Conical defects in minimal model holography

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based on

- Andrea Campoleoni, Tomáš Procházka, J.R., *A note on conical solutions in 3D Vasiliev theory*, arXiv:1303.0880 [hep-th].
- Eric Perlmutter, Tomáš Procházka, J.R., *The semiclassical limit of W_N CFTs and Vasiliev theory*, arXiv:1210.8452 [hep-th].
- Alejandra Castro, Rajesh Gopakumar, Michael Gutperle, J.R., *Conical Defects in Higher Spin Theories*, arXiv:1111.3381[hep-th]

Overview

- 3D higher spins and Chern-Simons theory

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- Outlook

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- Examples of tractable CFTs are the Virasoro minimal models and their generalizations with W-symmetry.
- Dual to 3D higher spin theories (Gaberdiel, Gopakumar 2010)
- Will explore space of smooth classical solutions obeying AdS boundary conditions, and their holographically dual CFT states.

3D HIGHER SPIN GRAVITY

- Simplest 3D higher spin theories: $sl(N)$ Chern-Simons. Describe spins $2, 3, \ldots, N$, and have classical W_N as asymptotic symmetry algebra (Campoleoni, Fredenhagen, Pfenniger, Theisen 2010).

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- 3D Vasiliev theories contain a parameter λ (Prokushkin, Vasiliev 1998). Higher spin algebra is $hs[\lambda]$, and consistent truncation to the massless sector gives $hs[\lambda]$ Chern-Simons theory (Ammon, Kraus, Perlmutter 2011). Asymptotic symmetry algebra is classical $W_\infty[\lambda]$ (Henneaux, Rey 2010, Gaberdiel, Hartman 2011, Campoleoni, Fredenhagen, Pfenniger 2011)

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- Vasiliev theory contains also a scalar with mass $m^2 = \lambda^2 - 1$.
- When $\lambda = \pm N$, $hs[\lambda]$ collapses to $sl(N)$.

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- Metric-like fields

$$
g_{\mu\nu} = \frac{12}{N(N^2-1)}\mathrm{tr}\left(e_\mu e_\nu\right), \quad \phi_{\mu\nu\rho} \sim \mathrm{tr}\left(e_{(\mu} e_\nu e_{\rho)}\right), \ldots
$$

W_N CHARGES

- Denote $sl(N)$ generators by $V^s_m,\ s = 2, \ldots N,\ |m| < s$. Global AdS in coords $r, z = \phi + it_E$:

$$\begin{aligned} A &= b^{-1}(a_{AdS}dz + d)b, \qquad b = e^{rV^2_0} \\ a_{AdS} &= \frac{1}{2}\left(V^2_1 + V^2_{-1}\right) \end{aligned}$$

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$$

- asymptotically AdS connections: $A - A_{AdS} \sim_{r\to\infty} \mathcal{O}(1)$. Can be brought to highest weight gauge

$$
\begin{aligned}
A &= b^{-1}(a(z)dz + d)b \qquad b = e^{rV_0^2} \\
a(z) &= \frac{1}{2}V_1^2 + \frac{12\pi}{c}\sum_{s=2}^{N}\frac{1}{N_s}W_s(z)V^s_{-(s-1)}
\end{aligned}
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- Fourier expand

$$W_s(z) = \frac{1}{2\pi} \sum (W_n^s - \frac{c}{24}) e^{-inz}.$$

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- Gauge-invariant expressions for lowest charges:

$$\begin{aligned} h &= W_0^2 = \frac{c}{2N(N^2-1)}\mathrm{tr}a^2 + \frac{c}{24}, \qquad c \equiv \frac{3l}{2G} \\ W_0^3 &= \frac{\sqrt{2}c}{3N(N^2-1)\sqrt{N^2-4}}\mathrm{tr}a^3, \\ W_0^4 &= \frac{c}{2N(N^2-1)(N^2-4)}\left(\mathrm{tr}a^4 - \frac{\mathrm{tr}a_{AdS}^4}{(\mathrm{tr}a_{AdS}^2)^2}(\mathrm{tr}a^2)^2\right) \end{aligned}$$

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- define

$$r_i^- = m_i - (N - i)$$

satisfying $r_1^- \geq r_2^- \geq \ldots \geq r_N^- = 0$

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- go to 'nothing gauge' $a=0$: possess an $sl(N,\mathbb{C})$ symmetry

