

**New Frontiers in
Lattice Gauge Theory**

Can the sextet gauge model hide the Higgs impostor ?

Lattice Higgs Collaboration (LHC)

**with Zoltan Fodor, Kieran Holland, Daniel Negradi,
Chris Schroeder, Chik Him Wong**

Julius Kuti

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**New Frontiers in Lattice Gauge Theory
Galileo Institute, Florence, September 17, 2012**

Outline

Two necessary conditions to hide the Higgs impostor
focus on $SU(3)$ sextet fermion representation with two flavors

Dilaton as Higgs impostor? (broken scale invariance)

Chiral and conformal tests of the sextet model

Running (walking) coupling

Summary and outlook

for more details of the discussions:

[arXiv:1209.0391](#) [[pdf](#), [other](#)]

Can the nearly conformal sextet gauge model hide the Higgs impostor?

[Zoltán Fodor](#), [Kieran Holland](#), [Julius Kuti](#), [Dániel Nógrádi](#), [Chris Schroeder](#), [Chik Him Wong](#)

Comments: 10 pages, 8 figures

Subjects: **High Energy Physics – Lattice (hep-lat)**; High Energy Physics – Phenomenology (hep-ph)

focus of talk

[arXiv:1208.1051](#) [[pdf](#), [ps](#), [other](#)]

The Yang–Mills gradient flow in finite volume

[Zoltan Fodor](#), [Kieran Holland](#), [Julius Kuti](#), [Daniel Nogradi](#), [Chik Him Wong](#)

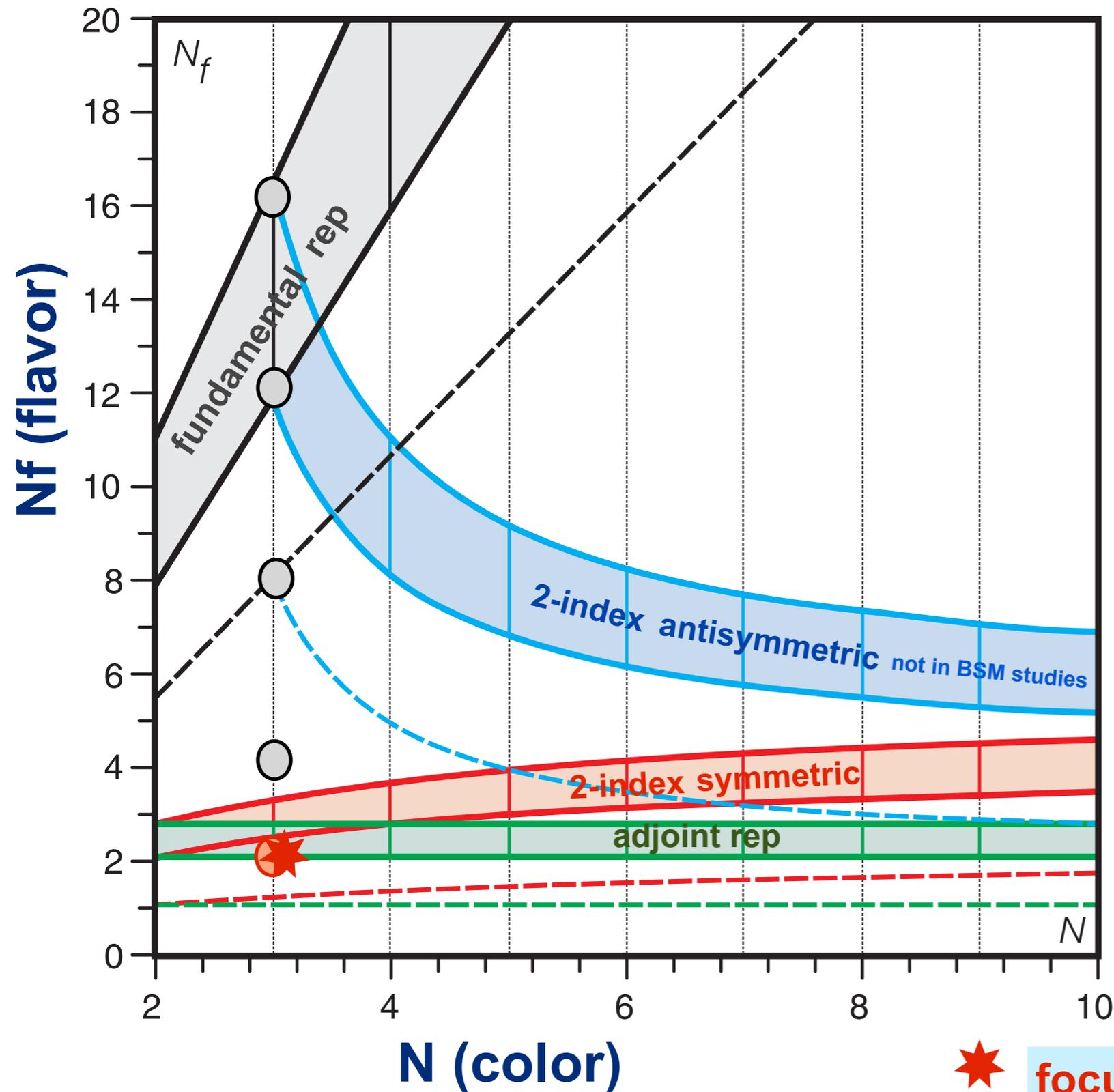
Comments: 16 pages, 8 figures, minor corrections, references added

Subjects: **High Energy Physics – Lattice (hep-lat)**

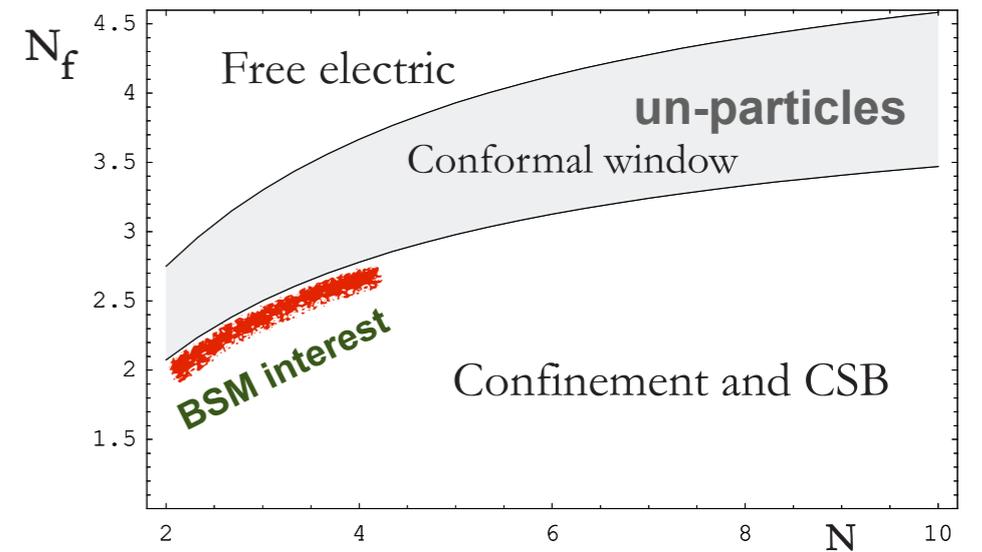
running coupling

why the sextet model?

theory space and conformal window
 color, flavor, and massless fermion representation



for each rep BSM interest is below conformal window but close to it:



earlier:
 arXiv:1205.1878 [hep-lat]

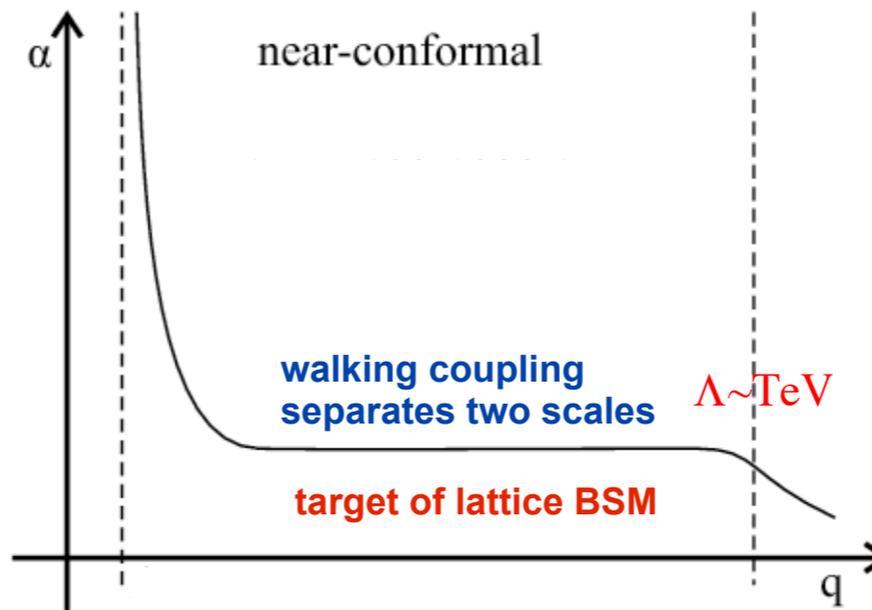
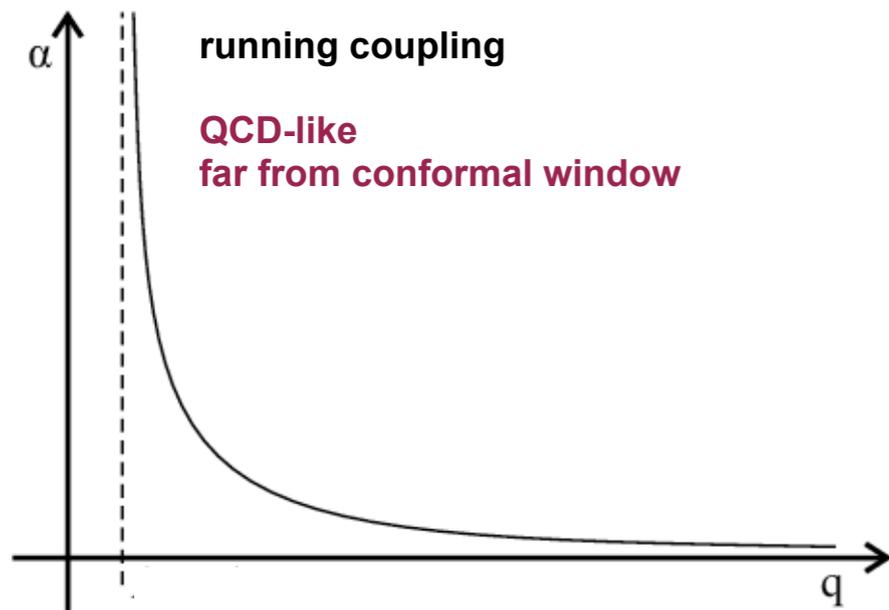
$N_f=12$ fundamental rep tests

physics prospect of sextet?

★ focus of talk

New extended data set and analysis

simple realization of composite Higgs: $N_f=2$ $SU(3)$ sextet representation



χ_{SB} on $\Lambda \sim \text{TeV}$ scale

walking gauge coupling

fermion mass generation not addressed

what composite Higgs mechanism ?

broken scale invariance (dilaton) and/or light non-SM composite Higgs particle?

original Technicolor paradigm replaced with sextet $SU(3)$ color rep:

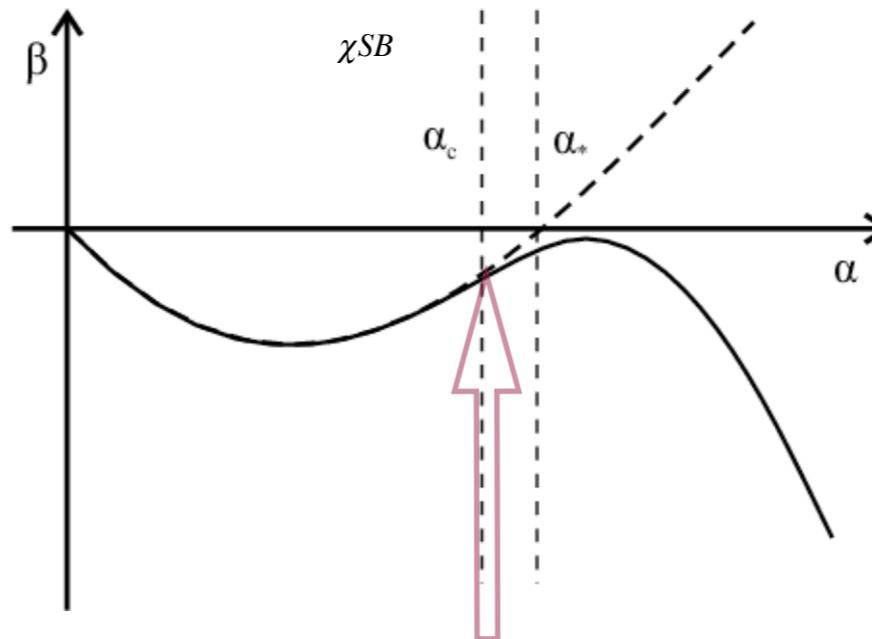
one massless fermion doublet χ_{SB} on $\Lambda \sim \text{TeV}$ scale

$$\begin{bmatrix} u \\ d \end{bmatrix}$$

three Goldstone pions become longitudinal components of weak bosons

composite Higgs mechanism scale of Higgs condensate $\sim F=250$ GeV

conflicts with EW precision constraints?



when chiral symmetry breaking turns conformal FP into walking

sextet model with two critical requirements: (1) χ_{SB} (2) walking gauge coupling?

Early work using sextet rep:

Marciano (QCD paradigm, 1980)

Kogut, Shigemitsu, Sinclair (quenched, 1984)

recent work:

DeGrand, Shamir, Svetitsky IRFP or walking gauge coupling

Lattice Higgs Collaboration χ_{SB}

Kogut, Sinclair finite temperature

role of the third fermion flavor?

dilaton as Higgs impostor?

The dilaton: pseudo-Goldstone particle of broken scale invariance

RG invariant

$$\partial_\mu \mathcal{D}^\mu = \Theta_\mu^\mu = \frac{\beta(\alpha)}{4\alpha} G_{\mu\nu}^a G^{a\mu\nu}$$

$$\mathcal{D}^\mu = \Theta^{\mu\nu} x_\nu \quad \text{Dilatation current symmetric energy-momentum tensor}$$

$$\langle 0 | \Theta^{\mu\nu}(x) | \sigma(p) \rangle = \frac{f_\sigma}{3} (p^\mu p^\nu - g^{\mu\nu} p^2) e^{-ipx}$$

$$\langle 0 | \partial_\mu \mathcal{D}^\mu(x) | \sigma(p) \rangle = f_\sigma m_\sigma^2 e^{-ipx}$$

$$[\Theta_\mu^\mu]_{NP} = \frac{\beta(\alpha)}{4\alpha} [G_{\mu\nu}^a G^{a\mu\nu}]_{NP}$$

$$[\Theta_\mu^\mu]_{NP} = \frac{\beta(\alpha)}{4\alpha} G_{\mu\nu}^a G^{a\mu\nu} - \langle 0 | \frac{\beta(\alpha)}{4\alpha} G_{\mu\nu}^a G^{a\mu\nu} | 0 \rangle_{PT}$$

Looking for PCDC relation among three unknowns:

1. dilaton mass m_σ
2. dilaton decay constant f_σ
3. non-perturbative gluon condensate

long history of PCDC relation
only non-perturbative part kept in derivation

recently:

Bai and Appelquist Phys.Rev. D82 (2010) 071701
Matsuzaki and Yamawaki arXiv:1206.6703[hep-ph]

$$m_\sigma^2 \simeq -\frac{4}{f_\sigma^2} \langle 0 | [\Theta_\mu^\mu(0)]_{NP} | 0 \rangle \quad \text{Partially Conserved Dilatation Current (PCDC)}$$

there are two different expectations on limit of right-hand side ratio when conformal window is approached:

1. dilaton mass parametrically vanishes when CW approached $m_\sigma^2 \simeq (N_f^c - N_f) \cdot \Lambda^2$ **Bai and Appelquist**

$g(\mu = \Lambda) = g_c$ where ChSB is triggered $\frac{m_\sigma}{f_\sigma} \rightarrow 0$

2. dilaton mass remains finite in the limit as measured in $f_\sigma \simeq \Lambda$ units **Yamawaki et al.** $\frac{m_\sigma}{f_\sigma} \rightarrow const$

Realistic BSM models have not been built with parametric tuning close to the conformal window. For example, the sextet model is at some intrinsically determined position near the conformal window and only non-perturbative lattice calculations can explore the physical properties of the scalar particle.

important role of $\frac{f_\pi}{f_\sigma}$ in electroweak phenomenology

both scenarios expect light Higgs-like dilaton

$$\lim_{a \rightarrow 0} \left(\frac{1}{a^4} \langle 1 - \frac{1}{3} \text{tr} U_P \rangle \right) = \frac{\pi^2}{36} \langle \frac{\alpha}{\pi} GG \rangle_{\text{lattice}}$$

non-perturbative lattice gluon condensate?

from current correlators?

