Intro	Tensorial SSP	Hspin eqs in TSSP	OSp(1 n) as AdS-TSSP	κ and $OSp(1 2n)$	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.

Tensorial superspace approach to higher spin theories

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Based on the papers with J. Lukierski, D. Sorokin, M. Tonin, P. Pasti, X. Bakaert, J. de Azcárraga, M. Tsulaia, C. Meliveo (time ordering - from 1998 till present- is used)

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May 9, 2013

Introduction

- Flat tensorial superspace $\Sigma^{(n(n+1)/2|n)}$
 - 4D Tensorial superspace $\Sigma^{(10|4)}$
 - Higher D tensorial superspace $\Sigma^{(\frac{n(n+1)}{2}|n)}$
 - Preonic superparticle in tensorial superspace $\Sigma^{(\frac{n(n+1)}{2}|n)}$
- ⁽³⁾ Higher spin equations in tensorial superspace $\Sigma^{(n(n+1)/2|n)}$
 - Higher spin equations in 4D tensorial superspace
 - Higher spin equations in 10D tensorial superspace
- **OSp**(1, *n*) as AdS generalization of $\Sigma^{(n(n+1)/2|n)}$
 - AdS HSpin equations on OSp(1|n) supergroup manifold
 - AdS HSpin equations on Sp(n) group manifold
 - Preonic superparticle on OSp(1, n) supergroup manifold
- Preonic properties and OSp(1|2n) superconformal symmetry of tensorial superparticle
 - κ symmetry and SUSY preserved by preonic BPS state
 - OSp(1|2n) symmetry of $\Sigma^{(n(n+1)/2|n)}$ and OSp(1|n) superparticles
- 6 Searching for the interacting theory: Supergravity in tensorial superspace
 - Preonic superparticle and SUGRA constraints in $\mathcal{M}^{(\frac{n(n+1)}{2}|n)}$
 - Supergravity in tensorial superspace
 - Higher spin equations in extended tensorial superspaces
 - Conclusions

Intro	Tensorial SSP	Hspin eqs in TSSP	<i>OSp</i> (1 <i>n</i>) as AdS-TSSP 00000	к and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.
Out	line						
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- 0
- Introduction
- Flat tensorial superspace $\Sigma^{(n(n+1)/2|n|)}$
 - 4D Tensorial superspace $\Sigma^{(10|4)}$
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- ⁽³⁾ Higher spin equations in tensorial superspace $\Sigma^{(n(n+1)/2|n|)}$
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 - Higher spin equations in 10D tensorial superspace
- OSp(1, n) as AdS generalization of $\Sigma^{(n(n+1)/2|n|)}$
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 - κ symmetry and SUSY preserved by preonic BPS state
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- 6 Searching for the interacting theory: Supergravity in tensorial superspace
 - Preonic superparticle and SUGRA constraints in $\mathcal{M}^{(\frac{n(n+1)}{2}|n)}$
 - Supergravity in tensorial superspace
 - Higher spin equations in extended tensorial superspaces

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Introduction

- The interacting theory of higher spin fields was constructed by Misha Vasiliev in late 80th. [Fradkin & Vasiliev 87, Vasiliev 88-89]
- Misha's interacting massless h-spin theory is formulated with the use of noncommutative star product and has quite a complicated structure.
- Not so many exact solutions of this theory are known. The known action principle [P. Sundell, N. Boulanger, N. Colombo] is quite unusual. Some properties are to be clarified.
- This stimulates not only its extensive study, but also a search for alternative frameworks to reformulate it/to construct interacting higher spin theories.
- One of such frameworks is provided by 'tensorial superspace'.

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- One of such frameworks is provided by 'tensorial superspace'.
- Its brief review will be the subject of the present talk.

	Tensorial SSP	Hspin eqs in TSSP 0000000	<i>OSp</i> (1 <i>n</i>) as AdS-TSSP 00000	κ and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.
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- 6 Searching for the interacting theory: Supergravity in tensorial superspace
 - Preonic superparticle and SUGRA constraints in $\mathcal{M}^{(\frac{n(n+1)}{2}|n)}$
 - Supergravity in tensorial superspace
 - Higher spin equations in extended tensorial superspaces

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4D Tei	nsorial superspac	e					

$\Sigma^{(10|4)}$

Fronsdal [1985]: tensorial space

$$\Sigma^{(10|0)} = \{x^m, y^{mn}\}, \quad y^{mn} = -y^{nm} \quad m, n = 0, 1, 2, 3$$

is the natural space for the 4D massless (=)conformal higher spin theories.

• The reason is clearer if we notice that

$$\begin{split} & \mathcal{L}^{(10|0)} = \{ X^{\alpha\beta} \}, \qquad X^{\alpha\beta} = X^{\beta\alpha}, \qquad \alpha, \beta = 1, .., 4 \\ & X^{\alpha\beta} = X^{\beta\alpha} \Rightarrow \quad X^{\alpha\beta} = x^m \gamma_m^{\alpha\beta} + \frac{1}{2} y^{mn} \gamma_{mn}^{\alpha\beta} \; . \end{split}$$

The first dynamical model in the superspace generalization of Σ^(10|0)

$$\Sigma^{(10|4)} = \{ \boldsymbol{x}^{m}, \boldsymbol{y}^{mn}, \boldsymbol{\theta}^{\alpha} \} = \{ \boldsymbol{X}^{\alpha\beta}, \boldsymbol{\theta}^{\alpha} \}, \qquad \alpha, \beta = 1, .., 4$$

was constructed in 1998 [I.B. + J. Lukierski MPLA 1999].

 Its quantization [I.B. + J. Lukierski + D. Sorokin 1999] gave the tower of conformal massless higher spin fields in D=4.

 Actually this 'generalized superparticle model' [I.B.+ J. Lukierski 1999] was formulated in

$$\Sigma^{(\frac{n(n+1)}{2}|n)} = \{ X^{\alpha\beta}, \theta^{\alpha} \}, \qquad \alpha, \beta = 1, .., n$$

where n is dim. of a min. spinor representation in D dimensions.

- It is *D* dimensional as far as $x^m = \propto \Gamma^m_{\alpha\beta} X^{\alpha\beta}$, m = 0, 1, ..., (D-1).
- The additional tensorial coordinates $y^{m_1...m_p} = \propto \Gamma_{\alpha\beta}^{m_1...m_p} X^{\alpha\beta}$
- correspond to tensorial central charges of most general D-dim SUSY algebra, {Q_α, Q_β} = P_{αβ} = Γ^m_{αβ}P_m + Γ^{m₁...m_p}_(αβ)Z_{m₁...m_p}.

• Only $Z_{m_1...m_p}$ with p, D obeying $\Gamma_{\alpha\beta}^{m_1...m_p} = \Gamma_{(\alpha\beta)}^{m_1...m_p}$ are present. Hence

• The action of [I.B.+ J.L. 1999]: $S = \int d\tau \lambda_{\alpha} \lambda_{\beta} (\dot{X}^{\alpha\beta}(\tau) - i\dot{\theta}^{(\alpha}\theta^{\beta)})$

• contains a huge amount of additional coordinate functions in $X^{\alpha\beta}(\tau)$.

	Tensorial SSP ○○●○	Hspin eqs in TSSP 0000000	<i>OSp</i> (1 <i>n</i>) as AdS-TSSP 00000	κ and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP 0000000	HSpin eqs in extended TSSPs	Concl.
Preon	ic superparticle						

Preonic superparticle action

• In addition to coordinate functions $X^{\alpha\beta} = X^{\alpha\beta}(\tau), \, \theta^{\alpha} = \theta^{\alpha}(\tau),$

$$\begin{split} \boldsymbol{S} &= \int \boldsymbol{d}\tau \mathcal{L} \quad = \quad \int \boldsymbol{d}\tau \lambda_{\alpha} \lambda_{\beta} (\dot{\boldsymbol{X}}^{\alpha\beta} - i\dot{\theta}^{(\alpha}\theta^{\beta)}) = \int \lambda_{\alpha} \lambda_{\beta} \Pi^{\alpha\beta} \\ \Pi^{\alpha\beta} &= \boldsymbol{d} \boldsymbol{X}^{\alpha\beta} - i \boldsymbol{d} \theta^{(\alpha} \theta^{\beta)} , \quad \Pi^{\alpha\beta}(\tau) := \boldsymbol{d}\tau \Pi^{\alpha\beta}_{\tau} \end{split}$$

contains auxiliary bosonic spinor $\lambda_{\alpha} = \lambda_{\alpha}(\tau)$.

• The canonical momentum $\mathcal{P}_{\alpha\beta} := \frac{\partial \mathcal{L}}{\partial \dot{X}^{\alpha\beta}}$ is expressed through λ_{α} ,

$$\mathcal{P}_{lphaeta} = \lambda_lpha\lambda_eta$$

- \Leftarrow 'twistorial dimensional reduction': momentum d.o.f.s: $\frac{n(n+1)}{2} \mapsto n$.
- $4D: 10 \mapsto 4, 6D: 36 \mapsto 8, 10D: 136 \mapsto 16, 11D: 528 \mapsto 32,$
- In D=4,6,10 (but not in D=11) we have also another two effects
- $p_m \propto \mathcal{P}_{\alpha\beta}\Gamma_m^{\alpha\beta} = \lambda \Gamma_m \lambda$ is light–like, $p_m p^m = 0$. \Leftarrow famous $\Gamma_m^{\alpha(\beta}\Gamma^{\gamma\delta)a} = 0$.
- *p_mp^m* = 0 suggests that the spectrum of the quantum states of the model consists of masseless particles.
- But to this end one has to prove the spectrum is discreet.

Intro	Tensorial SSP ○○○●	Hspin eqs in TSSP	<i>OSp</i> (1 <i>n</i>) as AdS-TSSP 00000	к and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.
Preon	ic superparticle						

Spectrum of D=4,6,10 preonic superparticle

• In D=4,6,10, this is the case due to 'twistorial compactification':

• the spaces
$$\{\lambda\}/\{p^m\}_{p_np^n=0} = \mathbb{S}^{2D-5}/\mathbb{S}^{D-2}$$
 is isomorphic to
 $\mathbb{S}^{D-3} = (\mathbb{S}^1, \mathbb{S}^3, \mathbb{S}^7)$ spheres (Hopf fibrations): $\{\lambda\}/\{p^m\}_{p_np^n=0} = \mathbb{S}^{2D-3}$

$${}^{m}\}_{p_{n}p^{n}=0}=\mathbb{S}^{D-3}$$

- In $D = 11 \ p_m p^m = 0$ is nonvanishing (arbitrary!) nor S^{31}/S^{11} (nor S^{31}/S^9) is known to be a sphere (or a compact space).
- The interest to this case was due to an M-theoretical perspective ('BPS preons' [I.B., J. de Azcárraga, J. Izquierdo, J. Lukierski, 2000]).
- In D = 4, 6, 10 the space of additional momentum variables is compact, S^{D-3} = (S¹, S³, S⁷), which implies that the spectrum of corresponding coordinate variables is discreet.

