ON A CANONICAL QUANTIZATION OF PURE 3D ADS GRAVITY

Work in progress with Jihun Kim

OUTLINE

- CLASSICAL ADS GRAVITY AS as CHERN-SIMONS SL(2,R)XSL(2,R)
- CLASSICAL PHASE SPACE OF SL(2,R) CONTAINS TEICHMULLER SPACE
- HOLOMORPHIC QUANTIZATION: WHICH NORM?
- A CONNECTION WITH CFTs

- GLOBAL DIFFEOMORPHISMS AND THE MODULAR GROUP ACTION ON WAVE FUNCTIONS
- NORMALIZABILITY OF THE WAVE FUNCTION
- AN IMPROVED CONNECTION WITH CFTs: HOLOGRAPHY AS SUPERSELECTION PROJECTION
- THE CASE OF $c<1$
- TENTATIVE CONCLUSIONS

CLASSICAL GRAVITY AS CHERN-SIMONS

$$A = e/l - \omega, \qquad \tilde{A} = e/l + \omega$$

$$S_E = S(A) - S(\tilde{A})$$

$$S = \frac{k}{4\pi}\int \mathrm{tr}\,(AdA + \frac{2}{3}AAA), \qquad k = \frac{l}{4G}$$

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IN CANONICAL QUANTIZATION 3D SPACE IS

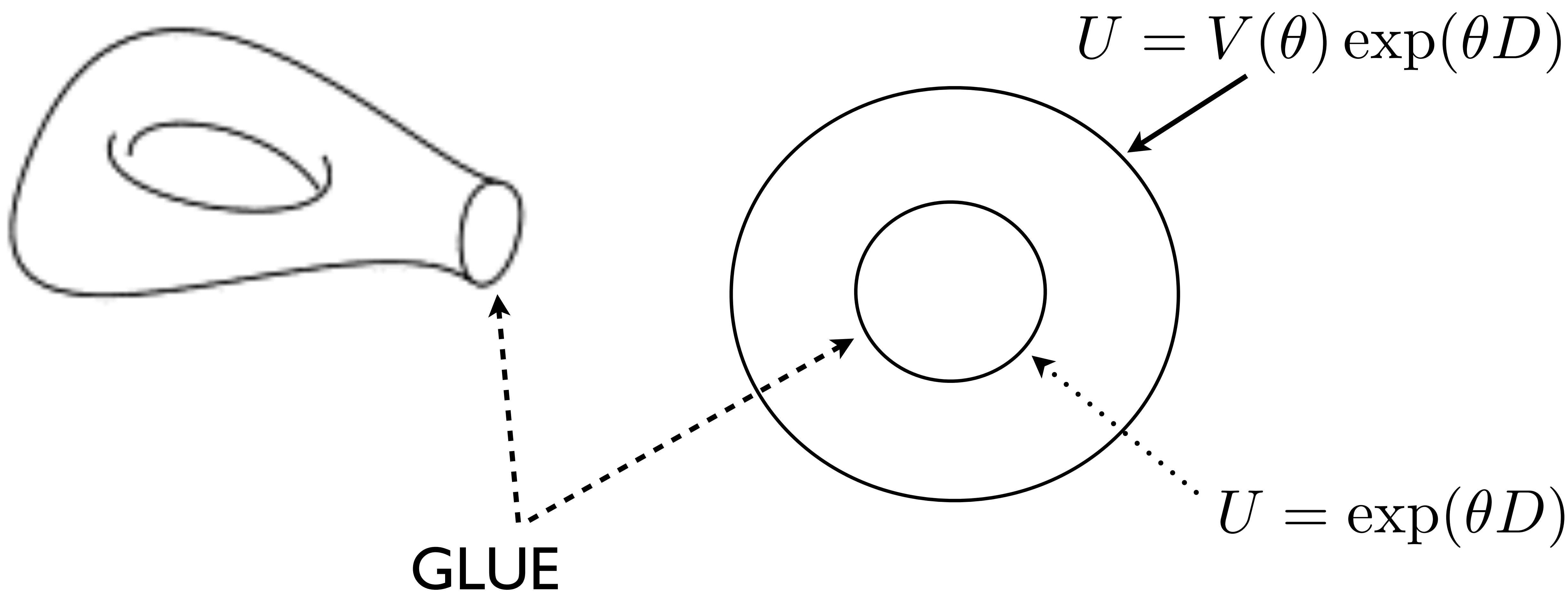
$$M = \Sigma \times R$$

CONSTRAINT EQUATION (GAUSS LAW)

$$F|_{\Sigma} = 0 \to A = dUU^{-1} \qquad \text{(locally)}$$

THE SPACE OF FLAT CONNECTIONS MODULO GAUGE TRANSFORMATIONS IS A DIRECT PRODUCT OF TWO SPACES: (EQUIVALENCE CLASSES OF) BOUNDARY GAUGE TRANSFORMATIONS TIMES A FINITE DIMENSIONAL SPACE

GEOMETRICALLY:



FINITE DIMENSIONAL SPACE WITH SEVERAL CONNECTED COMPONENTS.

