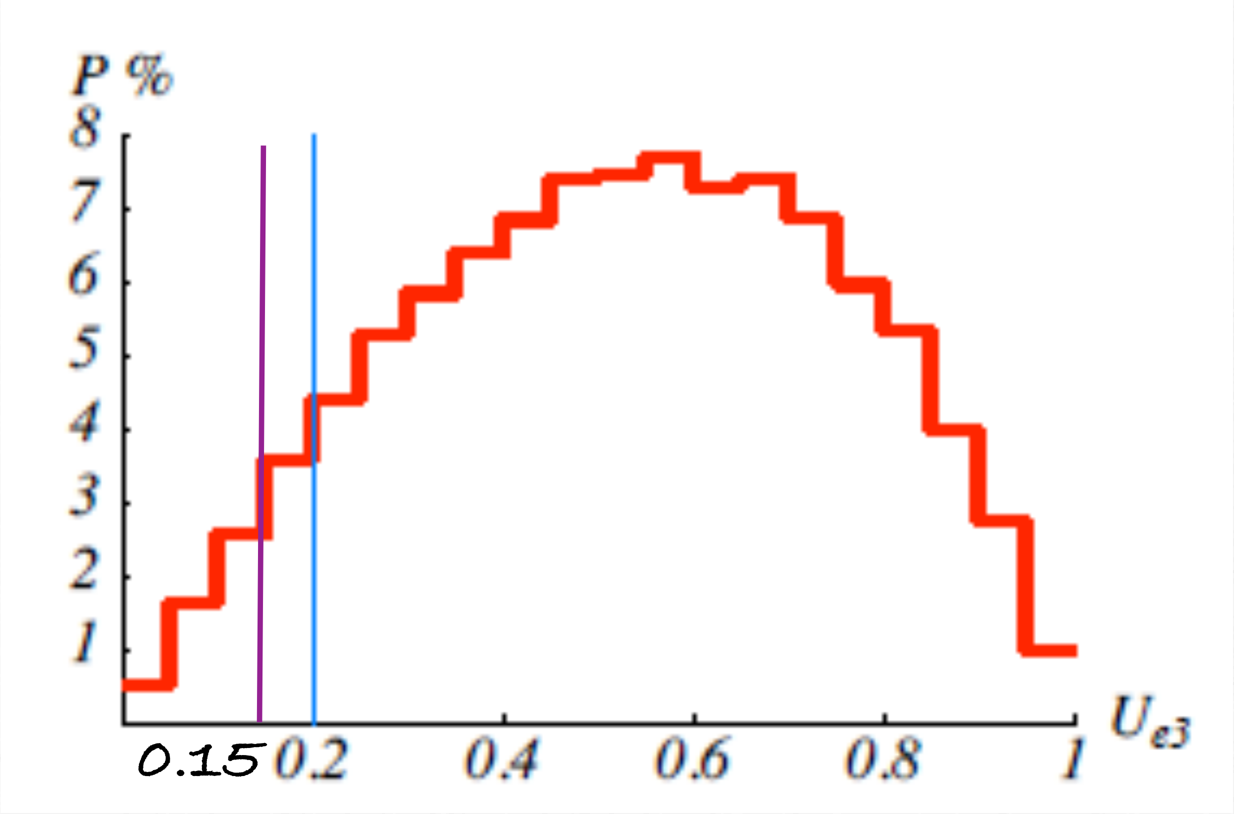
- Symmetry
  - Direct
  - Indirect
- Anarchy
  - Landscape

Anarchy

Hall, Murayama, de Gouvea

- Anarchy: all angles are "large" and unpredicted, so expect $\sin\theta_{13} \sim 0.5$
- Hence larger reactor angle is good news
- Problem is that reactor angle is not that large...
- Also Anarchy not very predictive c.f. landscape

