COLOR SUPERCONDUCTIVITY

Massimo Mannarelli

INFN-LNGS

massimo@lngs.infn.it

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“Compact Stars in the QCD Phase Diagram”, Copenhagen August 2001

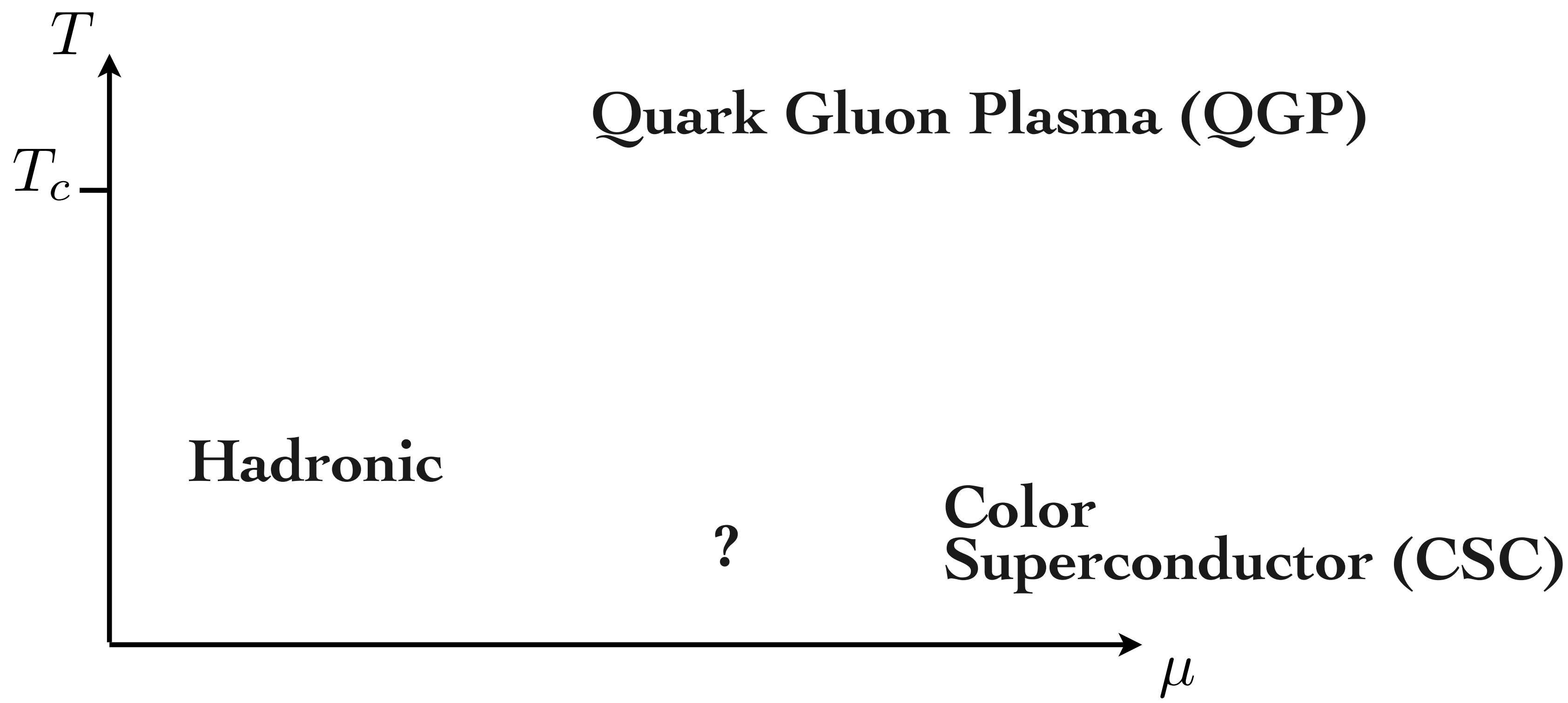
Outline

- Motivations
- Superconductors
- Color Superconductors
- Low energy degrees of freedom
- Crystalline color superconductors

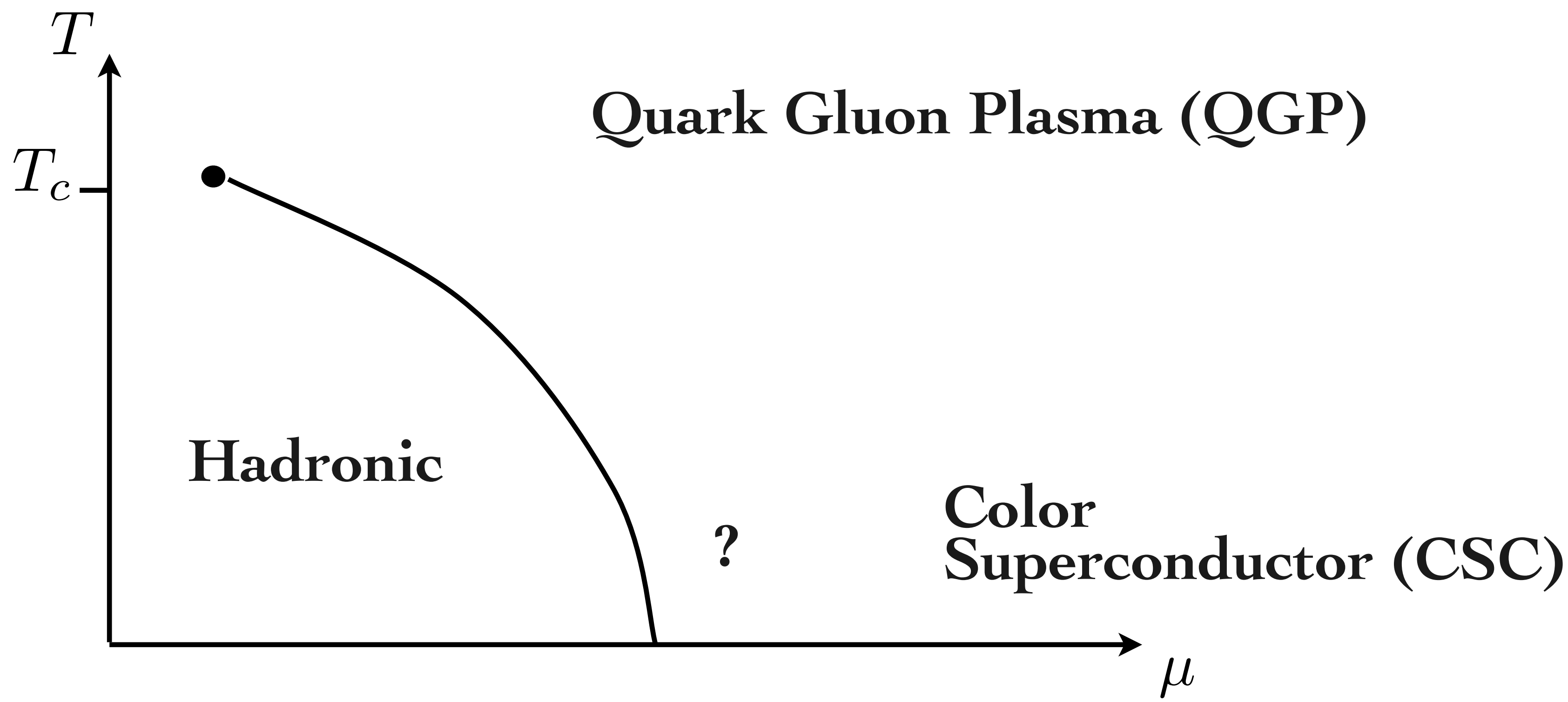
Reviews: hep-ph/0011333, hep-ph/0202037, 0709.4635
Lecture notes by Casalbuoni http://theory.fi.infn.it/casalbuoni/barcellona.pdf

MOTIVATIONS

QCD phase diagram



QCD phase diagram



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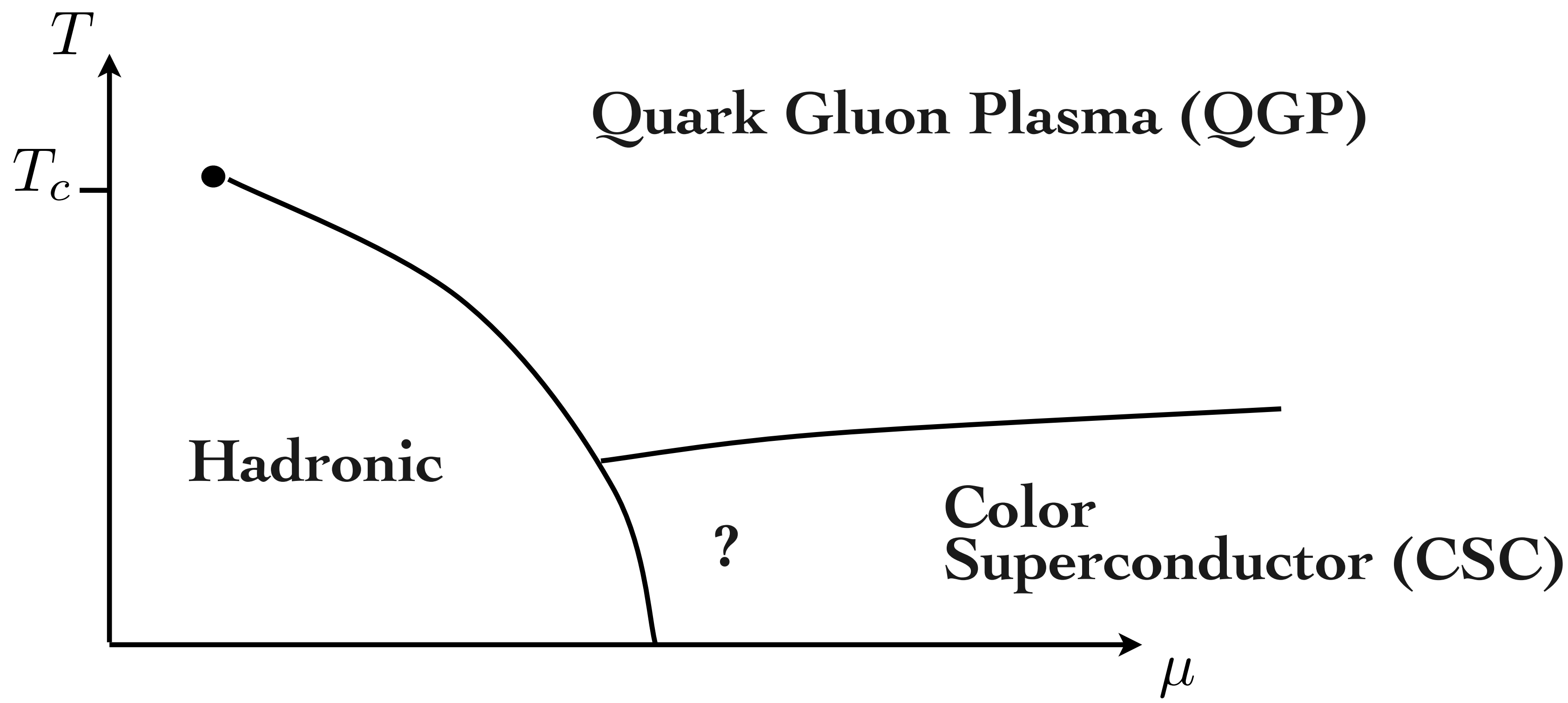

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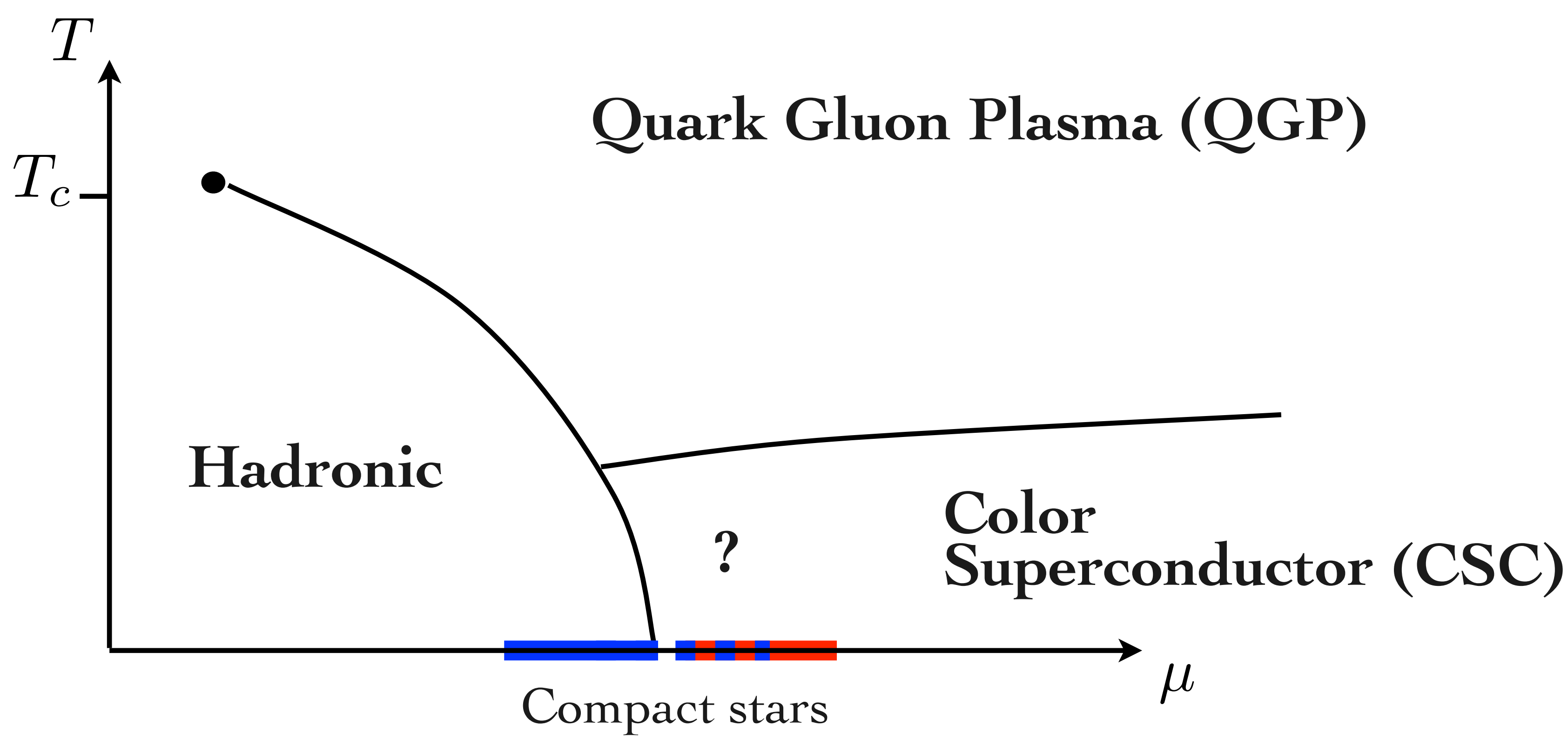

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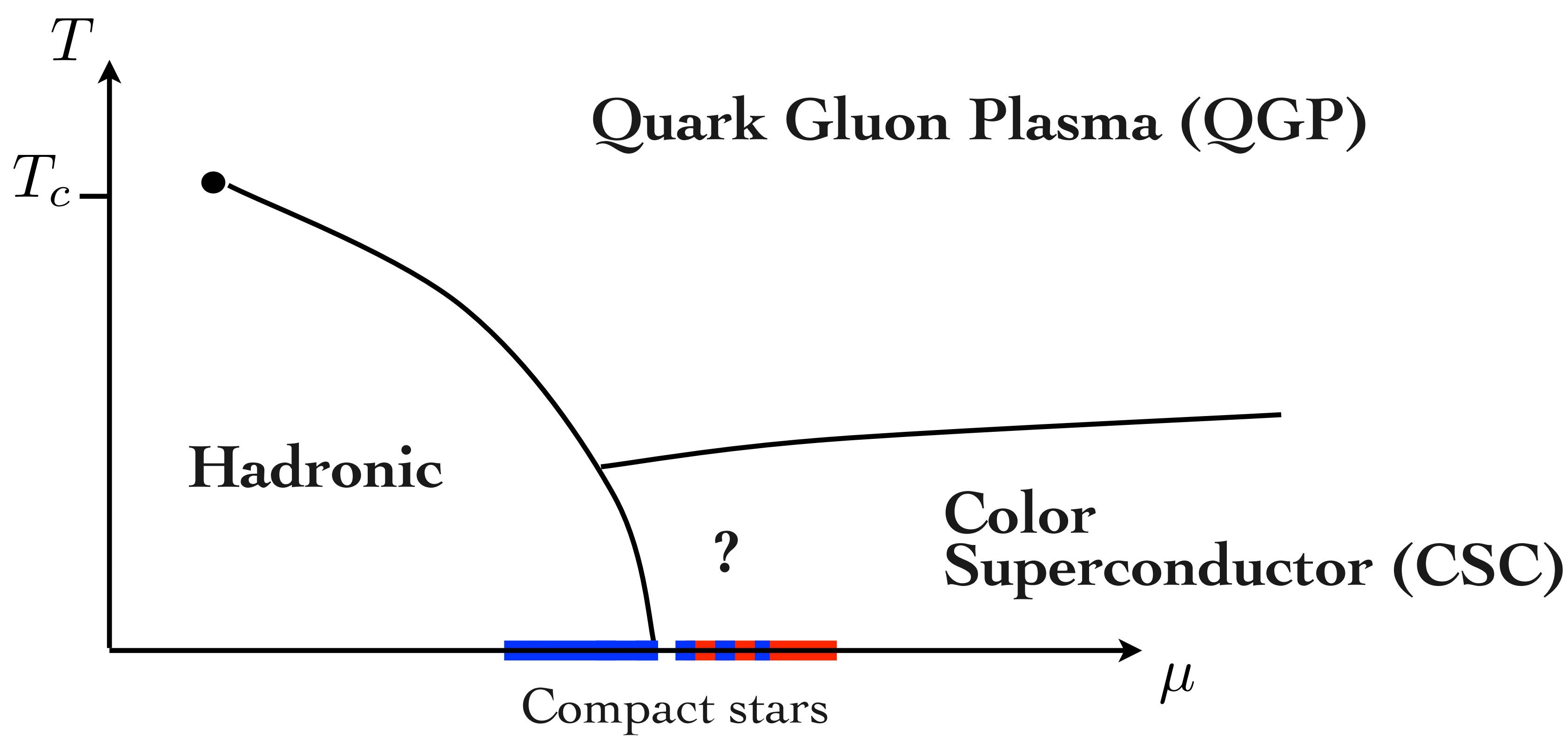

