Critical Solitons in Gauge Theories ⇔ Srings /D branes/Dualities

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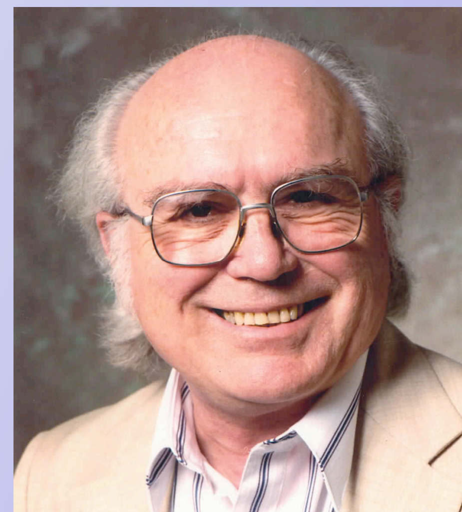
"New Directions Beyond the Standard Model in Field & String Theory" *

* The first GGI Workshop

A. Yung, ...

Can there be ANY symmetry between bosons and fermions? $\Longrightarrow$ 1970's: YES!

Golfand & Likhtman, 71

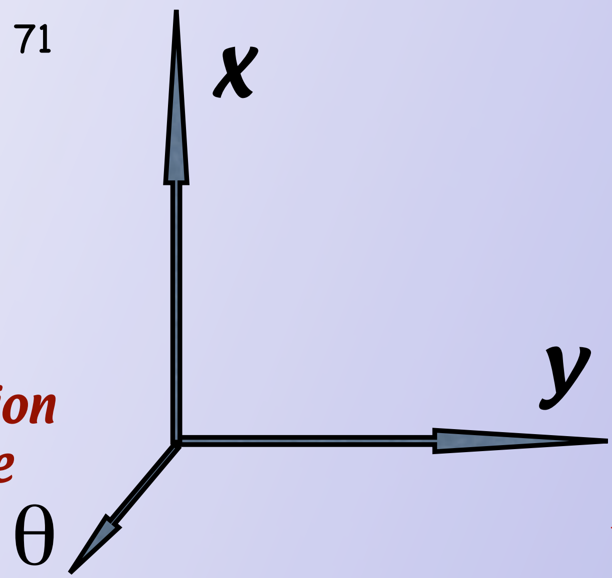


Wess & Zumino, 73

$$\theta^2 = 0$$

"fermion" direction of the superspace

$\longrightarrow$



In 1+3 dimensions

$$\{t, x, y, z\} \longrightarrow \{t, x, y, z;\ \theta^i_\alpha\}$$

E. Witten:

Supersymmetry, if it holds in nature, is part of the quantum structure of space and time. In everyday life, we measure space and time by numbers, “It is three o’clock, the elevation is ten meters,” and so on. Numbers are classical concepts, known to humans since long before quantum mechanics. The discovery of quantum mechanics changed our understanding of almost everything in physics, but our basic way of thinking about space and time has not yet been affected.

Showing that nature is supersymmetric would change that, by revealing a quantum dimension of space and time, not measurable by ordinary numbers. This quantum dimension would be manifested in the existence of new elementary particles, which would be produced in accelerators and whose behavior would be governed by supersymmetric laws.

$$\boxed{E = mc^2}$$

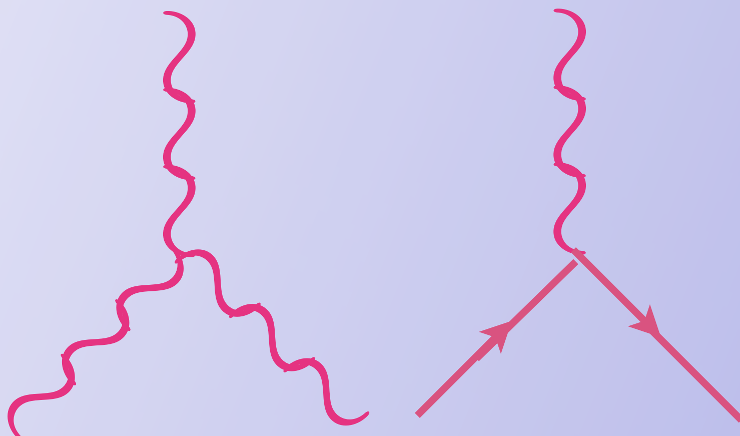
Cultural icon of the 20th century

$$\{\bar{Q}_{\dot{\alpha}}\,,\,Q_{\beta}\} = 2\,\sigma^{\mu}_{\dot{\alpha}\beta}\,P_{\mu} \longleftarrow \text{Of the 21st ?}$$

$$\mathcal{L} = -\frac{1}{4g^2} G^a_{\mu\nu} G^{\mu\nu\, a} + \frac{i}{2}\bar{\lambda} \not{D} \lambda$$

