One-loop quantum gravity and HS from a worldline viewpoint

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F.Bastianelli, R.B., O.Corradini, E.Latini, arXiv:1210.4649

F.Bastianelli, R.B., arXiv:1304.7135

Motivations

- The worldline formalism proved to be useful to compute various one-loop quantities of QFT's, such as n-point functions, propagators, effective actions and so on
- O(N) spinning particles in flat space describe quite efficiently higher spin fields (spin $\frac{N}{2}$ in $D = 4$),

  (Berezin and Marinov, Gershun and Tkach, Howe, Penati, Pernici and Townsend,...)

  and can be quantized on (A)dS spaces

  (Kuzenko, Yarevskaya,...)
- The O(4) model correctly describes massless spin 2 particles in $D = 4$ on (A)dS backgrounds (possible extension to conformally flat), but does not admit a coupling to general backgrounds
- Alternative route to give a worldline description of quantum gravity at one loop

Outline

- The O(N) spinning particle in flat space
- Quantized O(N) spinning particle in (A)dS and effective action
- One-loop Einstein's gravity
- Worldline description of gravity dof
- First quantized approach to the effective action

O(N) spinning particle in flat space

Consider, in D dimensional flat spacetime, the worldline action

$$S = \int dt \Big[p_\mu \dot{x}^\mu + \tfrac{i}{2} \psi_i^\mu \dot{\psi}_{\mu i} - \tfrac{1}{2} p^2 \Big]$$

where $x^\mu(t)$ are target space coordinates, $p_\mu(t)$ conjugate momenta and $\psi_i^\mu(t)$, $i = 1, .., N$ are real worldline fermions.

It enjoys O(N)-extended WL supersymmetry, generated by

$$H = \frac{p^2}{2}\;, \quad Q_i = \psi_i^\mu\, p_\mu\;, \quad J_{ij} = \tfrac{i}{2} [\psi_i^\mu, \psi_{\mu j}]$$

The generators obey the following supersymmetry algebra

$$\begin{aligned}
&\{Q_i, Q_j\} = 2\,\delta_{ij}\, H \\
&[J_{ij}, Q_k] = i\,\delta_{jk}\, Q_i - i\,\delta_{ik}\, Q_j \\
&[J_{ij}, J_{kl}] = i\,\delta_{jk}\, J_{il} - i\,\delta_{ik}\, J_{jl} - i\,\delta_{jl}\, J_{ik} + i\,\delta_{il}\, J_{jk}
\end{aligned}$$

- The algebra is first class $\rightarrow$ It can be gauged to make the (super)symmetries local
- One introduces WL gauge fields $e(\tau)$, $\chi_i(\tau)$ and $a_{ij}(\tau)$ to be coupled to the symmetry generators
- By doing so one ends up with the O(N) spinning particle action:

$$S = \int_0^1 dt \left[p_\mu \dot{x}^\mu + \frac{i}{2} \psi_i^\mu \dot{\psi}_{\mu i} - eH - i\chi_i Q_i - \frac{1}{2} a_{ij} J_{ij} \right]$$

- Dirac quantization $\rightarrow$ The constraints J_{ij}, Q_i and H have to annihilate physical states

Dirac constraints J_{ij}, Q_i and $H \Rightarrow$ physical Hilbert space consists of linear, trace-free (or gamma trace-free) HS curvatures

- In $D = 4$ and for $N = 2s$ they give Fronsdal equations for spin s fields with compensators

  Francia, Sagnotti; 2003

  Bastianelli, Corradini, Latini; 2008

- For general D one gets Fronsdal-Labastida equations for tensors with rectangular $(D/2 - 1) \times s$ Young tableau

  Bastianelli, Corradini, Latini; 2008

- For odd N we have Fang-Fronsdal equations for fermionic HS fields

  Corradini; 2010

HS on (A)dS backgrounds

- Trying to couple the O(N) spinning particle to a curved space, the SUSY algebra is not first class, obstructed by target space curvature
- On conformally flat spaces the algebra is first class, but with structure functions $\rightarrow$ Cumbersome to quantize (BV...)
- On Maximally symmetric spaces the algebra becomes quadratic in constraints $\rightarrow$ Simpler BRST quantization

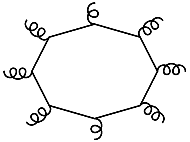
  Bastianelli, Corradini, Latini; 2008

$$R_{abcd} = b\,(\eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc}) \quad \rightarrow$$

$$\{Q_i, Q_j\} = 2\delta_{ij}H - \frac{b}{2}\,\left(J_{ik}J_{jk} + J_{jk}J_{ik} - \delta_{ij}J_{kl}J_{kl}\right)$$

