(More) Flavor Physics from Fermilab and MILC

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(MILC & Fermilab Lattice/MILC Collaborations)

New Frontiers in Lattice Gauge Theory
Galileo Galilei Institute, Florence
September 21, 2012

Possible Outline

- ✦ Claude's talk focused mainly on results that are usually considered "Standard Model" quantities:
  - leptonic decay constants (heavy-light, light-light)
  - heavy-light meson mixing
    - final results so far only for SM operator O_1 (actually ratio ξ)
    - BSM operators in progress
- ✦ He also prepared slides on two topics he did not get to:
  - $K \to \pi\ \ell\ \nu$
  - Electromagnetic effects on π, K masses
- ✦ He said I will talk about more "BSMy" quantities:
  - E.g., $B \to K \ell \ell$; $B \to D\, \tau\, \nu$;
    semileptonic ratio $(B_s \to D_s)/(B \to D)$ for $B_s \to \mu^+ \mu^-$; ...

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E&M Effects on Masses of π, K

- Disentangling electromagnetic and isospin-violating effects in the pions and kaons is long-standing issue.
- Crucial for determining light quark masses.
  - Fundamental parameters in Standard Model; important for phenomenology.
  - Size of EM contributions is largest uncertainty in determination of m_u/m_d.

	m_u [MeV]	m_d [MeV]	m_u/m_d
value	1.9	4.6	0.42
statistics	0.0	0.0	0.00
lattice syst.	0.1	0.2	0.01
perturbative	0.1	0.2	--
EM	0.1	0.1	0.04

MILC, RMP **82**, 1349 (2010), arXiv:0903.3598

  - Reduce error by calculating EM effects on the lattice.

E&M: Background

- ✦ EM error in m_u/m_d dominated by error in $(M^2_{K^+} - M^2_{K^0})^\gamma$, where γ indicates the EM contribution.
- ✦ Dashen (1960) showed that EM splittings same for K and π (to "leading order in chiral expansion").

$$(M^2_{K^+} - M^2_{K^0})^\gamma = (M^2_{\pi^+} - M^2_{\pi^0})^\gamma$$

- ✦ Parameterize higher order effects ("corrections to Dashen's theorem") by

$$(M^2_{K^+} - M^2_{K^0})^\gamma = (1+\epsilon)(M^2_{\pi^+} - M^2_{\pi^0})^\gamma$$

  - Note: ϵ not exactly same as quantity defined by FLAG (Colangelo, et al., arXiv:1011.4408), which uses experimental pion splittings. But EM splitting ≈ experimental splitting, since isospin violations in pions small. So difference negligible for us at this stage.

E&M: Background

- MILC calculations of m_u/m_d after 2004 assumed $\epsilon = 1.2(5)$.
  - Came from estimate by Donoghue of range of continuum phenomenology, based on: Bijnens and Prades, NPB 490 (1997) 239; Donoghue and Perez, PRD 55 (1997) 7075; B. Moussallam, NPB 504 (1997) 381.
- This now seems too large; FLAG (Colangelo, *et al*., arXiv:1011.4408) quote $\epsilon = 0.7(5)$, based largely on $\eta \rightarrow 3\pi$ decay (but also lattice results by several groups).
- Would like to improve on this value with direct lattice calculation of EM effects.
- Fortunately, Bijnens & Danielsson, PRD75 (2007) 014505 showed that EM contributions to (mass)^2 differences are calculable through NLO in SU(3) χPT with *quenched* photons (and full QCD).

MILC EM Project

- ✦ We have been accumulating a library of dynamical QCD plus quenched EM.
  - Improved staggered ("Asqtad") ensembles:
    - 2+1 flavors.
    - 0.12 fm ≥ a ≥ 0.06 fm.
    - ~1000-2000 configs for most ensembles.
    - valence quark charges 1, 2, or 3 × physical charges:
      - ✦ ±2/3e, ±4/3e, ±2e for u-like quarks.
      - ✦ ±1/3e, ±2/3e, ±e for d-like quarks.
  - Progress has been reported previously: PoS(LATTICE 2008)127, PoS(Lattice 2010)084, PoS(Lattice 2010)127.

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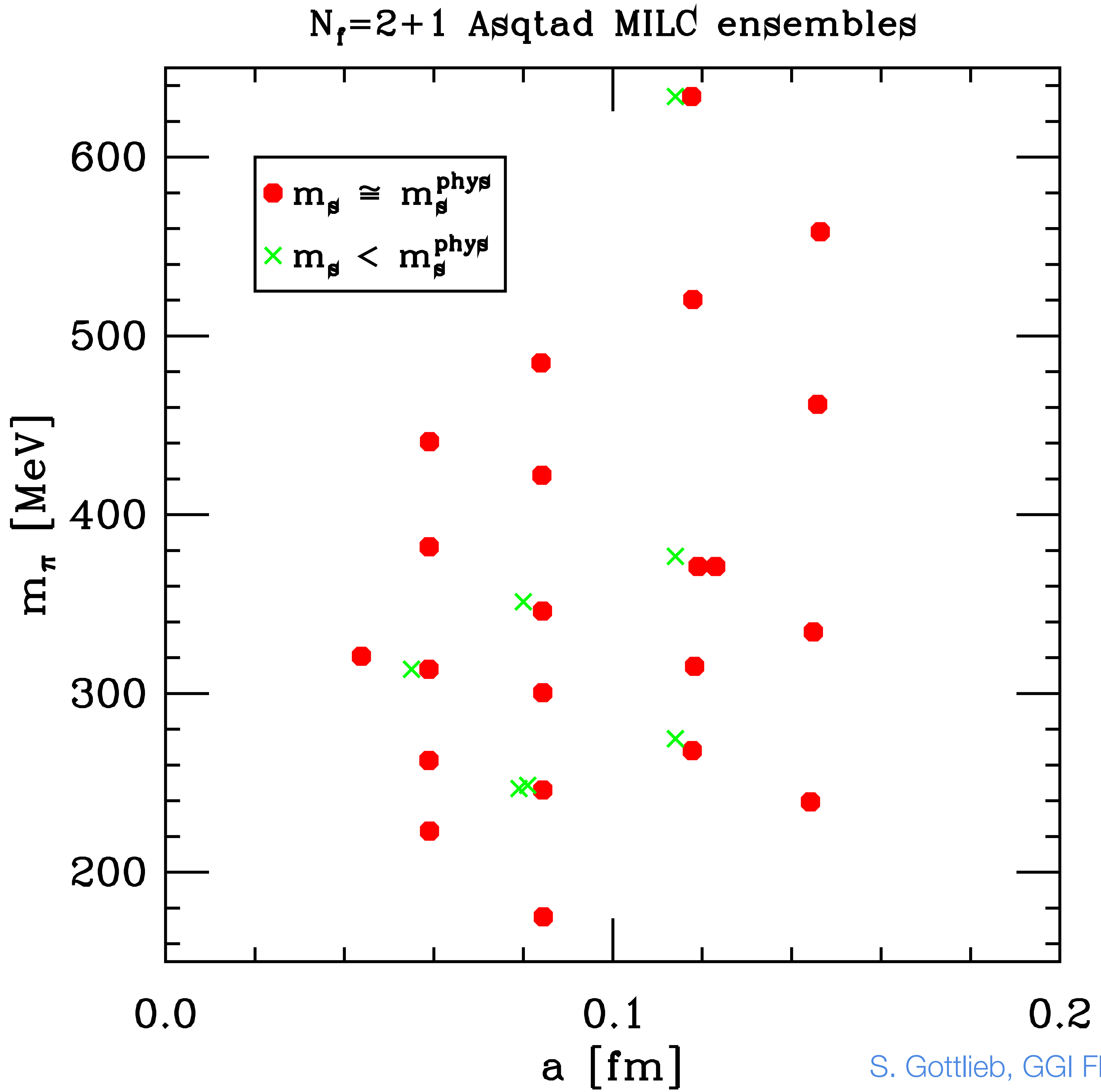
MILC
C. Bernard, L. Levkova, SG
[S. Basak, A. Torok]

Asqtad Ensembles



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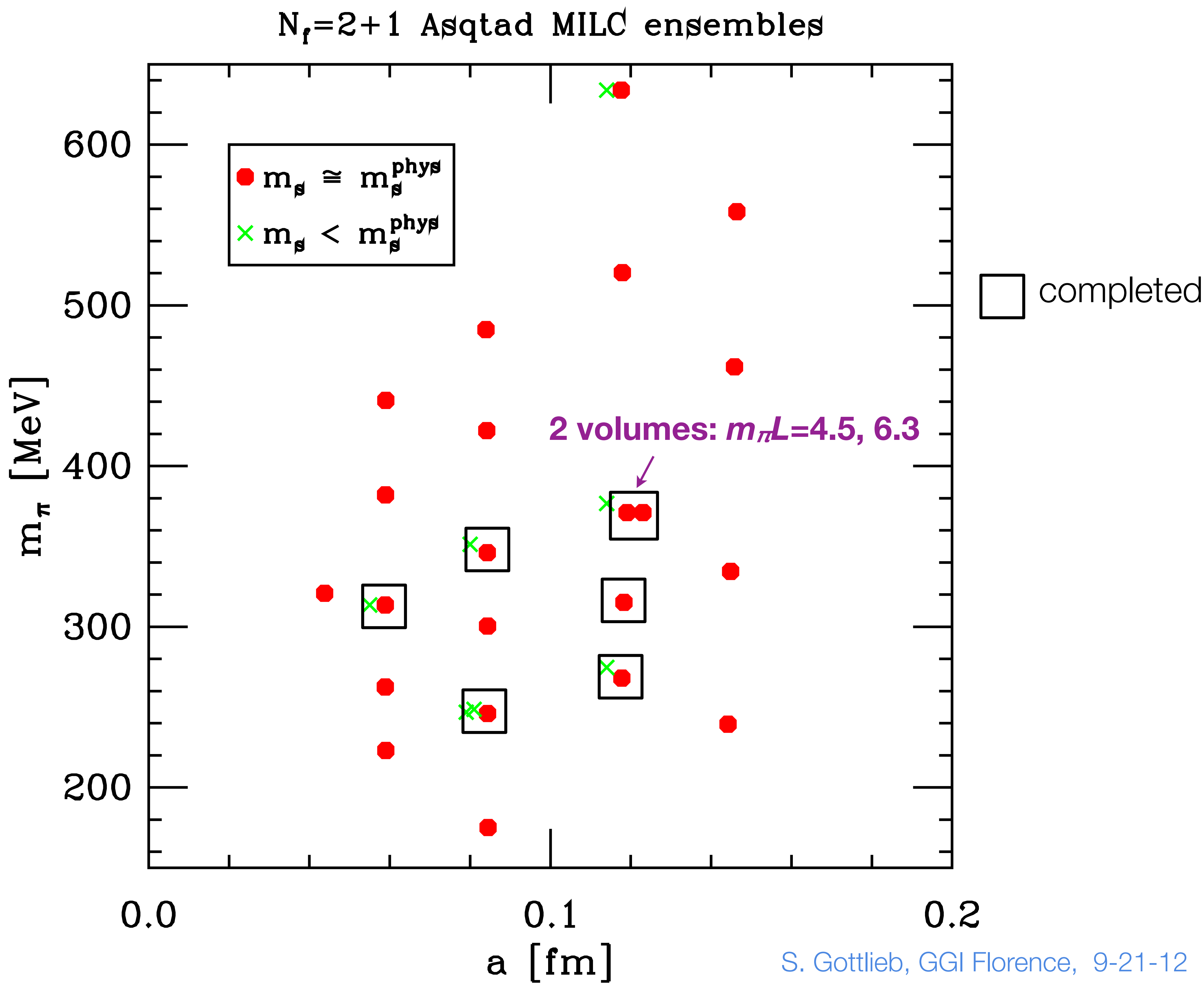

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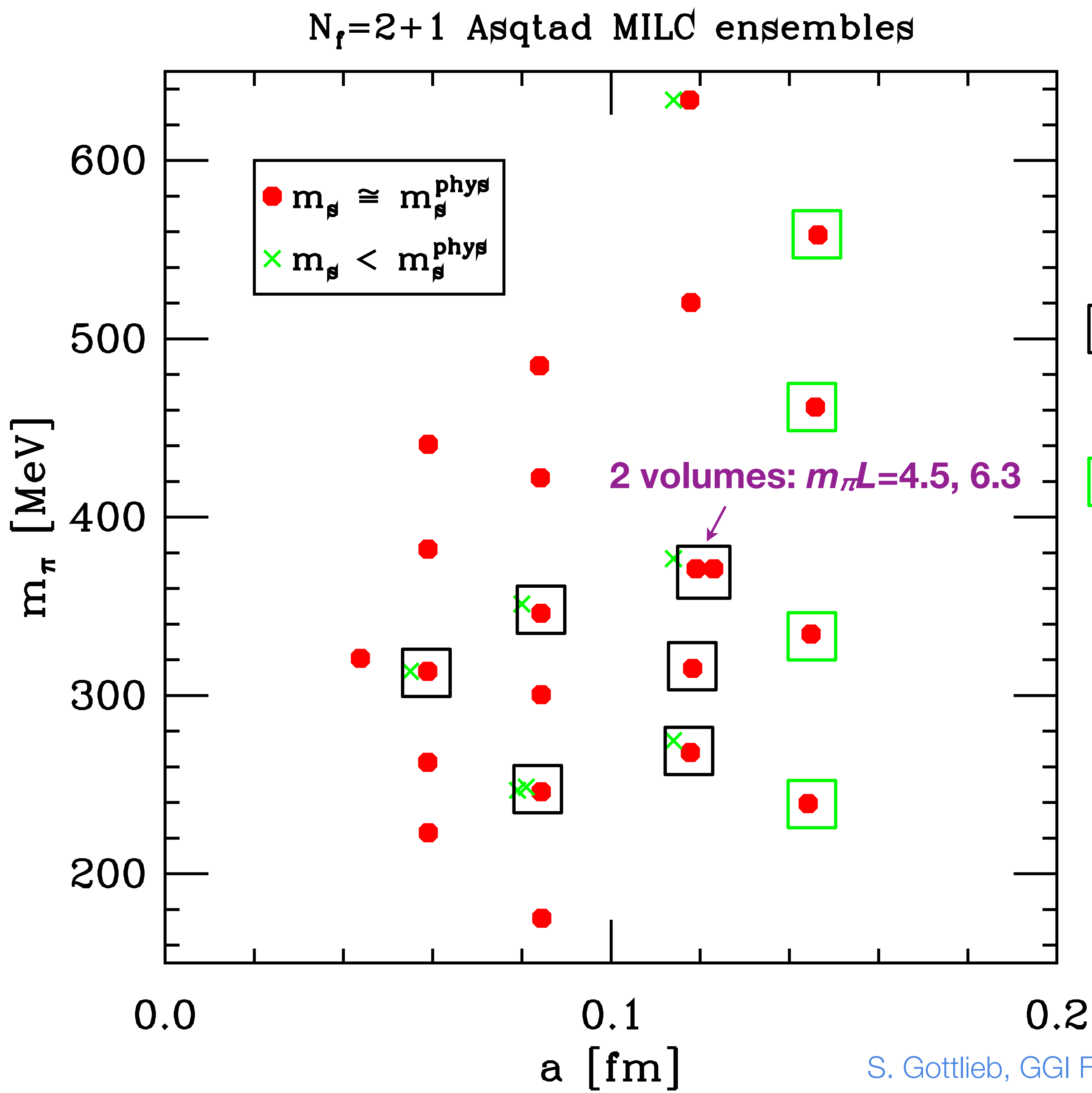
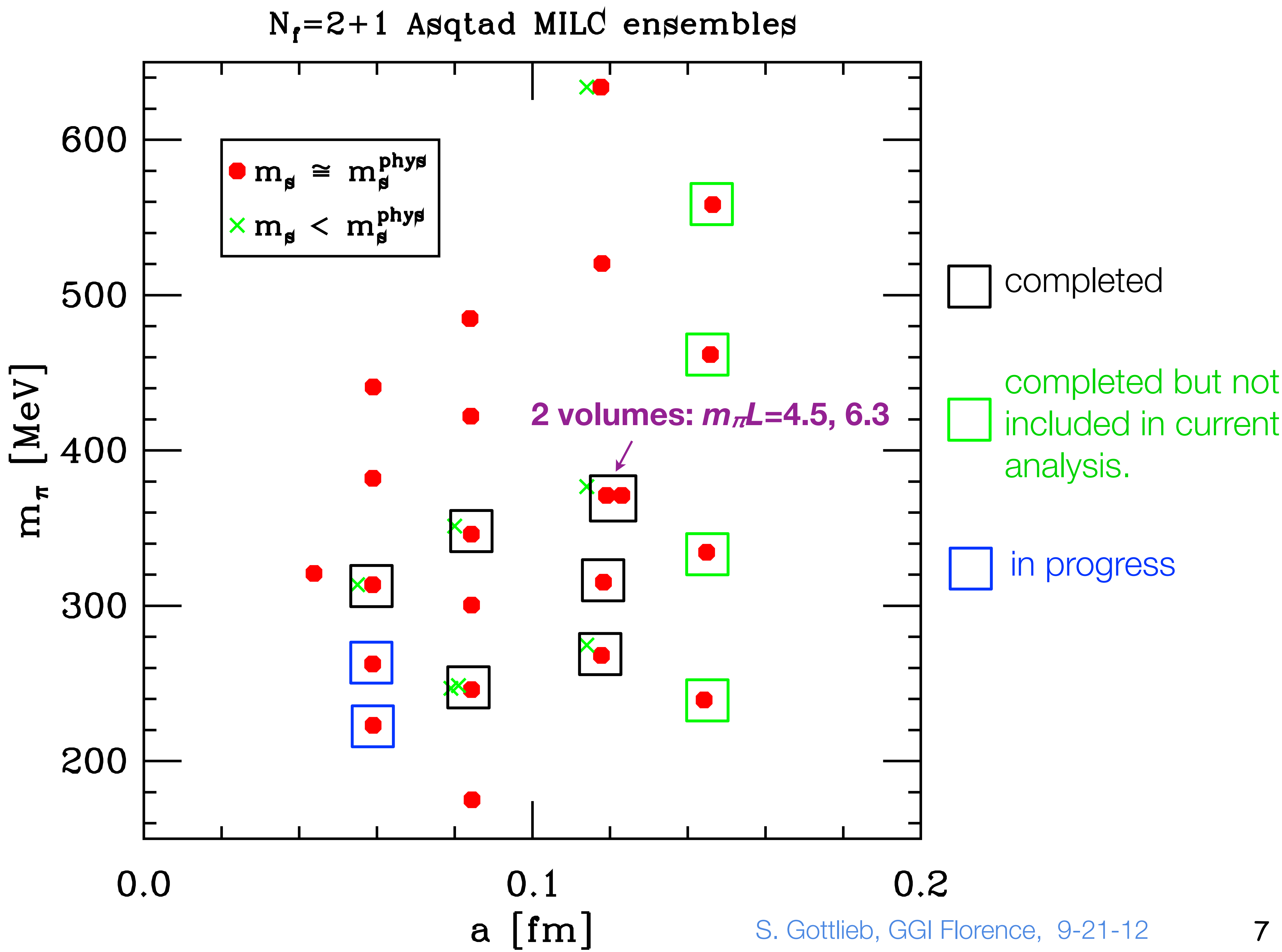


completed

completed but not included in current analysis.

Asqtad Ensembles



Some Definitions

- ✦ Lattice data includes many partially quenched points.
  - valence quarks called x and y, with charges q_x and q_y.
    - [Always talk of quark charges, not antiquark ones. A neutral meson has $q_x = q_y$.]
  - sea quarks are u, d, s.
    - Sea charges vanish in simulation, but physical charges can be restored at NLO in SU(3) χPT for (mass)^2 differences
      - i.e., difference with same valence masses, different valence charges
    - Other quantities may also be calculated, but they have an uncontrolled *electromagnetic* quenching error.

