From Gross-Neveu to Gross-Pitaevskii, and beyond

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Conference in honor of
ROBERTO CASALBUONI
70th Birthday

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Ultracold atoms in optical lattices as simulators for quantum physics

Michele Modugno

IKERBASQUE & Department of Theoretical Physics, UPV/EHU - Bilbao

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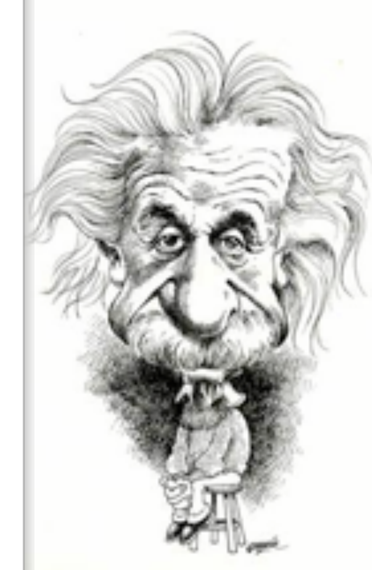
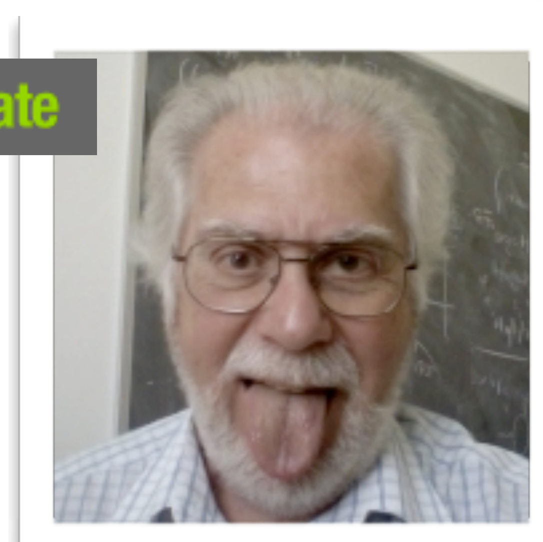
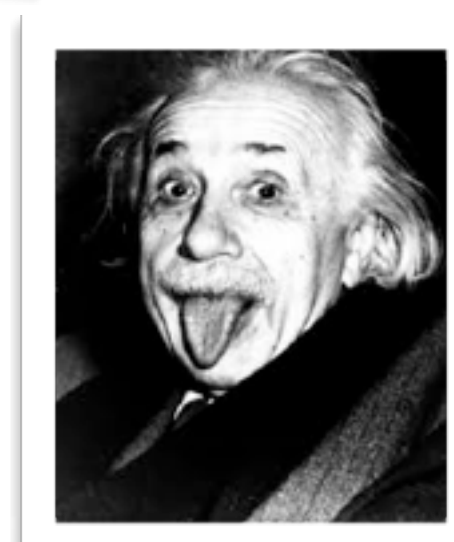
1993

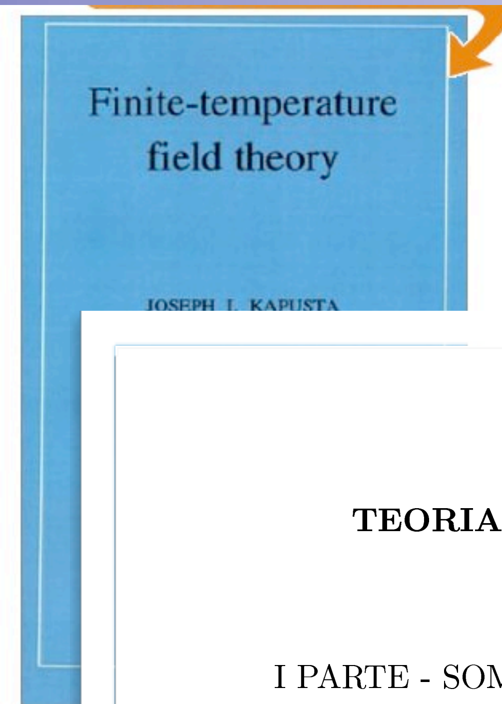


ResearchGate





summer 1992

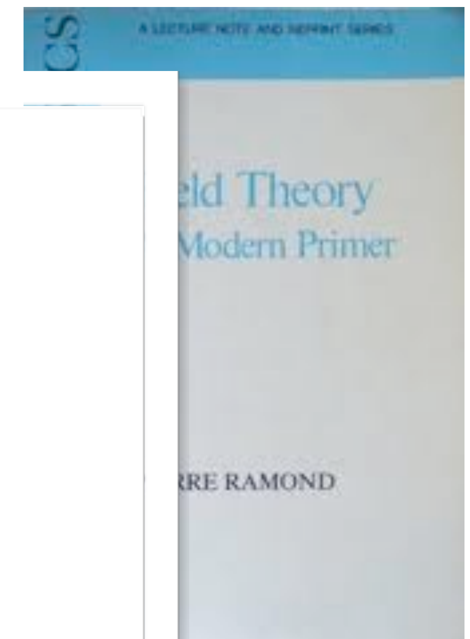


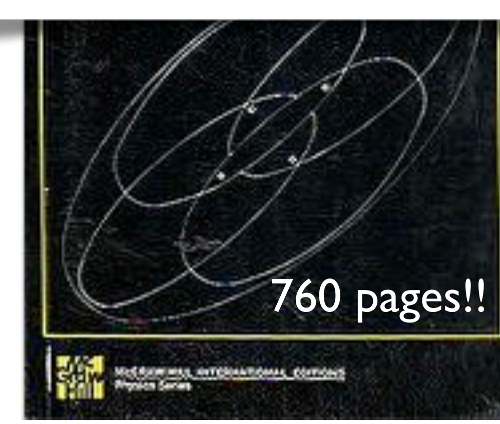
TEORIA DEI CAMPI

I PARTE - SOMMA SUI CAMMINI

Roberto Casalbuoni

Dipartimento di Fisica, Universita' di Firenze





Gross-Neveu

PHYSICAL REVIEW D VOLUME 10, NUMBER 10 15 NOVEMBER 1974

Dynamical symmetry breaking in asymptotically free field theories*

David J. Gross[†] and André Neveu
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 21 March 1974)

Two-dimensional massless fermion field theories with quartic interactions are analyzed. These models are asymptotically free. The models are expanded in powers of $1/N$, where N is the number of components of the fermion field. In such an expansion one can explicitly sum to all orders in the coupling constants. It is found that dynamical symmetry breaking occurs in these models for any value of the coupling constant. The resulting theories produce a fermion mass dynamically, in addition to a scalar bound state and, if the broken symmetry is continuous, a Goldstone boson. The resulting theories contain no adjustable parameters. The search for symmetry breaking is performed using functional techniques, the new feature here being that a composite field, say, $\bar{\psi}\psi$, develops a nonvanishing vacuum expectation value. The "potential" of composite fields is discussed and constructed. General results are derived for arbitrary theories in which all masses are generated dynamically. It is proved that in asymptotically free theories the dynamical masses must depend on the coupling constants in a nonanalytic fashion, vanishing exponentially when these vanish. It is argued that infrared-stable theories, such as massless-fermion quantum electrodynamics, cannot produce masses dynamically. Four-dimensional scalar field theories with quartic interactions are analyzed in the large-N limit and are shown to yield unphysical results. The models are extended to include gauge fields. It is then found that the gauge vector mesons acquire a mass through a dynamical Higgs mechanism. The higher-order corrections, of order $1/N$, to the models are analyzed. Essential singularities, of the Borel-summable type, are discovered at zero coupling constant. The origin of the singularities is the ultraviolet behavior of the theory.

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - M)\psi + \frac{\lambda}{2N}(\bar{\psi}\psi)^2$$

PHYSICAL REVIEW D VOLUME 51, NUMBER 6 15 MARCH 1995

Thermodynamics of the massive Gross-Neveu model

A. Barducci, R. Casalbuoni, M. Modugno, and G. Pettini
Dipartimento di Fisica, Università di Firenze, Istituto Nazionale di Fisica Nucleare, Sezione di Firenze, Firenze, Italy

R. Gatto
Département de Physique Théorique, Université de Génève, Génève, Switzerland
(Received 13 June 1994)

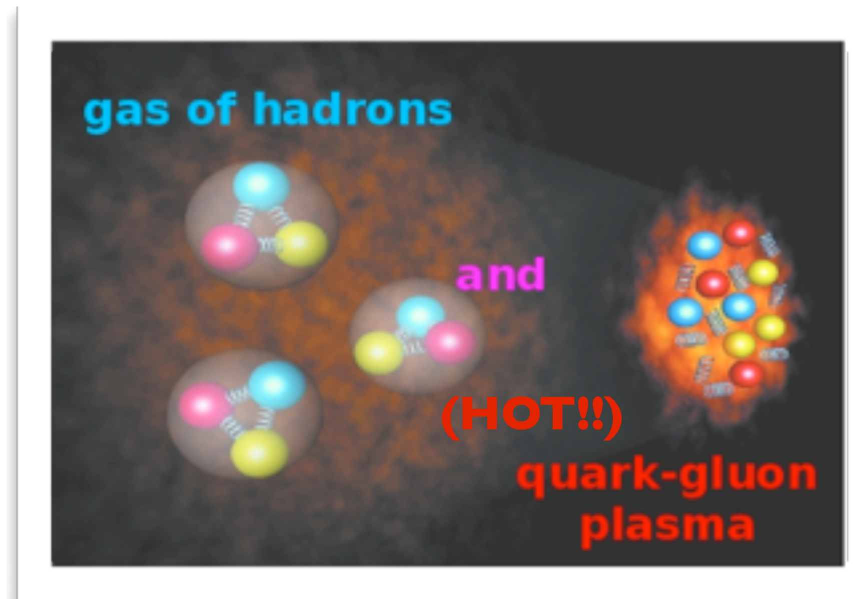
We study the thermodynamics of massive Gross-Neveu models with explicitly broken discrete or continuous chiral symmetries for finite temperature and fermion densities. The large N limit is discussed, paying attention to the no-go theorems for symmetry breaking in two dimensions which apply to the massless cases. The main purpose of the study is to serve as an analytical orientation for the more complex problem of the chiral transition in four-dimensional QCD with quarks. For any nonvanishing fermion mass, we find, at finite densities, lines of first-order phase transitions. For small mass values, traces of would-be second-order transitions and a tricritical point are recognizable. We study the thermodynamics of these models, and in the model with broken continuous chiral symmetry we examine the properties of the pionlike state.