• These are helicity in D = 4 and its generalizations in D = 6 and 10.

- This implies that quantum state spectrum of the D=4,6,10 'tensorial' superparticle $S = \int d\tau \lambda_{\alpha} \lambda_{\beta} (\dot{X}^{\alpha\beta}(\tau) i\dot{\theta}^{(\alpha}\theta^{\beta)})$ is given by the complete tower of massless higher spin fields. [I.B, J.Lukierski and D.Sorokin 99].
- Also the equations of motion for higher spin fields in $\sum \frac{n(n+1)}{2} |n\rangle$ can be obtained by quantizing $S = \int d\tau \lambda_{\alpha} \lambda_{\beta} (\dot{X}^{\alpha\beta} i\dot{\theta}^{(\alpha}\theta^{\beta}))$.
- But to lighten the representation, we will go another way.

Intro	Tensorial SSP	Hspin eqs in TSSP	OSp(1 n) as AdS-TSSP 00000	к and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.

Outline

- Introduction
- Flat tensorial superspace $\Sigma^{(n(n+1)/2|n]}$
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- ³ Higher spin equations in tensorial superspace $\Sigma^{(n(n+1)/2|n)}$
 - Higher spin equations in 4D tensorial superspace
 - Higher spin equations in 10D tensorial superspace
 - **OSp(1**, *n*) as AdS generalization of $\Sigma^{(n(n+1)/2|n)}$
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 - κ symmetry and SUSY preserved by preonic BPS state
 - OSp(1|2n) symmetry of $\Sigma^{(n(n+1)/2|n)}$ and OSp(1|n) superparticles
- 6 Searching for the interacting theory: Supergravity in tensorial superspace
 - Preonic superparticle and SUGRA constraints in $\mathcal{M}^{(\frac{n(n+1)}{2}|n)}$
 - Supergravity in tensorial superspace
 - Higher spin equations in extended tensorial superspaces

	Tensorial SSP 0000	Hspin eqs in TSSP ●000000	<i>OSp</i> (1 <i>n</i>) as AdS-TSSP 00000	к and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.
4D Hig	gher spin equatio	าร					

Higher spin equations in tensorial superspace

• Free equations of motion for all the field strengths of all bosonic and fermionic higher spin fields can be collected in [M. Vasiliev 2001]

$$\begin{array}{ll} \partial_{\alpha[\beta}\partial_{\gamma]\delta} \, b(X) = 0 & \Leftrightarrow & (\partial_{\alpha\beta}\partial_{\gamma\delta} - \partial_{\alpha\gamma}\partial_{\beta\delta}) \, b(X) = 0 \ , \\ \partial_{\alpha[\beta}f_{\gamma]}(X) = 0 & \Leftrightarrow & (\partial_{\alpha\beta}f_{\gamma}(X) - \partial_{\alpha\gamma}f_{\beta}(X)) = 0 \ . \end{array}$$

• where $\partial_{\alpha\beta} := \frac{1}{2} \frac{\partial}{\partial X^{\alpha\beta}}$ and $f_{\beta}(X) = f_{\beta}(X^{\alpha\beta})$ is fermionic.

• In D=4
$$\{X^{\alpha\beta}\} = \{x^m, y^{mn}\}$$

$$b(x, y) = \phi(x) + y^{m_1 n_1} F_{m_1 n_1}(x) + y^{m_1 n_1} y^{m_2 n_2} \hat{R}_{m_1 n_1, m_2 n_2}(x) + + \sum_{s=3}^{\infty} y^{m_1 n_1} \cdots y^{m_s n_s} \hat{R}_{m_1 n_1, \dots, m_s n_s}(x),$$

$$f^{\alpha}(x, y) = \psi^{\alpha}(x) + y^{m_1 n_1} \hat{\mathcal{R}}_{m_1 n_1}^{\alpha}(x) + + \sum_{s=\frac{5}{2}}^{\infty} y^{m_1 n_1} \cdots y^{m_{s-\frac{1}{2}} n_{s-\frac{1}{2}}} \hat{\mathcal{R}}_{m_1 n_1, \dots, m_{s-\frac{1}{2}} n_{s-\frac{1}{2}}}^{\alpha}(x).$$

• $\partial_{\alpha[\beta}\partial_{\gamma]\delta} b(X) = 0 \Rightarrow \begin{cases} \partial_{[m}F_{nk]} = 0, \ \partial_{[m_3}R_{m_1m_2]n_1n_2} = 0, \dots \\ \Box \phi(x) = 0, \ \partial^m F_{mn} = 0, \ \partial^{m_1}R_{m_1m_2n_1n_2} = 0 \dots, \end{cases}$ $\partial_{\alpha[\beta}f_{\gamma]}(X) = 0 \Rightarrow \{\partial \psi = 0, \dots$

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4D Hiç	gher spin equatior	IS					

Higher spin equations in 4D tensorial superspace

• In a more schematic notation $^{[2]} := {}^{[mn]} \equiv -{}^{[nm]}; {}^{[2]_1} := {}^{[m_1n_1]}$

$$\begin{split} b(x, y) &= \phi(x) + y^{[2]} \mathcal{F}_{[2]}(x) + y^{[2]_1} y^{[2]_2} \hat{\mathcal{R}}_{[2]_1[2]_2}(x) + \\ &+ \sum_{s=3}^{\infty} y^{[2]_1} \cdots y^{[2]_s} \hat{\mathcal{R}}_{[2]_1 \cdots [2]_s}(x) \,, \\ f^{\alpha}(x, y) &= \psi^{\alpha}(x) + y^{[2]} \hat{\mathcal{R}}_{[2]}^{\alpha}(x) + \sum_{s=\frac{5}{2}}^{\infty} y^{[2]_1} \cdots y^{[2]_{s-1/2}} \hat{\mathcal{R}}_{[2]_1 \cdots [2]_{s-1/2}}^{\alpha}(x) \end{split}$$

• and eqs. for higher spin curvatures are (with D = 4)

$$\partial_{[m_1} R_{m_2 n_1] n_2 [2]_3 \cdots [2]_s} = 0, \qquad \partial^{m_1} R_{m_1 m_2, [2]_2 \cdots [2]_s} = 0.$$
$$R_{[m_1 m_2 n_1] n_2 [2]_3 \cdots [2]_s} = 0 \quad \Leftrightarrow \quad R = \underbrace{\square \dots \square}_{s}$$

•
$$\Leftrightarrow$$
 $R_{m_1n_1,\cdots,m_sn_s} = \sigma_{m_1n_1}^{A_1A_{s+1}} \cdots \sigma_{m_sn_s}^{A_sA_{2s}} C_{A_1\cdots A_sA_{s+1}\cdots A_{2s}} + c.c.$

• where symmetric spin-tensor $C_{A_1 \cdots A_s A_{s+1} \cdots A_{2s}}$ and its c.c. obey the Bargmann–Wigner equations

$$\partial^{B\dot{B}}C_{BA_1\dots A_{2s-1}}(x) = 0, \quad \partial^{B\dot{B}}C_{\dot{B}\dot{A}_1\dots \dot{A}_{2s-1}}(x) = 0$$

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10D Higher spin equations							

Higher spin equations in 10D tensorial superspace

● ←

• In D=10 we denote $^{[5]} := ^{[mnk/p]}; ^{[5]_1} := ^{[m_1n_1k_1l_1p_1]}$

$$\begin{split} b(x, y) &= \phi(x) + y^{[5]} \mathcal{F}_{[5]}(x) + y^{[5]_1} y^{[5]_2} \hat{\mathcal{R}}_{[5]_1 5]_2}(x) + \\ &+ \sum_{s=3}^{\infty} y^{[5]_1} \cdots y^{[5]_s} \hat{\mathcal{R}}_{[5]_1 \cdots [5]_s}(x) , \\ f_{\alpha}(x, y) &= \psi_{\alpha}(x) + y^{[2]} \hat{\mathcal{R}}_{\alpha_{[5]}}(x) + \sum_{s=\frac{5}{2}}^{\infty} y^{[5]_1} \cdots y^{[5]_{s-1/2}} \hat{\mathcal{R}}_{\alpha_{[5]_1 \cdots [5]_{s-1/2}}}(x) . \end{split}$$

• and eqs. for higher spin curvatures are (with D = 10)

$$\partial_{[m_6} R_{m_1 \cdots m_5], \left[\frac{D}{2}\right]_2 \cdots \left[\frac{D}{2}\right]_s} = 0, \qquad \partial^n R_{n[4]_1, \left[5\right]_2 \cdots \left[5\right]_s} = 0.$$

 $\partial_{\alpha[\beta}\partial_{\gamma]\delta} b(X) = 0$; fermionic counterparts \leftarrow

$$\partial_{\alpha[\beta}f_{\gamma]}(X)=0$$
.

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10D H	liaher snin eauati	ons					

On relation with preonic superparticle

• How to see the relation with preonic (tensorial superparticle)?

$${\cal S}=\int {\it d} au {\cal L} ~=~ \int {\it d} au \lambda_lpha \lambda_eta (\dot{X}^{lphaeta}-i\dot{ heta}^{(lpha} heta^{eta})) =\int \lambda_lpha \lambda_eta \Pi^{lphaeta}$$

This produces the generalization of the Cartan-Penrose relation:

$$\mathcal{P}_{lphaeta} = \lambda_lpha\lambda_eta$$

- $\partial_{\alpha[\beta}\partial_{\gamma]\delta} b(X) = 0$ in the momentum representation reads $p_{\alpha[\beta}p_{\gamma]\delta} b(p) = 0$
- $\Rightarrow b(p) \neq 0$ when $rank(p_{\alpha\beta}) = 1 \Leftrightarrow p_{\alpha\beta} = \lambda_{\alpha}\lambda_{\beta}$ for some λ_{α} .
- $\Rightarrow \partial_{\alpha[\beta}\partial_{\gamma]\delta} b(X) = 0$ is solved by $b(X) = \int d^n \lambda \Phi(X, \lambda)$, where

$$(\partial_{\alpha\beta} - i\lambda_{\alpha}\lambda_{\beta})\Phi(X,\lambda) = 0$$

- Fermionic $\partial_{\alpha[\beta} f_{\gamma]}(X) = 0$ is solved by $f_{\alpha}(X) = \int d^n \lambda \, \lambda_{\alpha} \Phi(X, \lambda)$.
- "Preonic wave function" Φ(X, λ) = 0 is not exactly wavefunction: it depends on both coordinates and momenta variables (p_{αβ} = λ_αλ_β).