HS Effective action on (A)dS

- One-loop effective action given by the worldline path integral

$$\Gamma[g] =$$  $$= \int_{S^1} \frac{\mathcal{D}x D\psi_i D e D\chi_i D a_{ij}}{\mathrm{Vol(Gauge)}} \, e^{-S}$$

- After gauge fixing WL symmetryes one has the Heat Kernel expansion

$$\Gamma[g] = \int_0^\infty \frac{dT}{T} \int \frac{d^D x \sqrt{|g|}}{(2\pi T)^{D/2}} a_0 \left\langle\!\!\left\langle e^{-S_{\mathrm{int}}} \right\rangle\!\!\right\rangle$$

$$a_0 \left\langle\!\!\left\langle e^{-S_{\mathrm{int}}} \right\rangle\!\!\right\rangle = a_0 \left(1 + v_1 \, R \, T + v_2 \, R^2 \, T^2 + \ldots\right)$$

Simplest case: $D = 4$ and even $N = 2s$
The SDW coefficients are defined as

$$\Gamma[g] = \int_0^\infty \frac{dT}{T} \int \frac{d^4x\sqrt{|g|}}{(2\pi T)^2}\, a_0 \left(1 + v_1\, R\, T + v_2\, R^2\, T^2 + \ldots\right)$$

and we get

$$a_0 = 2 - \delta_{s,0}\ , \quad v_1 = -\frac{s^2}{6}\ , \quad v_2 = -\frac{1}{8640} + \frac{s^2}{288} - \frac{s^4}{144}$$

Bastianelli, R.B., Corradini, Latini; arXiv:1210.4649

- Explicit result also for half-integer spins and arbitrary even dimensions

One-loop quantum gravity

- Obstruction to describe quantum spin 2 on generic gravitational background
- Similar problem in coupling the O(2) particle (vector fields in spacetime) to a gauge background was treated with BRST techniques

  Dai, Huang, Siegel, 2008

  that look not suitable for the gravitational case
- We undertake an alternative route, starting directly from QFT

One-loop Einstein gravity

- Consider four dimensional (euclidian) Einstein gravity with cosmological constant

$$S[G] = -\frac{1}{\kappa^2}\int d^4x\,\sqrt{G}\left[R(G) - 2\Lambda\right]$$

- We employ background field method splitting $G_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$. At one-loop order we look for the part quadratic in quantum fluctuations $h_{\mu\nu}$:

$$S_2 = -\int d^4x\,\sqrt{g}\left\{\tfrac{1}{4}\,h^{\mu\nu}\left(\nabla^2 + 2\Lambda\right)h_{\mu\nu} - \tfrac{1}{8}\,h(\nabla^2 + 2\Lambda)h + \tfrac{1}{2}\left(\nabla^\nu h_{\nu\mu} - \tfrac{1}{2}\nabla_\mu h\right)^2 \right.$$
$$\left. + \tfrac{1}{2}\,h^{\mu\lambda}h^{\nu\sigma}R_{\mu\nu\lambda\sigma} + \tfrac{1}{2}\left(h^{\mu\lambda}h^\nu_\lambda - hh^{\mu\nu}\right)R_{\mu\nu} + \tfrac{1}{8}\left(h^2 - 2h^{\mu\nu}h_{\mu\nu}\right)R\right\}$$

One-loop Einstein gravity

We choose covariant gaussian averaged gauge fixing:

$$S_{\text{GF}} = \frac{1}{2}\int d^4x\sqrt{g}\,\left(\nabla^\nu h_{\nu\mu} - \tfrac{1}{2}\nabla_\mu h\right)^2$$

- The quadratic part of the ghost action then reads

$$S_{bc} = -\int d^4x\,\sqrt{g}\,b^\mu\left[\nabla^2 c_\mu + R_{\mu\nu}\,c^\nu\right]$$

- We split traceless and trace part of metric fluctuations: $h_{\mu\nu} = \bar{h}_{\mu\nu} + \frac{1}{4}g_{\mu\nu}h$, in order to avoid worldline non-perturbative vertices