from Gross-Neveu to Gross-Pitaevskii

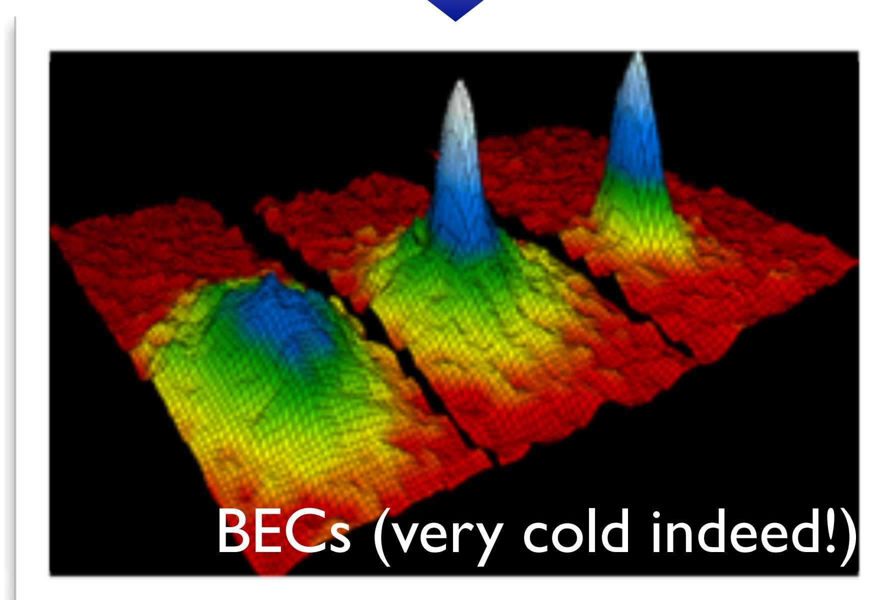
~~Gross-Neveu~~

Gross-Pitaevskii

$$E[\Phi]=\int d\mathbf{r}\left[\frac{\hbar^2}{2m}|\nabla\Phi|^2+V_{\text{ext}}(\mathbf{r})|\Phi|^2+\frac{g}{2}|\Phi|^4\right]$$



1998



ultracold atoms in optical lattices as simulators for quantum physics

Outline

◉ Overview (quantum simulators, ultracold atoms in optical lattices)

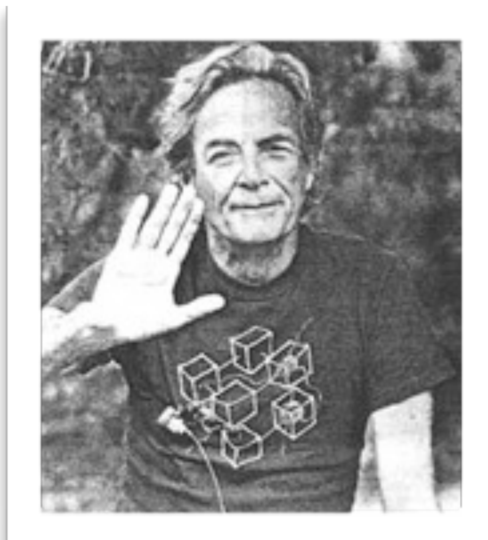
◉ <u>tight binding regime</u>: simulating condensed matter

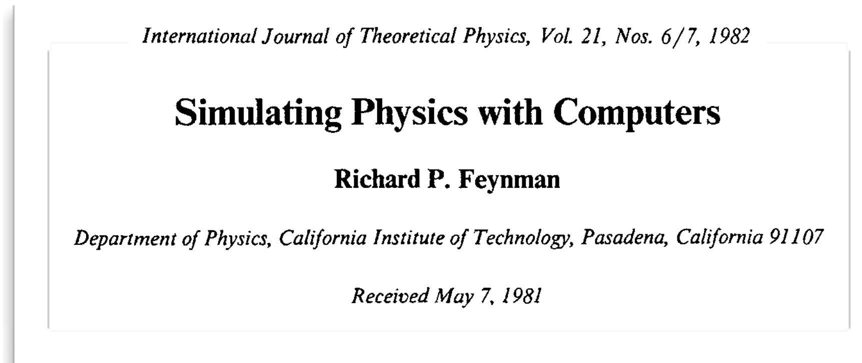
✔ *Maximally localized Wannier functions for ultracold atoms in 1D double-well periodic potentials* [MM and G. Pettini, NJP **14**, 055004 (2012)]

◉ <u>mean field regime</u>: simulating quantum mechanics

✔ *Anomalous Bloch oscillations in one-dimensional parity-breaking periodic potentials* [G. Pettini and MM, PRA **83**, 013619 (2011)]

quantum simulators





International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

Can physics be simulated by a universal computer?

"Universal Quantum Simulator": a certain class of quantum systems which would simulate other quantum systems

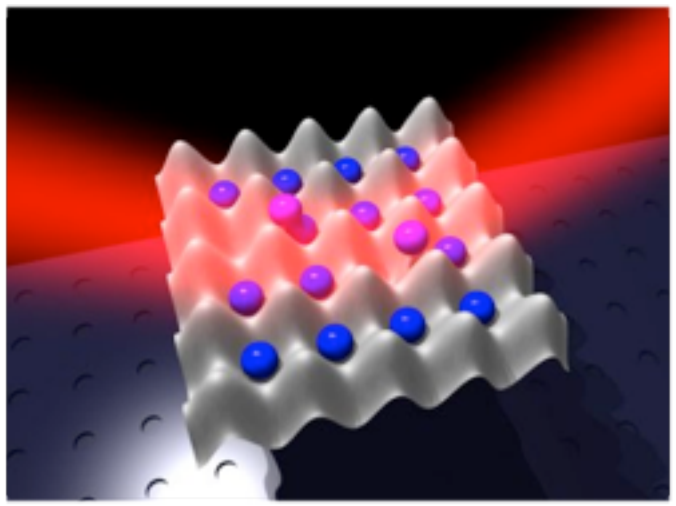
quantum simulators

- characteristics:
  - implement the hamiltonian of other physical systems
  - prepare the relevant quantum state
  - tunability and control of the parameters
  - precise measurements

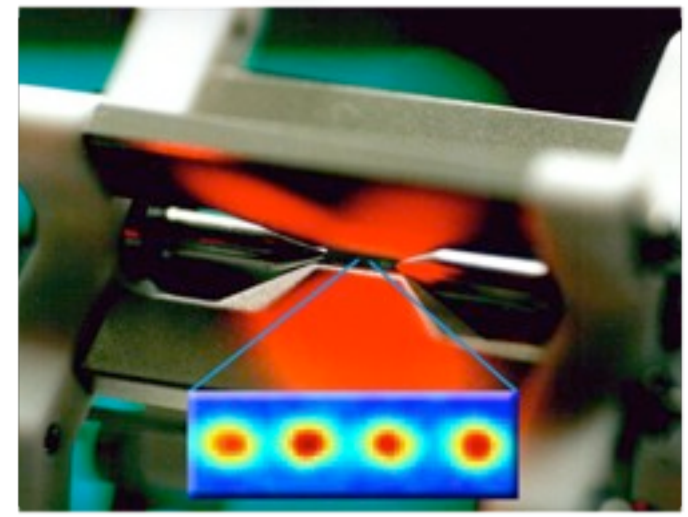
- scope:
  - reproducing the quantum behavior of systems that are difficult to access
  - control and analisys of specific effects (that could be hidden)
  - explore new parameter regimes (even unphysical)
  - substitute "classical" computation

quantum simulators

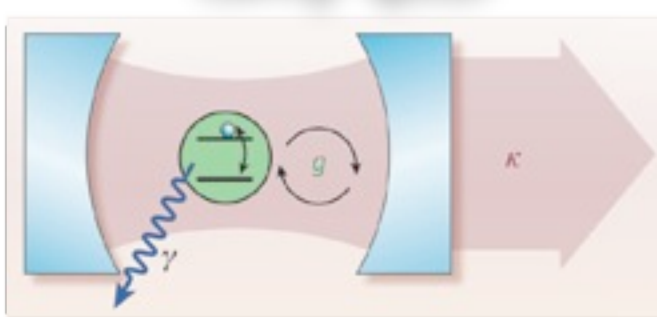
ultracold atoms



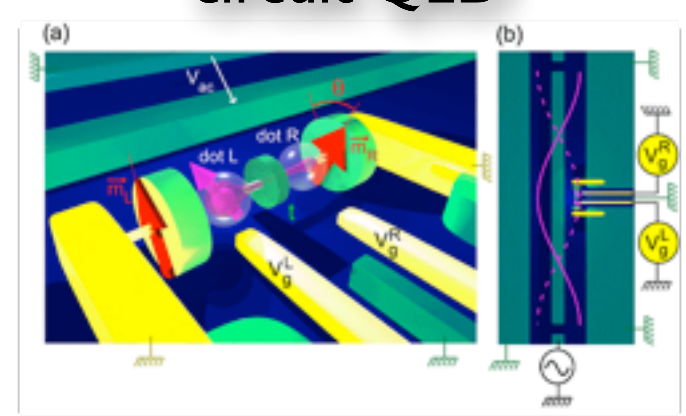
trapped ions

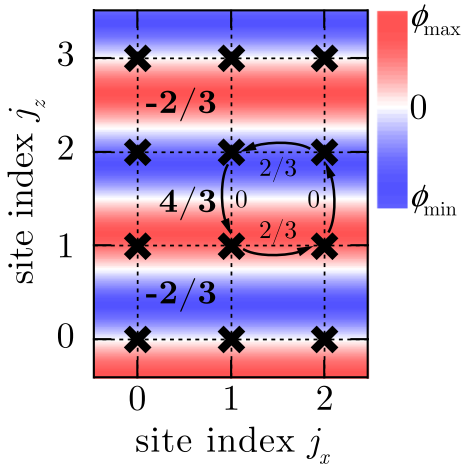


cavity QED

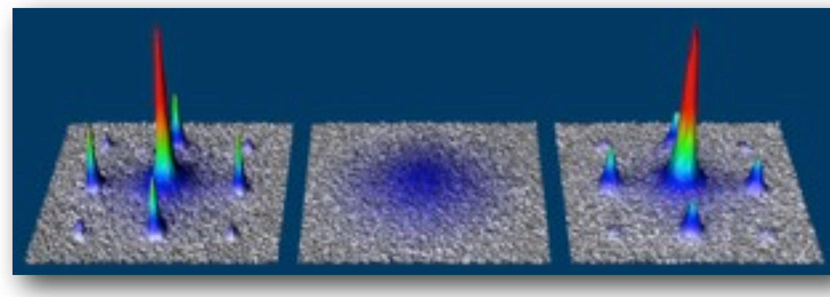


circuit QED

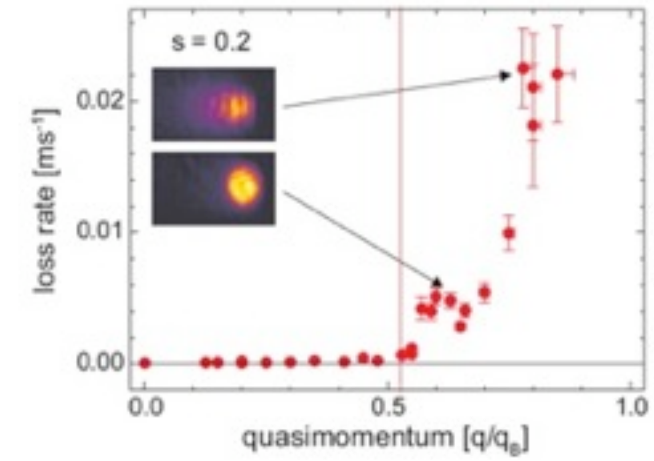




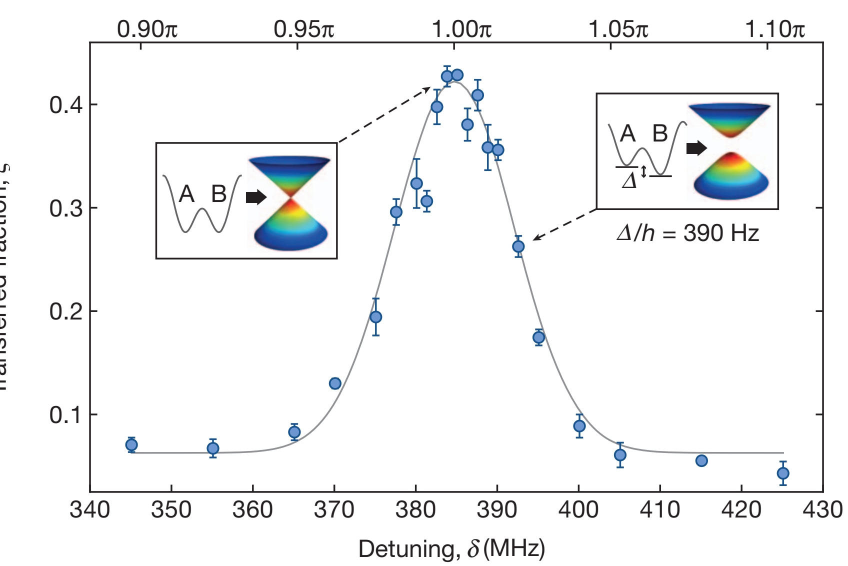
artificial **gauge fields** [NIST 2009-2012]