	Tensorial SSP 0000	Hspin eqs in TSSP ○○○○●○○	OSp(1 n) as AdS-TSSP 00000	κ and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.	
10D H	10D Higher spin equations							

4D. Preonic equation and unfolding equations.

• In D=4
$$\lambda_{\alpha} = (\lambda_{A}, \bar{\lambda}^{\dot{A}}), y^{mn} = \propto (\sigma^{[m} \tilde{\sigma}^{n]})_{AB} y^{AB} + c.c.$$
 and

$$\boxed{(\partial_{\alpha\beta} - i\lambda_{\alpha}\lambda_{\beta})\Phi(X, \lambda) = 0} \iff \begin{cases} \left(\frac{\partial}{\partial x^{AB}} - i\lambda_{A}\bar{\lambda}_{\dot{B}}\right)\Phi(x, y; \lambda, \bar{\lambda}) = 0, \\ \left(\frac{\partial}{\partial y^{AB}} - i\lambda_{A}\lambda_{B}\right)\Phi(x, y; \lambda, \bar{\lambda}) = 0, \\ \left(\frac{\partial}{\partial y^{AB}} - i\bar{\lambda}_{\dot{A}}\bar{\lambda}_{\dot{B}}\right)\Phi(x, y; \lambda, \bar{\lambda}) = 0, \end{cases}$$

• Misha Vasiliev prefers to work with a Fourier transform $C(X, y^{\alpha}) = \int d^n \lambda e^{i \lambda_{\alpha} y^{\alpha}} \Phi(X, \lambda)$ which obeys the unfolded eqs.

$$(\partial_{\alpha\beta} + i\frac{\partial}{\partial Y^{\alpha}}\frac{\partial}{\partial Y^{\beta}})C(X,y) = 0 \quad \Leftrightarrow \begin{cases} \left(\frac{\partial}{\partial x^{A\bar{B}}} + i\frac{\partial}{\partial Y^{A}}\frac{\partial}{\partial \bar{Y}^{B}}\right)C(x,y;Y,\bar{Y}) = 0, \\ \left(\frac{\partial}{\partial y^{A\bar{B}}} + i\frac{\partial}{\partial Y^{A}}\frac{\partial}{\partial Y^{B}}\right)C(x,y;Y,\bar{Y}) = 0, \\ \left(\frac{\partial}{\partial y^{A\bar{B}}} + i\frac{\partial}{\partial \bar{Y}^{A}}\frac{\partial}{\partial \bar{Y}^{B}}\right)C(x,y;Y,\bar{Y}) = 0, \end{cases}$$

• One can show [Vasiliev 2001] that in $C(X, y) = b(X) + f_{\alpha}(X) y^{\alpha} + \sum_{n=2}^{\infty} C_{\alpha_1 \cdots \alpha_n}(X) y^{\alpha_1} \cdots y^{\alpha_n}$ the only dynamical fields are scalar b(X) and spinor (or 'svector') $f_{\alpha}(X)$ which satisfy $\partial_{\alpha[\beta}\partial_{\gamma]\delta} b(X) = 0$ and $\partial_{\alpha[\beta}f_{\gamma]}(X) = 0$.

Intro	Tensorial SSP 0000	Hspin eqs in TSSP ○○○○○○○	OSp(1 n) as AdS-TSSP	к and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP	HSpin eqs in extended TSSPs 00000	Concl.
10D Higher spin equations							

Superfield generalization

• Let us introduce covariant Grassmann derivative in $\Sigma^{(\frac{n(n+1)}{2}|n)}$

$$D_{\alpha} = \partial/\partial \theta^{\alpha} + i \theta^{\beta} \partial_{\beta \alpha} , \qquad \{D_{\alpha}, D_{\beta}\} = 2i \partial_{\alpha \beta} .$$

The (manifestly) GL(n) covariant eq. [I.B., Pasti, Sorokin, Tonin 2004]

$$D_{[\alpha}D_{\beta]}\Phi(X,\theta)=0$$

- \Rightarrow in $\Phi(X^{\alpha\beta}, \theta^{\gamma}) = b(X) + f_{\alpha}(X) \theta^{\alpha} + \sum_{i=2}^{n} \phi_{\alpha_{1}\cdots\alpha_{i}}(X) \theta^{\alpha_{1}}\cdots\theta^{\alpha_{i}}$ the only dynamical fields are scalar b(X) and spinor (or 'svector') $f_{\alpha}(X)$ which satisfy $\partial_{\alpha[\beta}\partial_{\gamma]\delta} b(X) = 0$ and $\partial_{\alpha[\beta}f_{\gamma]}(X) = 0$.
- Actually this equation possesses OSp(1|2n) invariance (OSp(1|8) for D=4), like $S = \int \lambda_{\alpha} \lambda_{\beta} (dX^{\alpha\beta} id\theta^{(\alpha} \theta^{\beta)})$ does [I.B., Lukierski 98].

Intro	Tensorial SSP	Hspin eqs in TSSP	OSp(1 n) as AdS-TSSP	κ and $OSp(1 2n)$	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.
		000000					
	igher opin equatio						

Superfield generalization

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- Actually this equation possesses OSp(1|2n) invariance (OSp(1|8) for D=4), like $S = \int \lambda_{\alpha} \lambda_{\beta} (dX^{\alpha\beta} id\theta^{(\alpha} \theta^{\beta)})$ does [I.B., Lukierski 98].
- Its quantization [I.B., Lukierski, Sorokin 1999]: wave function is a Clifford superfield (χχ = 1, χθ = −θχ)

$$\Upsilon(X,\theta,\lambda,\chi) = g_0(X,\theta,\lambda) + i\chi g_1(X,\theta,\lambda) = \Upsilon(X,\theta,-\lambda,-\chi))$$

obeying $(D_{\alpha} - \chi \lambda_{\alpha})\Upsilon(X, \theta, \lambda, \chi) = 0.$

• $\Rightarrow (D_{\alpha}D_{\beta} + \lambda_{\alpha}\lambda_{\beta})g_0(X,\theta,\lambda) = 0 \Rightarrow D_{[\alpha}D_{\beta]}g_0(X,\theta,\lambda) = 0.$

•
$$\Phi(X,\theta) = \int d^n \lambda g_0(X,\theta,\lambda) = b(X) + f_\alpha(X) \theta^\alpha,$$

 $b(X) = \int d^n \lambda g_0(X,0,\lambda),$
 $f_\alpha(X) = \int d^n \lambda D_\alpha g_0(X,\theta,\lambda)|_{\theta=0},$

	Tensorial SSP 0000	Hspin eqs in TSSP 0000000	OSp(1 n) as AdS-TSSP	к and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.			
Out	line									
	1 Intro 2 Flat • 4[• Hi	duction tensorial sup D Tensorial s igher D tenso	perspace $\Sigma^{(n(n+1)}$ uperspace $\Sigma^{(10)}$	$\frac{)/2 n)}{2} \sum \left(\frac{n(n+1)}{2} n\right)$						
	 Preonic superparticle in tensorial superspace Σ^{(n(n+1)/2)} Higher spin equations in tensorial superspace Σ^{(n(n+1)/2)} Higher spin equations in 4D tensorial superspace Higher spin equations in 10D tensorial superspace 									
	 Ac Ac Pr 	dS HSpin eq dS HSpin eq reonic super	uations on <i>OSp</i> uations on <i>Sp(r</i> particle on <i>OSp</i>	(1 n) superg (1 n) group mai (1, n) superg	group manif nifold group manif	old				
	 5 Preosupe supe κ 	nic propertie prparticle symmetry ar	es and <i>OSp</i> (1 2) nd SUSY preser	n) supercon	formal symr nic BPS sta	netry of tensorial te				

- OSp(1|2n) symmetry of $\Sigma^{(n(n+1)/2|n)}$ and OSp(1|n) superparticles
- 6 Searching for the interacting theory: Supergravity in tensorial superspace
 - Preonic superparticle and SUGRA constraints in $\mathcal{M}^{(\frac{n(n+1)}{2}|n)}$
 - Supergravity in tensorial superspace
 - Higher spin equations in extended tensorial superspaces

	Tensorial SSP 0000	Hspin eqs in TSSP 0000000	OSp(1 n) as AdS-TSSP ●0000	κ and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP 0000000	HSpin eqs in extended TSSPs	Concl.		
AdS I	AdS HS eqs. on OSp(1 n)								

AdS higher spin equations. Superfield form

 Thus all the free massless conformal higher spin eqs. in D=4,6,10 can be collected in one scalar eq. in Σ^{(n(n+1)/2) n)} with n = 4,8,16:

$$\boxed{D_{[\alpha}D_{\beta]}\Phi(X,\theta)=0} \Rightarrow \begin{cases} \Phi(X^{\alpha\beta},\theta^{\gamma})=b(X)+f_{\alpha}(X)\,\theta^{\alpha},\\ \partial_{\alpha[\beta}\partial_{\gamma]\delta}\,b(X)=0, \partial_{\alpha[\beta}f_{\gamma]}(X)=0 \end{cases}$$

- Can we do this with (massless conformal) AdS higher spin equations?
- 1) What is the AdS generalization of the tensorial superspace $\Sigma^{(\frac{n(n+1)}{2}|n)}$?
- [I.B., Lukierski, Preitschopf, Sorokin 2000]: $AdS^{(\frac{n(n+1)}{2}|n)} = OSp(1|n)$
- In particular, $AdS^{(10|4)} = OSp(1|4)$
- Indeed, it is natural as far as $AdS_4 = Sp(4)/SO(1,3)$.
- N = 1 AdS superspace is $AdS^{(4|4)} = OSp(4)/SO(1,3)$.
- Abelian algebra of *Z_{mn}* can be considered as a contraction of *so*(1,3) algebra.

