# Chern-Simons-Matter Theories, High-Spin Gravity, and 3D Bosonization

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[Aharony, GA, Yacoby 2011, 2012] [GA, Yacoby 2012]

[Aharony, Giombi, GA, Maldacena, Yacoby 2012]

# Vector Models in 3D

$$\mathcal{L} = \partial^\mu \phi_i^\dagger \partial_\mu \phi^i + \frac{\lambda_6}{N^2}(\phi_i^\dagger \phi^i)^3$$

- Complex scalars $i = 1, \ldots, N$
- Singlet sector of $U(N)$
- Single-trace primaries:

$$J_s = \phi_i^\dagger \partial \cdots \partial \phi^i \qquad s = 0, 1, 2, \ldots$$

$$\Delta_s = s + 1$$

# Vector Models in 3D

$$\mathcal{L} = \bar{\psi}_i \gamma^\mu \partial_\mu \psi^i$$

- Singlet sector of $U(N)$
- Single-trace primaries:

$$J_0 = \bar{\psi}\psi$$

$$\Delta_0 = 2$$

$$J_s = \bar{\psi}\gamma\partial\cdots\partial\psi$$

$$s = 1, 2, \ldots$$

$$\Delta_s = s + 1$$

# Outline

- 3D vector models $\leftrightarrow$ high-spin gravity
- Matter + Chern-Simons interactions
- 3D bosonization

# Vasiliev's Theory

- Defined on $AdS_4$
- Scalar, photon, graviton, ...
- Consistent, classical interacting theory

$$d_x\hat{\mathcal{A}} + \hat{\mathcal{A}} * \hat{\mathcal{A}} = \left[\frac{1}{4} + e^{i\theta_0} B * K\right] dz^2 + \text{c.c.}$$

$$d_x B + \hat{\mathcal{A}} * B - B * \pi(\hat{\mathcal{A}}) = 0$$

# Holographic Duality

Boundary scalars:

$$J_0, J_s \to B, \hat{\mathcal{A}} \qquad \theta_0 = 0 \qquad G_N \sim \frac{1}{N}$$

Boundary fermions:

$$\theta_0 = \pi/2$$

[Sezgin, Sundell 2001]

# Critical Vector Model

$$\delta\mathcal{L} \sim (\phi^\dagger \phi)^2$$

- $J_0$ dimension: $\Delta_0^{\mathrm{IR}} = d - \Delta_0 = 2$
- Currents not renormalized at large *N*
- Bulk: change of scalar boundary conditions

[Klebanov, Polyakov 2002]

Free Scalars

$\downarrow$

Wilson-Fischer

# Detailed Evidence

- Conformal theory in flat 3D space + high-spin symmetry $\Rightarrow$ free theory
  [Maldacena, Zhiboedov 2011]
- All 3-point functions agree
  [Giombi, Yin 2010]

# Motivation

- Weak-weak duality
- Tensionless limit of string theory
  [Chang, Minwalla, Sharma, Yin 2012]
- Boundary scalars and fermions continuously connected

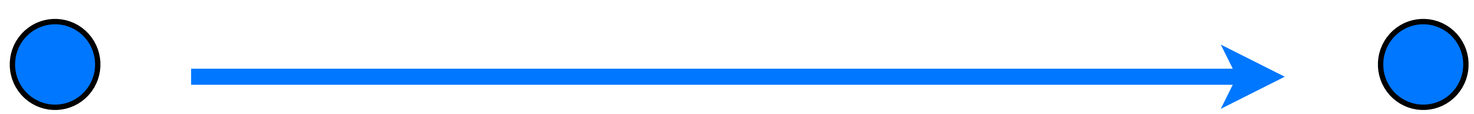
# Bulk Suggests Bosonization

Boundary scalars / fermions are connected

$$d_x \hat{\mathcal{A}} + \hat{\mathcal{A}} * \hat{\mathcal{A}} = \left[ \frac{1}{4} + e^{i\theta_0} B * K \right] dz^2 + \text{c.c.}$$