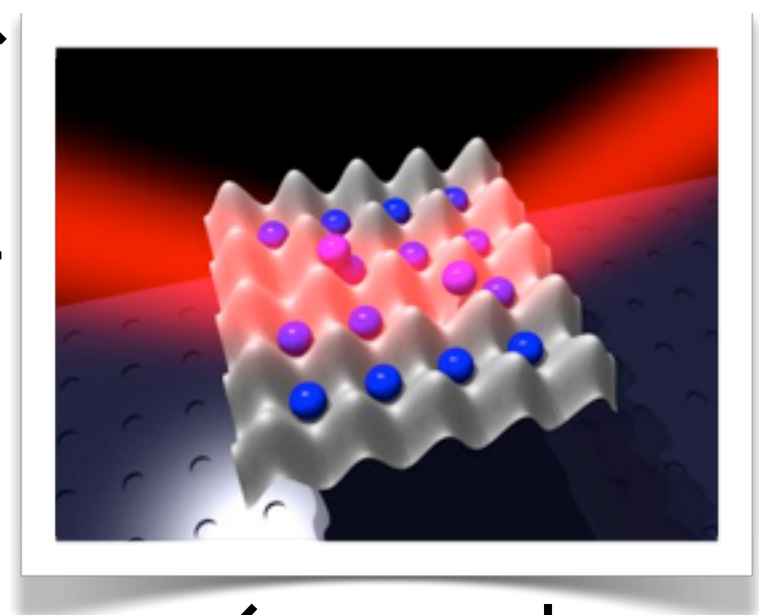
Mott insulator transition [Munich 2002]

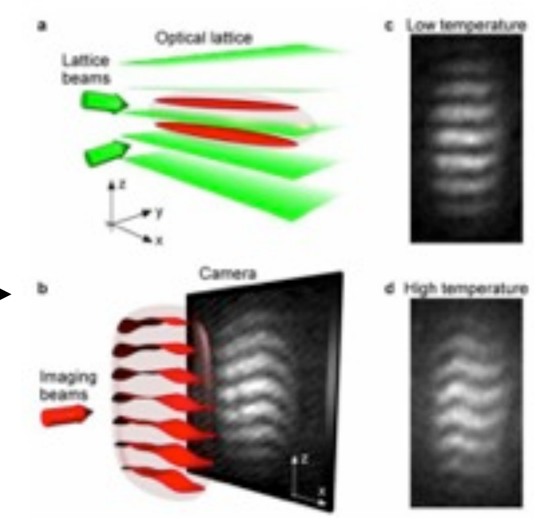


nonlinear **instabilities** [LENS 2003]

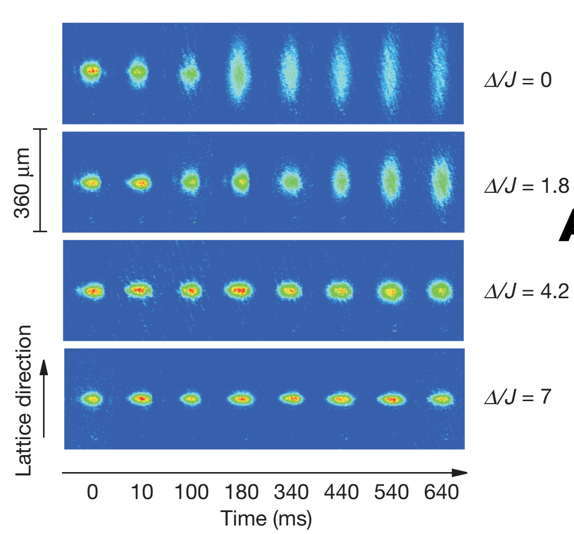


creation and manipulation of **Dirac points** [Zurich 2011]

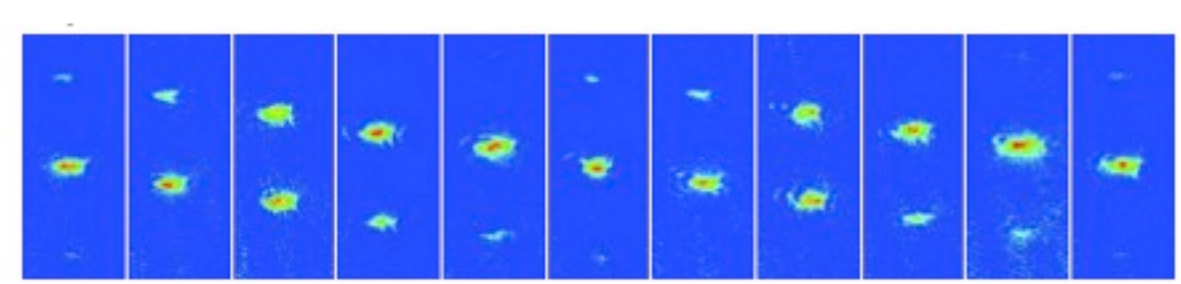




BKT transition [Paris 2006]



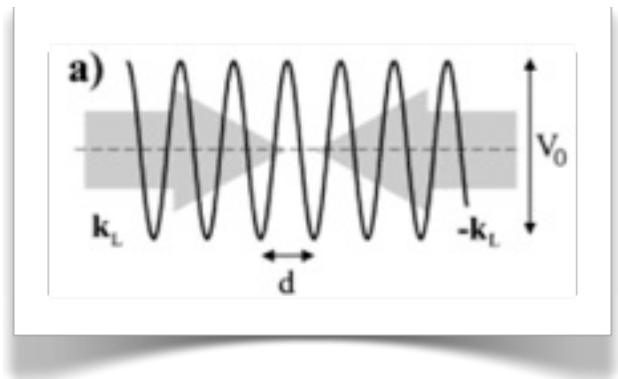
Anderson localization [LENS, Paris 2008]



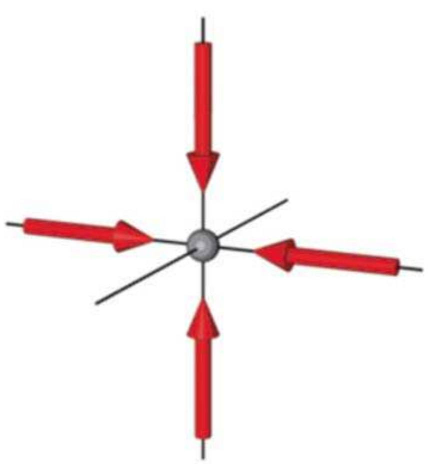
long lived **Bloch oscillations** [LENS, Innsbruck 2006-2007]