	Tensorial SSP 0000	Hspin eqs in TSSP	OSp(1 n) as AdS-TSSP ○●○○○	κ and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.		
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- Can we do this with (massless conformal) AdS higher spin equations?
- 1) [I.B., Lukierski, Preitschopf, Sorokin 2000]: $AdS^{(\frac{n(n+1)}{2}|n)} = OSp(1|n)$
- 2) Free conformal AdS higher spin equations can be collected in

$$\left(
abla_{[lpha}
abla_{eta]} + i rac{\varsigma}{4} C_{lphaeta}
ight) \Phi(X, heta) = \mathbf{0} \; .$$

where $\varsigma \propto \frac{1}{R_{AdS}}$, $C_{\alpha\beta} = -C_{\beta\alpha}$ is the Sp 'metric' and the OSp(1|n) covariant derivatives $\nabla_{\alpha}, \nabla_{\alpha\beta}$ obey the osp(1|n) superalgebra

$$\begin{aligned} \{\nabla_{\alpha}, \nabla_{\beta}\} &= 2i\nabla_{\alpha\beta} , \qquad [\nabla_{\alpha\alpha'}, \nabla_{\beta}] = \varsigma \, \mathcal{C}_{\beta(\alpha} \nabla_{\alpha'}), \\ &[\nabla_{\alpha\beta}, \nabla_{\gamma\delta}] = \varsigma \, \mathcal{C}_{\alpha(\gamma} \nabla_{\delta)\beta} + \varsigma \, \mathcal{C}_{\beta(\gamma} \nabla_{\delta)\alpha} . \end{aligned}$$

Intro	Tensorial SSP 0000	Hspin eqs in TSSP 0000000	OSp(1 n) as AdS-TSSP ○○●○○	к and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP	HSpin eqs in extended TSSPs 00000	Concl.	
AdS F	AdS HS eqs. on $Sp(n)$							

AdS higher spin equations. Component form on Sp(n) space

• $\left[\left(\nabla_{[\alpha} \nabla_{\beta]} + i \frac{\varsigma}{4} C_{\alpha\beta} \right) \Phi(X, \theta) = 0 \right] \Rightarrow$ 'component equations on Sp(n)[Sorokin, Plyushchay, Tsulaia 2003]

$$egin{aligned}
abla_{lpha[eta}
abla_{\gamma]\delta}b(X) &= rac{arsigma}{4}\left(C_{lpha[eta}
abla_{\gamma]\delta}+C_{\delta[eta}
abla_{\gamma]lpha}-C_{eta\gamma}
abla_{lpha\delta}
ight)b(X)+ \ &+rac{arsigma^2}{16}\left(C_{lpha\delta}C_{eta\gamma}-C_{lpha[eta}C_{\gamma]\delta}
ight)b(X), \ &
abla_{lpha[eta}f_{\gamma]}(X) &= -rac{arsigma}{4}\left(C_{lpha[\gamma}f_{eta]}(X)+C_{eta\gamma}f_{lpha}(X)
ight). \end{aligned}$$

where $[\nabla_{\alpha\beta}, \nabla_{\gamma\delta}] = \varsigma C_{\alpha(\gamma} \nabla_{\delta)\beta} + \varsigma C_{\beta(\gamma} \nabla_{\delta)\alpha}$.

The counterpart of the Clifford superfield eq. (D_α − χ λ_α)Υ = 0,

$$(
abla_{lpha} - \chi Y_{lpha})\Upsilon(X, heta, \lambda, \chi) = 0, \qquad Y_{lpha} = \lambda_{lpha} - rac{I\varsigma}{4} \, \mathcal{C}_{lphaeta} \, rac{\partial}{\partial\lambda_{eta}} \, .$$

was studied in [Didenko, Valsiliev 2003].

To be more precise, they studied its Fourier transform

$$(\nabla_{\alpha} - \chi Y_{\alpha})\Upsilon(X, \theta, y^{\beta}, \chi) = 0, \quad Y_{\alpha} \equiv i \frac{\partial}{\partial y^{\alpha}} + C_{\alpha\beta} \frac{\varsigma}{4} y^{\beta}.$$

	Tensorial SSP 0000	Hspin eqs in TSSP 0000000	OSp(1 n) as AdS-TSSP ○○○●○	к and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP 0000000	HSpin eqs in extended TSSPs 00000	Concl.
AdS H	HS eqs. on Sp(n)						

AdS higher spin equations. Component form on Sp(n) space

$$(\nabla_{\alpha} - \chi Y_{\alpha})\Upsilon(X, \theta, \lambda, \chi) = \mathbf{0}, \qquad Y_{\alpha} = \lambda_{\alpha} - \frac{I\varsigma}{4} C_{\alpha\beta} \frac{\partial}{\partial \lambda_{\beta}}$$

• It results in an 'AdS preonic' equation ($Y = g_0 + \chi g_1$)

$$\left[\nabla_{\alpha\beta}-iY_{(\alpha}Y_{\beta)}\right]g_{0}(X,\theta,\lambda)=0\,,\quad Y_{\alpha}\equiv\lambda_{\alpha}-\frac{i\varsigma}{4}\frac{\partial}{\partial\lambda_{\alpha}},$$

and in a more general

$$(
abla_lpha
abla_eta + Y_eta Y_lpha) \, g_0(X, heta,\lambda) = 0 \;, \qquad Y_lpha = \lambda_lpha - rac{i_\varsigma}{4} \, \mathcal{C}_{lphaeta} \; rac{\partial}{\partial\lambda_\lambda} \;.$$

which also includes $\left(\nabla_{[\alpha} \nabla_{\beta]} + i\frac{\varsigma}{4} C_{\alpha\beta}\right) g_0(X, \theta, \lambda) = 0$

• Then, $\Phi(X, \theta) = \int d^n \lambda g_0(X, \theta, \lambda)$ obeys the superfield version of the AdS higher spin equation

$$\left(\nabla_{[\alpha}\nabla_{\beta]}+i\frac{\varsigma}{4}C_{\alpha\beta}\right)\Phi(X,\theta)=0$$

	Tensorial SSP 0000	Hspin eqs in TSSP	OSp(1 n) as AdS-TSSP ○○○○●	к and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.
Preoni	ic superparticle o	n <i>OSp</i> (1 <i>n</i>)					

Preonic superparticle on OSp(1|n)

• $(\nabla_{\alpha} - \chi Y_{\alpha})\Upsilon(X, \theta, \lambda, \chi) = 0$ with $Y_{\alpha} = \lambda_{\alpha} - \frac{i_{\zeta}}{4} C_{\alpha\beta} \frac{\partial}{\partial \lambda_{\beta}} (= \lambda_{\alpha} * ...)$ can be obtained by quantization of OSp(1|n) superparticle

$${oldsymbol{\mathcal{S}}} = \int\limits_{W^1} \lambda_lpha \lambda_eta \hat{\mathcal{E}}^{lphaeta} = \int {oldsymbol{d}} au \lambda_lpha \lambda_eta \partial_ au \hat{\mathcal{Z}}^M \mathcal{E}_M^{lphaeta}(\hat{\mathcal{Z}}(au)) \ ,$$

• where $\mathcal{E}^{\alpha\beta} = dZ^M \mathcal{E}^{\alpha\beta}_M(Z)$ and $\mathcal{E}^{\alpha} = dZ^M \mathcal{E}^{\alpha}_M(Z)$ obey

$$\begin{aligned} d\mathcal{E}^{\alpha\beta} &= -i\mathcal{E}^{\alpha}\wedge\mathcal{E}^{\beta}-\zeta\mathcal{E}^{\alpha\gamma}\wedge\mathcal{E}^{\delta\beta}\mathcal{C}_{\gamma\delta} \ , \\ d\mathcal{E}^{\alpha} &= -\zeta\mathcal{E}^{\alpha\gamma}\wedge\mathcal{E}^{\delta}\mathcal{C}_{\gamma\delta} \ , \end{aligned}$$

Z^M = (X^{ά,β}, θ^ά) are local coordinates of OSp(1|n) supergroup manifold
and Z^M = Â^M(τ) defines the embedding of W¹ in OSp(1|n).

This action possesses rigid OSp(1|2n) symmetry (generalized conformal symm.) and also (n – 1) local fermionic κ–symmetries (3 in D=4):

$$\delta_{\kappa} \hat{Z}^{M} \mathcal{E}_{M}^{\alpha\beta}(\hat{Z}) = 0 , \qquad \delta_{\kappa} \hat{Z}^{M} \mathcal{E}_{M}^{\alpha}(\hat{Z}) \lambda_{\alpha} = 0 .$$

	Tensorial SSP 0000	Hspin eqs in TSSP	<i>OSp</i> (1 <i>n</i>) as AdS-TSSP 00000	к and <i>OSp</i> (1 2 <i>n</i>)	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.			
Out	line									
	 Intro Flat f 4[Hi 	duction tensorial sup D Tensorial s gher D tenso	verspace $\Sigma^{(n(n+1)}$ uperspace $\Sigma^{(10)}$ prial superspace)/2 n) 4) $2\sum (\frac{n(n+1)}{2} n)$						
	 Pr High Hi Hi 	 Preonic superparticle in tensorial superspace Σ^{(n(n+1)/2 n)} Higher spin equations in tensorial superspace Σ^{(n(n+1)/2 n)} Higher spin equations in 4D tensorial superspace Higher spin equations in 10D tensorial superspace 								
	 OSp Ac Ac Pr 	 OSp(1, n) as AdS generalization of Σ^{(n(n+1)/2 n)} AdS HSpin equations on OSp(1 n) supergroup manifold AdS HSpin equations on Sp(n) group manifold Preopic superparticle on OSp(1, n) supergroup manifold 								
	 5 Preo supe • κ • Ο 	nic propertie rparticle symmetry ar Sp(1 2n) syr	as and $OSp(1 2)$ and SUSY presert metry of $\Sigma^{(n(n+1))}$	n) supercond wed by preo $(1)^{(2 n)}$ and $(2)^{(2 n)}$	formal symi nic BPS sta DSp(1 n) su	metry of tensorial ate uperparticles				
	SearPr	ching for the eonic super	interacting theo particle and SU	ory: Supergr GRA constra	avity in tensatints in $\mathcal{M}^{(l)}$	sorial superspace $\frac{n(n+1)}{2} n)$				

- Supergravity in tensorial superspace
- Higher spin equations in extended tensorial superspaces Conclusions

Intro	Tensorial SSP 0000	Hspin eqs in TSSP 0000000	<i>OSp</i> (1 <i>n</i>) as AdS-TSSP 00000	κ and OSp(1 2n) ●000	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.
κ svm	nm. & preserved	SUSY					