Chiral Perturbation Theory

✦ Staggered version of NLO SU(3) χPT has been calculated (C.B. & Freeland, arXiv:1011.3994):

$$\begin{aligned}\Delta M^2_{xy,5} \;=\;& q^2_{xy}\delta_{EM} - \frac{1}{16\pi^2}e^2 q^2_{xy} M^2_{xy,5}\left[3\ln(M^2_{xy,5}/\Lambda^2_\chi) - 4\right] \\ & -\frac{2\delta_{EM}}{16\pi^2 f^2}\frac{1}{16}\sum_{\sigma,\xi}\left[q_{x\sigma}q_{xy}M^2_{x\sigma,\xi}\ln(M^2_{x\sigma,\xi}) - q_{y\sigma}q_{xy}M^2_{y\sigma,\xi}\ln(M^2_{y\sigma,\xi})\right]\end{aligned}$$

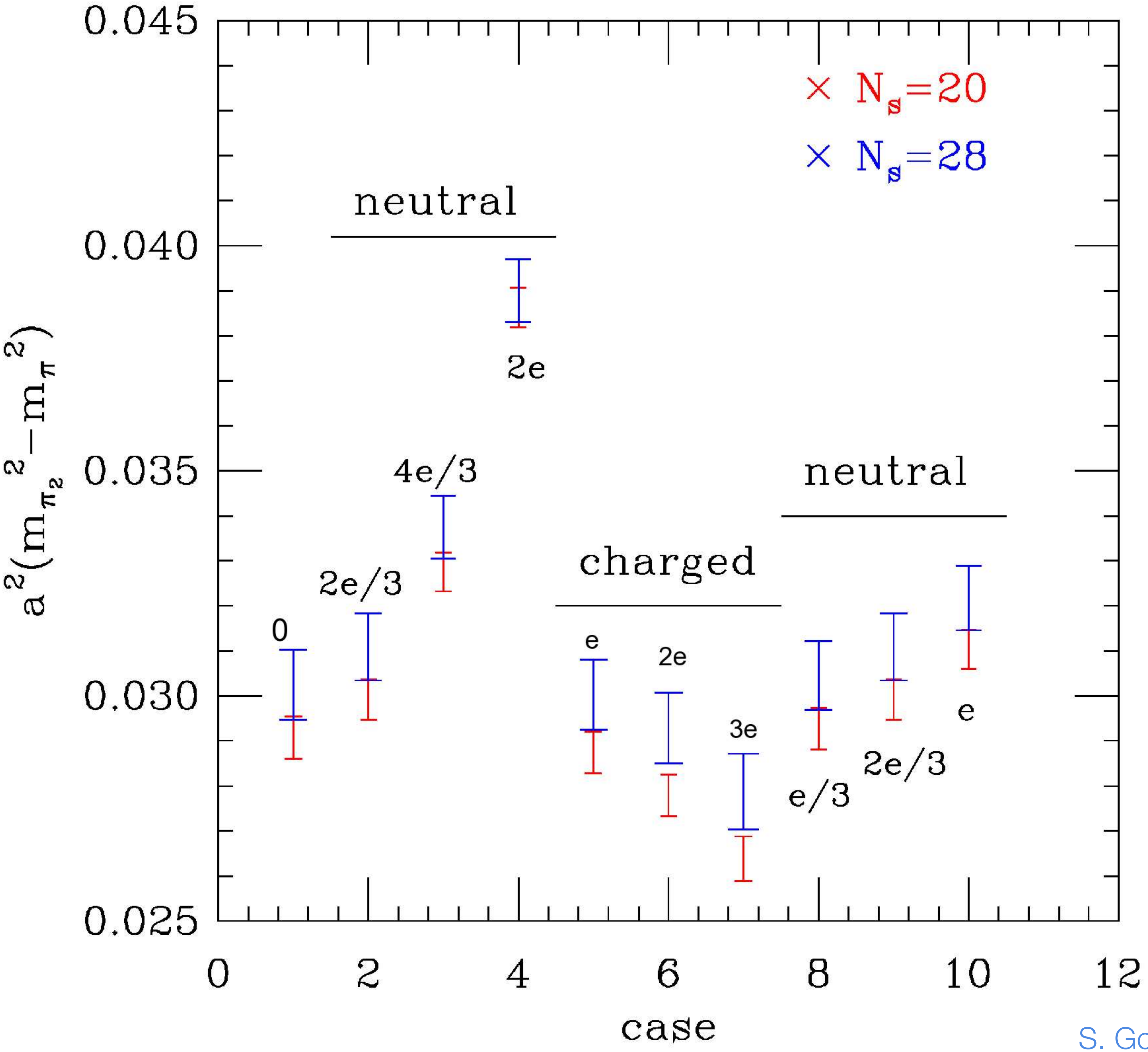
$$+c_1 q^2_{xy}a^2 + c_2 q^2_{xy}(2m_\ell + m_s) + c_3(q^2_x + q^2_y)(m_x + m_y) + c_4 q^2_{xy}(m_x + m_y) + c_5(q^2_x m_x + q^2_y m_y)$$

- x,y are the valence quarks.
- q_x, q_y are quark charges; $q_{xy} \equiv q_x - q_y$ is meson charge.
- δ_{EM} is the LO LEC; ξ is the staggered taste
- σ runs over sea quarks (m_u, m_d, m_s, with $m_u = m_d \equiv m_\ell$)

✦ Errors in $\Delta M^2_{xy} \equiv M^2_{xy}(q_x, q_y) - M^2_{xy}(0,0)$ are ~ 0.3% for charged mesons, ~1% for neutrals.

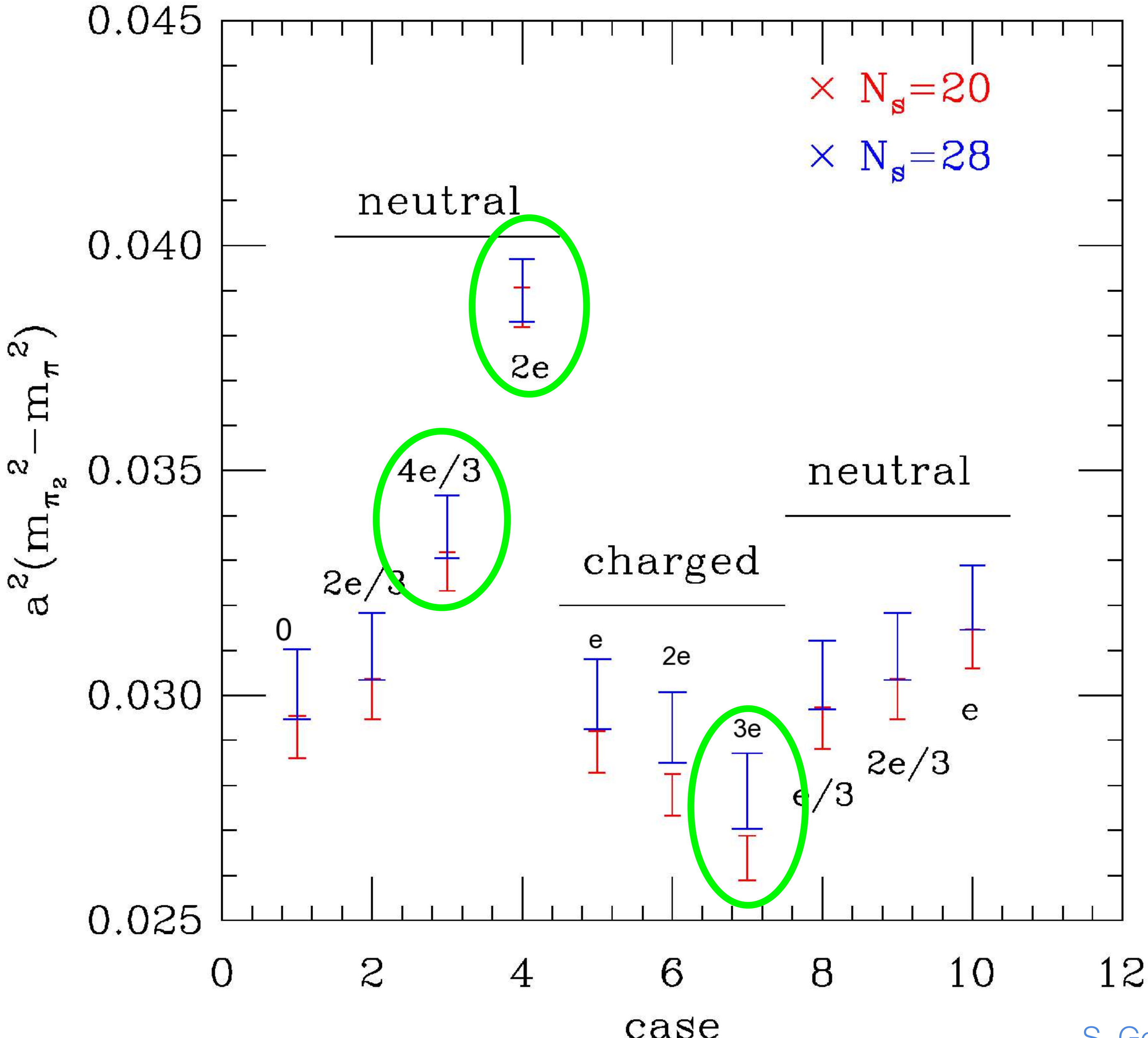
- Need NNLO: but only analytic terms are available.
- May need $O(\alpha^2)$ too.

Taste Splitting



- As charges increase, EM taste-violating effects start to become evident.

Taste Splitting



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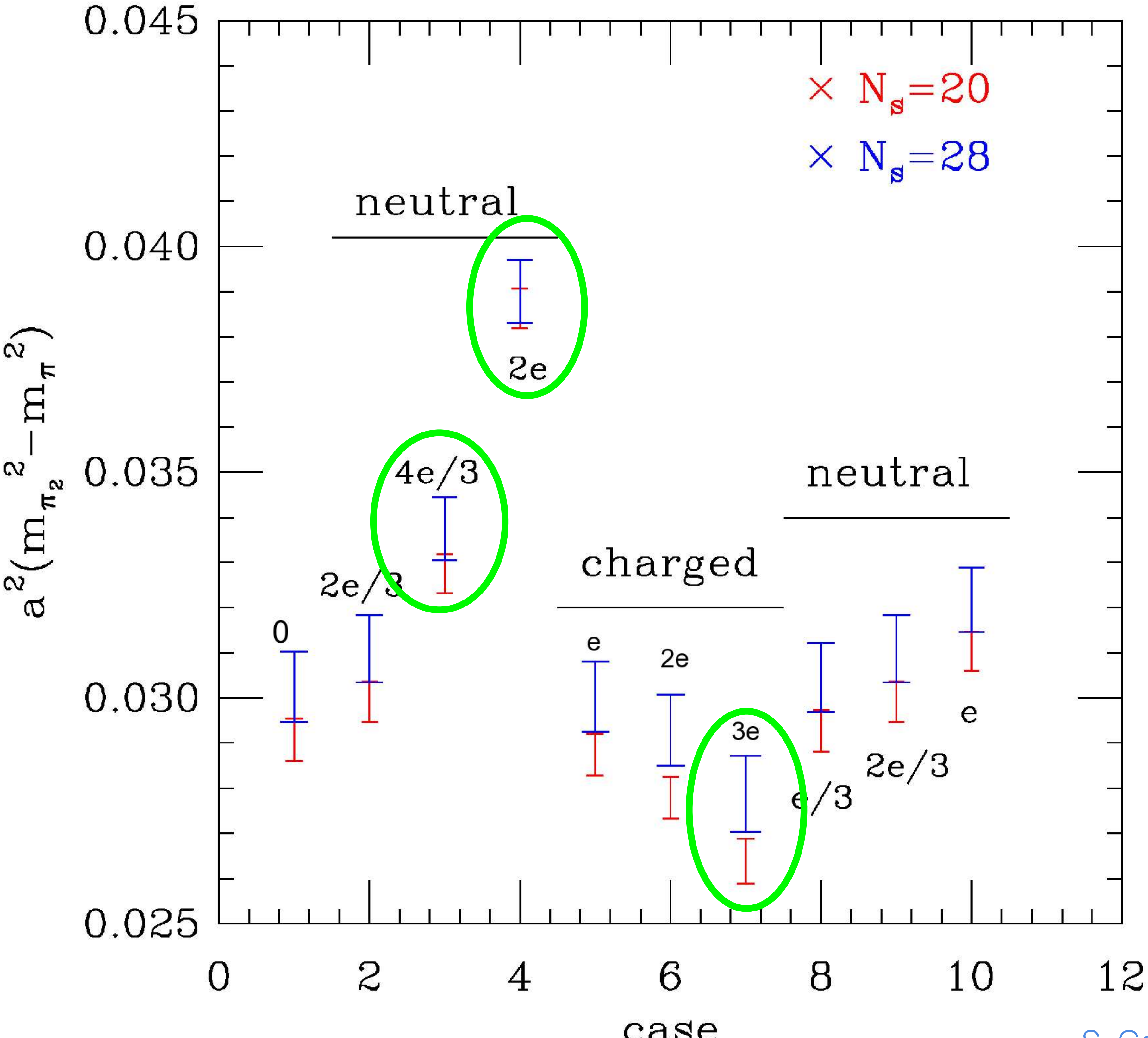

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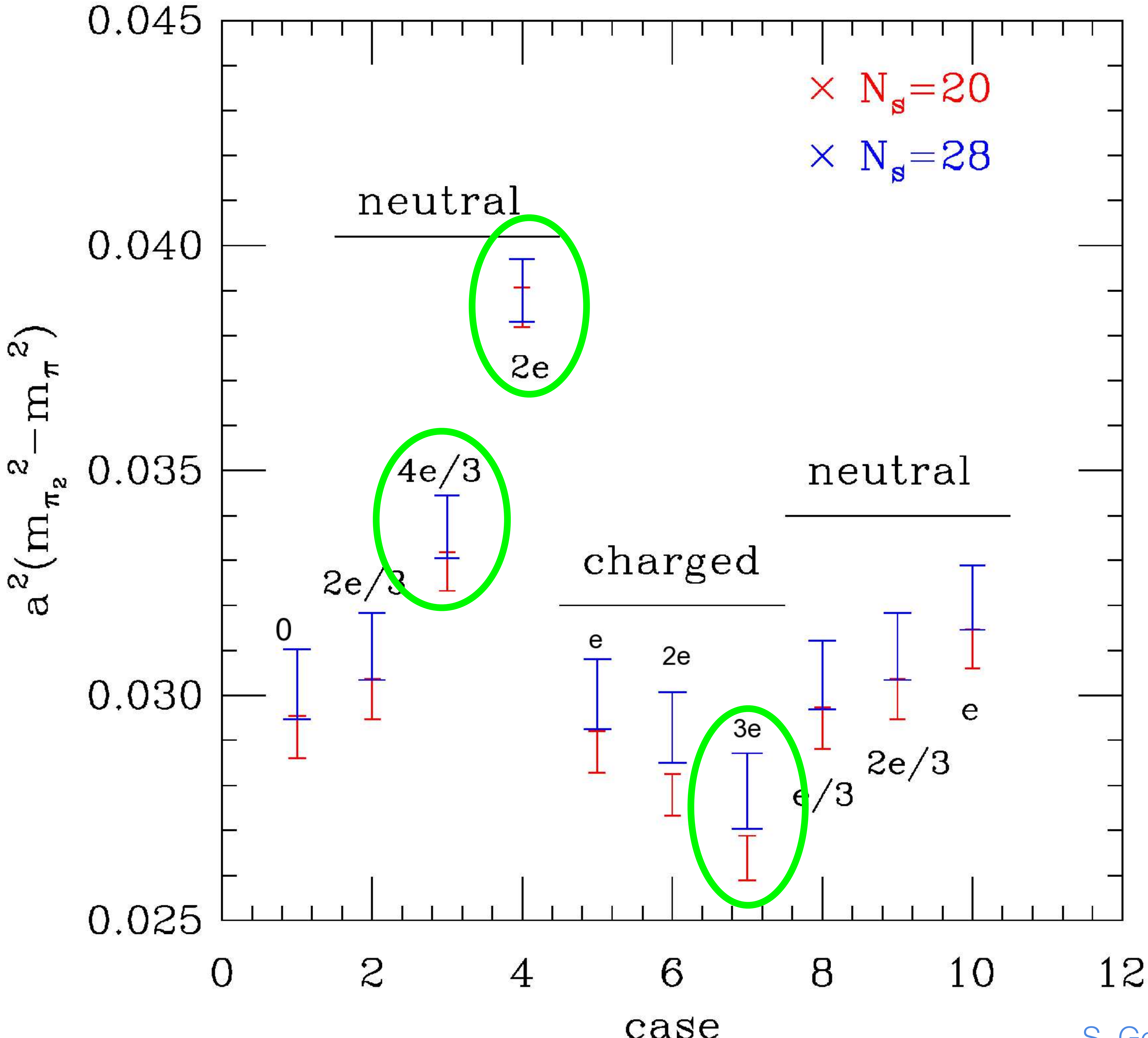
  EM taste-violations not included in the χPT.

Taste Splitting



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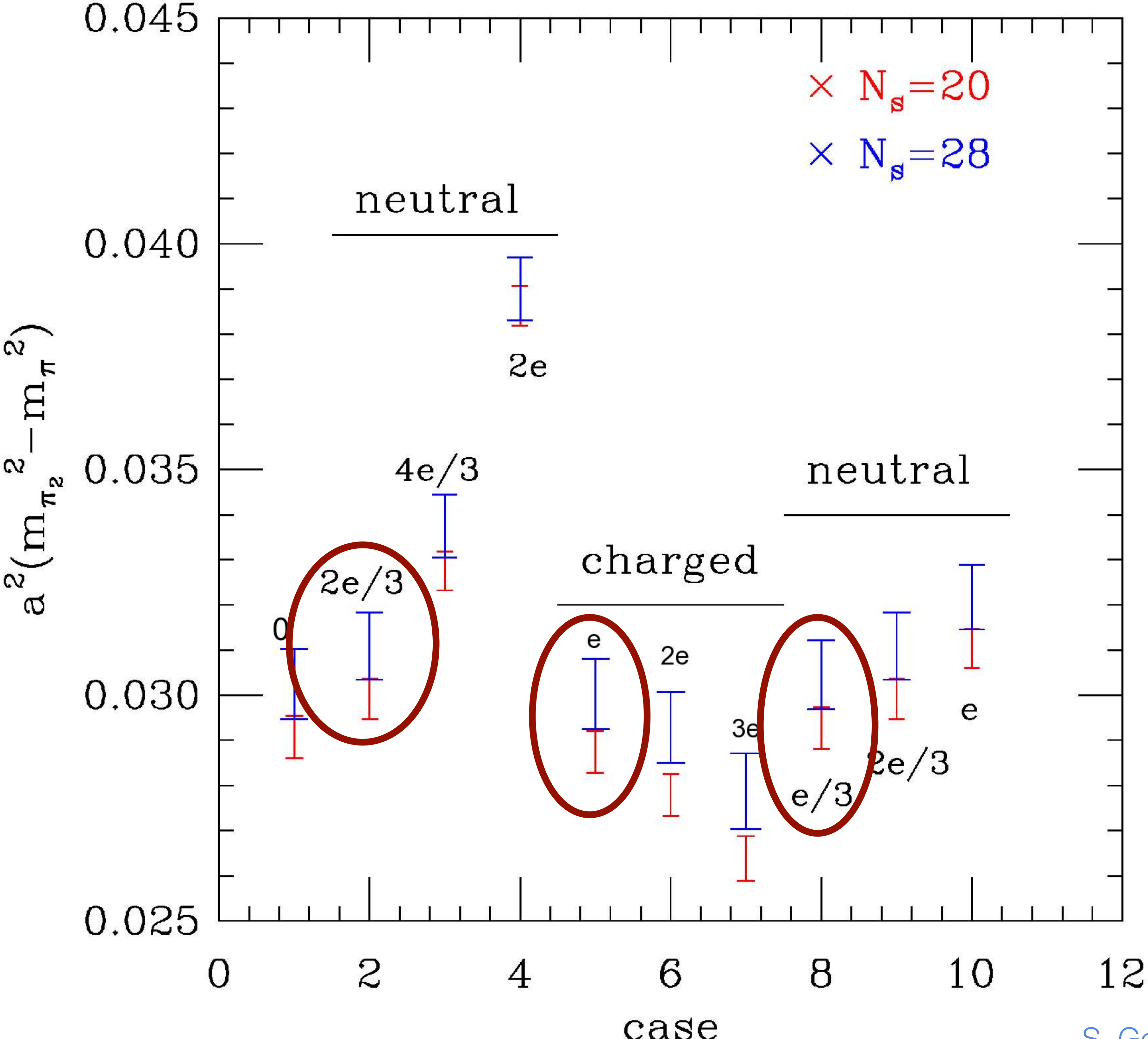


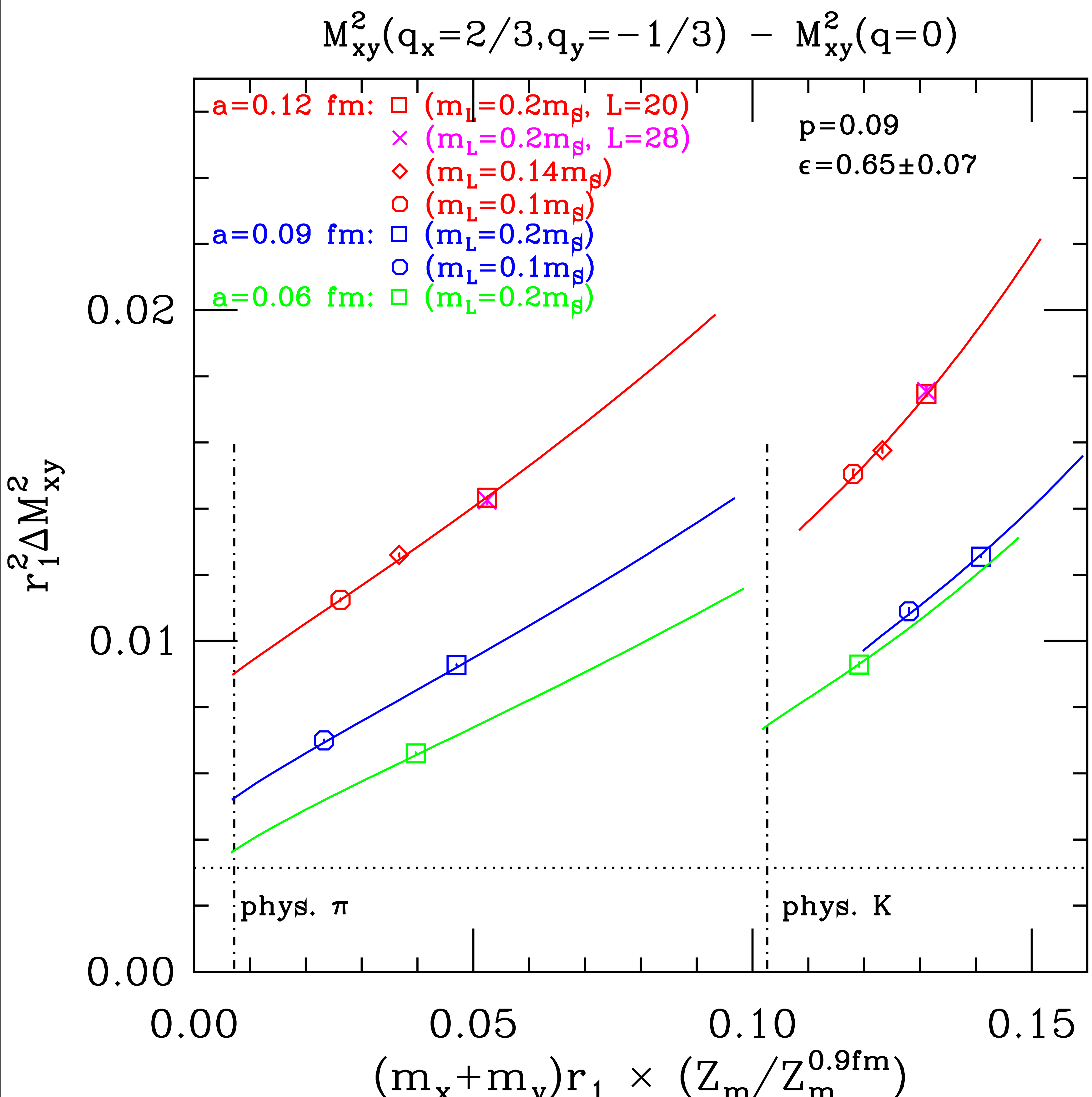
- As charges increase, EM taste-violating effects start to become evident.