Warning: QCD is perturbative only at asymptotic energy scales

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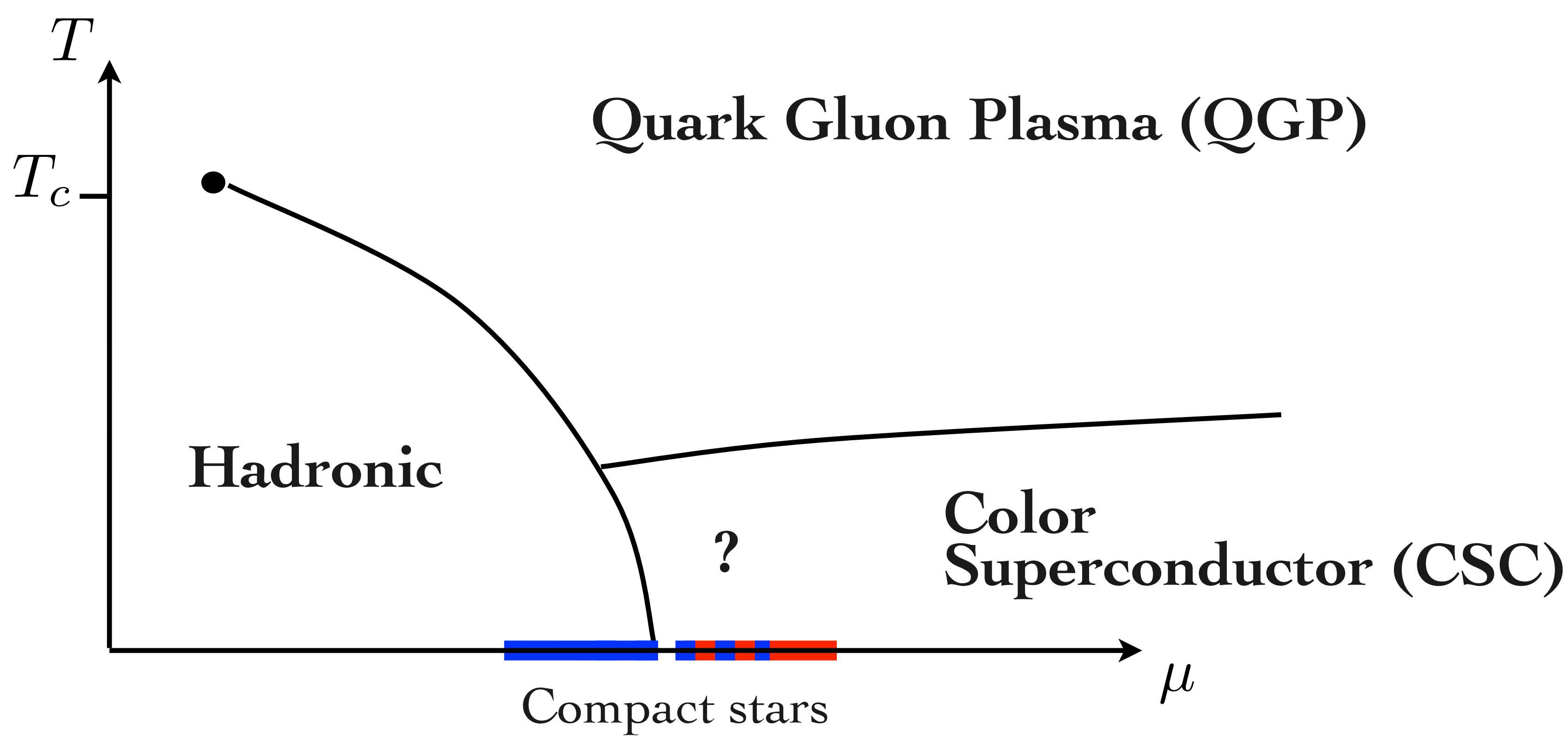

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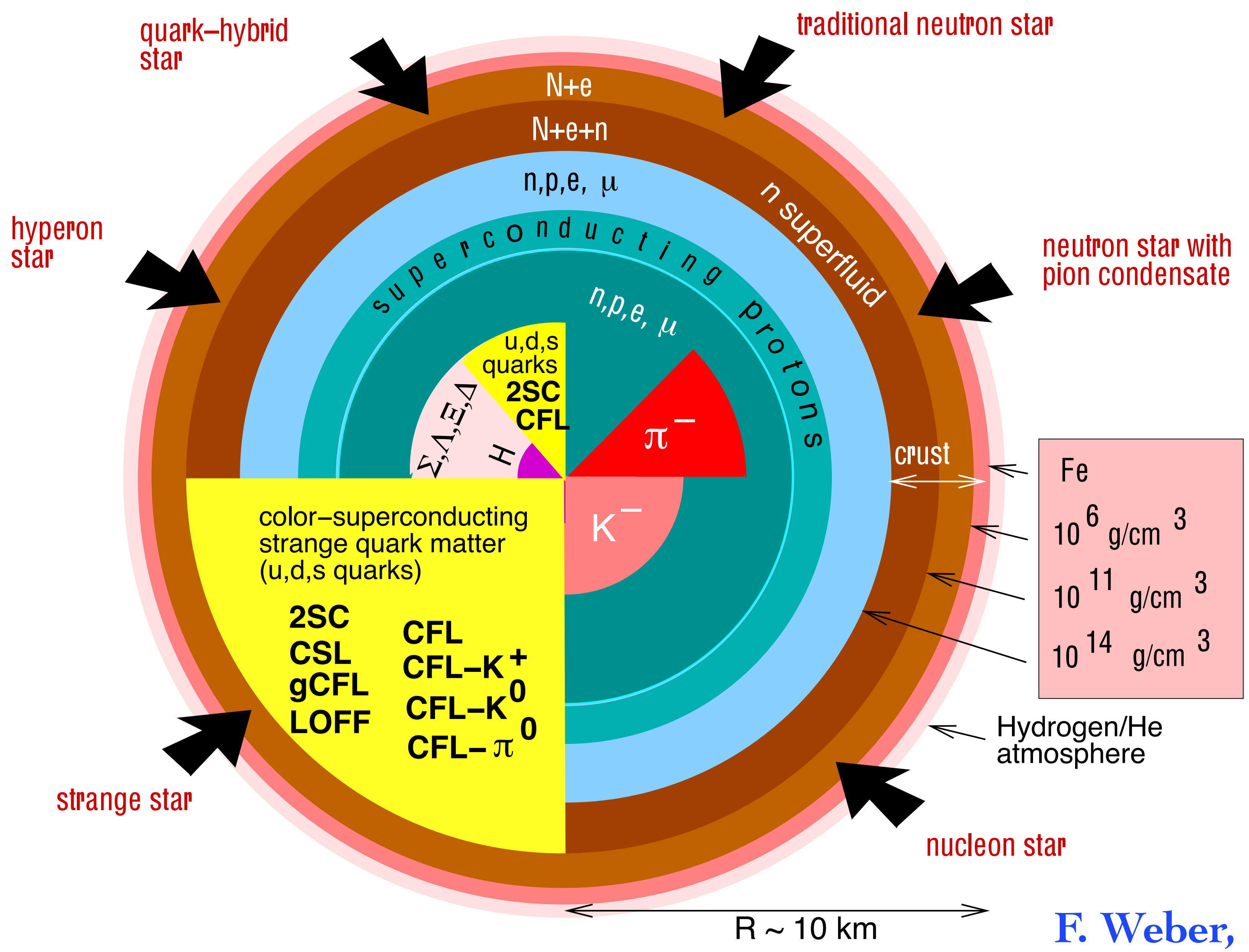
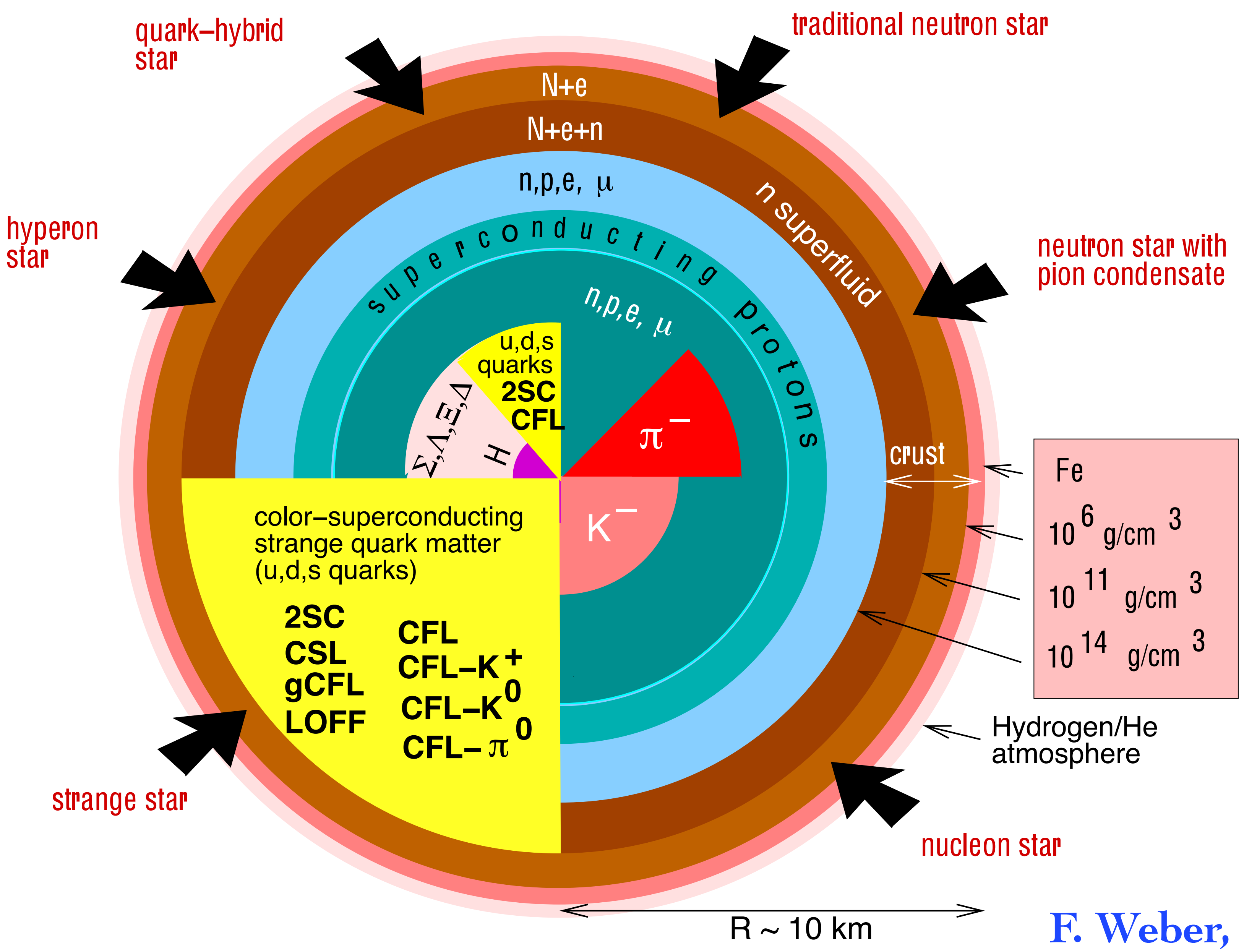
EXPERIMENTS	**HOT MATTER** RHIC LHC	**ENERGY-SCAN** RHIC NA61/SHINE@CERN-SPS CBM@FAIR/GSI MPD@NICA/JINR	**EMULATION** Ultracold fermionic atoms

Compact stars



F. Weber, Prog.Part.Nucl.Phys. 54 (2005) 193

Compact stars



"Probes"
cooling
glitches
instabilities
mass-radius
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......

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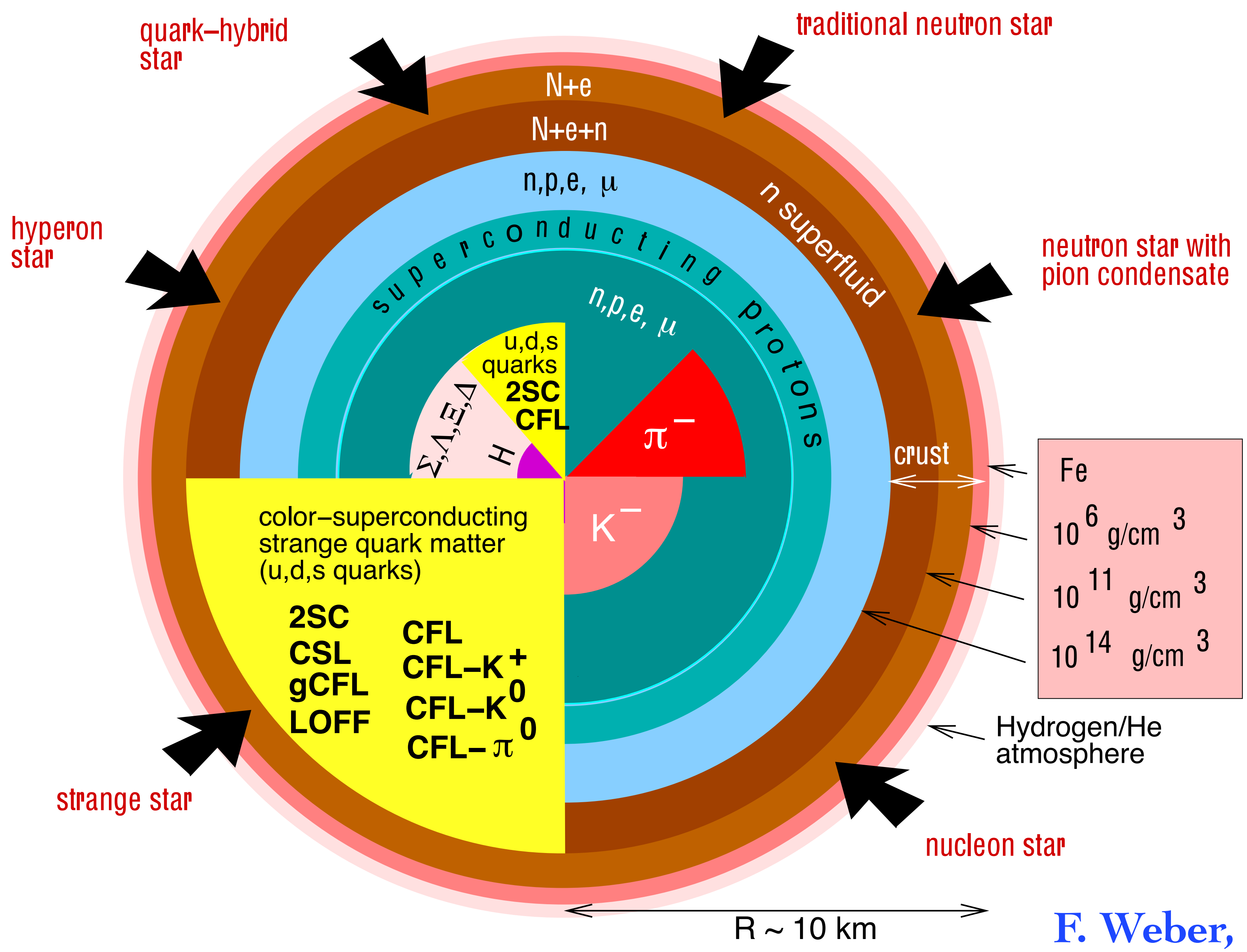

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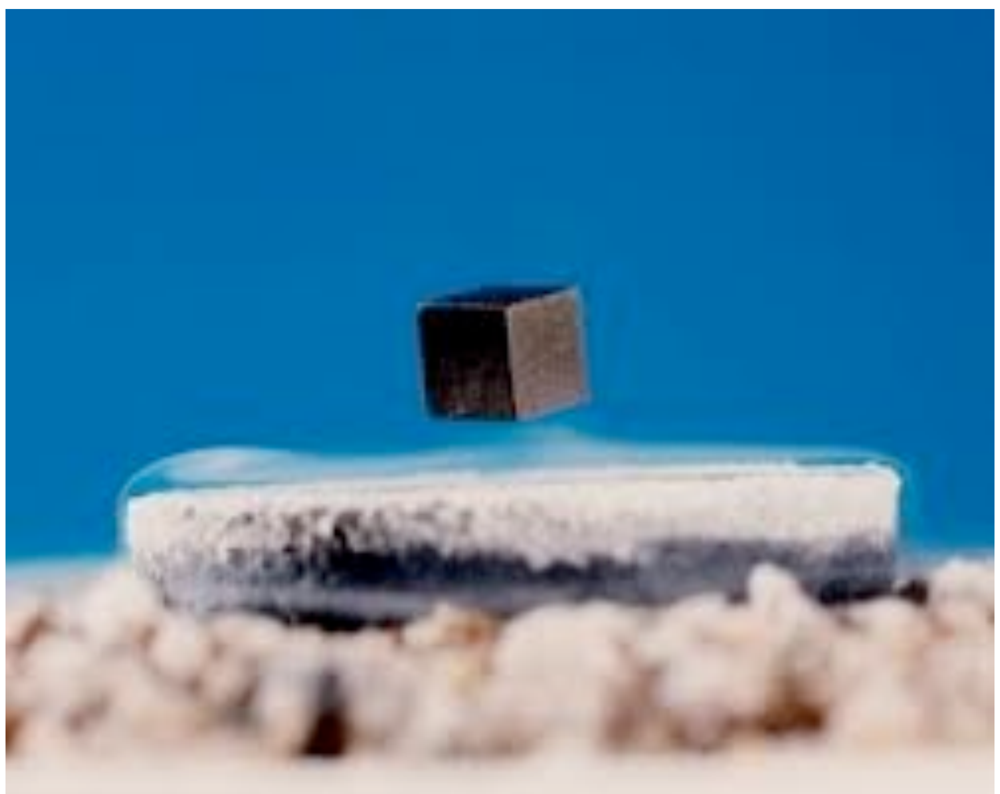
Example

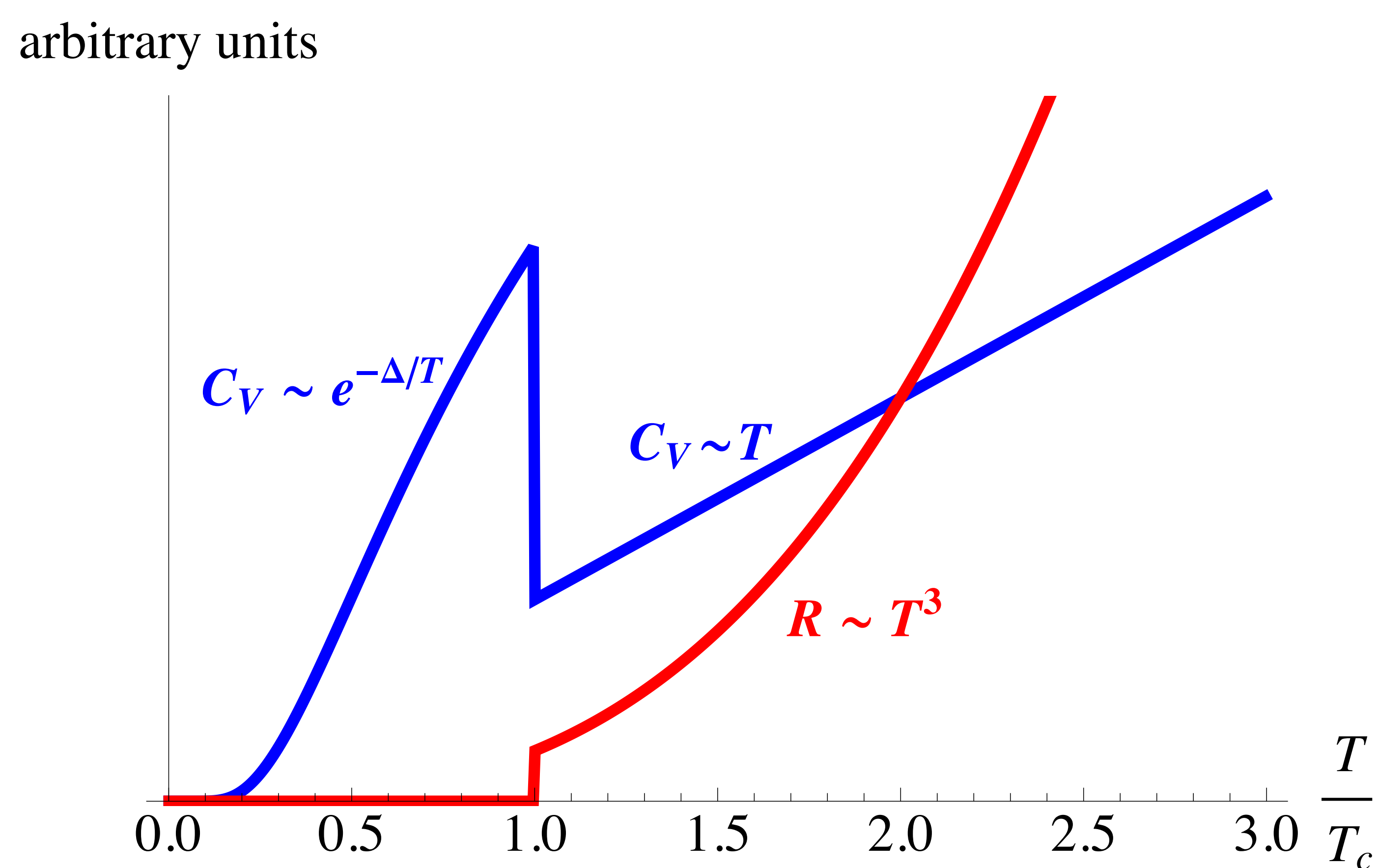
PSR J1614-2230 mass M ~ 2 M$_\odot$ Demorest et al Nature 467, (2010) 1081

hard to explain with quark matter models Bombaci et al. Phys. Rev. C 85, (2012) 55807

SUPERCONDUCTORS

In 1911, H.K. Onnes, cooling mercury, found almost no resistivity at T = 4.2 K.