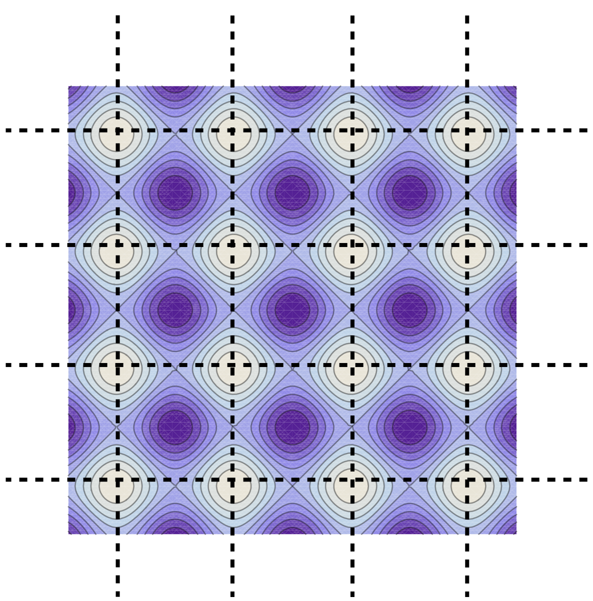
optical lattices

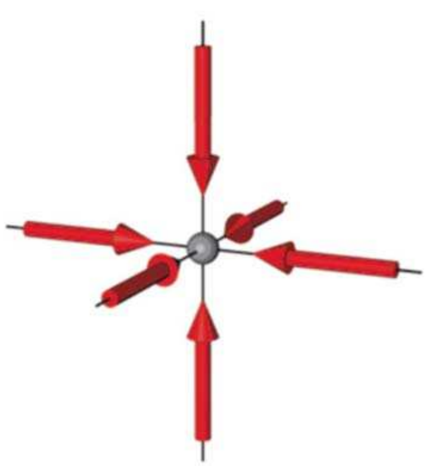
atom-laser electric dipole interaction

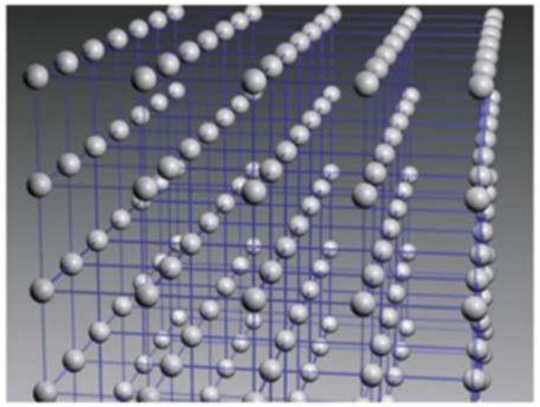


two counterpropagating laser beams → (periodic) standing wave



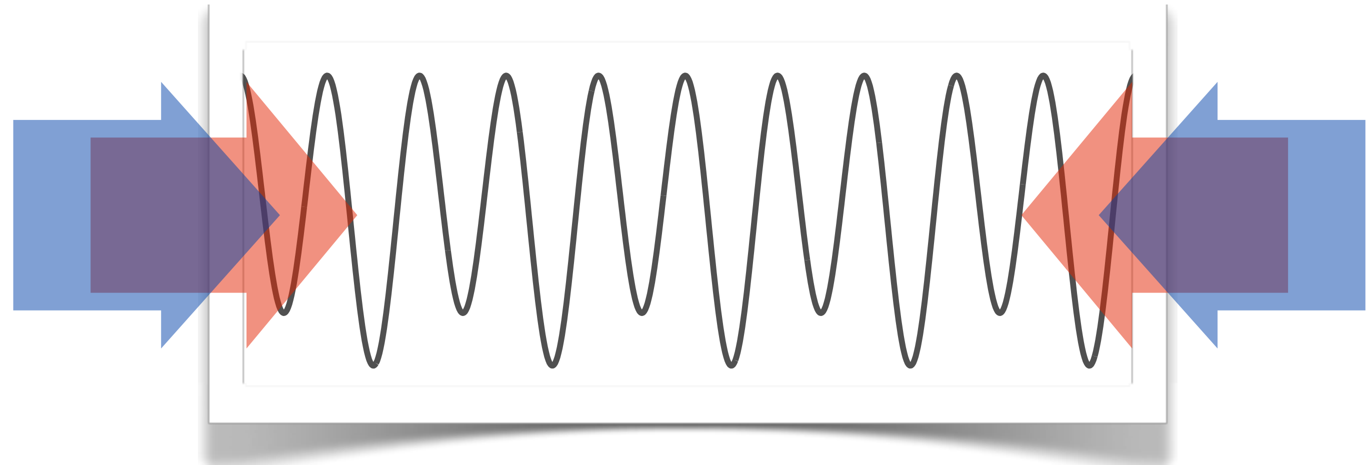




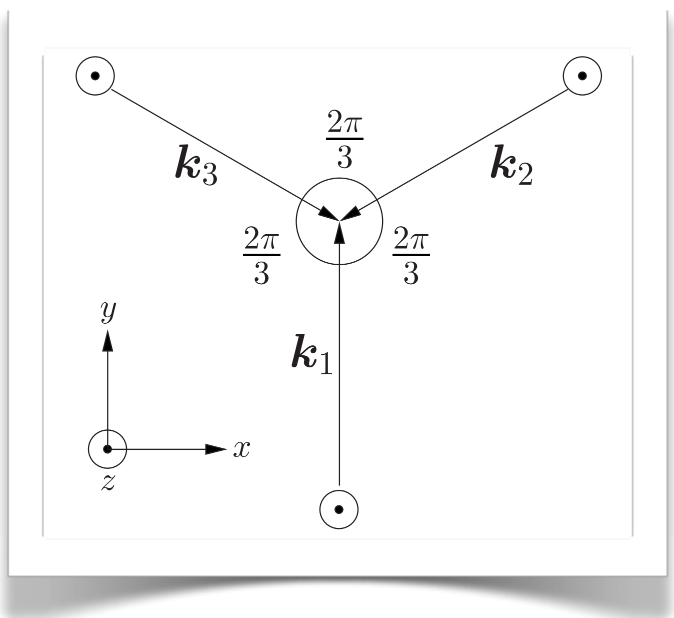


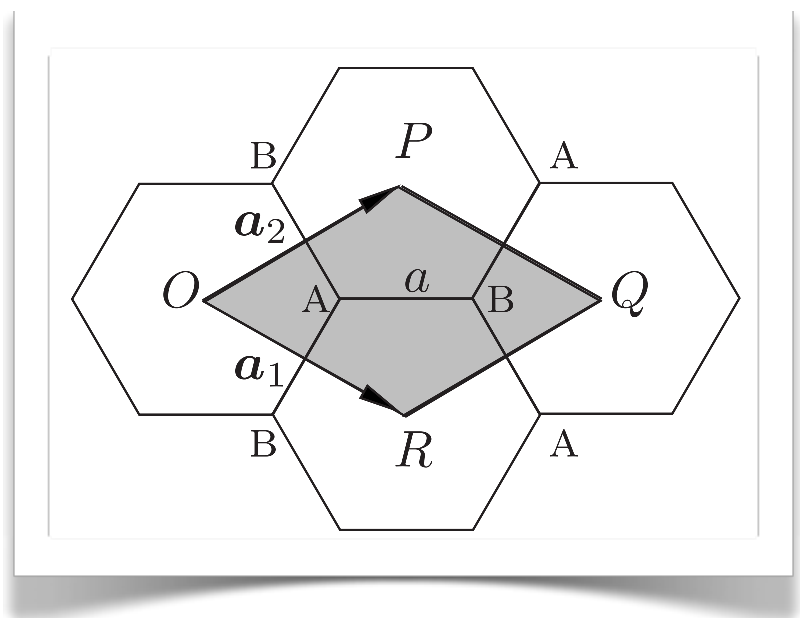
optical lattices

periodic lattices with **multiple wells per unit cell**



graphene-type optical lattice [Lee et al., PRA **80**, 043411 (2009)]





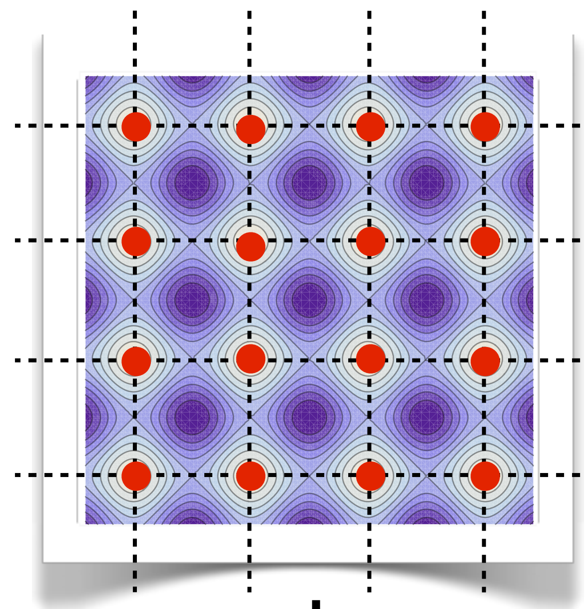
+

quasiperiodic structures, **disorder**

tight-binding regime:
simulating condensed matter

tight-binding regime

tight-binding: lattice intensity sufficiently high to localize the atoms in the lowest vibrational states of the potential wells



discrete lattice

it is convenient to map the system Hamiltonian onto a **tight-binding model**

tight-binding models

Bose–Hubbard model (bosons)

$$\hat{H}_{HB} = -J \sum_{\langle j,j' \rangle} \hat{a}_j^\dagger \hat{a}_{j'} + \sum_j \varepsilon_j \hat{n}_j + \frac{U}{2} \sum_j \hat{n}_j \left(\hat{n}_j - 1 \right)$$

on-site **energy**

tunneling between neighboring sites

on-site **interaction**

Hubbard model (fermions)

$$H = -J \sum_{\langle \mathbf{R}, \mathbf{R}' \rangle, \sigma} (\hat{c}^\dagger_{\mathbf{R},\sigma} \hat{c}_{\mathbf{R}',\sigma} + \mathrm{H.c.}) + U \sum_{\mathbf{R}} \hat{n}_{\mathbf{R}\uparrow} \hat{n}_{\mathbf{R}\downarrow}$$

the actual values of **J, U, ε depend on the parameters of the underlying continuous model** (those directly accessed in the experiment)

tight-binding models

expansion over a basis of **localized functions** at each potential well

$$\hat{\psi}(x) \equiv \sum_{nj} \hat{a}_{nj} f_{nj}(x)$$

$$[\hat{a}_{nj}, \hat{a}^{\dagger}_{n'j'}] = \delta_{jj'}\delta_{nn'}$$

precise knowledge of these basis functions important to connect the actual experimental parameters with the coefficients of the discrete model

Wannier functions

Bloch functions: $\psi_{nk}(x) = \mathrm{e}^{\mathrm{i}kx} u_{nk}(x)$

$$w_n(x - R_j) = \sqrt{\frac{d}{2\pi}} \int_{\mathcal{B}} \mathrm{d}k\, \mathrm{e}^{-\mathrm{i}kR_j} \psi_{nk}(x) \equiv w_{nj}(x)$$

not uniquely defined, their form depends on the (arbitrary) phase of Bloch functions

<u>simple sinusoidal potential</u>:
exponentially decaying Wannier functions discussed by Kohn [Phys. Rev. **115**, 809 (1959)]

maximally localized Wannier functions

for a generic potential the Kohn–Wannier recipe is not sufficient

(e.g. symmetric double well: KW functions display the same symmetry as the local potential structure and cannot be associated with a single lattice site)

- **tight binding Wannier functions** obtained from a specific **ansatz** based on linear combinations of **"atomic orbitals"** localized in each potential well of the primitive cell

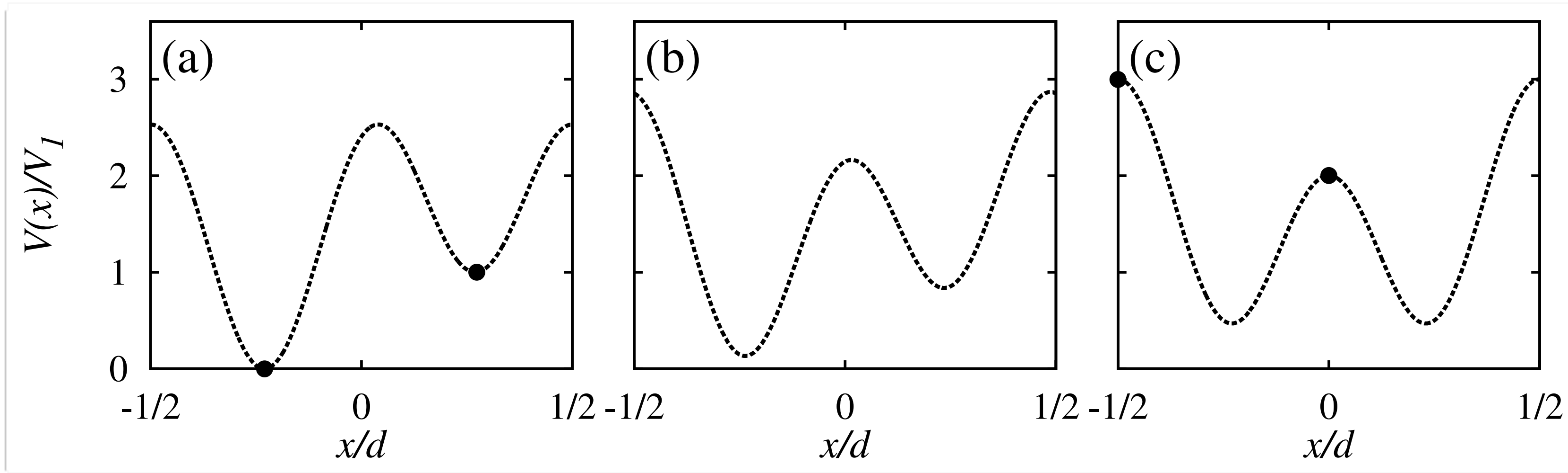
(e.g., for the case of a symmetric double-well unit cell of 2D graphene-like optical lattices [Lee et al., PRA 80, 043411 (2009)])

- **maximally localized Wannier functions** (**MLWFs**) by Marzari and Vanderbilt [PRB 56, 12847 (1997)], obtained by minimizing the spread of a set of Wannier functions via a **gauge transformation of Bloch eigenfunctions**

$$\tilde{w}_n(x - R_j) = \sqrt{\frac{d}{2\pi}} \int_{\mathcal{B}} \mathrm{d}k \, \mathrm{e}^{-\mathrm{i}kR_j} \sum_m U_{nm}(k)\psi_{mk}(x)$$