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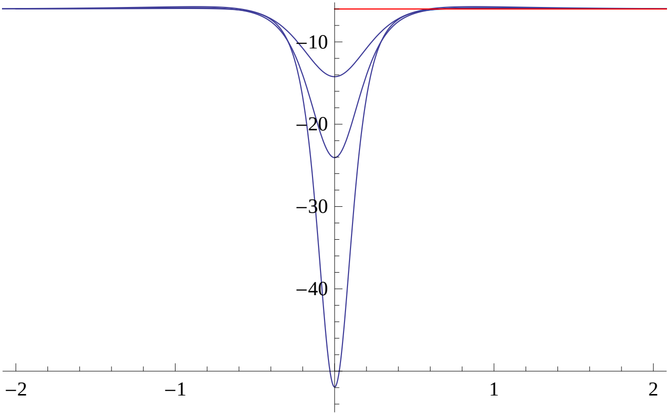
- conical singularity resolved by higher spins



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- charges are continuations of those of $sl(N)$ solutions

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- Higher spin-minimal model holography conjecture (Gaberdiel, Gopakumar 2010): expect that dual CFT is *semiclassical limit of W_N minimal models, where we take c large at fixed N*. Indeed, then $k \to -N-1$ and $\frac{N}{N+k} \to -N$ to be identified with λ in bulk Vasiliev theory.

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- Is a *different large c limit from 't Hooft limit* consisting of $N, k, c \to \infty,\ \lambda = \frac{N}{N+k}$ fixed

Degenerate representations

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- for Young tableau with r_i boxes in the i-th row (and $r_N \equiv 0$)

$$\Lambda_i = r_i - \frac{\sum_j r_j}{N} \qquad i = 1, \ldots, N$$

Define

$$\begin{aligned} n^{\pm} &= \Lambda^{\pm} + \rho \\ \rho_i &= \frac{N+1}{2} - i \qquad \text{Weyl vector} \end{aligned}$$

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- have $N-1$ independent null vectors at levels

$$(r_j^+ - r_{j+1}^+ + 1)(r_j^- - r_{j+1}^- + 1) \qquad j = 1, \ldots, N-1$$

Semiclassical limit

- in semiclassical limit $c \to \infty$ with N fixed, charges behave as

$$w_s(\Lambda^+, \Lambda^-) = w_s^{(-1)}(\Lambda^-)\, c + w_s^{(0)}(\Lambda^+, \Lambda^-) + \mathcal{O}(1/c).$$

e.g.

$$\begin{aligned} h^{(-1)}(\Lambda^-) &= -\frac{1}{2N(N^2-1)}(n^-)^2 + \frac{1}{24} \\ w_3^{(-1)}(\Lambda^-) &= -\frac{\sqrt{2}}{3N(N^2-1)\sqrt{4-N^2}} \sum_i (n_i^-)^3 \end{aligned}$$

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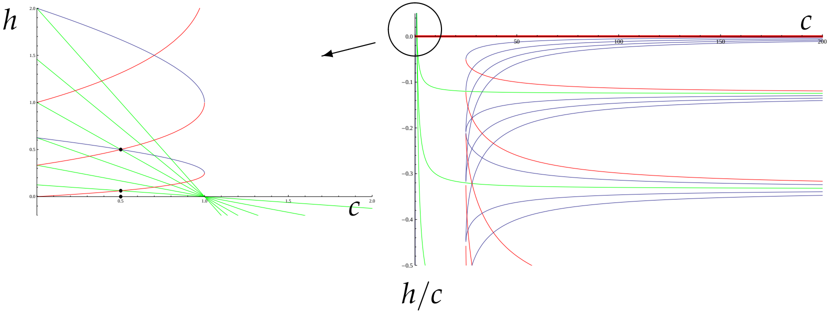
- Λ^- label nonperturbative sectors with energy of order c, Λ^+ label perturbative excitations with energy of order 1

Perturbative vs. nonperturbative states

Example: Virasoro case, $N = 2$:



in red: $|0, \Lambda^-\rangle$ primaries, in green: $|\Lambda, \Lambda\rangle$ primaries, in blue: other $|\Lambda^+, \Lambda^-\rangle$ primaries

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  - the conical solutions represent the primaries $|0, \Lambda^-\rangle$, not $|\Lambda^-, \Lambda^-\rangle$ as was believed earlier
  - general $|\Lambda^+, \Lambda^-\rangle$ primaries correspond to (single- and multiparticle) excitations of the Vasiliev scalar around conical defects.