Altarelli, Feruglio, Masina

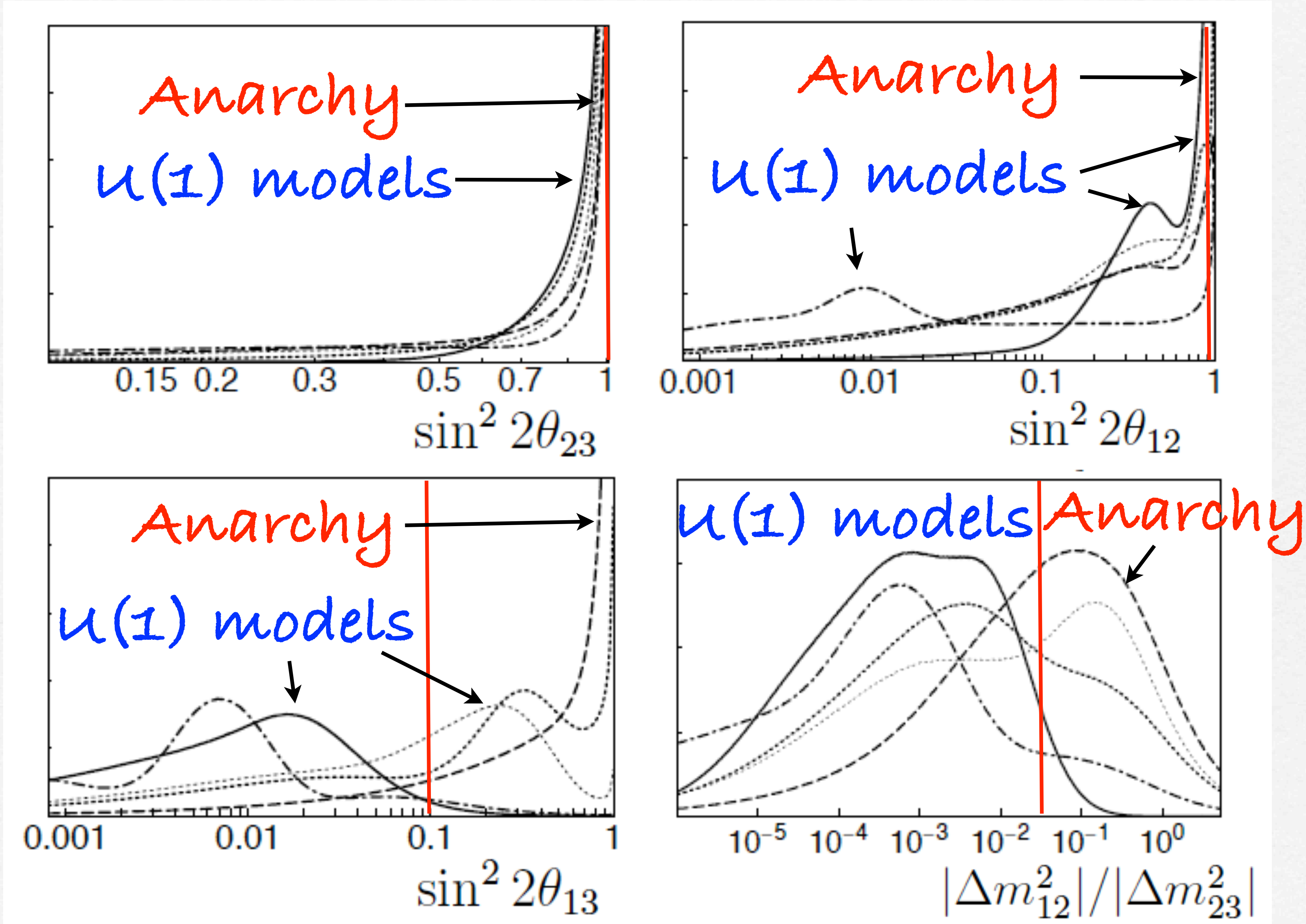


$$U_{e3} = 0.15 \pm 0.02$$

U(1) family symmetry helps...