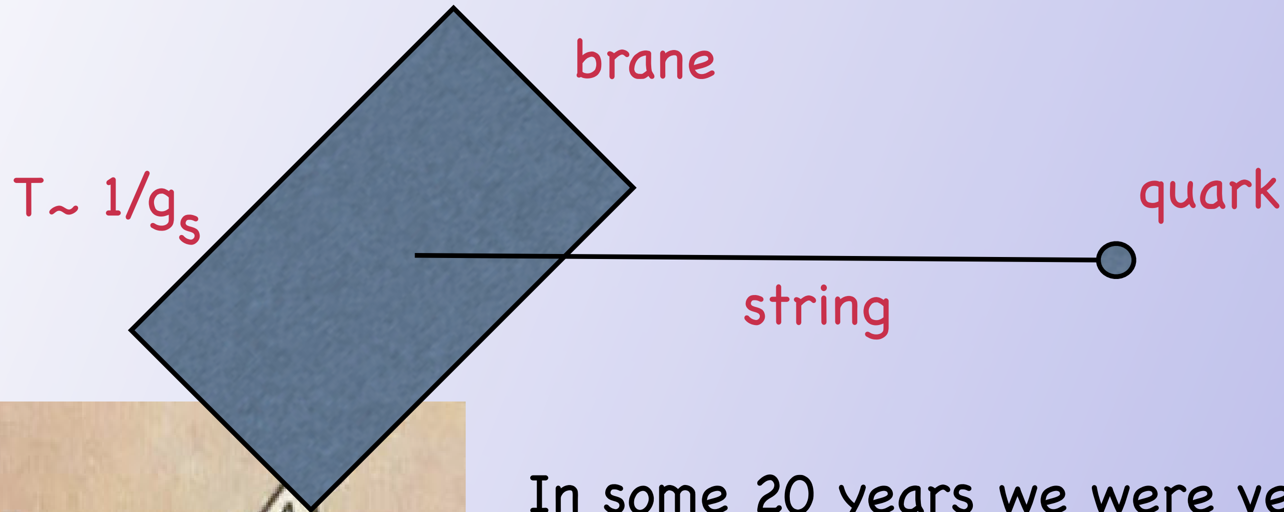
gluino

gluon



SUSY Yang–Mills

supersymmetric gluodynamics



In some 20 years we were very successful in producing a raw first draft of the world from string theory. It turned out to be notoriously difficult to pass to the second draft. This has not yet been done. -- E.Witten

Duality between ST & YM

↓ ↓ ↓

YM must support domain walls of D-brane type & non-Abelian strings ending on the walls and trapping flux sources !!!!

✶✶✶ Why do we need SUSY & "stringy" ideas? ✶✶✶

A tool for solving otherwise unsolvable problems of strong coupling dynamics

Costs nothing;
Enormous progress since mid-1990's!

▲▲▲▲▲▲▲▲▲

Comes at a price: not quite QCD, but close ...

- Critical = BPS saturated (Bogomol'nyi, Prasad, Sommerfeld)

  BEFORE SUSY

- Topological charges = central charges (Witten, Olive, 1976)

$$\boxed{\{Q,\, Q\} = P + C}$$

- If C=0, all Q's are broken.

  If $C \neq 0$, some Q's may survive!

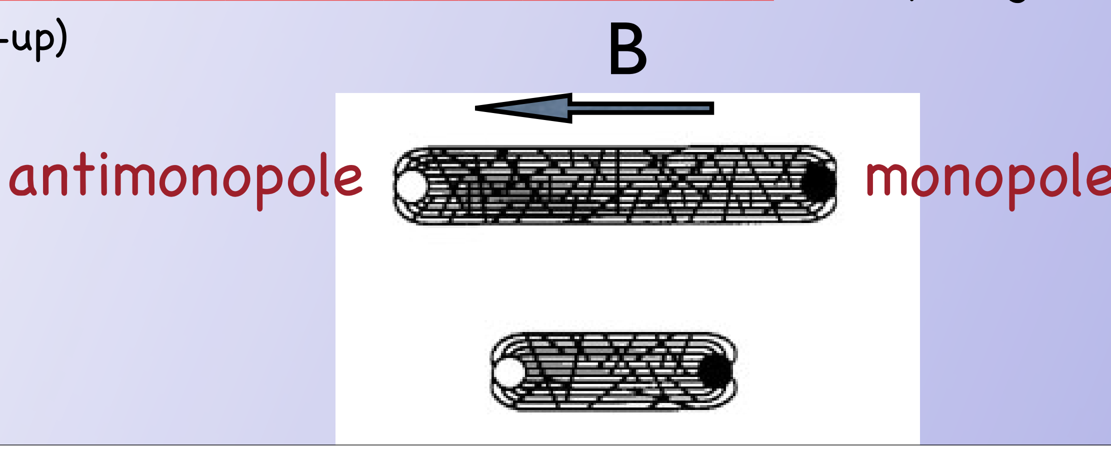
- 1/2 BPS, 1/4 BPS, ...   M (or T) $\equiv$ C

- In many instances C's are exactly calculable

✱✱✱ Non-Abelian Strings ✱✱✱

✳ Abrikosov-Nielsen-Olesen string: Abelian
Gauge group = U(1);
Electric charge condenses;
Magnetic flux is trapped in a tube and quantized;
No internal degrees of freedom besides position of the tube center (e.g. Seiberg-Witten solution ⇒ ANO string)

✳ Non-Abelian strings: Assume the BULK theory has a global symmetry G unbroken in the vacuum.
Assume G→H on the string;
Coset G/H of orientational moduli. (Hanany-Tong, nongauge; Auzzi et al. gauge set-up)



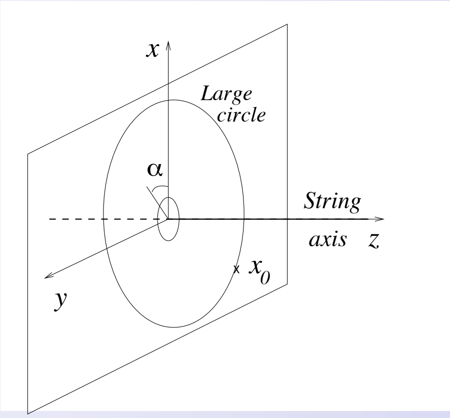
✷ Basic bulk theory: $\mathcal{N}=2$ SQCD with $U(N)_{gauge}$

and $N_f = N$ ✷

✷ Example: U(2), two flavors;
Parameters: $m_1 = m_2$, Fayet-Iliopoulos ξ

$$S = \int d^4x \left\{ \frac{1}{4g^2} F_{\mu\nu}^2 + \frac{1}{g^2} |\partial_\mu a|^2 + \bar{\nabla}_\mu \bar{q}_A \nabla_\mu q^A + \bar{\nabla}_\mu \tilde{q}_A \nabla_\mu \bar{\tilde{q}}^A \right.$$

$$\left. + \frac{g^2}{8} \left(|q^A|^2 - |\tilde{q}_A|^2 - \xi \right) + \frac{g^2}{2} \left| \tilde{q}_A q^A \right|^2 + \frac{1}{2} \left(|q^A|^2 + |\tilde{q}^A|^2 \right) \left| a + \sqrt{2} m_A \right|^2 \right\},$$



$$q^A{}_k = \sqrt{\xi/2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$e^{i\alpha}$