MLWFs for 1D double-well periodic potentials

[MM and G. Pettini, NJP **14**, 055004 (2012)]



$$V(x) = V_1 \sin^2(k_B x + \phi_0) + V_2 \sin^2(2k_B x + \theta_0 + 2\phi_0)$$

ultracold bosons in 1D optical lattices:

$$\hat{\mathcal{H}} = \int \mathrm{d}x \; \hat{\psi}^\dagger \hat{H}_0 \hat{\psi} + \frac{g}{2} \int \mathrm{d}x \; \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi} \equiv \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\mathrm{int}}$$

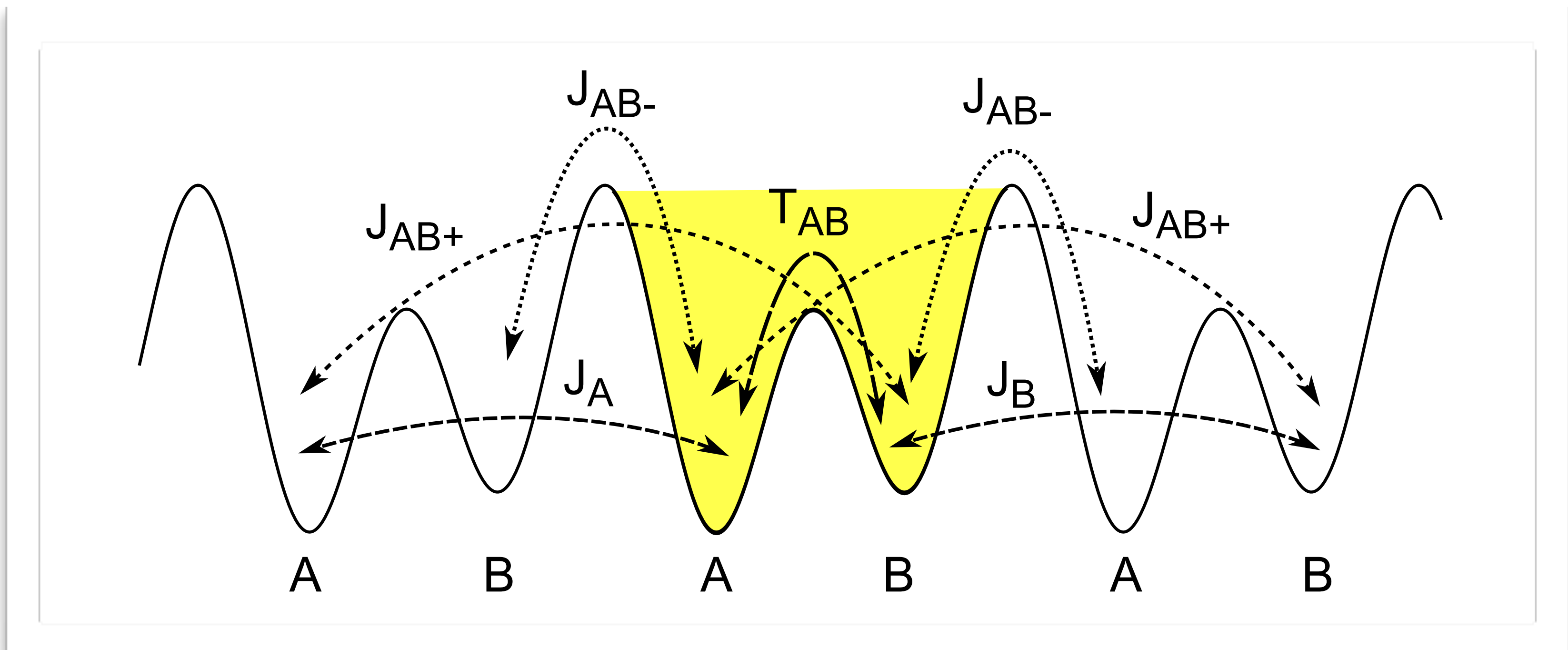
$$\hat{H}_0 = -(\hbar^2/2m)\nabla^2 + V(x)$$

tight-binding model

$$\hat{\psi}(x) \equiv \sum_{nj} \hat{a}_{nj} f_{nj}(x)$$

two wells: two-band **approximation** (~single-band approximation)

$$\hat{\mathcal{H}}_0 \simeq \sum_{\nu\nu'=\mathrm{A,B}} \sum_{jj'} \hat{a}^\dagger_{j\nu} \hat{a}_{j'\nu'} \langle f_{j\nu} | \hat{H}_0 | f_{j'\nu'} \rangle$$



nearest neighboring cell ($\neq$ well) **approximation**

MLWFs: minimizing the spread

Marzari and Vanderbilt:

$$\Omega = \sum_n [\langle x^2 \rangle_n - \langle x \rangle_n^2]$$

generalized spread of the Wannier functions

$$\Omega_{\mathrm{I}} + \Omega_{\mathrm{D}} = \sum_n \sum_{j \neq 0} |\langle w_{nj} | \hat{x} | w_{n0} \rangle|^2 + \Omega_{\mathrm{OD}} = \sum_{m \neq n} \sum_j |\langle w_{mj} | \hat{x} | w_{n0} \rangle|^2$$

- the method is implemented by a software package (**Wannier90**) adapted for computing MLWFs of real condensed matter systems
- in 1D it is possible to design **analytically a two-step gauge transformation** in terms of a set of ODEs (to be integrated numerically)

MLWFs: Berry formulation

generalized **Berry vector potentials**

$$A_{nm}(k) = \mathrm{i}\frac{2\pi}{d}\langle u_{nk}|\partial_k|u_{mk}\rangle$$

$$\gamma_n = \mathrm{i}\frac{2\pi}{d}\int_{\mathcal{B}}\langle u_{nk}|\partial_k|u_{nk}\rangle = \int_{\mathcal{B}} A_{nn}(k) \equiv \frac{2\pi}{d}\langle A_{nn}\rangle_{\mathcal{B}}$$

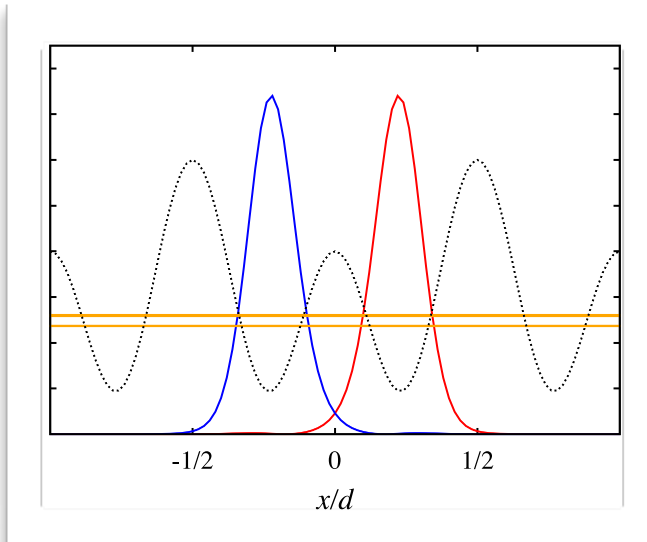
Zak–Berry phase

$$\Omega_{\mathrm{D}} = \sum_n \langle (A_{nn}(k) - \langle A_{nn}\rangle_{\mathcal{B}})^2\rangle_{\mathcal{B}} = \sum_n \Omega_{\mathrm{D}n}$$

$$\Omega_{\mathrm{OD}} = \sum_{m\neq n}\langle |A_{nm}|^2\rangle_{\mathcal{B}}$$

in 1D these two terms can be made strictly vanishing
(parallel transport gauge, $A_{nm}(k)$ diagonal)

gauge transformations



specific gauge transformation for a **composite two-band** system (we have two wells)

$$U_{nm}(k) = \mathrm{e}^{\mathrm{i}\phi_n(k)} S_{nm}(k)$$

$$\partial_k \phi_n = A_{nn} - \langle A_{nn} \rangle_{\mathcal{B}}$$

$$\begin{pmatrix} \cos\frac{\alpha}{2} + \mathrm{i}\sin\frac{\alpha}{2}\cos\theta & \mathrm{i}\,\mathrm{e}^{\mathrm{i}(\chi-\varphi)}\sin\theta\sin\frac{\alpha}{2} \\ \mathrm{i}\,\mathrm{e}^{\mathrm{i}\varphi}\sin\theta\sin\frac{\alpha}{2} & \mathrm{e}^{\mathrm{i}\chi}\left(\cos\frac{\alpha}{2} - \mathrm{i}\sin\frac{\alpha}{2}\cos\theta\right) \end{pmatrix}$$

$\in$ U(2)=SU(2)$\rtimes$ U(1)

technicalities

1. some math:

$$\frac{\partial_k \alpha}{2} = -\frac{\cos 2\theta}{\sin\theta}\left(A^{\mathrm{R}}_{12}\cos\eta + A^{\mathrm{I}}_{12}\sin\eta\right) - \mathrm{cotg}\frac{\alpha}{2}\mathrm{cotg}\theta\left(A^{\mathrm{R}}_{12}\sin\eta - A^{\mathrm{I}}_{12}\cos\eta\right) + \cos\theta\,(A_{11} - A_{22}),$$