Boundary Scalars

Boundary Fermions



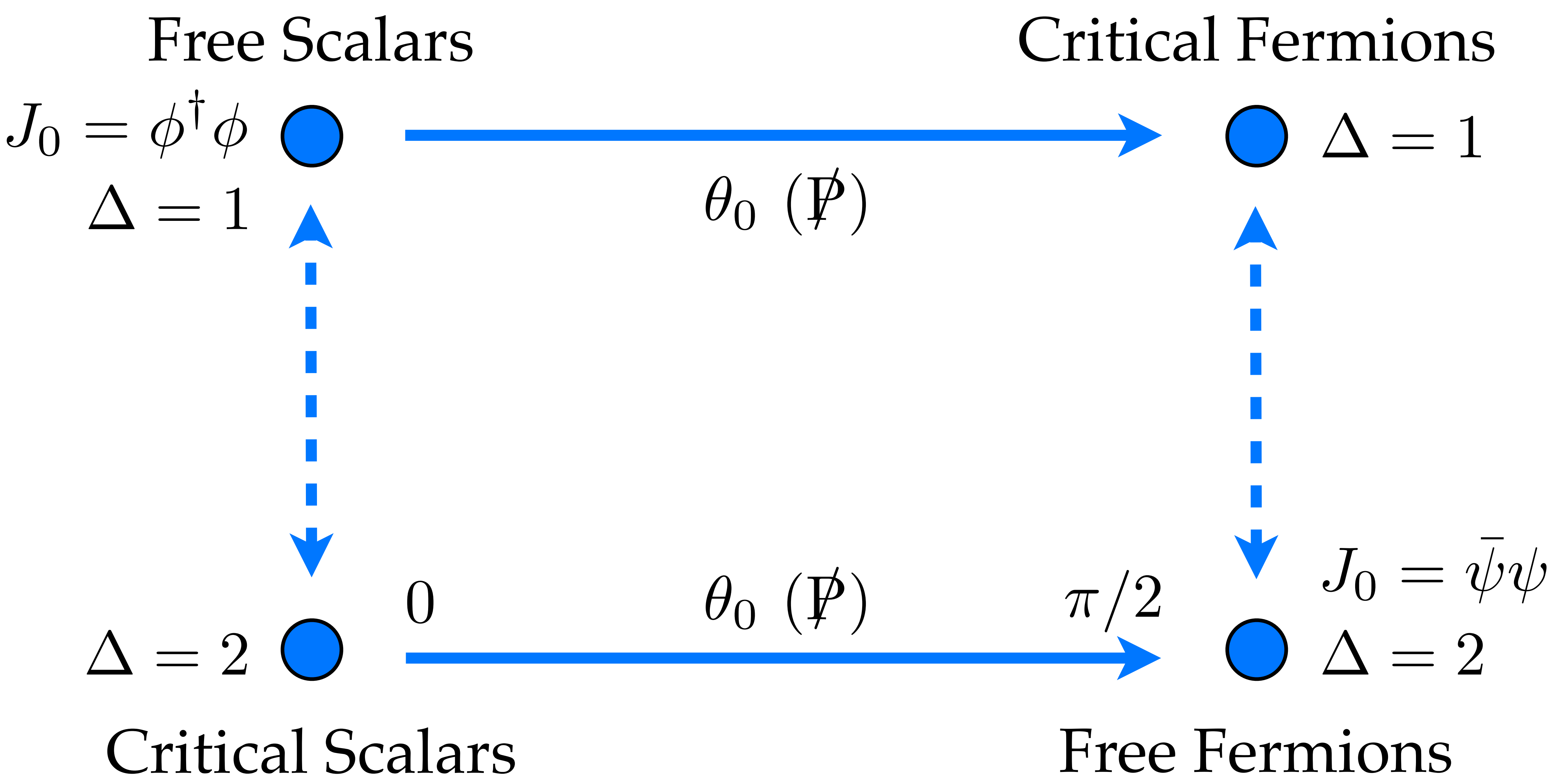
0

$\theta_0$

(~~Parity~~)

$\pi/2$

# Bulk Suggests Bosonization



# Matter + Chern-Simons Interactions

# Why Chern-Simons?

$$S_{\mathrm{matter}} + \frac{ik}{4\pi}\int \mathrm{Tr}\left(A \wedge dA + \frac{2}{3}A^3\right)$$

$$N, k \to \infty \qquad\qquad \lambda = \frac{N}{k}$$

- $U(N)$ symmetry already gauged
- No additional primary operators

# Conformal Symmetry

$$S_{\text{matter}} + \frac{ik}{4\pi}\int \mathrm{Tr}\left(A \wedge dA + \frac{2}{3}A^3\right)$$

- Chern-Simons level $k$ is quantized
- $(\phi^\dagger \phi)^3$ deformation marginal at large $N$
- Theory is conformal at large $N$

[Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin 2011]
[Aharony, GA, Yacoby 2011]