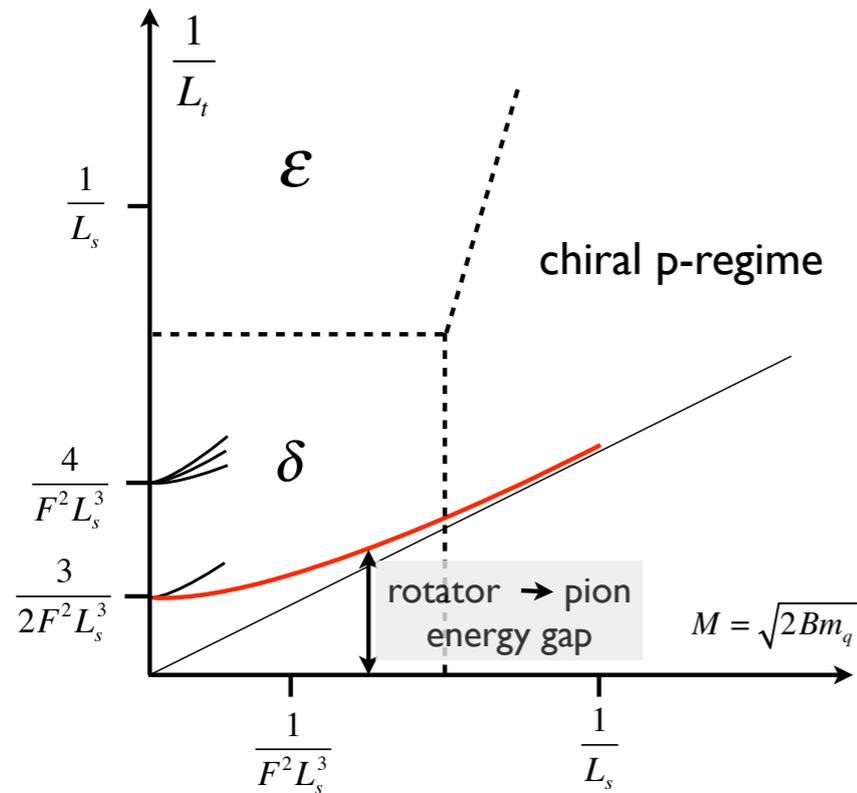
$$\langle 1 - \frac{1}{3} \text{tr} U_P \rangle = \sum_n c_n \cdot g_0^{2n} + a^4 \frac{\pi^2}{36} \left(\frac{b_0}{\beta(g_0)} \right) \langle \frac{\alpha}{\pi} GG \rangle_{\text{lattice}} + O(a^6)$$

from gradient flow of GG composite operator?

is this separation meaningful?

chiral symmetry breaking in the sextet model

mass deformed chiral SB in finite volume below conformal window:

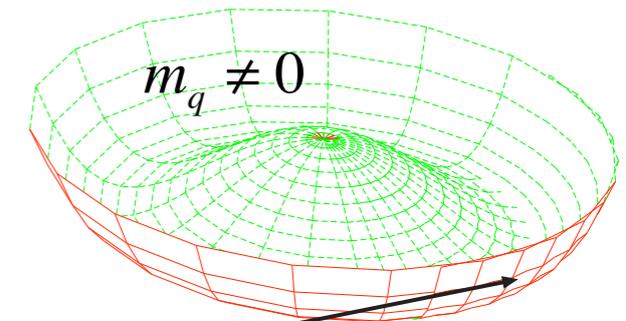
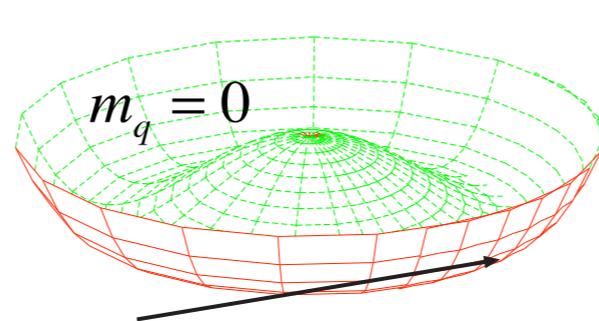


Goldstone dynamics is different in each regime

We study δ and ϵ -regimes (RMT) and p-regime (probing chiral loops)

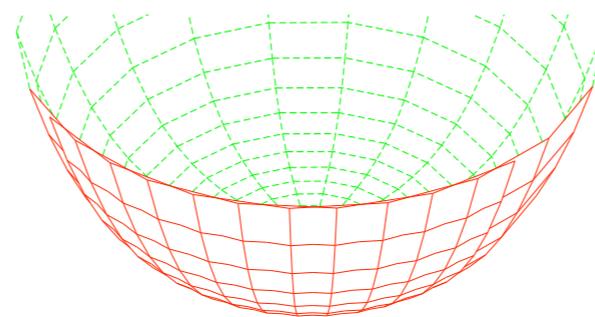
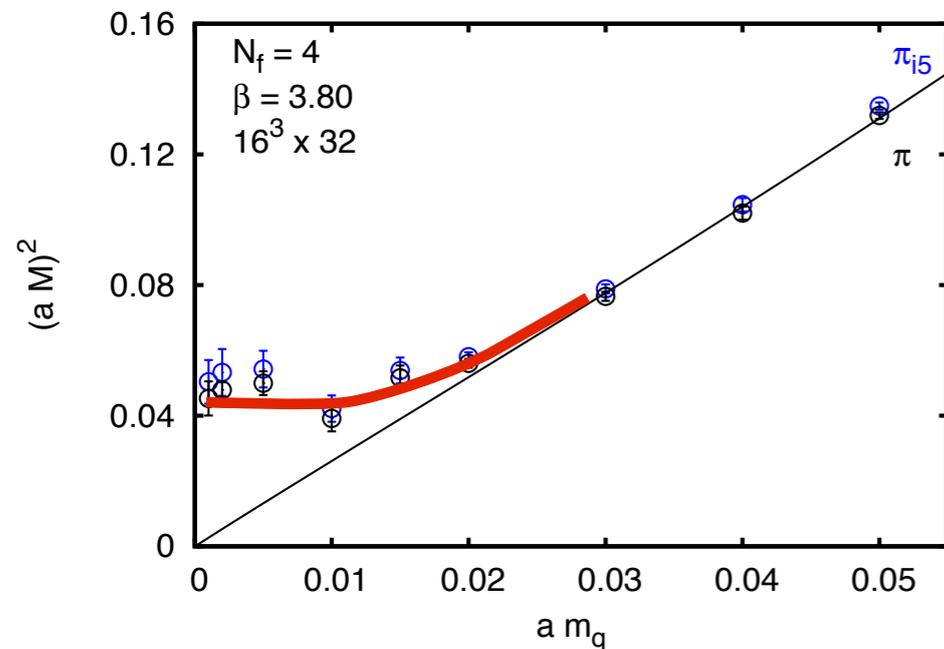
complement each other

interpretation of rotator levels in $m_q \rightarrow 0$ limit:



$m_q = 0$
 V_{eff} : chiral condensate in flavor space
 arbitrary orientation of condensate

$m_q \neq 0$
 tilted condensate



Not to misidentify rotator gaps as evidence of chirally symmetric phase !

Our sextet simulations are in the p-regime $\beta=3.2$ and $\beta=3.25$

simulation details:

tree level improved Symanzik gauge action; $\beta=6/g^2$ normalization

smearing in staggered fermions: 2 stout steps

rooting with two flavors (follow-up work without rooting if model will pass first tests)

RHMC

multiple time scales and Omelyan integrator

$\beta=3.20$ $m=0.003-0.010$ mass range $24^3 \times 48$, $28^3 \times 56$, $32^3 \times 64$, $48^3 \times 96$ lattices

$\beta=3.25$ $m=0.004-0.008$ mass range $24^3 \times 48$, $28^3 \times 56$, $32^3 \times 64$ lattices

error analysis: mass fits with double Jackknife procedure on covariance matrices

Nf=2 sextet bulk phase structure ?

are we sitting in the weak coupling phase at $\beta=3.2$?

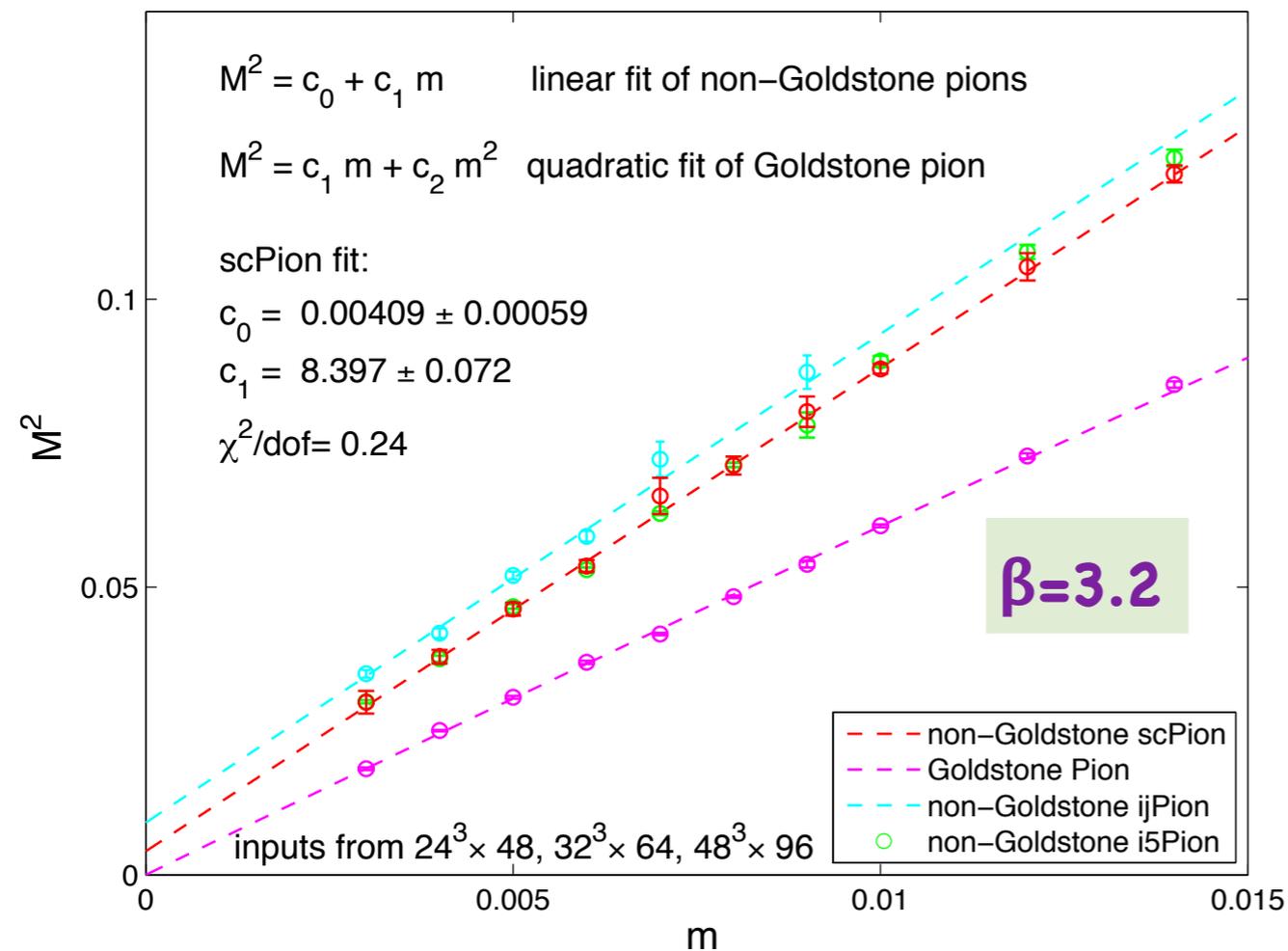
(most of the results)

new data: $\beta=3.2 \rightarrow \beta=3.25$

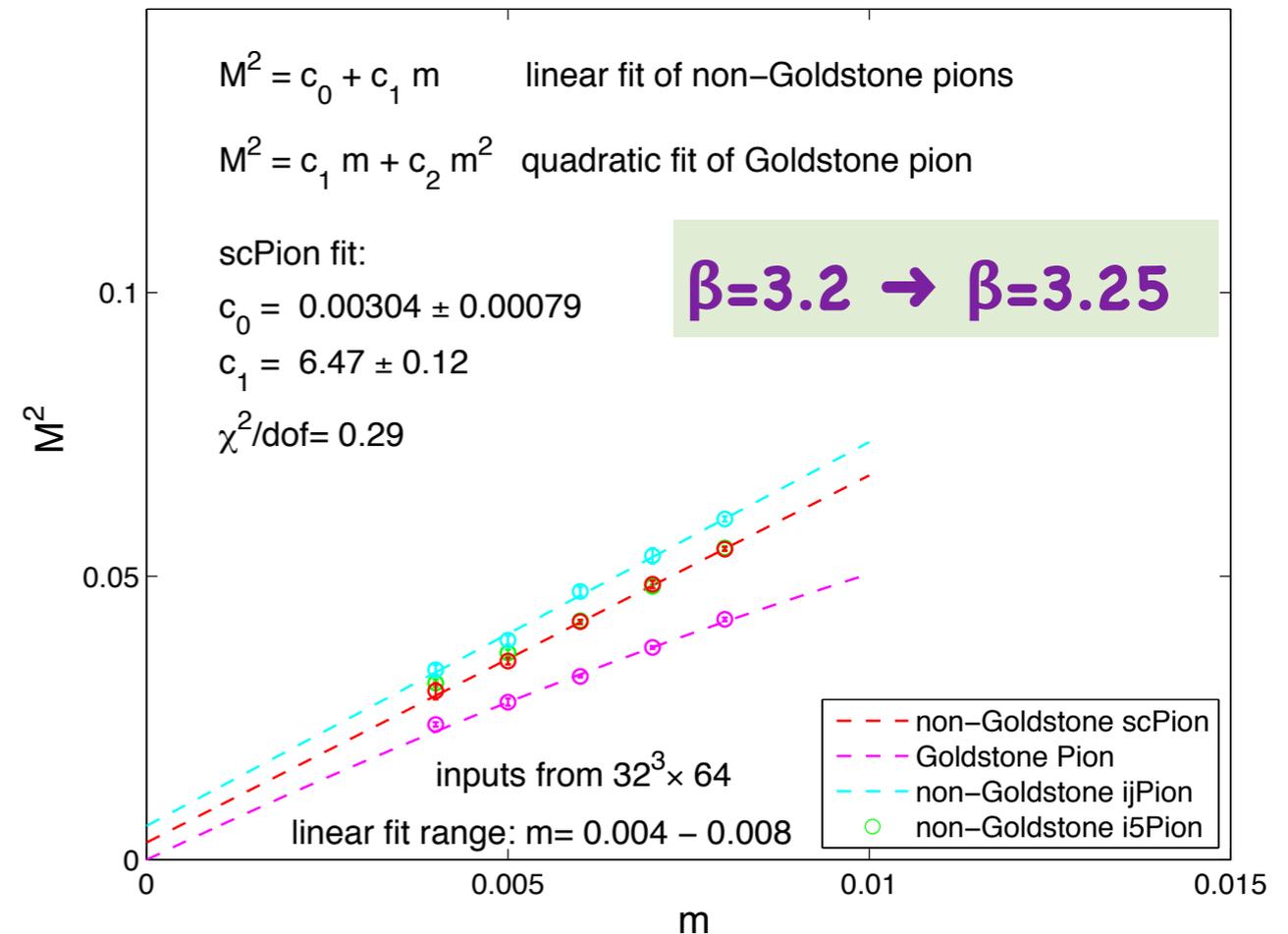
(non)Goldstone splittings and spectroscopy respond like in weak coupling QCD