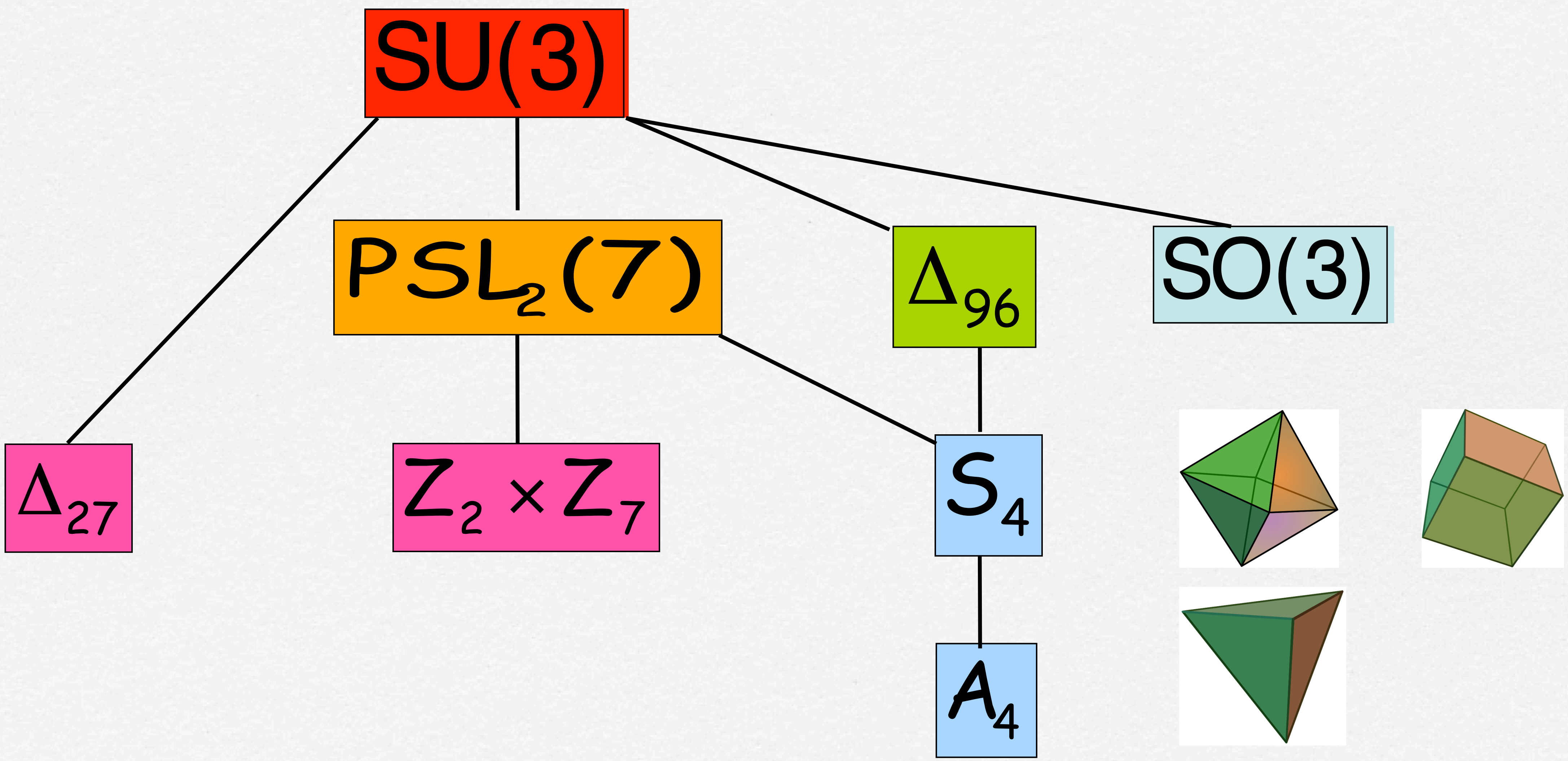
Hirsch and King ('01)

- lepton (Quark) generations labelled by U(1) family symmetry
- $\sin^2 2\theta_{13}$ may peak at lower values



Family Symmetry

Family Symmetries G_F
which contain triplet reps
(three families in a triplet)