Superconductivity is a quantum phenomenon at the macroscopic scale

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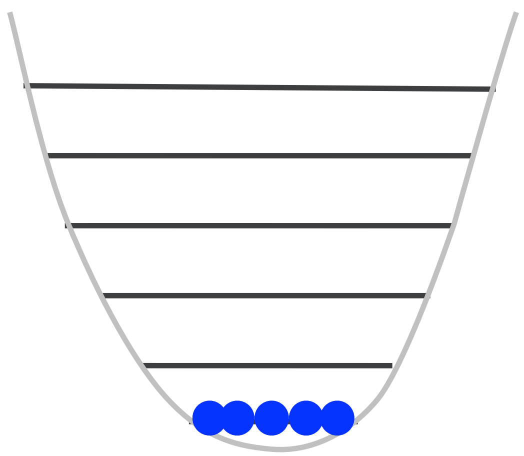
T=0

BOSONS



Bosons occupy the same quantum state: They "like" to move together, no dissipation

^{4}He becomes superfluid at
T ≃ 2.17 K, Kapitsa et al (1938)

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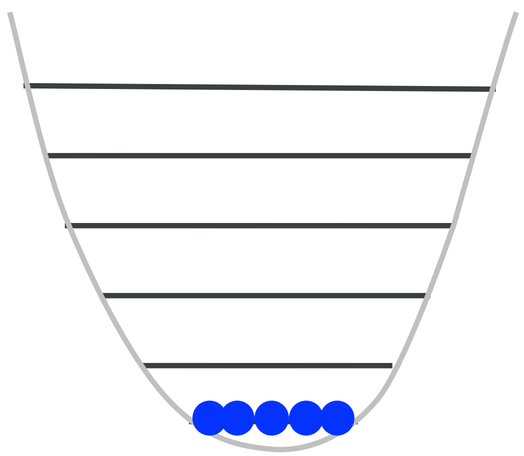
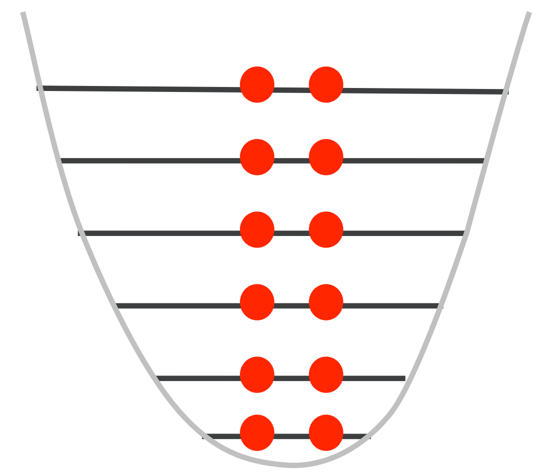


FERMIONS



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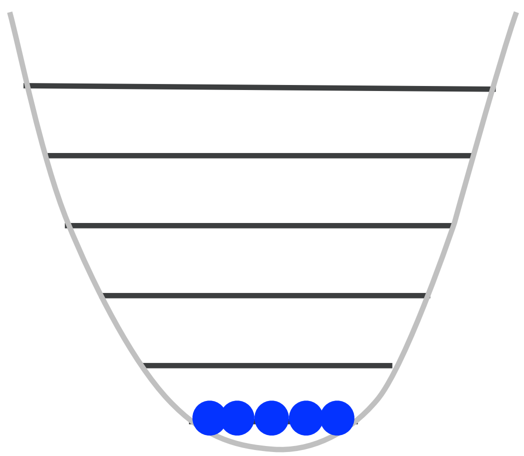

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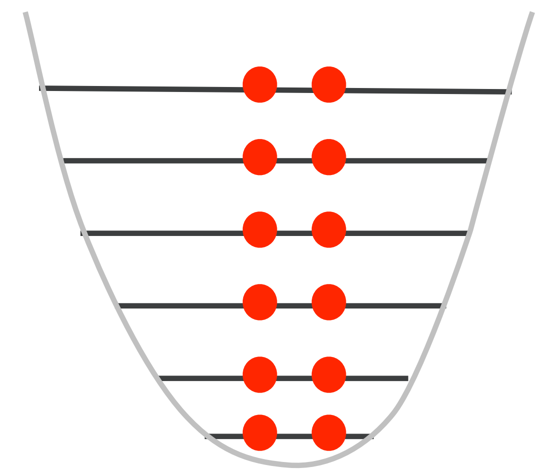

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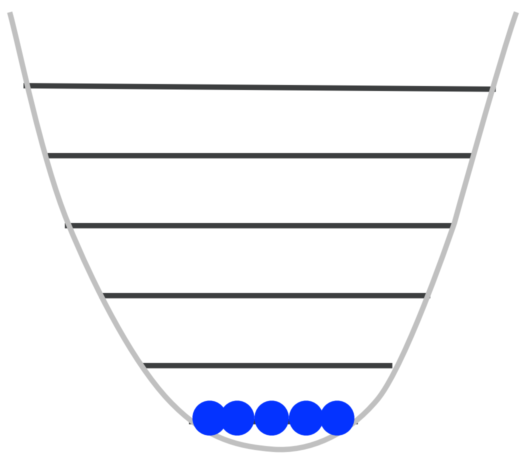

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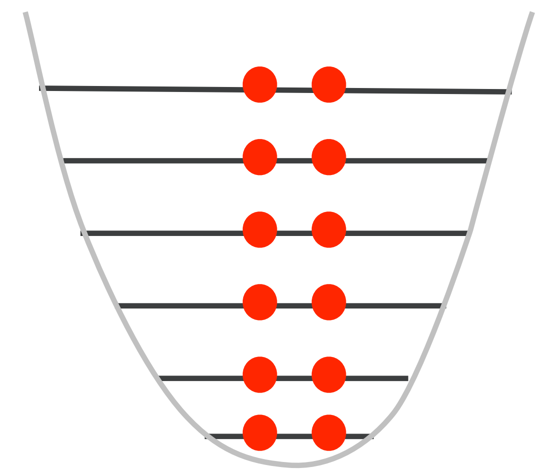

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BCS Theory

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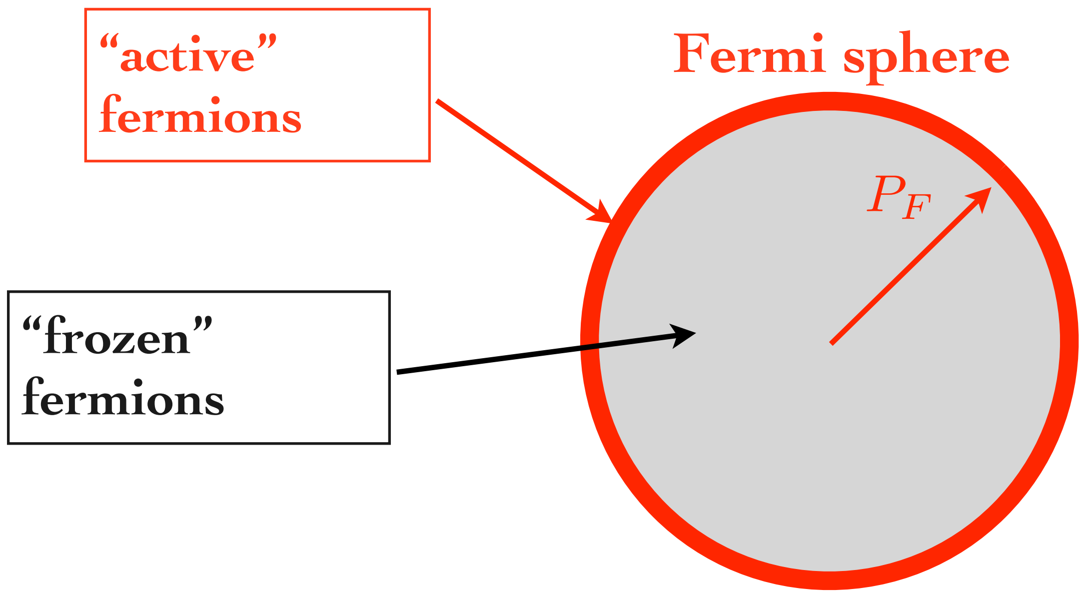

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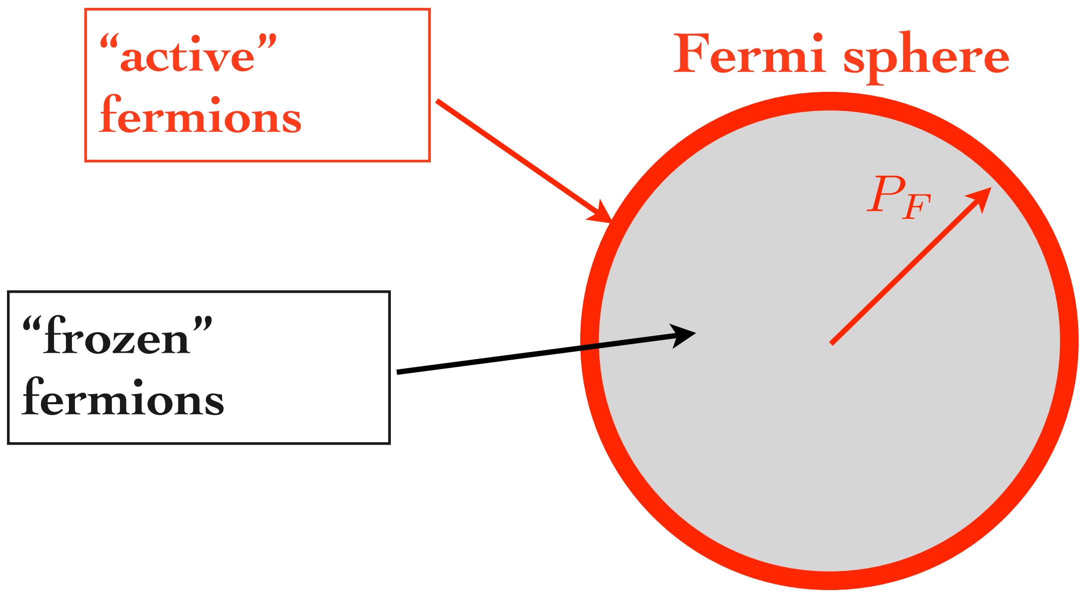

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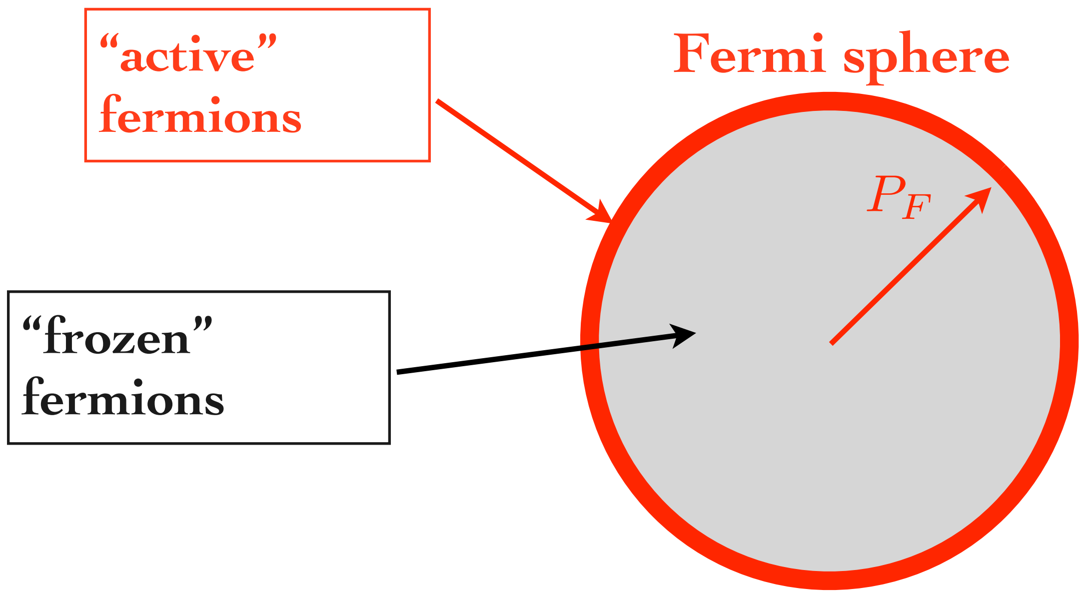

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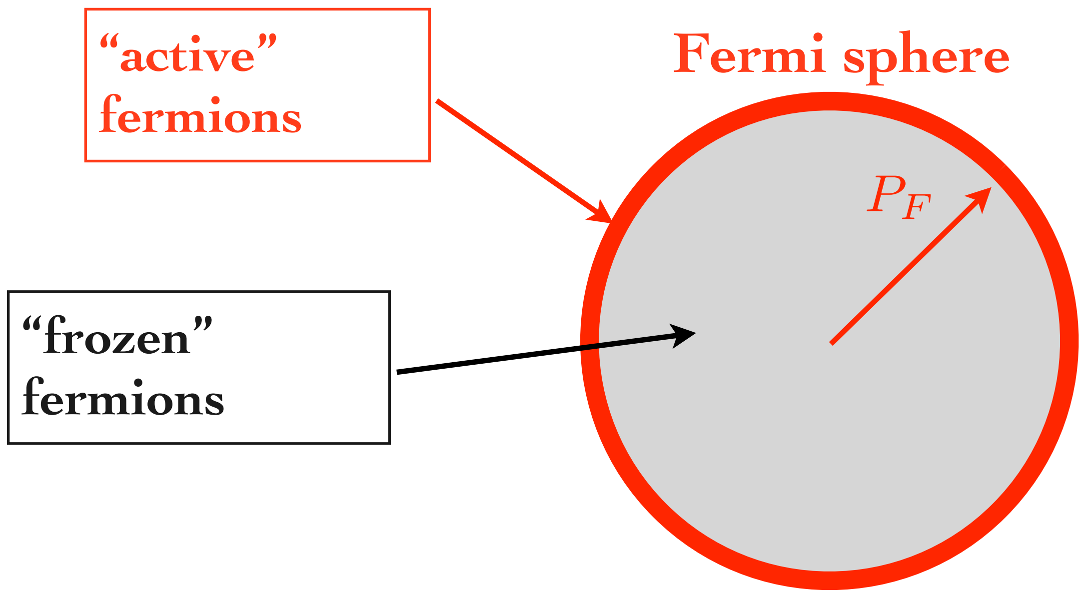

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Increasing the temperature the coherence is lost at $$T_c \simeq 0.3\,\Delta_0$$

Superfluid vs Superconductors

Definitions

Superfluid: frictionless fluid with potential flow $\boldsymbol{v} = \boldsymbol{\nabla}\varphi$. Irrotational: $\boldsymbol{\nabla}\times \boldsymbol{v} = 0$

Superconductor: perfect diamagnet (Meissner effect)

Cooper pairing is at the basis of both phenomena (for fermions)

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Superconductor

Broken gauge symmetry

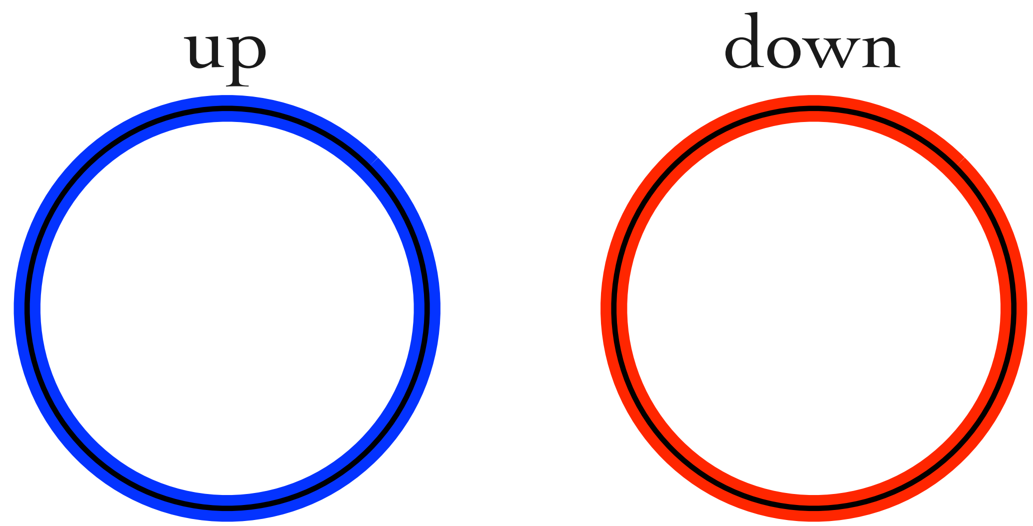
Higgs mechanism $\longrightarrow$ Broken gauge fields with mass, M, penetrates for a length $\lambda \propto 1/M$