full scan of bulk phase to reconfirm chSB phase

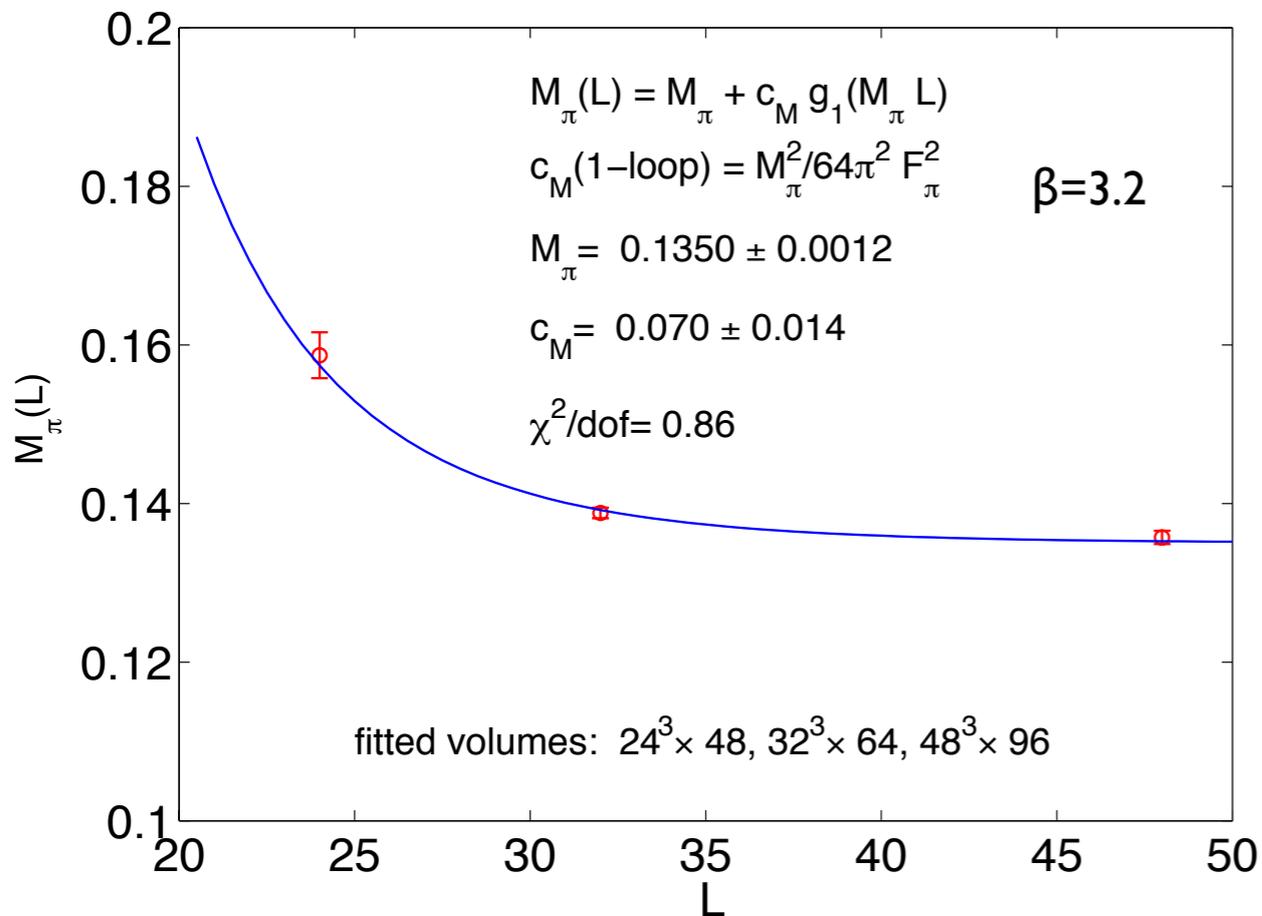
$\beta=3.20$ Goldstone and non-Goldstone pion spectra



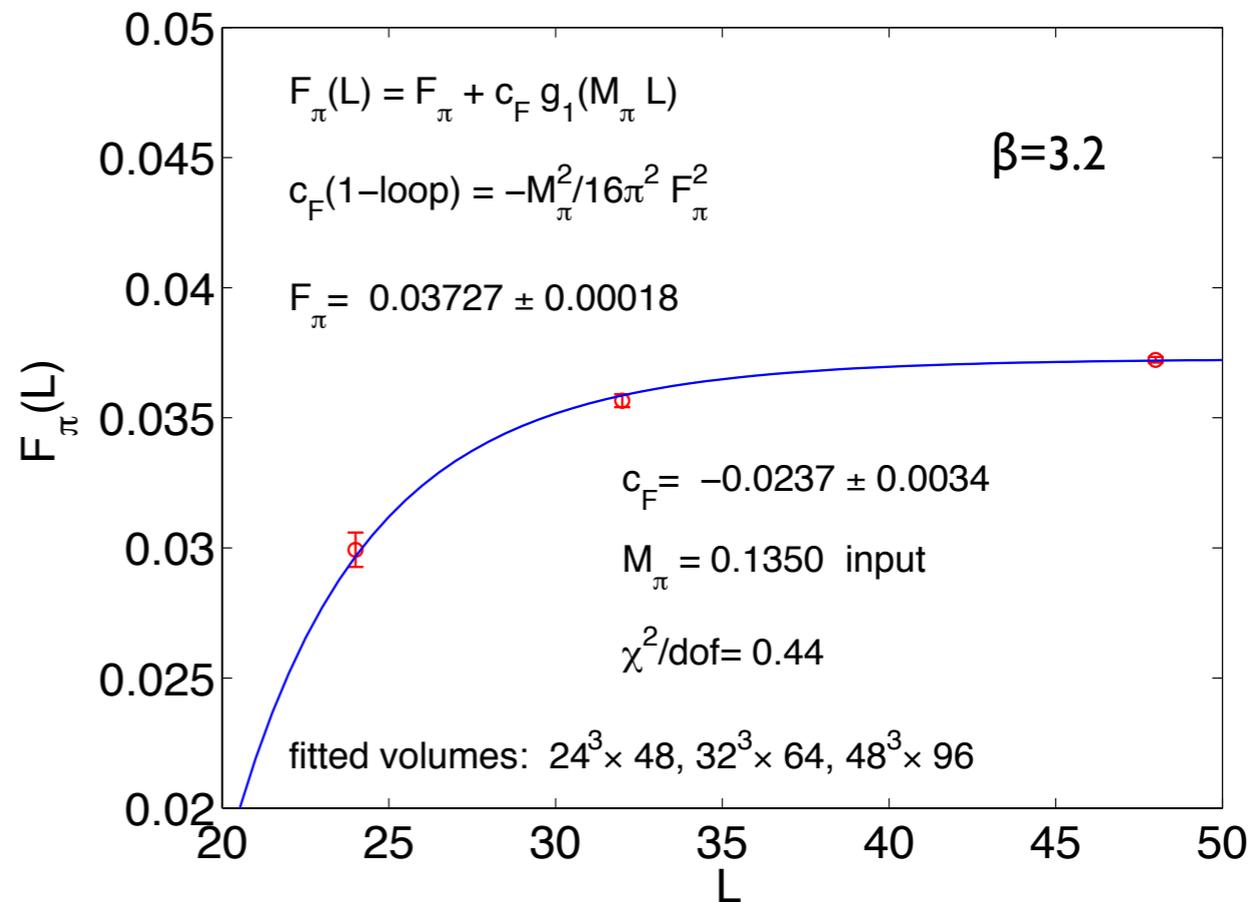
$\beta=3.25$ Goldstone and non-Goldstone pion spectra



M_π $\beta = 3.2$ $m=0.003$

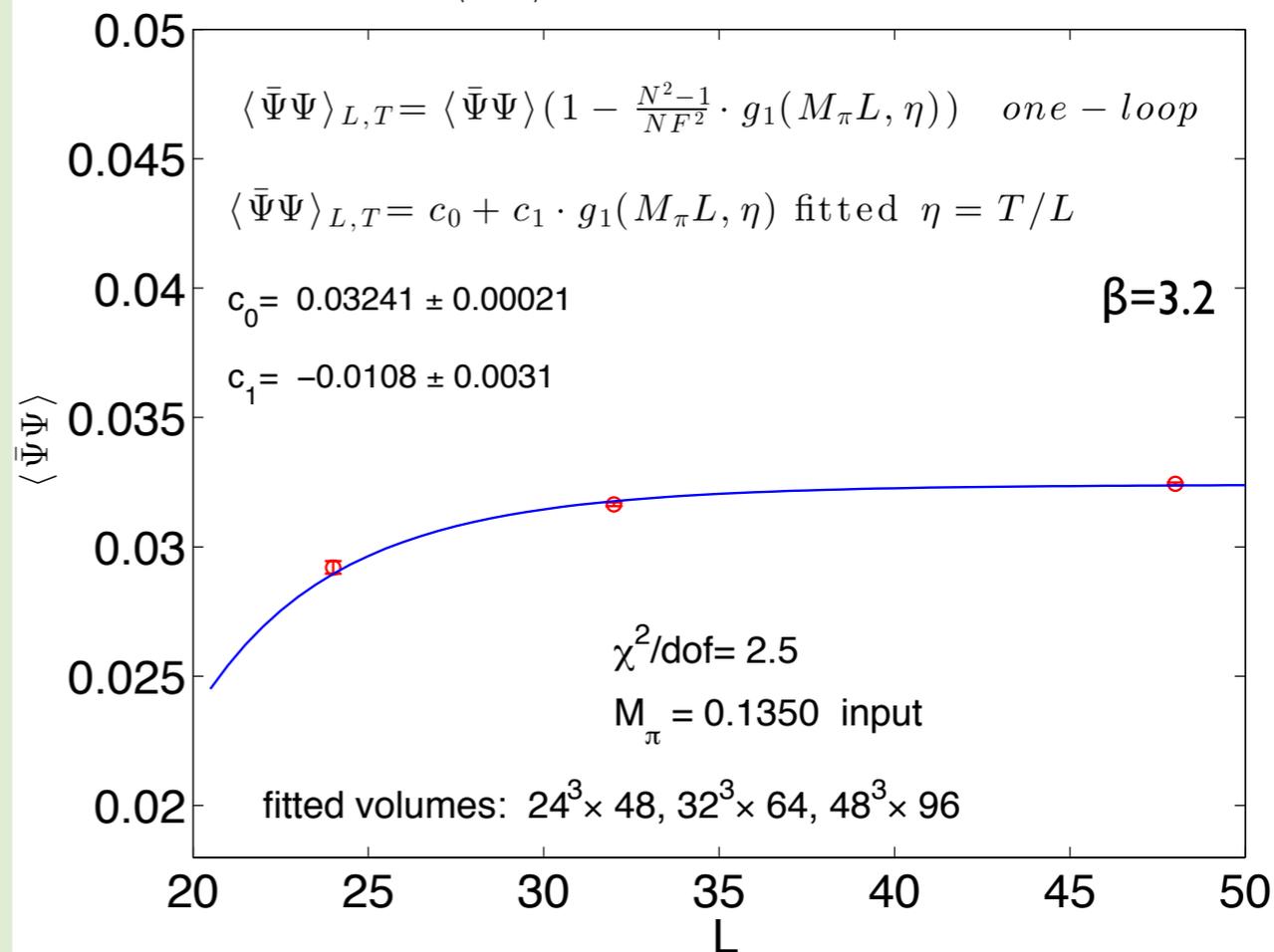


F_π $\beta = 3.2$ $m=0.003$



strategy: $L \rightarrow \infty$ extrapolation first
mass-deformed theory
close to $m=0$ critical surface
 $L \rightarrow \infty$ extrapolated chiral and conformal
scaling tests in sextet model
for $L \cdot M_\pi > 5$ less than one percent
 L correction left
 C_M and C_F signs correct, numerically off

$\langle \bar{\Psi}\Psi \rangle$ $m=0.003$ finite volume fit



Strategy I: $L=\infty$ extrapolation first and then scaling tests in m

Chiral hypothesis

(in)complete analysis on both sides

Conformal hypothesis

chiral logs not resolved yet for $N_f=8$, or $N_f=12$!

