Matter matters in higher spin holography

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Based on:

1209.4937, w/P. Kraus

1210.8452, w/T. Prochazka, J. Raeymaekers

1302.6113, w/E. Hijano, P. Kraus

1305.xxxx, w/M. Gaberdiel, K. Jin

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Duality checklist

- Classical symmetries
- Spectrum of perturbative states (vacuum descendants; scalar primaries)
- Spectrum of nonperturbative states (classical geometries, e.g. conical defects and black holes)
  - Modular invariance: relates operators with Δ >>1 to those with $\Delta \sim O(1)$
- Interactions among these states: correlation functions
  - On the plane, on the torus
- Many higher order questions

- **Beyond symmetry**: we compute, from gravity **and** CFT:
  1. Thermal correlators in the presence of higher spin charge
  2. 4-point functions of W_N scalar primary operators
     - Highly nontrivial check of structure of interactions in CFT!

Basics: scalars in 3d Vasiliev gravity

- Linearizing around flat connections, matter equation is:

$$dC + A \star C - C \star \overline{A} = 0$$

- C = spacetime 0-form containing scalar + derivatives
- Physical scalar field is identity piece of C: $\boxed{\Phi = \mathrm{Tr}[C] = C_0^1}$
- Locally, all is pure gauge: $C = e^{-\Lambda} \star c \star e^{\overline{\Lambda}}\ ,\ \text{where } dc = 0$

For bulk-boundary propagators, take c to be a highest weight state of hs[λ]; then AdS/CFT says

$$\langle \bar{\mathcal{O}}(z,\bar{z})\mathcal{O}(0,0)\rangle \sim \Phi|_{\rm bndy}$$

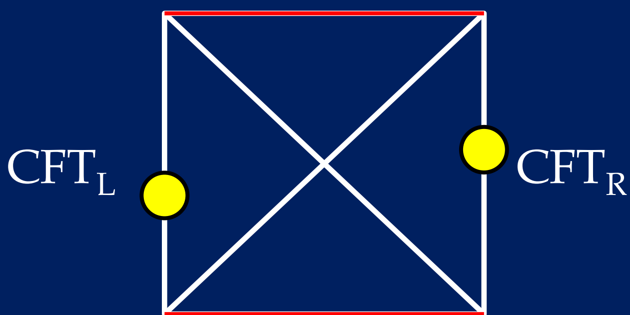
[Note: c is a projector!]

- For λ=-N, this is simple:

$$G(z,\overline{z}) = \left|\langle N|e^{-\Lambda_0}|1\rangle\right|^2\ ,\ \text{where } \Lambda_0 = A_z z + A_{\overline{z}}\overline{z}$$

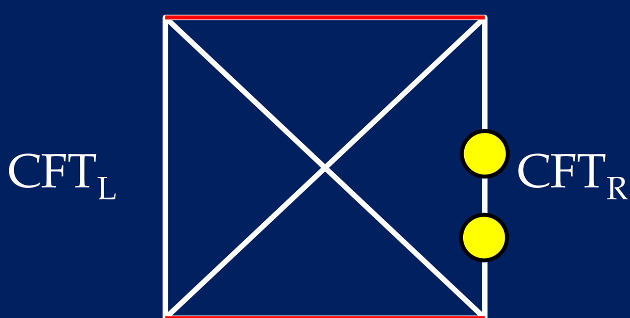
Probing higher spin black holes

- Starting from BTZ, turn on spin-3 chemical potential, α
  - Infinite tower of nonzero charges
- "Smoothness" ~ $\boxed{H = e^{\oint_\tau A} = H_{BTZ} = e^{i\pi(1+\lambda)}\mathbf{1}}$
  - Strong evidence for black hole interpretation despite non-invariant notions of geometry!
- Probe with a scalar: what does it see?



- Mixed correlator: nonsingular

$$\langle \overline{\mathcal{O}}(t,\phi)\mathcal{O}(0,0)\rangle \sim \frac{\sum_n \alpha^n \times \text{Finite}(t,\phi)}{(\cosh r_+ t + \cosh r_+ \phi)^{1+\lambda}}$$



- Thermal correlator in spin-3 perturbed CFT

Computed through O(α^2) in bulk

→ **Match to CFT**

Probing higher spin black holes from CFT

- Bulk higher spin chemical potential ←→ Perturbation of CFT action

$$\delta S_{CFT} = -\mu \int d^2v W(v) \ , \ \text{where } \mu = \alpha/\overline{\tau}$$

- Deformed correlators:

$$\langle \overline{\mathcal{O}}(w,\overline{w})\mathcal{O}(0,0)\rangle_\alpha = \langle \overline{\mathcal{O}}(w,\overline{w})\mathcal{O}(0,0)e^{-\mu \int d^2v W(v)}\rangle$$

  - e.g. O(α) result fixed by 3-pt function, integrated over the torus
  - In fact, O(α) result is universal at high T; on the plane,

$$\frac{\langle \overline{\mathcal{O}}(x,\overline{x})\mathcal{O}(1,1)W(z)\rangle}{\langle \overline{\mathcal{O}}(x,\overline{x})\mathcal{O}(1,1)\rangle} = f(\lambda)\left(\frac{x-1}{(z-1)(z-x)}\right)^3$$

where f(λ) = spin-3 eigenvalue of O.