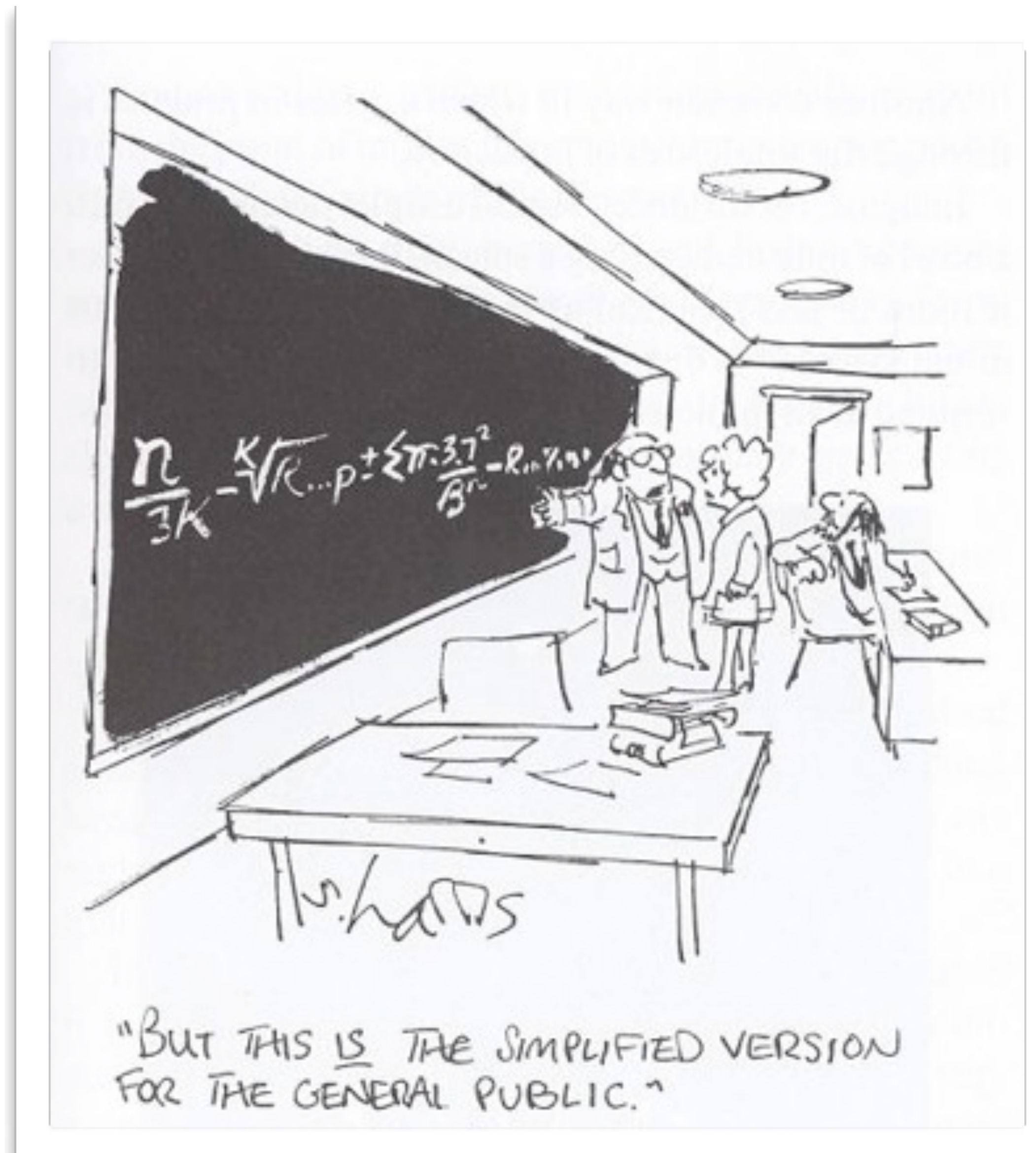
$$\partial_k \theta = \frac{\cos\theta\sin\alpha}{\sin^2(\alpha/2)}\left(A^{\mathrm{R}}_{12}\cos\eta + A^{\mathrm{I}}_{12}\sin\eta\right) + \frac{\cos\alpha}{\sin^2(\alpha/2)}\left(A^{\mathrm{R}}_{12}\sin\eta - A^{\mathrm{I}}_{12}\cos\eta\right) - \mathrm{cotg}\frac{\alpha}{2}\sin\theta\,(A_{11} - A_{22}),$$

$$\partial_k \eta = 0$$

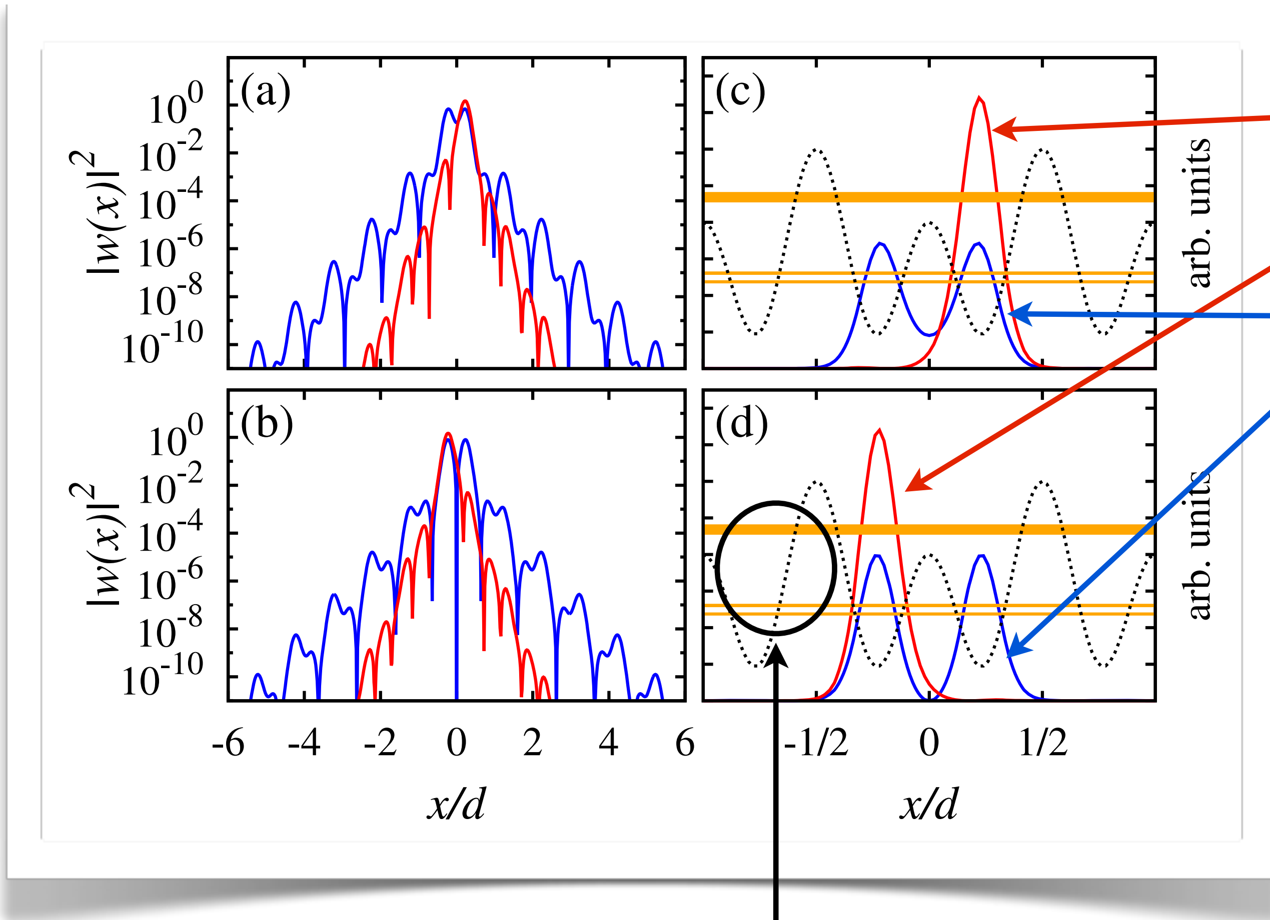
2. compute the Bloch eigenfunctions:
   - ✓ Fourier representation (k-space)
   - ✓ diagonalization of H_0
   - ✓ ensure smoothness in k

3. integrate the above set of ODEs with periodic boundary conditions

   (nested shooting algorithm / iterative method)



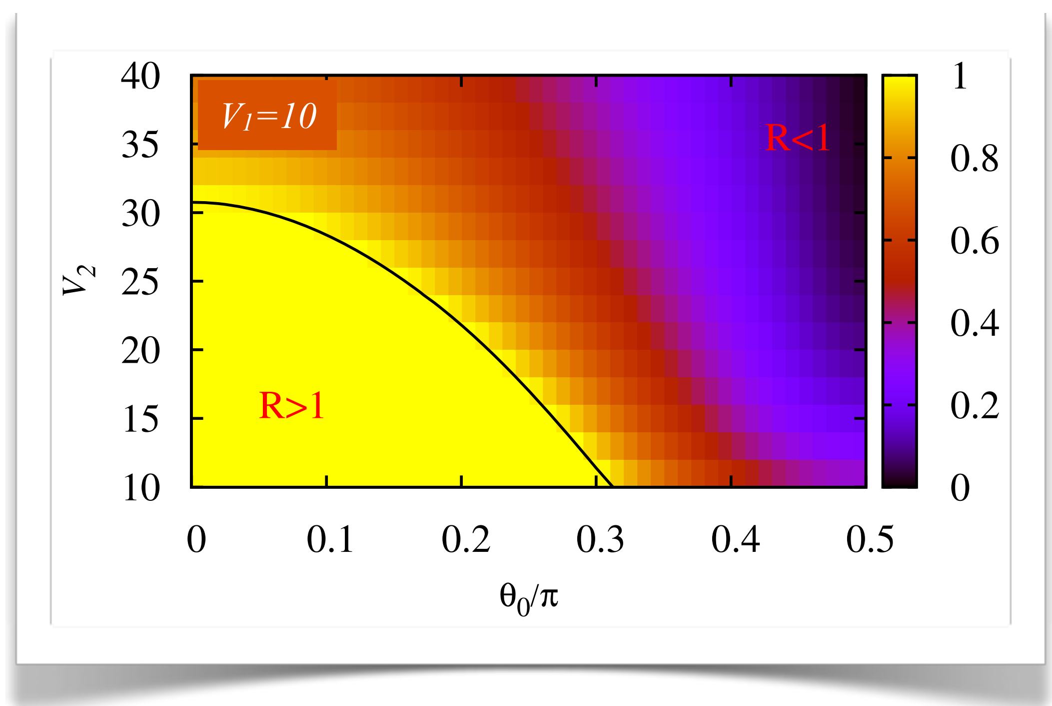
example of MLWFs



MLWFs

Wannier (single band)

regime of validity of the **composite band approach**:
band gap < distance between the 2nd and 3rd band



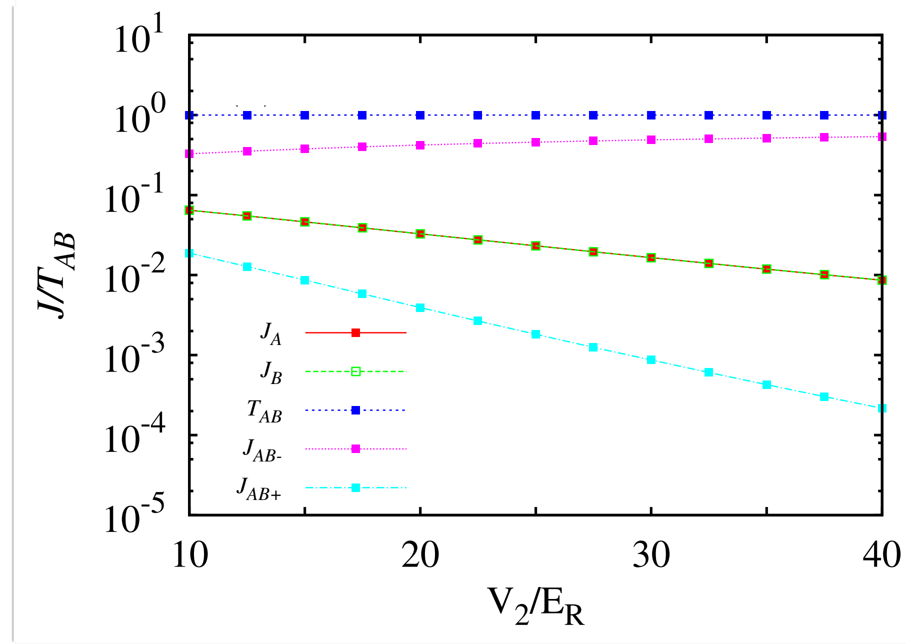
tunneling coefficients

$$E_\nu = \frac{d}{2\pi}\int_{\mathcal{B}} \mathrm{d}k \sum_{m=1}^{2} |S_{\nu m}(k)|^2 \varepsilon_m(k),$$