$N_f=2$ sextet easier to reach chiral log regime

$$(M_\pi^2)_{NLO} = (M_\pi^2)_{LO} + (\delta M_\pi^2)_{1-loop} + (\delta M_\pi^2)_{m^2} + (\delta M_\pi^2)_{a^2 m} + (\delta M_\pi^2)_{a^4}$$

$$\sim m^2 \quad \sim a^2 m \quad \sim a^4$$

$$(M_\pi^2)_{LO} = 2B \cdot m + a^2 \Delta_B$$

kept cutoff term in B see LO a^2 term

would require more data

$$(\delta M_\pi^2)_{1-loop} = [(M_\pi^2)_{LO} + a^2]^2 \ln(M_\pi^2)_{LO}$$

$$M_\pi^2 = c_1 m + c_2 m^2 + \text{logs}$$

fitted function for all Goldstones

$$M_{nuc} = c_0 + c_1 m + \text{logs}$$

nucleon states, rho, a1, higgs, ...

$$(F_\pi)_{LO} = F, \quad (\delta F_\pi)_{1-loop} = [(M_\pi^2)_{LO} + a^2] \ln(M_\pi^2)_{LO}$$

chiral log regime was not reached in fermion mass range

$$(\delta F_\pi)_{m^2} \sim m, \quad (\delta F_\pi)_{a^2 m} = a^2$$

kept cutoff term in F

$$F_\pi = F + c_1 m + \text{logs}$$

fitted function

$$\langle \bar{\psi} \psi \rangle = \langle \bar{\psi} \psi \rangle_0 + c_1 m + c_2 m^2 + \text{logs}$$

chiral condensate

$$M_\pi = c_\pi \cdot m^{1/y_m}, \quad y_m = 1 + \gamma$$

leading conformal scaling

functional form for all hadron masses

$$F_\pi = c_F \cdot m^{1/y_m}, \quad y_m = 1 + \gamma$$

same critical exponent

$$\langle \bar{\psi} \psi \rangle = c_\gamma \cdot m^{(3-\gamma)/y_m} + c_1 m$$

Del Debbio and Zwicky

recent improvements (Patella) from Dirac spectrum

Asymptotic infinite volume limit has not been reached yet in important candidate models for conformal window

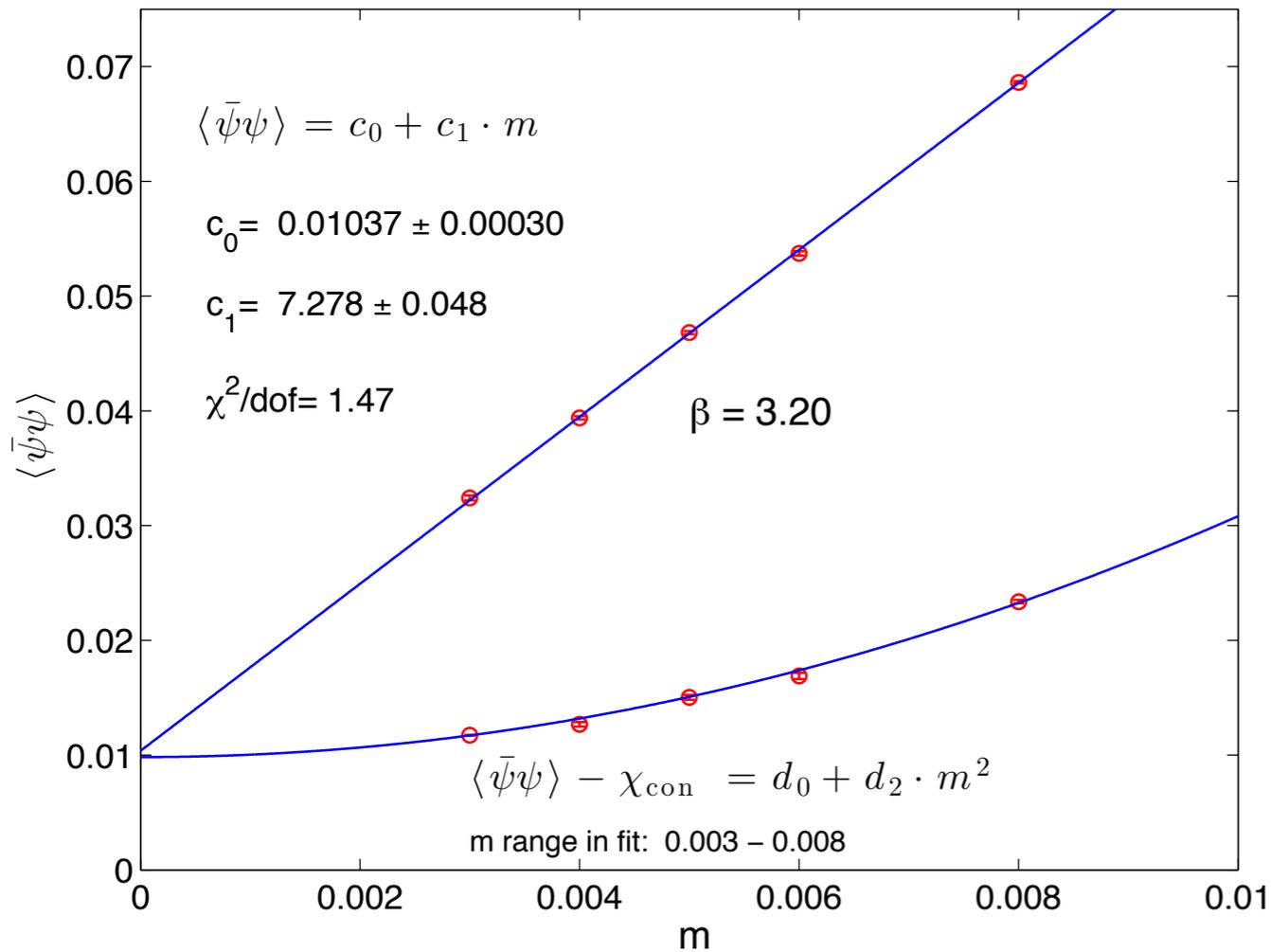
infinite volume conformal scaling violation analysis ?

conformal finite size scaling analysis and its scaling violations ?

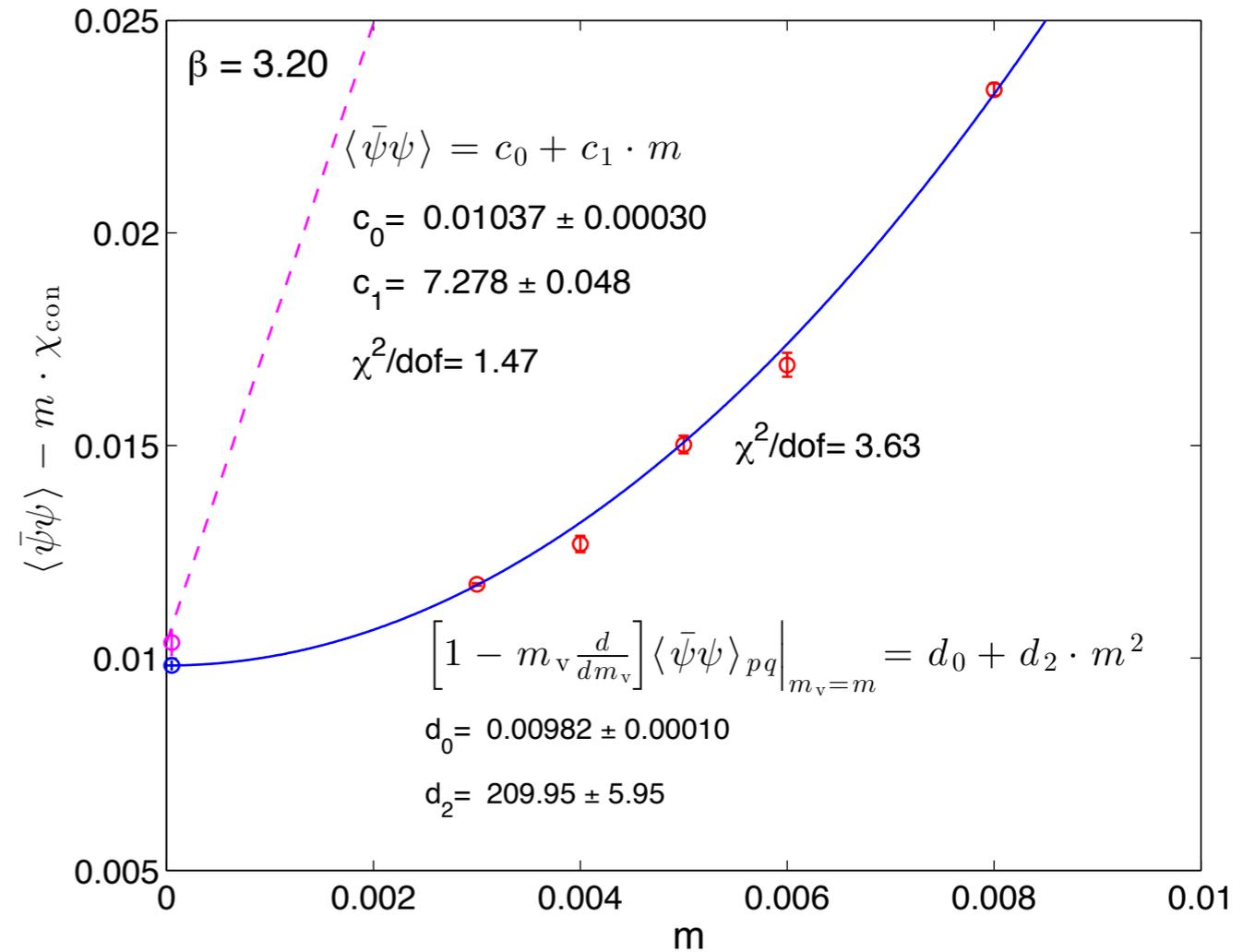
related criticisms did not change our conclusions

Nf=2 SU(3) sextet chiral condensate

chiral condensate and its subtracted form



subtracted chiral condensate



two independent determinations of the chiral condensate

consistently non-vanishing in chiral limit

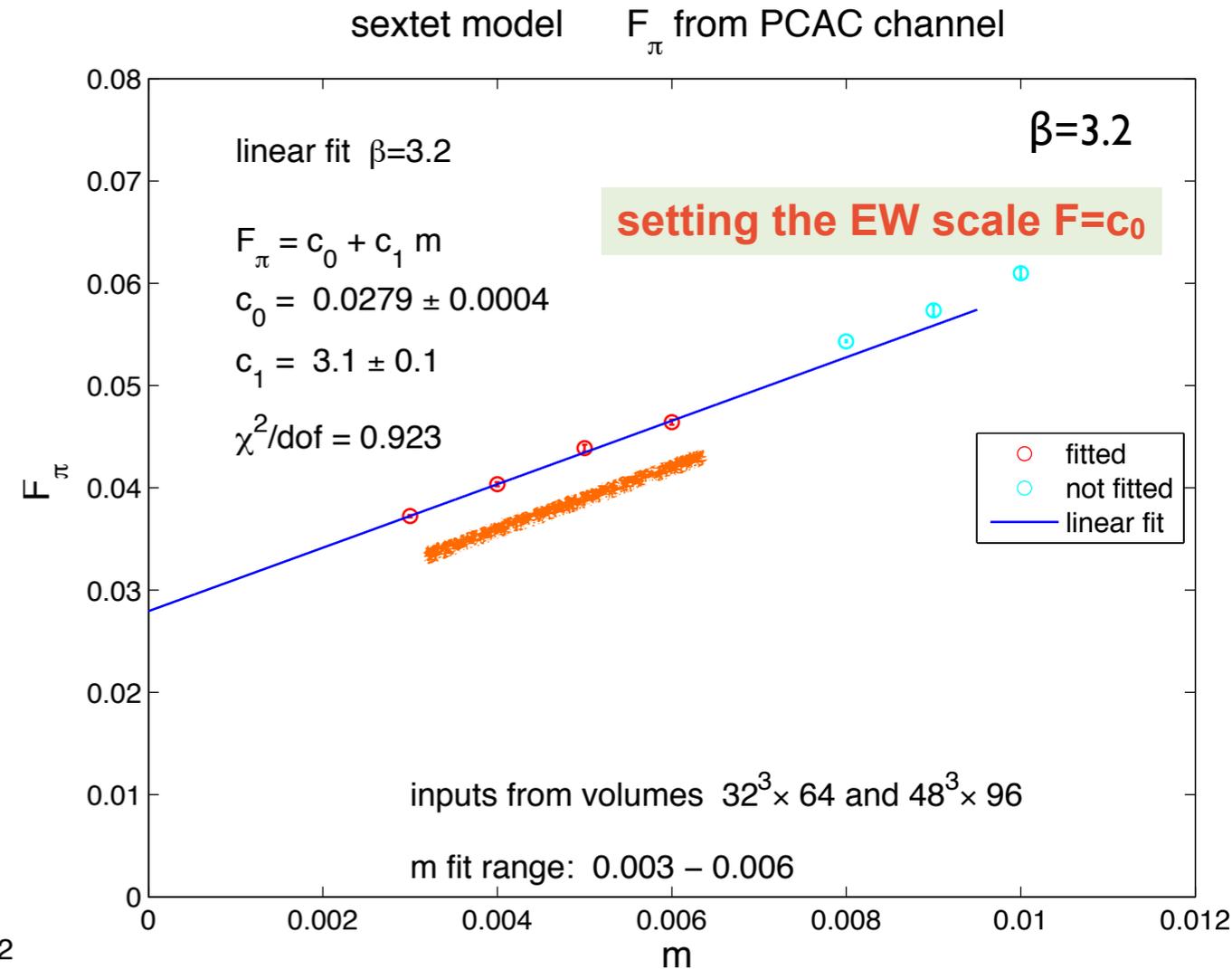
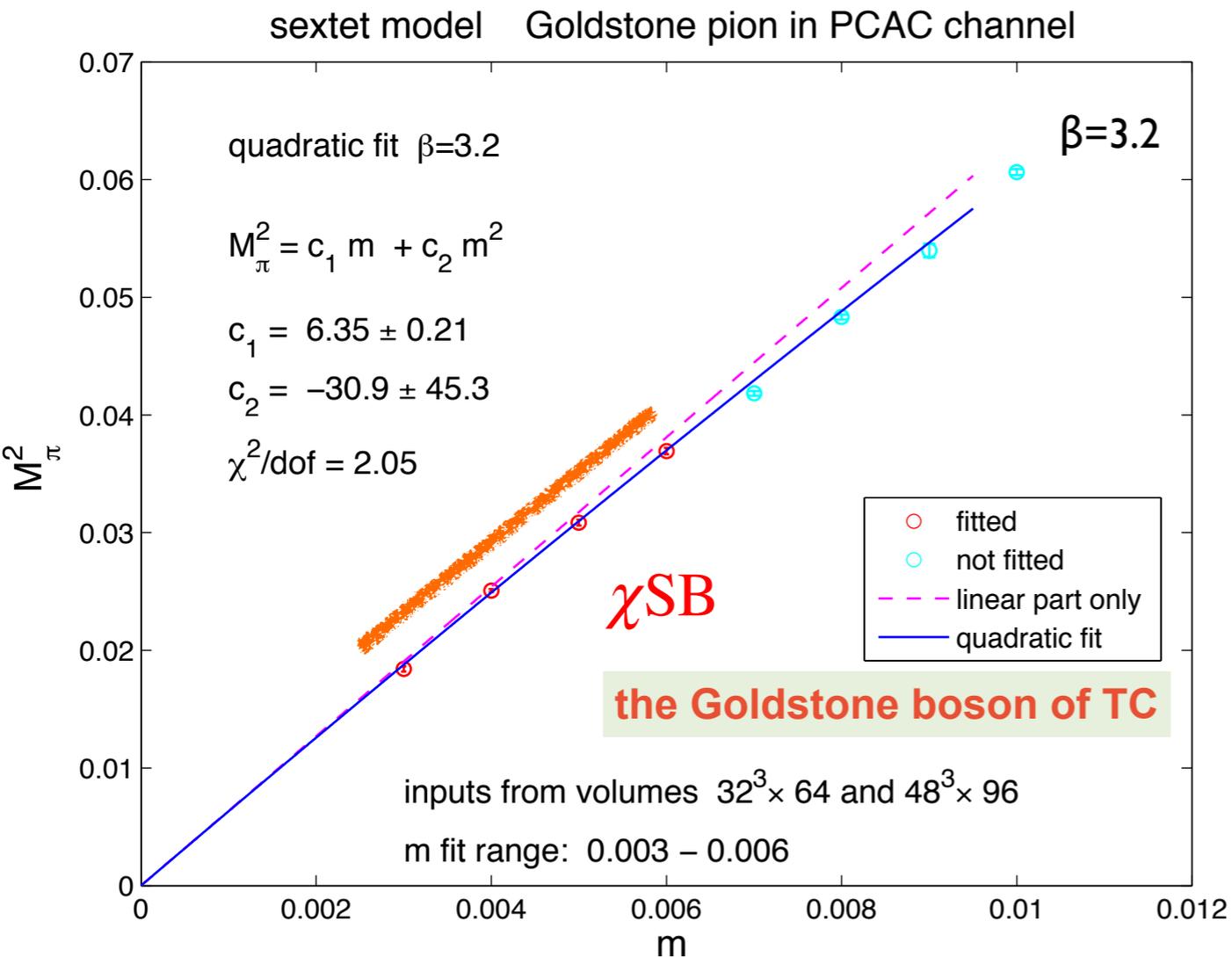
all sextet results are treated as inf volume (only $m=0.003$ is truly extrapolated)

new run set will have full finite volume analysis without relying on $L \cdot M\pi > 5$ less

than one percent L correction

spectral density analysis in the works (Giusti and Luscher)