$SU(2) \rightarrow U(1)$

$$U(N)_{gauge} \times SU(N)_{flavor} \to SU(N)_{global}$$

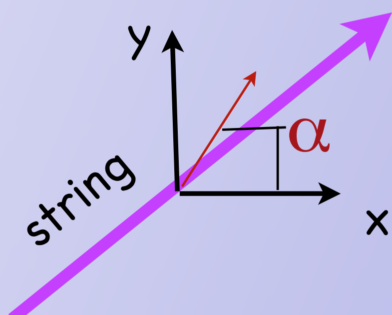
Color-flavor locked vacuum

✷ Weak coupling in the bulk ! $(\mathrm{If} \quad \xi \gg \Lambda^2)$

✷ Non-Abelian Strings:

$$\boxed{\pi_1\left[SU(N) \times U(1)/Z_N\right] \neq 0}$$

$$\Phi_{string} = \sqrt{\xi} \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & e^{i\alpha} \end{pmatrix}$$

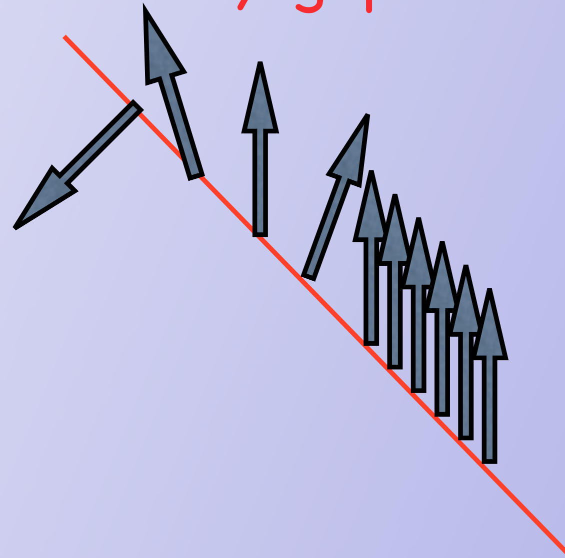


Flux =1/N Abrikosov

Tension =1/N Abrikosov

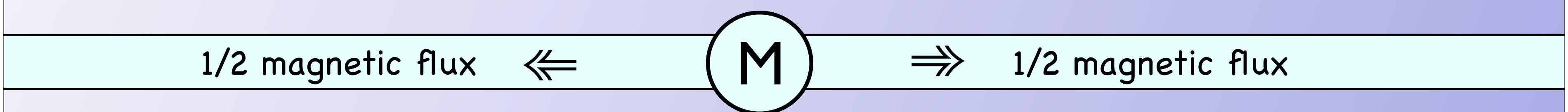
SU(2)/U(1) = CP(1)~O(3) sigma model

classically gapless excitation



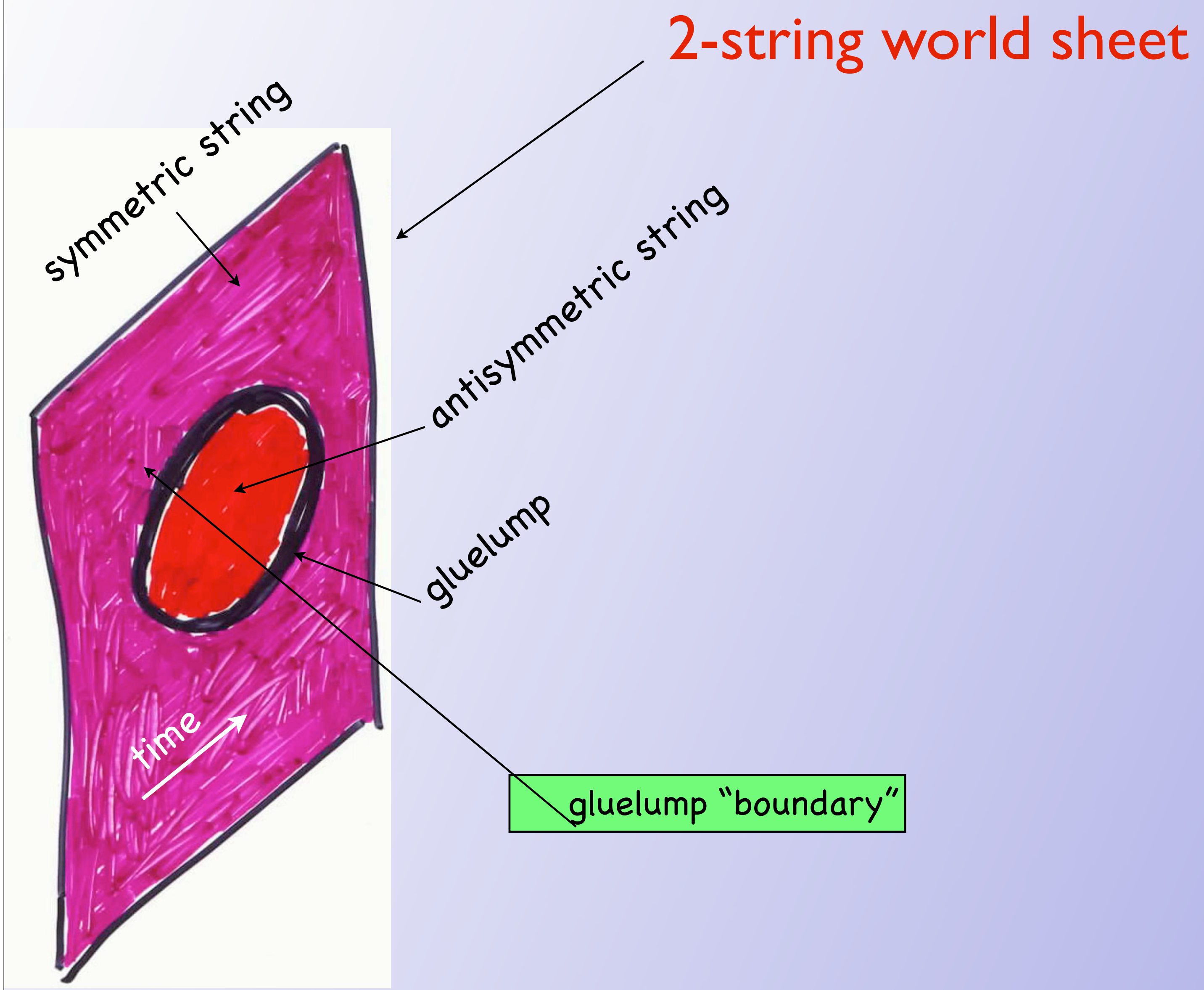
g^2 of the bulk theory is matched by g^2 of the 2D sigma model, and so do Λ's; 2D theory gets strongly coupled; mass gap generated; 2 vacua.