# Fate of the High-Spin Symmetry

$$\partial \cdot J_s \sim 0 + \frac{1}{N} J_{s-2} J_0 + \cdots$$

$\Delta - s = 3$

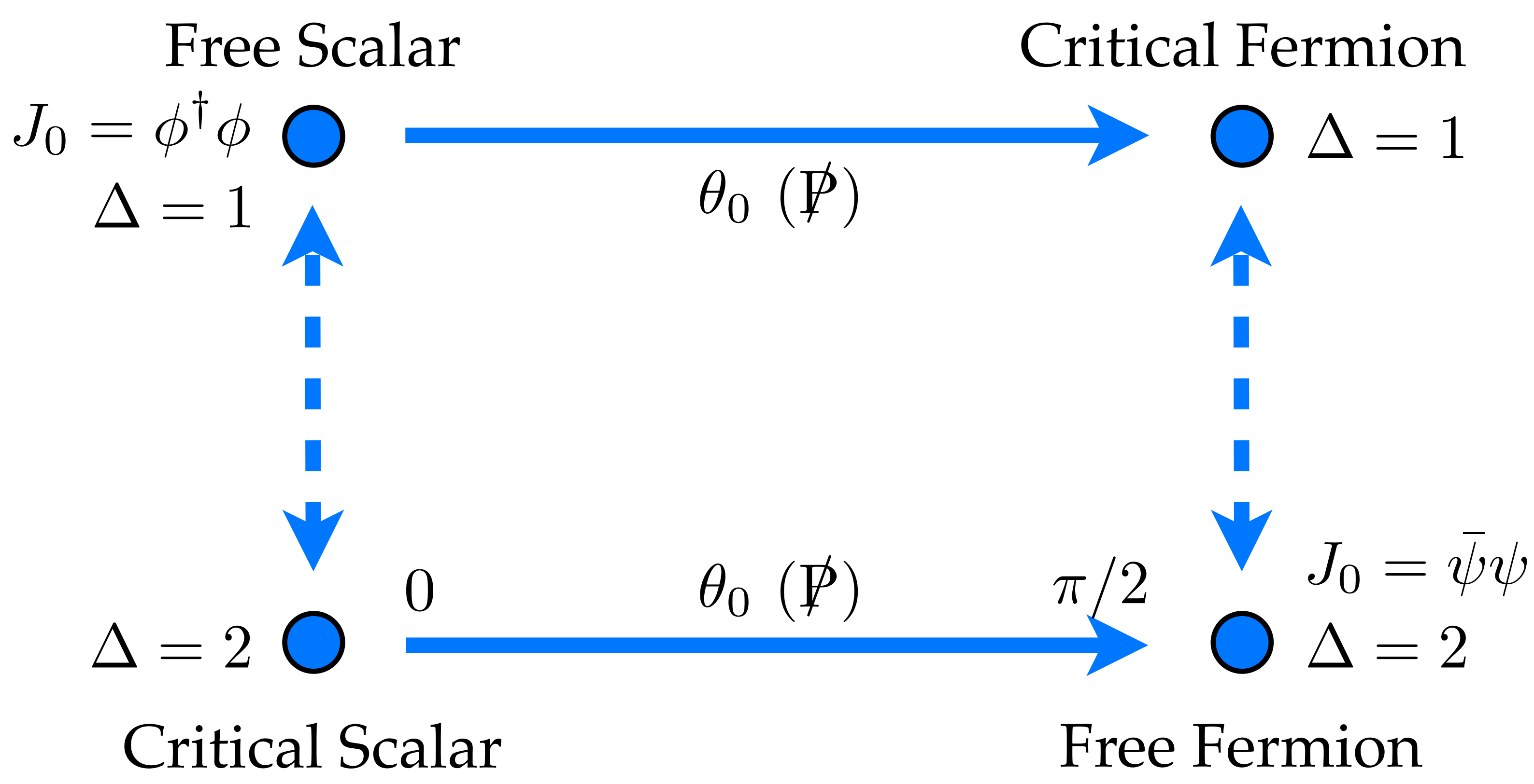
$$\partial \cdot \langle J_s J_{s'} \rangle = 0$$

Currents not renormalized at large $N$

$$\partial \cdot \langle J_s J_{s'} J_{s''} \rangle \neq 0$$

Symmetry is broken, $J_0$ not renormalized

# 3D Bosonization



# CFT Perspective

- General theory, high-spin symmetry 'slightly' broken: $\partial \cdot J_s \sim J_{s-2} J_0 / N + \cdots$
- Broken Ward identities constrain correlators
- 3-point functions of single-trace operators fixed by two parameters $\tilde{N}, \tilde{\lambda}$