Preonic superparticle on OSp(1|n) and on $\Sigma^{(n(n+1)/2|n)}$

• These symmetries survive - and become simpler- in flat SSP limit $OSp(1|n)_{\overrightarrow{c} \to 0} \Sigma^{(\frac{n(n+1)}{2}|n)}$, $\mathcal{E}^{\alpha\beta}_{\overrightarrow{c} \to 0} \Pi^{\alpha\beta} = dX^{\alpha\beta} - id\theta^{(\alpha} \theta^{\beta)}$, $\mathcal{E}^{\alpha}_{\overrightarrow{c} \to 0} d\theta^{(\alpha}$,

$$\begin{array}{ll} d\mathcal{E}^{\alpha\beta} &=& -i\mathcal{E}^{\alpha}\wedge\mathcal{E}^{\beta}-\zeta\mathcal{E}^{\alpha\gamma}\wedge\mathcal{E}^{\delta\beta}\mathcal{C}_{\gamma\delta} \ , \\ d\mathcal{E}^{\alpha} &=& -\zeta\mathcal{E}^{\alpha\gamma}\wedge\mathcal{E}^{\delta}\mathcal{C}_{\gamma\delta} \ , \end{array} \right\} \underset{\zeta\mapsto 0}{\longmapsto} \left\{ \begin{array}{l} d\Pi^{\alpha\beta} = -id\theta^{\alpha}\wedge d\theta^{\beta} \ , \\ dd\theta^{\alpha} \equiv 0 \ , \end{array} \right.$$

• The κ -symmetry of the $\Sigma^{(n(n+1)/2|n)}$ superparticle action $S = \int_{W^1} \lambda_{\alpha} \lambda_{\beta} \hat{\Pi}^{\alpha\beta} = \int d\tau \lambda_{\alpha} \lambda_{\beta} (\partial_{\tau} X^{\alpha\beta} - i \partial_{\tau} \theta^{(\alpha} \theta^{\beta)})$ reads

$$\delta_{\kappa} X^{lpha eta} = i \delta_{\kappa} heta^{(lpha} \, heta^{eta}) \;, \qquad \delta_{\kappa} heta^{lpha} \, \lambda_{lpha} = \mathbf{0} \;.$$

- δ_κθ^α λ_α = 0 can be solved in terms of (n − 1) bosonic spinors 'orthogonal' to λ_α: δ_κθ^α = κ^Iu^α_I, u^α_I λ_α = 0, I = 1,..., 15.
- This makes clear that we can gauge away all but one component of $\theta^{\alpha}(\tau)$: $\eta = \theta^{\alpha}(\tau)\lambda_{\alpha}$ which is κ -invariant.
- This is related to global OSp(1|2n) symmetry of the system
- but also shows that its ground state preserves all but one SUSY
- is a BPS preon [I.B., de Azcárraga, Izquierdo, Lukierski, 2001].

κ symm. & preserved SUSY

κ symmetry and preserved SUSY, or Why tensorial superparticle is preonic?

•
$$S = \int_{W^1} \lambda_{\alpha} \lambda_{\beta} \hat{\Pi}^{\alpha\beta} = \int d\tau \lambda_{\alpha} \lambda_{\beta} (\partial_{\tau} X^{\alpha\beta} - i \partial_{\tau} \theta^{(\alpha} \theta^{\beta)})$$
 is invariant

- under κ -symmetry $\delta_{\kappa} X^{\alpha\beta} = i \delta_{\kappa} \theta^{(\alpha} \theta^{\beta)}$, $\delta_{\kappa} \theta^{\alpha} = \kappa^{\alpha}$, $\kappa^{\alpha} \lambda_{\alpha} = 0$
- and under rigid SUSY $\delta_{\epsilon} X^{\alpha\beta} = -i\delta_{\epsilon}\theta^{(\alpha} \,\theta^{\beta)} , \qquad \delta_{\epsilon}\theta^{\alpha} = \epsilon^{\alpha}$
- Thus $\delta\theta^{\alpha} = \delta_{\kappa}\theta^{\alpha} + \delta_{\epsilon}\theta^{\alpha} = \kappa^{\alpha} + \epsilon^{\alpha}, \ \kappa^{\alpha}\lambda_{\alpha} = 0$
- In the ground state of the system fermions are equal to zero: $\theta^{\alpha} = 0$
- so that it can be preserved by symmetries which preserve $\theta^{\alpha} = 0$, i.e.
- which obey $0 = \delta \theta^{\alpha} = \kappa^{\alpha} + \epsilon^{\alpha}, \ \kappa^{\alpha} \lambda_{\alpha} = 0$.
- This identifies all but one SUSY parameters with nontrivial parameters of κ -symmetry, $\epsilon^{\alpha} = -\kappa^{\alpha}$ and set to zero only one linear combination of the components of ϵ^{α} : $\epsilon^{\alpha}\lambda_{\alpha} = 0$.
- Thus all but one target space supersymmetry are preserved by the ground state of the tensorial superparticle.
- This ground state is $\frac{n-1}{n}$ BPS, i.e. $\frac{3}{4}$ BPS in D=4, $\frac{15}{16}$ BPS in D=10 and $\frac{31}{32}$ BPS in D=11;
- this is to say it is a BPS preon [I.B., de Azcárraga, Izquierdo, Lukierski, 2001].

	Tensorial SSP 0000	Hspin eqs in TSSP	<i>OSp</i> (1 <i>n</i>) as AdS-TSSP 00000	κ and <i>OSp</i> (1 2 <i>n</i>) ○○●○	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.
OSp(1	2n) symmetry						

OSp(1|2n) symmetry of $\sum_{n=1}^{n} \frac{n(n+1)}{2} |n|$ superparticle

• $S = \int_{W^1} \lambda_{\alpha} \lambda_{\beta} \hat{\Pi}^{\alpha\beta} = \int \lambda_{\alpha} \lambda_{\beta} (d\hat{X}^{\alpha\beta} - id\hat{\theta}^{(\alpha} \hat{\theta}^{\beta)})$ can be rewritten as

$$\mathcal{S} = \int_{W^1} (\lambda_{lpha} d\mu^{lpha} - \mu^{lpha} d\lambda_{lpha} - i d\chi \chi) , \qquad \begin{cases} \mu^{lpha} = \hat{X}^{lpha\beta} \lambda_{eta} - rac{i}{2} \hat{ heta}^{lpha} \chi , \ \chi = \hat{ heta}^{lpha} \lambda_{lpha} \end{cases}$$

[I.B.+Lukierski 98] or equivalently as

$$S = \int_{W^1} d\Upsilon^{\Sigma} \Xi_{\Sigma\Omega} \Upsilon^{\Omega} \;, \quad \Upsilon^{\Sigma} = \begin{pmatrix} \mu^{lpha} \\ \lambda_{lpha} \\ \chi \end{pmatrix} \;, \quad \Xi_{\Sigma\Omega} = \begin{pmatrix} 0 & \delta_{lpha}{}^{eta} & 0 \\ -\delta^{lpha}{}_{eta} & 0 & 0 \\ 0 & 0 & -i \end{pmatrix} \;,$$

[I.B.+Lukierski 98]. Here $\Xi_{\Sigma\Omega}$ is the OSp(1|2n) 'metric' Υ^{Σ} is orthosymplectic supertwistor

• carrying the index of the fundamental representation of OSp(1|2n).

• Thus
$$S = \int_{W^1} \lambda_{\alpha} \lambda_{\beta} \hat{\Pi}^{\alpha\beta} = \int_{W^1} d\Upsilon^{\Sigma} \Xi_{\Sigma\Omega} \Upsilon^{\Omega}$$
 is manifestly $OSp(1|2n)$ invariant.

Intro	Tensorial SSP 0000	Hspin eqs in TSSP 0000000	OSp(1 n) as AdS-TSSP 00000	κ and <i>OSp</i> (1 2 <i>n</i>) ○○○●	SUGRA in TSSP	HSpin eqs in extended TSSPs 00000	Concl.
OSp(1	1 2n) symmetry						

OSp(1|2n) symmetry of OSp(1|n) superparticle from GL flatness of OSp(1|n) supergroup manifold

• The simplest way to show the OSp(1|2n) symmetry of OSp(1|n)superparticle $S = \int_{W^1} \lambda_{\alpha} \lambda_{\beta} \hat{\mathcal{E}}^{\alpha\beta}$ is to use the GL(n) flatness of OSp(1|n)

• [Plyushchay, Sorokin, Tsulaia 2003]:

$$\begin{split} \overline{\mathcal{E}^{\alpha\beta} &= \Pi^{\alpha\beta} \, \mathcal{G}_{\gamma}^{\ \alpha}(X,\theta) \, \mathcal{G}_{\delta}^{\ \beta}(X,\theta)} \ , \qquad \mathcal{E}^{\alpha} &= e^{\rho(x,\theta)} (\mathcal{D}\theta^{\alpha} - \theta^{\alpha} \mathcal{D}\rho) \\ \mathcal{G}_{\beta}^{\ \alpha}(x,\theta) &= \mathcal{G}_{\beta}^{\ \alpha}(x) - \frac{i_{\varsigma}}{4} \left(\Theta_{\beta} - 2 \mathcal{G}_{\beta}^{\ \gamma}(x) \Theta_{\gamma} \right) \Theta^{\alpha}, \\ \mathcal{G}_{\beta}^{-1\alpha}(x) &= \delta_{\beta}^{\ \alpha} + \frac{\varsigma}{2} x_{\beta}^{\ \alpha}, \\ \theta^{\alpha} &= \Theta^{\beta} \, \mathcal{G}_{\beta}^{-1\alpha}(x) e^{-\rho(\Theta)}, \qquad e^{\rho(\Theta)} = \sqrt{1 + \frac{i_{\varsigma}}{4} \Theta^{\beta} \Theta_{\beta}}, \\ \mathcal{D}\theta^{\alpha} &= d\theta^{\alpha} - \varsigma \theta^{\gamma} \mathcal{C}_{\gamma\beta} \mathcal{E}^{\beta\alpha}(X,0) = d\theta^{\alpha} - \varsigma \theta^{\gamma} \mathcal{C}_{\gamma\beta} (\mathcal{G}^{T} X \mathcal{G})^{\beta\alpha} \ . \end{split}$$
Hence $S = \int_{W^{1}} \lambda_{\alpha} \lambda_{\beta} \hat{\mathcal{E}}^{\alpha\beta} = \int_{W^{1}} \tilde{\lambda}_{\alpha} \tilde{\lambda}_{\beta} \hat{\Pi}^{\alpha\beta} \text{ with } \tilde{\lambda}_{\alpha} = \mathcal{G}_{\alpha}^{\ \beta}(X,\theta) \lambda_{\beta} \\ \bullet \text{ and } OSp(1|2n) \text{ superconformal invariance of the } OSp(1|n) \text{ superparticle follows from the } OSp(1|2n) \text{ superconformal invariance of the } \Sigma^{(\frac{n(n+1)}{2}|n)} \\ \bullet \text{ superparticle.} \end{split}$