$$J_\nu = -\frac{d}{2\pi}\int_{\mathcal{B}} \mathrm{d}k\, \mathrm{e}^{-\mathrm{i}kd} \sum_{m=1}^{2} |S_{\nu m}(k)|^2 \varepsilon_m(k),$$

$$T_{\mathrm{AB}} = -\frac{d}{2\pi}\int_{\mathcal{B}} \mathrm{d}k\, \mathrm{e}^{\mathrm{i}\Delta\phi(k)} \sum_{m=1}^{2} S_{1m}^*(k) S_{2m}(k) \varepsilon_m(k),$$

$$J_{\mathrm{AB}_\pm} = -\frac{d}{2\pi}\int_{\mathcal{B}} \mathrm{d}k\, \mathrm{e}^{\mathrm{i}(\Delta\phi(k)\mp kd)} \sum_{m=1}^{2} S_{1m}^*(k) S_{2m}(k) \varepsilon_m(k),$$



dependence on **gauge transformation**
& **potential parameters**

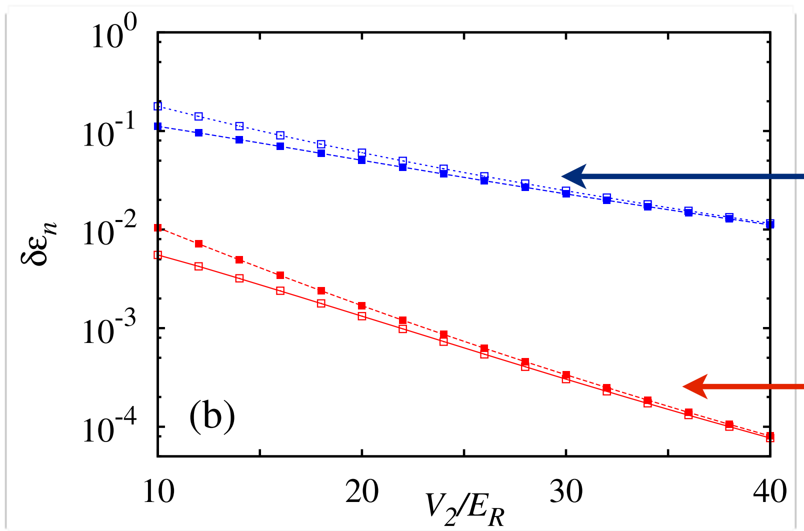
“fidelity” of the tb model

quadratic **energy spread**

$$\delta\varepsilon_n \equiv \frac{1}{\Delta\varepsilon_n}\sqrt{\frac{d}{2\pi}\int_{\mathcal{B}} \mathrm{d}k\,[\varepsilon_n(k)-\varepsilon_n^{\mathrm{tb}}(k)]^2}$$

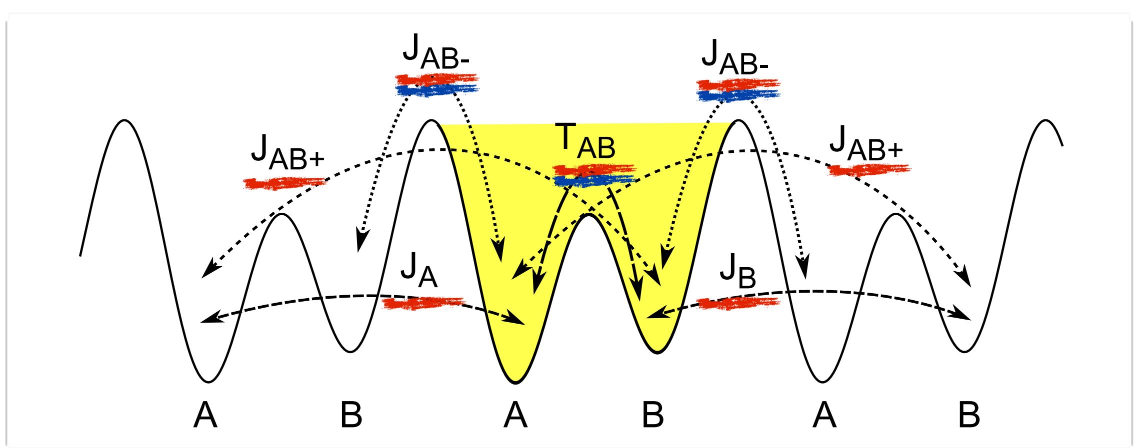
Bloch spectrum (exact)

tb model

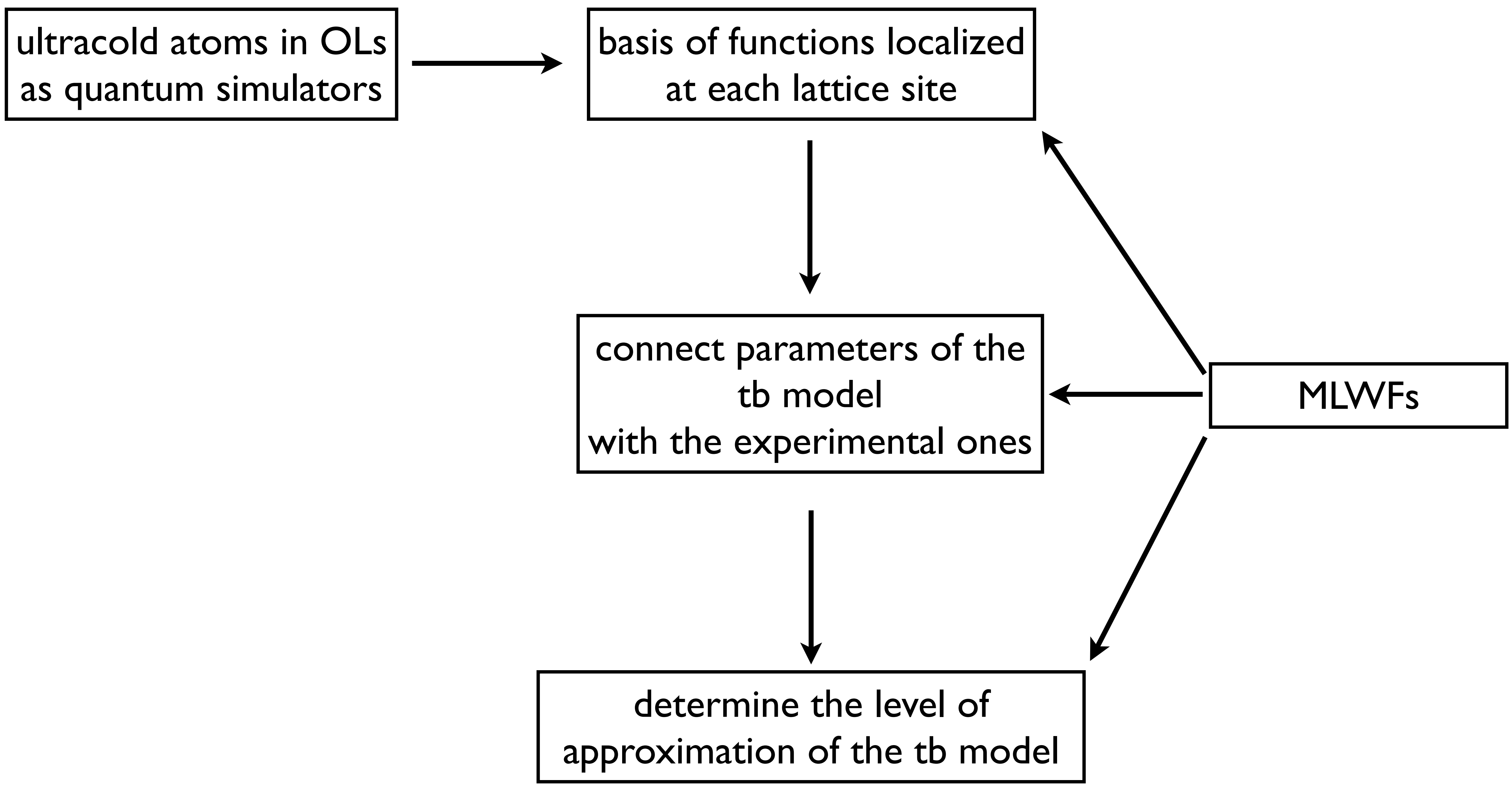


nearest neighboring **wells**

nearest neigboring **cells**



summarizing



mean field regime:
simulating quantum mechanics

Gross-Pitaevskii theory

$$i\hbar \frac{\partial}{\partial t}\Phi(\mathbf{r},t) = \left(-\frac{\hbar^2\nabla^2}{2m} + V_{\text{ext}}(\mathbf{r}) + \cancel{g|\Phi(\mathbf{r},t)|^2} \right)\Phi(\mathbf{r},t)$$

tuned by Feschbach resonances

periodic potential

simulate a single electron in a (perfect) crystal lattice

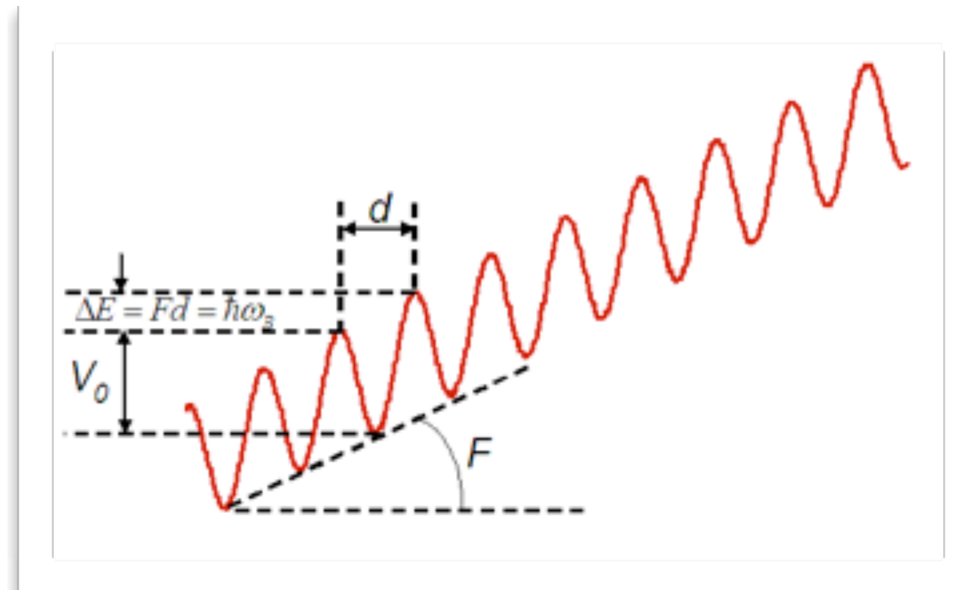
Bloch oscillations

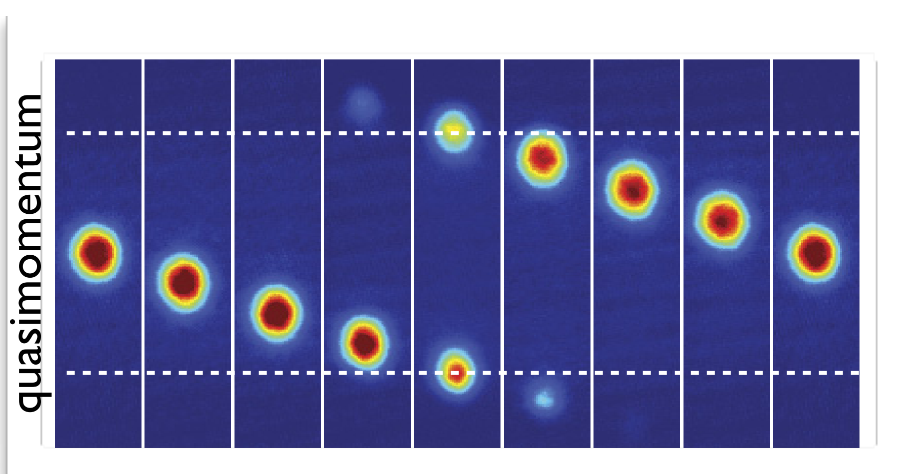
semiclassical eqs: particle dynamics described in terms of the wave packet centers x_c and k_c in coordinate and quasimomentum space