Partial list of authors who have worked on symmetry approach to large θ_{13}

A. Adulpravitchai, Y. H. Ahn, C. H. Albright, G. Altarelli, S. Antusch, A. Aranda, T. Araki, F. Bazzocchi, W. Buchmuller, P. S. Bhupal Dev, G. C. Branco, Q.-H. Cao, H.-Y. Cheng, I. K. Cooper, S. Dev, G. Blankenburg, C. Bonilla, F. Gonzalez Canales, W. Chao, J.-M. Chen, M.-C. Chen, X. Chu, A. Datta, K. N. Deepthi, M. Dhen, D. A. Dicus, G.-J. Ding, P. V. Dong, V. Domcke, L. Dorame, B. Dutta, D. A. Eby, L. Everett, R. P. Feger, F. Feruglio, P. Ferreira, P. H. Frampton, M. Fukugita, R. R. Gautam, S. -F. Ge, D. K. Ghosh, R. Gonzalez Felipe, S. Gollu, S. Gupta, W. Grimus,C. Gross, N. Haba, C. Hagedorn, T. Hambye, J. Kersten, J. E. Kim, Y. Koide, K. Hashimoto, K. Harigaya, H. -J. He, X. -G. He, J. Heek, D. Hernandez, M. Holthausen, R. S. Hundi, M. Ibe, H. Ishimori, F. R. Joaquim, A. S. Joshipura, S. K. Kang, T. W. Kephart, S. Khalil, S. F. King, T. Kobayashi, S. Kumar, L. Lavoura, X.-Q. Li, H. N. Long, P. O. Ludl, C. Luhn, B. Q. Ma, E. Ma, S. K. Majee, K.T. Mahanthappa, D. Marzocca, V. Maurer, D. Meloni, A. Merle, A. Meroni, R. Mohanta, R. N. Mohapatra, E. Molinaro, A. Mondragon, M. Mondragon, S. Morisi, C. H. Nam, H. Nishiura, S. Oh, H. Okada, K. M. Patel, K. M. Parattu, E. Peinado, S. T. Petcov, N. Qin, A. Rashed, W. W. Repko, A. D. Rojas, W. Rodejohann, A. Romanino, G. G. Ross, S. Rigolin, M. A. Schmidt, K. Schmitz, M. Severson, M.-S. Seo, H. Serodio, Y. Shimizu, J. I. Silva-Marcos, L. Singh, K. Siyeon, C. Sluka, A. Yu. Smirnov, M. Spinrath, E. Stamou, A. J. Stuart, R. Takahashi, M. Tanimoto, R. d. A. Toorop, J. W. F. Valle, I. d. M. Varzielas, L. Velasco, V. V. Vien, B. Wang, Q. Wang, A. Watanabe, D. Wegman, A. Wingerter, Yue-Liang Wu, Z. -Z. Xing, T. T. Yanagida, W.-M. Yang, B. Zaldívar, F. -R. Yin, A. Zee, H. Zhang, Y. -j. Zheng, J.-J. Zhong, S. Zhou, R. Zwicky, ...

(sincere apologies for incompleteness)

Altarelli, Feruglio, Ma, Hagedorn, Merlo, Luhn, ...

The Direct Approach

Family Symmetry G_F broken in special way

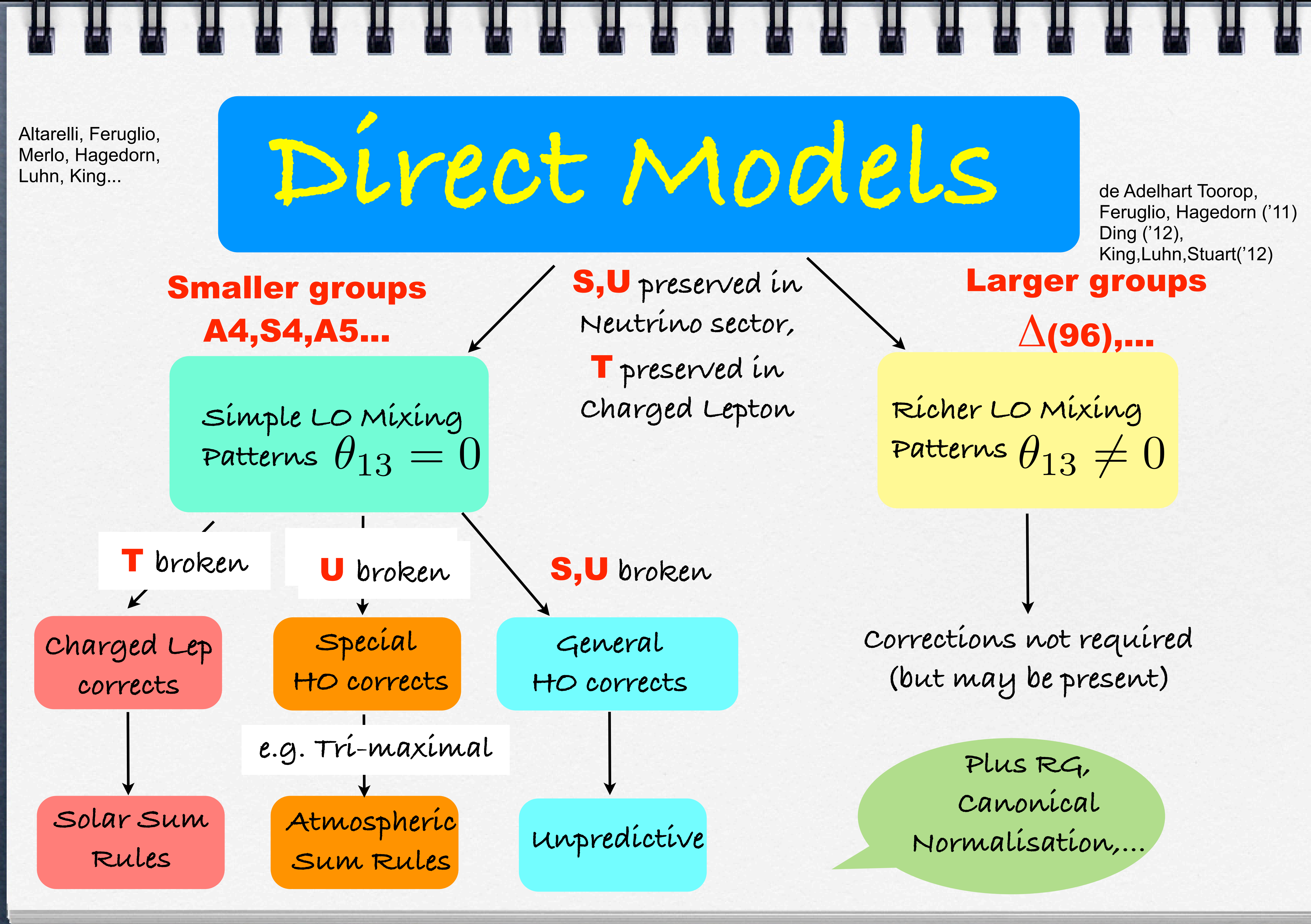
Subgroups preserved in neutrino/charged lepton sectors