	Tensorial SSP 0000	Hspin eqs in TSSP	OSp(1 n) as AdS-TSSP 00000	к and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl
Outl	ine						
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- OSp(1|2n) symmetry of Σ^{(n(n+1)/2(n)} and OSp(1|n) superparticles
 Searching for the interacting theory: Supergravity in tensorial superspace
 - Preonic superparticle and SUGRA constraints in $\mathcal{M}^{(\frac{n(n+1)}{2}|n)}$
 - Supergravity in tensorial superspace
 - Higher spin equations in extended tensorial superspaces

Intro	Tensorial SSP 0000	Hspin eqs in TSSP	OSp(1 n) as AdS-TSSP 00000	к and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP ●OOOOOO	HSpin eqs in extended TSSPs 00000	Concl.			
SUGR	SUGRA constraints in curved tensorial SSP									

Geometry of curved tensorial superspace

- Tensorial supergravity, a theory dynamical curved tensorial superspace $\mathcal{M}^{(\frac{n(n+1)}{2}|n)}$ is a natural candidate for interacting higher spin theory.
- Z^M = (X^{ă,β}, θ^š)= are local coordinates of M^{(n(n+1)/2|n)}. We construct the theory from the objects which are invariant under superdiffeomorphisms, Z'^M = f^M(Z^N): (sdet(∂f^M/∂Z^N) ≠ 0):
- Supervielbein one forms $E^{\mathcal{A}} = (E^{\alpha\beta}, E^{\alpha}) = dZ^{M}E^{\mathcal{A}}_{M}(Z)$

$$E^{\alpha\beta}(Z) = E^{\beta\alpha}(Z) = dZ^M E_M^{\ \alpha\beta}(Z) , \qquad E^{\alpha}(Z) = dZ^M E_M^{\ \alpha}(Z)$$

• And GL(n) connection $\Omega_{\beta}{}^{\alpha} := dZ^{M}\Omega_{M\beta}{}^{\alpha} \equiv E^{\mathcal{A}}\Omega_{\mathcal{A}\beta}{}^{\alpha}$,

• The torsion and GL(n) curvature 2-forms

$$T^{\alpha\beta} := \mathcal{D}E^{\alpha\beta} \equiv dE^{\alpha\beta} - 2E^{(\alpha|\gamma} \wedge \Omega_{\gamma}{}^{|\beta)} =: \frac{1}{2}E^{\mathcal{B}} \wedge E^{\mathcal{A}} T_{\mathcal{AB}}{}^{\alpha\beta} ,$$

$$T^{\alpha} := \mathcal{D}E^{\alpha} \equiv dE^{\alpha} - E^{\beta} \wedge \Omega_{\beta}{}^{\alpha} =: \frac{1}{2}E^{\mathcal{B}} \wedge E^{\mathcal{A}} T_{\mathcal{AB}}{}^{\alpha} ,$$

$$\mathcal{R}_{\beta}{}^{\alpha} := d\Omega_{\beta}{}^{\alpha} - \Omega_{\beta}{}^{\gamma} \wedge \Omega_{\gamma}{}^{\alpha} =: \frac{1}{2}E^{\mathcal{B}} \wedge E^{\mathcal{A}} \mathcal{R}_{\mathcal{AB}}{}^{\alpha} .$$

Superparticle in curved tensorial superspace

- As in the case of SUGRA in usual SSP we need to restrict the supervielbein and connection by superspace constriants.
- Their essential part can be obtained by the condition of preservation of the κ-symmetry of the preonic superparticle in curved tensorial SSP [I.B., Pasti, Sorokin, Tonin JHEP 2004]
- Its action $S = \int\limits_{W^1} \lambda_{lpha} \lambda_{eta} \hat{E}^{lphaeta}$ possesses the κ -symmetry

$$\begin{split} \delta_{\kappa} \hat{Z}^{M} E^{\alpha \alpha'}_{M}(\hat{Z}) &= 0 , \quad \delta_{\kappa} \lambda_{\alpha} = 0 , \qquad \delta_{\kappa} Z^{M} E^{\alpha}_{M}(\hat{Z}) \lambda_{\alpha} = 0 , \quad \Leftrightarrow \\ \Leftrightarrow \qquad \delta_{\kappa} \hat{Z}^{M} E^{\alpha}_{M}(\hat{Z}) &= \mu^{\alpha I} \kappa_{I}(\tau) , \quad I = 1, .., 15, \quad \mu^{\alpha I} \lambda_{\alpha} = 0 \end{split}$$

provided supervielbein is restricted by torsion constraints

$$T^{lphaeta} = -i E^{lpha} \wedge E^{eta} - 2 E^{(lpha} \wedge E^{eta)\gamma} t_{\gamma}(Z) + 2 E^{\gamma(lpha} \wedge E^{eta)\delta} R_{\gamma\delta}(Z) \;,$$

with some fermionic $t_{\gamma}(Z)$ and bosonic $R_{\gamma\delta}(Z) = -R_{\delta\gamma}(Z)$.

• As usually, the theory is still reducible and we need to impose also a number of conventional constraints, counterparts of $T_{cb}{}^a = 0$ in General Relativity. One of this can be $t_{\gamma}(Z) = 0$, but there are a number of others.

Intro Tensorial SSP Hspin eqs in TSSP OSp(11)n) as AdS-TSSP r and OSp(112n) SUGRA in TSSP Hspin eqs in extended TSSPs Concl.

SUGRA constraints and their consequences

 After imposing the essential and conventional constraints and studying their consistency conditions given by Bianchi identities

$$egin{array}{lll} \mathcal{D}T^{lphaeta}+E^{lpha\gamma}\wedge\mathcal{R}_{\gamma}{}^{eta}+E^{eta\gamma}\wedge\mathcal{R}_{\gamma}{}^{lpha}&\equiv&0\ ,\ \mathcal{D}T^{lpha}+E^{eta}\wedge R_{eta}{}^{lpha}&\equiv&0\ ,\ \mathcal{D}\mathcal{R}_{eta}{}^{lpha}&\equiv&0\ \end{array}$$

the torsion and curvature 2-forms are expressed by

$$\begin{array}{lll} T^{\alpha\beta} &=& -i E^{\alpha} \wedge E^{\beta} + 2 E^{\gamma(\alpha} \wedge E^{\beta)\delta} R_{\gamma\delta}(Z) \;, \\ T^{\alpha} &=& 2 E^{\alpha\beta} \wedge E^{\gamma} R_{\beta\gamma} + E^{\alpha\beta} \wedge E^{\gamma\delta} U_{\beta\gamma\delta} \;, \\ \mathcal{R}_{\beta}^{\alpha} &=& i E^{\gamma\delta} \wedge E^{\alpha} U_{\beta\gamma\delta} - E^{\alpha\gamma} \wedge E^{\delta} (F_{\delta\beta\gamma} + \mathcal{D}_{\delta} R_{\beta\gamma}) - \\ &-& E^{\alpha\gamma} \wedge E^{\delta\epsilon} (\mathcal{D}_{(\beta} U_{\gamma)\delta\epsilon} + \mathcal{D}_{\delta\epsilon} R_{\beta\gamma}) \;. \end{array}$$

• in terms of 'main superfields' $R_{\beta\alpha} = -R_{\alpha\beta}$, $U_{\beta\gamma\delta}U_{\beta(\gamma\delta)}$ and $F_{\alpha\beta\gamma} = 2iU_{(\beta\gamma)\alpha} - iU_{\alpha\beta\gamma} - 2D_{(\beta}R_{\gamma)\alpha}$ which obey a number of relations

$$\mathcal{D}_{[\alpha} U_{\beta]\gamma\delta} = -\mathcal{D}_{\gamma\delta} R_{\alpha\beta} , \quad \mathcal{D}_{(\alpha} U_{\beta)\gamma\delta} = -i\mathcal{D}_{(\gamma} F_{\delta)\alpha\beta}$$

 $\mathcal{D}_{\alpha\beta} U_{\gamma\delta\sigma} - \mathcal{D}_{\delta\sigma} U_{\gamma\alpha\beta} + 2U_{\gamma\alpha(\sigma} R_{\delta)\beta} + 2U_{\gamma\beta(\sigma} R_{\delta)\alpha} = 0 ,$

Intro	Tensorial SSP 0000	Hspin eqs in TSSP	OSp(1 n) as AdS-TSSP 00000	к and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP	HSpin eqs in extended TSSPs 00000	Concl.		
Supergravity in tensorial superspace									

SUGRA constraints and their sulutions

• The constraints $T^{\alpha\beta} = -iE^{\alpha} \wedge E^{\beta} + 2E^{\gamma(\alpha} \wedge E^{\beta)\delta}R_{\gamma\delta}(Z)$,

$$egin{array}{rcl} T^lpha&=&2E^{lphaeta}\wedge E^\gamma R_{eta\gamma}+E^{lphaeta}\wedge E^{\gamma\delta}U_{eta\gamma\delta}\ ,\ {\cal R}_{eta}{}^lpha&=&iE^{\gamma\delta}\wedge E^lpha U_{eta\gamma\delta}-E^{lpha\gamma}\wedge E^\delta(F_{\deltaeta\gamma}+{\cal D}_\delta R_{eta\gamma})-\ &-&E^{lpha\gamma}\wedge E^{\delta\epsilon}({\cal D}_{(eta}U_{\gamma)\delta\epsilon}+{\cal D}_{\delta\epsilon}R_{eta\gamma})\ . \end{array}$$

• have $\Sigma^{(\frac{n(n+1)}{2}|n)}$ solution: $R_{\gamma\delta} = 0$ and $U_{\gamma\delta\epsilon} = 0$ (} $\Rightarrow F_{\gamma\delta\epsilon} = 0$).