Kink = trapped monopole



What does that mean in the dual (QCD) language?

Gluelump (non-SUSY version):



✶✶✶ Branes/ Domain walls ✶✶✶

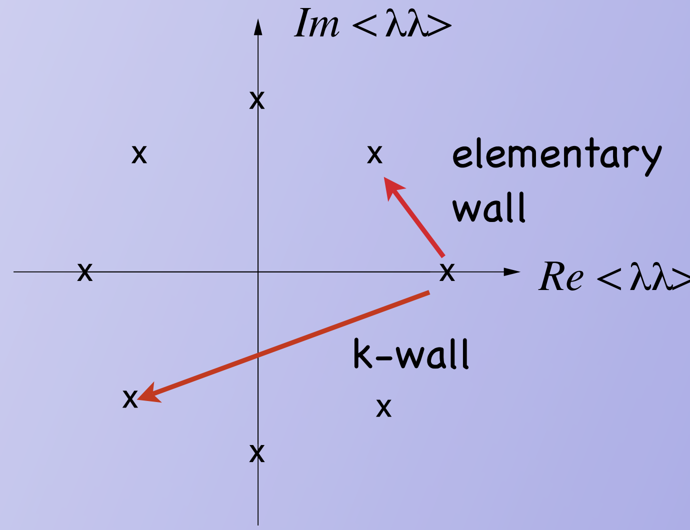
SUSY gluodynamics (1996, G. Dvali +MS)

✳ N vacua labeled by

$\langle\lambda\lambda\rangle = -6N\Lambda^3 \exp(2\pi i k/N)$

$\{Q_\alpha Q_\beta\} = \Sigma_{\alpha\beta}\, N(\Delta\langle\lambda\lambda\rangle)/8\pi^2$

quantum anomaly



N vacua for SU(N)

$T_{wall} = N\Lambda^3 \sim 1/g_s$ ⟵ D brane, Witten '97

Acharya & Vafa, from wrapped D brane + duality:

- World-volume theory = U(k) gauge theory (k=1 for elementary wall);
- Field content of $\mathcal{N} = 2$; Level-N Chern-Simons term breaks $\mathcal{N} = 2$ to $\mathcal{N} = 1$;
- # of distinct k-walls = N!/k!(N-k)!

Confirmed in field theory (Ritz, Vainshtein+MS)

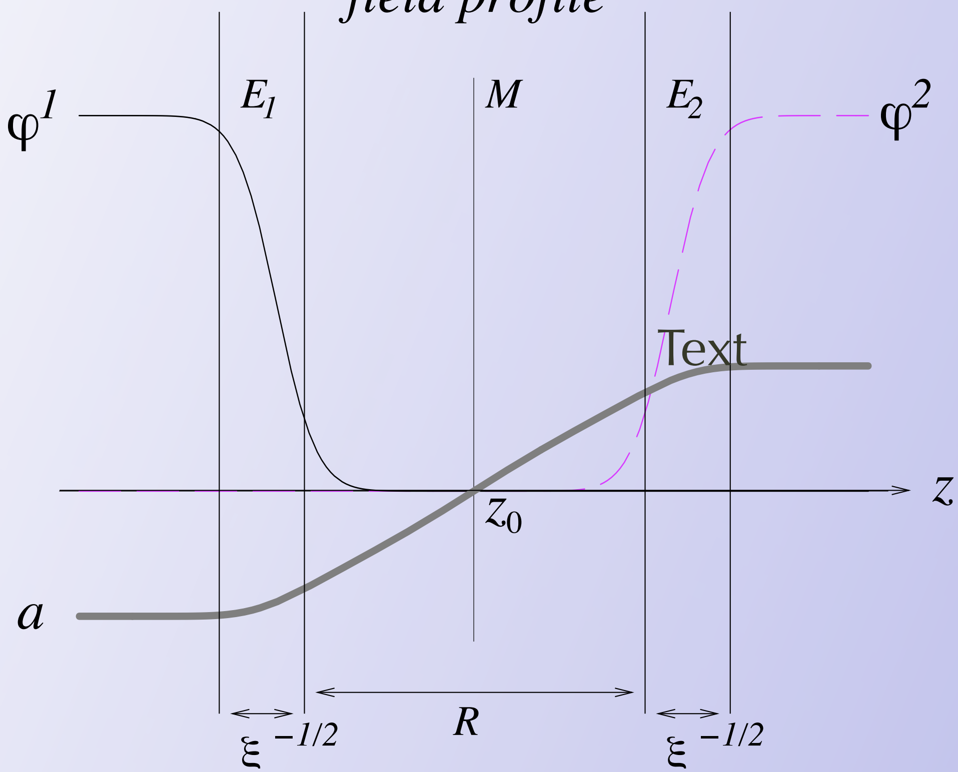
<u>Stack of k noninteract. walls **must** support U(k) gauge fields!</u>

Basic Elements of the Construction ($\mathcal{N}$=2 bulk):

Elementary Domain wall * ($m_1 \neq m_2$)

implementation of DS idea

field profile



Two edges (domains *E*) of the width ~ √1/ξ are separated by a broad middle band *M* of the width $R \sim \Delta m/(g^2\ \xi)$.