WHEN ALL HOLONOMIES ARE HYPERBOLIC AND MANIFOLD HAS ONE BOUNDARY COMPONENT THIS SPACE IS

$$T_\Sigma \times \widehat{SL}(2,R)/S_1$$

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RESTRICT $$A|_{\partial\Sigma} = \begin{pmatrix} 0 & L(t+\phi) \\ 1 & 0 \end{pmatrix}$$

MODULI SPACE IS $$T_\Sigma \times Diff(S_1)/S_1$$

THIS SPACE ADMITS A KAHLER STRUCTURE AND A KAHLER FORM: THE WEYL-PETERSSON FORM

THE WEIL-PETERSSON FORM HAS A KAHLER POTENTIAL WHICH ALLOWS TO QUANTIZE THE TEICHMULLER SPACE IN HOLOMORPHIC QUANTIZATION.
THE RESULT IS JUST A (SUM OF PRODUCT) HILBERT SPACES

$$\mathcal{H} = \sum_L V_L \otimes H_T,$$

$$H_T = \text{holomorphic functions on } T, \qquad \mathrm{tr}\exp D = 2\cosh L$$

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HOW DO STATE VECTORS

$$\psi \in H_T$$

TRANSFORM UNDER THE MODULAR GROUP?

NORM:

$$\langle \psi_I | \psi_J \rangle = \int_T \bar{\psi}_I \exp K \psi_J$$

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SINCE

$$K \to K + F + \bar{F}, \qquad F = \text{holomorphic}$$

THEN

$$\psi_I \to U_I^J e^{-F} \psi_J, \qquad U_I^L U_L^J = \delta_I^J$$

THE KAHLER POTENTIAL FOR THE WEYL PETERSSON FORM OF THE TEICHMULLER SPACE FOR PUNCTURED SURFACE IS KNOWN: ZOGRAF AND TAKHTAJAN PROVED THAT IT EQUALS THE (REGULARIZED) LIOUVILLE ACTION COMPUTED ON SHELL:

$$K_0 = S_L|_{on\ shell} + H(t,\bar{t})$$

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FOR THE PUNCTURED SPHERE THIS IS THE POLYAKOV CONJECTURE (TAKTAJAN, ZOGRAF, MENOTTI,..)

THE SYMPLECTIC FORM FOR A RIEMANN SURFACE WITH A BOUNDARY OF LENGTH L IS (CO-HOMOLOGOUS TO) THE SUM OF THE SYMPLECTIC FORM FOR A SURFACE WITH A PUNCTURE PLUS THE FIRST CHERN CLASS OF THE TANGENT BUNDLE AT THE PUNCTURE (MIRZAKHANI, USING DUISTERMAAT-HECKMAN)

$$\omega_L \approx \omega_0 + \frac{1}{2}L^2 c_1(T)$$

ITS KAHLER POTENTIAL IS THEN (SEE ALSO DIJKGRAAF, VERLINDE, VERLINDE, 1990)

$$K_L = K_0 + \frac{1}{2}L^2 \log g_{z\bar{z}}|_{z=w}$$

THE MEASURE THAT WE GET FROM THE KAHLER NORM THAT WE FIND IS

$$e^{K} = e^{-S_L^{-6k}+2h\phi+H(t,\bar{t})}\Big|_{\phi} |F|^{-2}$$

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NOTICE:
TIMELIKE
LIOUVILLE

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NOTICE: TIMELIKE LIOUVILLE

$$h \equiv \frac{1}{2}kL^2$$

SQUARE OF HOLOMORPHIC FUNCTION

$$g_{z\bar{z}} \equiv e^{2\phi}$$

THIS NORM CAN BE PROMOTED TO A NORM VALID FOR ANY VALUE OF c=3l/2G

$$e^{-S_L^{26-c}+2h\phi+H(t,\bar{t})}\Big|_{\phi}|F|^{-2}\rightarrow\langle\int_\Sigma e^{2\alpha\phi}\rangle^L_{26-c}Z^{bc}(\det\Im\Omega_{ij})^{c/2}Z^S_c$$

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ONE-POINT VEV IN (TIMELIKE) LIOUVILLE WITH CENTRAL CHARGE 26-c

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$$\alpha(\alpha - Q) = -h(L) + 1, \qquad 26 - c = 1 - 6Q^2$$

CONFORMAL WEIGHT OF IRREP V_L

THIS NORM CAN BE PROMOTED TO A NORM VALID FOR ANY VALUE OF c=31/2G

$$e^{-S_L^{26-c}+2h\phi+H(t,\bar{t})}\Big|_\phi |F|^{-2} \to \langle \textstyle\int_\Sigma e^{2\alpha\phi}\rangle^L_{26-c} Z^{bc} (\det \Im\Omega_{ij})^{c/2} Z^S_c$$

THIS PRODUCT FACTORIZES INTO

$$e^{H(t,\bar{t})}|F|^{-2}$$

(QUILLEN)

REDEFINE THE WAVE FUNCTION

$$\chi_I = \psi_I / F,$$

UNDER MODULAR TRANSFORMATIONS:

$$dz^{kL^2/2}\chi_I \to dz^{kL^2/2} U_I^J \chi_J$$

SCALAR PRODUCT

$$\langle \chi_I | \chi_J \rangle = \int_T \langle \int_\Sigma e^{2\alpha\phi} \rangle_{26-c}^L Z^{bc} e^{H(t,\bar{t})} \bar{\chi}_I \chi_J$$