EM taste-violations not included in the χPT.

But if effect stays relatively small, should be describable by α^2 analytic terms.

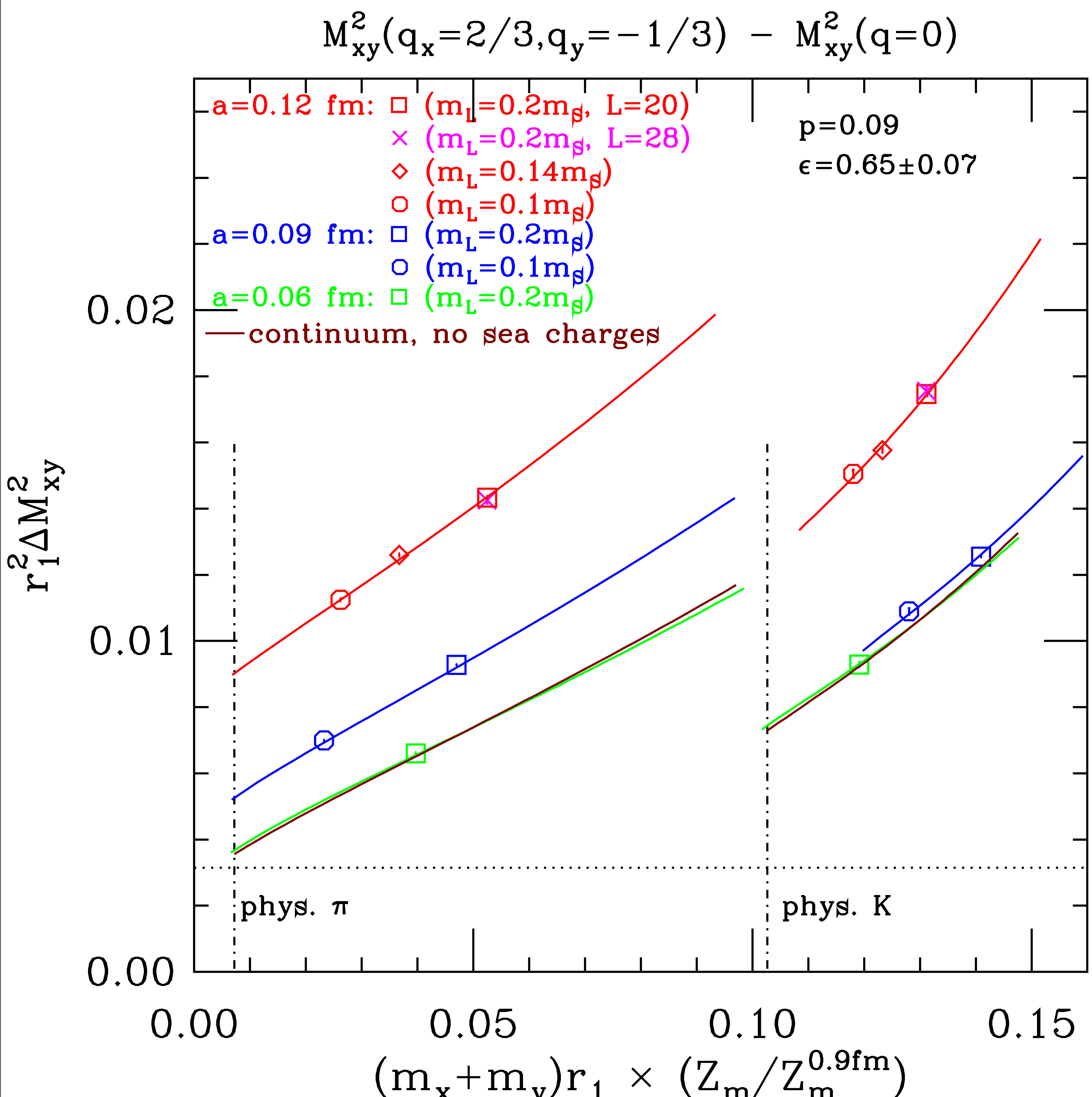
Results below use only physical charges, however.

Chiral Fit and Extrapolation



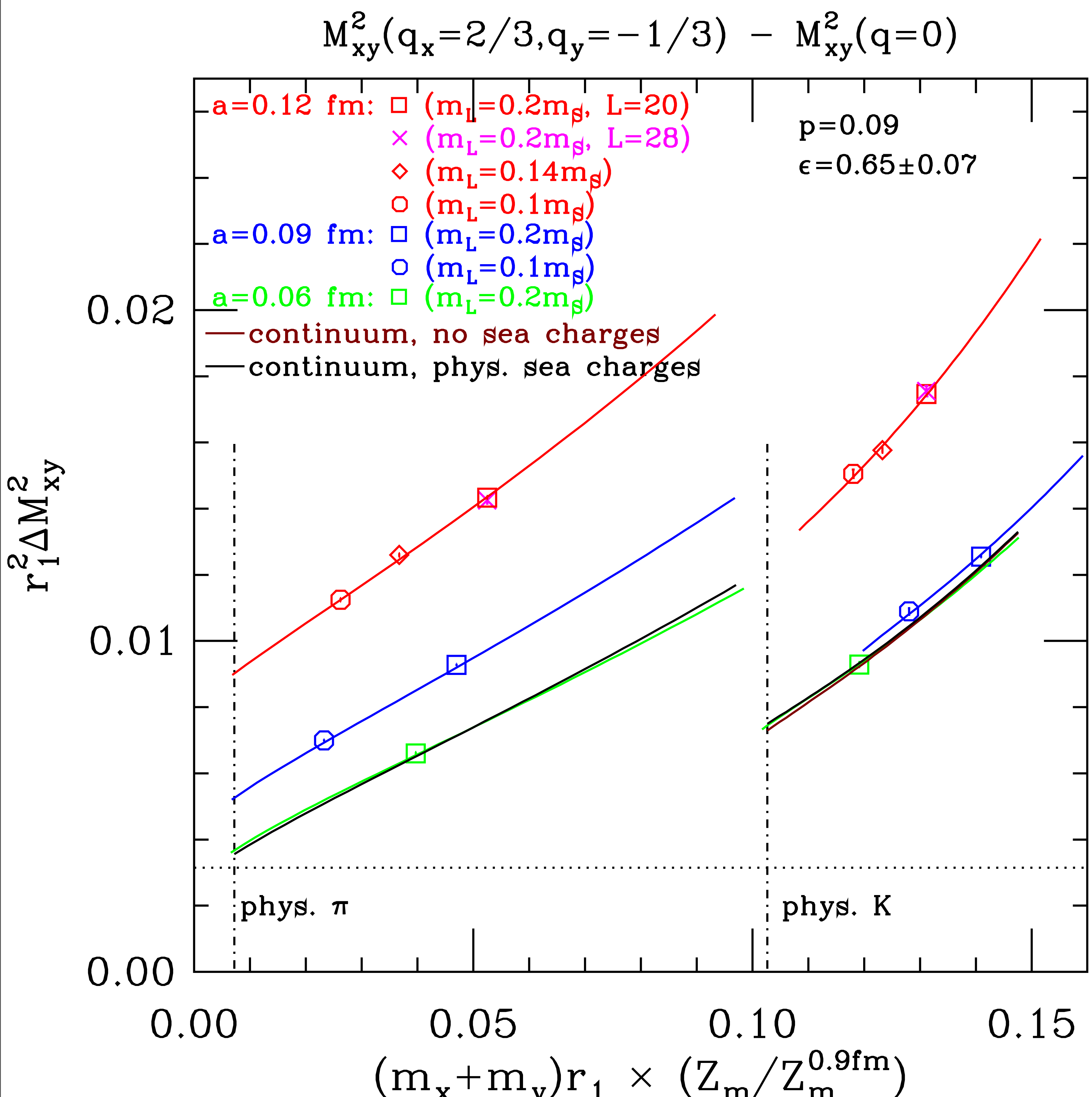
- Only unitary π^+ & K^+ shown, but fit is to all partially quenched points, charged and neutral.
- Different masses & charges for same ensembles are highly correlated, leading to nearly singular covariance matrix.
- This fit is non-covariant (neglects correlations).
- Covariant fits generally have very poor p values; a few of better ones are included in systematic error estimate.

Chiral Fit and Extrapolation



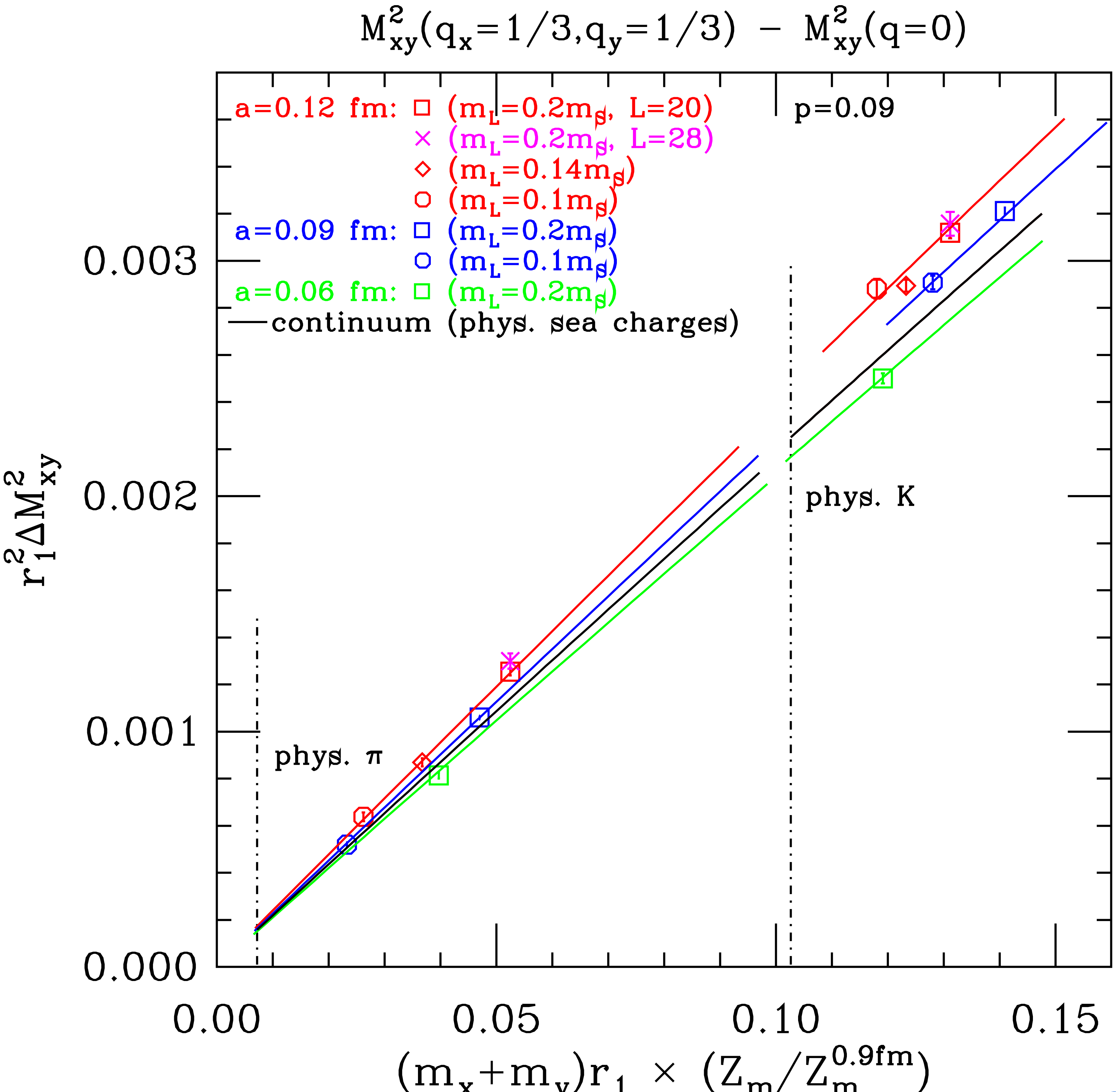
- Extrapolate to continuum, and set valence, sea masses equal.
- Adjust m_s to physical value.
- Keep sea charges = 0.

Chiral Fit and Extrapolation



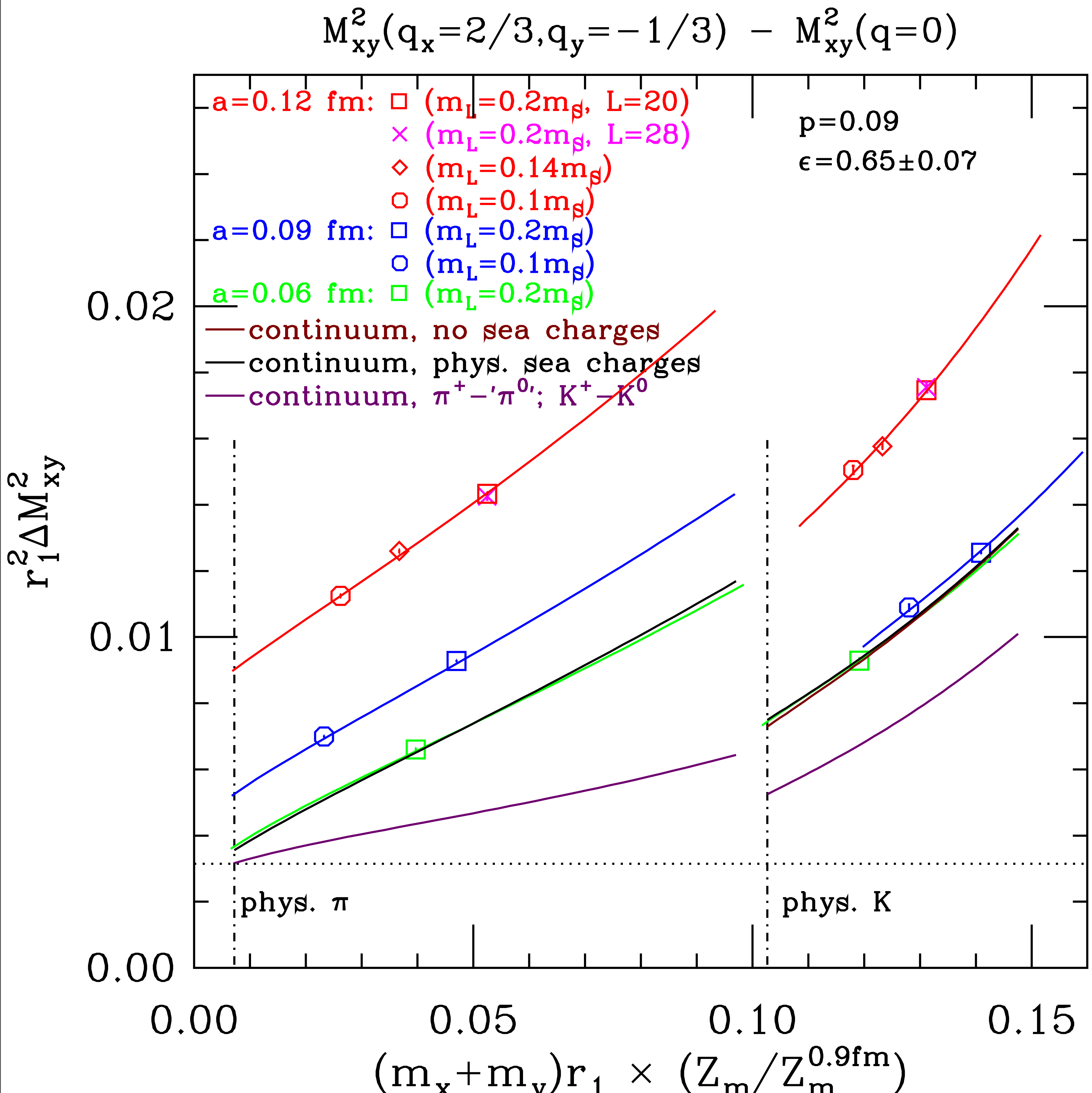
- Set sea quark charges to their physical values, using NLO chiral logs.
- Difference with previous case is very small for kaon; vanishes identically for pion.

Chiral Fit and Extrapolation



- Neutral $d\bar{d}$-like mesons ($q_x = q_y$ =1/3) for same fit.
- Note difference in scale from charged meson plot.
- ~Function of (m_x+m_y) only (π and K line up).
- Nearly linear: chiral logs vanish for neutrals.

Chiral Fit and Extrapolation



- Now subtract neutral masses.
- Perfect agreement of π splitting with physical value is an accident:
  - systematic errors are larger than the difference of purple & black lines: difference between "π^0" and $\pi(q_x=q_y=0)$.
- Can now read off ratio of π and K splittings:

$$\epsilon = 0.65(7)$$

Preliminary Results

$$
\begin{aligned}
(M^2_{\pi^+} - M^2_{``\pi^0"})^\gamma &= 1270(90)(230)\ \mathrm{MeV}^2 \\
(M^2_{K^+} - M^2_{K^0})^\gamma &= 2100(90)(250)\ \mathrm{MeV}^2
\end{aligned}
$$

$$
\epsilon = 0.65(7)(14)
$$

$$
\left.\begin{aligned}
(M^2_{``\pi^0"})^\gamma &= 157.8(1.4)(1.7)\ \mathrm{MeV}^2 \\
(M^2_{K^0})^\gamma &= 901(8)(9)\ \mathrm{MeV}^2
\end{aligned}\right\}
$$

uncontrolled EM quenching error

- Finite volume errors not yet included: seem relatively small at present, but need to be studied more, and quantified.
- The quantity $(M^2_{``\pi^0"})^\gamma$ may give rough estimate of size of effect of neglecting disconnected EM diagrams in the "π_0".
    - Keeping that in mind, and neglecting effects of isospin violation in the π^0, $(M^2_{\pi^+} - M^2_{``\pi^0"})^\gamma$ may be compared with expt. π^+-π^0 splitting: $1261\ \mathrm{MeV}^2$.

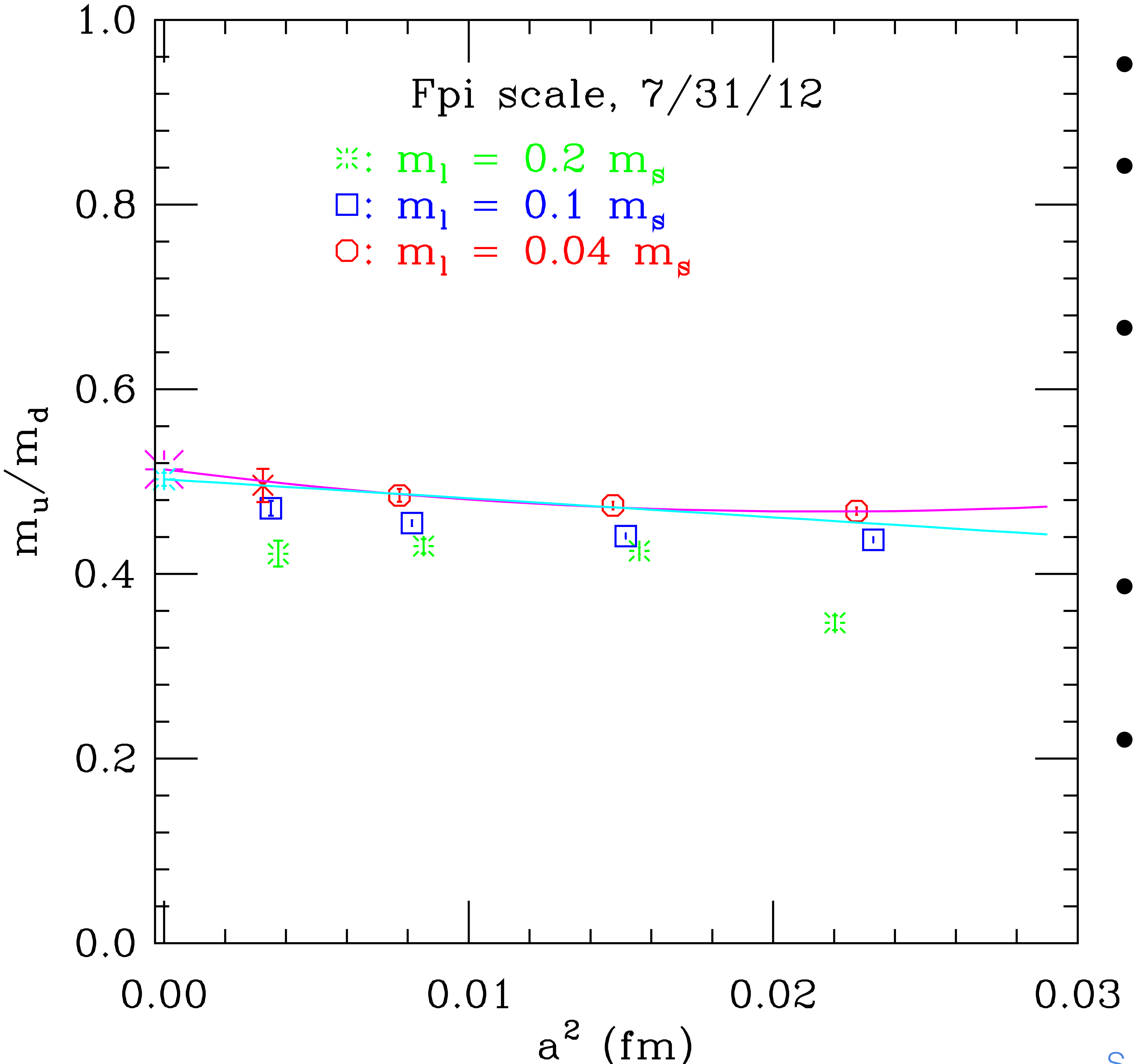
Comparison with Other Work

- ✦ ε = 0.60(14) [statistics only], Portelli et al. (2010), arXiv:1011.4189.
- ✦ ε = 0.628(59) [statistics only], Blum et al. (2010), arXiv:1006.1311.
- ✦ ε = 0.70(4)(8)(??), Portelli et al. (2012), arXiv:1201.2787.
- ✦ ε = 0.65(7)(14)(?), this work.