$G_F \supset S,T,U$ ← Group generators

"Klein symmetry" **S,U** only preserved → **Neutrino Mass Matrix**

T only preserved → **Charged lepton Mass Matrix**

Neutrino Mass Matrix ↔ U_{PMNS} ↔ **Charged lepton Mass Matrix**



Altarelli, Feruglio, Merlo, Hagedorn, Luhn, King...
Direct Models
de Adelhart Toorop, Feruglio, Hagedorn ('11) Ding ('12), King,Luhn,Stuart('12)
Smaller groups
A4,S4,A5...
S,U preserved in Neutrino sector, T preserved in Charged Lepton
Larger groups
Δ(96),...
Simple LO Mixing Patterns $\theta_{13} = 0$
Richer LO Mixing Patterns $\theta_{13} \neq 0$
T broken
U broken
S,U broken
Charged Lep corrects
Special HO corrects
General HO corrects
Corrections not required (but may be present)
e.g. Tri-maximal
Solar Sum Rules
Atmospheric Sum Rules
Unpredictive
Plus RG, Canonical Normalisation,...

Simple LO Mixing Patterns

$$\theta_{13} = 0 \quad \theta_{23} = 45^o$$

- Bimaximal

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} P \quad \theta_{12} = 45^o$$

V. Barger, S. Pakvasa, T. Weiler and K. Whisnant

- Tri-bimaximal

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

$$\theta_{12} = 35.26^o$$

Harrison, Perkins and Scott

- Golden ratio

$$U_{GR} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

Datta, Ling, Ramond; Kajirama, Raidal, Strumia; Everett, Stuart,Ding: Feruglio, Paris

$$\phi = \frac{1+\sqrt{5}}{2} \qquad \tan\theta_{12} = \frac{1}{\phi} \qquad \theta_{12} = 31.7^o$$

Large Charged Lepton Corrections to the rescue

$\theta_{12}^e \approx \theta_C$

- BM, TBM, GR might only apply to neutrino mixing and $U_{PMNS} = U_e U_\nu^\dagger$ implies $\theta_{13} \approx \frac{\theta_{12}^e}{\sqrt{2}}$
- Solar Sum Rules

Sum Rule: King ('05); Masina ('05); Antusch, King ('05)