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THE WAVE FUNCTIONS χ_I TRANSFORM AS ONE-POINT CONFORMAL BLOCKS OF LIOUVILLE THEORY (VERLINDE, TESCHNER)

QG IN 3D ADS IS DEFINED BY SL(2,R)XSL(2,R) SO WE NEED TO COMBINE TOGETHER THE MODULI SPACES OF BOTH CS

$$\Psi_{QG} = \chi_I(t)\chi_J(\bar{t}')?$$

NO: THE WAVE FUNCTION OF QG MUST BE INVARIANT ALSO UNDER GLOBAL DIFFEOMORPHISMS

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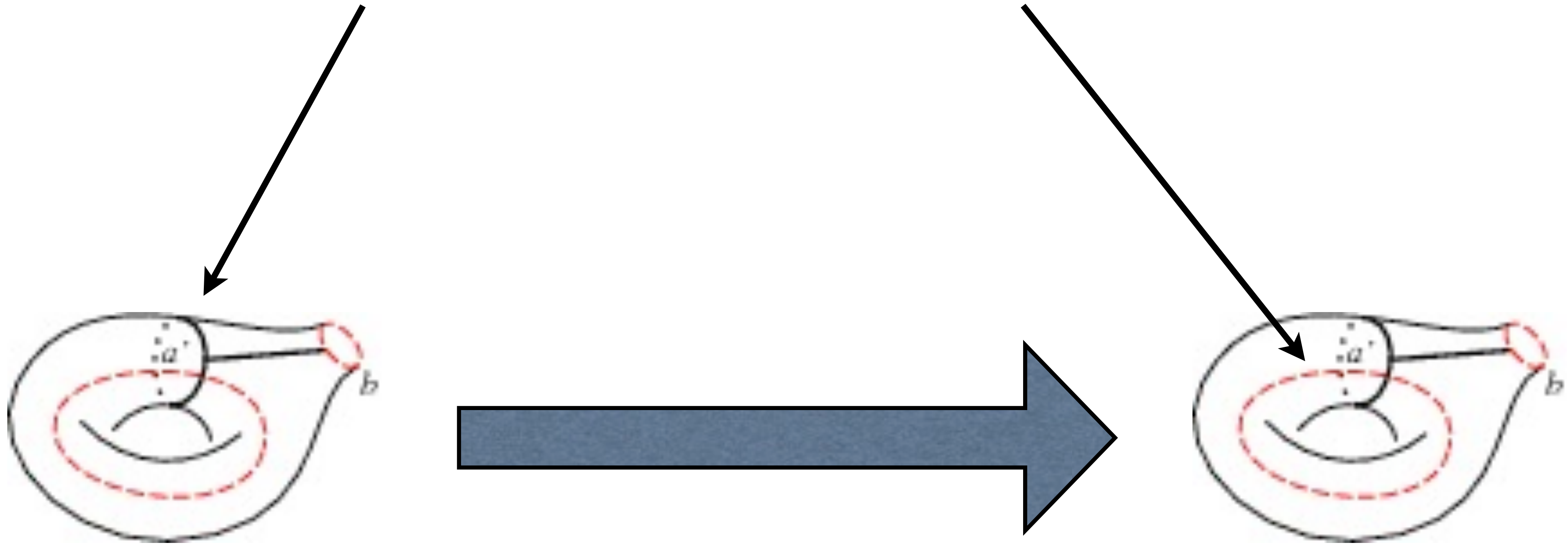
UNDER GLOBAL DIFFS $\phi : \Sigma \to \Sigma$

$$\operatorname{tr} Pe^{\oint_\gamma A} \in T \to \operatorname{tr} Pe^{\oint_{\phi^{-1}(\gamma)} \phi_* A}$$

$$\operatorname{tr} Pe^{\oint_\gamma \tilde{A}} \in \bar{T}' \to \operatorname{tr} Pe^{\oint_{\phi^{-1}(\gamma)} \phi_* \tilde{A}}$$

DIAGONAL ACTION ON $T \times \bar{T}'$

THE BLACK CYCLE MAPS INTO THE RED CYCLE UNDER A GLOBAL DIFFEOMORPHISM



$$\mathrm{tr}\, P e^{\oint_\gamma A} \in T \to \mathrm{tr}\, P e^{\oint_{\phi^{-1}(\gamma)} \phi_* A}$$