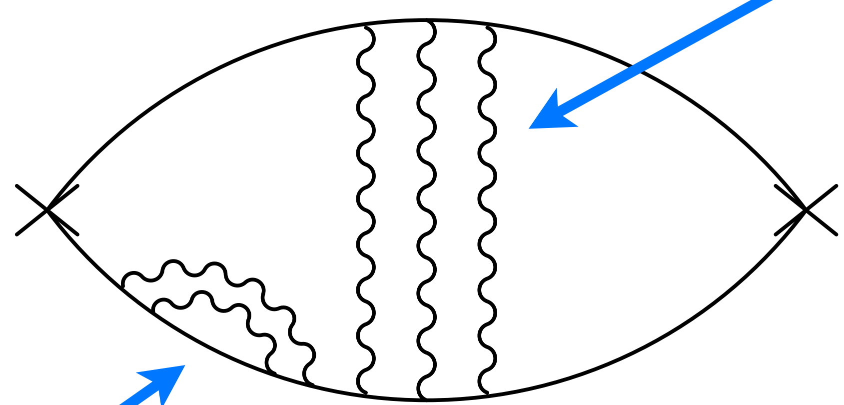
$$\langle JJJ \rangle_{\Delta=1} = \tilde{N} \left[ \frac{1}{1+\tilde{\lambda}^2} \langle \cdot \rangle_{\mathrm{bos.}} + \frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2} \langle \cdot \rangle_{\mathrm{fer.}} + \frac{\tilde{\lambda}}{1+\tilde{\lambda}^2} \langle \cdot \rangle_{\mathrm{odd}} \right]$$

[Maldacena, Zhiboedov 2012]

$$\boxed{\begin{array}{c} \tilde{N}, \tilde{\lambda} \leftrightarrow N, \lambda = N/k \\ ? \end{array}}$$