- Bulk result is reproduced in CFT via the contour integral

$$\frac{3\sin\frac{w}{\tau} + (2+\cos\frac{w}{\tau})(\frac{\overline{w}}{\overline{\tau}} - \frac{w}{\tau})}{2\sin^2\frac{w}{2\tau}} \propto \oint dz\ z^2 \log|z|^2 \left(\frac{x-1}{(z-1)(z-x)}\right)^3\Bigg|_{x=e^{-iw/\tau}}$$

- Computations done through O(α^2); total agreement [See Kewang's talk]

Interlude

- These calculations do not uniquely select W_N minimal models as CFT dual.

- Now we compute something that does:

Scalar primary 4-pt functions at large c and T=0

AdS3/CFT2: W_N CFTs at large c

- W_N minimal models: $$\frac{SU(N)_k \oplus SU(N)_1}{SU(N)_{k+1}}$$ [Gaberdiel, Gopakumar]

$$c_{N,\lambda} = -(N-1)(\lambda-1)\left(1+\frac{N\lambda}{N+\lambda}\right) \quad \text{where} \quad \lambda = \frac{N}{N+k}$$

- Currents of spin-s=2, 3, …, N, and tower of scalar primaries
- Minimal model reps labeled by pair of affine SU(N) highest weights: (Λ^+, Λ^-)

Consider two very different large c limits at fixed λ:

1. <u>'t Hooft limit:</u> $N \to \infty$

Dual to 3d Vasiliev gravity with $0 \le \lambda < 1$, assuming certain operator spectrum

Theory has classical $W_\infty[\lambda]$ symmetry in this limit

2. <u>"Semiclassical limit":</u> $N \to -\lambda$

Dual to Vasiliev gravity with λ =-N

c > N-1 implies non-unitarity, e.g. Δ<0

<u>**At linear level, sl(N) Chern-Simons theory + matter**</u>

State map in semiclassical limit

- In semiclassical limit,

[See Joris' talk]

$$h_{|\Lambda^+,0\rangle} \sim O(1)$$
$$h_{|0,\Lambda^-\rangle} \sim O(c)$$
$$\Downarrow$$

1. Perturbative excitations: $|\Box, 0\rangle \sim \phi$ ("Single trace")
   $|\otimes^n \Box, 0\rangle \sim \phi^n$ ("Multi-trace")

2. Classical backgrounds ("conical defects"): $|0, \Lambda^-\rangle$

`Conical defect' = Smooth, asymptotically AdS solution of sl(N,C) Chern-Simons theory with nonzero higher spin charges and contractible spatial cycle

[Castro, Gopakumar, Gutperle, Raeymaekers]

$$A = (e^\rho L_1 + \sum_{n=1}^{N-1} Q_{n+1} e^{-n\rho} (L_{-1})^n) dw + L_0 d\rho$$

Higher spin charges

Smoothness fixes {Q} to contain precisely the information in an SU(N) Young diagram, viz. that of Λ^-

Beyond symmetry

- Goals:
  1. Compute 4-point functions from bulk Vasiliev theory
  2. Match to a boundary calculation, in the semiclassical limit

- Obvious question: how does one compute 4-pt functions in the bulk without pain?

- One answer: Choose a simple correlator!

Matching 4-pt functions in higher spin gravity

$$\langle D|\overline{\phi}(1,1)\phi(z,\overline{z})|D\rangle = \langle 0|D(\infty)\overline{\phi}(1,1)\phi(z,\overline{z})\overline{D}(0)|0\rangle$$

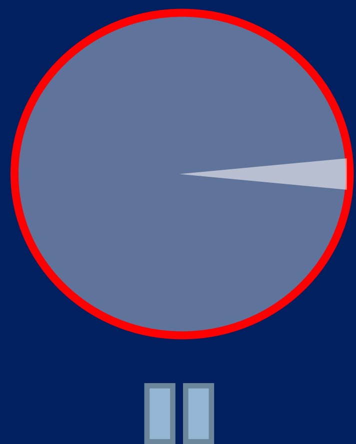
(Defect background) ⟸ $D \equiv |0,\Lambda^-\rangle\ ,\quad \phi \equiv |f,0\rangle$ ⟹ (Perturbative state)

FREE SCALARS IN λ=-N VASILIEV THEORY

W_N CORRELATORS FROM COULOMB GAS

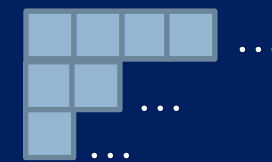
I. Free scalars in λ=-N Vasiliev theory

- Simple manipulations, simple result



$\Longleftrightarrow$

$|0, \Lambda^{-}\rangle$, where $\Lambda^{-} =$



$$\left[n_i = r_i - \frac{B}{N} + \frac{N+1}{2} - i \right]$$

Flat, diagonalizable connection, a_z: $\mathrm{eig}(a_z) = i(n_1, n_2, \ldots, n_N)$