Nf=2 SU(3) sextet chiral fits of M_π and F_π

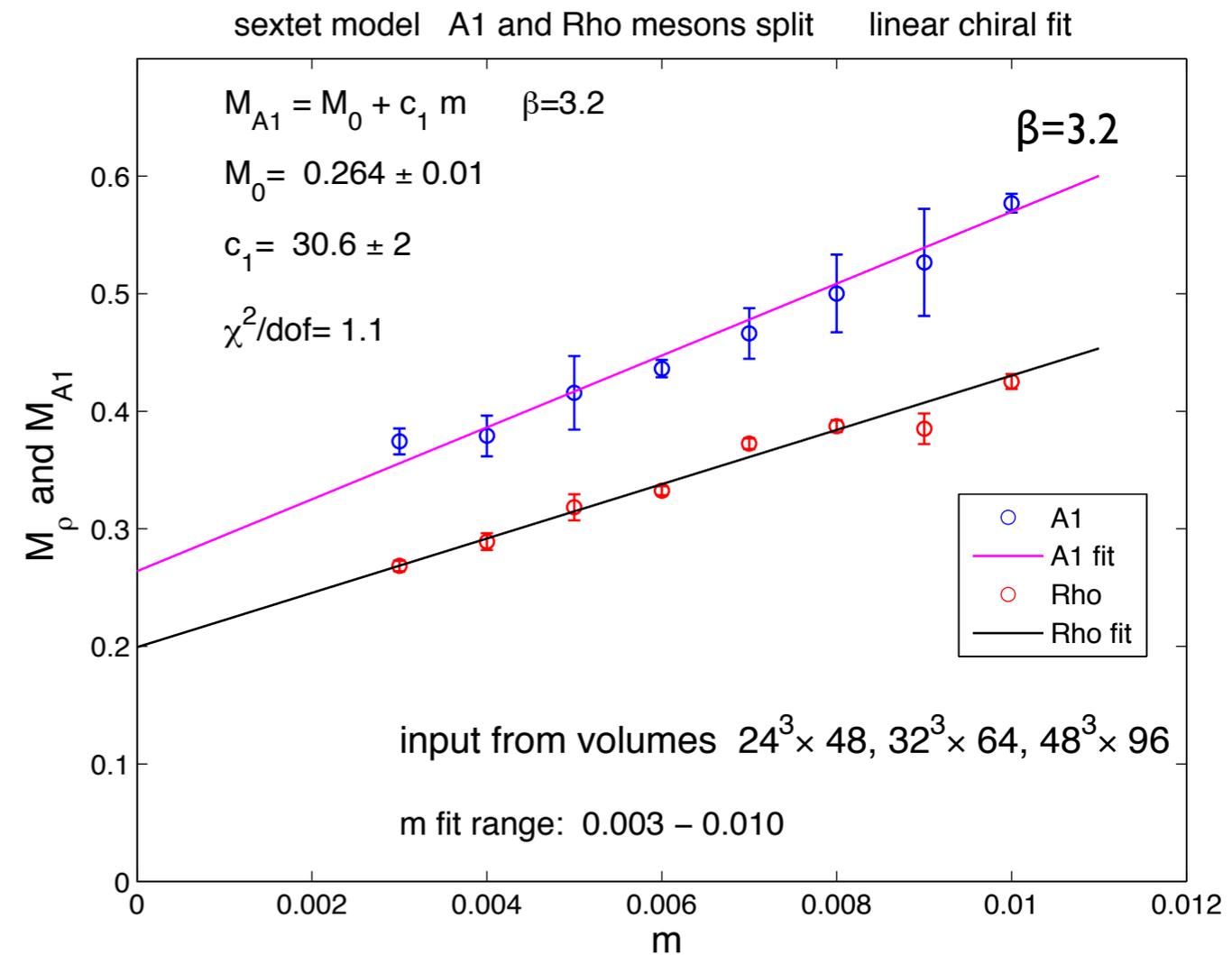
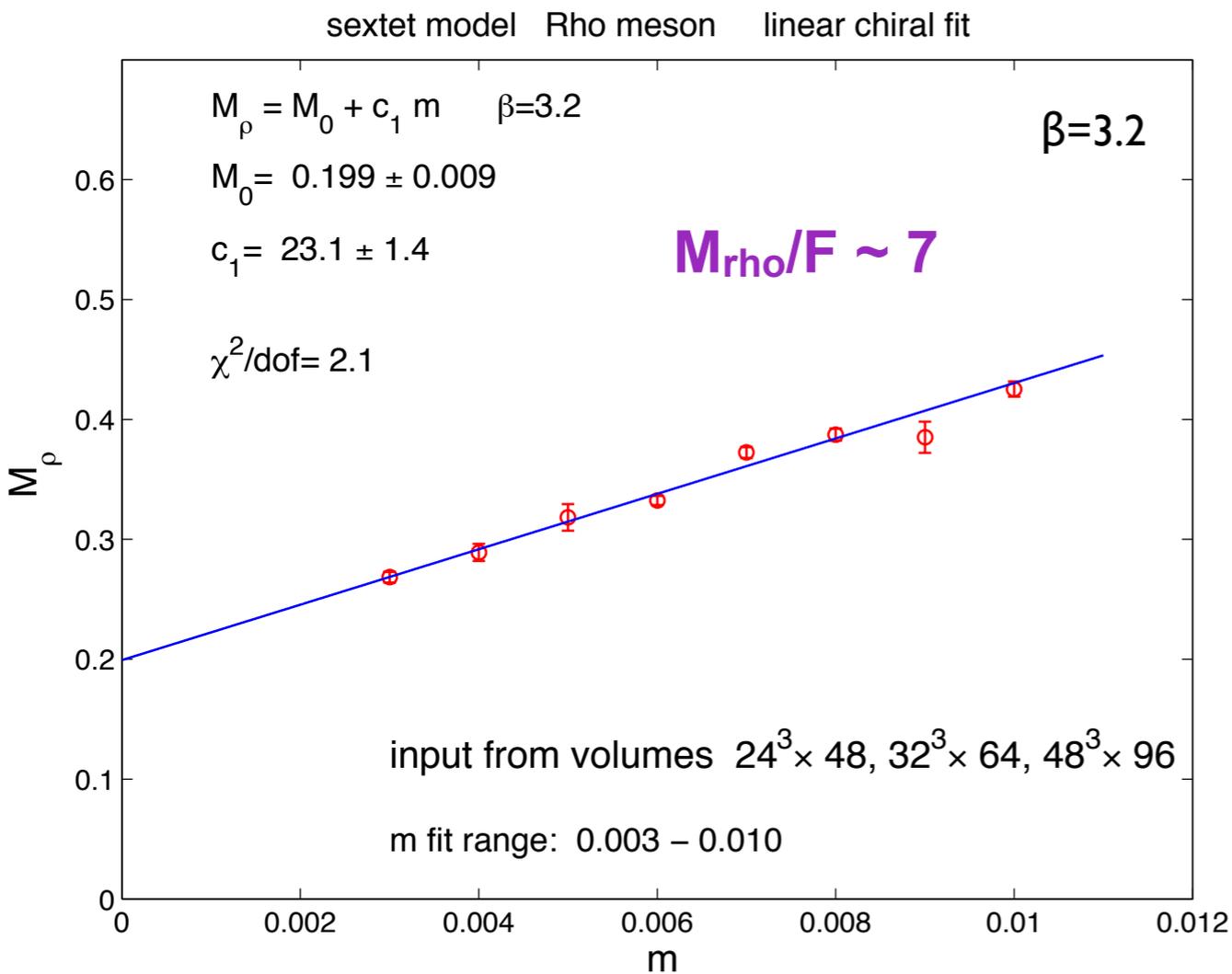


$m=0.003-0.006$ range close to chiral log regime?
 log detection will require even more precise data

Nf=2 helps, more QCD-like

consistency with partially quenched staggered chiral perturbation theory?

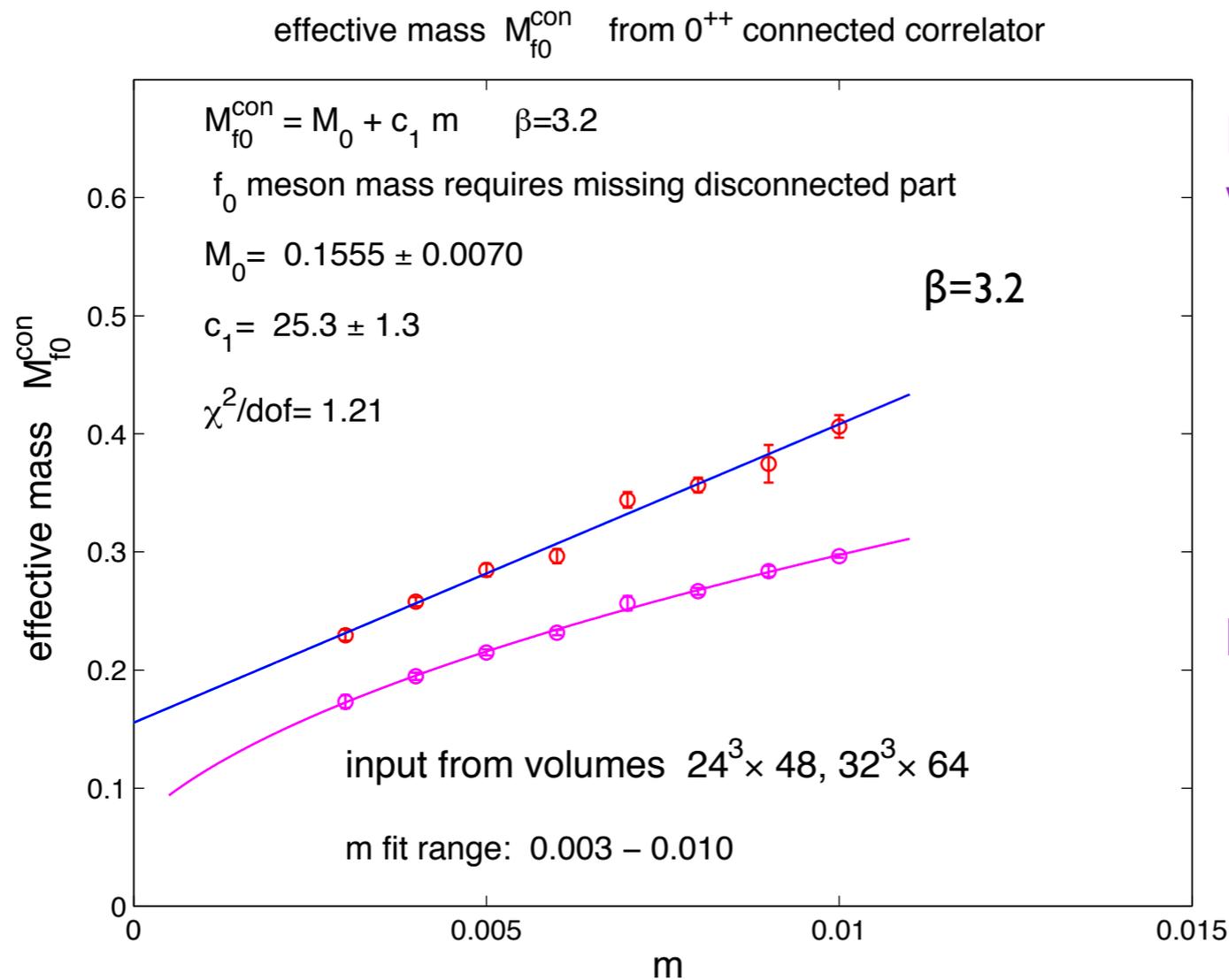
Nf=2 SU(3) sextet chiral fits M_ρ and $M(A_1)$



M_ρ remains heavy in massless fermion limit

parity partners remain split in massless fermion limit

Nf=2 SU(3) sextet chiral fits: f_0 state with 0^{++} quantum numbers:



$M(f_0)/F \sim 6$
without disconnected diagram



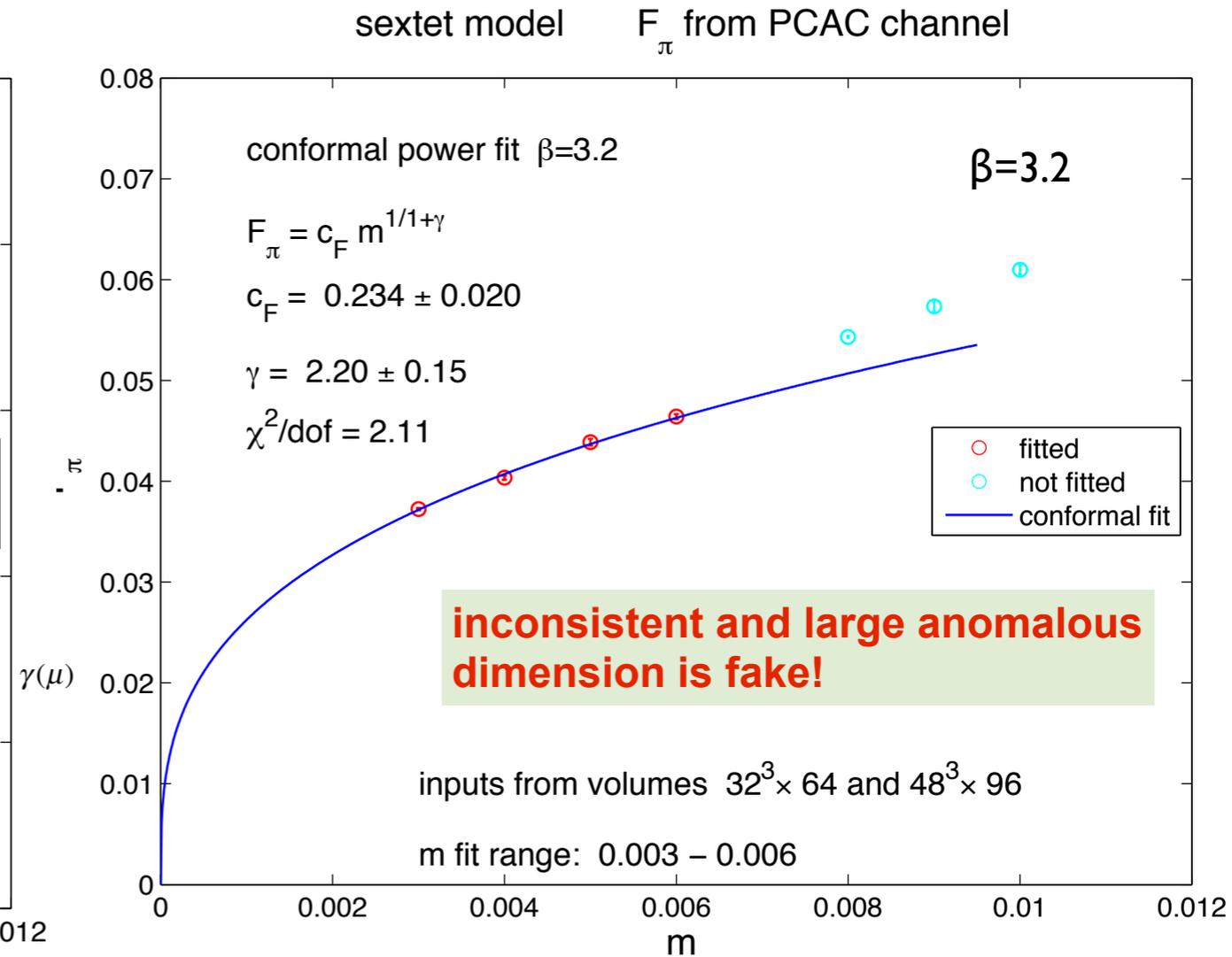
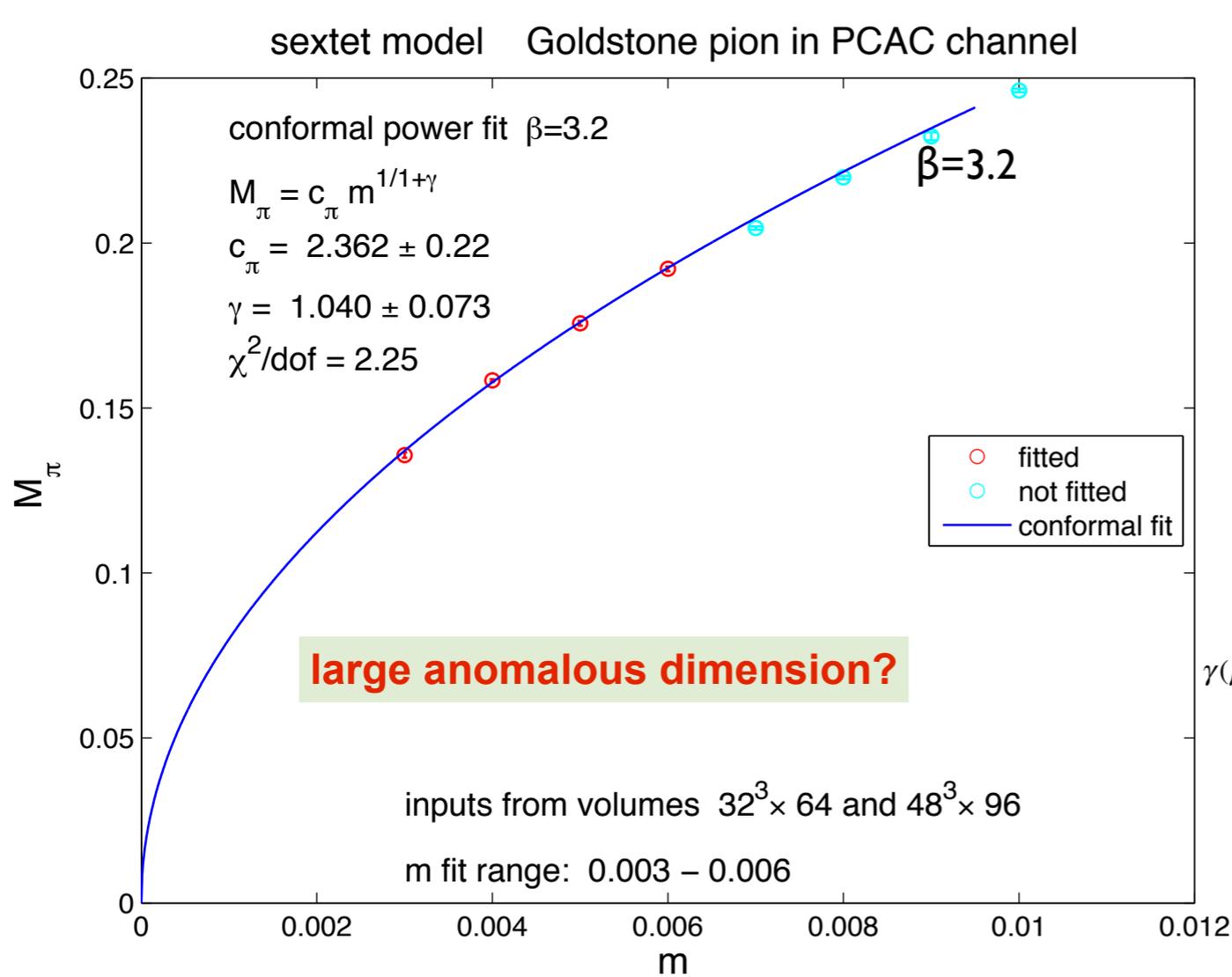
Higgs impostor in this channel?

to include disconnected diagram with good signal/noise: major undertaking

staggered fermions with rooting presents further complication Bernard et al.

mixed action is being considered

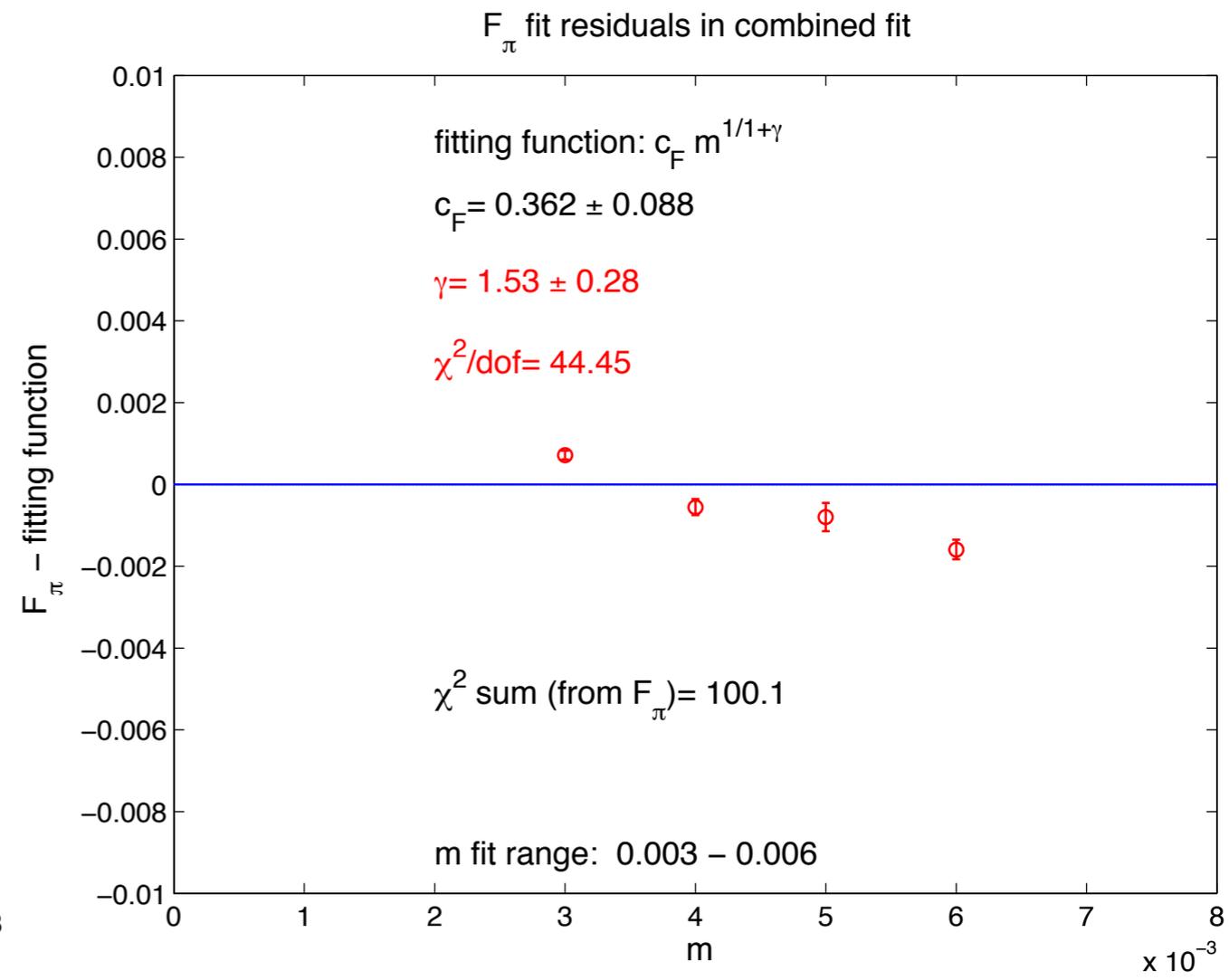
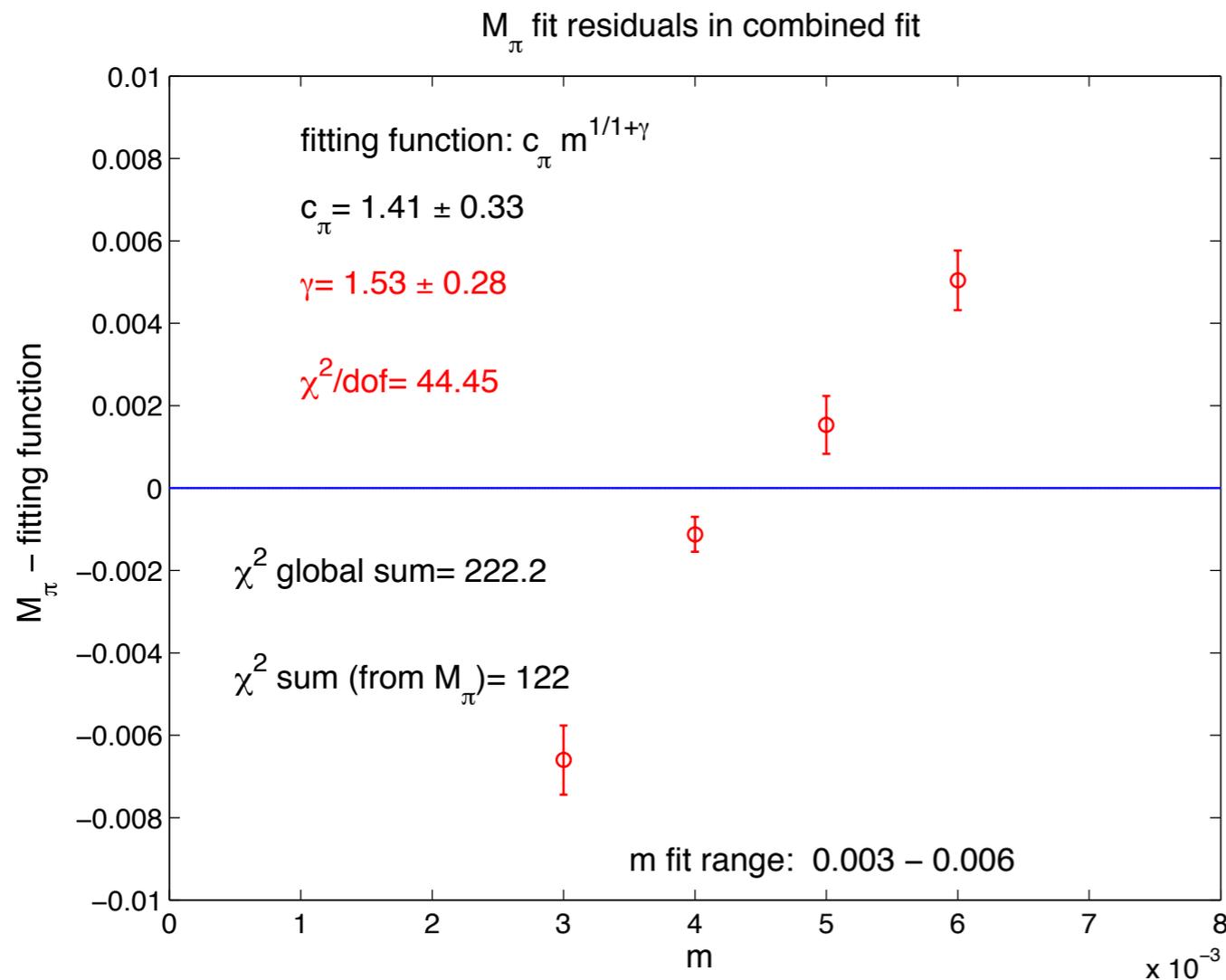
conformal hypothesis breaks down in global fits:



inconsistent large critical exponents γ forced by chiral behavior in far infrared

it is not the running $\gamma(\mu)$ at scale μ !

conformal hypothesis breaks down in global fits:



large and inconsistent critical exponents γ

are we close enough to the critical surface?

fix with scaling violation terms? don't think so

further $N_f=2$ $SU(3)$ sextet model tests ?

$L=\infty$ conformal scaling tests ✓

conformal FSS tests

illustrated by $N_f=12$ model

confining force in chiral limit ?

Kieran Holland's talk next week

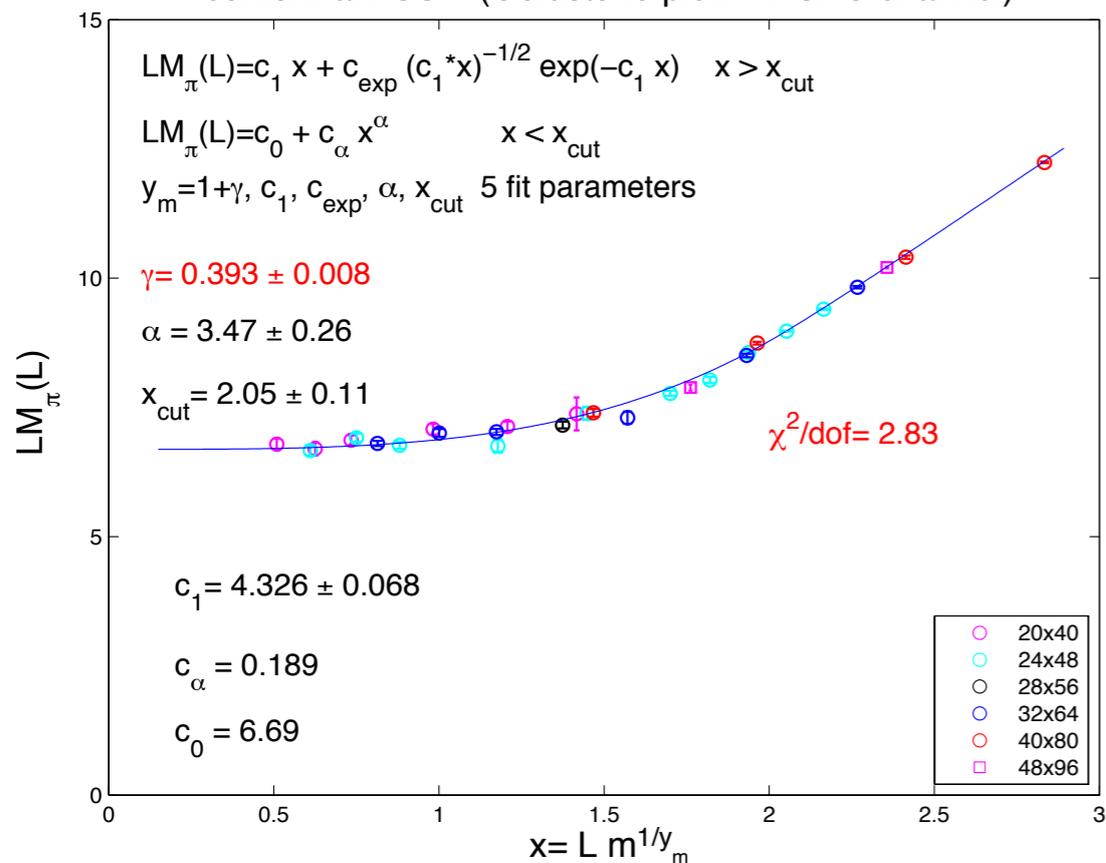
Electroweak phase transition

(and dark matter from third flavor)