# Computing Exact Correlators

$$\langle J_0 J_0 \rangle = \quad J_0$$



$$J_0 \quad + \cdots$$

$\langle \phi\phi^\dagger \phi\phi^\dagger \rangle$

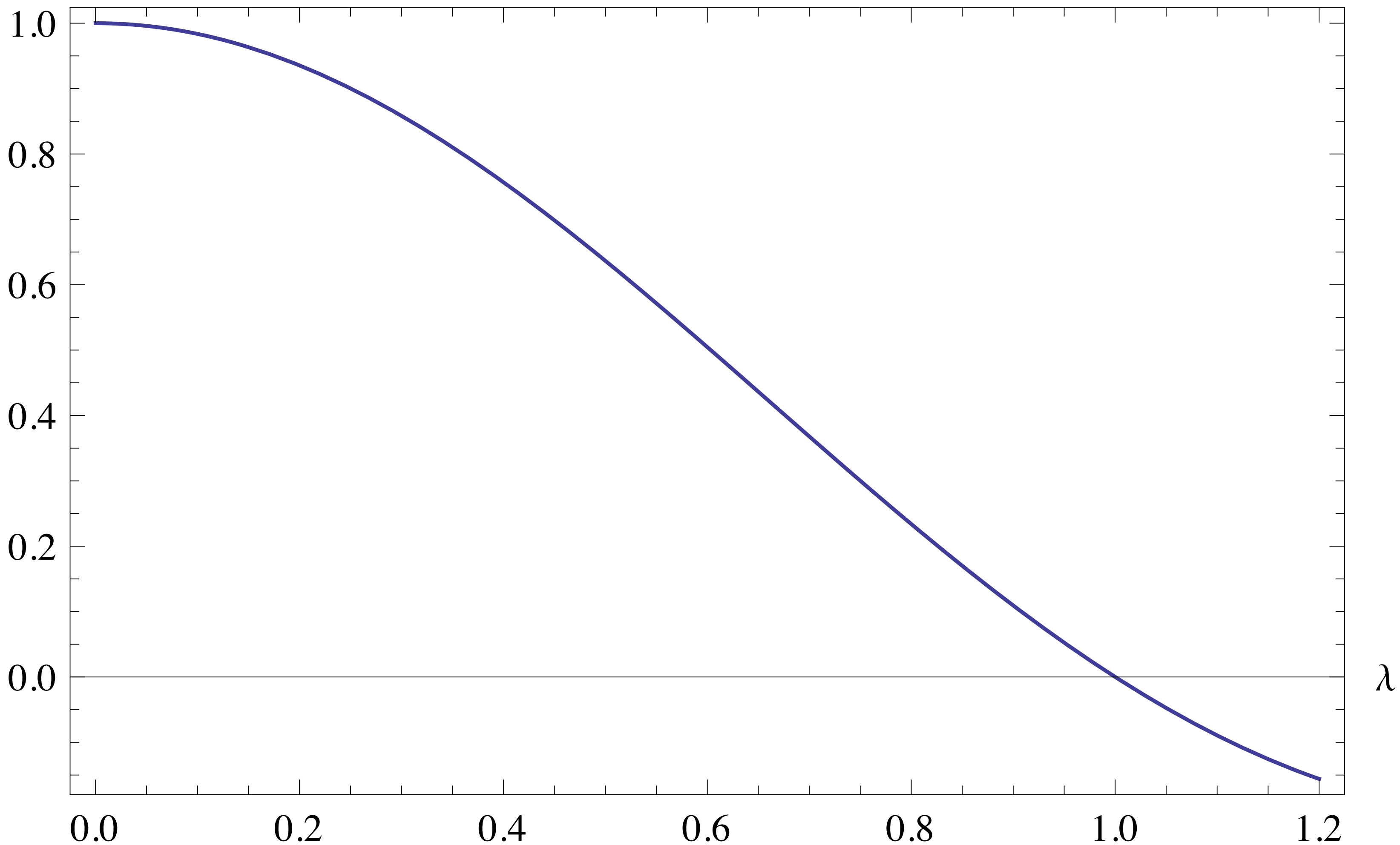
$\langle \phi\phi^\dagger \rangle$

$$\langle TT \rangle \, , \; \langle J_0 J_0 J_0 \rangle \, , \; \cdots$$

[Aharony, GA, Yacoby 2012] [GA, Yacoby 2012]

# Degrees of Freedom

$$\frac{\langle TT \rangle}{\langle TT \rangle_{\text{free}}} = \frac{\sin(\pi\lambda)}{\pi\lambda} \qquad (\lambda = N/k)$$




[Aharony, GA, Yacoby 2012] [GA, Yacoby 2012]

# Exact Correlators

$$\langle JJJ \rangle_{\Delta=1} = \tilde{N} \left[ \frac{1}{1+\tilde{\lambda}^2} \langle \cdot \rangle_{\text{bos.}} + \frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2} \langle \cdot \rangle_{\text{fer.}} + \frac{\tilde{\lambda}}{1+\tilde{\lambda}^2} \langle \cdot \rangle_{\text{odd}} \right]$$

$$= \frac{2N \sin(\pi\lambda)}{\pi\lambda} \left[ \cos^2\left(\frac{\pi\lambda}{2}\right) \langle \cdot \rangle_{\text{bos.}} + \sin^2\left(\frac{\pi\lambda}{2}\right) \langle \cdot \rangle_{\text{fer.}} \right.$$

$$\left. + \frac{\sin(\pi\lambda)}{2} \langle \cdot \rangle_{\text{odd}} \right] \qquad (\lambda = N/k)$$

$\langle J_0 J_0 J_0 \rangle, \ldots$

Bulk coupling: $\quad \theta_0 = \dfrac{\pi\lambda}{2}$

[Chang, Minwalla, Sharma, Yin 2012]

[Aharony, GA, Yacoby 2012] [GA, Yacoby 2012]

# Bosonization Duality

- Comparing correlators gives relation

$$\underset{\text{(Scalars)}}{N, \lambda} \quad \leftrightarrow \quad \underset{\text{(Fermions)}}{N, \lambda} \qquad (\lambda = N/k)$$

- Level-rank duality in terms of $k_0 = k - \mathrm{sign}(k)N$

$\lambda = 0$ Scalar + $U(N)_{k_0}$ $\lambda = 1$

$\lambda = 1$ Critical Fermion + $U(k_0)_{-N}$ $\lambda = 0$

# Thermal Free Energy

- Thermal cycle can support non-trivial holonomy

$$\oint A \neq 0$$

- Eigenvalues hold gauge-invariant info

$$F/T : \quad NVT^2 \gg N^2$$

Matter sector
Attractive potential

Pure Chern-Simons

[Aharony, Giombi, GA, Maldacena, Yacoby 2012]