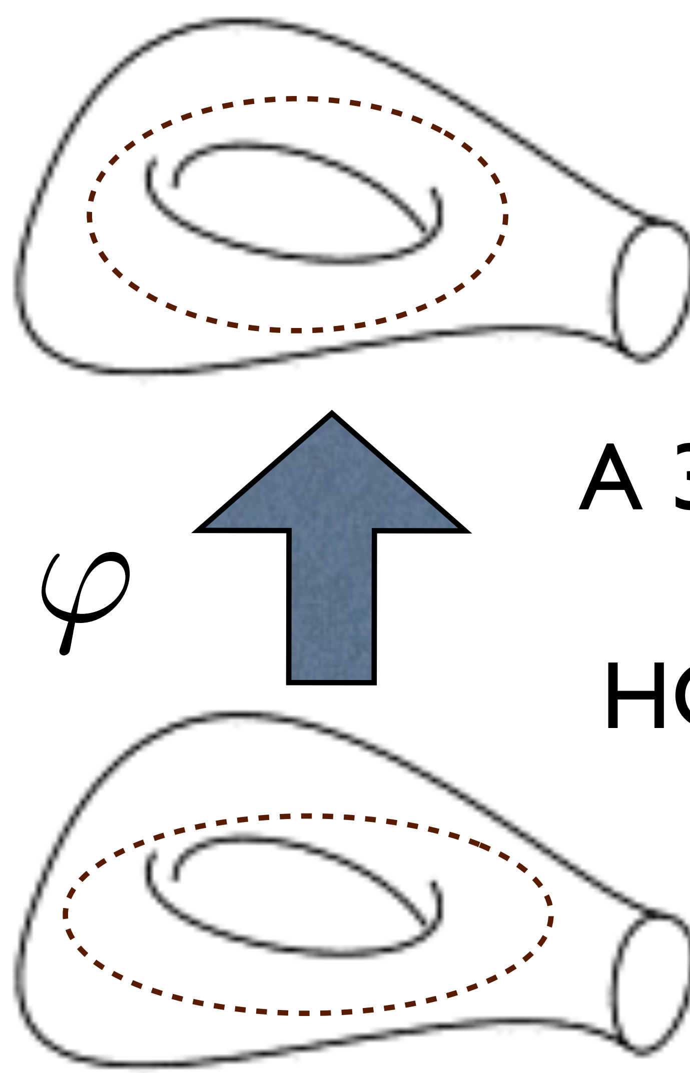
$$\mathrm{tr}\, P e^{\oint_\gamma \tilde{A}} \in \bar{T}' \to \mathrm{tr}\, P e^{\oint_{\phi^{-1}(\gamma)} \phi_* \tilde{A}}$$

$$T \times \bar{T}' \to T \times \bar{T}'/M$$

IN SECOND ORDER (METRIC) FORMULATION, THE TEICHMULLER SPACES LABEL INITIAL AND FINAL 2D METRICS.

HOMOLOGY CANNOT CHANGE UNDER A 3D DIFFEOMORPHISM THAT EXTENDS TO

$$\Sigma \times R$$



A 3D DIFF CANNOT CHANGE THE HOLONOMY BASIS

SO THE WAVE FUNCTION IS

$$\Psi = |v\rangle \sum_{IJ} \chi_I^h(t) \bar{\chi}_J^{\bar{h}}(\bar{t}') N^{IJ}, \qquad |v\rangle \in V_h \otimes V_{\bar{h}}$$

$dz^h d\bar{z}^{\bar{h}} \Psi$ INVARIANT UNDER DIAGONAL MAPPING CLASS GROUP

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ACTUALLY ANALYTIC CONTINUATION TO INDEPENDENT LEFT AND RIGHT MODULI (SEE WITTEN '07, SEGAL)

NEW RESTRICTIONS COME FROM NORMALIZABILITY

$$\Psi = |v\rangle \sum_{IJ} \chi_I^h(t)\bar{\chi}_J^{\bar{h}}(\bar{t}')N^{IJ}, \qquad |v\rangle \in V_h \otimes V_{\bar{h}}$$

$$\langle \sum_{IJ} \chi_I \bar{\chi}_J N^{IJ} | \sum_{KL} \chi_K \bar{\chi}_L N^{KL} \rangle < \infty$$