BCS-BEC crossover

correlation length $\xi \sim \frac{v_F}{\Delta}$

vs

average distance $n^{-1/3}$



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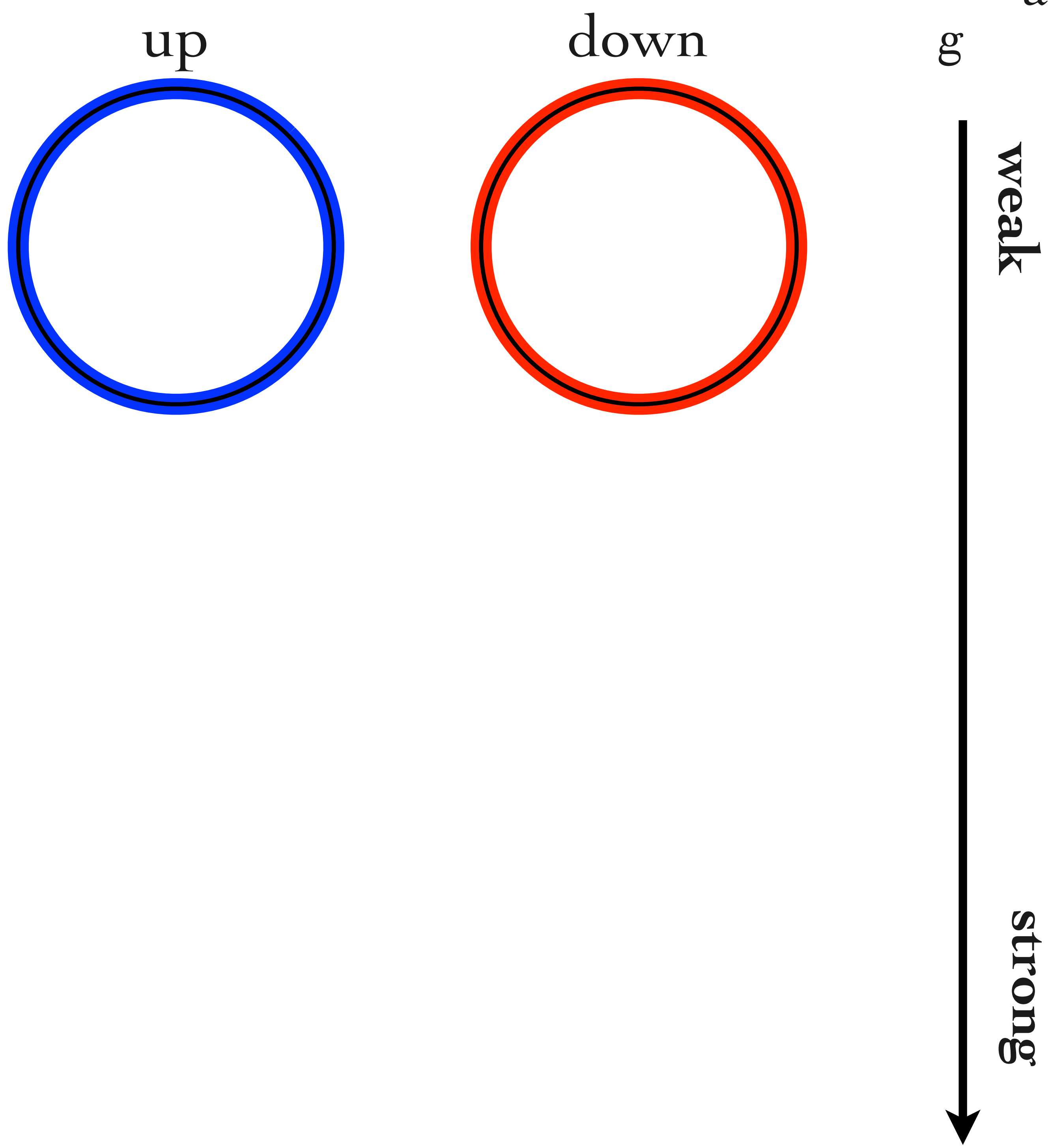

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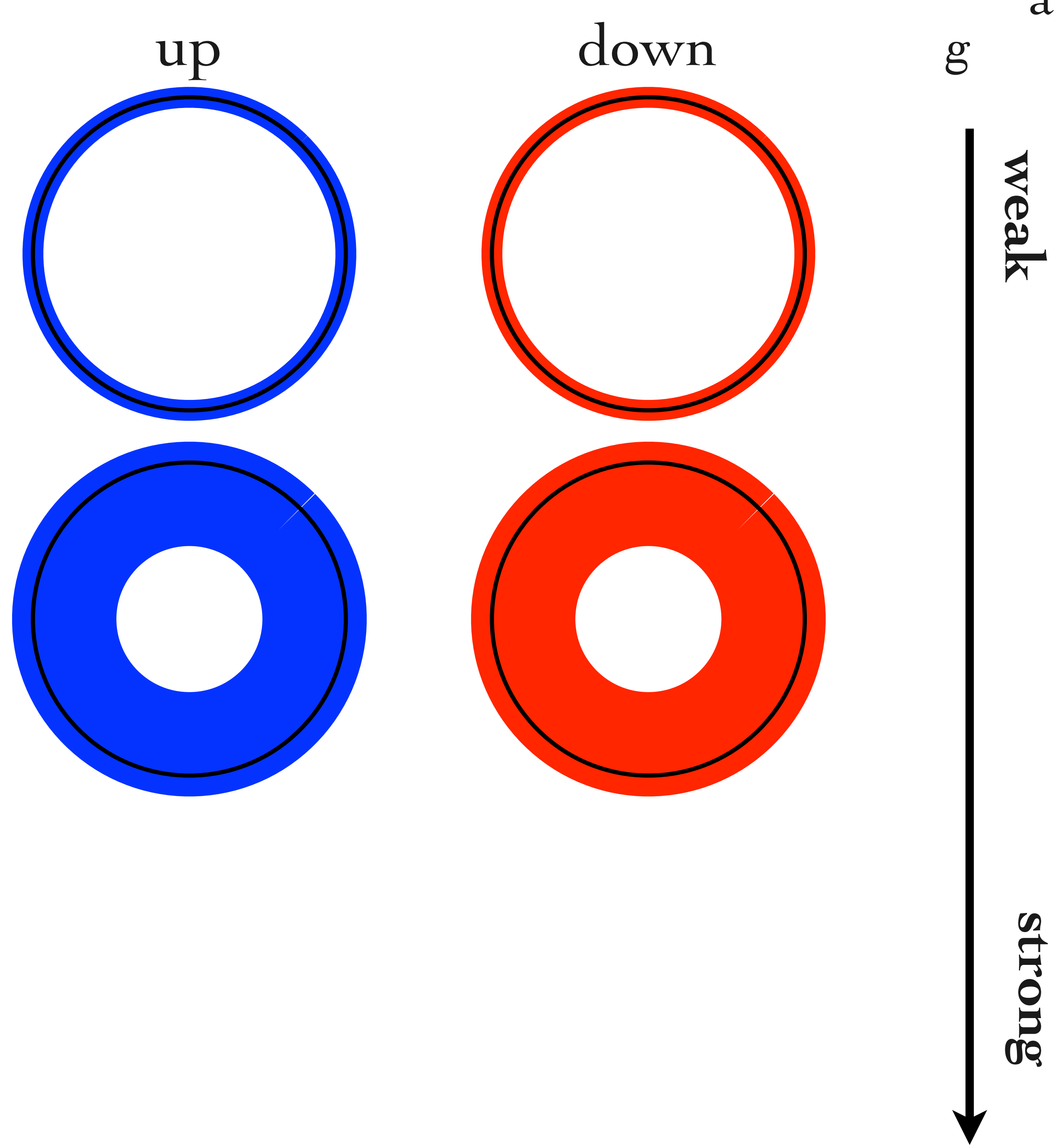

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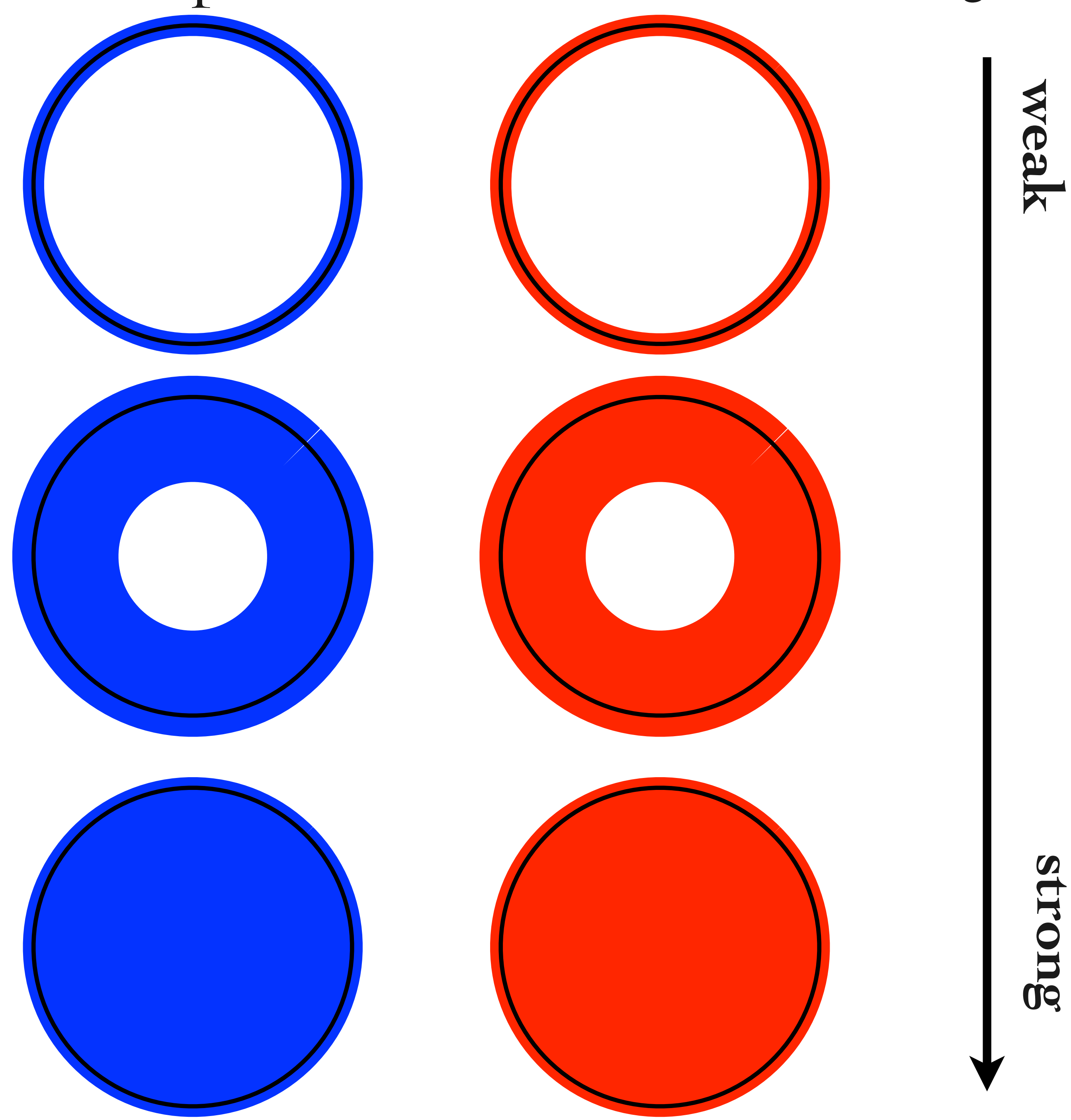
up | down | g



BCS
fermi surface phenomenon — $\xi \gg n^{-1/3}$

BCS-BEC crossover
depleting the Fermi sphere — $\xi \sim n^{-1/3}$

BEC
equivalent to ^{4}He — $\xi \ll n^{-1/3}$

COLOR SUPERCONDUCTIVITY

A bit of history

- Quark matter inside compact stars, Ivanenko and Kurdgelaidze (1965), Paccini (1966) ...
- Quark Cooper pairing was proposed by Ivanenko and Kurdgelaidze (1969)
- With asymptotic freedom (1973) more robust results by Collins and Perry (1975), Baym and Chin (1976)
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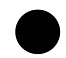
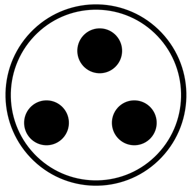
- A large gap with instanton models by Alford et al. (1998) and by Rapp et al. (1998)
- The color flavor locked (CFL) phase was proposed by Alford et al. (1999)

The idea with a cartoon

“particle”	quark	baryon	diquark
			
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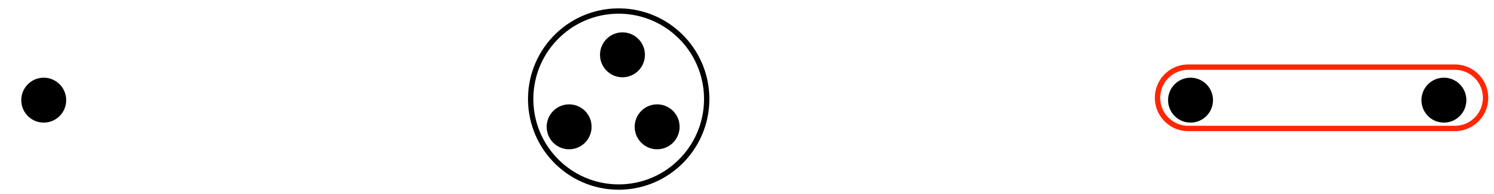
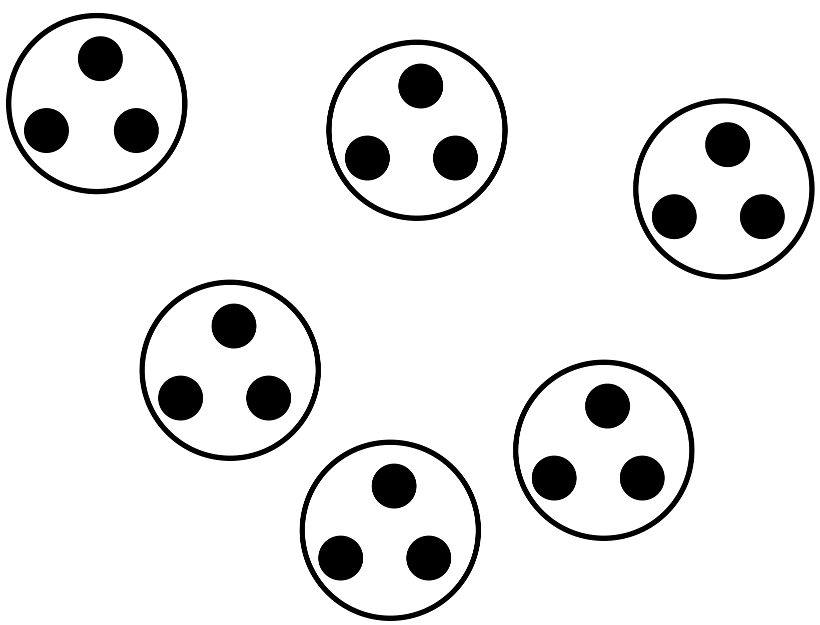


High density



Liquid of neutrons

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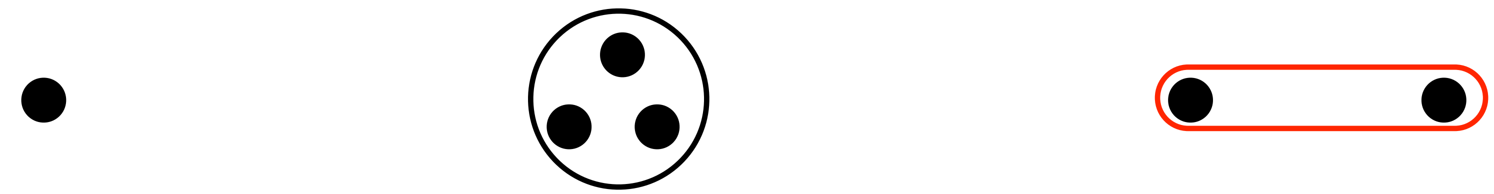

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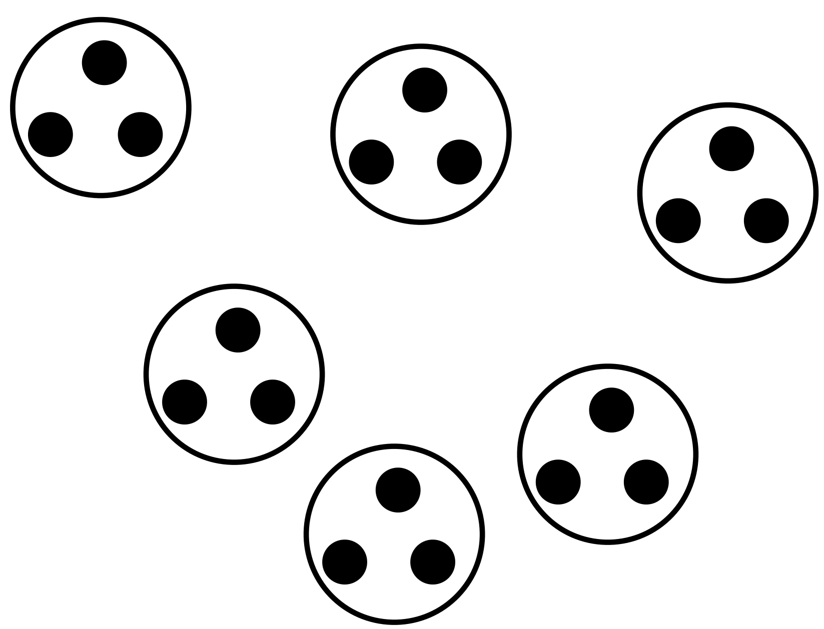

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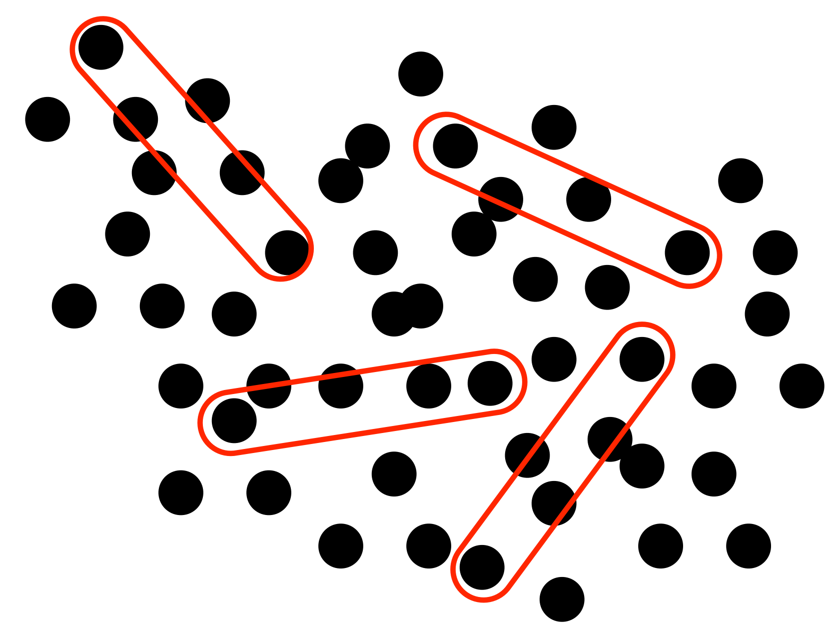

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	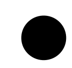	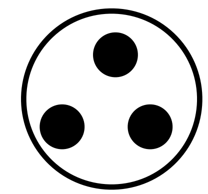	
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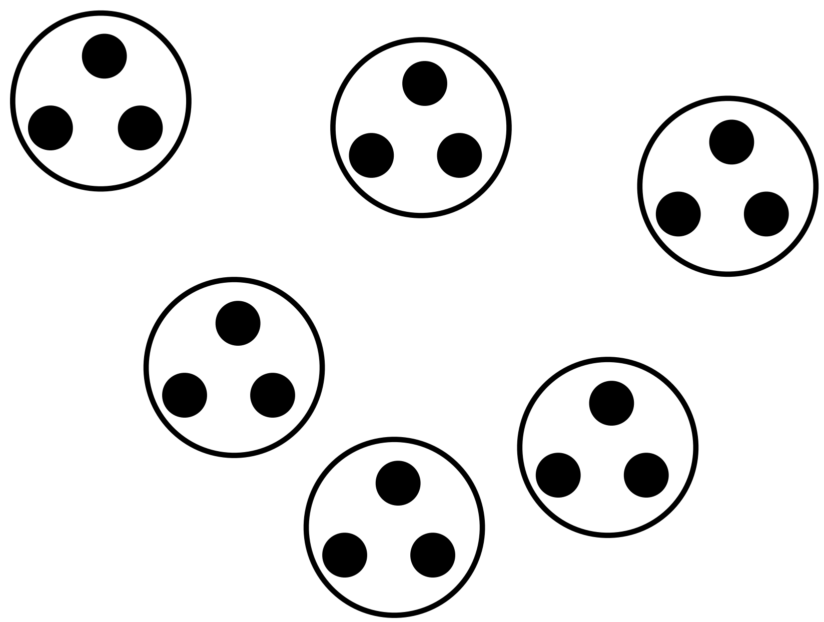

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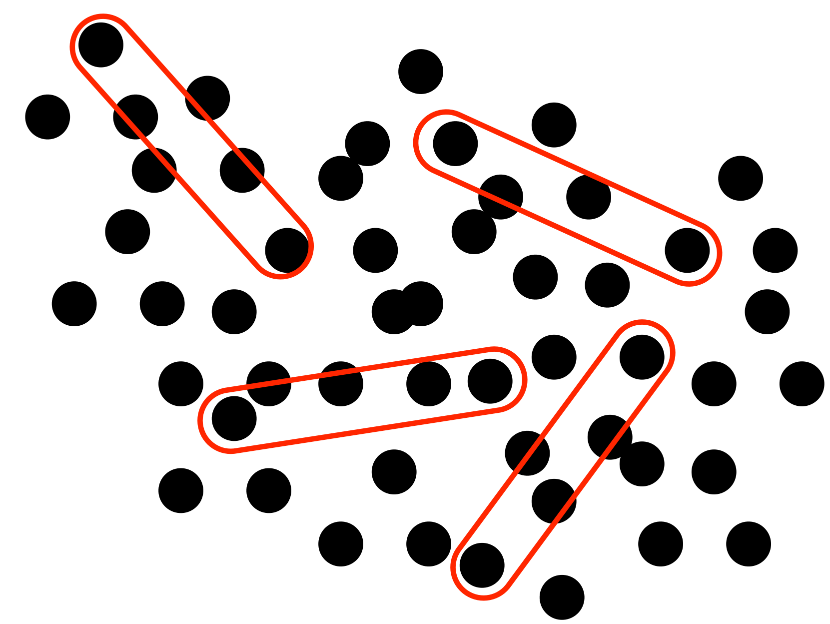
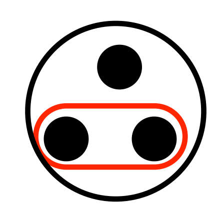


Liquid of quarks with correlated diquarks

Models for the lowest-lying baryon excited states with **diquarks**
Anselmino et al. Rev Mod Phys 65, 1199 (1993)



Do we have the ingredients?