?? = discretization errors; ? = finite volume errors

- Good agreement between the groups.
- Errors still need work...

Preliminary Effect on m_u/m_d



- From HISQ lattices.
- Extrapolations omit $m_\ell = 0.2\ m_s$ ensembles.
- Preliminary analysis, not including staggered χPT.
  - Is upward curvature believable?
- Get:

$$m_u/m_d = 0.508(10)(22)$$

- EM error reduced by ~factor of 2 (but still the main source of error).

CKM Matrix

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ & K \to \pi l \nu & B \to \pi l \nu \\ \pi \to l \nu & K \to l \nu & B \to l \nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \pi l \nu & D \to K l \nu & B \to D^{(*)} l \nu \\ D \to l \nu & D_s \to l \nu & \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d | \bar{B}_d \rangle & \langle B_s | \bar{B}_s \rangle & \end{pmatrix}$$

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CKM Matrix

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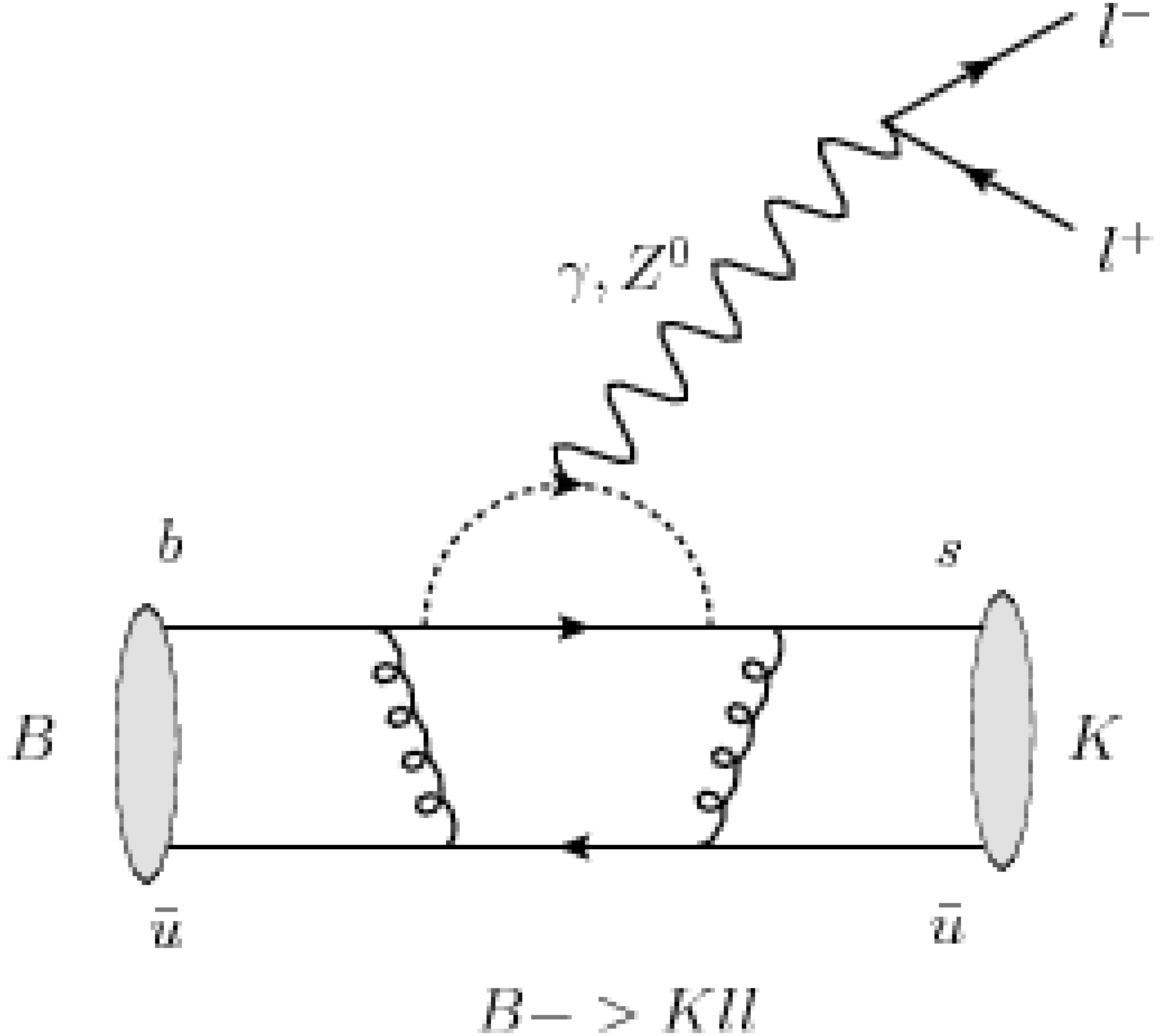
\begin{pmatrix}
\mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\
 & K \to \pi l\nu & B \to \pi l\nu \\
\pi \to l\nu & K \to l\nu & B \to l\nu \\
\mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\
D \to \pi l\nu & D \to K l\nu & B \to D^{(*)} l\nu \\
D \to l\nu & D_s \to l\nu & \\
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
\langle B_d | \bar{B}_d \rangle & \langle B_s | \bar{B}_s \rangle &
\end{pmatrix}
$$

$B \to K l l$ $\quad$ $B_s \to \mu^+ \mu^-$ $\quad$ Rare decays

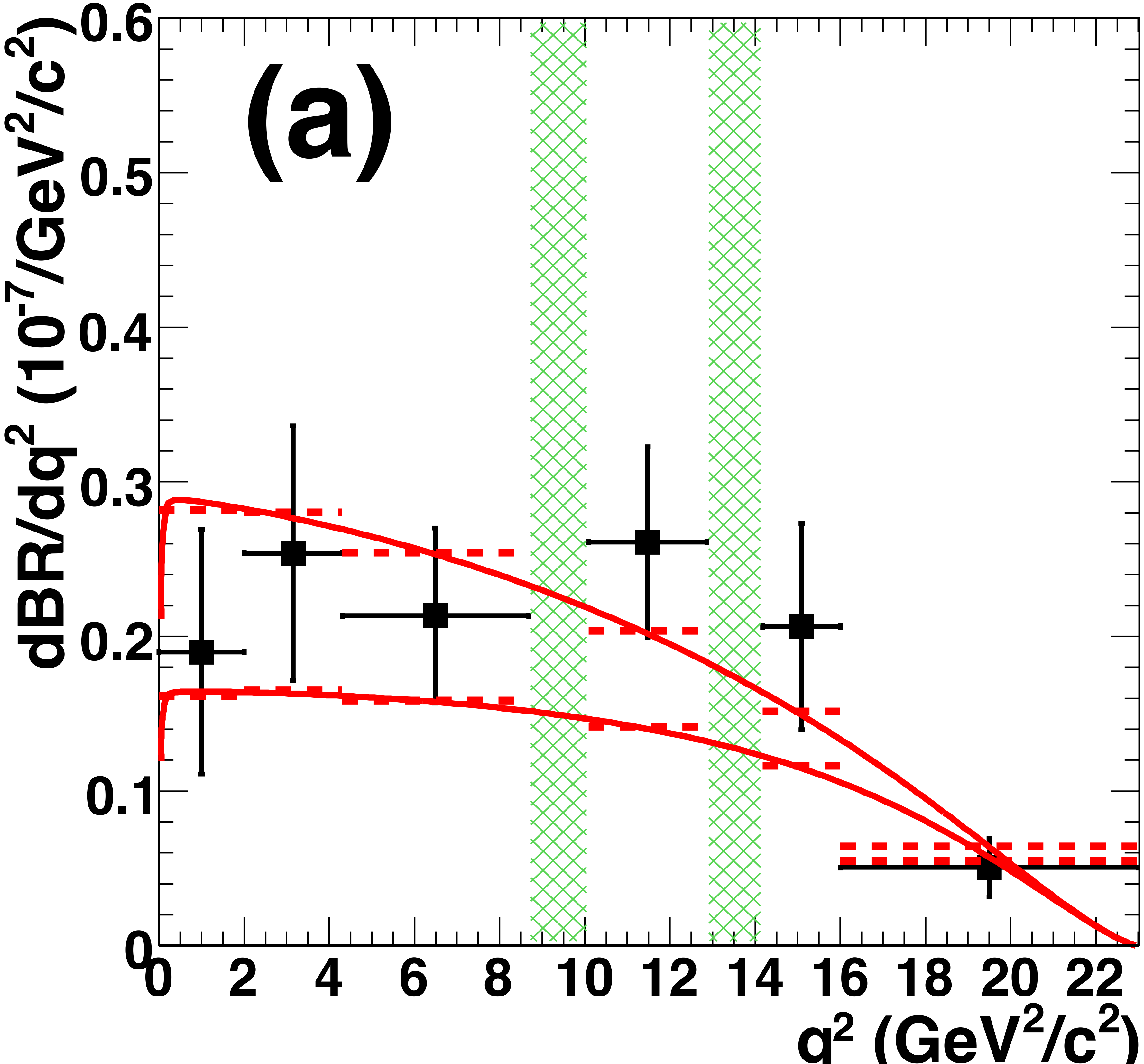
$B \to Kll$ Motivation

- $B \to Kll$ is a rare decay mediated by a flavor changing neutral current (FCNC)
- Standard model (SM) contribution occurs through penguin diagrams (b->s l l)
- Since SM contribution is small there is an opportunity to detect BSM physics
- Studied by BABAR, Belle, CDF, LHCb, etc.
- LHCb, SuperB, and SuperKEKB will improve experimental precision

Typical Penguin Diagram

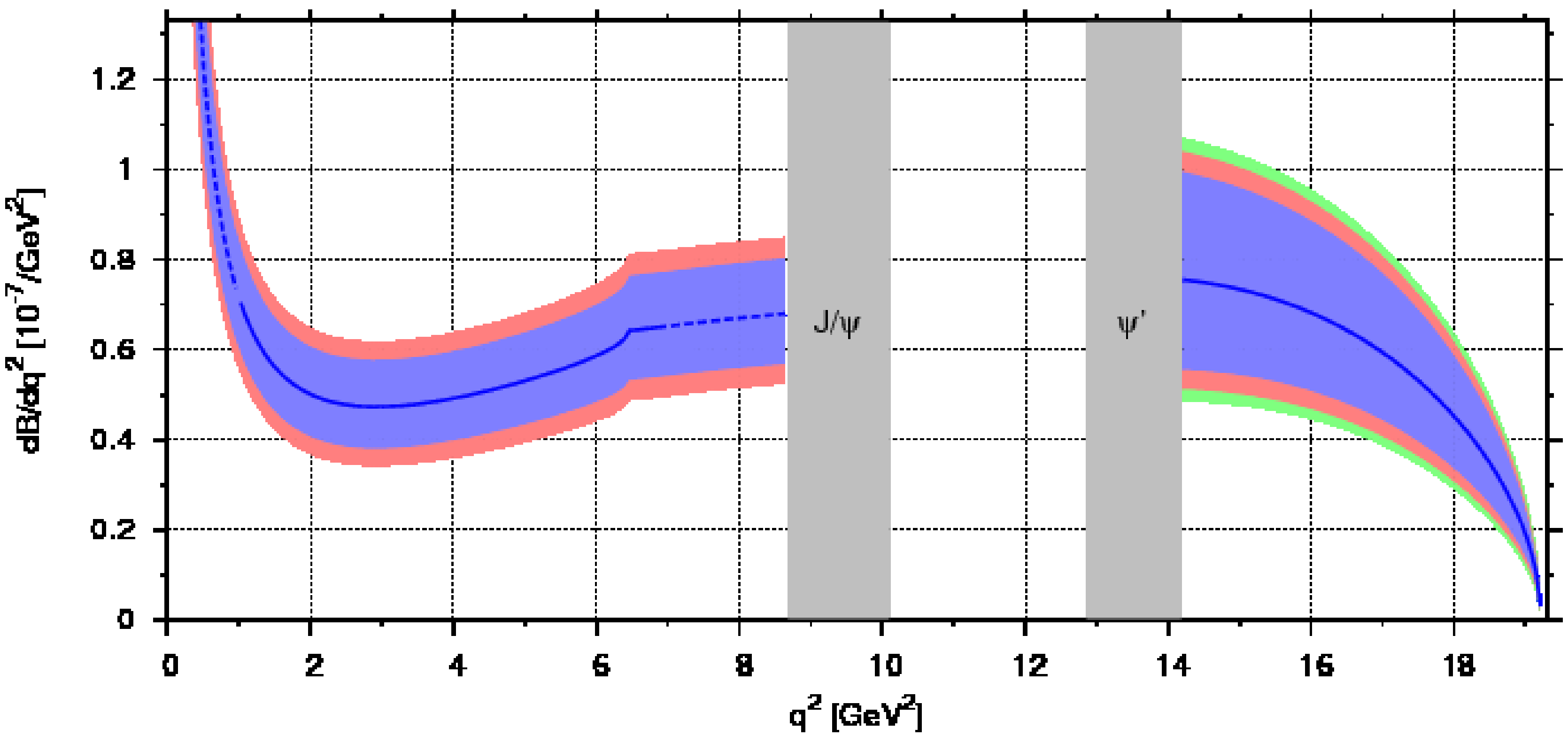


Example of observable in $B \to Kll$



- differential branching ratio from CDF, PRL **106**, 161801 (2011)
- Red lines are based on the max. and min. allowed form factors from light cone sum rules (LCSR); Ali et al., PRD **61**, 074024 (2000).

$B^+ \to K^* l^+ l^-$



- differential branching ratio from Bobeth, Hiller and Dyk, arXiv: 1006.5013
- Blue band shows uncertainty due to form factor.
- Green band shows uncertainty due to Lambda/Q expansion of improved Isgur-Wise relations

- Red band show uncertainty from subleading terms of order alpha_s lambda/Q (low recoil); or lambda/m_b and lambda/E_K* terms (high recoil).

LQCD Studies of B→Kll form factors

- ✦ Quenched lattice QCD:
  - A. Abada et al. Phys. Lett. B 365, 275 (1996)
  - L. Del Debbio et al. Phys. Lett. B 416, 392 (1998)
  - D. Becirevic et al. Nucl. Phys. B 769, 31 (2007)
  - A. Al-Haydari et al. (QCDSF) Eur. Phys. J. A 43, 107120 (2010)
- ✦ Recent studies with dynamical N_f=2+1 flavors:
  - FNAL/MILC: (B→Kll), hep-lat/1111.0981
  - Cambridge/W&M/Edinburgh: (B→K/K*ll), hep-ph/1101.2726

Asqtad Ensembles used in B→Kll

- Four time sources are used on each configuration

~a(fm)	size	am_l/am_s	N_{meas}
0.12	$20^3 \times 64$	0.02/0.05	2052
0.12	$20^3 \times 64$	0.01/0.05	2259
0.12	$20^3 \times 64$	0.007/0.05	2110
0.12	$20^3 \times 64$	0.005/0.05	2099
0.09	$28^3 \times 96$	0.124/0.031	1996
0.09	$28^3 \times 96$	0.0062/0.031	1931
0.09	$32^3 \times 96$	0.00465/0.031	984
0.09	$40^3 \times 96$	0.0031/0.031	1015
0.09	$64^3 \times 96$	0.00155/0.031	791
0.06	$48^3 \times 144$	0.0036/0.018	673
0.06	$64^3 \times 144$	0.0018/0.018	827
0.045	$64^3 \times 192$	0.0028/0.014	800

Form Factors in B→Kll decays: I

- Two matrix elements are needed:

$$\langle B|\bar{b}\gamma^\mu s|\{K(k)\rangle \qquad \langle B|\bar{b}\sigma^{\mu\nu} s|\{K(k)\rangle$$

- Vector current:

$$\langle B|\bar{b}\gamma^\mu s||K(k)\rangle = (p^\mu + k^\mu - \frac{m_B^2 - m_K^2}{q^2} q^\mu) \underline{f_+(q^2)} + \frac{m_B^2 - m_K^2}{q^2} q^\mu \underline{f_0(q^2)}$$

- Tensor current:

$$\langle B|\bar{b}\sigma^{\mu\nu} s|K(k)\rangle = \frac{i\underline{f_T}}{m_B + m_K}\left[(p^\mu + k^\mu)q^\nu - (p^\nu + k^\nu)q^\mu\right]$$

Form Factors in B→Kll decays: II

- For LQCD convenient to work in B rest frame. We define:

$$\langle B(p)|\bar{b}\gamma^\mu s|K(k)\rangle = \sqrt{2m_B}\left[f_\| \frac{p^\mu}{m_B} + f_\perp p_\perp^\mu\right]$$

- Form factors considered to be functions of kaon energy:

$$\begin{cases} f_\|(E_K) = \dfrac{\langle B(p)|\bar{b}\gamma^0 s|K(k)\rangle}{\sqrt{2m_B}} \\ f_\perp(E_K) = \dfrac{\langle B(p)|\bar{b}\gamma^i s|K(k)\rangle}{2\sqrt{m_B}} \dfrac{1}{k_i} \end{cases}$$

and

$$f_T = \frac{m_B + m_K}{\sqrt{2m_B}} \frac{\langle B(p)|i\bar{(b)}\sigma^{0i} s|K(k)\rangle}{\sqrt{2m_B} k^i}$$

NLO Staggered χPT

$$f_{\|} = \frac{C_0}{f}(1 + \text{logs} + C_1 m_x + C_2 m_y + C_3 E + C_4 E^2 + C_5 a^2)$$

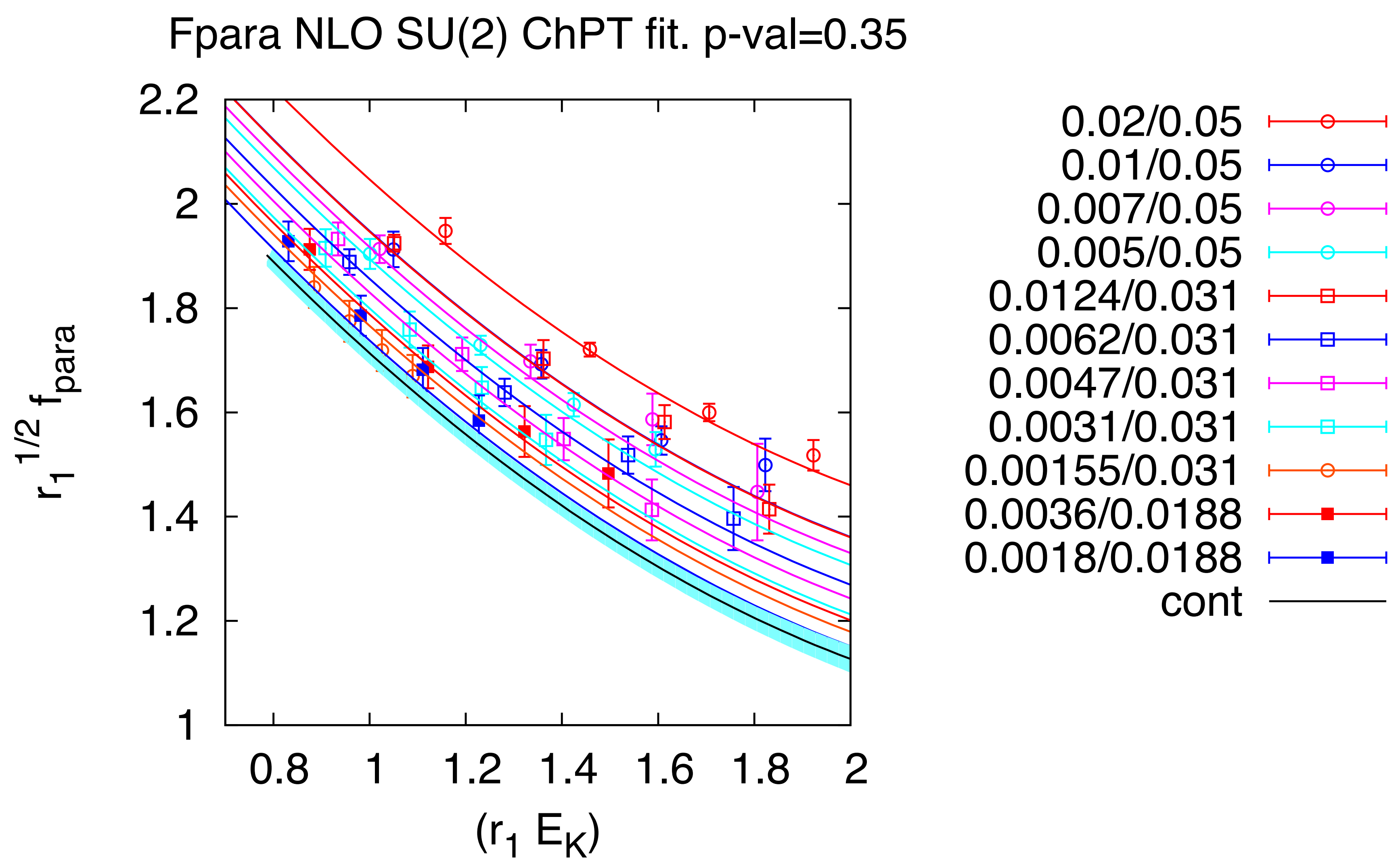
$$f_{\perp} = \frac{C_0}{f}\left[\frac{g}{E + \Delta_B^* + D}\right]$$

$$+ \frac{(C_0/f)g}{E + \Delta_B^*}(\text{logs} + C_1 m_x + C_2 m_y + C_3 E + C_4 E^2 + C_5 a^2)$$

✦ where $\Delta_{B^*} = m_{Bs^*} - m_B$; D and logs are chiral log terms.

- we use SU(2) chiral logs in the chiral fit
- the expression for f_T and $f_\perp$ are the same at this order in the $1/m_B$ expansion (Becirevic et al., PRD **68**, 074003 (2003)).