Charged Lepton Corrections: King ('02), Frampton, Petcov, Rodejohann ('04), Altarelli, Feruglio, Masina ('04), Antusch, King ('04), Ferrandis, Pakvasa ('04), Feruglio ('05), Datta, Everett, Ramond ('05), Mohapatra, Rodejohann ('05) Antusch, Maurer ('11) Mazocca, Petcov, Romanino, Spinrath ('11)

- Bimaximal $\theta_{12} = 45^o + \theta_{13}\cos\delta \rightarrow \delta \approx \pi$
- Tri-bimaximal $\theta_{12} = 35^o + \theta_{13}\cos\delta \rightarrow \delta \approx \pm\frac{\pi}{2}$
- Golden ratio $\theta_{12} = 32^o + \theta_{13}\cos\delta$

Experiment $\theta_{12} = 34^o \pm 1^o$ $\theta_{13} = 9^o \pm 1^o$

King; Parke; Pakvasa, Rodejohann, Weiler

Tri-Bimaximal Parametrisation

$$U_{\rm PMNS} \approx \begin{pmatrix} \frac{2}{\sqrt{6}}(1-\frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix} P$$

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s)\ , \qquad \sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a)\ , \qquad \sin\theta_{13} = \frac{r}{\sqrt{2}}$$

$$s = -0.03 \pm 0.03 \qquad a = -0.02 \pm 0.10 \qquad r = 0.22 \pm 0.02$$

s = solar   a = atmospheric   r = reactor

Allows for deviations from TB mixing

E.g. TB solar sum rule recast as $s = r.\cos\delta$

Tri-bimaximal Hydras

- Tri-bimaximal (s=a=r=0)

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

Harrison, Perkins, Scott

- Tri-bimaximal-reactor (s=a=0)

$$U_{TBR} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}}(1-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

King; Antusch,Boudjemaa,King; Morisi,Patel,Peinado; Luhn, King

- Tri-maximal 1 (s=0, a=r.cosδ)

$$U_{\mathrm{TM}_1} = P' \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} re^{-i\delta} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}(1-\frac{3}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+re^{-i\delta}) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}(1+\frac{3}{2}re^{i\delta}) & -\frac{1}{\sqrt{2}}(1-re^{-i\delta}) \end{pmatrix} P$$

Lam; Albright,Rodejohann; Antusch, King, Luhn, Spinrath

Haba,Watanabe,Yoshioka; He,Zee; Grimus,Lavoura; Albright,Rodejohann; King, Luhn

- Tri-maximal 2 (s=0, a=-r/2.cosδ)

$$U_{\mathrm{TM}_2} = P' \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+\frac{3}{2}re^{i\delta}) & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}(1-\frac{1}{2}re^{-i\delta}) \\ -\frac{1}{\sqrt{6}}(1-\frac{3}{2}re^{i\delta}) & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}(1+\frac{1}{2}re^{-i\delta}) \end{pmatrix} P$$

N.B. Atmospheric sum rules: a=r.cosδ, a=-r/2.cosδ

Indirect Models

King, Ross, de Medeiros Varzielas, Antusch, Malinsky,...

Starting point is type I see-saw

$$m_{LR} = \begin{pmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{pmatrix} \quad M_{RR} = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix} \rightarrow m^{\nu} = \frac{AA^T}{M_1} + \frac{BB^T}{M_2} + \frac{CC^T}{M_3}$$

$$A^T = (A_1, A_2, A_3) \quad B^T = (B_1, B_2, B_3)$$

Promote the columns (A,B,C) to dynamical fields

G_F yields special vacuum alignments, for example:

- (A,B,C) proportional to columns of PMNS called Form Dominance (FD)
  Chen, King('09)
- $AA^T/M_1 \rightarrow 0$ gives hierarchy ($m_1 \rightarrow 0$) called Sequential Dominance (SD)
  King('98,'02)
- SD with B~(1,1,-1) and C~(0,1,1) called Constrained SD gives TB Mixing
  King('05)
- SD with B~(1,1,-1) and C~(r,1,1) called Partially CSD gives TBR mixing
  King('09), King,Luhn('11)
- SD with B~(1,2,0) and C~(0,1,1) called CSD2 gives TM1 mixing
  Antusch, King, Luhn, Spinrath ('11)