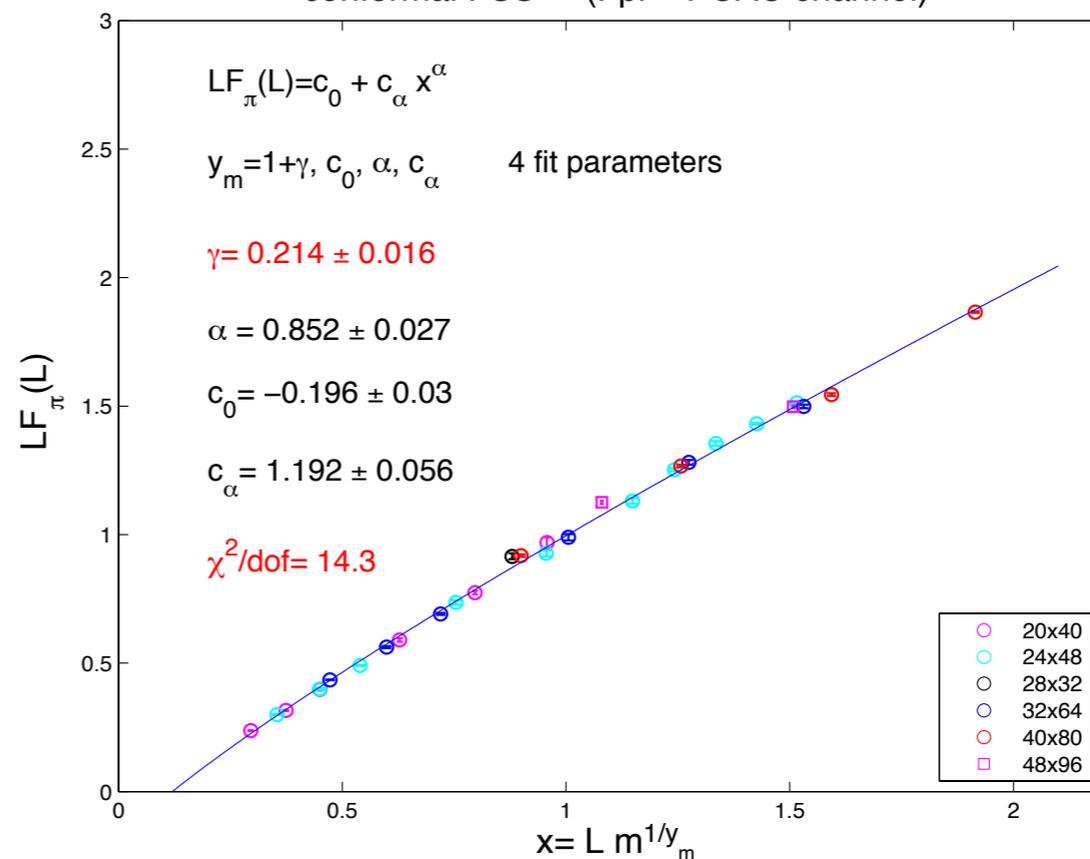
running (walking?) gauge coupling

conformal scaling test with FSS - physical model fit (spline fit similar)

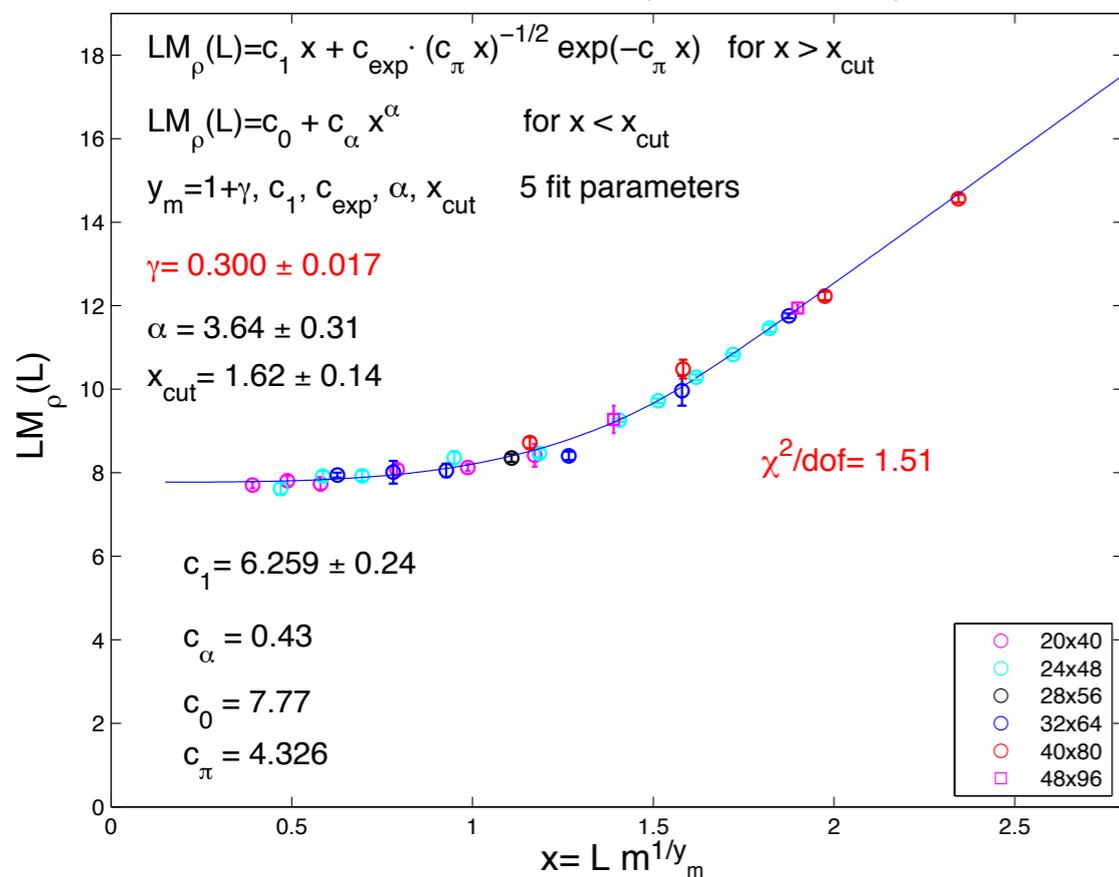
conformal FSS (Goldstone pion – PCAC channel)



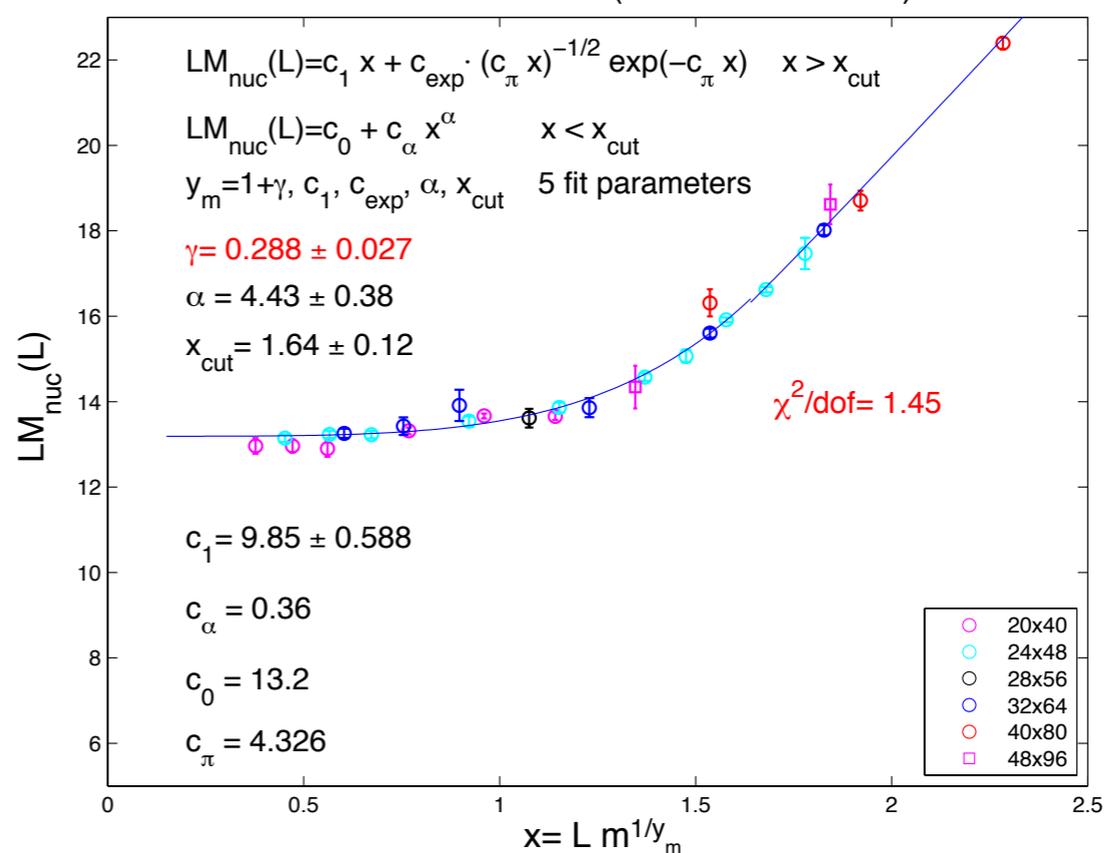
conformal FSS (Fpi – PCAC channel)



conformal FSS (cRho2 channel)



conformal FSS (cNucleon channel)



further $N_f=2$ $SU(3)$ sextet model tests ?

$L=\infty$ conformal scaling tests ✓

conformal FSS tests

illustrated by $N_f=12$ model

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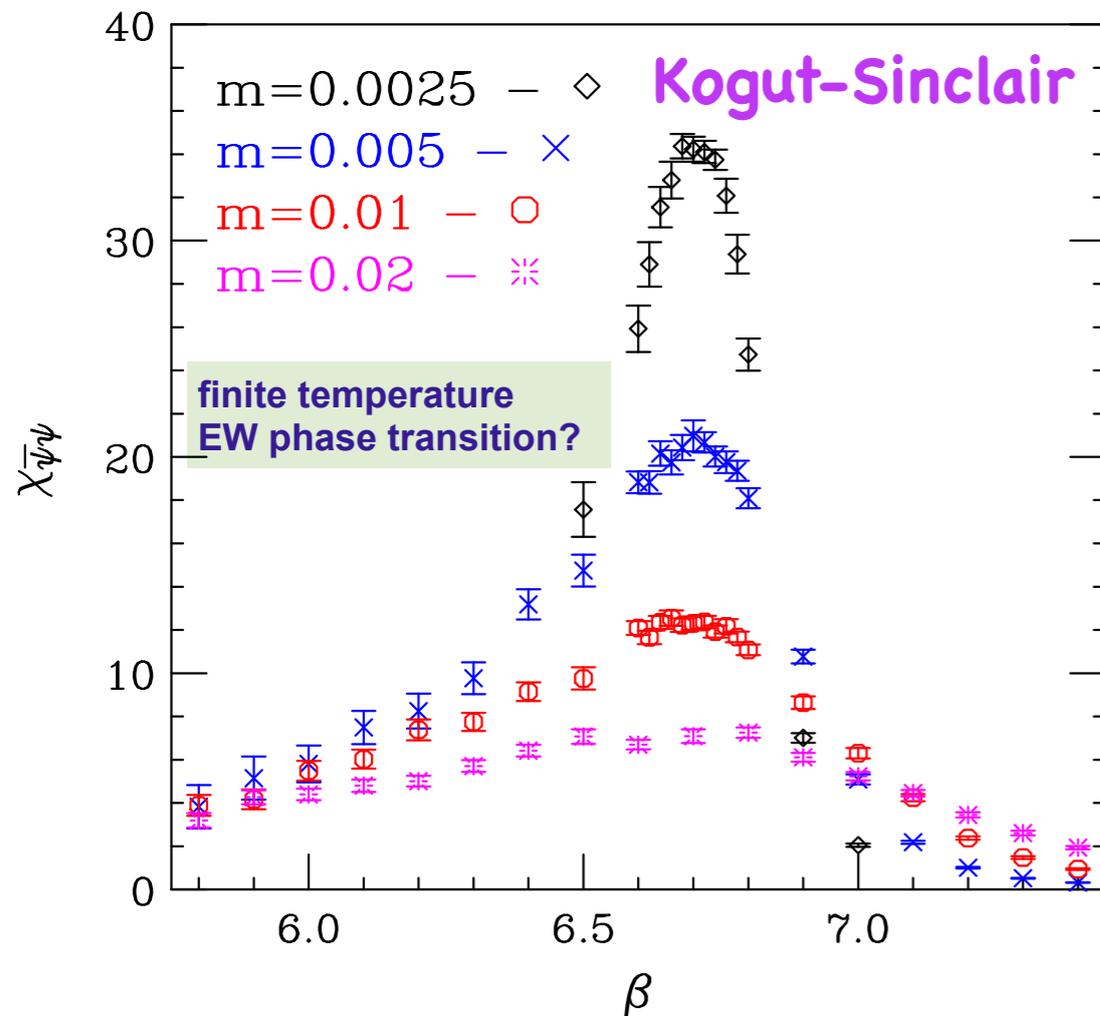
EW phase transition in sextet model - early universe

Kogut-Sinclair consistent with χ SB phase at T=0
relevance in early cosmology

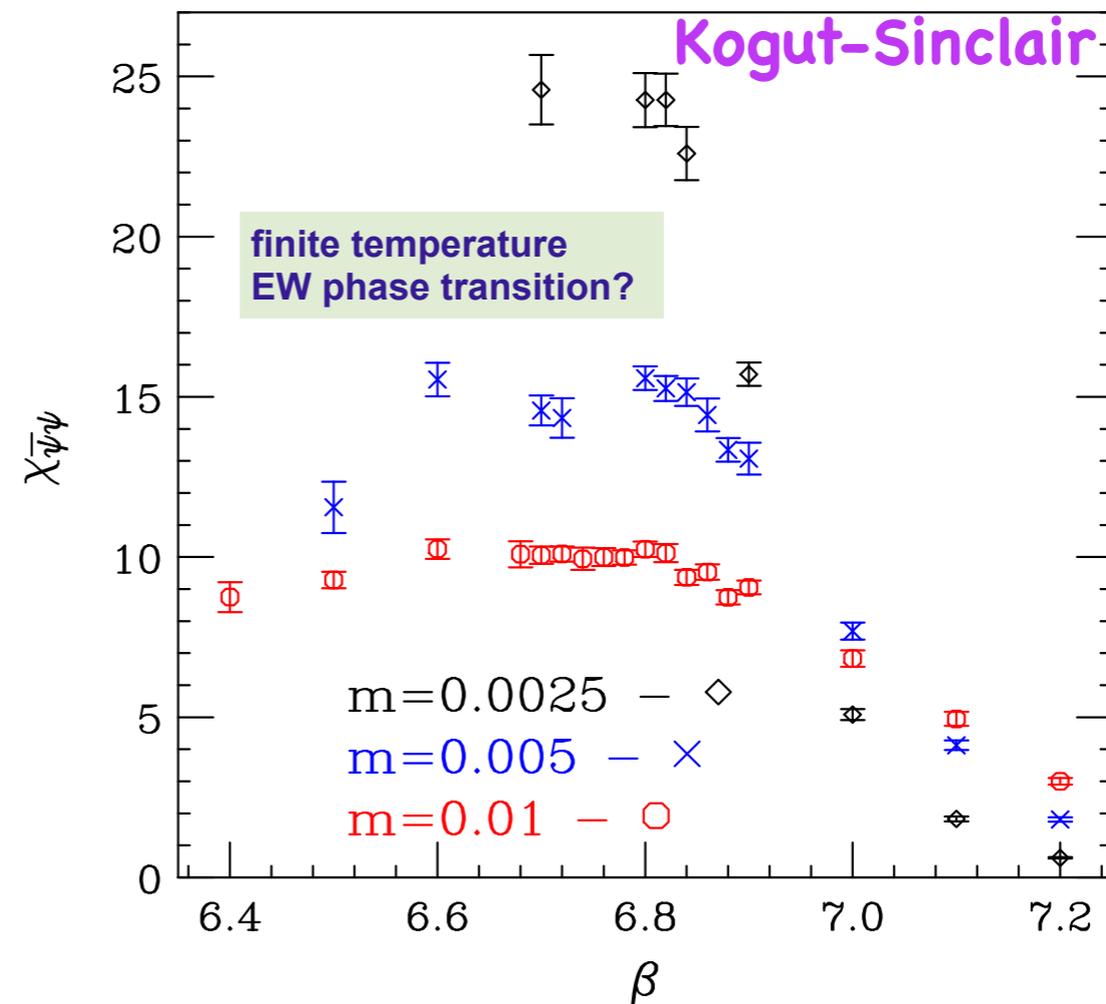
We are planning to run sextet thermodynamics

Third massive fermion flavor (electroweak singlet) dark matter?