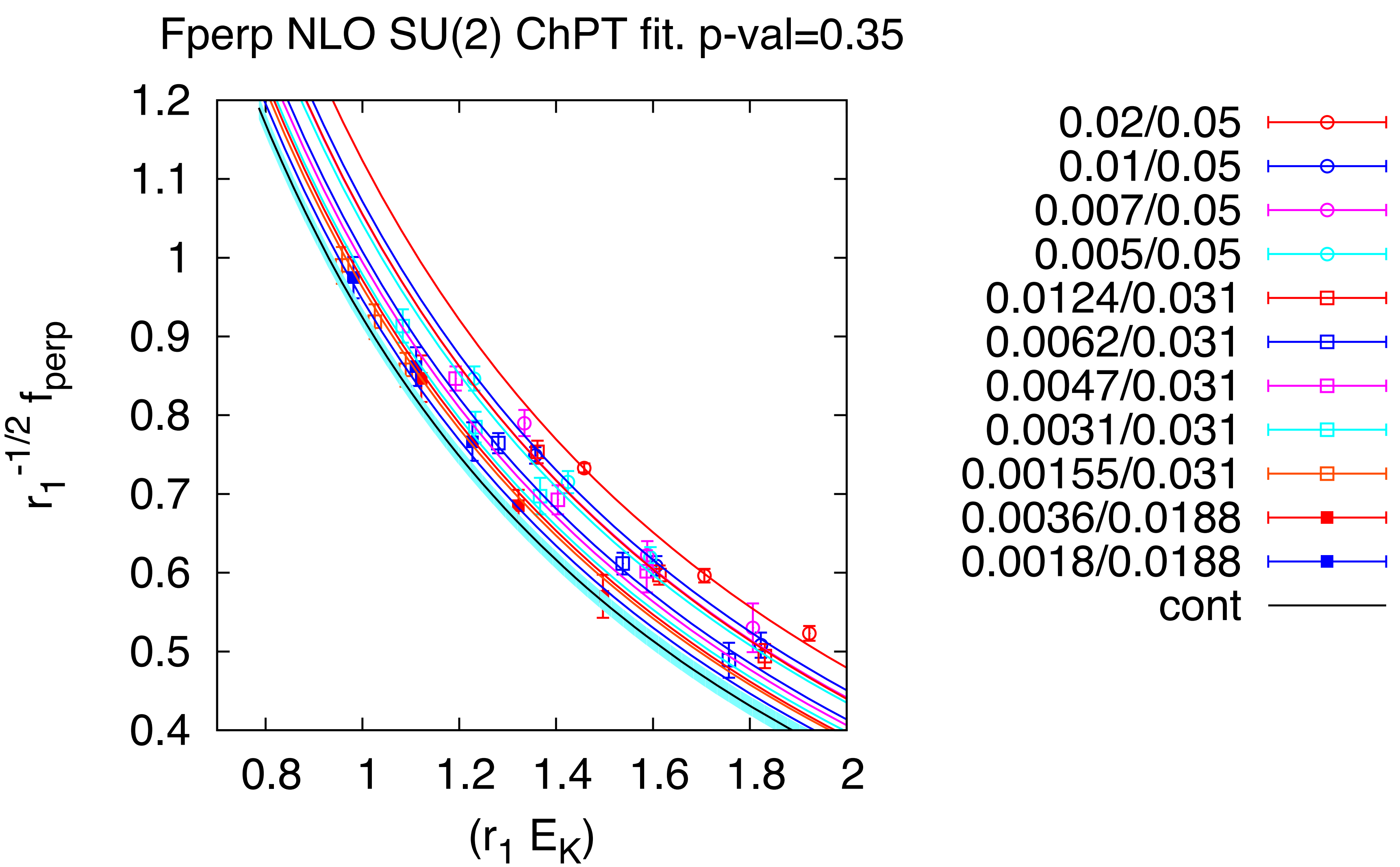
$f_{||}$ chiral-continuum extrapolation



Chiral-continuum extrapolations give form factors at small E_K (large q^2).

$$q^2 = (p_B - p_K)^2 = m_B^2 + m_K^2 - 2m_B E_K$$

f_T chiral-continuum extrapolation



z-expansion for B→Kll form factors

✦ *z*-expansion is based on field theoretic principles: analyticity, crossing symmetry, unitarity. It is systematically improvable by adding more orders.

- *z*-expansion maps q^2 to z by:

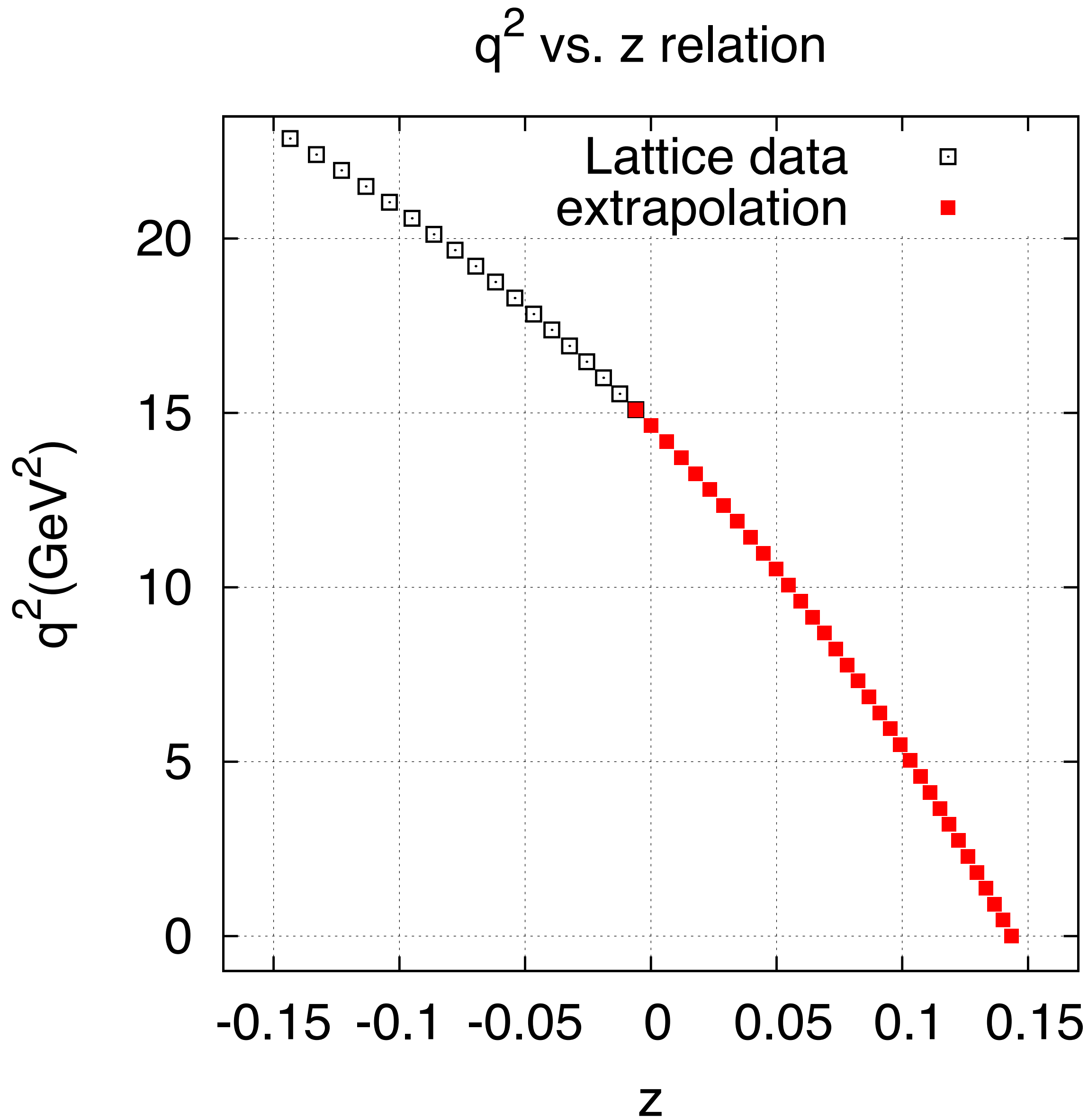
$$z(q^2,t_0) \quad = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \qquad t_\pm = (m_B \pm m_K)^2$$

- choose $t_0 = t_+\left(1 - \sqrt{1 - \frac{t_-}{t_+}}\right)$ such that *z* « 1
- expand form factors as a function of z

$$f(q^2) = \frac{1}{B(z)\phi(z)} \sum_{k=0}^{\infty} a_k z^k$$

- where *B(z)* is used to account for the pole structure and **Φ**(z) assures $\sum_{k=0}^{\infty} a_k^2 \leq 1$

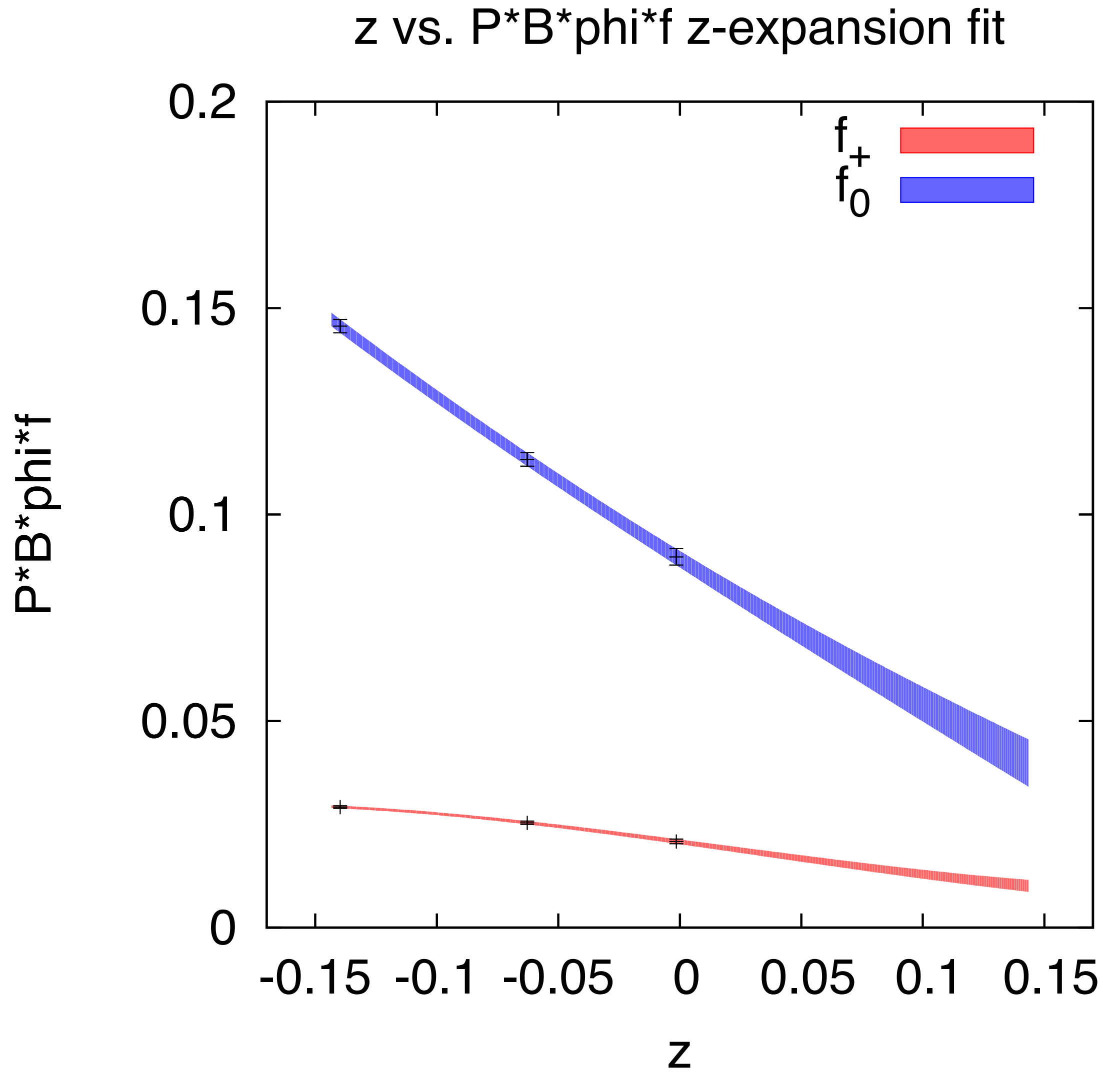
z-expansion continued

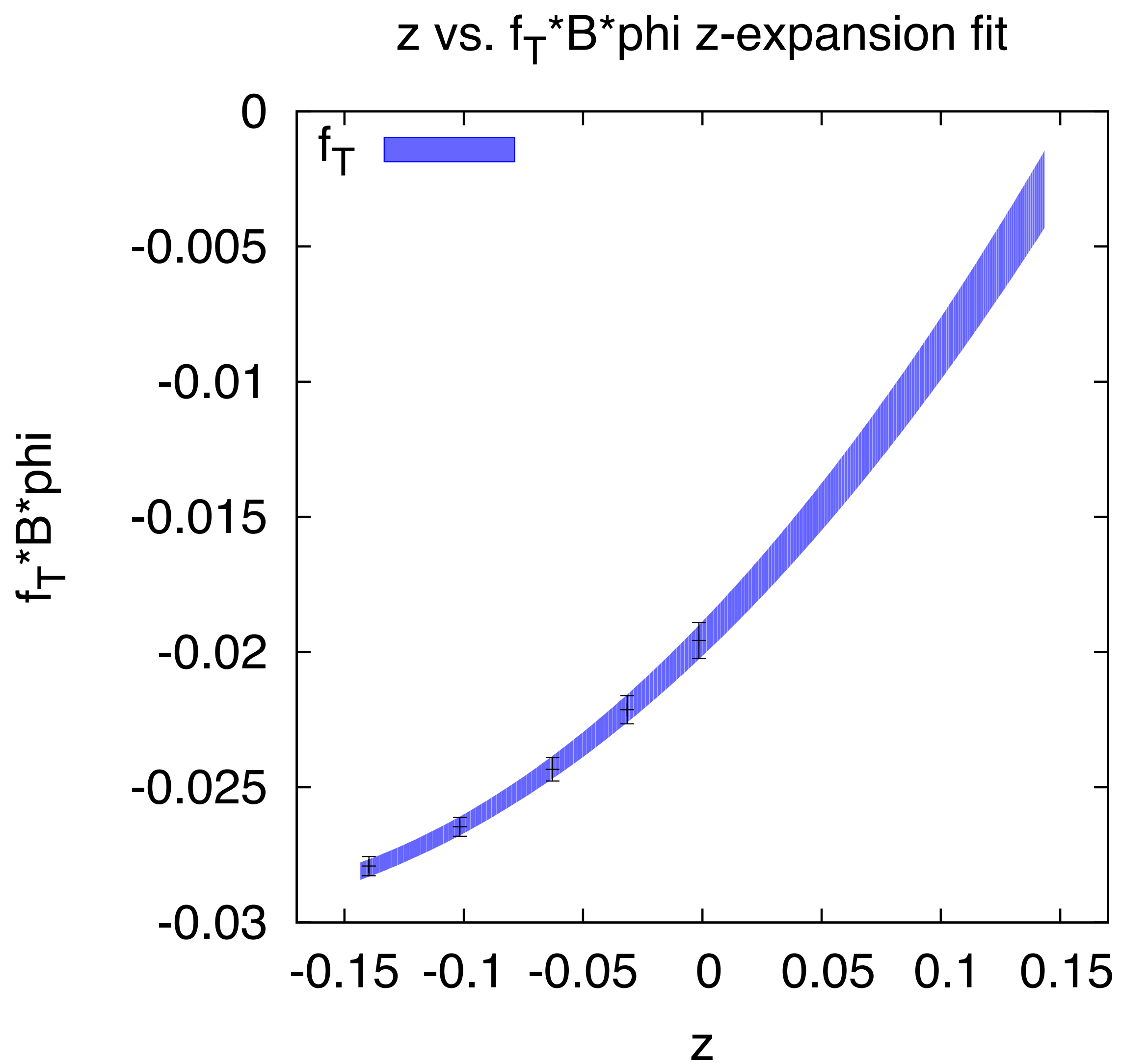


$$f(q^2) = \frac{1}{B(z)\phi(z)} \sum_{k=0}^{\infty} a_k z^k$$

- $q^2 \in (0,23) \Rightarrow z \in (-0.15, 0.15)$
- B_s^* pole corresponds to =0.367
- Fit $f(q^2)\ B(z)\ \Phi(z)$ as a polynomial in z

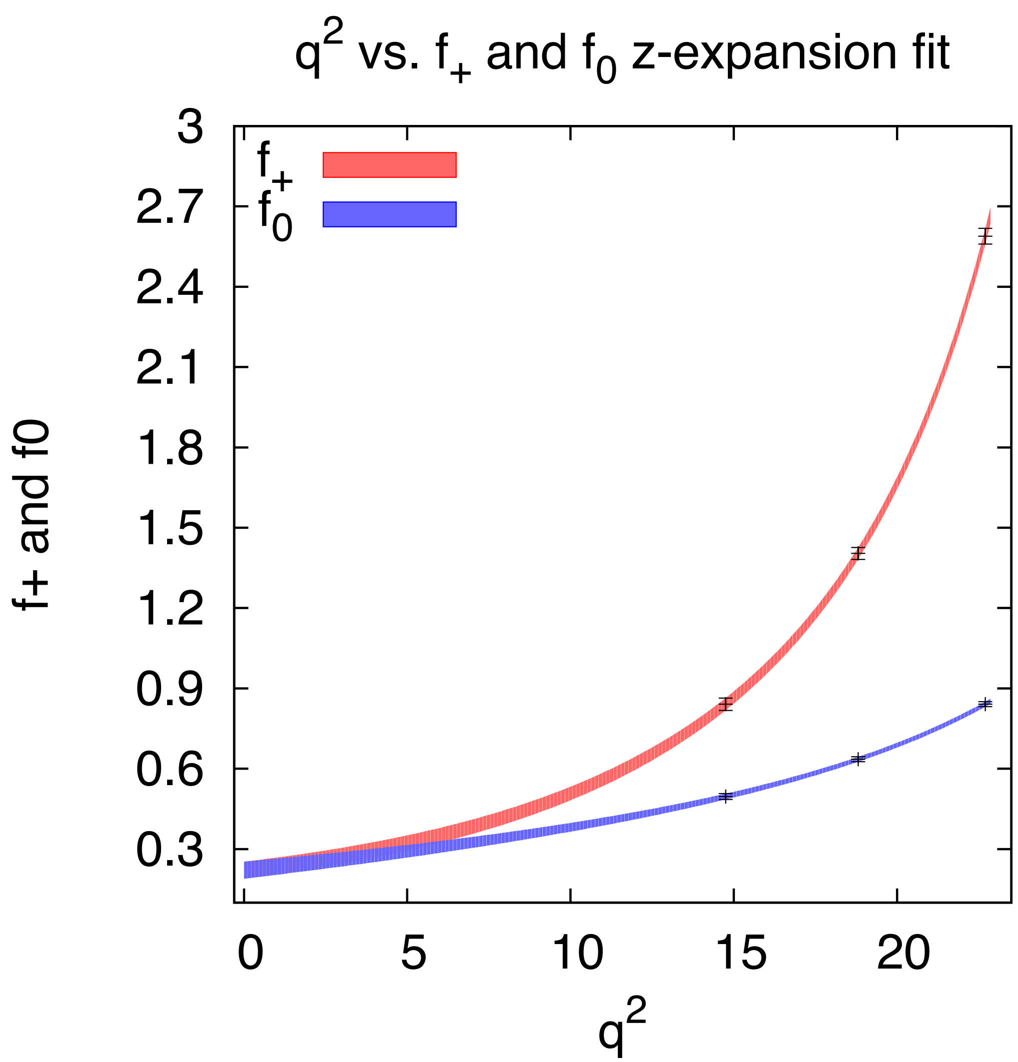
z-expansion fitting

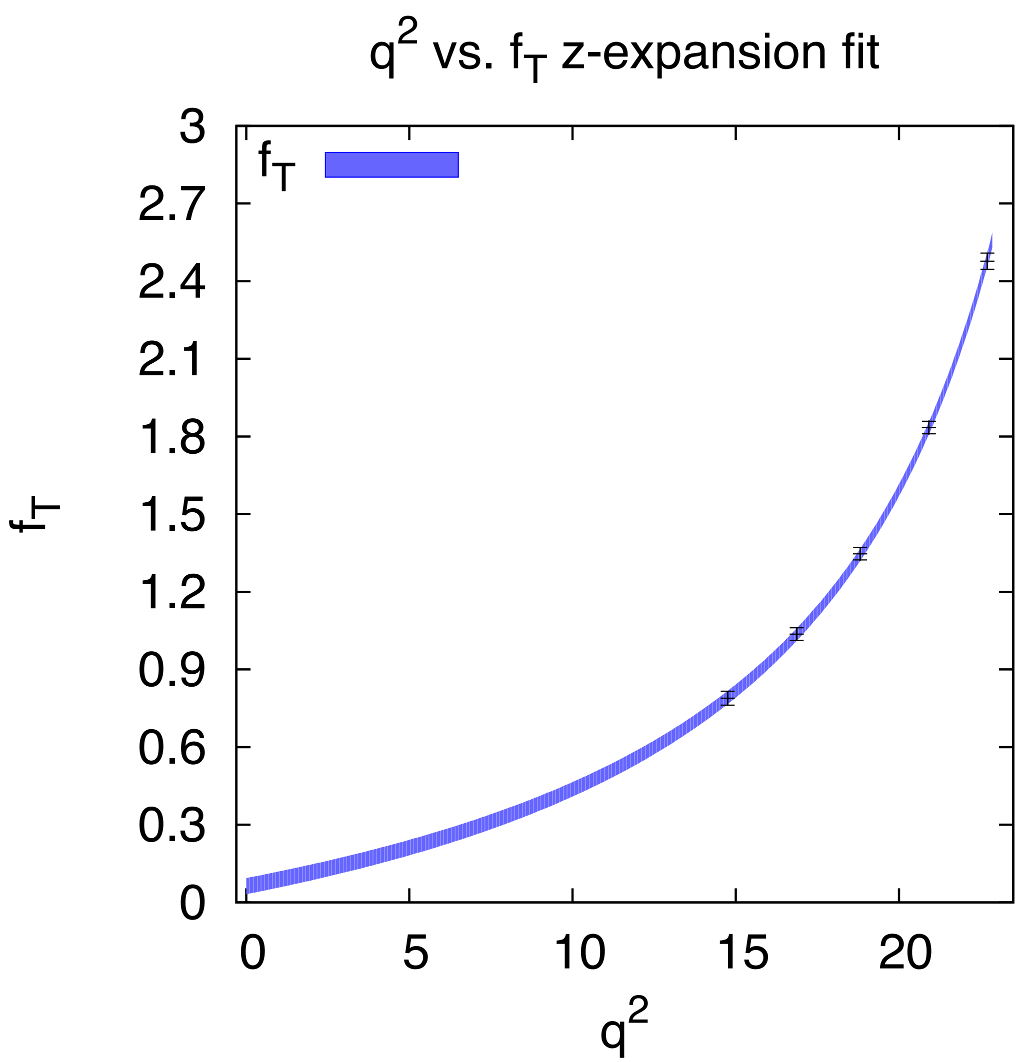




- Synthetic data points are selected from the chiral-continuum fit of the form factors. They are multiplied by appropriate factors and fit as a polynomial in *z.*
- Only statistical errors are shown here.