The tension $T = \Delta m\ \xi$

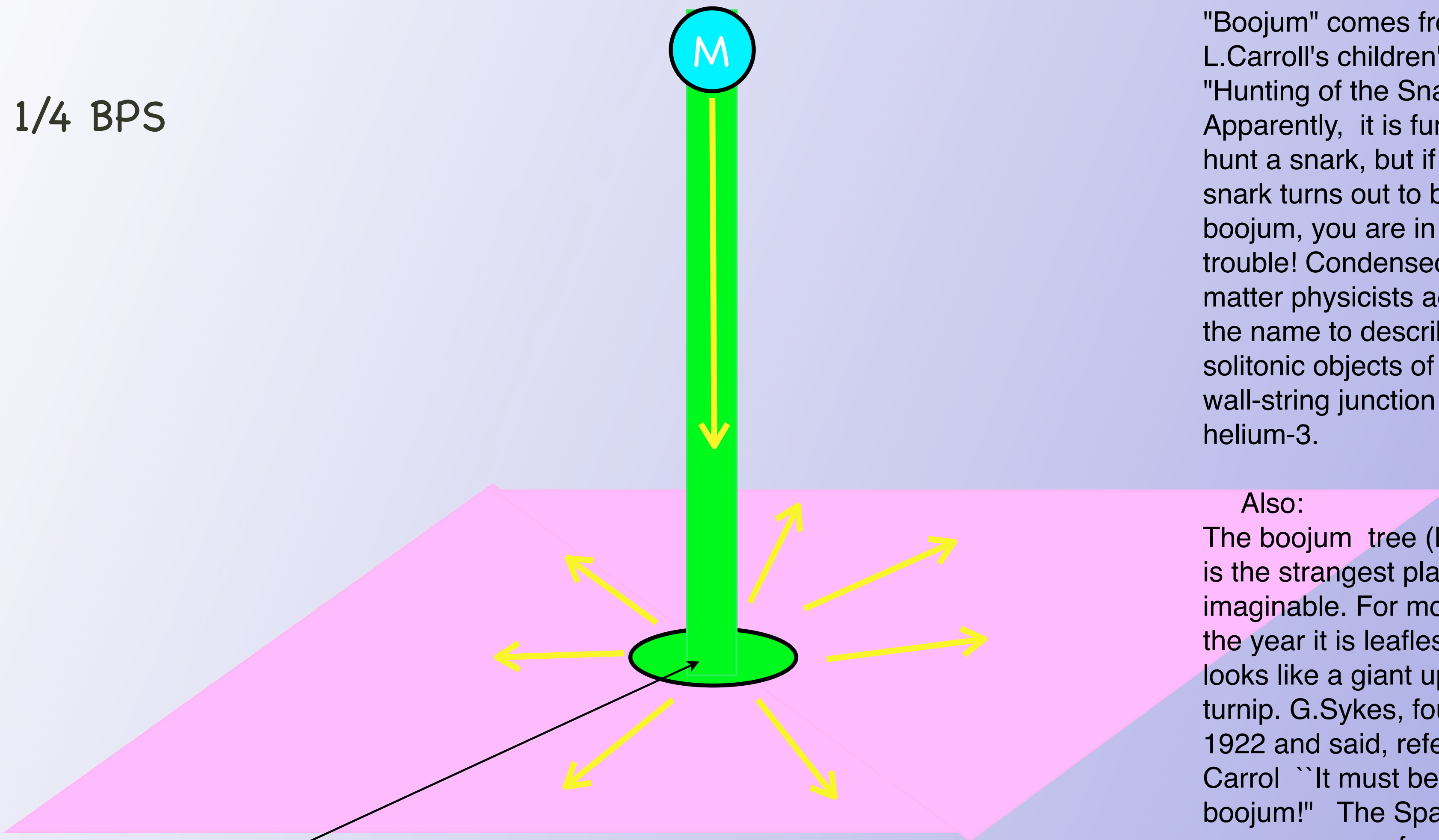
Moduli:

z_0 and σ ⇐ relative phase between ϕ^1 and ϕ^2

σ dualizes 3D photon a la Polyakov

(boojums) Wall-string junctions **(SY,ST,ASY)

1/4 BPS



Monopole=dual charge

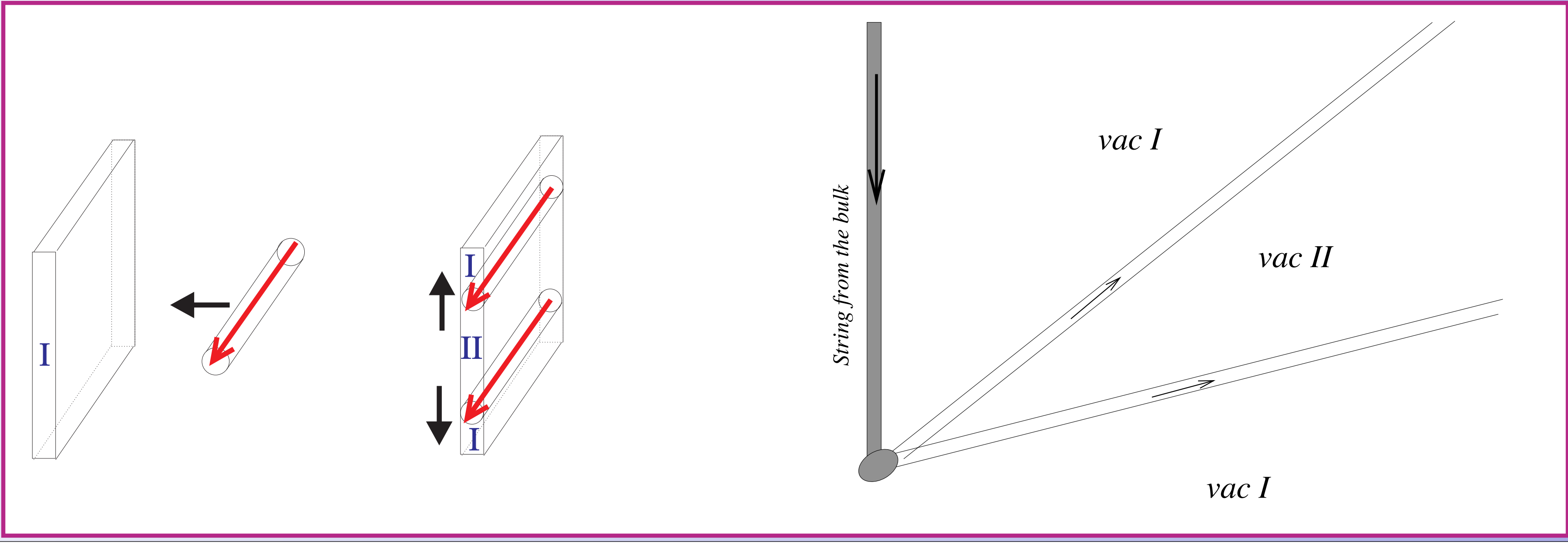
"Boojum" comes from L.Carroll's children's book "Hunting of the Snark." Apparently, it is fun to hunt a snark, but if the snark turns out to be a boojum, you are in trouble! Condensed matter physicists adopted the name to describe solitonic objects of the wall-string junction type in helium-3.