Setting R_{αβ} = -^c/₂C_{αβ} and U_{αβγ}(Z) = 0 we find R_α^β = 0 ⇒ we can gauge away GL(n) connection (Ω_α^β = 0) and arrive at

$$\begin{aligned} d\mathcal{E}^{\alpha\beta} &= -i\mathcal{E}^{\alpha}\wedge\mathcal{E}^{\beta}-\zeta\mathcal{E}^{\alpha\gamma}\wedge\mathcal{E}^{\delta\beta}\mathcal{C}_{\gamma\delta} , \\ d\mathcal{E}^{\alpha} &= -\zeta\mathcal{E}^{\alpha\gamma}\wedge\mathcal{E}^{\delta}\mathcal{C}_{\gamma\delta} . \end{aligned}$$

which are the Maurer–Cartan eqs. for OSp(1|n).

 Thus OSp(1|n) supergroup manifold is also a solution of the TSSP SUGRA constraints.

	Tensorial SSP	Hspin eqs in TSSP	OSp(1 n) as AdS-TSSP	к and <i>OSp</i> (1 2 <i>n</i>)	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.
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Supero	ravity in tensorial	superspace					

Scalar superfield eq. in SUGRA background and reduction of the holonomy group to $\ensuremath{\mathsf{SL}}(n)$

- A natural generalization of the free superfield equations for higher spin fields to curved TSSP is $\mathcal{D}_{[\alpha}\mathcal{D}_{\beta]}\Phi = \frac{i}{2}R_{\alpha\beta}\Phi$.
- Its integrability condition results in a quite complicated equation
- the known solution of which reduces the holonomy group from $GL(n, \mathbb{R})$ to $SL(n, \mathbb{R})$ i.e. implies $\mathcal{R}_{\alpha}^{\ \alpha} = 0$.
- Such a reduction simplifies a bit equations for main superfields,
- but also makes possible to prove [I.B., Pasti, Sorokin, Tonin 2004]:
- the general solution of supergravity constraints is superconformally equivalent either to OSp(1|n) or to the flat Σ^{(n(n+1)/2)}/2).
- Namely, they can be obtained by

$$\begin{aligned} E^{\prime\alpha\beta} &= E^{\alpha\beta}, \quad E^{\prime\alpha} = E^{\alpha} + E^{\alpha\beta} W_{\beta} \\ \Omega^{\prime\alpha}_{\beta} &= \Omega_{\beta}^{\ \alpha} - i E^{\alpha} W_{\beta} - E^{\alpha\gamma} (\mathcal{D}_{\gamma} W_{\beta} + i W_{\gamma} W_{\beta}) \end{aligned}$$

with $W_{\beta} = -i\mathcal{D}_{\beta}W$ (and GL(n) gauge transformations, $e^{kW}\delta_{\alpha}{}^{\beta}$, if convenient) from flat or OSp supervielbein and (trivial) GL(n) connection.

Intro Tensorial SSP Hspin eqs in TSSP OSp(1)n) as AdS-TSSP r and OSp(1)2n) SUGRA in TSSP Hspin eqs in extended TSSPs Concl.

Solution of SL(n) SUGRA in TSSP are superconformally OSp or superconformally flat

• The general solution for the main superfields

$$\begin{aligned} \boldsymbol{R}_{\alpha\beta} &= i \, \boldsymbol{e}^{-\frac{2W}{n}} \left[i \frac{5}{2} \boldsymbol{C}_{\alpha\beta} + \nabla_{[\alpha} \nabla_{\beta]} \, \boldsymbol{W} + \frac{1}{2} \, \nabla_{\alpha} \, \boldsymbol{W} \, \nabla_{\beta} \, \boldsymbol{W} \right] \\ \boldsymbol{U}_{\beta\gamma\delta} &= \boldsymbol{e}^{-\frac{3W}{n}} \left[-i \nabla_{\gamma\delta} \nabla_{\beta} \, \boldsymbol{W} + \nabla_{(\gamma} \, \boldsymbol{W} \, \nabla_{\delta)} \nabla_{\beta} \, \boldsymbol{W} \right] \,. \end{aligned}$$

• Note: the original OSp solution, $R_{\alpha\beta} = -\frac{5}{2}C_{\alpha\beta}$ and $U_{\alpha\beta\gamma}(Z) = 0$ is preserved by superWeyl (supplemented by certain GL(n) gauge) transformations if the superfield parameter *W* obeys

$$abla_{[lpha}
abla_{eta]} W + rac{1}{2}
abla_{lpha} W
abla_{eta} W = -rac{i_{\varsigma}}{2} C_{lphaeta} \left(1 - e^{-rac{W}{2}}
ight)$$

 which is an equivalent form of the free eqs, for the free higher spin fields in AdS: W = 2 ln (Φ+a) where a = const > 0 and Φ(X, θ) obeys

$$\left(
abla_{[lpha}
abla_{eta]} + i rac{\varsigma}{4} C_{lpha eta}
ight) \Phi(X, heta) = \mathbf{0} \; .$$

 However, the conclusion is that supergravity in tensorial superspace is super-Weyl trivial: It does not describe dynamical potentials of higher spin supergravity and describes, at most, the free higher spin eqs.

Intro	Tensorial SSP 0000	Hspin eqs in TSSP	OSp(1 n) as AdS-TSSP 00000	к and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP ○○○○○○●	HSpin eqs in extended TSSPs	Concl.			
Super	Supergravity in tensorial superspace									

super-Weyl triviality of SUGRA in TSSP and possible ways out

- Supergravity in tensorial superspace, as it has been formulated, is super-Weyl trivial ⇒ It does not describe dynamical potentials of higher spin SUGRA and describes, at most, the free higher spin eqs.
- Some deformation of the theory or introduction of new elements are needed to continue the search for interacting HSpin theory on this basis.
- Ex.: current project with Dima Sorokin and Per Sundel. To start form an a-deformed 4D tensorial superparticle [I.B., J. Lukierski and D. Sorokin 1999]. SUGRA constraints from preservation of its 2 (not 3) κ–symmetries.
- Other basis to construct interacting higher spin theories in tensorial SSP. Tensorial SYM?
- YM field appears in the multiplets of extended SUSY. ⇒ some interest superfield theories in extended TSSP. These were studied in [I.B., J. de Azcárraga, C. Meliveo, 2011].

	Tensorial SSP 0000	Hspin eqs in TSSP 0000000	<i>OSp</i> (1 <i>n</i>) as AdS-TSSP 00000	к and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP 0000000	HSpin eqs in extended TSSPs	Concl.
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- Supergravity in tensorial superspace
- Higher spin equations in extended tensorial superspaces

 Intro
 Tensorial SSP
 Hspin eqs in TSSP
 OSp(1|n) as AdS-TSSP
 κ and OSp(1|2n)
 SUGRA in TSSP
 HSpin eqs in extended TSSPs
 Concl.

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HSpin equations in \mathcal{N} -extended tensorial superspaces

Higher spin equations in extended tensorial superspaces

$$\Sigma^{\left(\frac{n(n+1)}{2}|\mathcal{N}n\right)} = \{Z^M\} = \{(X^{\alpha\beta}, \theta^{\alpha I})\}, \quad \begin{cases} \alpha, \beta = 1, ..., n, \\ I = 1, ..., N \end{cases}$$

for even \mathcal{N} : [I.B., J. de Azcárraga, C. Meliveo, 2011]

is convenient to write in terms of complex fermionic coordinates

$$\begin{split} \Theta^{\alpha q} &= \frac{1}{2} (\theta^{\alpha q} - i \theta^{\alpha (q + \mathcal{N}/2)}) = (\bar{\Theta}^{\alpha}_{q})^{*} , \qquad q = 1, \dots, \mathcal{N}/2 , \\ \partial_{\alpha q} &:= \frac{\partial}{\partial \Theta^{\alpha q}} = \frac{\partial}{\partial \theta^{\alpha q}} + i \frac{\partial}{\partial \theta^{\alpha (q + \mathcal{N}/2)}} , \end{split}$$

and complex fermionic covariant derivatives,

$$\begin{aligned} \mathcal{D}_{\alpha q} &= \partial_{\alpha q} + 2i \partial_{\alpha \beta} \bar{\Theta}_{q}^{\beta} , \quad \bar{\mathcal{D}}_{\alpha}{}^{q} = \bar{\partial}_{\alpha}{}^{q} + 2i \partial_{\alpha \beta} \Theta^{\beta q} , \quad \partial_{\alpha q} := \frac{\partial}{\partial \Theta^{\alpha \bar{q}}} , \\ \hline \left\{ \mathcal{D}_{\alpha q}, \bar{\mathcal{D}}_{\beta}^{p} \right\} &= 4i \partial_{\alpha \beta} \delta_{q}^{p} \\ , \quad \left\{ \mathcal{D}_{\alpha q}, \mathcal{D}_{\beta}^{p} \right\} = \mathbf{0} = \left\{ \bar{\mathcal{D}}_{\alpha q}, \bar{\mathcal{D}}_{\beta}^{p} \right\} . \end{aligned}$$

The free higher spin equations with extended SUSY read

$$ar{\mathcal{D}}^q_{lpha} \Phi(X,\Theta^{q'},ar{\Theta}_{
ho'}) = 0 \;, \qquad \mathcal{D}_{q[eta}\mathcal{D}_{\gamma]
ho} \Phi(X,\Theta^{q'},ar{\Theta}_{
ho'}) = 0 \;.$$

Intro	Tensorial SSP	Hspin eqs in TSSP	OSp(1 n) as AdS-TSSP	κ and $OSp(1 2n)$	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.
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HSpin equations with $\mathcal{N}=2$ -extended supersymmetry

The superfield equations

$$\bar{\mathcal{D}}^q_{\alpha} \Phi(X, \Theta, \bar{\Theta}) = 0 , \qquad \mathcal{D}_{q[\beta} \mathcal{D}_{\gamma]p} \Phi(X, \Theta, \bar{\Theta}) = 0 .$$

• For $\mathcal{N} = 2$: \Rightarrow $\Phi(X, \Theta, \overline{\Theta}) = \phi(X_L) + i\Theta^{\alpha}\psi_{\alpha}(X_L)$ with $X_L^{\alpha\beta} = X^{\alpha\beta} + 2i\Theta^{(\alpha}\overline{\Theta}^{\beta)}$ (notice that n=4 for D=4) and $\partial_{\alpha[\gamma}\partial_{\delta]\beta}\phi(X) = 0$, $\partial_{\alpha[\beta}\psi_{\gamma]}(X) = 0$.