Recipe for superconductivity

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N.b. Quarks have color, flavor as well as spin degrees of freedom: complicated dishes.
A long menu of colored dishes.

Two good dishes ...

$$\langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \sim \epsilon_{I\alpha\beta} \epsilon_{Iij} \Delta_{\mathrm{I}}$$

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CFL

Color superconductor
Baryonic superfluid
“e.m.” insulator

$$\Delta_3 = \Delta_2 = \Delta_1 > 0$$

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_{\mathrm{Q}}} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{\mathrm{Q}}}} \times Z_2$$

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2SC

Color superconductor
"e.m." conductor

$$\Delta_3 > 0\,,\ \Delta_2 = \Delta_1 = 0$$

$$SU(3)_c \times \underbrace{SU(2)_L \times SU(2)_R \times U(1)_B \times U(1)_S}_{\supset U(1)_{\rm Q}} \to SU(2)_c \times \underbrace{SU(2)_L \times SU(2)_R \times U(1)_{\tilde{B}} \times U(1)_S}_{\supset U(1)_{\tilde{\rm Q}}}$$

The main course: Color-flavor locked phase

Condensate

(Alford, Rajagopal, Wilczek hep-ph/9804403)

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Using instantons or NJL models

$$\Delta_{\rm CFL} \simeq (10 - 100)\ \text{MeV}$$

$\mu \simeq 400\,\text{MeV}$ $\qquad n^{1/3} \propto \mu$

$\xi \gtrsim n^{-1/3}$ **in between BCS and BEC**

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Symmetry breaking

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- Higgs mechanism: All gluons acquire “magnetic” mass
- χSB: 8 (pseudo) Nambu-Goldstone bosons (NGBs)
- $U(1)_B$ breaking: 1 NGB
- “Rotated” electromagnetism mixing angle $\cos\theta = \dfrac{g}{\sqrt{g^2 + 4e^2/3}}$ (analog of the Weinberg angle)

Quark-hadron complementarity

Mapping of the NGBs of the hadronic phase with the NGBs of the CFL phase

Lagrangian Casalbuoni and Gatto, Phys. Lett. B 464, (1999) 111

$$\mathcal{L}_{\text{eff}} = \frac{f_\pi^2}{4}\text{Tr}[\partial_0\Sigma\partial_0\Sigma^\dagger - v_\pi^2\partial_i\Sigma\partial_i\Sigma^\dagger]$$

where $\Sigma = e^{i\phi^a\lambda_a/f_\pi}$ ϕ^a describes the octet $(\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta)$

$$f_\pi^2 = \frac{21 - 8\log 2}{18}\frac{\mu^2}{2\pi^2} \qquad v_\pi^2 = \frac{1}{3}$$

Quark-hadron complementarity

Mapping of the NGBs of the hadronic phase with the NGBs of the CFL phase

Lagrangian

Casalbuoni and Gatto, Phys. Lett. B 464, (1999) 111

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Masses

Son and Sthephanov, Phys. Rev. D 61, (2000) 74012

$$m^2_{\pi^\pm} = A\,(m_u + m_d)m_s$$

$$m^2_{K^\pm} = A\,(m_u + m_s)m_d$$

$$m^2_{K^0,\bar{K}^0} = A\,(m_d + m_s)m_u$$

kaons are lighter than mesons!

$$A = \frac{3\Delta^2}{\pi^2 f_\pi^2}$$

$$\pi^+ \sim (\bar{d}\bar{s})(us)$$

$$K^+ \sim (\bar{d}\bar{s})(ud)$$

"Phonons"

There is an additional massless NGB, φ, associated to $U(1)_B$ breaking to Z_2

Quantum numbers φ ~ <Λ Λ > like the **H-dibaryon** of **Jaffe, Phys. Rev. Lett. 38, 195 (1977)**

Effective Lagrangian up to quartic terms

Son, hep-ph/0204199

$$\mathcal{L}_{\rm eff}(\varphi) = \frac{3}{4\pi^2}\left[(\mu - \partial_0\varphi)^2 - (\partial_i\varphi)^2\right]^2$$

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bulk **"sound" or phonon**

$$\varphi(x) = \bar{\varphi}(x) + \phi(x)$$

classical field **long-wavelength fluctuations**

Phenomenology

MM et al., Phys. Rev. Lett. 101, 241101 (2008)

Dissipative processes due to vortex-phonon interaction **damp r-mode oscillation** for CFL stars rotating at frequencies < 1 Hz

Mismatched Fermi spheres
(3 flavor quark matter)

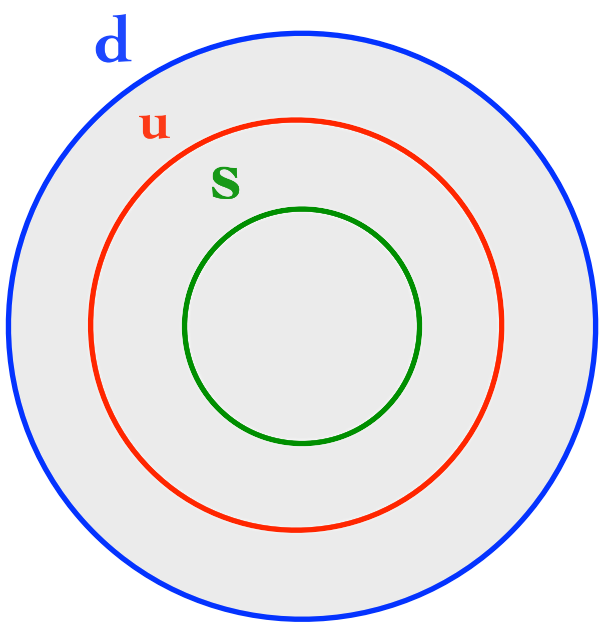
More realistic conditions

sizable strange quark mass
+
weak equilibrium
+
electric neutrality

→

mismatch of the Fermi momenta around

$$\mu = \frac{\mu_u + \mu_d + \mu_s}{3}$$



Fermi spheres of u,d, s quarks

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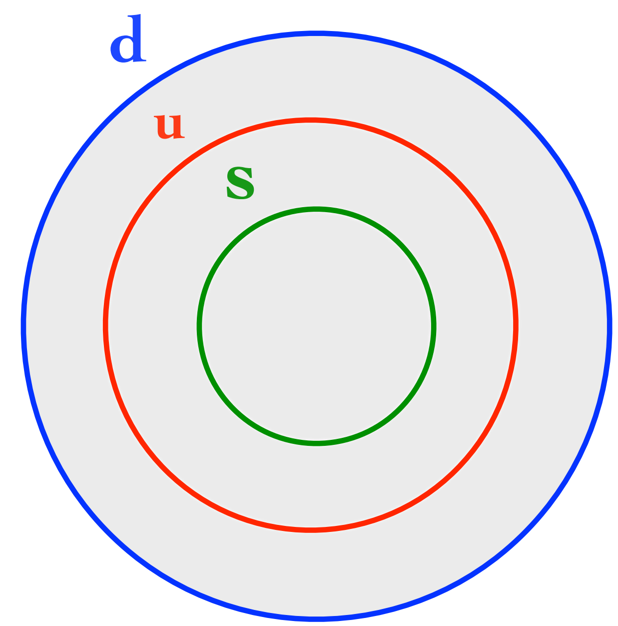
No pairing case

Fermi momenta $p_u^F = \mu_u \quad p_d^F = \mu_d \quad p_s^F = \sqrt{\mu_s^2 - m_s^2}$



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weak decays

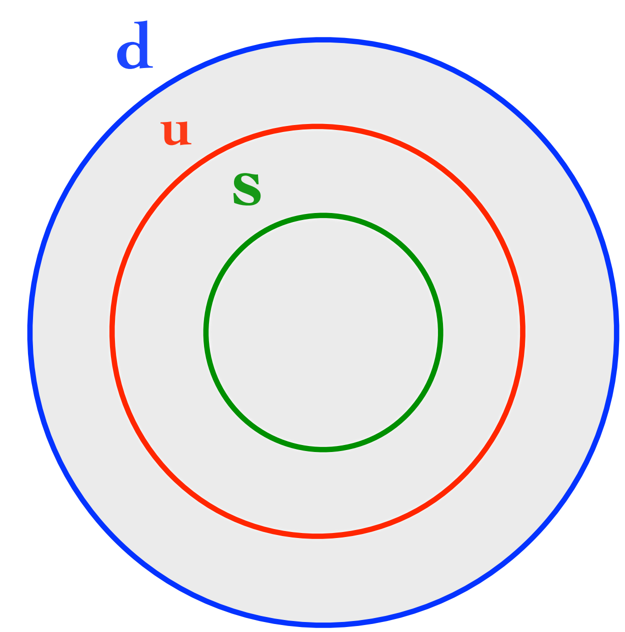
$u \to d + \bar{e} + \nu_e$
$u \to s + \bar{e} + \nu_e$
$u + d \leftrightarrow u + s$

→

$$\mu_u = \mu_d - \mu_e$$
$$\mu_d = \mu_s$$

electric neutrality →

$$\frac{2}{3}N_u - \frac{1}{3}N_d - \frac{1}{3}N_s - N_e = 0$$



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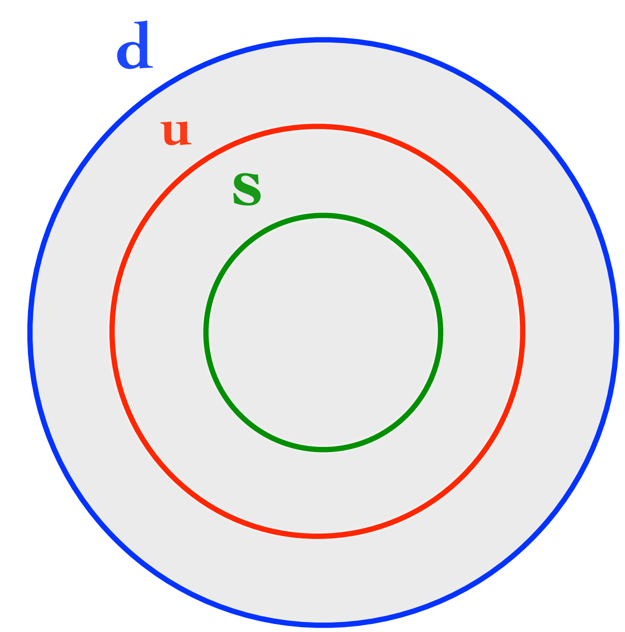
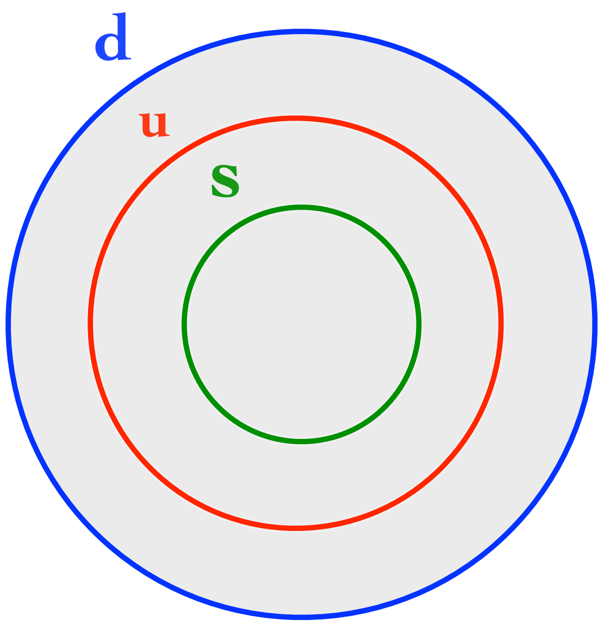


Fermi spheres of u,d, s quarks

$$\mu_e \simeq \frac{m_s^2}{4\mu} \qquad p_d^F = \mu + \frac{1}{3}\mu_e \quad p_u^F = \mu - \frac{2}{3}\mu_e \quad p_s^F \simeq \mu - \frac{5}{3}\mu_e$$

Alford, Rajagopal, JHEP 0206 (2002) 031

Mismatch vs Pairing



- Energy gained in pairing $\sim 2\Delta_{CFL}$
- Energy cost of pairing $\sim \delta\mu \sim \dfrac{m_s^2}{\mu}$

The CFL phase is favored for $\dfrac{m_s^2}{\mu} \lesssim 2\Delta_{CFL}$

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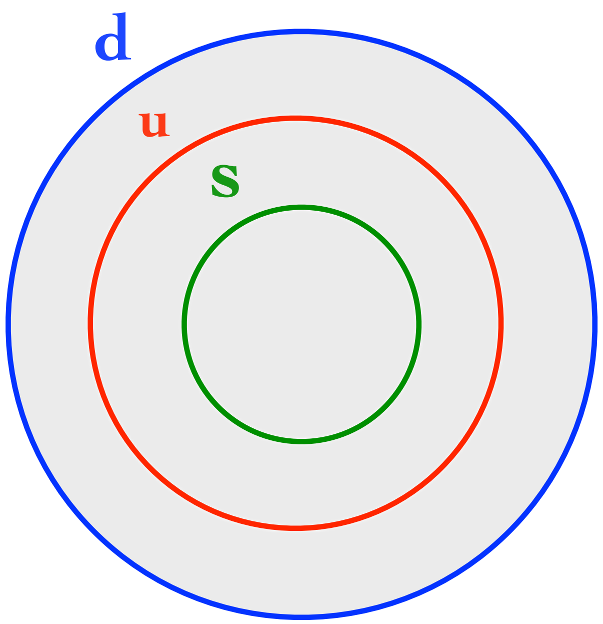

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Casalbuoni, MM et al. Phys.Lett. B605 (2005) 362

Forcing the superconductor to a homogenous gapless phase $E(p) = -\delta\mu + \sqrt{(p-\mu)^2 + \Delta^2}$

Leads to the "chromomagnetic instability" $M^2_{\text{gluon}} < 0$

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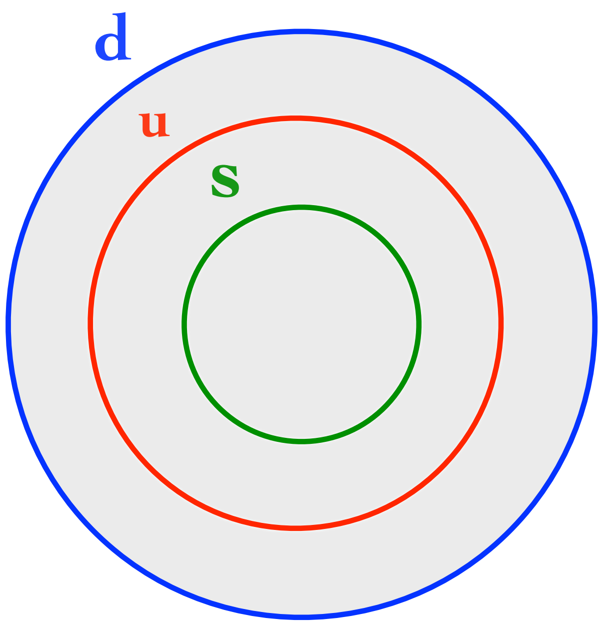

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For $\dfrac{m_s^2}{\mu} \gtrsim 2\Delta_{CFL}$ some less symmetric CSC phase should be realized

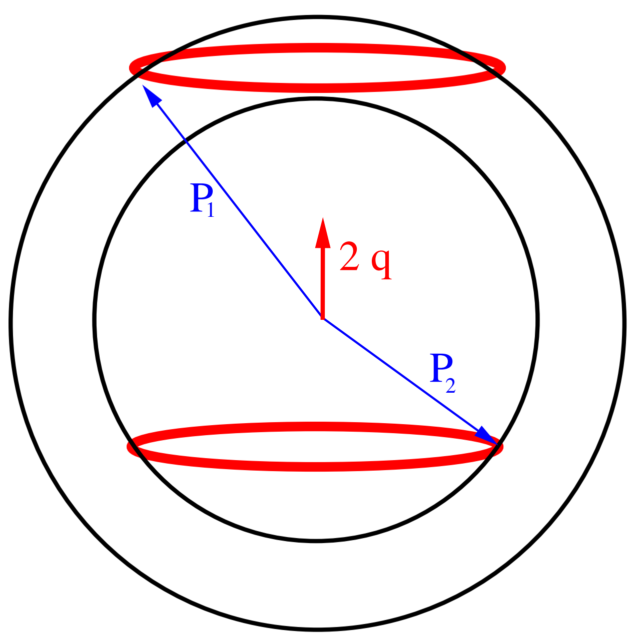
LOFF-phase

For $\delta\mu_1 < \delta\mu < \delta\mu_2$ the superconducting phase named LOFF is favored with Cooper pairs of non-zero total momentum

LOFF: Larkin-Ovchinnikov and Fulde-Ferrel

For two flavors

$$\delta\mu_1 \simeq \frac{\Delta_0}{\sqrt{2}} \qquad \delta\mu_2 \simeq 0.75\,\Delta_0$$