Also:
The boojum tree (Mexico) is the strangest plant imaginable. For most of the year it is leafless and looks like a giant upturned turnip. G.Sykes, found it in 1922 and said, referring to Carrol ``It must be a boojum!" The Spanish common name for this tree is Cirio, referring to its candle-like appearance.

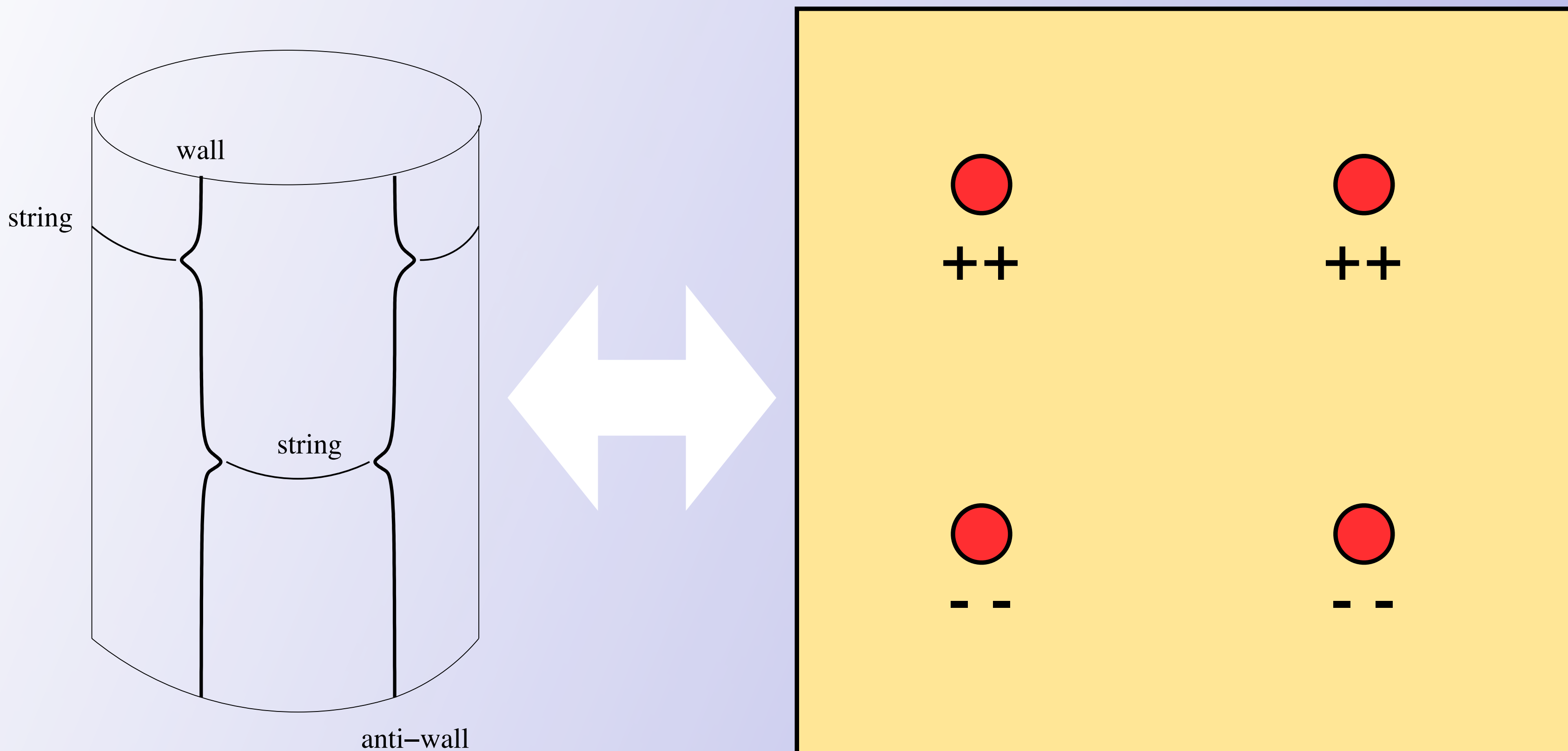
Polyakov's 3D confinement ***(ASY)

breaks $\mathcal{N}$ = 2 to $\mathcal{N}$ = 1

$$\Delta\mathcal{W} = \frac{\beta}{\sqrt{2}}\left(Q^1\tilde{Q}_2 - \tilde{Q}_1 Q^2\right) \longrightarrow V = \frac{\beta^2\xi}{m}\cos^2\sigma + O(\beta^3)$$



4D   3D

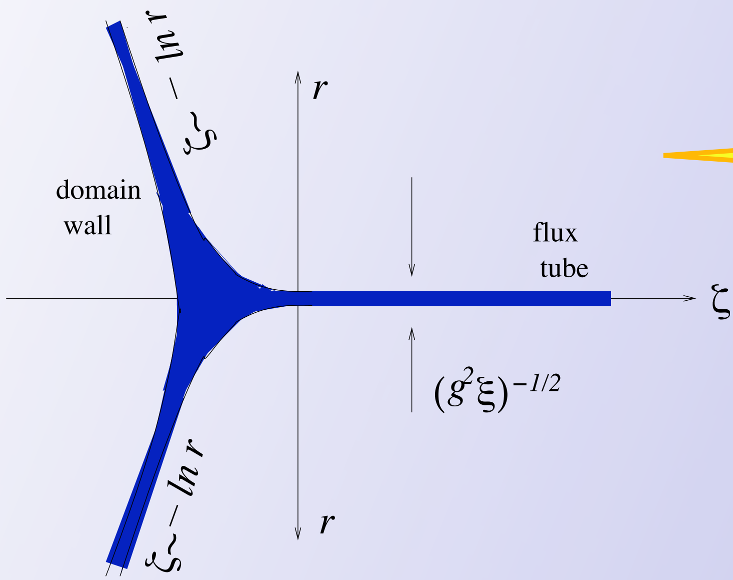


8 supercharges
walls & flux tubes

4 supercharges
SQED; CS

World-volume theory on the wall:

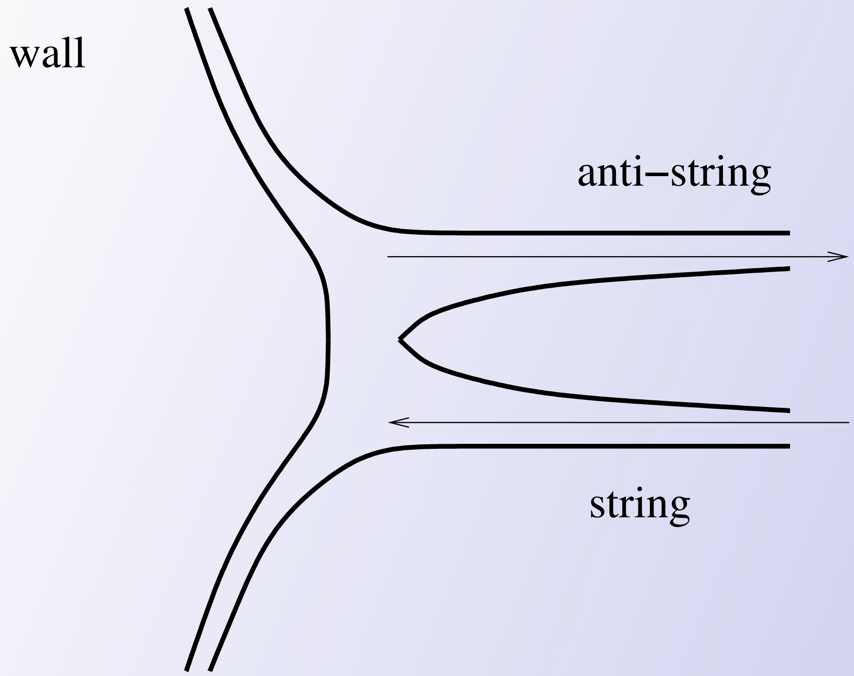
$$\frac{T_w}{2}\left(\partial_n z_0\right)^2 - \frac{1}{4e^2}\left(F_{mn}^{(2+1)}\right)^2 = \frac{1}{2e^2}\left(\partial_n a_{2+1}\right)^2 - \frac{1}{4e^2}\left(F_{mn}^{(2+1)}\right)^2$$



In addition, the same logarithm

☺ ☺ $$F_{0i}^{2+1} = \frac{e_{2+1}^2}{2\pi}\frac{x_i}{r^2}$$

$$E_{(2+1)}^G = \int_{r_0}^{r_f} \frac{1}{2e_{2+1}^2}\left(F_{0i}\right)^2 2\pi r\, dr = \frac{\pi\xi}{\Delta m}\int_{r_0}^{r_f}\frac{dr}{r} = \frac{\pi\xi}{\Delta m}\ln\frac{r_f}{r_0}.$$