One-loop Einstein gravity

The gauge fixed action reads

$$S_q = \int d^4x\,\sqrt{g}\,\Big\{ -\tfrac{1}{4}\,\bar{h}^{\mu\nu}\big(\nabla^2 + 2\Lambda\big)\bar{h}_{\mu\nu}$$
$$-\frac{1}{2}\,\bar{h}^{\mu\lambda}\bar{h}^{\nu\sigma}\,R_{\mu\nu\lambda\sigma} - \frac{1}{2}\,\bar{h}^{\mu\lambda}\bar{h}^{\nu}_{\lambda}\,R_{\mu\nu} - \frac{1}{4}\,\bar{h}^{\mu\nu}\bar{h}_{\mu\nu}\,R$$
$$+\frac{1}{16}\,h\big(\nabla^2 + 2\Lambda\big)h - b^{\mu}\Big[\nabla^2 c_{\mu} + R_{\mu\nu}\,c^{\nu}\Big]\Big\}$$

- As it is well known, the scalar piece has the wrong sign. We choose the prescription given by Gibbons and Hawking to Wick rotate its path integral
- The one-loop effective action splits in three contributions:

$$\Gamma[g] = \tfrac{1}{2}\Big\{\mathrm{Tr}_{TT}\ln\big[K_{\mu\nu,\lambda\sigma} + M_{\mu\nu,\lambda\sigma}\big] + \mathrm{Tr}_S\ln\Big[-\tfrac{1}{2}\big(\nabla^2 + 2\Lambda\big)\Big] - 2\mathrm{Tr}_V\ln\big[G^{\mu}_{\nu}\big]\Big\}$$

Worldline description

- Use Schwinger's proper time integral to exponentiate logs:

$$\Gamma[g] = -\frac{1}{2}\int_0^\infty \frac{dT}{T}\left\{\mathrm{Tr}_{TT}\left[e^{-T(\hat{K}+\hat{M})}\right] + \mathrm{Tr}_S\left[e^{\frac{T}{2}(\nabla^2+2\Lambda)}\right] - 2\mathrm{Tr}_V\left[e^{-T\hat{G}}\right]\right.$$
$$- \frac{1}{2}\left\{\Gamma_2 + \Gamma_0 - 2\Gamma_1\right\}$$

- Γ_0 is represented by an ordinary bosonic QM path integral
- For Γ_2 and Γ_1 we need two worldline actions containing traceless symmetric tensors and vectors, respectively, in their Hilbert spaces
- Their quantum hamiltonians have to be $\hat{K} + \hat{M}$ and $\hat{G}$, respectively

Tensor model

Graded phase space: bosonic coordinates and momenta $x^\mu(t)$, $p_\mu(t)$ and WL fermions that are symmetric traceless tensors in target space $\psi^{ab}(t)$, $\bar\psi^{ab}(t)$

- Hilbert space:

$$|\bar h\rangle = h(x) + \bar h_{ab}(x)\psi^{ab} + \tfrac{1}{2}\,\phi_{(ab)(cd)}(x)\psi^{ab}\psi^{cd} + \ldots$$

  we need to project away the unwanted fields $\to$ gauge the $U(1)$ fermion number

- Covariant momenta:

$$\pi_\mu = p_\mu - i\omega_{\mu ab}\psi^{ac}\bar\psi^{b}_{c}$$

- Hamiltonian:

$$\hat H = \tfrac{1}{2}\,g^{-1/4}\pi_\mu\,g^{\mu\nu}\,g^{1/2}\pi_\nu\,g^{-1/4} - \Lambda - \tfrac{1}{2}\,R_{abcd}\,\psi^{ac}\bar\psi^{bd} - \tfrac{1}{2}\,R_{ab}\,\psi^{a}\cdot\bar\psi^{b} + \tfrac{1}{2}\,R$$

  it is indeed $\hat K + \hat M$

Tensor model

Worldline action

$$S_{\text{TT}} = \int_0^1 d\tau \Big[\tfrac{1}{2T}\, g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu + \tfrac{1}{2T}\, \bar{\psi}_{ab}\left(D_\tau + iA\right)\psi^{ab} - \tfrac{1}{2}\, R_{abcd}\, \psi^{ac}\bar{\psi}^{bd}$$

$$- \tfrac{1}{2}\, R_{ab}\, \psi^a \cdot \bar{\psi}^b - T\Big(\tfrac{3}{8}\, R + \Lambda\Big)\Big] - \tfrac{7i}{2}\int_0^1 d\tau\, A$$