Symmetries and null states

- Fluctuations of the higher spin fields are gauge transformations that preserve the boundary conditions: 'boundary gravitons'. Let ζ_m^s the gauge parameter such that the associated charge is W_m^s, i.e.

$$\delta_{\zeta_m^s} F = \{W_m^s, F\}_{PB}$$

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- Example: symmetries of AdS background correspond to wedge modes

$$\delta_{\zeta^s_m} a_{AdS} = 0, |m| < s \quad \Longleftrightarrow \quad W^s_m |0,0\rangle = 0, |m| < s$$

Wedge modes form an $sl(N)$ algebra at large c (Bowcock-Watts 1993).

- nontrivial example: $\Lambda^- = f$ surplus has symmetries at level 1 and 2 generated by $\zeta^3_{-1} + \frac{i\sqrt{N^2-4}}{\sqrt{2}N}\zeta^2_{-1}$, $\zeta^3_{-2} - \frac{2\sqrt{2}i}{N\sqrt{N^2-4}}\zeta^2_{-2}$.

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$$N_{1,(0,f)} = \left(W^{(3)}_{-1} - \frac{\sqrt{4-N^2}}{\sqrt{2}N} L_{-1} + \mathcal{O}(1/c) \right) |0,f\rangle$$

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$$N_{2,(0,f)} = \left(W^{(3)}_{-2} - \frac{2\sqrt{2}}{N\sqrt{4-N^2}} L_{-2} + \mathcal{O}(1/c) \right) |0, f\rangle$$

- analyzing invariance equation $\delta_\zeta a = \zeta' + [a, \zeta] = 0$ in diagonal gauge, one finds $N-1$ symmetries at levels $r^-_i - r^-_{i+1} + 1$, in agreement with levels of $N-1$ null vectors of $|0, \Lambda^-\rangle$.

TWISTED WEDGE ALGEBRA AT LARGE c

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Twisted wedge algebra at large c

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- e.g. $N = 2$:

$$e^{\pm} = \frac{i}{r_1^- + 1}\zeta_{\pm(r_1^-+1)}$$
$$h = -\frac{2}{r_1^- + 1}\zeta_0$$

$N = 3$:

$$e_1^{\pm} = \frac{1}{\mathcal{N}\sqrt{r_1^- - r_2^- + 1}}\left(\sqrt{\frac{5}{2}}\zeta^3_{\pm(r_1^- - r_2^- + 1)} - \frac{i}{6}(r_1^- + r_2^- + 3)\zeta_{\pm(r_1^- - r_2^- + 1)}\right)$$
$$e_2^{\pm} = \frac{1}{\mathcal{N}\sqrt{r_2^- + 1}}\left(\sqrt{\frac{5}{2}}\zeta^3_{\pm(r_2^- + 1)} + \frac{i}{6}(2r_1^- - r_2^- + 3)\zeta_{\pm(r_2^- + 1)}\right)$$
$$h_1 = \frac{1}{\mathcal{N}^2}\left(\frac{1}{3}\left((r_1^-)^2 - 4r_1^- r_2^- + (r_2^- - 6)r_2^- - 3\right)\zeta_0^2 - i\sqrt{\frac{5}{2}}(r_1^- + r_2^- + 3)\zeta_0^3\right)$$
$$h_2 = \frac{1}{\mathcal{N}^2}\left(\frac{1}{3}\left(2r_1^- r_2^- - 2r_1^-(r_1^- + 3) + (r_2^-)^2 + 6r_2^- - 3\right)\zeta_0^2 + i\sqrt{\frac{5}{2}}(2r_1^- - r_2^- + 3)\zeta_0^3\right)$$

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- for $\Lambda^- = 0$, this is the wedge algebra of $W^s_m, \ |m| < s$ leaving the vacuum invariant (Bowcock-Watts 1993)
- verified this for $N = 2$ and $N = 3$

Matter fluctuations

- Vasiliev theory contains a matter field C in twisted adjoint representation. For $\lambda = -N$: a complex $N \times N$ matrix satisfying

$$dC + AC - C\bar{A} = 0.$$

Physical scalar is

$$\Phi = \mathrm{Tr} C,$$

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- Around AdS, one finds N^2 discrete solutions of Klein-Gordon with $M^2 = N^2 - 1$. Built on primary with $h = \bar{h} = \frac{1-N}{2}$, span $\mathbf{N} \times \mathbf{N}$ nonunitary representation of the $sl(N) \times \overline{sl(N)}$ isometries of AdS.

