Hadronic light-by-light contribution to the muon anomalous magnetic moment

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Collaborators

Work on g-2 done in collaboration with

HVP	HLbL
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Outline

Introduction

The hadronic light-by-light (HLbL) contribution (O(α^3))

Summary/Outlook

The magnetic moment of the muon

Interaction of particle with static magnetic field

$$V(\vec{x}) \;=\; -\vec{\mu}\cdot\vec{B}_{\rm ext}$$

The magnetic moment $\vec{\mu}$ is proportional to its spin $(c = \hbar = 1)$

$$\vec{\mu} \;=\; g\left(\frac{e}{2m}\right)\vec{S}$$

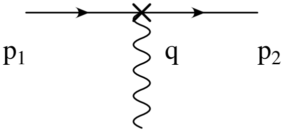
The Landé g-factor is predicted from the free Dirac eq. to be

$$g \;=\; 2$$

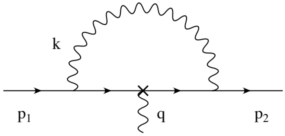
for elementary fermions

The magnetic moment of the muon

In interacting quantum (field) theory g gets corrections



$+$



$+ \dots$

$$\gamma^\mu \;\rightarrow\; \Gamma^\mu(q) = \left(\gamma^\mu F_1(q^2) + \frac{i\,\sigma^{\mu\nu}\,q_\nu}{2m} F_2(q^2)\right)$$

which results from Lorentz and gauge invariance when the muon is on-mass-shell.

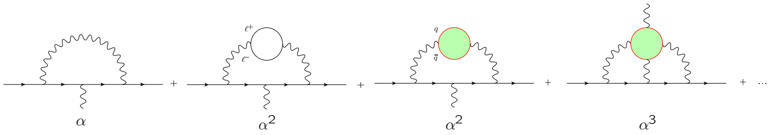
$$F_2(0) = \frac{g-2}{2} \equiv a_\mu \qquad (F_1(0) = 1)$$

(the anomalous magnetic moment, or anomaly)

The magnetic moment of the muon

Compute these corrections order-by-order in perturbation theory by expanding $\Gamma^\mu(q^2)$ in QED coupling constant

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137} + \ldots$$

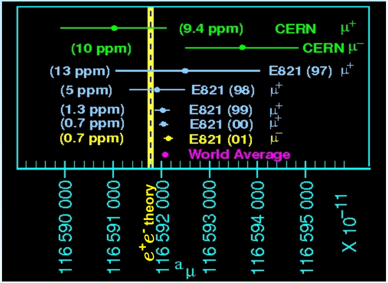


Corrections begin at $\mathcal{O}(\alpha)$; Schwinger term $= \frac{\alpha}{2\pi} = 0.0011614\ldots$

hadronic contributions $\sim 6 \times 10^{-5}$ times smaller (leading error).

Experimental value (L. Roberts, INT WS) Dominated by BNL E821

E821 achieved ± 0.54 ppm. The e^+e^- based theory is at the ~0.4 ppm level. Difference is ~3.6 σ



$$a_\mu^{exp} = 116\,592\,089(63) \times 10^{-11} \;\; (0.54\,\text{ppm})$$
$$\Delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM} = (287 \pm 80) \times 10^{-11}$$

Theory: arXiv:1010.4180v1 [hep-ph] Davier, Hoecker, Malaescu, and Zhang, Tau2010

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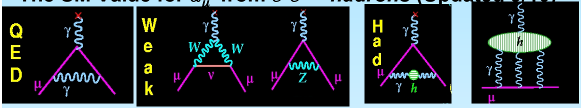
Theory value (standard model) (L. Roberts, INT WS)