DANGEROUS REGIONS: DEGENERATING RIEMANN SURFACES

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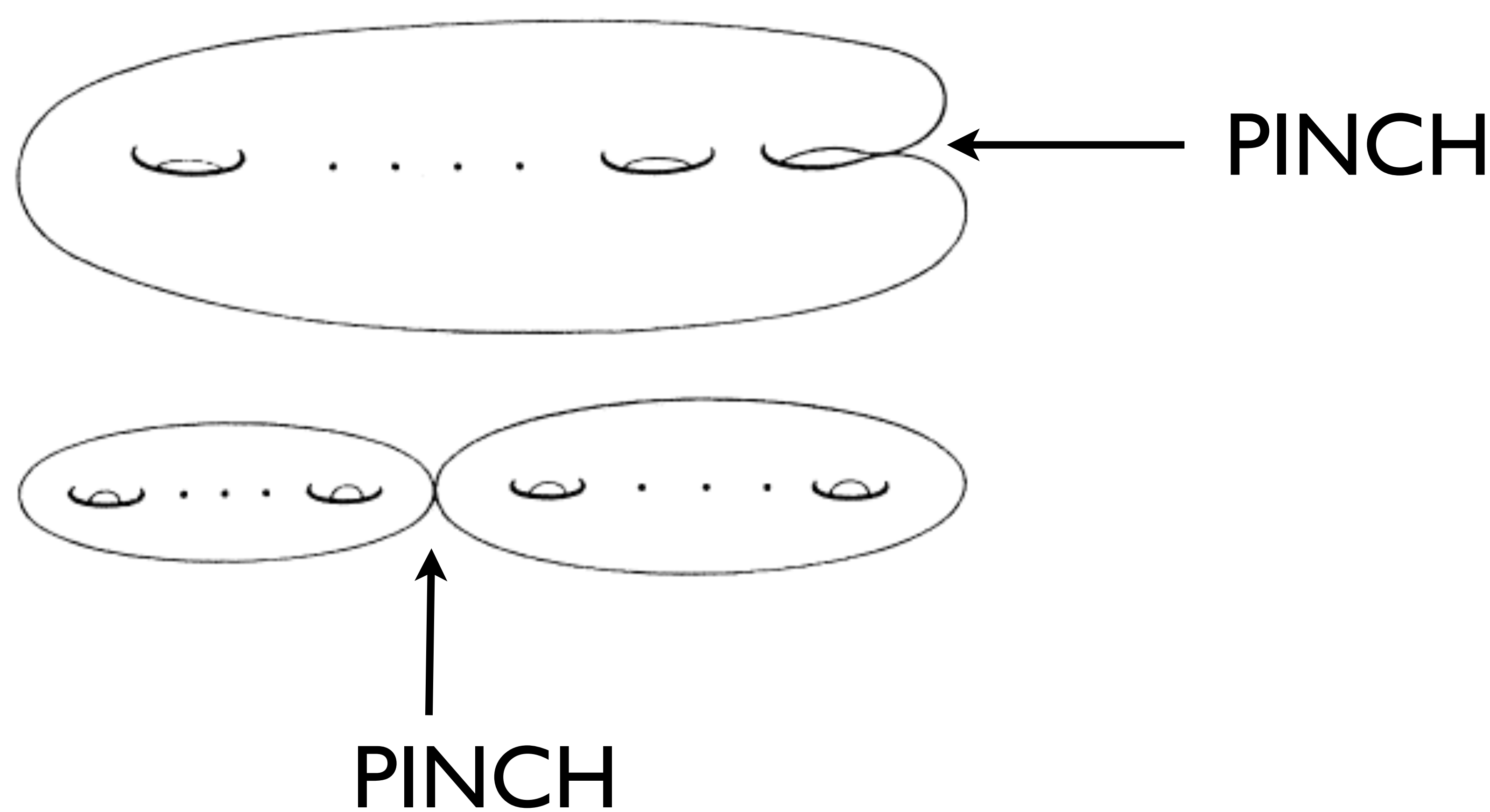
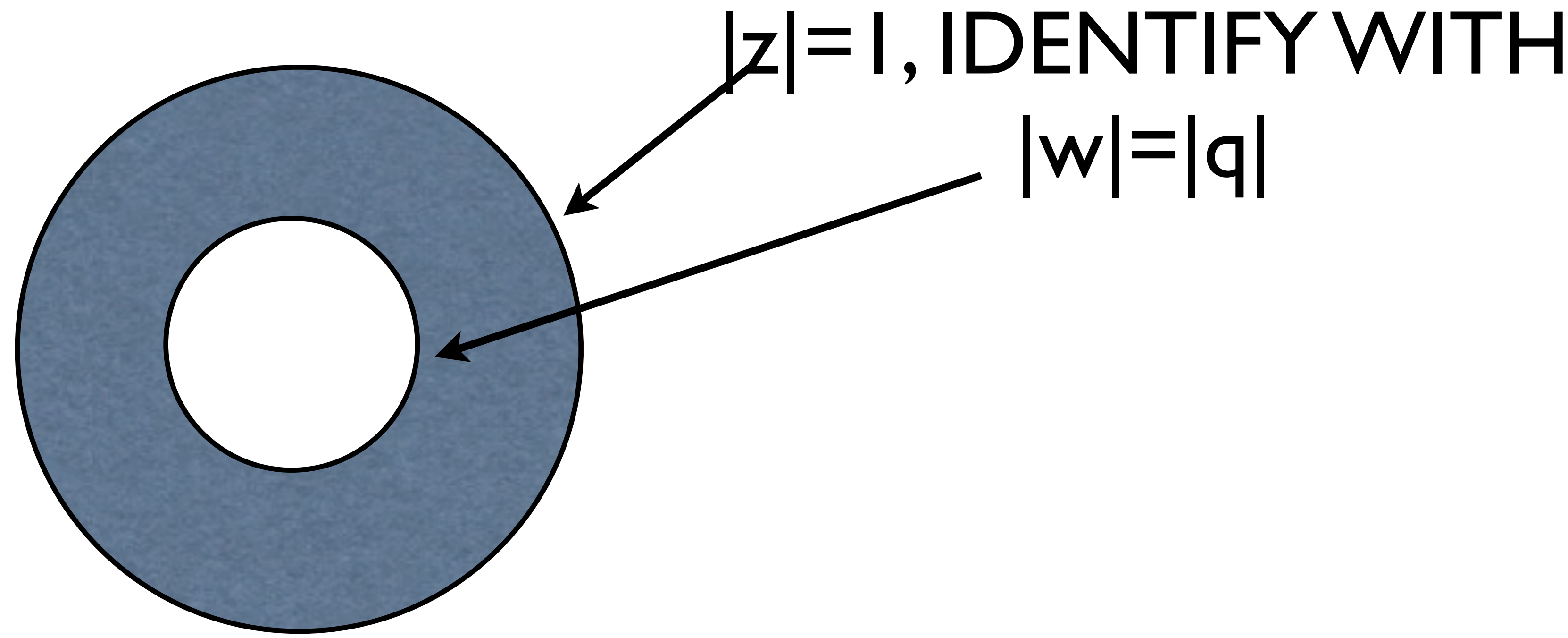
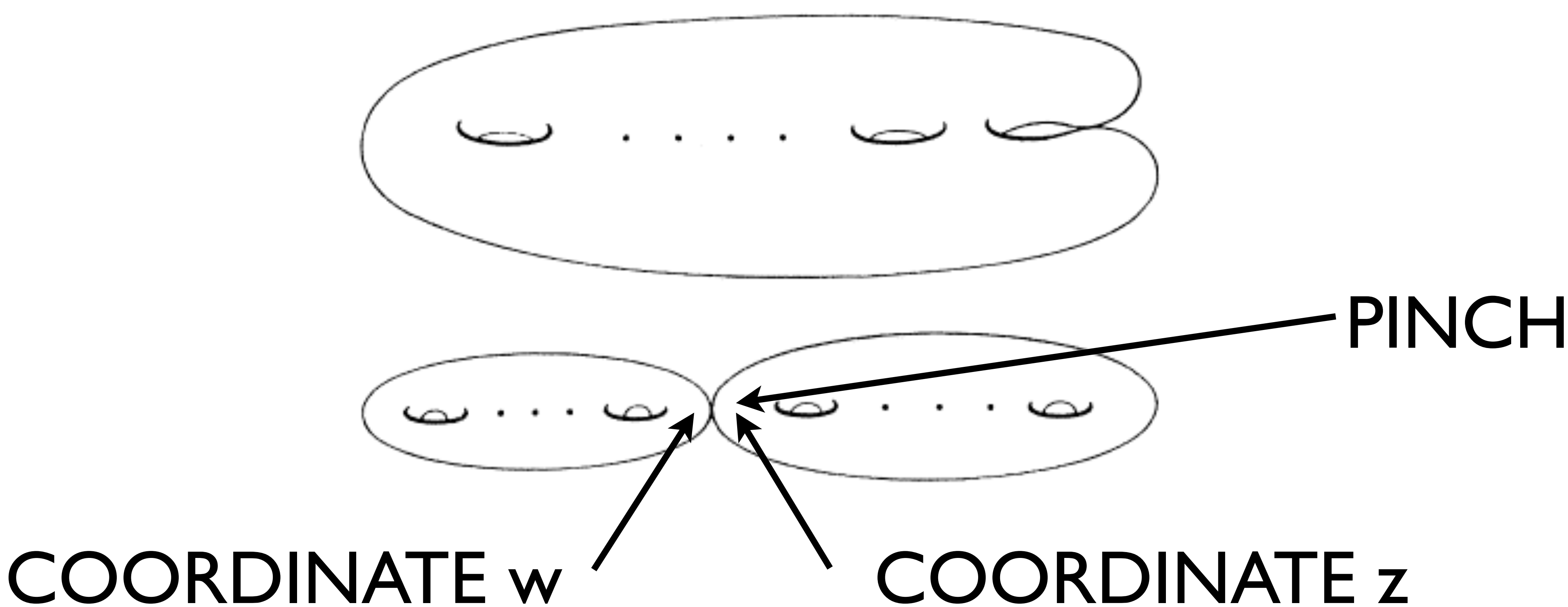