King 1205.0506

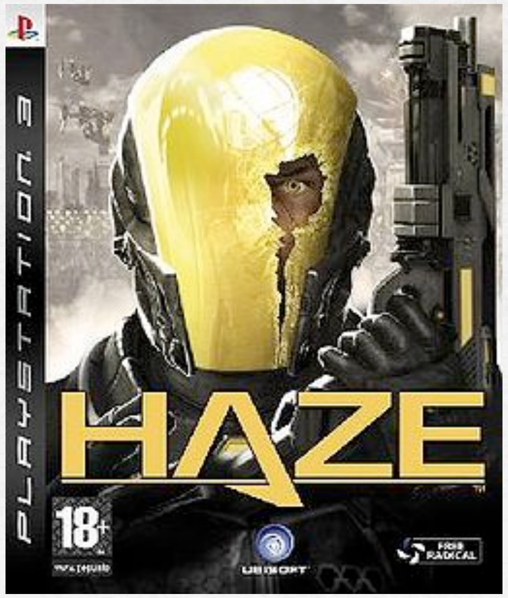
Tri-bimaximal-Cabibbo Mixing

See also: Antusch,Gross, Maurer, Sluka 1205.1051; Relation also appears in "Quark Lepton Complementarity" ... : Minakata, Smirnov ('04); and "Cabibbo Haze" Everett, Ramond ('05)

Combine TB mixing with $\theta_{13} \approx \frac{\theta_C}{\sqrt{2}} \approx 9.2^o$

$$s_{13} = \frac{\lambda}{\sqrt{2}}, \quad s_{12} = \frac{1}{\sqrt{3}}, \quad s_{23} = \frac{1}{\sqrt{2}} \qquad \lambda = 0.2253 \pm 0.0007$$

$$U_{TBC} \approx \begin{pmatrix} \sqrt{\frac{2}{3}}(1-\frac{1}{4}\lambda^2) & \frac{1}{\sqrt{3}}(1-\frac{1}{4}\lambda^2) & \frac{1}{\sqrt{2}}\lambda e^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+\lambda e^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}\lambda e^{i\delta}) & \frac{1}{\sqrt{2}}(1-\frac{1}{4}\lambda^2) \\ \frac{1}{\sqrt{6}}(1-\lambda e^{i\delta}) & -\frac{1}{\sqrt{3}}(1+\frac{1}{2}\lambda e^{i\delta}) & \frac{1}{\sqrt{2}}(1-\frac{1}{4}\lambda^2) \end{pmatrix}$$



Describes all current data!

Obtained from PCSD with B~(1,1,-1) and C~(λ ,1,1)

Conclusions

- With large θ_{13}, still two theory approaches: Symmetry or Anarchy
- Family Symmetry may be implemented Directly or Indirectly
- Simplest Direct models A4,S4,A5 with S,U and T conservation predict Bimaximal, Tri-bimaximal, Golden Ratio at LO
- However T broken in GUT models due to Charged Lepton corrections, (Cabibbo-like charged lepton angle required) imply solar sum rules
- U breaking at HO leads to TM2 mixing, atmospheric sum rules
- Larger Finite Groups such as Delta(96) predict e.g. $\theta_{13} \sim 12^{\circ}$ at LO
- Indirect family symmetry models can lead to TM1 or TBC mixing
- Vital to measure the mixing angles and CP phase delta to good precision to test the sum rules, hence discriminate between models, decide if the Universe is based on Symmetry or if Anarchy Rules

Summary of Sum Rule Predictions

- Quark-Lepton Complementarity $\theta_{12} + \theta_C = 45^o$
- Solar sum rules
  - Bimaximal $\theta_{12} = 45^o + \theta_{13}\cos\delta$
  - Tri-bimaximal $\theta_{12} = 35^o + \theta_{13}\cos\delta$
  - Golden Ratio $\theta_{12} = 32^o + \theta_{13}\cos\delta$

Plus HO corrections…

- Atm. sum rules
  - Tri-bimaximal-Cabibbo $\theta_{12} = 35^o \quad \theta_{23} = 45^o$ $\theta_{13} = \theta_C/\sqrt{2} = 9.2^o$
  - Trimaximal1 $\theta_{23} = 45^o + \sqrt{2}\theta_{13}\cos\delta$
  - Trimaximal2 $\theta_{23} = 45^o - \frac{\theta_{13}}{\sqrt{2}}\cos\delta$

Plus Charged Lepton Corrections…

Now that θ_{13} is measured these predict $\cos\delta$