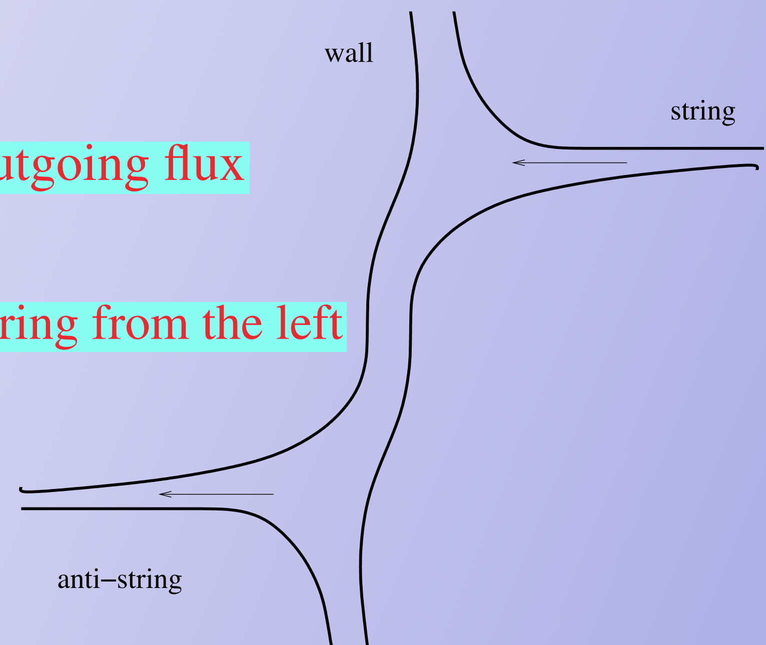


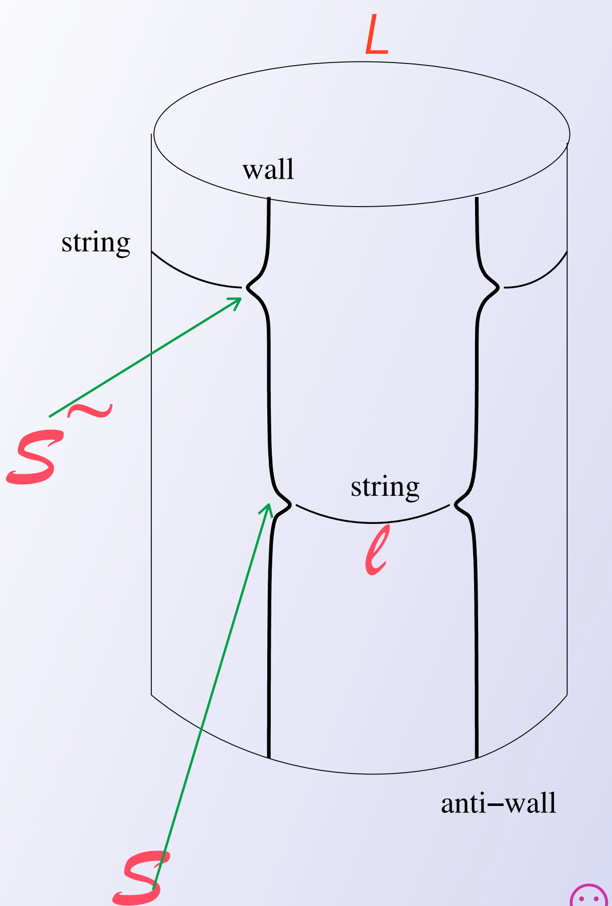
☺ String: $(n_e\ ,\ n_s)=(+1\ ,+1)$

☺ Antistring: $(n_e, n_s)=(-1,+1)$

$n_e = +1$, incoming flux, $n_e = -1$, outgoing flux

$n_s = +1$, string from the right, $n_s = -1$, string from the left





$L \gg \ell$

$$a_- \equiv \frac{1}{\sqrt{2}}\left(a^{(2)}_{2+1} - a^{(1)}_{2+1}\right) = \frac{2\pi\xi}{\sqrt{2}}\,\ell$$

$$A^-_n \equiv \frac{1}{\sqrt{2}}\left(A^{(1)}_n - A^{(2)}_n\right)$$

Crucial tests:

☺ $m_s = \sqrt{2}\,\langle a_- \rangle = 2\pi\xi\,\ell$

☺ If m= $2\pi\xi L/\sqrt{2}$ then $m_{\tilde{s}} = 2\pi\xi\,(L-\ell)$

"real mass"

$$-\frac{1}{4e^2}F^-_{mn}F^{-\,mn} + \frac{1}{2e^2}\left(\partial_n a_-\right)^2 + |D_n s|^2 + \left|\tilde{D}_n \tilde{s}\right|^2 - 2a_-^2\,\bar{s}s - 2(m-a_-)^2\,\bar{\tilde{s}}\tilde{s} - e^2\left(|s|^2 - |\tilde{s}|^2\right)^2$$

Physics of the world-volume theory

3D ferm. part

$$\frac{1}{e^2}\,\bar\lambda_-\, i\not\partial\lambda_- + \bar\psi\, i\not{D}\,\psi + \bar{\tilde\psi}\, i\tilde{\not{D}}\,\tilde\psi - \sqrt{2}\left(a_-\bar\psi\psi + (m-a_-)\bar{\tilde\psi}\tilde\psi\right)$$

4 supercharges

$$\frac{1}{4\pi}\Big[\mathrm{sign}(a)+\mathrm{sign}(m-a)\Big]\,\varepsilon_{nmk}A_n^-\partial_m A_k^-$$

Induced CS

SUSY

$$\frac{D}{2\pi}\Big[|m-a_-|-|a_-|\Big] = \frac{D}{2\pi}(m-2a_-)$$

After integrating out S and S~

$$\frac{1}{2e^2}(\partial_n a_-)^2 - \frac{1}{4e^2}(F_{mn}^-)^2 + \frac{1}{2\pi}\varepsilon_{nmk}A_n^-\partial_m A_k^- + \frac{e^2}{8\pi^2}(2a_- - m)^2$$

Approximation not applic. if ℓ is close to 0 or L

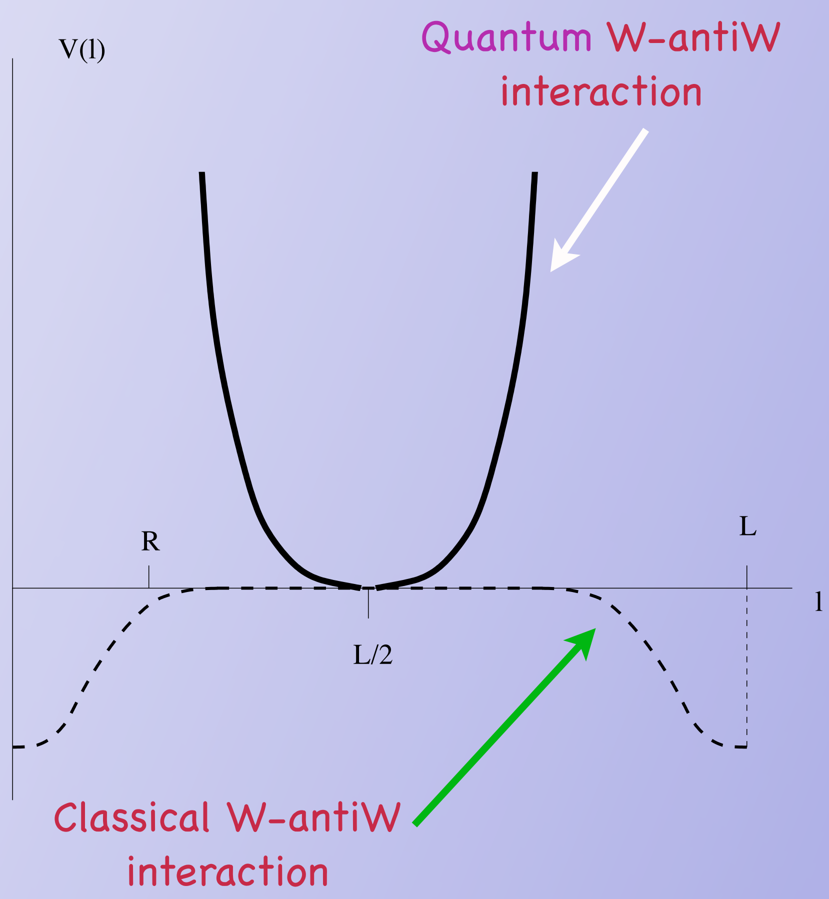
<u>Approximation applic. on plateau</u>

$$\langle a_- \rangle = \frac{m}{2}\,, \qquad \ell = \frac{L}{2}$$

Stabilization!

$$m_a = \frac{e^2}{\pi} \ll m_s$$

Infinite rigidity
of strings;
induces CS



Conclusions:

☺ Domain walls (branes), non-Abelian strings, confined non- Abelian monopoles are well understood **in field theory**

☺ A wealth of junctions

☺ Dualities

☺ ☺ ☺ Practical applications beginning to emerge