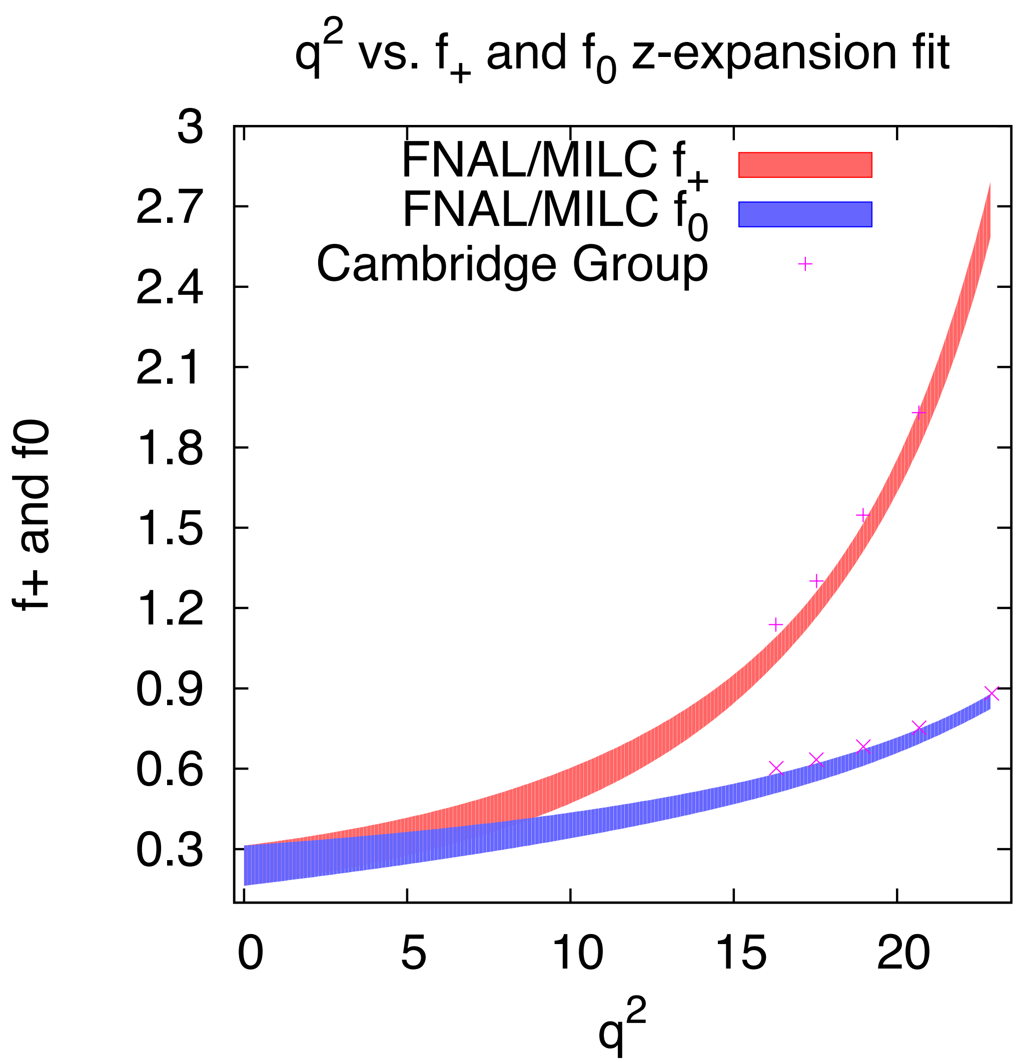
form factors from *z*-expansion I

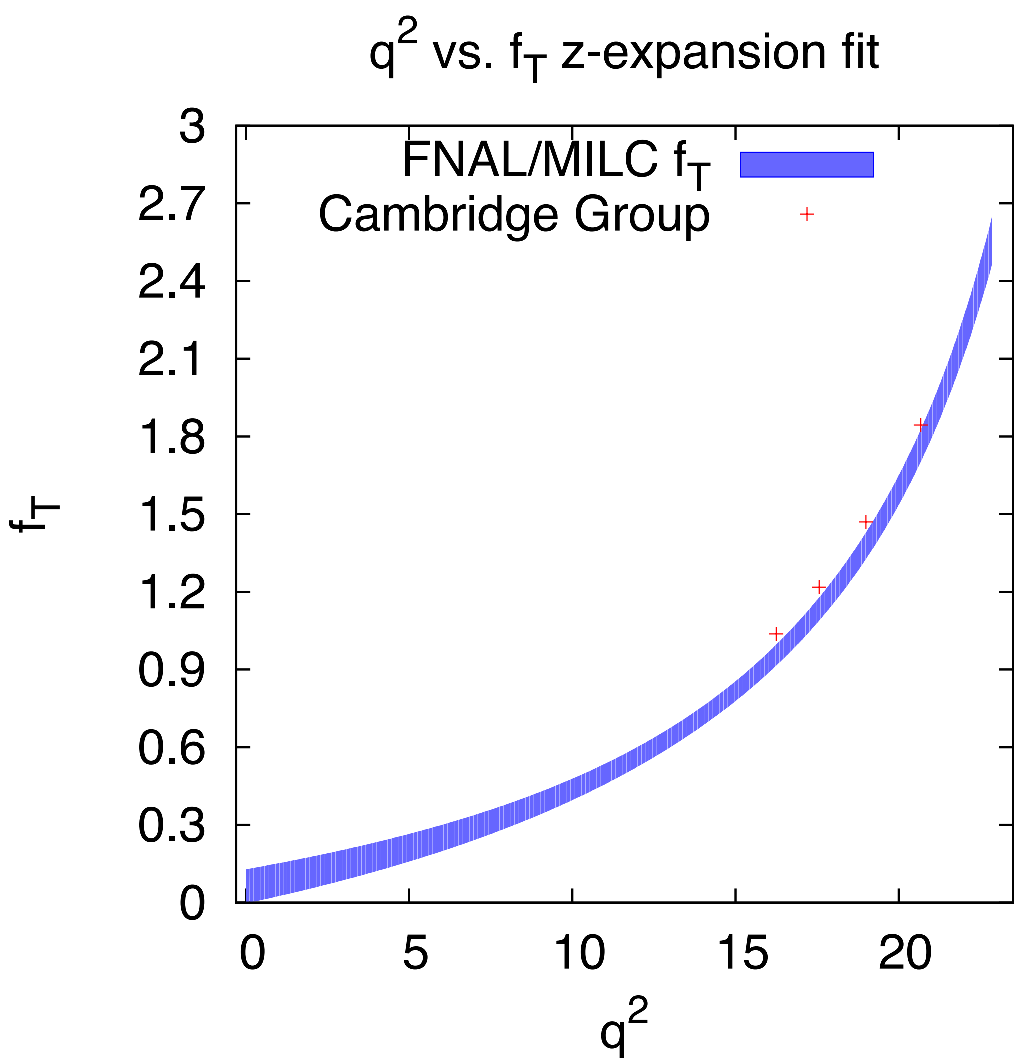




- Kinematic constraint on *z*-expansion assures $f_+(q^2=0)=f_0(q^2=0)$
- Only statistical errors are shown here.

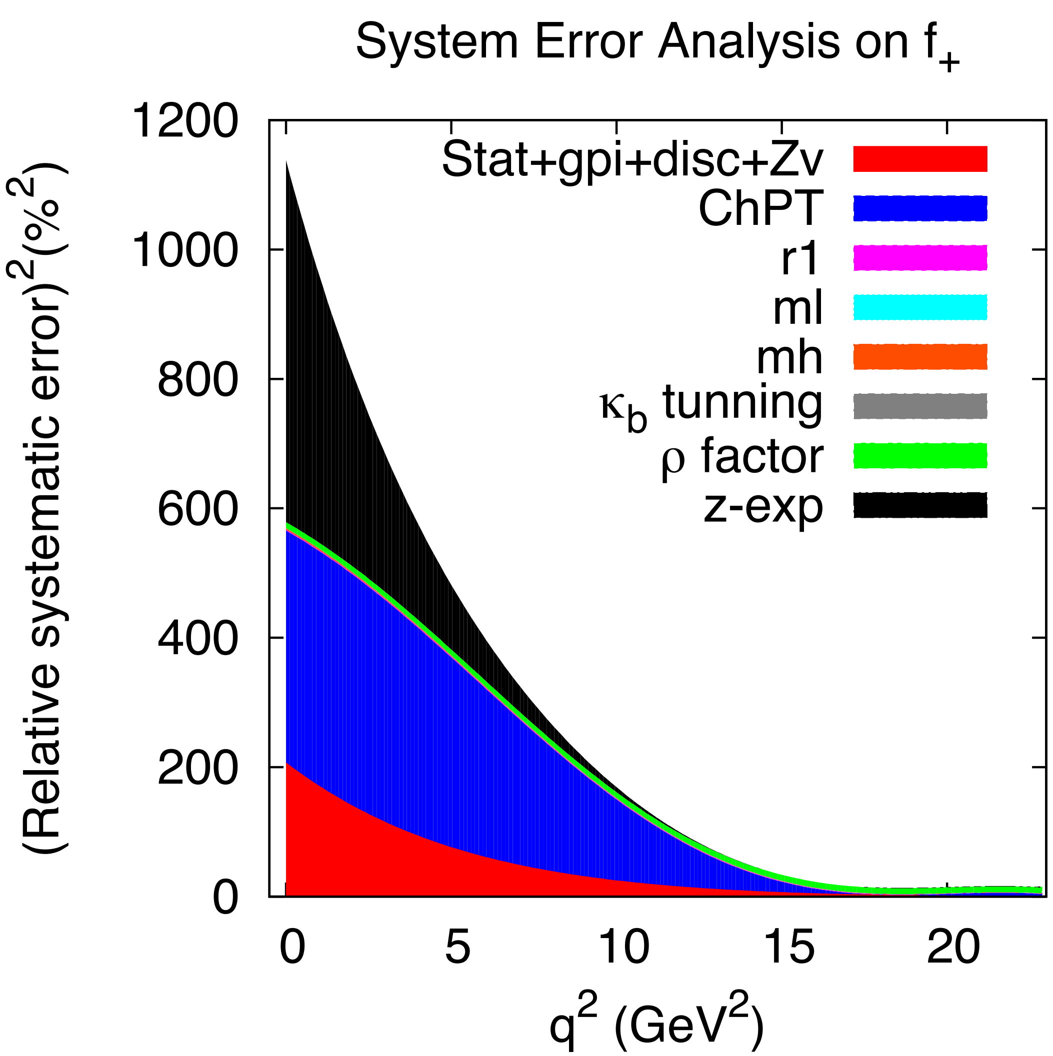
form factors from z-expansion II

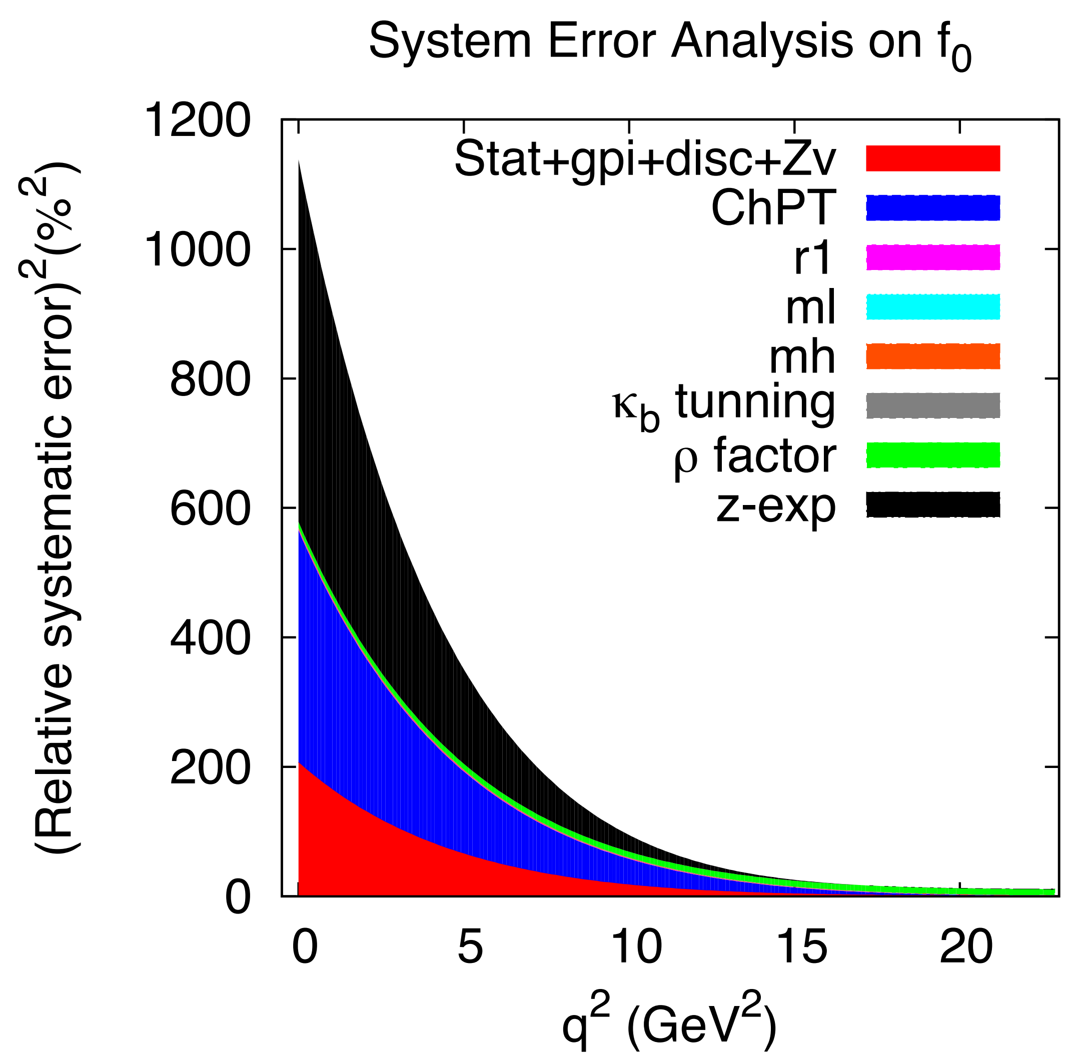




- Systematic and statistical errors are shown here
- Breakdown of systematic error on next slides
- Reasonable agreement with Cambridge/W&M/Edinburgh calculation

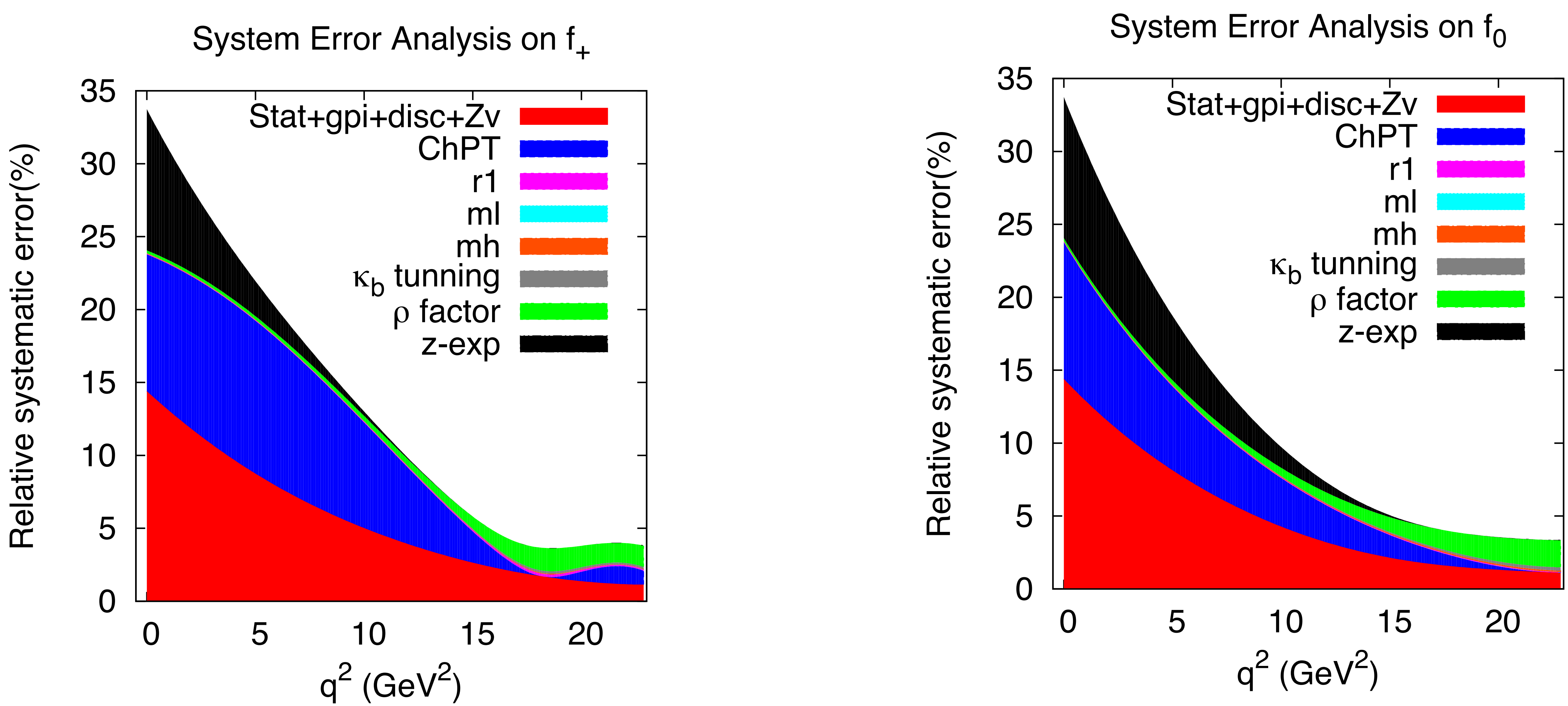
Systematic error budget I





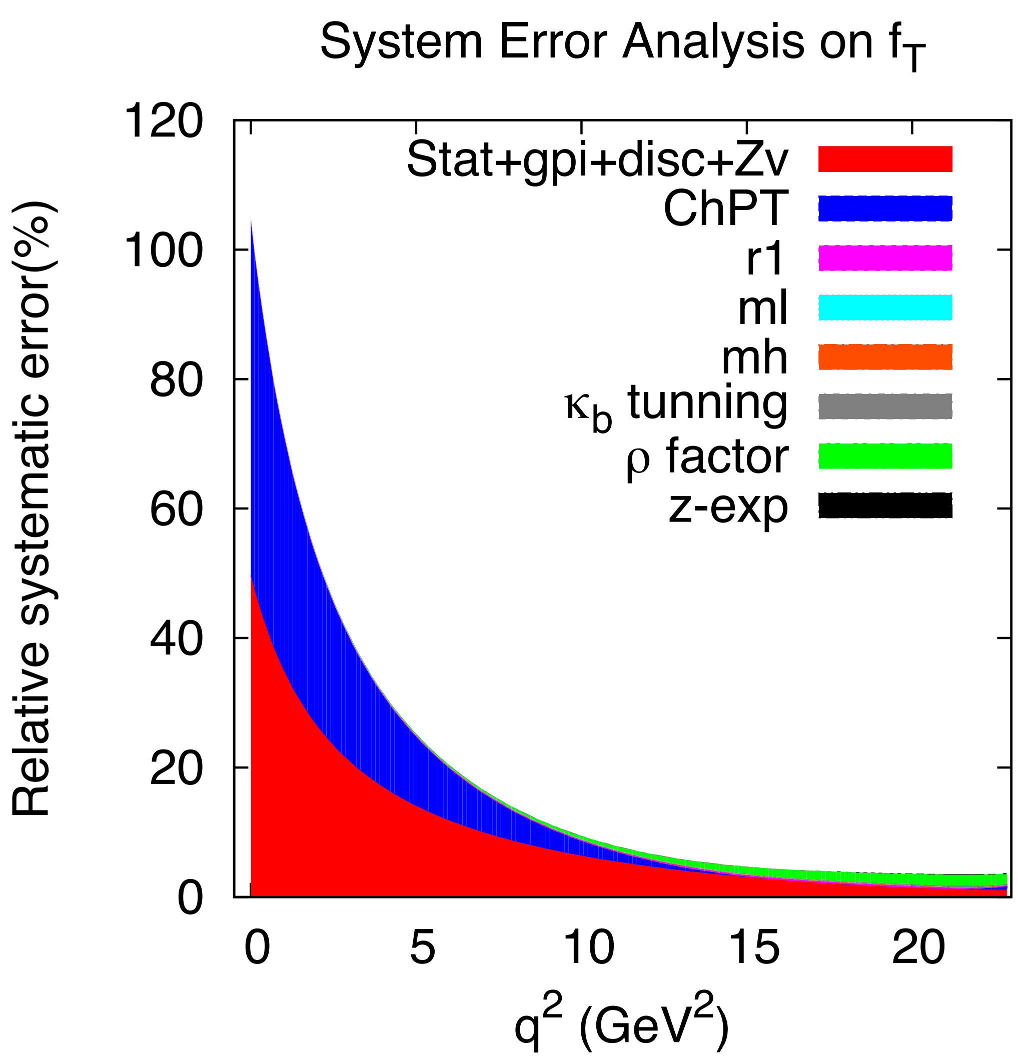
- Systematic and statistical errors are shown here
- Errors shown in quadrature
- Chiral extrapolation error and *z*-expansion are most significant
- Results preliminary

Systematic error budget II



- Errors shown as per cent
- For large q^2, error about 5%
- Relative error is large at small q^2 partially because form factor is small

Systematic error budget III



- Errors shown as per cent
- For large q^2, error about 5%
- Relative error is large at small q^2 partially because form factor is small

$B \to D\tau\nu$ Probing New Physics

- $B \to D\tau\nu$ is sensitive to a scalar current such as mediated by a charged Higgs boson.
- *BABAR* recently reported first observation of the decay at a rate about 2σ above the standard model rate for $R(D)=BR(B \to D\tau\nu)/BR(B \to Dl\nu)$. PRL 109 (2012) 101802
- However, the SM prediction was not based on ab initio LQCD form factors using dynamical quark ensembles.
- *BABAR:* $R(D)=0.440\pm0.058\pm0.042$ (consistent with Belle)
- FNAL/MILC: $R(D)=0.316\pm0.012\pm0.007$ PRL 109 (2012) 071802
- Previous SM: $R(D)=0.297\pm0.017$
- Difference reduced to 1.7σ

Talk ended with previous slide

- Rest of material on $B \rightarrow D\tau\nu$ and R(D) was prepared by me.
- Material that follows on Kaon semi-leptonic decay was prepared by Claude Bernard.