- In momentum space

$$< \psi(\mathbf{p_1})\psi(\mathbf{p_2}) > \sim \Delta\,\delta(\mathbf{p_1} + \mathbf{p_2} - \mathbf{2q})$$

- In coordinate space

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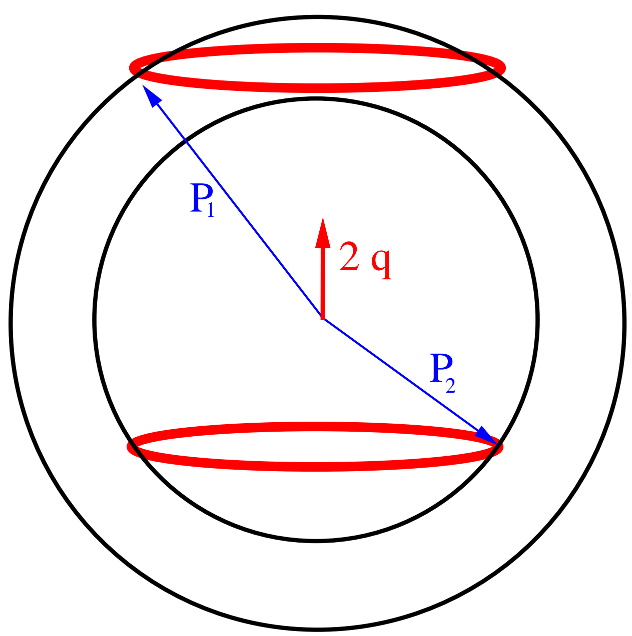

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The LOFF phase corresponds to a non-homogeneous superconductor, with a spatially modulated condensate in the spin 0 channel

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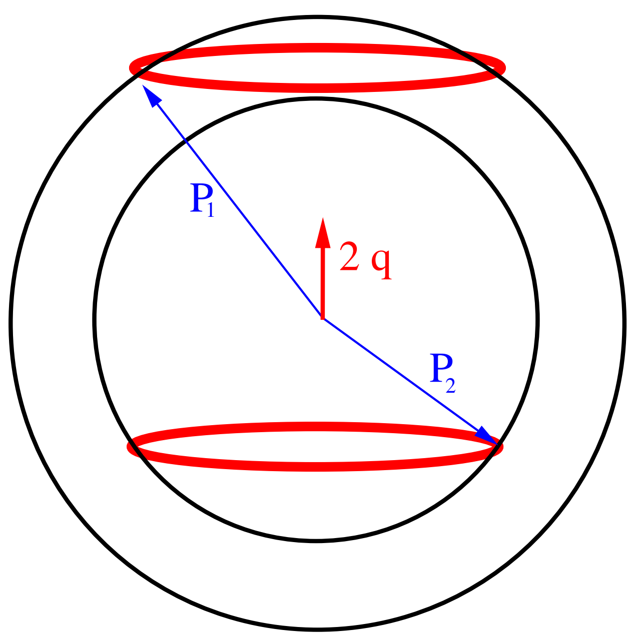

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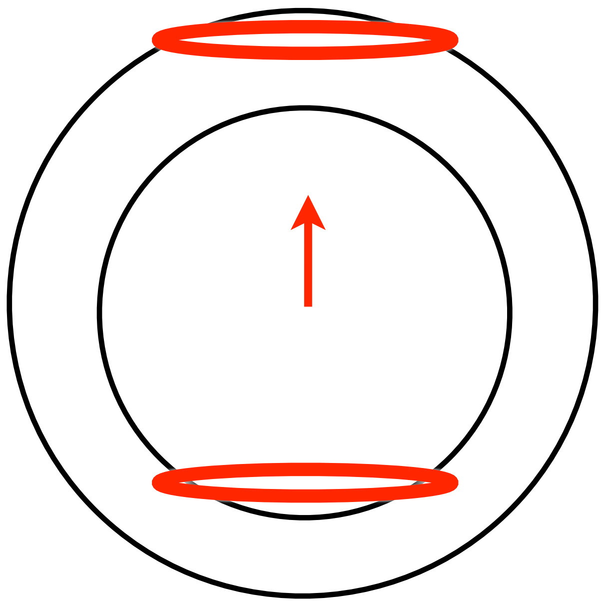
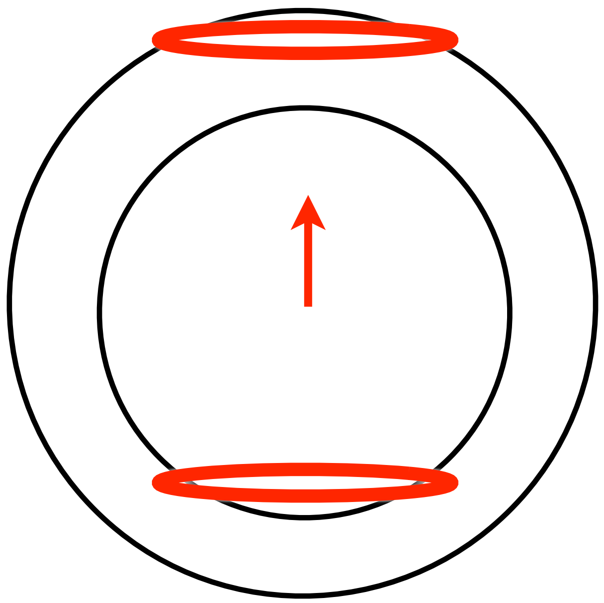
The dispersion law of quasiparticles is gapless in some specific directions. No chromomagnetic instability.

Crystalline structures



Crystalline structures



- Structures combining more plane waves

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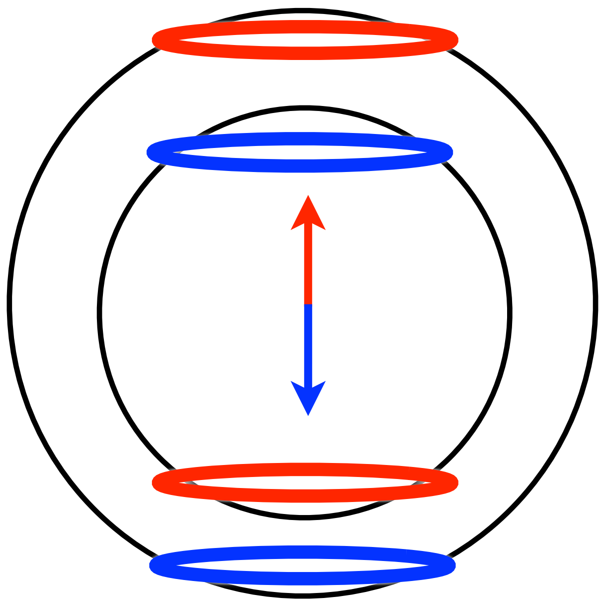

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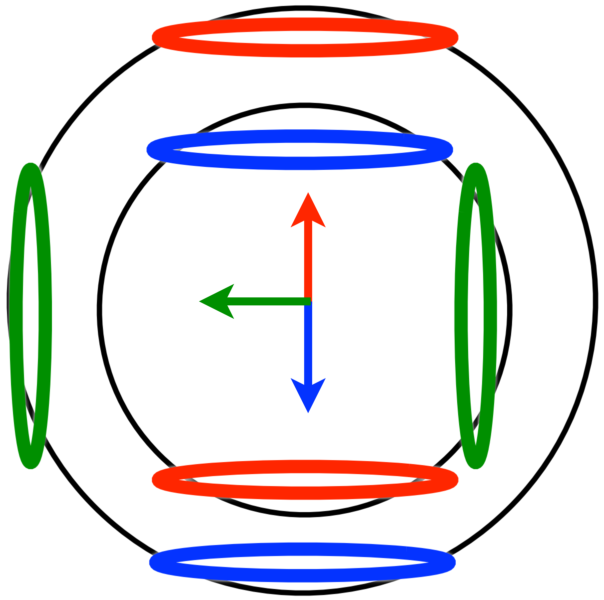

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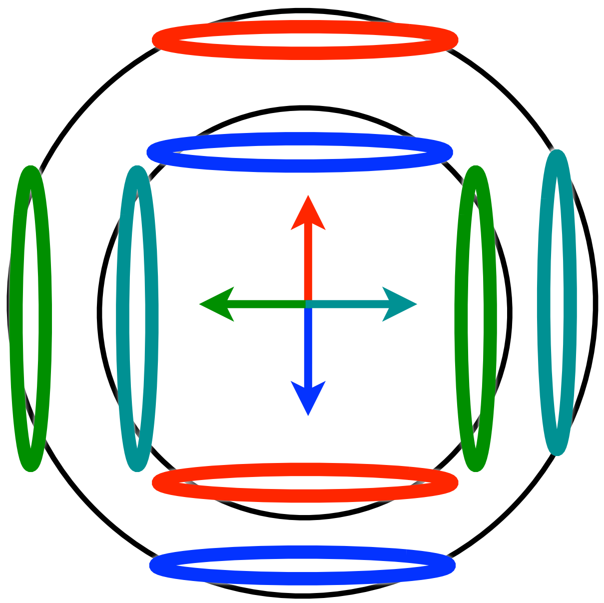

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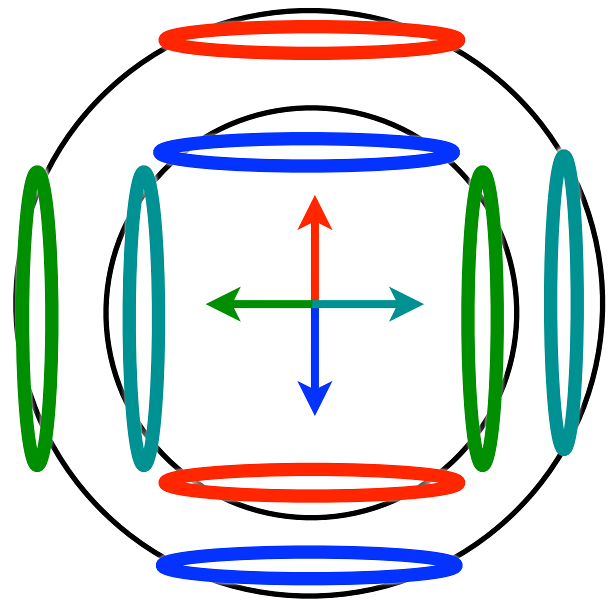

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- From GL studies: “no-overlap” condition between ribbons

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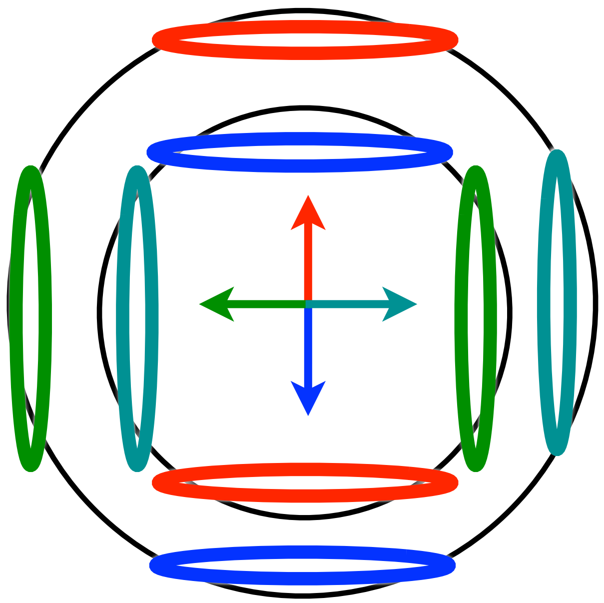

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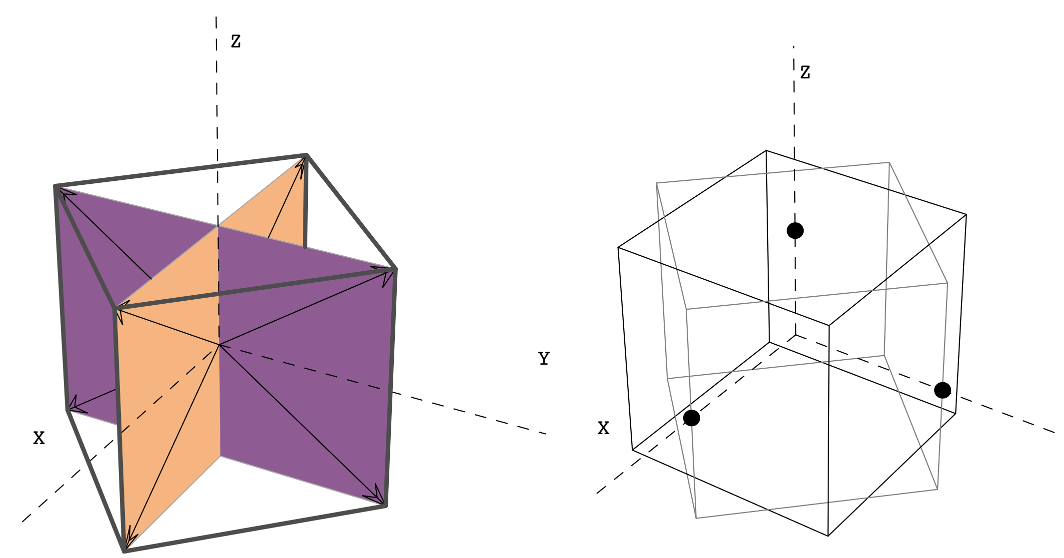
Three flavors

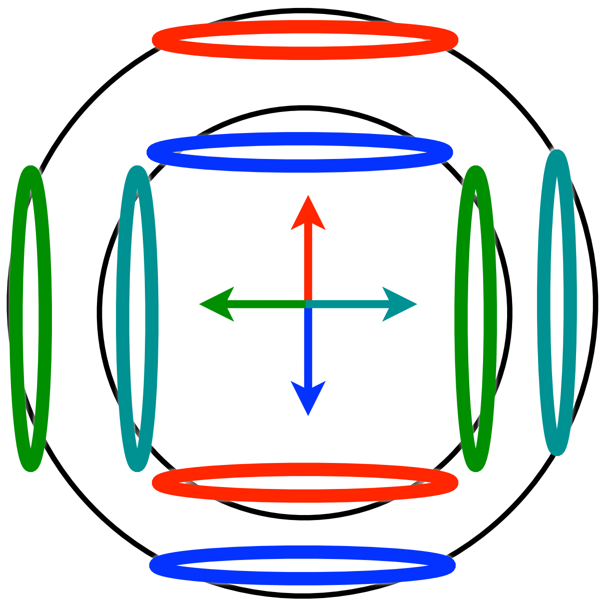
$$< \psi_{\alpha i} C\gamma_5 \psi_{\beta j} > \sim \sum_{I=2,3} \Delta_I \sum_{\mathbf{q}_I^a \in \{\mathbf{q}_I^a\}} e^{2i\mathbf{q}_I^a\cdot\mathbf{r}} \epsilon_{I\alpha\beta}\epsilon_{Iij}$$

CX **2cube45z**



Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

Crystalline structures



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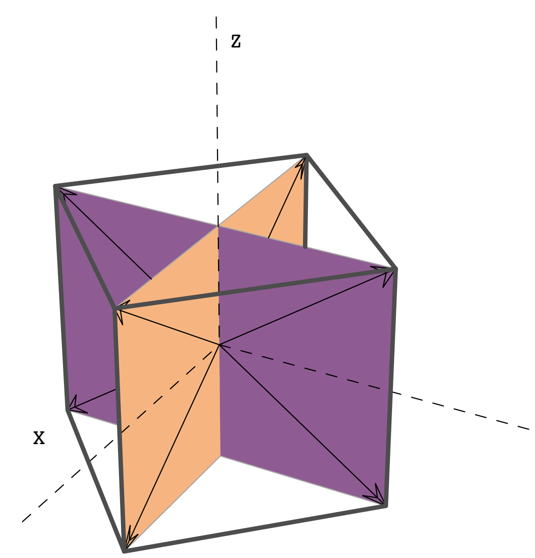

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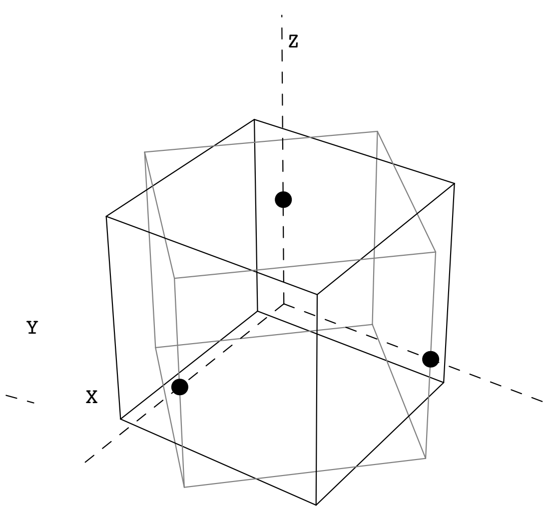


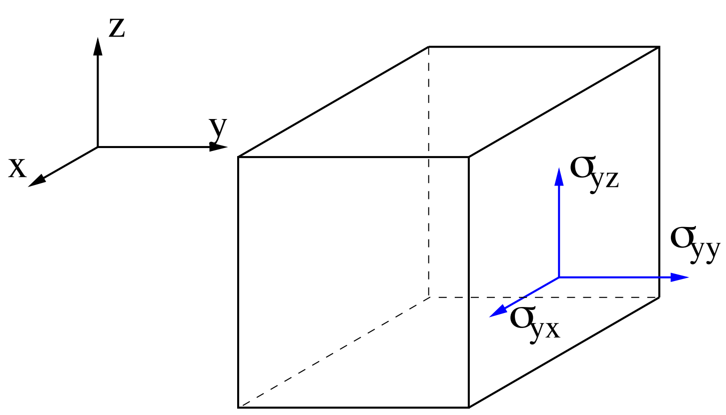
Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

- Crystal oscillations

Casalbuoni, MM et al. Phys.Rev. D66 (2002) 094006

MM, Rajagopal and Sharma Phys.Rev. D76 (2007) 074026

Shear modulus



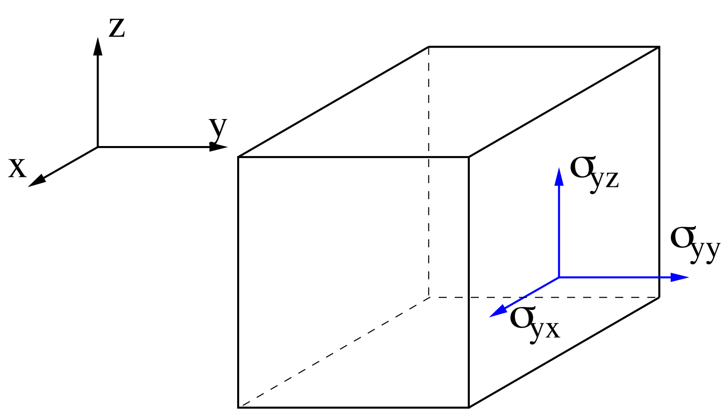
The shear modulus describes the response of a crystal to a shear stress

$$\nu^{ij} = \frac{\sigma^{ij}}{2s^{ij}} \qquad \text{for} \quad i \neq j$$

σ^{ij} stress tensor acting on the crystal

s^{ij} strain (deformation) matrix of the crystal

Shear modulus



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- **It is this pattern of modulation that is rigid (and oscillates)**