$$z = qw, \qquad |q| < |z| < 1, \qquad |q| < |w| < 1, \qquad q = e^{2\pi i \tau}$$



NEAR A NODE THE KAHLER FORM AND CERTAIN MODULAR TRANSFORMATIONS ARE KNOWN AND SIMPLE

$$(\tau, \tau') \to (\tau + 1, \tau' + 1) \ (\text{Dehn Twist})$$

$$\omega_{WP} \propto \frac{d\tau \wedge d\bar{\tau}}{(\Im\tau)^3} \quad \Longrightarrow \quad K \propto \frac{1}{\Im\tau}$$

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INVARIANCE UNDER THE DIAGONAL DEHN TWIST:

$$\Psi = |v\rangle \times \langle V^{h,\bar{h}}\rangle, \qquad \langle V^{h,\bar{h}}\rangle = \sum_n \int d\mu_n(\Delta) q^{\Delta - c/24} \bar{q}'^{\Delta + n - c/24}$$

NORMALIZABILITY OF THE WAVE FUNCTION IMPOSES NEW CONSTRAINTS

DEFINE:

$$\tau = i\rho + (\theta + \zeta)/2\pi, \qquad \tau' = i\rho' + (\theta - \zeta)/2\pi$$

NORM:

$$||\langle V^{h,\bar{h}}\rangle||^2 \approx \sum_n \int^{\infty} d\rho \int^{\infty} d\rho' \int_{-\infty}^{\infty} d\zeta \int d\mu_n(\Delta) d\mu_n(\Delta') e^{-2\pi\rho(\Delta+\Delta'-\frac{c-1}{12})} e^{-2\pi\rho'(\Delta+\Delta'+2n-\frac{c-1}{12})} e^{i\zeta(\Delta-\Delta')}$$