Formalism

✦ Define lepton helicity in virtual W rest frame

$$\frac{d\Gamma_-}{dq^2} = \frac{1}{24\pi^3}\left(1-\frac{m_\ell^2}{q^2}\right)^2 |p_D|^3 \left| G_V^{\ell cb} f_+(q^2) - \frac{m_\ell}{M_B} G_T^{\ell cb} f_2(q^2)\right|^2 ,$$

$$\frac{d\Gamma_+}{dq^2} = \frac{1}{16\pi^3}\left(1-\frac{m_\ell^2}{q^2}\right)^2 \frac{|p_D|}{q^2} \left\{ \frac{1}{3}|p_D|^2 \left| m_\ell G_V^{\ell cb} f_+(q^2) - \frac{q^2}{M_B} G_T^{\ell cb} f_2(q^2)\right|^2 \right.$$

$$\left. + \frac{\left(M_B^2 - M_D^2\right)^2}{4M_B^2}\left|\left(m_\ell G_V^{\ell cb} - \frac{q^2}{m_b - m_c} G_S^{\ell cb}\right) f_0(q^2)\right|^2 \right\} ,:$$

$$\Gamma_{\rm tot} = (\Gamma_+ + \Gamma_-)$$

Formalism

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- In SM, $G_S = G_T = 0$.

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Formalism

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$$\Gamma_{\text{tot}} = (\Gamma_+ + \Gamma_-)$$

✦ In SM, $G_S = G_T = 0$.

✦ $\Gamma_+ \propto m_l$, so negligible for e, **μ**, but not for **τ**

Formalism

- Define lepton helicity in virtual W rest frame

$$\frac{d\Gamma_-}{dq^2} = \frac{1}{24\pi^3}\left(1-\frac{m_\ell^2}{q^2}\right)^2 |p_D|^3 \left| G_V^{\ell cb} f_+(q^2) - \cancel{\frac{m_\ell}{M_B} G_T^{\ell cb} f_2(q^2)}\right|^2 ,$$

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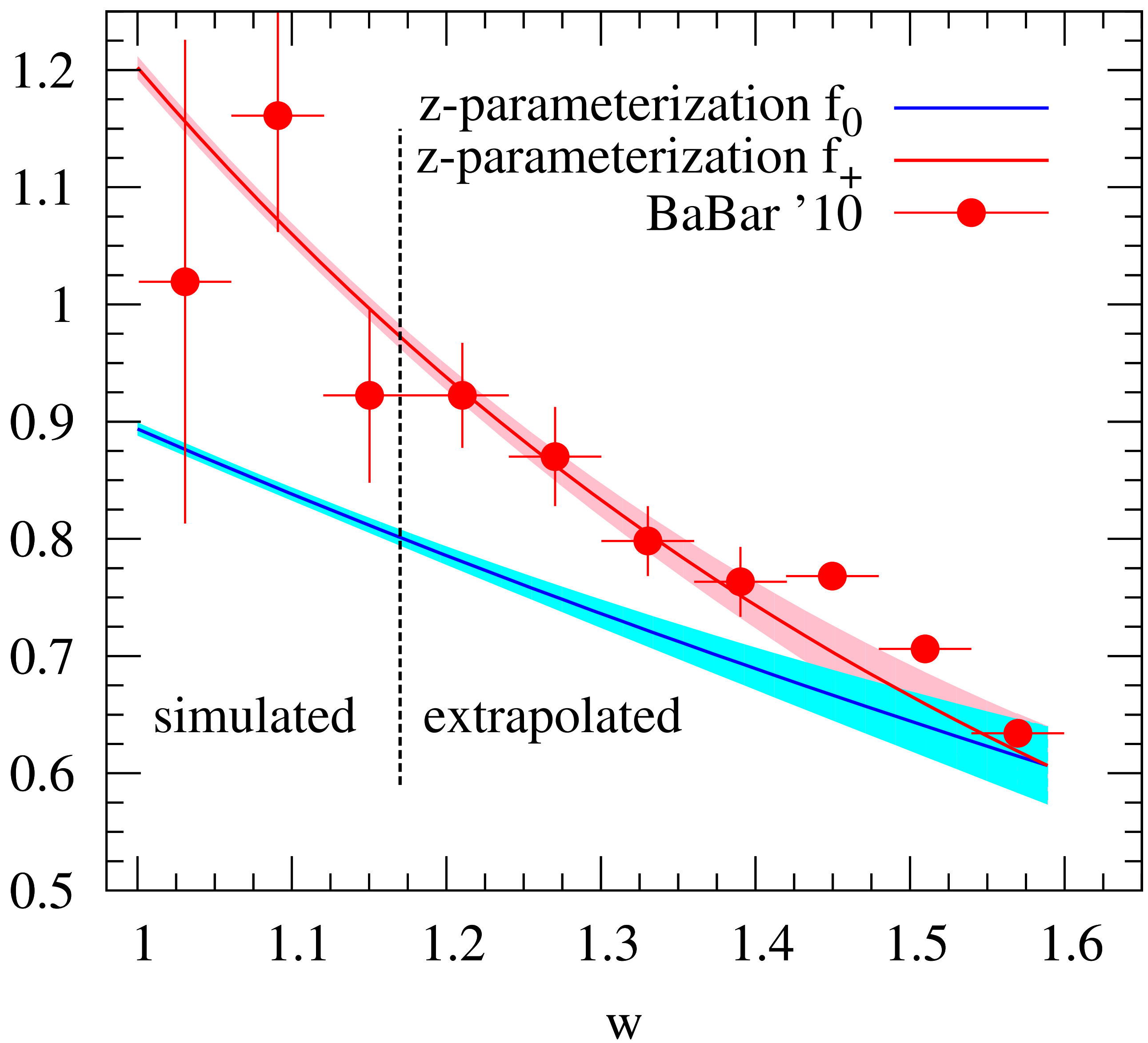
$$\Gamma_{\text{tot}} = (\Gamma_+ + \Gamma_-)$$

- In SM, $G_S = G_T = 0$.
- $\Gamma_+ \propto m_l$, so negligible for e, μ, but not for τ
- We won't need tensor coupling, but G_S needed for charged Higgs

Lattice Calculation

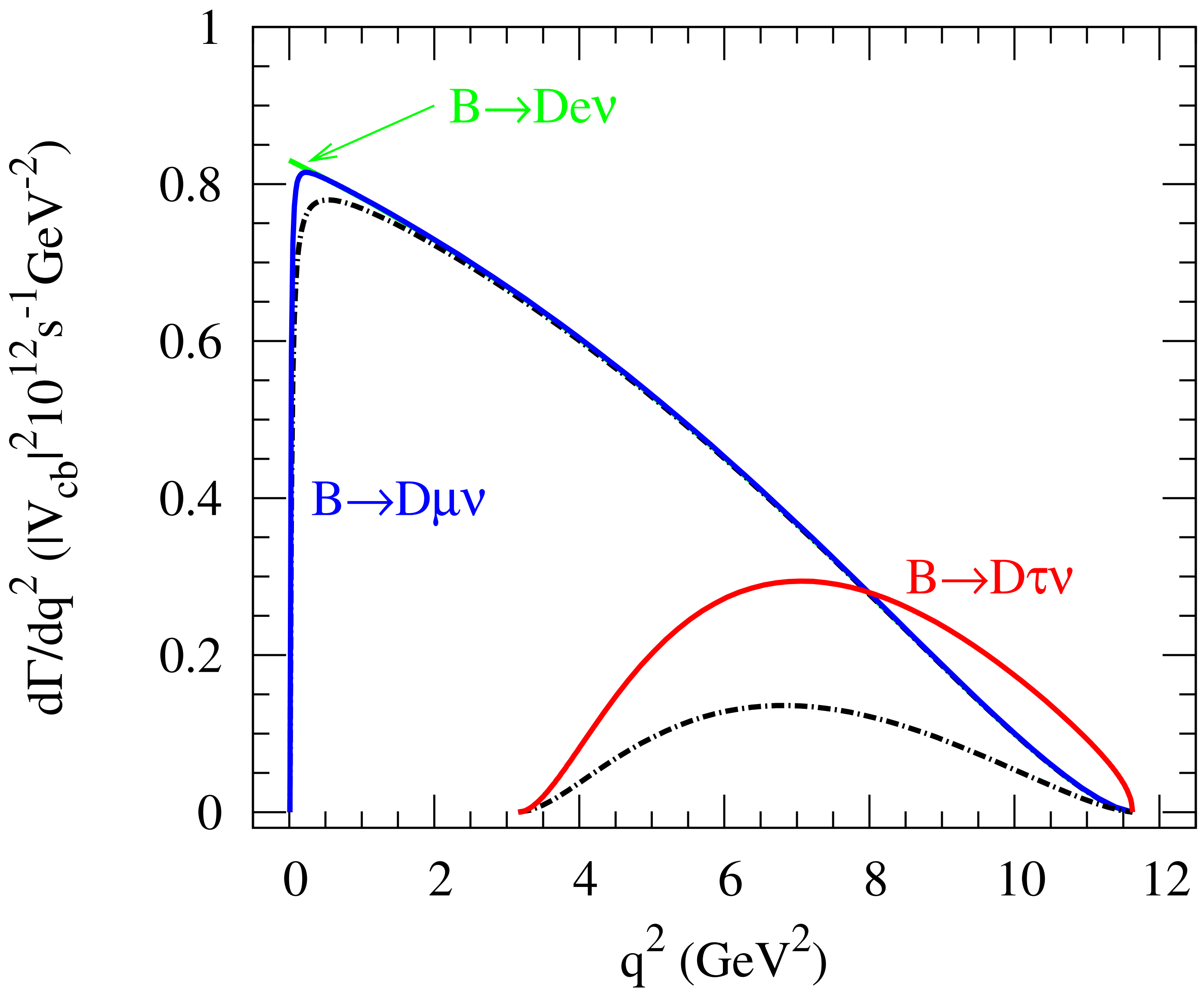
- *Ab initio* calculation of form factors based on two lattice spacings a=0.12 and 0.09 fm.
- Should be sufficient for ratio needed here, but we plan to analyze additional ensembles to improve precision of form factors.
- See J. Bailey *et al*. [FNAL/MILC], PRD **85** (2012)114502, arXiv:1202.6346 [hep-lat] for all the details of form factor calculation
- See J. Bailey *et al*. [FNAL/MILC], PRL **109** (2012) 071802, arXiv:1206.4992 [hep-ph] for all the details of application to R(D) and polarization ratio.

Form Factors



- Comparison of f_+ with BaBar 2010 data for light lepton decay
- Left part of curve comes from lattice kinematic range, right part from z-parameterization
- expt'l errors large where LCD is more precise
- f_0 result is prediction

Differential Decay Rates



- SM rate based on our form factors
- Dash-dotted black line shows the rate when $f_0(q^2)=0$.
- Clearly, τ decay mode is most sensitive to f_0 and probes range of f_+ differently from light lepton modes.

Error Budget

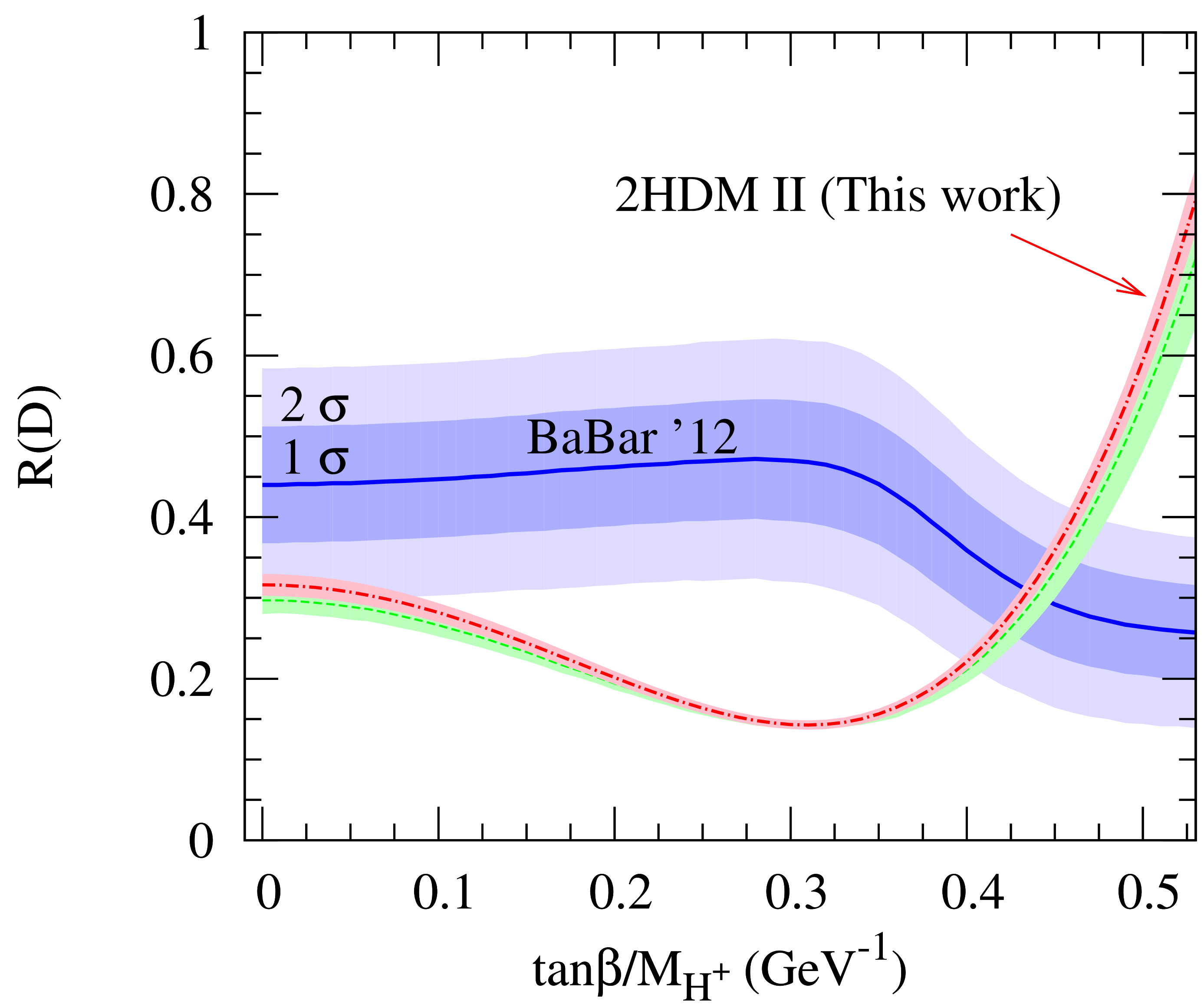
- Also determined $P_L(D) = 0.325(4)(3)$ where

$$P_L(D) = \left(\Gamma_+^{B\to D\tau\nu} - \Gamma_-^{B\to D\tau\nu}\right)/\Gamma_{\text{tot}}^{B\to D\tau\nu}$$

- Error budget in percent

Source	$R(D)$	$P_L(D)$
Monte-Carlo statistics	3.7	1.2
Chiral-continuum extrapolation	1.4	0.1
z-expansion	1.5	0.1
Heavy-quark mass (κ) tuning	0.7	0.1
Heavy-quark discretization	0.2	0.3
Current $\rho_{V^i_{cb}}/\rho_{V^0_{cb}}$	0.4	0.7
total	4.3%	1.5%

Charged Higgs Bounds



- R(D) measurement can be used to constrain parameters of two Higgs doublet model
- Show are BaBar result (blue), bound based on prior form factor estimates (green) and our result (red).
- Note at LH edge of graph comparison of SM predictions with BaBar.

Fermilab Lattice/MILC Collaboration

J. Bailey	Seule National U.
A. Bazavov	BNL
C. Bernard	Washington U.
C. Bouchard	Ohio State
C. DeTar	U. of Utah
A.X. El-Khadra	U. of Illinois
R.T. Evans	U. of Illinois, North Carolina State U.
E.D. Freeland	U. of Illinois, Benedictine U.
W. Freeman	George Washington U.
E. Gamiz	Fermilab, U. de Granada
S. Gottlieb	Indiana U.
J. Komijani	Washington U.
U.M. Heller	APS
J.E. Hetrick	U. of the Pacific
J. Kim	U. of Arizona
A.S. Kronfeld	Fermilab
J. Laiho	U. of Glasgow
L. Levkova	U. of Utah
M. Lightman	Washington U.
P.B. Mackenzie	Fermilab
E. Neil	Fermilab
M.B. Oktay	U. of Utah
J. Simone	Fermilab
R. Sugar	U.C. Santa Barbara
D. Toussaint	U. of Arizona
R.S. Van de Water	BNL→ Fermilab

$K \to \pi$ semileptonic decay

- ✦ Focus at $q^2=0$, where we can use the method HPQCD proposed for semileptonic D decay:
  - Full matrix element of vector current V_μ is hard because conserved current is complicated and local current needs renormalization.
  - Instead use $\partial^\mu V_\mu = (m_b - m_a)\, S$
    - S is local, and product $(m_b - m_a)S$ not renormalized.
  - This is sufficient for $f_+(q^2=0) = f_0(q^2=0)$.
- ✦ Two-part program:
  - HISQ valence on 2+1 Asqtad ensembles (close to completion).
  - HISQ valence on 2+1+1 HISQ ensembles (early stage).
    - ultimately to include D → K, and $q^2 \neq 0$