- Computation requires only the matrix element <N | exp(a_z z) | 1>

$$\langle D|\overline{\phi}(1,1)\phi(z,\overline{z})|D\rangle = \left| z^{\frac{N-1}{2}} \sum_{j=1}^{N} \frac{z^{n_j}}{\underline{\prod_{l\neq j}(n_l - n_j)}} \right|^2$$

[Note: det(Vandermonde^{-1})]

W_N vs. Virasoro minimal models

- Generally, 4-pt functions not fixed by conformal symmetry
- Recall some facts about Virasoro:
  - 3-pt functions w/descendants fixed by those of primaries
  - Minimal model representations contain null states, e.g.

$$L_{-2}\mathcal{O}_1 = \frac{3}{2(2h+1)}\partial^2\mathcal{O}_1$$

  - Null state differential equations hugely constrain correlators: e.g.

$$G_n \equiv \langle \mathcal{O}_1(z_1,\overline{z}_1)\ldots\mathcal{O}_n(z_n,\overline{z}_n)\rangle$$

obeys
$$\frac{3}{2(2h+1)}\partial_{z_1}^2 G_n = \sum_{j\neq 1}\left(\frac{h_j}{(z_1-z_j)^2}+\frac{1}{z_1-z_j}\partial_{z_j}\right)G_n$$

  - e.g. G_4 ~ (2,1) hypergeometric functions
- In W_N, more null states needed for closed form answers!
- <u>We will compute correlators involving φ=(f,0). Many null states!</u>

$$\chi_{(f,0)} \approx q^h(1+q+2q^2+\ldots)$$

[Fateev, Litvinov; Papadodimas Raju; Chang, Yin]

II. W_N minimal model 4-pt functions

- Compute using Coulomb gas of N-1 free bosons, at finite (N,k)

Hard: For 4-pt correlator of φ with generic field O,

$$\langle \mathcal{O}(\infty)\overline{\phi}(1)\phi(z)\overline{\mathcal{O}}(0)\rangle = \sum_{j=1}^{N} \mathcal{M}_{jj}(\{n_i^{\mathcal{O}}\})|H_j(z)|^2 \ , \quad \text{where}$$

$$H_j(z) = {}_NF_{N-1}(\{n_i^{\mathcal{O}}\}|z)$$

(Monodromy matrix, function of Dynkin data for O)

Easier: Now specify even more: take O = D = (0,Λ). Only one block contributes!

$$\langle D(\infty)\overline{\phi}(1)\phi(z)\overline{D}(0)\rangle \propto \left|(1-z)^{\frac{2\alpha_+^2}{N}} z^{v_{N-1}-\frac{B}{N}-2\alpha_+^2}\, {}_NF_{N-1}\left(\begin{matrix} 2\alpha_+^2, 2\alpha_+^2\mathbf{1}-\mathbf{v} \\ \mathbf{1}-\mathbf{v}\end{matrix}\middle|\frac{1}{z}\right)\right|^2$$

$$2\alpha_+^2 = \frac{N+k+1}{N+k} \ , \quad v_k = \sum_{j=1}^{k}(1+d_j-2\alpha_+^2)$$

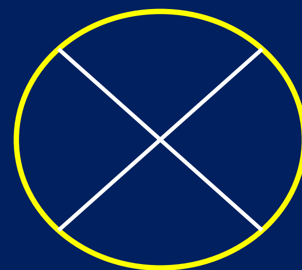
Easiest: Take semiclassical limit: huge simplification!

$$\boxed{\lim_{\alpha_+\to 0}\langle D(\infty)\overline{\phi}(1)\phi(z)\overline{D}(0)\rangle \propto \left| z^{\frac{N-1}{2}} \sum_{j=1}^{N} \frac{z^{n_j}}{\prod_{l\neq j}(n_l-n_j)}\right|^2}$$

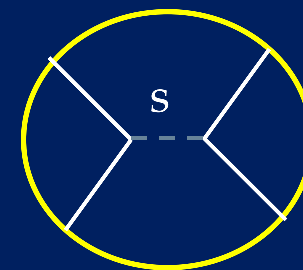
Open questions

- 3d Vasiliev perturbation theory
  - "Witten diagrams":

$$\langle \bar{\phi}\phi\bar{\phi}\phi \rangle =$$



$$+ \ \Sigma_s$$



  - Backreaction: Can we form a black hole?
  - The CFT has a global hs[λ] symmetry. Can we see this in the bulk, beyond linearized order?
- Next order questions for the duality:
  - Quantum corrections in $1/c \sim G_N$: many predictions from CFT
  - Black hole formation