- U(1) gauge field $A(\tau)$ to constrain fermion number J, CS term undoes normal ordering constant in J
- DR $\rightarrow$ we added the corresponding counterterm $V = -\frac{7}{8}\, R$
- Path integral on the circle gives Γ_2

$$\Gamma_2 = \int_0^\infty \frac{dT}{T} \int_0^{2\pi} \frac{d\phi}{2\pi} \int_P \mathcal{D}x \int_A D\bar{\psi} D\psi\, e^{-S_{\text{TT}}}$$

$$= \int_0^\infty \frac{dT}{T} e^{T\Lambda} \int \frac{d^4x\sqrt{g}}{(2\pi T)^2} \left\langle\!\left\langle e^{-S_{\rm int}} \right\rangle\!\right\rangle$$

Vector model

Graded phase space: coordinates and momenta $x^\mu(t)$, $p_\mu(t)$ and vector WL fermions $\lambda^a(t)$, $\bar\lambda^a(t)$

- Hilbert space:

$$|v\rangle = v(x) + v_a(x)\lambda^a + \frac{1}{2} v_{ab}(x)\lambda^a\lambda^b + \ldots$$

  project away the unwanted fields $\rightarrow$ gauge the $U(1)$ fermion number

- Covariant momenta:

$$\pi_\mu = p_\mu - i\omega_{\mu ab}\lambda^a\bar\lambda^b$$

- Hamiltonian:

$$\hat H = \hat G = \frac{1}{2} g^{-1/4}\pi_\mu g^{\mu\nu} g^{1/2}\pi_\nu g^{-1/4} - \frac{1}{2} R_{ab}\lambda^a\bar\lambda^b$$

Vector model

Worldline action

$$S_{\mathrm{v}} = \int_0^1 d\tau \Big[\tfrac{1}{2T} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + \tfrac{1}{T} \bar{\lambda}_a \left(D_\tau + ia \right) \lambda^a - \tfrac{1}{2} R_{ab} \lambda^a \bar{\lambda}^b - \tfrac{3T}{8} R \Big]$$
$$- i \int_0^1 d\tau \, a$$

- U(1) gauge field $a(\tau)$ constrains fermion number to one, CS undoes normal ordering constant in it
- DR $\rightarrow$ counterterm $V = -\frac{3}{8} R$
- Path integral on the circle gives Γ_1

$$\Gamma_1 = \int_0^\infty \frac{dT}{T} \int_0^{2\pi} \frac{d\theta}{2\pi} \int_P \mathcal{D}x \int_A D\bar{\lambda} D\lambda \, e^{-S_{\mathrm{v}}}$$
$$= \int_0^\infty \frac{dT}{T} \int \frac{d^4x \sqrt{g}}{(2\pi T)^2} \left\langle\!\!\left\langle e^{-S_{\mathrm{int}}} \right\rangle\!\!\right\rangle$$

Gravity effective action

Proper time perturbative expansion, assembling the three contribution we get

$$\Gamma[g] \propto \int_0^\infty \frac{dT}{T} \int \frac{d^4x\sqrt{g}}{(2\pi T)^2} \Big\{ 2 + T\Big(-\tfrac{23}{6} R + 10\Lambda \Big)$$
$$T^2 \Big[\tfrac{53}{180} R^2_{\mu\nu\lambda\sigma} - \tfrac{361}{360} R^2_{\mu\nu} + \tfrac{43}{144} R^2 - \tfrac{19}{60} \nabla^2 R + 5\Lambda^2 - \tfrac{13}{6} R\Lambda \Big] + \mathcal{O}(T^3) \Big\}$$

It reproduces well known one-loop log divergencies of pure gravity:

$$\mathcal{L}_{div} \propto \frac{1}{8\pi^2}\frac{1}{\epsilon} \left\{ \frac{53}{90} E + \frac{7}{20} R^2_{\mu\nu} + \frac{1}{120} R^2 + 10\Lambda^2 - \frac{13}{3} R\Lambda \right\}$$

Bastianelli, R.B.; arXiv:1304.7135

Conclusions and outlook

- O(N) spinning particle $\to$ spin $N/2$ field in spacetime
- Results for first SDW coefficients of quantum HS in AdS
- Possibly useful in HS/CFT duality?
- Extension to HS on conformally flat backgrounds?
- The O(4) particle describes massless spin 2 fields but does not admit coupling to arbitrary background
- The alternative description does reproduce known SDW coefficients in D=4
- Desirable a gauge invariant description, not directly related to QFT

THANKS FOR YOU ATTENTION!