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Fermilab/MILC
E. Gámiz

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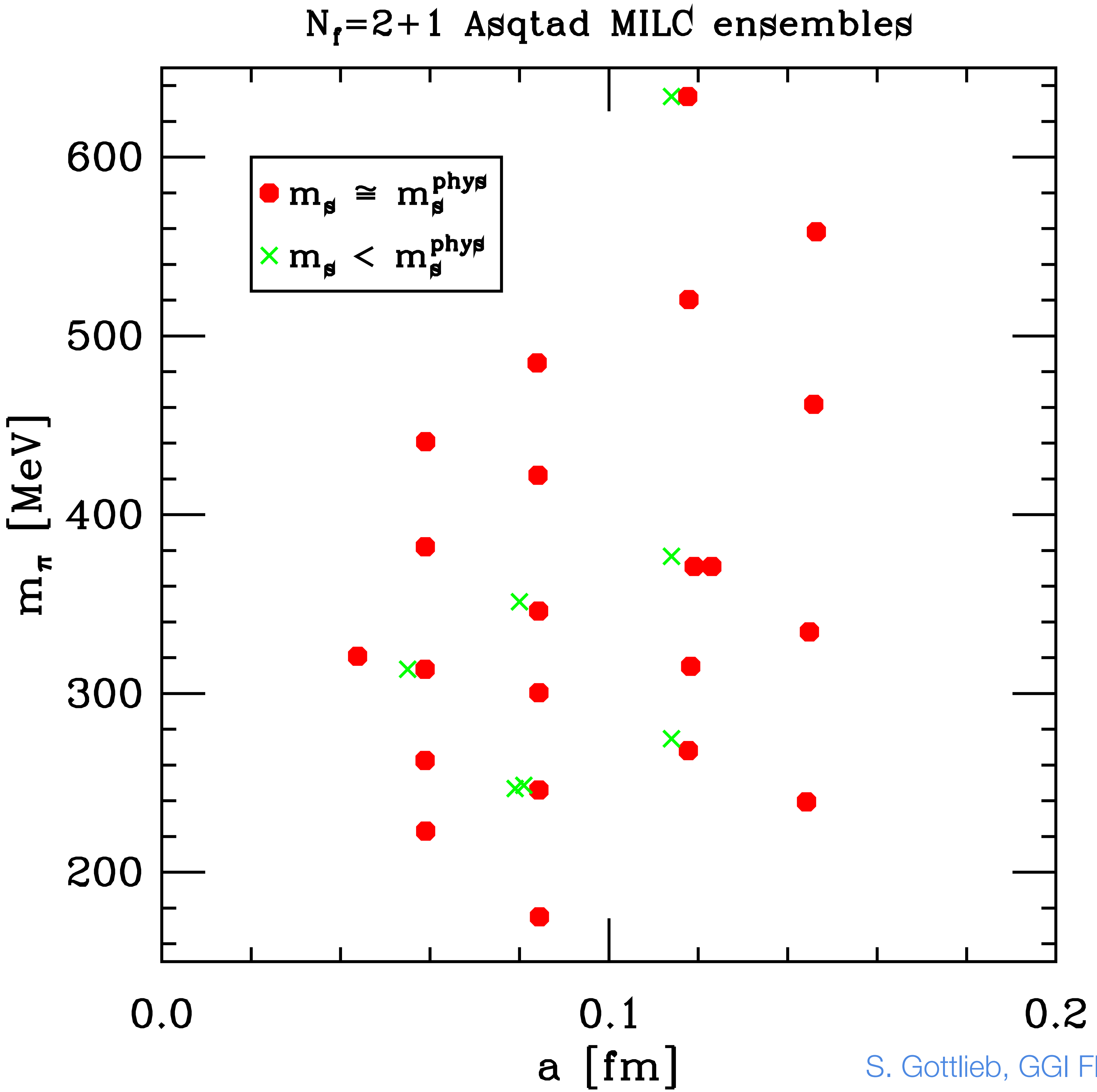
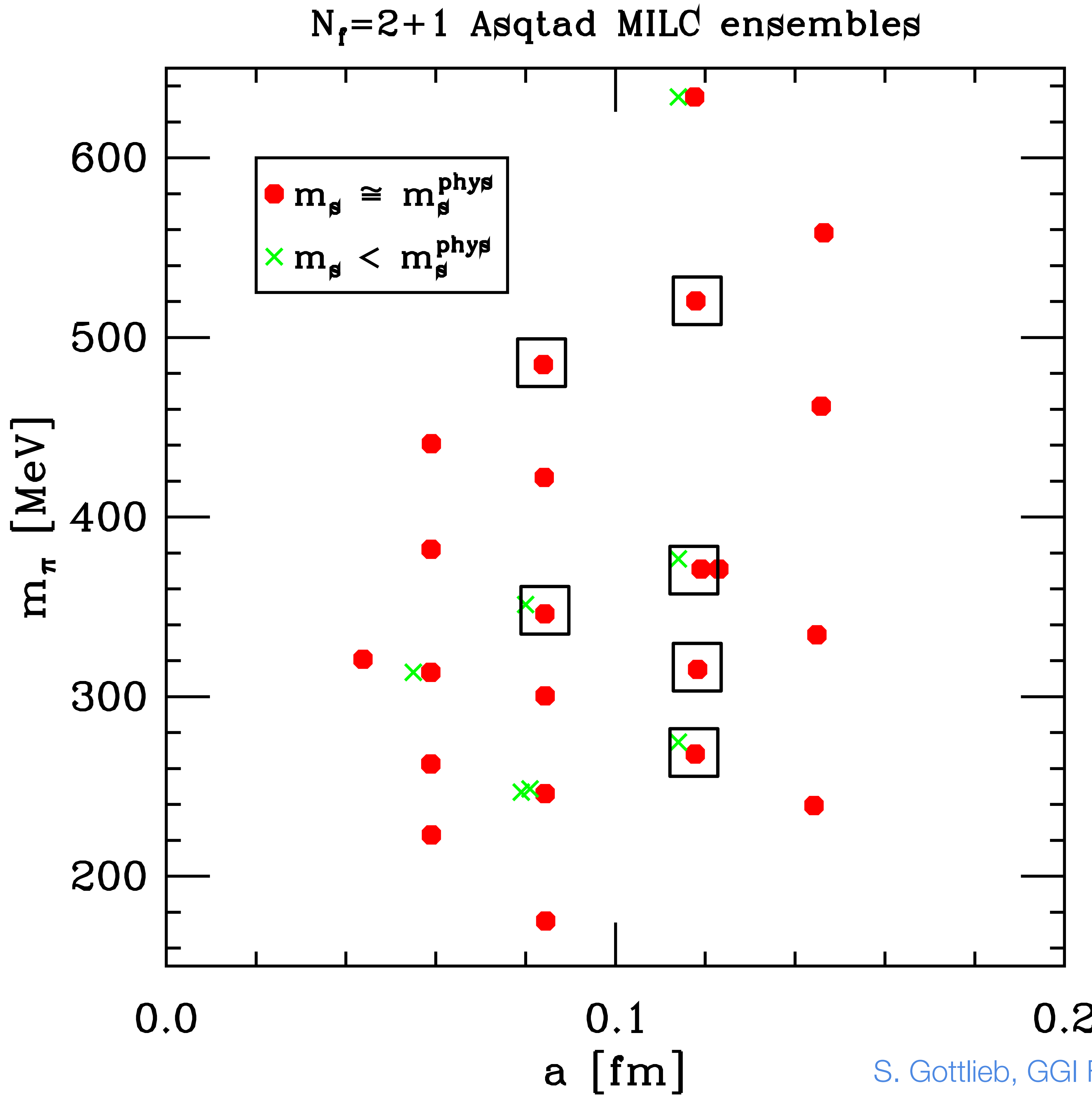
Asqtad Ensembles



Asqtad Ensembles



$K \to \pi$ project (HISQ on Asqtad)

$K \to \pi$; HISQ on Asqtad

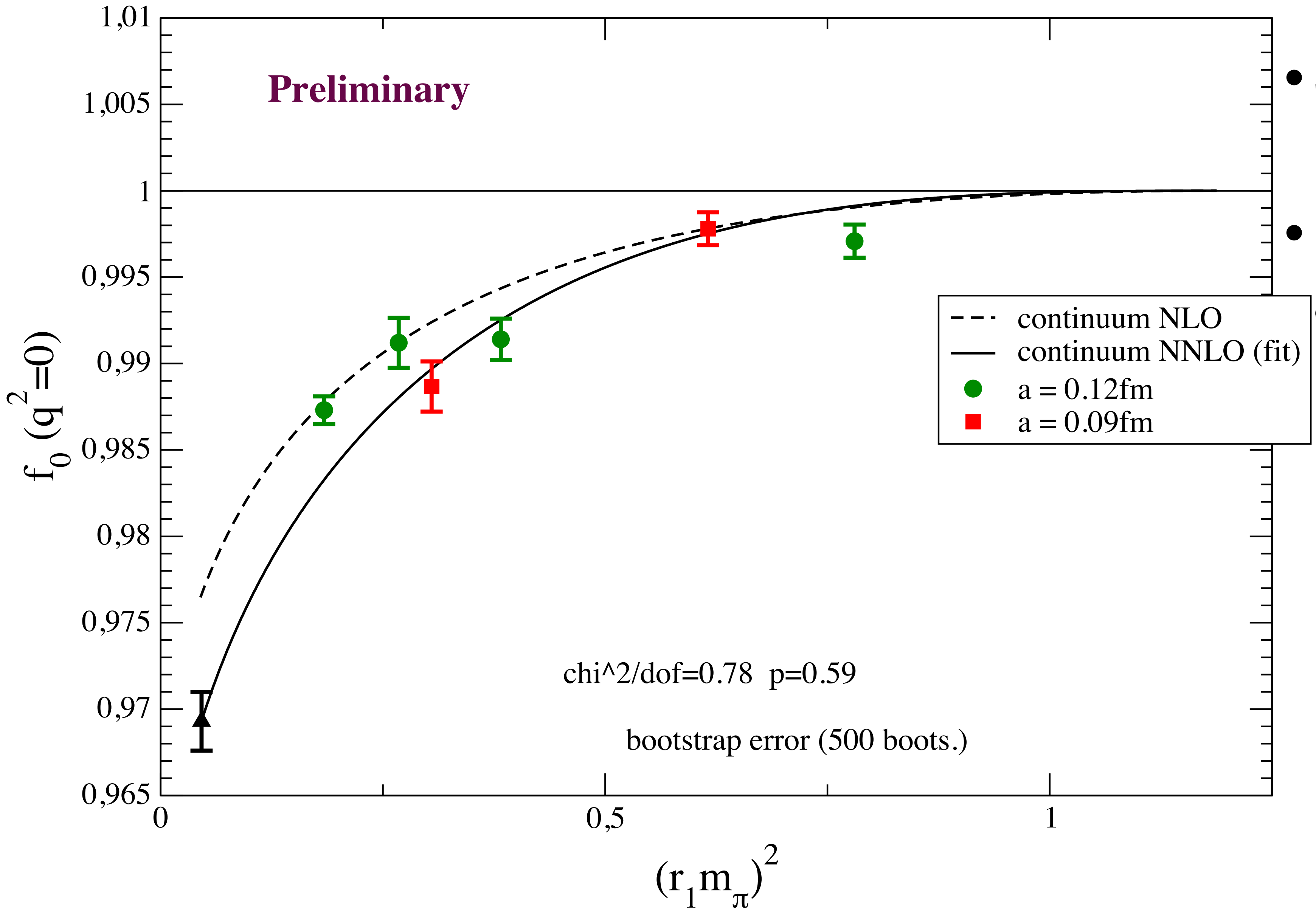
- Strange HISQ valence mass tuned to its physical value [from Davies, et al, PRD 81 (2010) 034506, using the "η_s"].
- Light HISQ valence mass tuned to Asqtad sea by:

$$\frac{m_l^{\text{val}}(\text{Hisq})}{m_s^{\text{phys}}(\text{Hisq})} = \frac{m_l^{\text{sea}}(\text{Asqtad})}{m_s^{\text{phys}}(\text{Asqtad})}$$

- So as close to "unitary" as possible for m_l in this mixed-action theory.
- Mixed-action SChPT at 1-loop has been calculated [E. Gámiz and CB], but still needs checking.

$K \rightarrow \pi$; HISQ on Asqtad

Sample Chiral Fit



- Statistical errors: ~0.2% -- 0.3%
- Different chiral fits tried so far agree within 1 stat. σ. E.g.:
  - 1-loop SChPT + 2-loop continuum ChPT.
  - 1-loop SChPT + higher order analytic.

- Need to understand the size of a^2 effects better; check SChPT.

$K \to \pi$; HISQ on Asqtad

- ✦ Expected error budget:
  - Statistical: 0.2--0.3%
  - Chiral extrapolation, fitting function: 0.1%
  - Discretization: 0.15%
  - Mistuning of m_s in the sea: 0.2%
- ✦ Total: 0.35%--0.5%, should be competitive with state of the art: RBC/UKQCD.

$K \rightarrow \pi$ semileptonic decay

- Focus at $q^2=0$, where we can use the method HPQCD proposed for semileptonic D decay:
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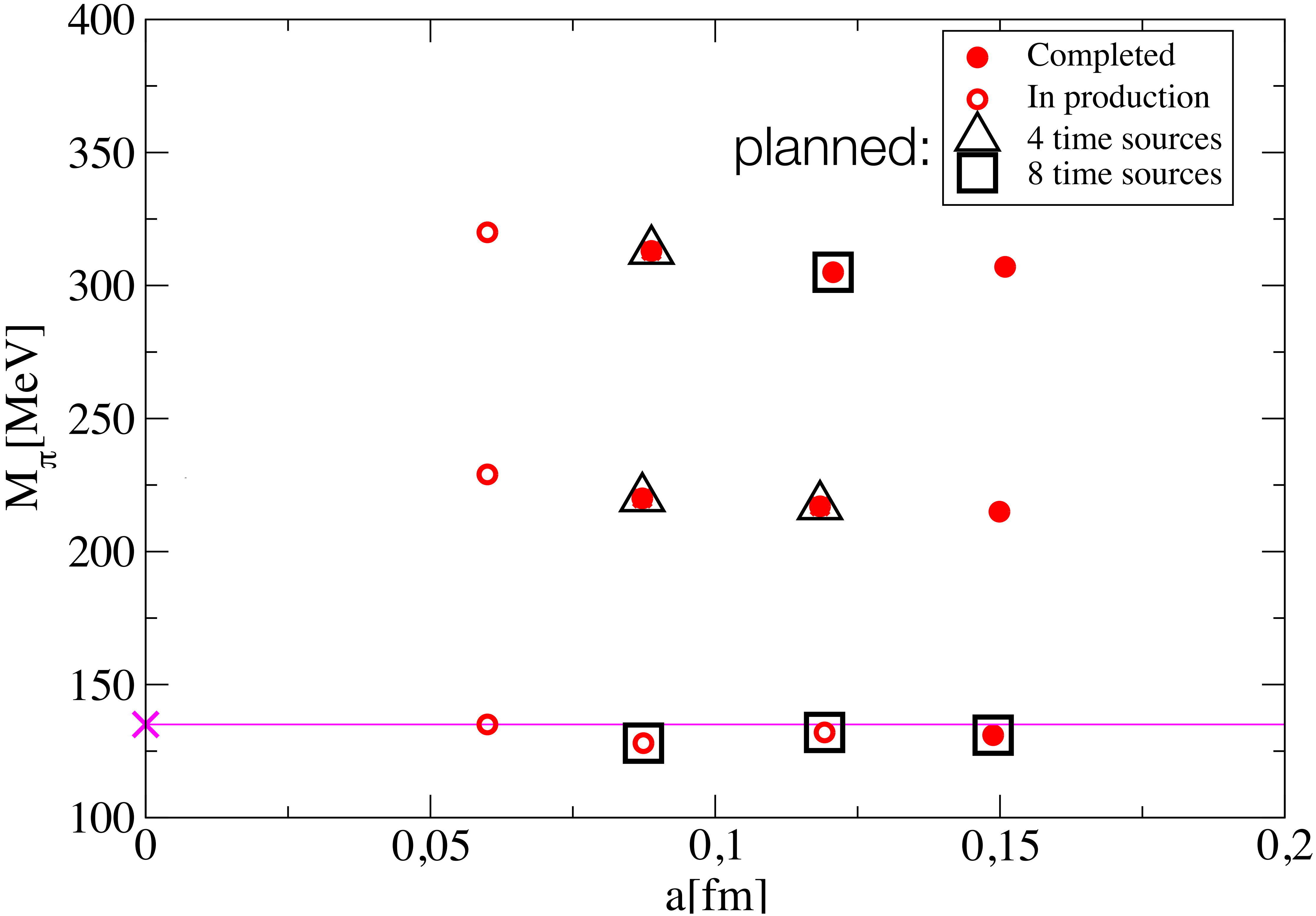
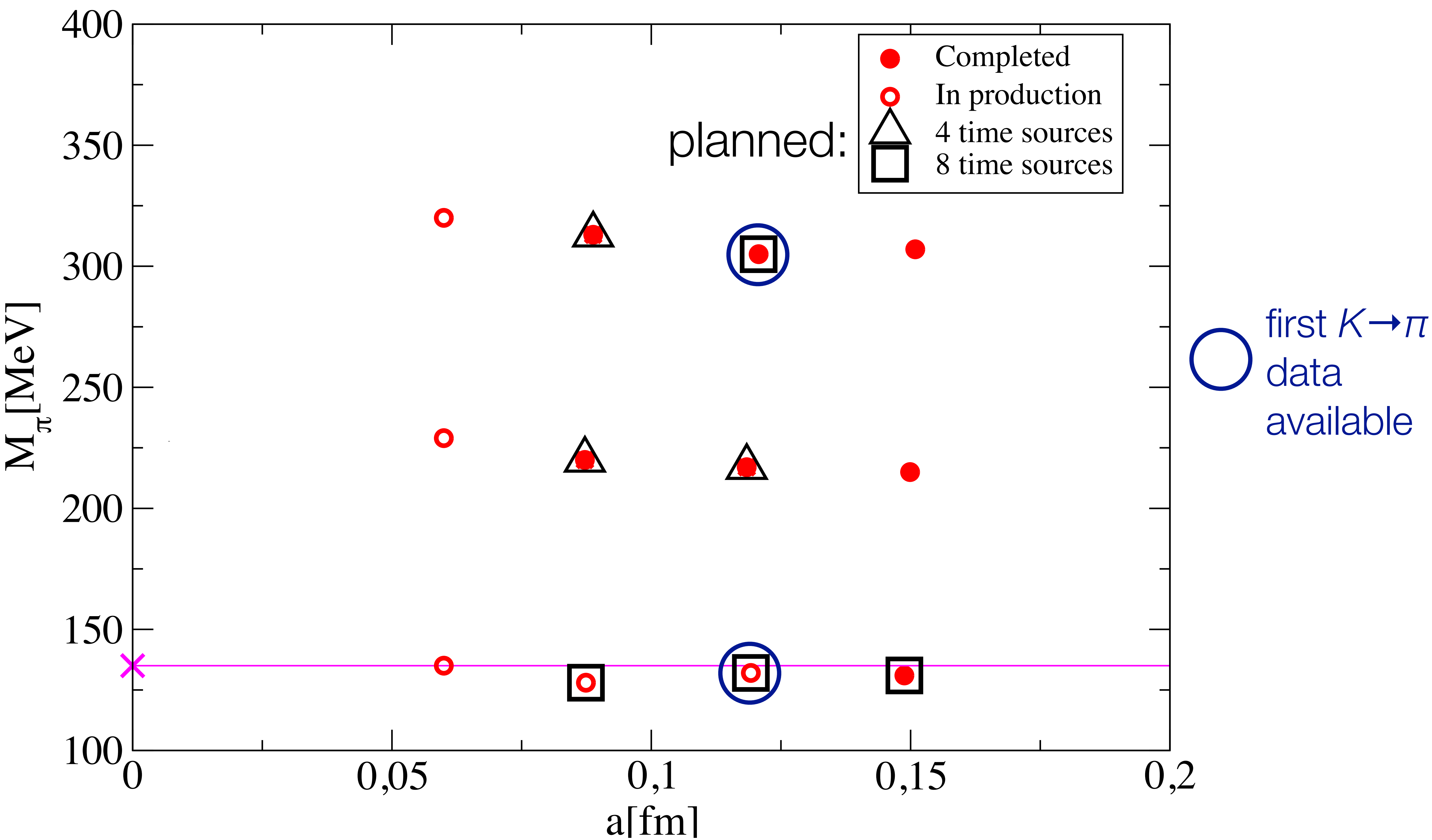
Fermilab/MILC

E. Gámiz

HISQ Ensembles

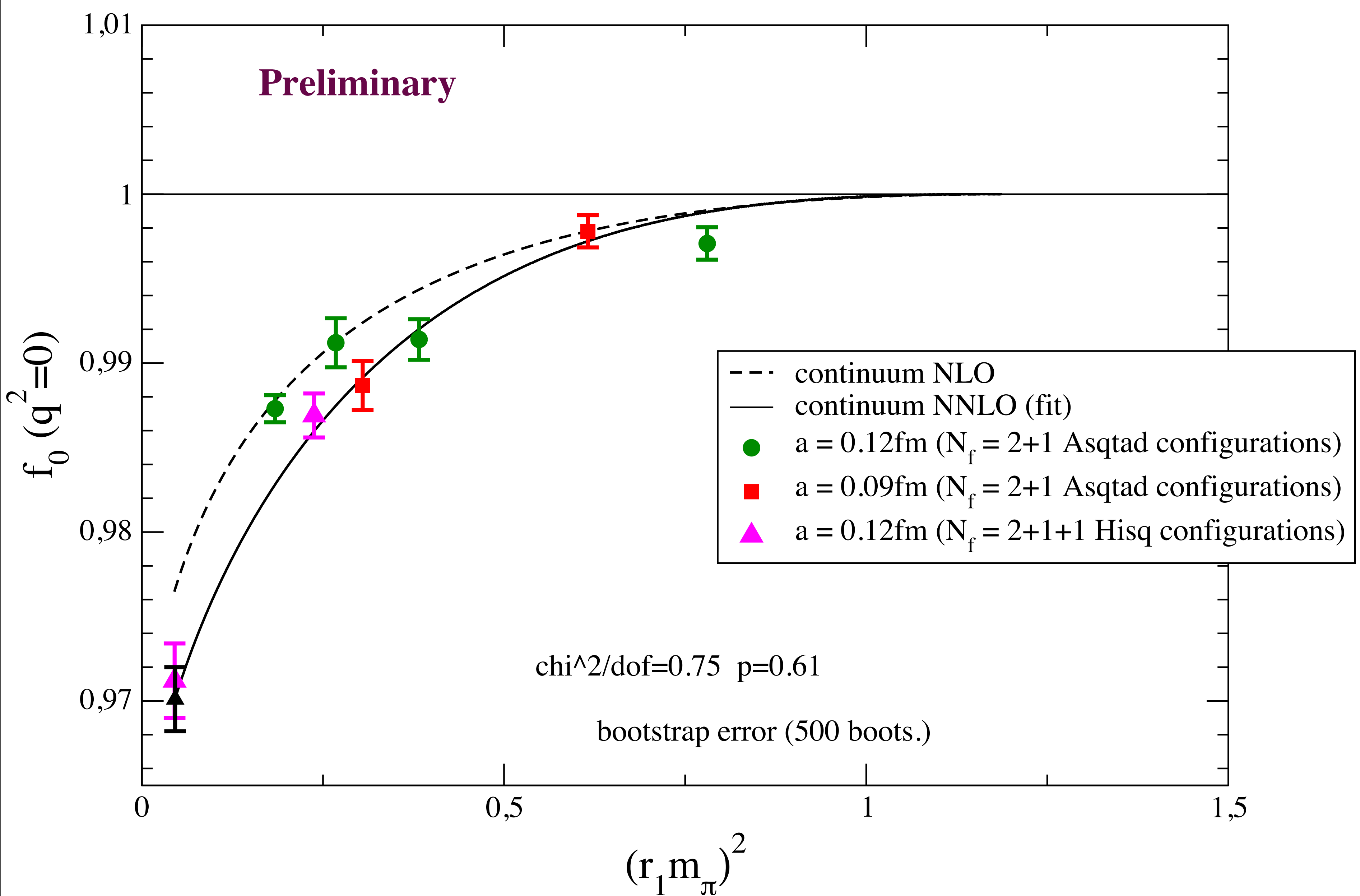


HISQ Ensembles



K→π : including HISQ on HISQ

Sample Chiral Fit



- Consistency with extrapolated HISQ on Asqtad results.
- Stat. errors larger on physical mass ensemble; momentum needed for *q*=0 is larger.
- Ensembles with heavier-than-physical u,d mass important for reducing final error.

- D→K being done in parallel, but fits not analyzed yet...