# Non-Trivial Holonomy

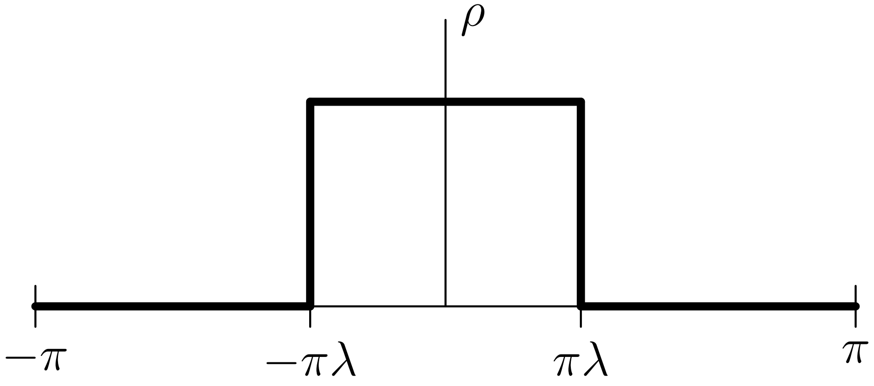
Reduce on $T^2 \times \mathbb{R}$

$$S_{\rm CS} \to \frac{k}{2\pi}\int dt\, a\, \dot{b} \qquad a, b = \oint_{\Sigma_{1,2}} A = \begin{pmatrix} \ddots \end{pmatrix}_{1,2}$$

$$\pi_b = \frac{k}{2\pi} a \qquad b \cong b + 2\pi \qquad \Rightarrow \qquad \Delta a = \frac{2\pi}{k}$$

Eigenvalues are quantized & fermionic, [Douglas, 94]

Spread evenly: $$\frac{2\pi}{k} N = 2\pi\lambda$$




[Aharony, Giombi, GA, Maldacena, Yacoby 2012]

# Thermal Free Energy

- Free energy computed by summing planar diagrams
- Consistent with bosonization

$$F_{\rm f} = -\frac{NV_2T^3}{2\pi^2 i\lambda}\left[\frac{\mu^2}{3}\mathrm{Li}_2\left(-e^{-\mu+\pi i\lambda}\right) + \int_\mu^\infty dy\, y\, \mathrm{Li}_2\left(-e^{-y+\pi i\lambda}\right) - \text{c.c.}\right]$$

Thermal mass

$$(1-\lambda)\mu = \frac{1}{\pi i}\left[\mathrm{Li}_2(-e^{-\mu-\pi i\lambda}) - \text{c.c.}\right]$$

[Aharony, Giombi, GA, Maldacena, Yacoby 2012]

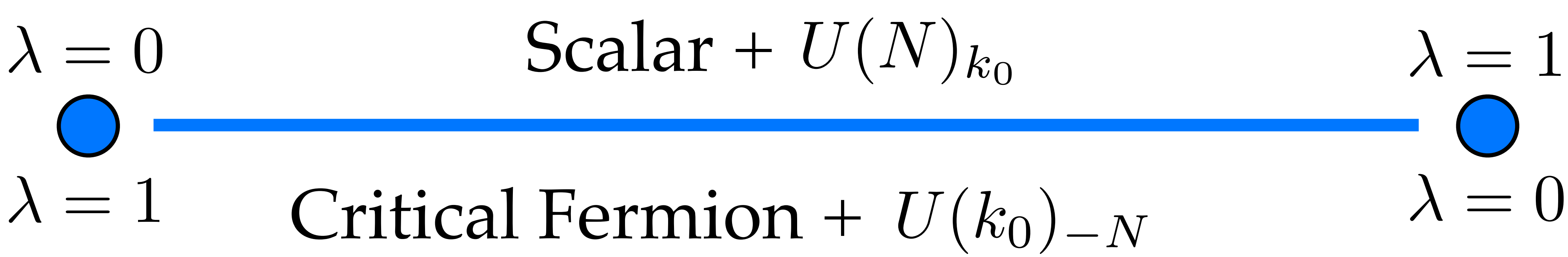
# Open Questions

- Better understand bosonization
  - $1/N$
  - Mapping of fundamental fields
- Better understand holographic duality
  - Singlet sector implementation
  - Pure Chern-Simons contributions
    [Banerjee, Hellerman, Maltz, Shenker 2012]

# Summary

- Vector models + Chern-Simons are dual to Vasiliev's theory with broken parity

- 3D bosonization




  - Single-trace 3-point functions, thermal free energy

Thank You