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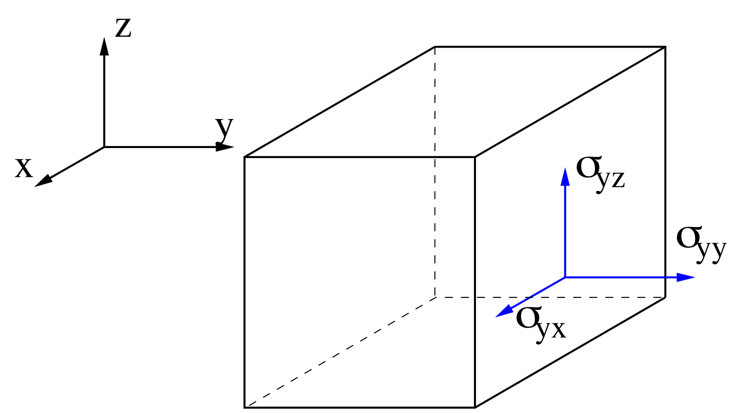

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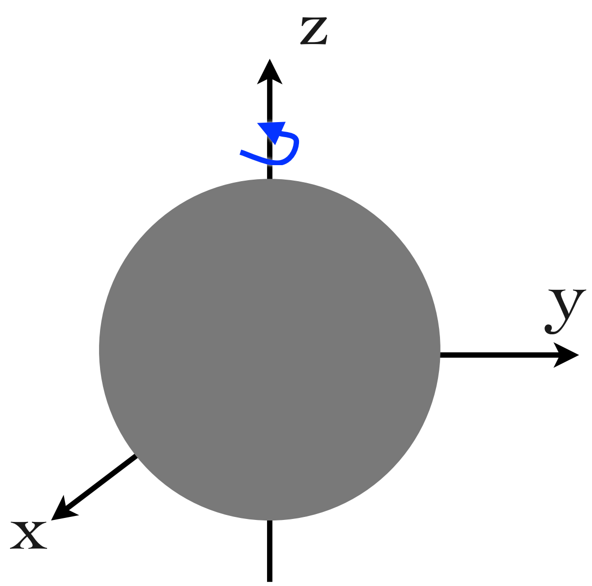
$$\nu = 2.47\,\frac{\mathrm{MeV}}{\mathrm{fm}^3}\left(\frac{\Delta}{10\mathrm{MeV}}\right)^2\left(\frac{\mu}{400\mathrm{MeV}}\right)^2$$

More rigid than diamond!!
20 to 1000 times more rigid than the crust of neutron star

MM, Rajagopal and Sharma Phys.Rev. D76 (2007) 074026

Gravitational waves from “mountains”



If the star has a non-axial symmetric deformation (mountain) it can emit gravitational waves

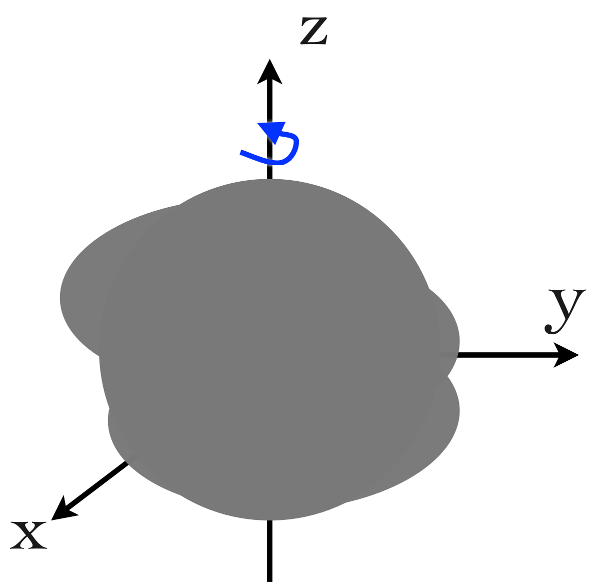
ellipticity

$$\epsilon = \frac{I_{xx} - I_{yy}}{I_{zz}}$$

GW amplitude

$$h = \frac{16\pi^2 G}{c^4}\frac{\epsilon I_{zz}\nu^2}{r}$$

Gravitational waves from "mountains"



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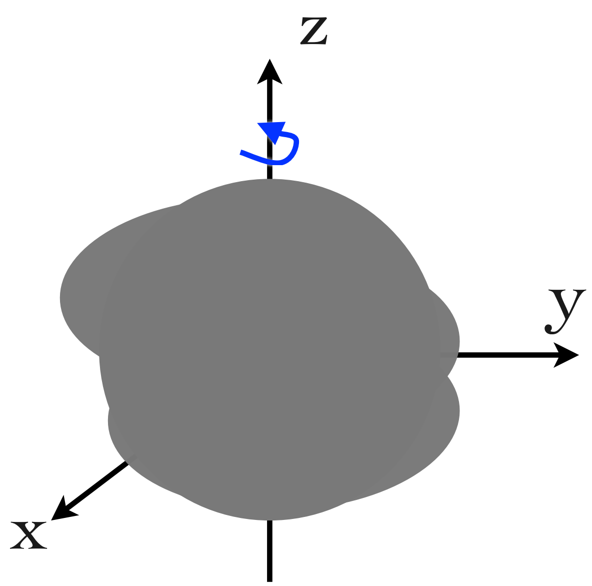

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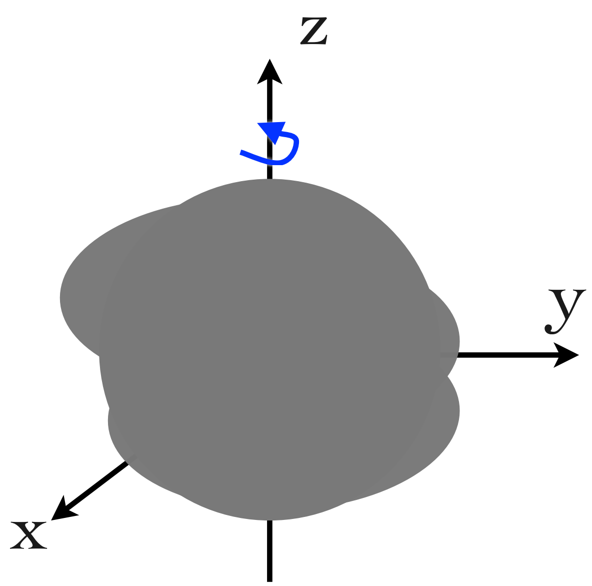

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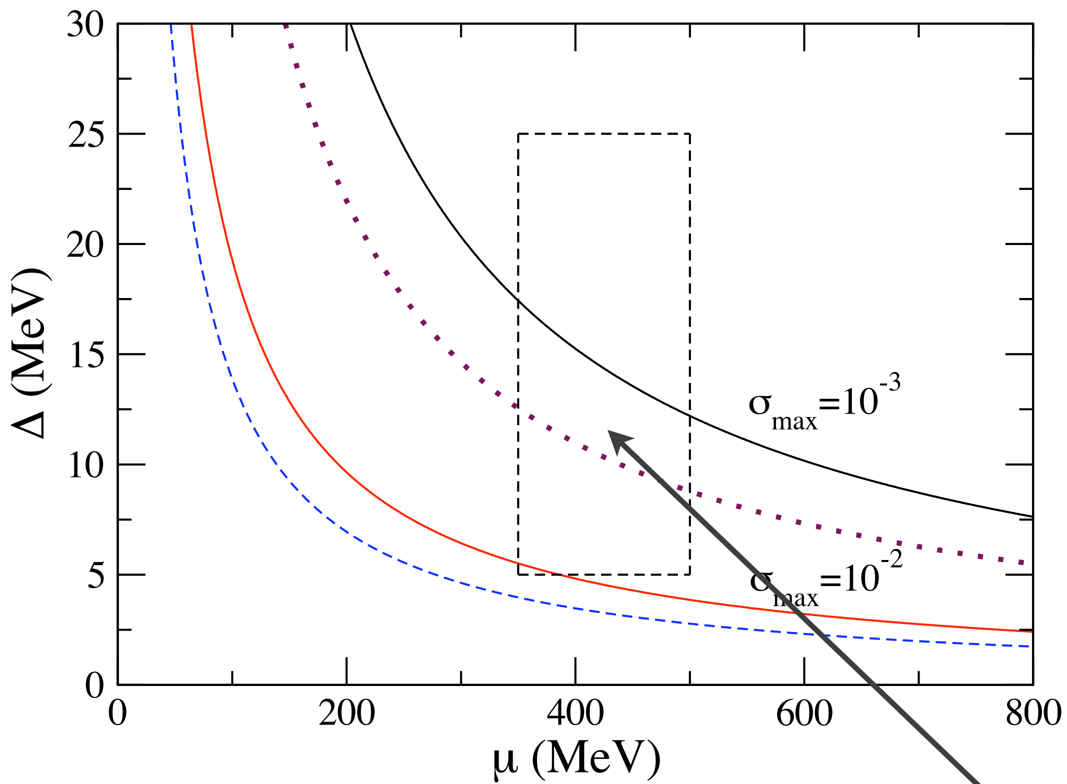
To have a "large" GW amplitude

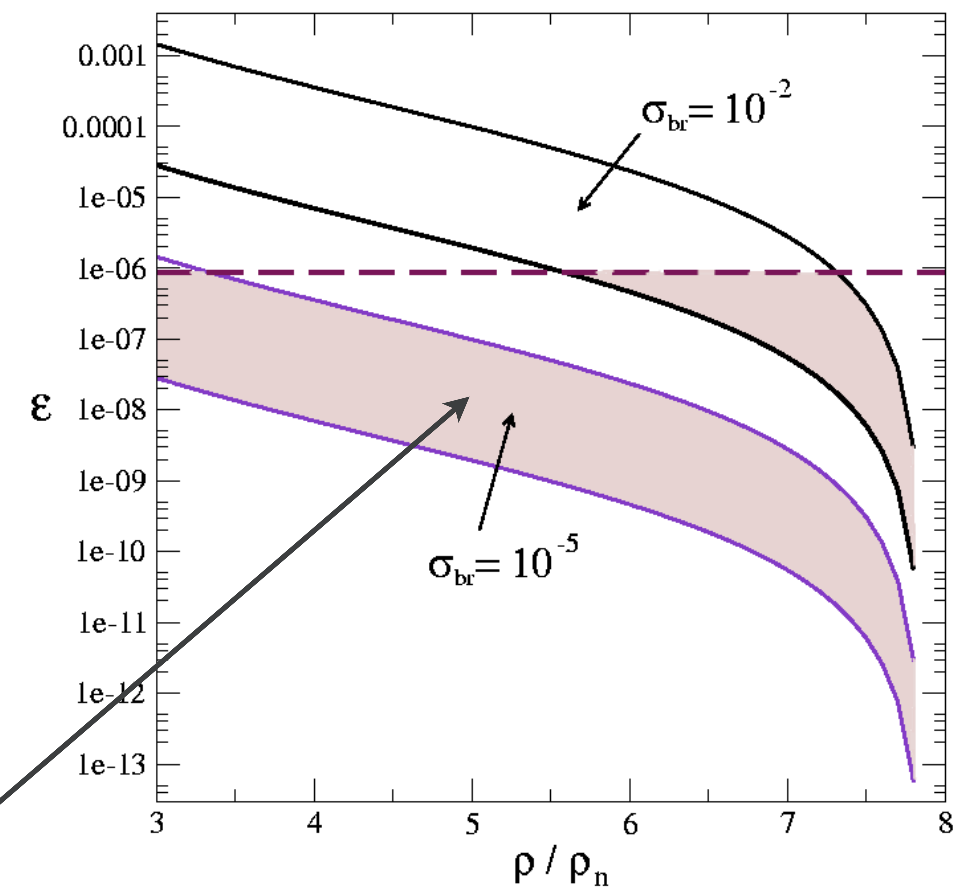
- **Large shear modulus**
- **Large breaking strain**

Gravitational waves

Using the non-observation of GW from the Crab by the LIGO experiment



Lin, Phys.Rev. D76 (2007) 081502



Andersson et al. Phys.Rev. Lett.99. 231101 (2007)

allowed regions

Gravitational waves

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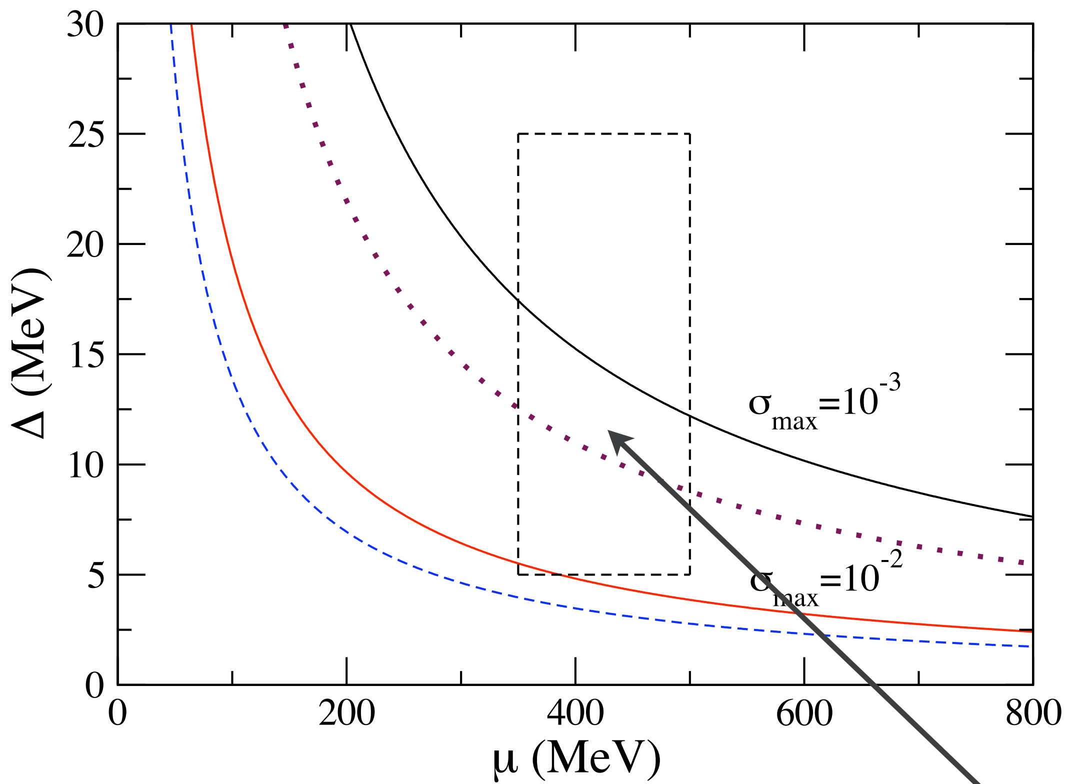

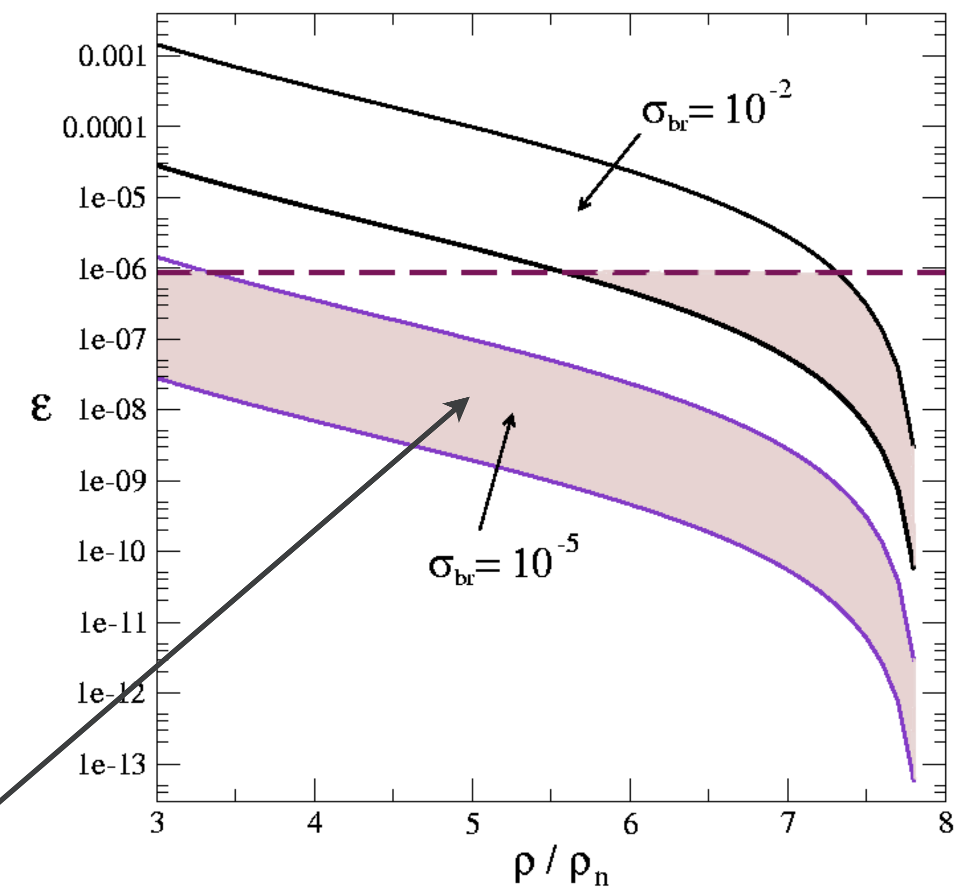

Lin, Phys.Rev. D76 (2007) 081502

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Andersson et al. Phys.Rev. Lett.99. 231101 (2007)

...we can restrict the parameter space!