The SM Value for a_μ from $e^+e^- \to$ *hadrons* (Updated 9/10)



well known — significant work ongoing

CONTRIBUTION	RESULT ($\times 10^{-11}$) UNITS
QED (leptons)	$116\ 584\ 718.09 \pm 0.14 \pm 0.04_\alpha$
HVP(lo)	$6\ 914 \pm 42_{\rm exp} \pm 14_{\rm rad} \pm 7_{\rm pQCD}$
HVP(ho)	$-98 \pm 1_{\rm exp} \pm 0.3_{\rm rad}$
HLxL	105 ± 26
EW	$152 \pm 2 \pm 1$
Total SM	$116\ 591\ 793 \pm 51$

A. Höcker Tau 2010, U. Manchester September 2010

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New experiments + new theory = new physics (?)

- Fermilab E989, $\sim$ 5 years away, 0.14 ppm
- J-PARC E34 ? (recently, lower priority than $\mu \to e$)
- $a_\mu(\mathrm{Expt})$-$a_\mu(\mathrm{SM}) = 287(63)(51)$ $(\times 10^{-11})$, or $\sim$ 3.6σ
- If both central values stay the same,
  - E989 ($\sim 4\times$ smaller error) $\to\sim 5\sigma$
  - E989+new HLBL theory (models+lattice, 10%) $\to\sim 6\sigma$
  - E989+new HLBL +new HVP (50% reduction) $\to\sim 8\sigma$
- Big discrepancy! (New Physics $\sim 2\times$ Electroweak)
- Lattice calculations crucial
- a_μ good for constraining and explaining BSM physics
- New 10th order $a_\mu(\mathrm{QED}) = 116\,584\,718\,951\,(80) \times 10^{-14}$

arXiv:1205.5370

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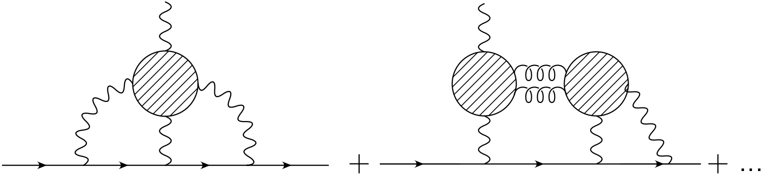
Summary/Outlook

HLbL



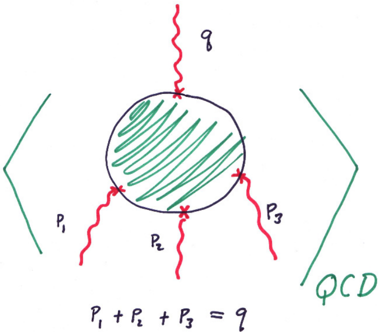
Blobs: all possible hadronic states

Model estimates put this $\mathcal{O}(\alpha^3)$ contribution at about $(10-12)\times 10^{-10}$ with a 25-40% uncertainty

No dispersion relation *a'la* vacuum polarization

Lattice regulator: model independent, approximations systematically improvable

Lattice QCD: conventional approach



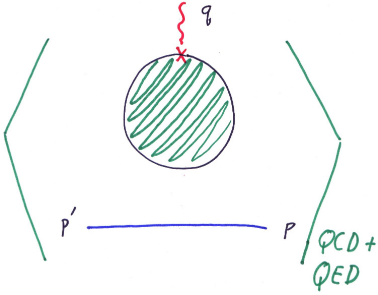
Correlation of 4 EM currents $\Pi^{\mu\nu\rho\sigma}(q, p_1, p_2)$

Two independent momenta $+$external mom q

Compute for all possible values of p_1 and p_2, ($O(V^2)$) four index tensor (32 Lorentz structures for g-2!)

several q,(extrap $q \to 0$), fit, plug into perturbative QED two-loop integrals

New approach (QCD+QED on the lattice)