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- Match with the large c properties of state $|f, 0\rangle$.

MATTER FLUCTUATIONS

- Around surplus specified by Λ^-: solutions span $\mathbf{N} \times \mathbf{N}$ built on primary with conformal weight and spin 3-charge

$$
\begin{aligned}
h &= \bar{h} = -n_1^- \\
w^3 &= \bar{w}^3 = i\sqrt{\frac{2}{N^2-4}}\left((n_1^-)^2 - \frac{n^- \cdot n^-}{N}\right)
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- Matches with large c properties of of CFT primary $|f, \Lambda^-\rangle$.
- More general primaries $|\Lambda^+, \Lambda^-\rangle$ come from multiparticle states around the Λ^- conical background

Bulk partition function

- Bulk partition function is sum over surplus backgrounds

$$Z^{grav} = \sum_{\Lambda^-} Z_{cl}(\Lambda^-) Z^{hs}_{1-loop}(\Lambda^-) Z^{scalar}_{1-loop}(\Lambda^-)$$

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- From fluctuation analysis, we know the single particle spectrum (positive frequency modes). 1-loop contributions using free boson formula

$$Z_{1-loop} = \exp\left[\sum_{n=1}^{\infty} \frac{Z_{1-part.}(q^n, \bar{q}^n)}{n}\right]$$

- higher spin fields

$$
\begin{aligned}
Z^{hs}_{1-part}(\Lambda^-) &= \left((N-1)(q+q^2+q^3+\ldots) - \sum_{1\le i<j\le N} q^{n_i^- - n_j^-}\right) \\
Z^{hs}_{1-loop}(\Lambda^-) &= \frac{|q|^{\frac{N-1}{12}} \prod_{1\le i<j\le N} |1-q^{n_i^- - n_j^-}|^2}{|\eta|^{2(N-1)}}
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- matter field

$$\begin{aligned} Z^{scalar}_{1-part.} &= (q^{-n_1^-}+\dots+q^{-n_N^-})(\bar q^{-n_1^-}+\dots+\bar q^{-n_N^-}) \\ &= \mathrm{tr}_f U \mathrm{tr}_f \bar U \qquad U = e^{-2\pi i\tau \mathrm{diag}(n^-)} \\ Z^{scalar}_{1-loop}(\Lambda^-) &= \sum_{\Lambda^+} \mathrm{tr}_{\Lambda^+} U \mathrm{tr}_{\Lambda^+} \bar U \\ &= \sum_{\Lambda^+} |\chi_{\Lambda^+}(-2\pi i\tau n^-)|^2 \end{aligned}$$

- Compare with CFT result in semiclassical limit (Niedermaier 1990)

$$Z_{\Lambda^-} \sim \frac{|q|^{-\frac{2C_2(n^-)}{N(N^2-1)}(c-(N-1))+2(\Lambda^-,n^-)}}{|\eta|^{2(N-1)}} \prod_{1\leq i<j\leq N} |1-q^{n_i^- - n_j^-}|^2 \times \sum_{\Lambda^+} |\chi_{\Lambda^+}(-2\pi i \tau n^-)|^2$$

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$$\sum_{\Lambda^+} |\chi_{\Lambda^+}(-2\pi i\tau n^-)|^2$$

- Missing only factor

$$(q\bar{q})^{h^{(0)}(0,\Lambda^-)},$$

coming from 1-loop correction to the surplus energy. Should come from careful treatment of path integral measure.

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- Open issues, e.g. is the semiclassical partition function modular invariant and does it contain black hole saddle points?
- 't Hooft limit of W_N minimal models is unitary but contains light states which likely have to be added as extra fields to minimal Vasiliev theory (Chang, Yin 2011, 2013, Jevicki, Yoon 2013). How are these two bulk theories related?