$16^3 \times 8$ lattice



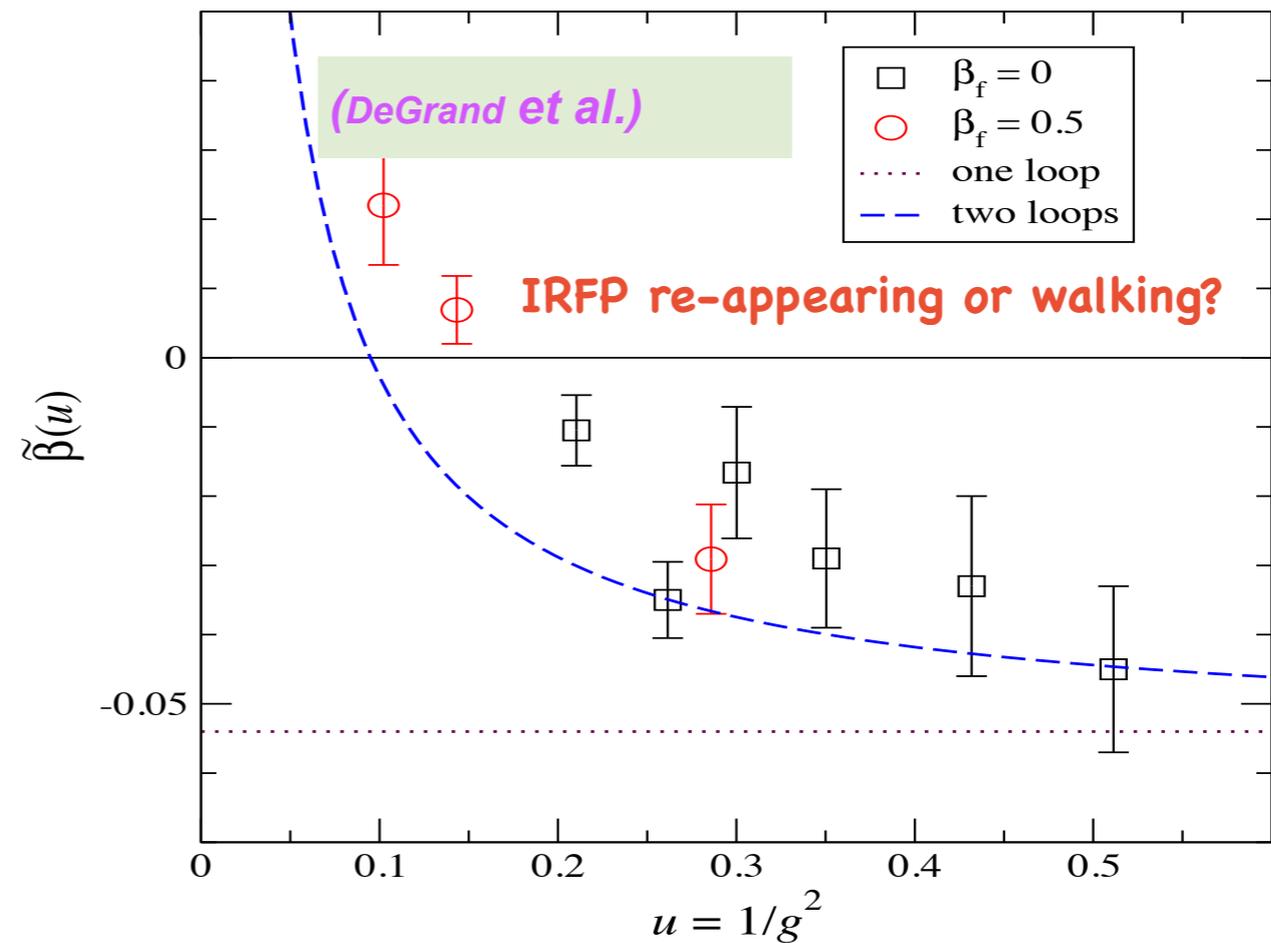
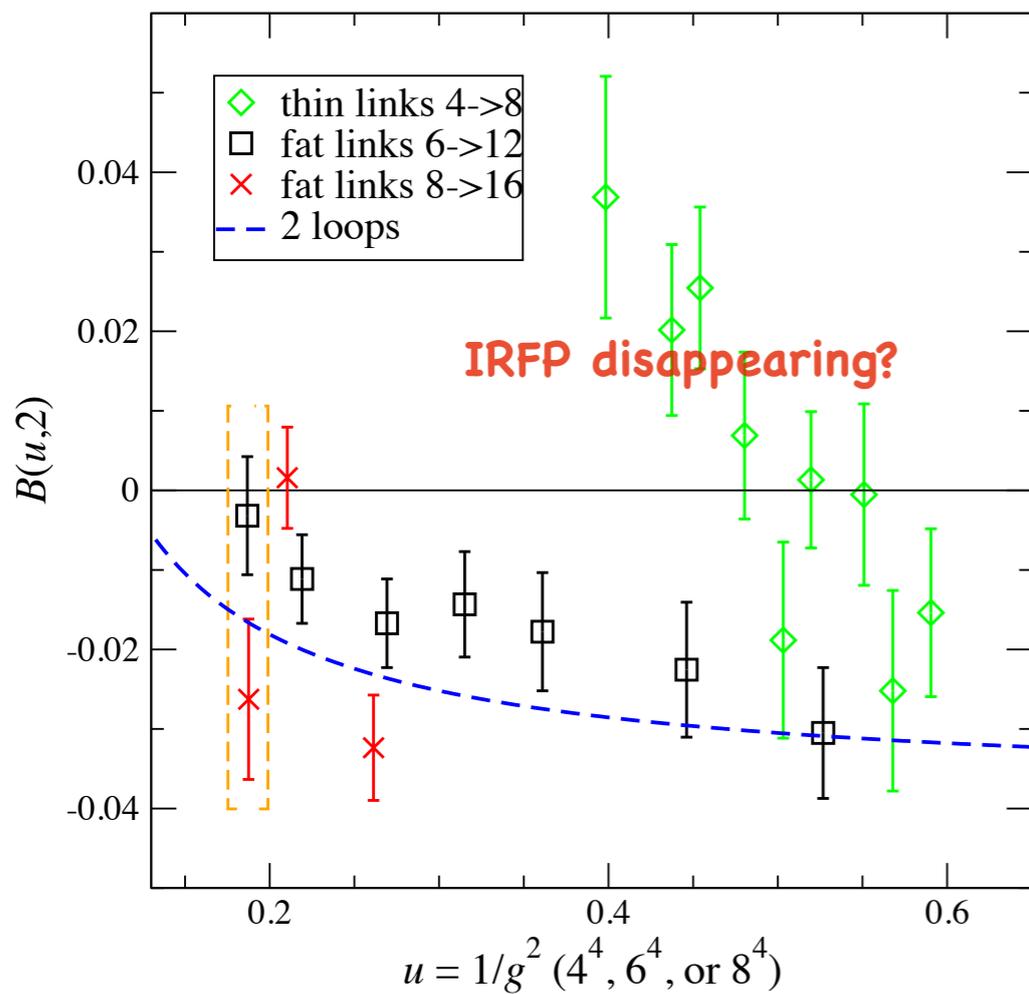
$24^3 \times 12$ lattice



running (walking?) gauge coupling

DeGrand et al. find: $N_f=2$ sextet beta function may have an IRFP zero or walks?
 model has small anomalous dimension ?

$\gamma(\mu) < 0.45$ controversy, if conformal; if χ^{SB} what is $\gamma(\mu)$?



Running coupling definition from gauge field gradient flow

$$\langle E(t) \rangle = \frac{3}{4\pi t^2} \alpha(q) \{ 1 + k_1 \alpha(q) + O(\alpha^2) \}, \quad q = \frac{1}{\sqrt{8t}}, \quad k_1 = 1.0978 + 0.0075 \times N_f$$

t is the gradient flow time
Running coupling definition (range is $(8t)^{1/2}$):

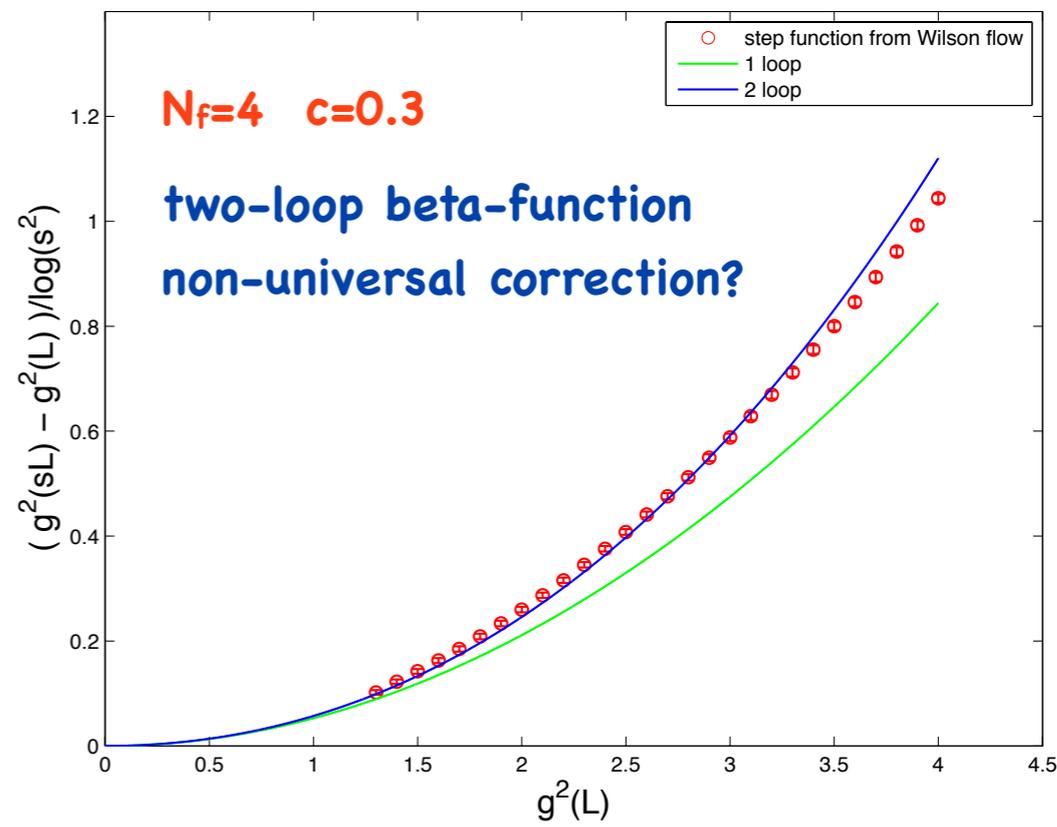
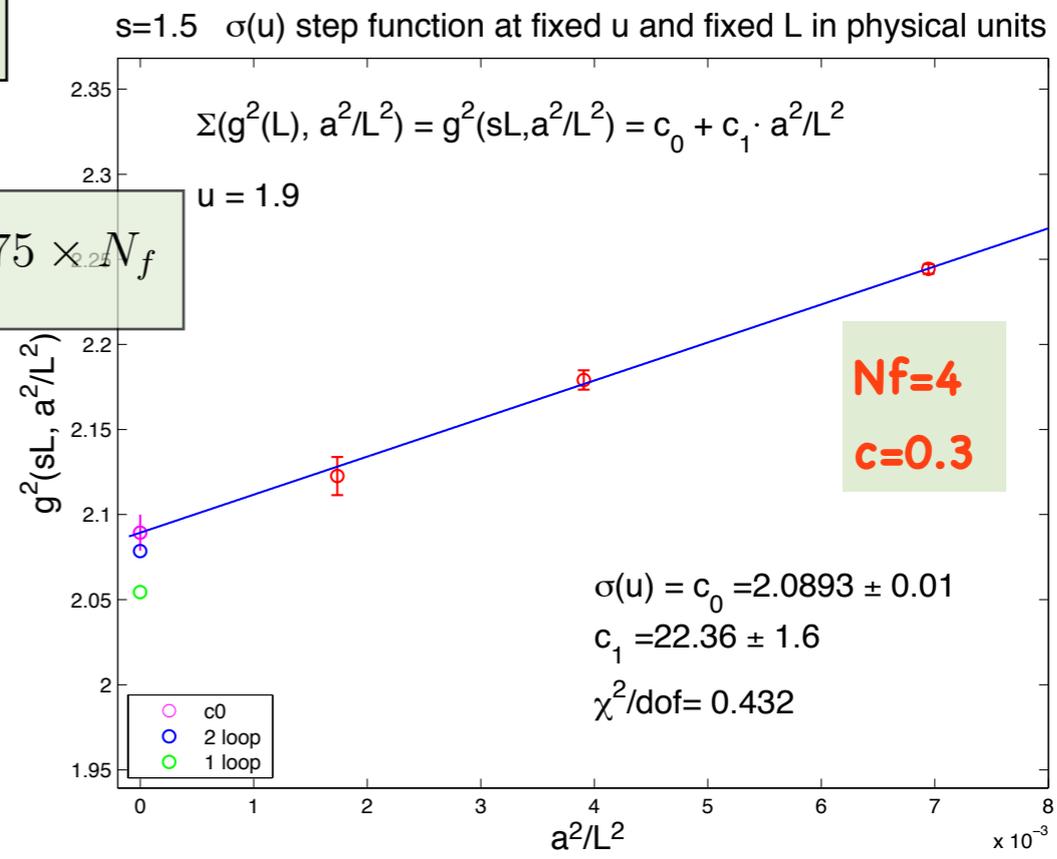
while holding $c = (8t)^{1/2}/L$ fixed: $\alpha_c(L) = \frac{4\pi}{3} \frac{\langle t^2 E(t) \rangle}{1 + \delta(c)}$

$$\delta(c) = \mathcal{V}_3^4(e^{-1/c^2}) - 1 - \frac{c^4 \pi^2}{3}$$

3rd Jacobi function

massless fermions; antiperiodic all directions $s=1.5$ step
 $N_f=4$ staggered fermions; 4-stout; $L=12-18; 16-24; 24-36$

run production now in sextet model



Summary and Outlook

Consistency with χ_{SB} in $N_f=2$ $SU(3)$ sextet model

Inconsistency with conformal symmetry and IRFP in all $L=\infty$ like tests

Results of DeGrand et al. reconciled if walking coupling

Scalar spectrum from disconnected correlator is highest priority

S-parameter and size of anomalous dimension remain unresolved

("effective" γ in 1-2 range)

Electroweak phase transition?