Average over combined gluon *and* photon gauge configurations

Quarks coupled to gluons and photons

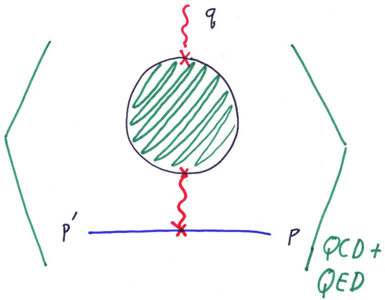
muon coupled to photons

[Hayakawa *et al.* (2005), hep-lat/0509016;

Chowdhury *et al.* (2008);

Chowdhury Ph. D. thesis (2009)]

New approach (QCD+QED on the lattice)



Attach one photon by hand (see why in a minute)

Correlation of hadronic loop and muon line

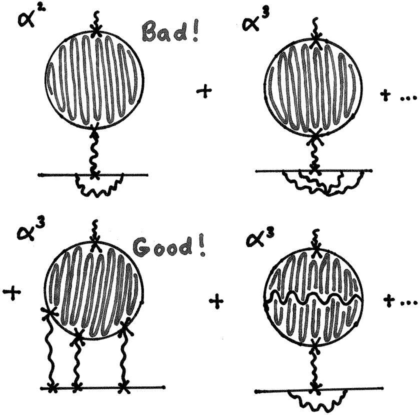
[Hayakawa *et al.* (2005), hep-lat/0509016;

Chowdhury *et al.* (2008);

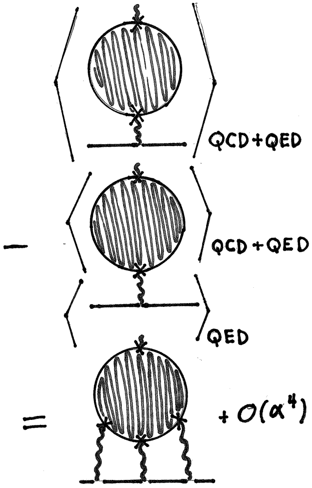
Chowdhury Ph. D. thesis (2009)]

New approach: Formally expand in α

The leading and next-to-leading contributions in α to magnetic part of correlation function come from



New approach: Subtraction of lowest order piece

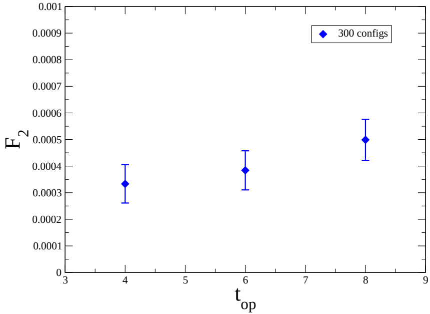


Subtraction term is product of separate averages of the loop and line

Gauge configurations identical in both, so two are highly correlated

In PT, correlation function and subtraction have same contributions except the light-by-light term which is absent in the subtraction

F_2 ($m_\mu/m_e = 40$, QED only) (Chowdhury Ph. D. thesis, UConn, 2009)



$F_2 = (3.96 \pm 0.70) \times 10^{-4}$

- quenched QED (Feynman gauge)
- $e = 1$
- $16^3 \times 32$ lattice size
- lowest non-zero momentum only
- **stat error only** (300 meas)

- Expected size of enhancement (compared to $m_\mu/m_e = 1$)
- Continuum PT result: $\approx 10(\alpha/\pi)^3 = 1.63 \times 10^{-4}$ ($e = 1$)
- roughly consistent with PT result, large finite volume effect

F_2, $m_\mu/m_e = 40$, finite volume study (QED only)

- Repeat calculation with 24^3 lattice volume
- Bigger box $F_2 = (1.19 \pm 0.32) \times 10^{-4}$
- Small box $F_2 = (3.96 \pm 0.70) \times 10^{-4}$
- finite volume effects manageable
- Continuum PT result: $\approx 10(\alpha/\pi)^3 = 1.63 \times 10^{-4}$ $(e = 1)$
- Roughly consistent with PT result