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CONVERGENCE IMPLIES $\Delta, \Delta + n > (c-1)/24$

INTEGRATION OVER REAL LINE IMPLIES $\Delta = \Delta'$

NORM PROPORTIONAL TO DIRAC DELTA: CONTINUOUS SPECTRUM

$$||\langle V^{h,\bar{h}}\rangle||^2 \approx \sum_n \int d\mu_n(\Delta) d\mu_n(\Delta') F(\Delta, n) \delta(\Delta - \Delta')$$

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SO HILBERT SPACE OF PURE 3D ADS GRAVITY
IS TARGET SPACE FOR CFTs WITH CONTINUOUS
SPECTRUM AND STATES WITH CONFORMAL WEIGHT

$$\Delta > (c-1)/24$$

SPACE OF SUCH CFTs IS NONEMPTY
ONE SOLUTION:
LIOUVILLE CFT

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IN PARTICULAR THEY ARE THE SAME FOR THE UNCOMPACTIFIED BOSON (CONTINUOUS SPECTRUM) AND FOR THE COMPACT BOSON (DISCRETE SPECTRUM)

HILBERT SPACE IS TOO LARGE

$$H_{CQG} = V_h \otimes V_{\bar{h}} \otimes \sum_g H_g$$

INFINITE MULTIPLICITY FOR ANY VIRASORO IRREP.

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PROPOSAL: CFTs AS SUPERSELECTION SECTORS

CLOSED UNDER OPE

$$O_\alpha : V_\beta \to V_\gamma$$

$$O_\alpha v_\beta = \sum_{v_\gamma} C_{\alpha\ v_\beta\ v_\gamma} v_\gamma$$

COEFFICIENTS OF OPE (FUISON RULES)

COEFFICIENTS OF FUSION RULES FIXED UP TO AN OVERALL CONSTANT

$$V_{h,\bar{h}} \to \lambda V_{h,\bar{h}} \Rightarrow C_{\alpha\ v_\beta\ v_\gamma} \to \lambda C_{\alpha\ v_\beta\ v_\gamma}$$

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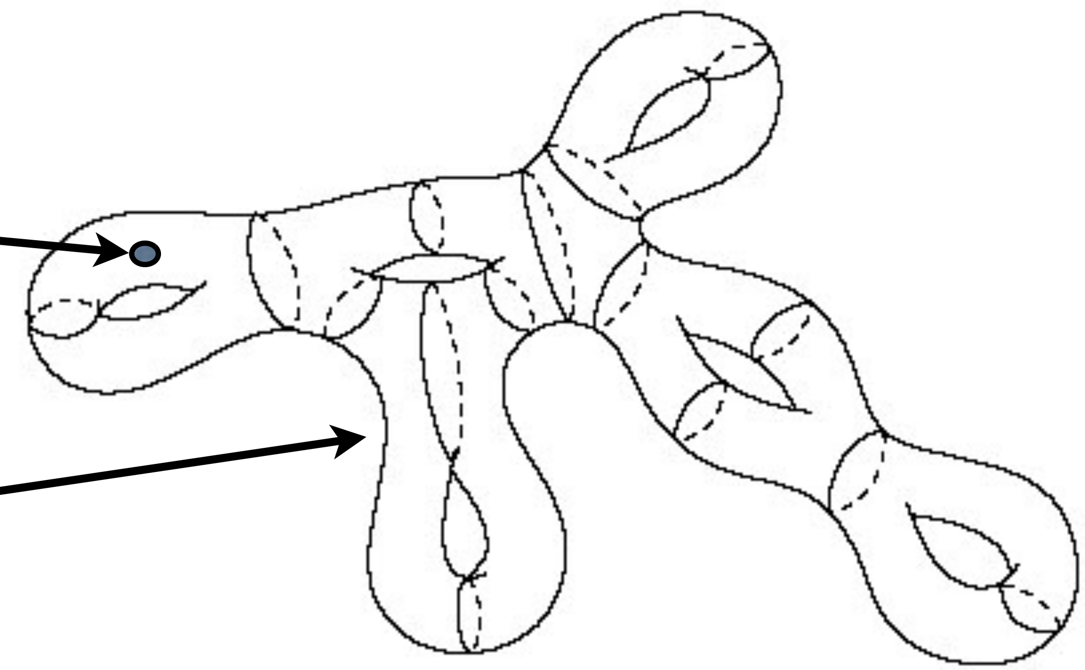