Tensorial SSP	Hspin eqs in TSSP	OSp(1 n) as AdS-TSSP	κ and $OSp(1 2n)$	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.
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HSpin equations with $\mathcal{N} = 4$ -extended supersymmetry

The superfield equations

$$ar{\mathcal{D}}^q_{\alpha} \Phi(X,\Theta,ar{\Theta}) = \mathbf{0} \;, \qquad \mathcal{D}_{q[\beta} \mathcal{D}_{\gamma]p} \Phi(X,\Theta,ar{\Theta}) = \mathbf{0} \;.$$

• For
$$\mathcal{N} = \mathbf{4}$$
: $\Rightarrow \\ \Phi(X, \Theta^q, \Theta_q) = \phi(X_L) + i\Theta^{\alpha q}\psi_{\alpha q}(X_L) + \epsilon_{pq}\Theta^{\alpha q}\Theta^{\beta p}\mathcal{F}_{\alpha \beta}(X_L) \\ \partial_{\alpha[\gamma}\partial_{\delta]\beta}\phi(X) = \mathbf{0} , \qquad \partial_{\alpha[\beta}\psi_{\gamma]q}(X) = \mathbf{0} ,$

and
$$\mathcal{F}_{\alpha\beta} = \mathcal{F}_{\beta\alpha}$$
 obeying $\partial_{\alpha[\gamma}\mathcal{F}_{\delta]\beta}(X) = 0$

 It might seem that we have found a tensorial superspace counterparts of the usual Maxwell equations and Bianchi identities

$$\partial_{\dot{A}[B}F_{C]D} = 0$$
, $\partial_{A[\dot{B}}F_{\dot{C}]\dot{D}} = 0$. (1)

However, the general solution of this tensorial space equation is

$$\mathcal{F}_{\alpha\beta} = \partial_{\alpha\beta} \tilde{\phi}(X) , \qquad \partial_{\alpha[\gamma} \partial_{\delta]\beta} \tilde{\phi}(X) = \mathbf{0} .$$

• Peccei-Quinn-like symmetry acting on the second scalar field $\tilde{\phi}(X)$: $\tilde{\phi}(X) \mapsto \tilde{\phi}(X) + const$.

Intro	Tensorial SSP	Hspin eqs in TSSP	OSp(1 n) as AdS-TSSP	κ and $OSp(1 2n)$	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.
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$\mathcal{N}>$ 4. HSpin equations with $\mathcal{N}=$ 8–extended supersymmetry

• In the generic case $\bar{\mathcal{D}}^{q}_{\alpha}\Phi(X,\Theta,\bar{\Theta}) = 0, \mathcal{D}_{q[\beta}\mathcal{D}_{\gamma]p}\Phi(X,\Theta,\bar{\Theta}) = 0 \} \Rightarrow$

$$\Phi(X,\Theta,\bar{\Theta}) = \phi(X_L) + i\Theta^{\alpha q}\psi_{\alpha q} + \sum_{k=2}^{N/2} \frac{1}{k!} \Theta^{\alpha_k q_k} \dots \Theta^{\alpha_1 q_1} \mathcal{F}_{\alpha_1 \dots \alpha_k q_1 \dots q_k}),$$

$$\mathcal{F}_{\alpha_1\dots\alpha_k} q_1\dots q_k(X_L) = \mathcal{F}_{(\alpha_1\dots\alpha_k)} [q_1\dots q_k](X_L) , \ X_L^{\alpha\beta} = X^{\alpha\beta} + 2i\Theta^{q(\alpha}\bar{\Theta}_q^{\beta)} ,$$

the higher components satisfy $\partial_{\alpha[\gamma} \mathcal{F}_{\delta]\beta_2...\beta_k} q_{1...q_k}(X_L) = 0$ which is solved in terms of derivatives of new scalar and spinor fields defined up to Peccei-Quinn-like symmetries.

 Intro
 Tensorial SSP
 Hspin eqs in TSSP
 OSp(1|n) as AdS-TSSP
 c and OSp(1|2n)
 SUGRA in TSSP
 HSpin eqs in extended TSSPs
 Concl.

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\mathcal{N} > 4. HSpin equations with \mathcal{N} = 8–extended supersymmetry

• $\bar{\mathcal{D}}^{q}_{\alpha}\Phi(X,\Theta,\bar{\Theta}) = 0$, $\mathcal{D}_{q[\beta}\mathcal{D}_{\gamma]\rho}\Phi(X,\Theta,\bar{\Theta}) = 0$ for $\mathcal{N} = 8$ is solved by $\Phi(X,\Theta,\bar{\Theta}) = \phi(X_{L}) + i\Theta^{\alpha q}\psi_{\alpha q} + \sum_{k=2}^{4} \frac{1}{k!}\Theta^{\alpha_{k}q_{k}}\dots\Theta^{\alpha_{1}q_{1}}\mathcal{F}_{\alpha_{1}\dots\alpha_{k}} {}_{q_{1}\dots q_{k}}(X_{L})$ the higher components $\mathcal{F}_{\alpha_{1}\dots\alpha_{k}} {}_{q_{1}\dots q_{k}}(X_{L}) = \mathcal{F}_{(\alpha_{1}\dots\alpha_{k})} {}_{[q_{1}\dots q_{k}]}$ satisfy $\partial_{\alpha[\gamma}\mathcal{F}_{\delta]\beta}{}_{q_{1}q_{2}}(X) = 0$, $\partial_{\alpha[\gamma}\psi_{\delta]\beta_{2}\beta_{3}}{}^{q}(X) = 0$, $\partial_{\alpha[\gamma}\mathcal{F}_{\delta]\beta_{2}\beta_{3}\beta_{4}}(X) = 0$. • which implies $\mathcal{F}_{\alpha\beta}{}_{q_{1}q_{2}}(X) = \partial_{\alpha\beta}\phi_{q_{1}q_{2}}(X)$,

$$\psi^q_{lpha_1lpha_2lpha_3}(X) = \partial_{(lpha_1lpha_2} ilde{\psi}^q_{lpha_3)}(X) \,, \quad \mathcal{F}_{lpha_1\dotslpha_4}(X) = \partial_{(lpha_1lpha_2} \partial_{lpha_3lpha_4)} ilde{\phi}(X) \,.$$

• so that the $\mathcal{N} = 8$ multiplet contains only scalar and s-vector fields

$$\begin{array}{l} \partial_{\alpha[\gamma}\partial_{\delta]\beta}\phi(X) = 0 \ , \qquad \partial_{\alpha[\beta}\psi_{\gamma]}(X) = 0 \ , \\ \partial_{\alpha[\gamma}\partial_{\delta]\beta}\phi_{qp}(X) = 0 \ , \qquad \partial_{\alpha[\beta}\tilde{\psi}_{\gamma]}{}^{q}(X) = 0 \ , \qquad \partial_{\alpha[\gamma}\partial_{\delta]\beta}\tilde{\phi}(X) = 0 \ . \end{array}$$

and defined up to a more complicated P-Q like symmetries:

$$\phi_{qp}(X) \mapsto \phi_{qp}(X) + a_{qp} , \qquad \tilde{\psi}_{\alpha}{}^{q}(X) \quad \mapsto \quad \tilde{\psi}_{\alpha}{}^{q}(X) + \beta_{\alpha}{}^{q} , \\ \tilde{\phi}(X) \quad \mapsto \quad \tilde{\phi}(X) + a + X^{lpha\beta} a_{lpha\beta} ,$$

	Tensorial SSP 0000	Hspin eqs in TSSP	OSp(1 n) as AdS-TSSP	к and <i>OSp</i> (1 2 <i>n</i>) 0000	SUGRA in TSSP	HSpin eqs in extended TSSPs	Concl.
Out	line						
	 Intro Flat 4[duction tensorial sup) Tensorial s	erspace $\Sigma^{(n(n+1))}$ uperspace $\Sigma^{(10)}$)/2 <i>n</i>) 4)			
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	5 Preo supe	nic propertie rparticle	is and $OSp(1 2)$	n) supercont	formal symr	metry of tensorial	

- κ symmetry and SUSY preserved by preonic BPS state
 OSp(1|2n) symmetry of Σ^{(n(n+1)/2|n)} and OSp(1|n) superparticles
- 6 Searching for the interacting theory: Supergravity in tensorial superspace
 - Preonic superparticle and SUGRA constraints in $\mathcal{M}^{(\frac{n(n+1)}{2}|n)}$
 - Supergravity in tensorial superspace
 - Higher spin equations in extended tensorial superspaces
 - Conclusions

Intro Tensorial SSP Hspin eqs in TSSP OSp(1|n) as AdS-TSSP κ and OSp(1|2n) SUGRA in TSSP HSpin eqs in extended TSSPs Concl. 0000 0000000 00000 000000 000000 000000 000000 000000 000000 000000 000000 000000 000000 000000 000000 000000 00000000 00000000 0000000</

$\mathcal{N} >$ 4. HSpin equations with $\mathcal{N} =$ 8–extended supersymmetry

- Tensorial superspace provides a beautiful basis to describe free conformal higher spin fields in D = 4, 6, 10.
- AdS Flat $\leftrightarrow OSp(1|n)$, flat $\leftrightarrow \Sigma^{(\frac{n(n+1)}{2}|n)}$. $(n = 4, 8, 16) D_{[\alpha}D_{\beta]}\Phi = 0$.
- (and also exotic BPS states in M-theory- BPS preons (n=32)).
- OSp(1|2n) superconformal symmetry (OSp(1|8) in D=4, OSp(1|32) in D=10, OSp(1|64) in D=11).
- The attempts to describe the HSpin interactions have not succeed (yet?)
- SUGRA in tensorial superspace, formulated with preservation of manifest GL(n) symmetry (SL(n) holonomy) was shown to be superconformally equivalent to either Σ^{(n(n+1)/2)n)} or OSp(1|n).
- A possible way is to search for a deformation which breaks (deforms) the *GL*(*n*) and *OSp*(1|2*n*) symmetry.
- Probably the suggestion will come from studies [Vasiliev, Gelfond, 10, 13] of the currents in Σ^{(n(n+1)/2) |n)} through the hypothetical
- tensorial AdS/CFT = $\Sigma^{(\frac{n(n+1)}{2}|n]} \leftrightarrow OSp(1|2n)$ correspondence.
- \Leftarrow Flat = $\Sigma^{(\frac{n(n+1)}{2}|n]}$. Its superconf. group = $OSp(1|2n) = AdS^{(n(2n+1)|2n)}$.