$a_\mu(\mathrm{HLbL})$ in 2+1f QCD+QED (PRELIMINARY)

- Same as before, but with $U = U(1) \times SU(3)$ [Duncan, *et al.*, 1996]
- QCD in the loop only (same in subtraction)
- QED in both loop and line
- 2+1 flavors (u, d, s) of DWF (RBC/UKQCD)
- $a = 0.114$ fm, $16^3 \times 32\,(\times 16)$, $a^{-1} = 1.73$ GeV
- $m_q \approx 0.013$, $m_\pi \approx 420$ MeV
- $m_\mu \approx 692$ MeV ($m_\mu^{\mathrm{phys}} = 105.658367(4)$ MeV)
- 100 configurations (one QED conf. for each QCD conf.)
- $(N_s/4)^3 = 64$ (loop) propagator calculations/configuration

$a_\mu(\mathrm{HLbL})$ in 2+1f lattice QCD+QED (PRELIMINARY)

- $a_\mu(\mathrm{HLbL}) = (-15.7 \pm 2.3) \times 10^{-5}$ (lowest non-zero mom, $e = 1$)
- HLBL amplitude depends strongly on m_μ (m_μ^2 in models)
- Magnitude 5-10 times bigger, sign opposite from models
- models not expected to be accurate in this regime
- Check subtraction is working by varying $e = 0.84, 1.19$
  - HLbL amplitude ($\sim e^4$) changes by ~ 0.5 and 2 ✓
  - while unsubtracted amplitude stays the same ✓

$a_\mu(\mathrm{HLbL})$ in 2+1f lattice QCD+QED (PRELIMINARY)

- Easy to lower muon mass (muon line is cheap)
- Try $m_\mu \approx 190$ MeV
- $a_\mu(\mathrm{HLbL}) = (-2.2 \pm 0.8) \times 10^{-5}$

  (lowest non-zero mom, $e = 1$)

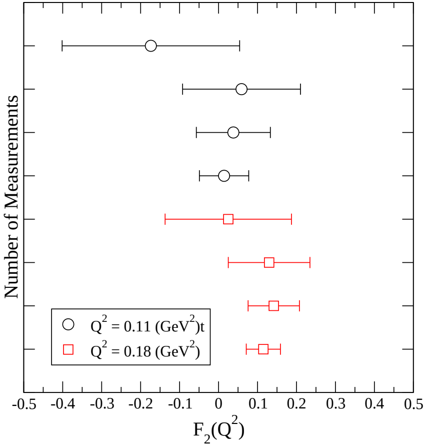
  Right direction...

$a_\mu(\mathrm{HLbL})$ in 2+1 flavor lattice QCD+QED

- Try larger lattice, 24^3 ($(2.7\text{ fm})^3$)
- Pion mass is smaller too, $m_\pi = 329$ MeV
- Same muon mass, $m_\mu \approx 190$ MeV
- two lowest values of Q^2 (0.11 and 0.18 GeV^2)
- Use All Mode Averaging (AMA)
  - 6^3 point sources/configuration (216)
  - AMA approximation:
    1. "sloppy CG" $r_{\mathrm{stop}} = 10^{-4}$
    2. 400 low modes

$a_\mu(\mathrm{HLbL})$ in 2+1f lattice QCD+QED (PRELIMINARY)

$F_2(Q^2)$ stable with additional configurations ($20 \to 40 \to 80 \to 160$)



24^3

$Q^2 = 0.11$ and 0.18 GeV^2

$m_\pi \approx 329$ MeV

$m_\mu \approx 190$ MeV

$a_\mu(\mathrm{HLbL})$ in 2+1f lattice QCD+QED (PRELIMINARY)

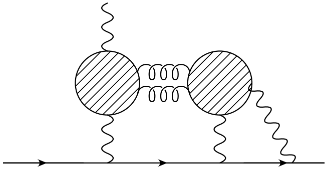
Signal may be emerging in the model ballpark:

- $F_2(0.18\ \mathrm{GeV}^2) = (0.115 \pm 0.044) \times \left(\frac{\alpha}{\pi}\right)^3$
- $F_2(0.11\ \mathrm{GeV}^2) = (0.014 \pm 0.063) \times \left(\frac{\alpha}{\pi}\right)^3$
- $a_\mu(\mathrm{HLbL/model}) = (0.084 \pm 0.020) \times \left(\frac{\alpha}{\pi}\right)^3$

Lattice size 24^3, $m_\pi = 329$ MeV, $m_\mu \approx 190$ MeV

model value/error is "Glasgow Consensus" (arXiv:0901.0306 [hep-ph])

HLbL systematic error



"Disconnected" diagrams (quark loops connected by gluons) not calculated yet (not suppressed).

Conceptually straightforward:

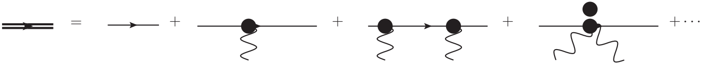
- quenched QED $\rightarrow$ dynamical QED
- Re-weight in α (T. Ishikawa, *et al.* 2012)
- or dynamical QED in HMC

A-slash sequential source (Izubuchi)

Aslash SeqSrc

- Quark propagator with QED charge, S(e)

$$S(e) = S + ieS\not{A}S - e^2 S(0)\not{A}S\not{A}S - e^2 S\not{A}\not{A}S + \cdots$$



- Each term could be computed by the sequential source method :

$$DX_0 = b, \quad DX_1 = \sum_x \not{A}(x) X_0(x) \rightarrow X_1 = S\not{A}Sb$$

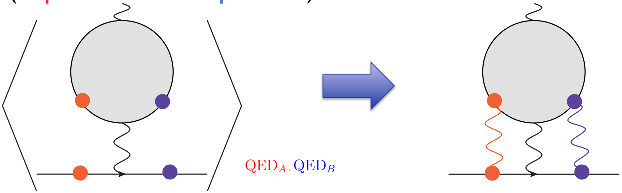
- $\not{A}$ is the conserved vector current with photon field contracted :

$$\not{A}(x) = \sum_\mu \mathcal{V}_\mu(x) A_\mu(x)$$

A-slash sequential source (Izubuchi)

Aslash SeqSrc for LbL

- Insert two Aslash for each of quark and lepton
- Use statistically independent photon field (A-photon and B-photon)



- An alternative to the subtraction method
- Explicitly free from lower/higher orders in alpha
- Could recycle low modes of pure QCD propagators

HLbL systematics

Need to address

- Finite volume
- $q^2 \to 0$ extrapolation
- $m_q \to m_{q,\,\mathrm{phys}}$
- $m_\mu \to m_{\mu,\,\mathrm{phys}}$
- excited states/"around the world" effects
- $a \to 0$
- QED renormalization
- ...

Outline

Summary/Outlook

- Demanding, but straightforward calculation
- Early HLbL lattice calculation encouraging
- Intermediate lattice calculations to check models (four-point, $\pi \to \gamma^*\gamma$, chiral susceptibility, ...)
- Optimistic lattice+models+expt can reach 10% goal in $\sim$ 5 years (INT WS on HLbL, Feb. 2011)
- White papers, prospects for lattice QCD:
  - USQCD white-paper (http://www.usqcd.org/collaboration.html)
  - Fundamental Physics at the Intensity Frontier white-paper (arXiv:1205.2671 [hep-ex])
- Expected precision
  - E989: 0.14 PPM (factor of 3-4 better than E821)
  - SM theory, HVP: 0.3% (factor of 2)
  - SM theory, HLbL 10% or better (?)
  - Same central values, a_μ discrepancy $\to$ 5-8 σ

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  - Ds cluster at FNAL
  - q-series clusters at JLab