in the presence of a **constant force F** (single-band approximation)

$$\dot{x}_c = \frac{1}{\hbar}\frac{\partial \varepsilon}{\partial k}\bigg|_{k=k_c}$$

$$\dot{k}_c = F/\hbar$$





first Brillounin zone

time

Anomalous Bloch oscillations in 1D parity-breaking potentials

for **time-dependent, parity-breaking** potentials:

$$\dot{x}_c = \frac{1}{\hbar}\frac{\partial \varepsilon}{\partial k}\bigg|_{k=k_c} + \underbrace{\frac{\partial A_k}{\partial t}\bigg|_{k=k_c} - \frac{\partial \chi_k}{\partial k}\bigg|_{k=k_c}}$$

$$\dot{k}_c = F/\hbar$$

normal velocity

anomalous velocity

$$A_k(t) = i\,\frac{2\pi}{a}\langle u_k|\frac{\partial}{\partial k}|u_k\rangle \qquad \chi_k(t) = i\,\frac{2\pi}{a}\langle u_k|\frac{\partial}{\partial t}|u_k\rangle$$

simulates the effect of an electric field in quasimomentum space

(higher dimensions: also a Lorentz-like term,

k-space magnetic field given by the Berry curvature)

an experimental proposal

[G. Pettini and MM, PRA **83**, 013619 (2011)]

$$\mathcal{V}(\xi,\tau) = \mathcal{V}_1(\tau)\cos^2(\xi) + \mathcal{V}_2(\tau)\cos^2(2\xi+\theta)$$

Ingredients:

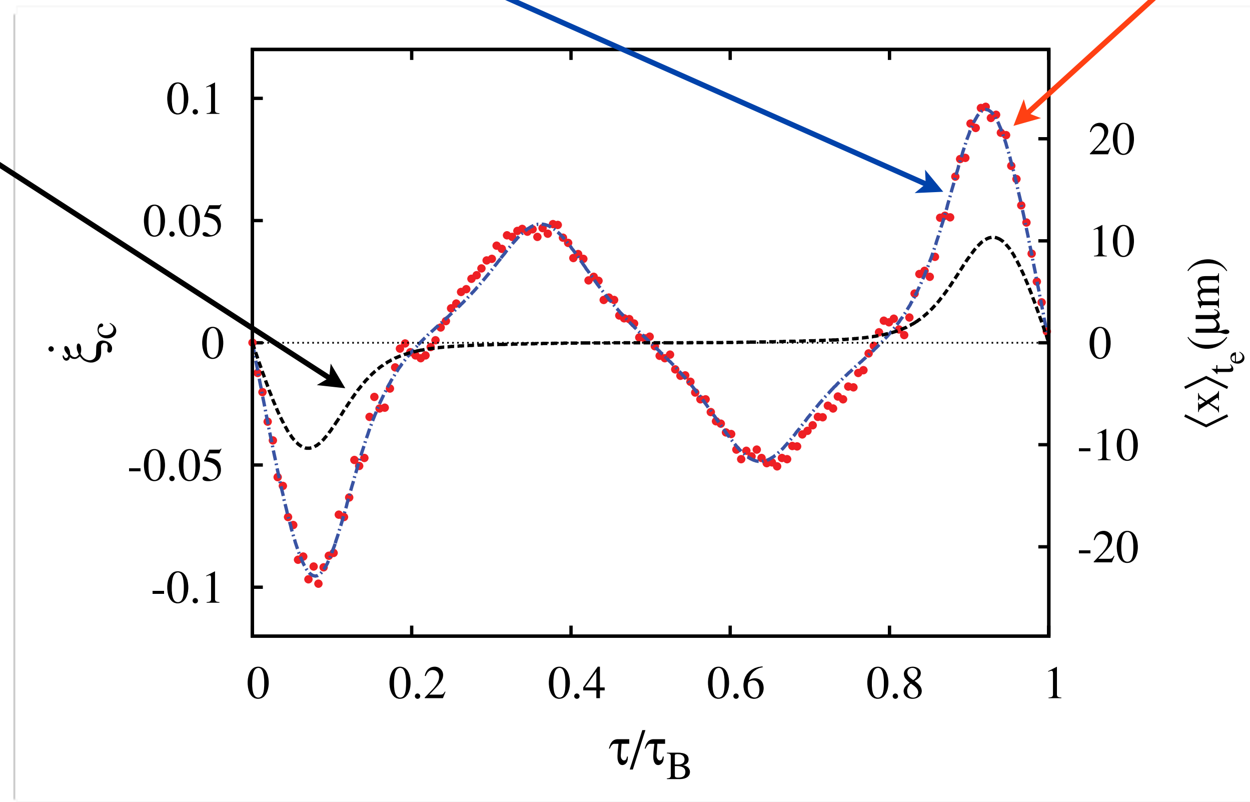
- parity breaking $\theta \neq n\pi/2$
- time modulation $\mathcal{V}_i(\tau) = \mathcal{V}_i(1 + A_i \sin^2(\Omega_i \tau))$

an experimental proposal

Berry semiclassical eqs.

full Schrödinger eq.

normal velocity



(in situ) CM velocity of the wavepaket

CM position after time-of-flight

$$\dot{\xi}_c(\tau) = \frac{2}{\hbar q}\frac{m}{t_e}\langle x\rangle_{t_e}$$

$$\lim_{t_e\to\infty} |\psi(x,t;t_e)|^2 \propto |\Psi(mx/t_e,t)|^2$$

t.o.f. = FT

final remarks

nature physics

INSIGHT | REVIEW ARTICLES

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Quantum simulations with ultracold quantum gases

Immanuel Bloch[1,2]*, Jean Dalibard[3] and Sylvain Nascimbène[1,3]

Ultracold quantum gases offer a unique setting for quantum simulation of interacting many-body systems. The high degree of controllability, the novel detection possibilities and the extreme physical parameter regimes that can be reached in these 'artificial solids' provide an exciting complementary set-up compared with natural condensed-matter systems, much in the spirit of Feynman's vision of a quantum simulator. Here we review recent advances in technology and discuss progress in a number of areas where experimental results have already been obtained.

Gross-Neveu back again!

PRL **105,** 190403 (2010) PHYSICAL REVIEW LETTERS week ending 5 NOVEMBER 2010

Cold Atom Simulation of Interacting Relativistic Quantum Field Theories

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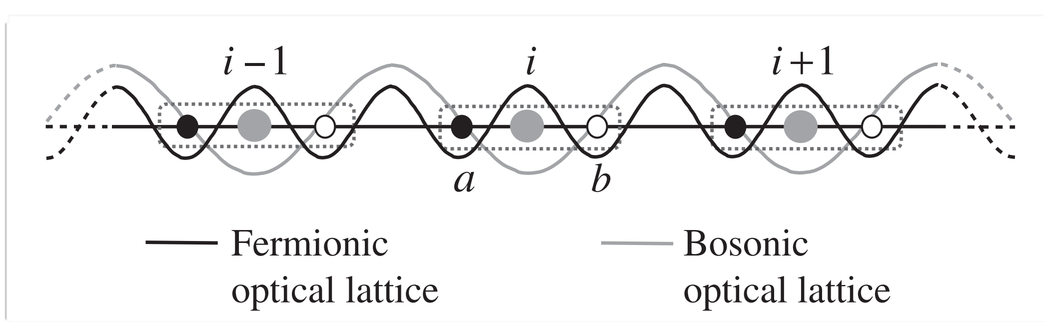
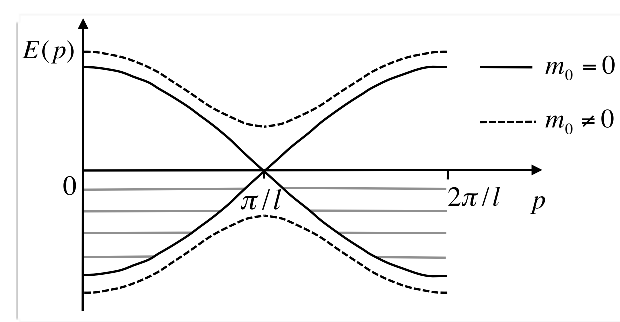
[2]*School of Economics and Management, Free University of Bozen-Bolzano, via Sernesi 1, 39100 Bolzano, Italy*
[3]*School of Physics and Astronomy, University of Leeds, Woodhouse Lane, Leeds LS2 9JT, UK*

(Received 15 June 2010; published 4 November 2010)

We demonstrate that Dirac fermions self-interacting or coupled to dynamic scalar fields can emerge in the low energy sector of designed bosonic and fermionic cold atom systems. We illustrate this with two examples defined in two spacetime dimensions. The first one is the self-interacting Thirring model. The second one is a model of Dirac fermions coupled to a dynamic scalar field that gives rise to the Gross-Neveu model. The proposed cold atom experiments can be used to probe spectral or correlation properties of interacting quantum field theories thereby presenting an alternative to lattice gauge theory simulations.

$$\frac{H_\Phi}{\hbar} = \int dx(v_s \bar{\Psi}_n \gamma_1 p \Psi_n + gm\Phi\bar{\Psi}_n\Psi_n + \frac{m^2}{2}\Phi^2)$$





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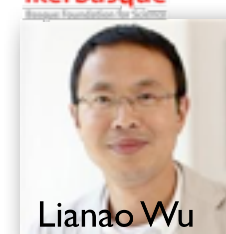
Quantum diffusion and localization
in quasiperiodic lattices

Universidad del Pais Vasco

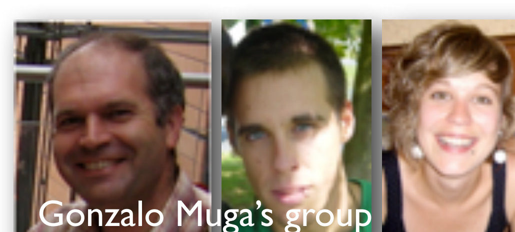
Ìñigo Egusquiza
Manuel Valle
Aitor Bergara
Asier Eiguren

ikerbasque



Quantum information
with ultracold atoms in OLs

Quantum Information, Science & Technology - UPV/EHU



Quantum control, transport and engineering
of ultracold atoms

Conference in honor of

ROBERTO CASALBUONI

Happy 70th Birthday !!!

GGI Arcetri, Firenze - September 21st, 2012