Summary

- **The study of matter in extreme conditions allows to shed light on the basic properties of QCD**

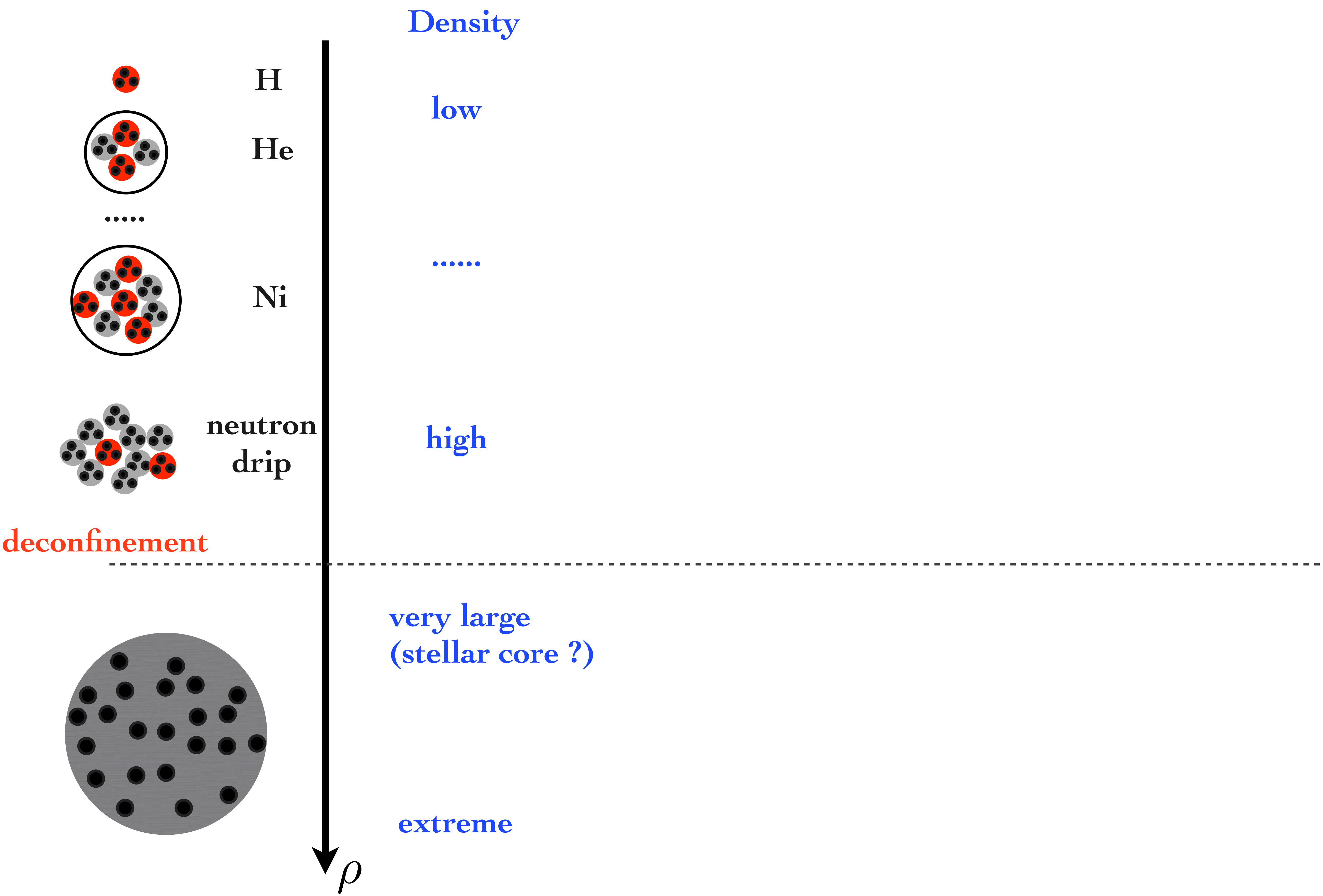
- **Color superconductivity is a phase of matter predicted by QCD**

- **At asymptotic densities matter should be color-flavor locked**

- **In realistic conditions a crystalline rigid color superconducting phase should be favored**

Back-up slides

Increasing the baryonic density

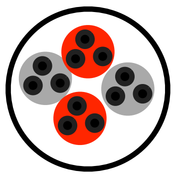


Increasing the baryonic density

Density $\alpha_s \equiv \alpha_s(\mu)$

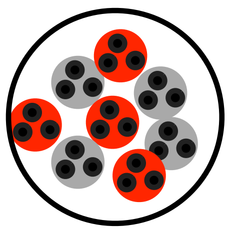
H

low



He

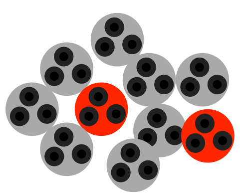
.....



......

Confining

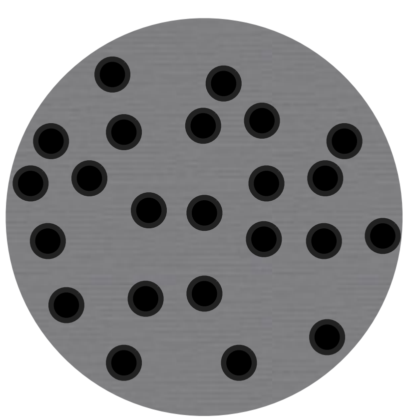
Ni



neutron drip

high

deconfinement



very large (stellar core ?)

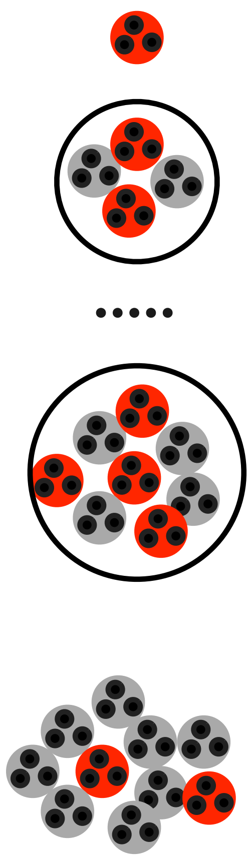
Strong coupling

extreme

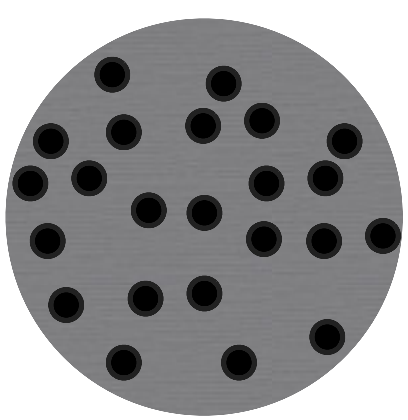
Weak coupling

ρ

Increasing the baryonic density



		Density	$\alpha_s \equiv \alpha_s(\mu)$	Degrees of freedom
	H	low		light nuclei
	He			.....
.....		......	Confining	heavy nuclei
	Ni			
	neutron drip	high		neutrons and protons
deconfinement				
		very large (stellar core ?)	Strong coupling	quarks and gluons Cooper pairs of quarks? quarkyonic phase?....
		extreme	Weak coupling	Cooper pairs of quarks NGBs
	ρ			

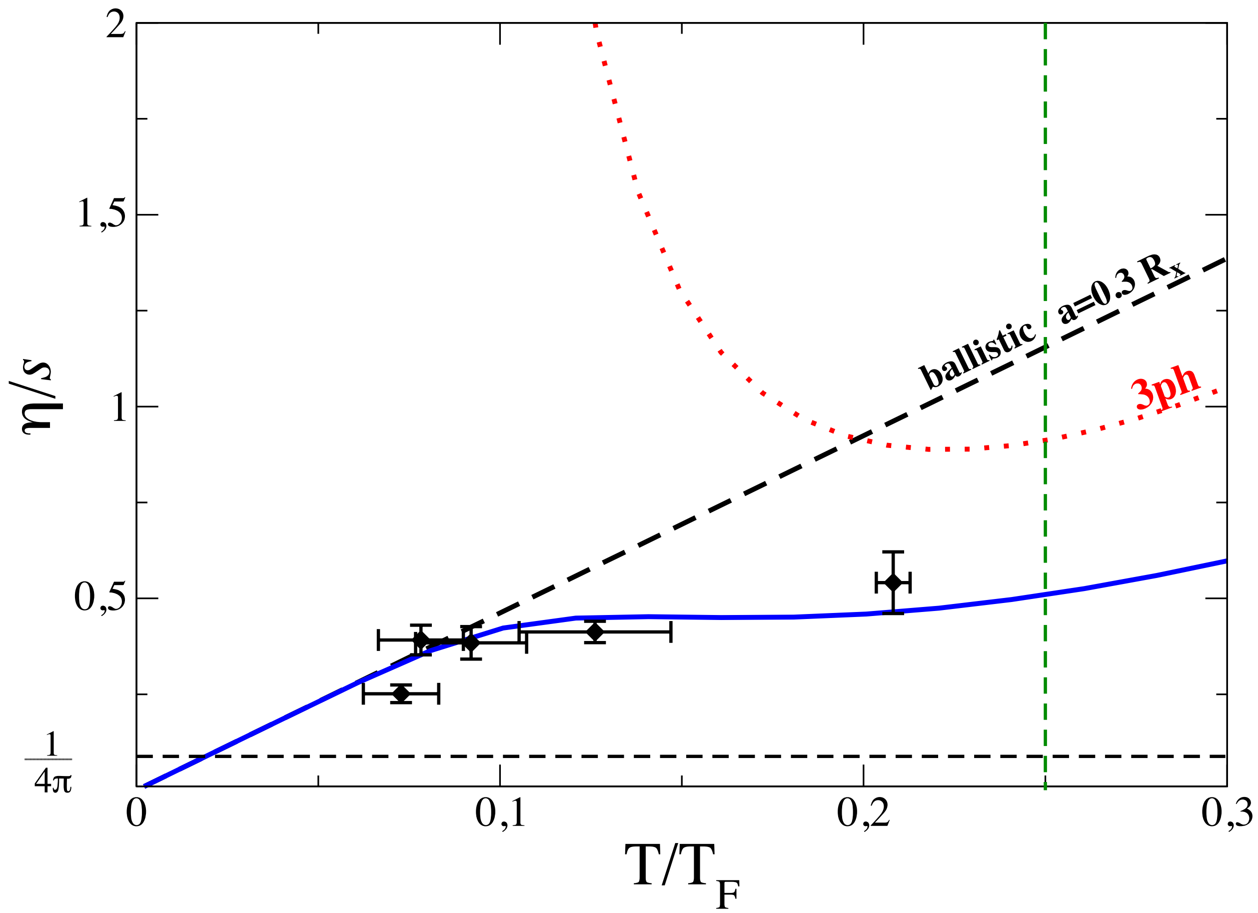


Phonons in cold atom experiments

Experiments with ultracold fermionic atoms in an optical trap helpful to understand properties of NGBs

Phonons originate from the breaking of particle number
At low temperature they should dominate the thermodynamics and the dissipative processes

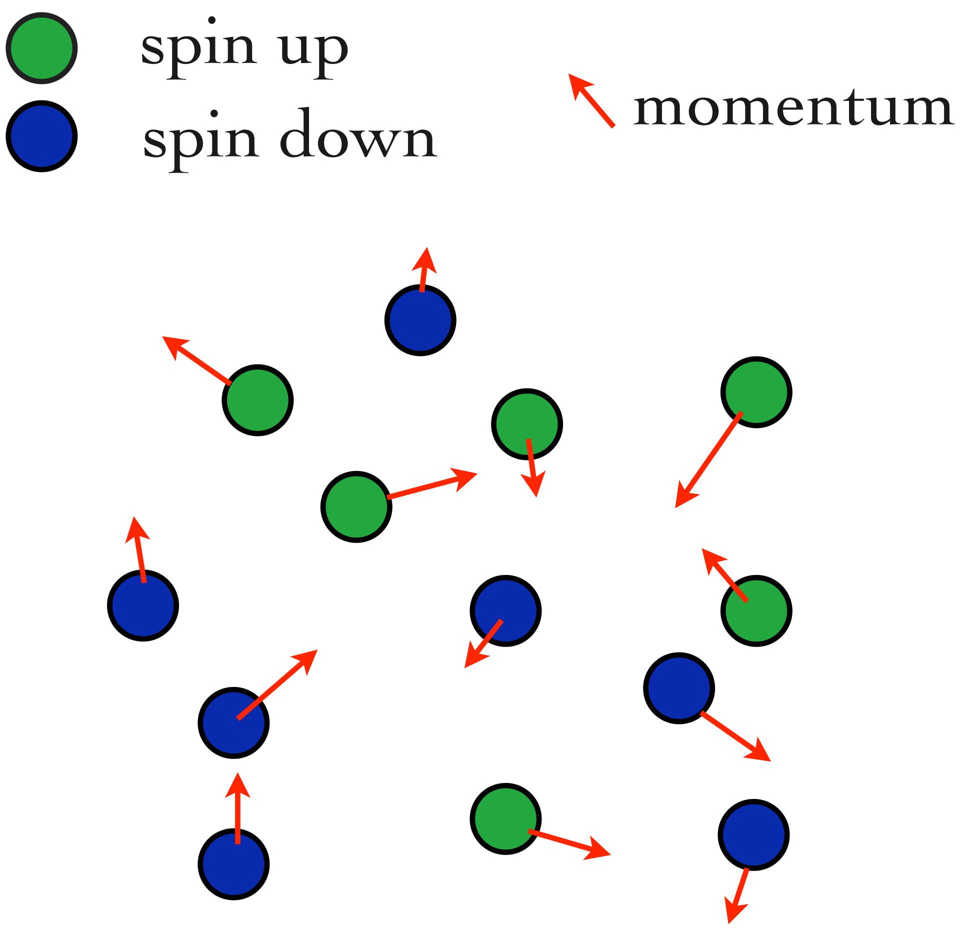
At very low temperature they are ballistic (but still produce dissipation)



MM, Manuel, Tolos 1201.4006

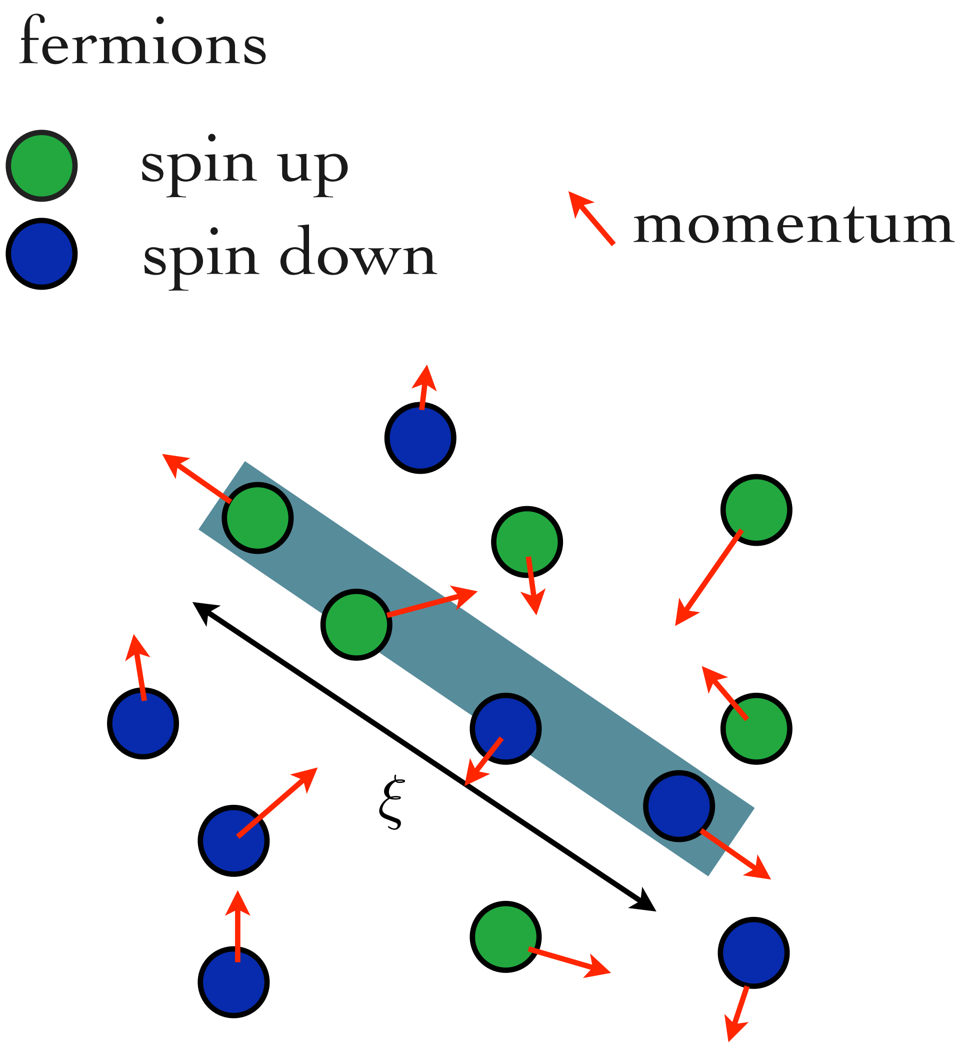
Pairing

fermions



- **Cooper pairs**: di-fermions with total spin 0 and total momentum 0

Pairing



- **Cooper pairs**: di-fermions with total spin 0 and total momentum 0

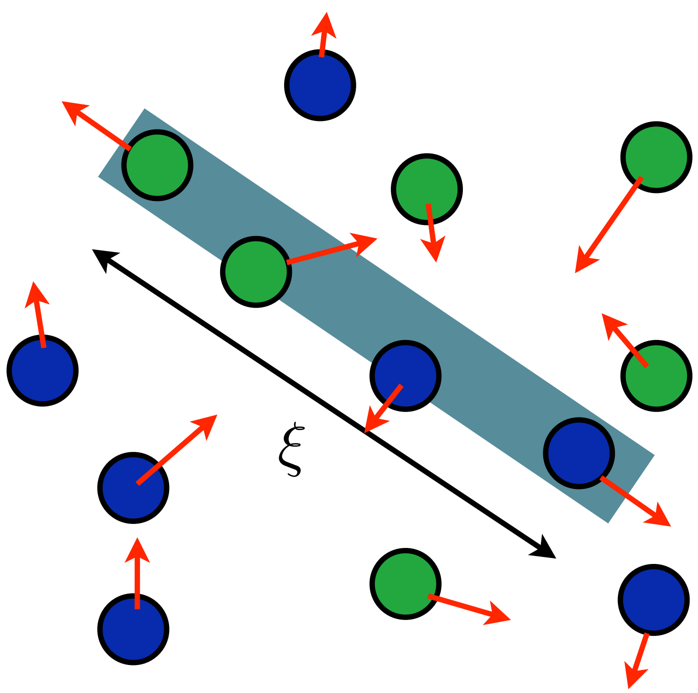
Pairing

fermions

spin up

spin down

momentum



- **Cooper pairs**: di-fermions with total spin 0 and total momentum 0

$$\xi \sim \frac{v_F}{\Delta}$$

BCS: loosely bound pairs $\xi \gtrsim n^{-1/3}$

BEC: tightly bound pairs $\xi \lesssim n^{-1/3}$

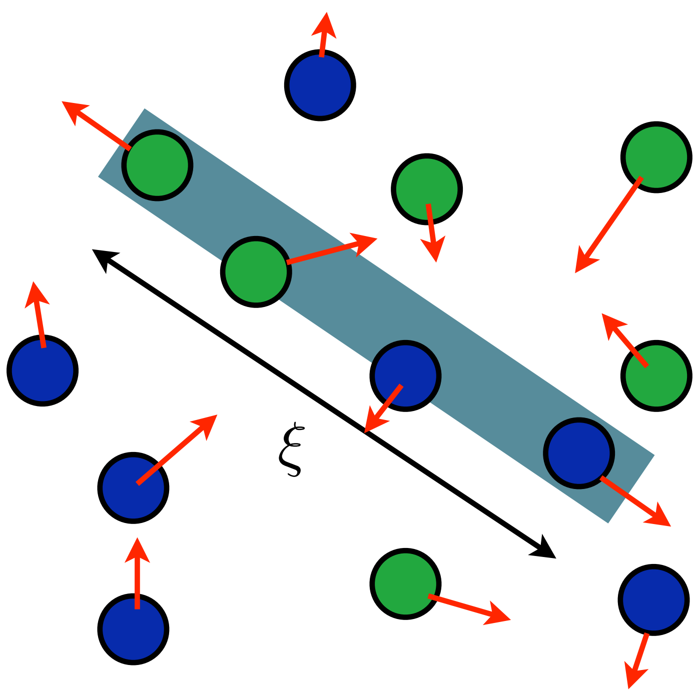
Pairing

fermions

spin up

spin down

momentum



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BCS: loosely bound pairs $\xi \gtrsim n^{-1/3}$

BEC: tightly bound pairs $\xi \lesssim n^{-1/3}$

Type I (Pippard): $\lambda \ll \xi$ first order phase transition to the normal phase

Type II (London): $\lambda \gg \xi$ second order phase transition to the normal phase

Chiral symmetry breaking

At low density the χSB is due to the condensate that locks left-handed and right-handed fields $\langle\bar{\psi}\,\psi\rangle$

In the CFL phase we can write the condensate as

$$\langle\psi^L_{\alpha i}\psi^L_{\beta j}\rangle = -\langle\psi^R_{\alpha i}\psi^R_{\beta j}\rangle = \kappa_1\delta_{\alpha i}\delta_{\beta j} - \kappa_2\delta_{\alpha j}\delta_{\beta i}$$

Color is locked to both left-handed and right-handed rotations.