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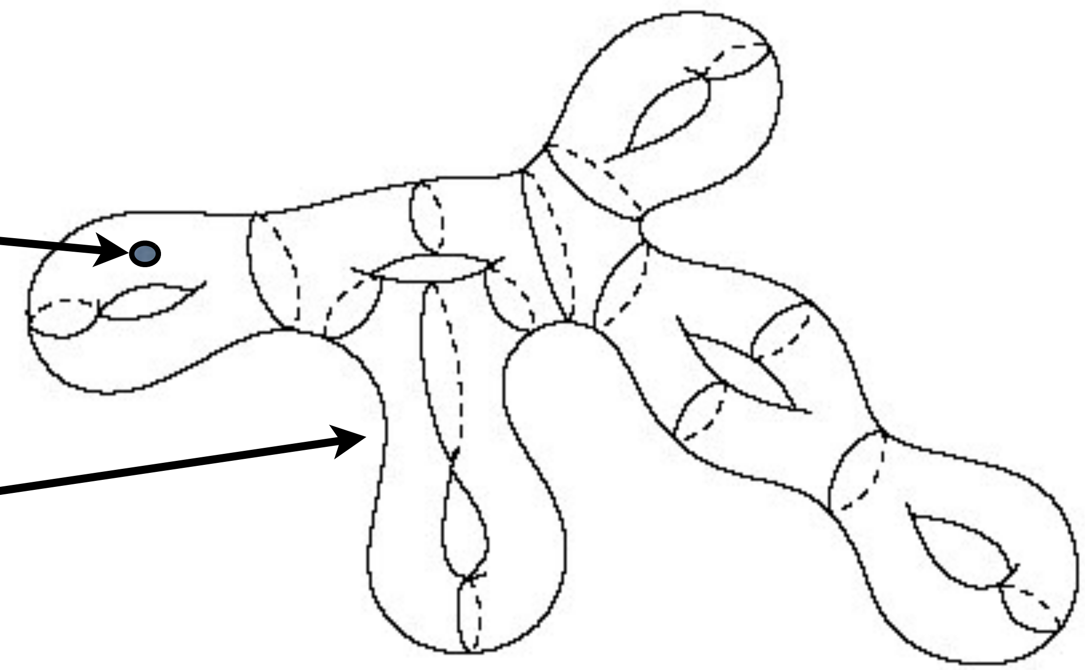

PANT $\approx C_{\alpha\ v_\beta\ v_\gamma}$

SO $\langle V_{h,\bar{h}} \rangle_{\Sigma_g}$ SCALES AS $\lambda^{2g-1} \langle V_{h,\bar{h}} \rangle_{\Sigma_g}$

NEW RULE: ASSOCIATE STATE IN CFT TO SPECIAL VECTOR IN HILBERT SPACE OF CQG BY:

$$w \in H^{CFT}_{h,\bar{h}} \to \Psi = v \times (\langle V_{h\bar{h}} \rangle_{\Sigma_1}, \langle V_{h\bar{h}} \rangle_{\Sigma_2}, \langle V_{h\bar{h}} \rangle_{\Sigma_3}, \ldots.), \qquad v \in V_h \otimes V_{\bar{h}}$$

$$||\Psi||^2 = \sum_g ||\langle V_{h\bar{h}} \rangle_{\Sigma_g}||^2 \propto \sum_g a_g \lambda^{4g-2} \qquad (\text{finite near } \lambda = 0)$$

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ANY DISCRETIZATION GIVES EFFECTIVE CENTRAL CHARGE

$$c_{eff} = c - 24\Delta_{min} = c - 24\frac{c-1}{24} = 1$$

MULTIPLICITY OF STATES NOT ENOUGH TO GIVE BEKENSTEIN-HAWKING ENTROPY

LASCIATE OGNI SPERANZA?

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$$T/\mathcal{N} = \text{ finite cover of } \mathcal{M}$$

$(T/\mathcal{N} \times \bar{T}'/\mathcal{N})/M$ HAS FINITE VOLUME= DISCRETE SPECTRUM

FOR $c<1$ MINIMAL MODELS WE CAN GET A REASONABLE THEORY, WITH DISCRETE SPECTRUM AND A DUAL TO THE ADS VACUUM IF WE IMPOSE A NEW NON-GEOMETRICAL GAUGE SYMMETRY THAT ACTS ON THE MODULI SPACE AS THE NORMAL, FINITE INDEX KERNEL OF THE MODULAR GROUP

(SEE CASTRO ET AL.)

TENTATIVE CONCLUSIONS

- A CANONICAL QUANTIZATION OF PURE GRAVITY IN 3D ADS YIELDS A HILBERT SPACE THAT CAN BE INTERPRETED AS THE TARGET SPACE FOR LIOUVILLE-LIKE CFTs
- CFTs APPEAR IN THE HILBERT SPACE AS SUPERSELECTION SECTORS CLOSED UNDER FUSION RULES
- FOR $c>1$ THERE ARE SEVERAL PROBLEMS WITH THIS PICTURE SUCH AS ABSENCE OF THE DUAL TO ADS SPACE AND CONTINUOUS SPECTRUM
- FOR $c<1$ WE FOUND THAT IMPOSING A NEW, NON GEOMETRIC GAUGE SYMMETRY GIVES A REASONABLE PICTURE (DISCRETE SPECTRUM, VACUUM)

MANY OPEN QUESTIONS

- DOES ALL THIS MAKE SENSE?
- CAN ONE EXTEND THE MINIMAL MODEL PICTURE TO HIGH SPIN 3D ADS THEORIES, OR SL(N,R)XSL(N,R) CS? (ANY RCFT ADMITS A NONTRIVIAL KERNEL OF THE MODULAR GROUP)
- IS $c>1$ DOOMED? MAYBE WE CAN MAKE SENSE OF LIOUVILLE-LIKE CFTs: LIOUVILLE THEORY POPS UP TOO MANY TIMES IN 3D PURE GRAVITY TO BE JUST BACKGROUND NOISE!