SO(10) Yukawa unification after the first run of the LHC

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Outline

- SO(10) Yukawa unification
- m_{top}, m_{bottom}, m_{tau}
- Large $\tan\beta \sim 50$
- Boundary conditions at M_{GUT}
  1. Universal gaugino masses
  2. effective "Mirage" mediation
- Flavor violation (B -> s gamma, $B_s \to \mu^+ \mu^-$)
- Higgs masses
- Soft SUSY breaking masses

Hierarchical SO(10) Yukawas

$$W \supset 16_3 \, 10 \, 16_3 + 16_3 \, 10 \, \frac{45}{M} \, 16_2 + \cdots$$

Albright, Ananthanarayan, Anderson, Babu, Barr, Barbieri, Berezhiani, Blazek, Carena, Chang, Dermisek, Dimopoulos, Hall, Lazarides, Masiero, Murayama, Pati, Raby, Romanino, Rossi, Shafi, Starkman, Tavartkiladze, Wagner, Wilczek, Wiesenfeldt, Willenbrock

Effective higher dimension operators,

Small rep's + Many predictions !!

Possible UV completion to strings !!

Yukawa Unification & Soft SUSY breaking

Blazek, Dermisek & Raby PRL 88, 111804 (2002)
PRD 65, 115004 (2002)
Baer & Ferrandis, PRL 87, 211803 (2001)
Auto, Baer, Balazs, Belyaev, Ferrandis & Tata
JHEP 0306:023 (2003)
Tobe & Wells NPB 663, 123 (2003)
Baer, Kraml, Sekmen & Summy
JHEP 0909 :005 (2009)
Badziak, Olechowski & Pokorski
JHEP 1108 :147 (2011)
Gogoladze, Shafi & Saleh Un JHEP 1208 :028 (2012)
Anandakrishnan, Raby & Wingerter arXiv:1212.0542
Anandakrishnan & Raby arXiv:1303.5125

$$\lambda \; 16_3 \; 10 \; 16_3$$

$$\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau} \equiv \lambda$$

Note, CANNOT predict top mass due to large SUSY threshold corrections to bottom and tau mass

Hall, Rattazzi & Sarid

Carena, Olechowski, Pokorski & Wagner

So instead use Yukawa unification to predict soft SUSY breaking masses !!

Bottom mass corrections

$$\delta m_b / m_b \propto \frac{\alpha_3 \mu M_{g} \tan\beta}{m_{\tilde{b}}^2} + \frac{\lambda_t^2 \mu A_t \tan\beta}{m_{\tilde{t}}^2} + \log corr.$$

$$\delta m_b / m_b \leq -2\%$$

Needed to fit data

$$\mu M_g > 0 \quad \Rightarrow \quad \mu A_t < 0$$

Anandakrishnan, Raby & Wingerter

arXiv:1212.0542

Anandakrishnan, Bryant, Raby & Wingerter

in preparation

Global χ^2 analysis

Free parameters - w/ Universal gaugino masses

Sector	Third Family Analysis	#
gauge	α_G, M_G, ϵ_3	3
SUSY (GUT scale)	m_{16}, $M_{1/2}$, A_0, m_{H_u}, m_{H_d}	5
textures	λ	1
neutrino		0
SUSY (EW scale)	$\tan\beta$, μ	2
Total #		11

Radiative EWSB requires

$$\Delta m_H^2 \equiv \frac{\left(m_{H_d}^2 - m_{H_u}^2\right)}{2m_{10}^2} \approx 13\%$$

Roughly ½ comes from RG running from $M_G \to m_{\nu_\tau}$

Blazek, Dermisek & Raby

"Just so" = "NUHM"

Low energy observables

Observable	Exp. Value	Ref.	Program	Th. Error
$\alpha_3(M_Z)$	0.1184 ± 0.0007	[23]	maton	0.5%
$\alpha_{\rm em}$	$1/137.035999074(44)$	[23]	maton	0.5%
G_μ	$1.16637876(7) \times 10^{-5}$ GeV^{-2}	[23]	maton	1%
M_W	80.385 ± 0.015 GeV	[23]	maton	0.5%
M_Z	91.1876 ± 0.0021	[23]	Input	0.0%
M_t	173.5 ± 1.0 GeV	[23]	maton	0.5%
$m_b(m_b)$	4.18 ± 0.03 GeV	[23]	maton	0.5%
M_τ	1776.82 ± 0.16 MeV	[23]	maton	0.5%
M_h	$125.3 \pm 0.4 \pm 0.5$ GeV	[24]	Ref. [25]	3 GeV
$\mathrm{BR}(b \to s\gamma)$	$(343 \pm 21 \pm 7) \times 10^{-6}$	[26]	SuperIso	$(181 - 505) \times 10^{-6}$
$\mathrm{BR}(B_s \to \mu^+\mu^-)$	3.2×10^{-9}	[27]	susy_flavor	1.5×10^{-9}

Yukawa Unification

$$\lambda \; 16_3 \, 10 \, 16_3$$

Universal Gaugino Masses

Fit t,b,tau requires

$$A_0 \approx -2m_{16} \quad m_{10} \approx \sqrt{2}m_{16}$$

$$m_{16} > \textit{few} \text{ TeV} \qquad \mu, M_{1/2} << m_{16}$$

$$\tan\beta \approx 50$$

Inverted scalar mass hierarchy

Bagger, Feng, Polonsky & Zhang
PLB473, 264 (2000)

Third family scalars lighter than first two !

Suppresses flavor & CP violation

$$A_0 \approx -2m_{16} \quad m_{10} \approx \sqrt{2}m_{16}$$

$$m_{16} > \textit{few} \text{ TeV} \qquad \mu, M_{1/2} << m_{16}$$

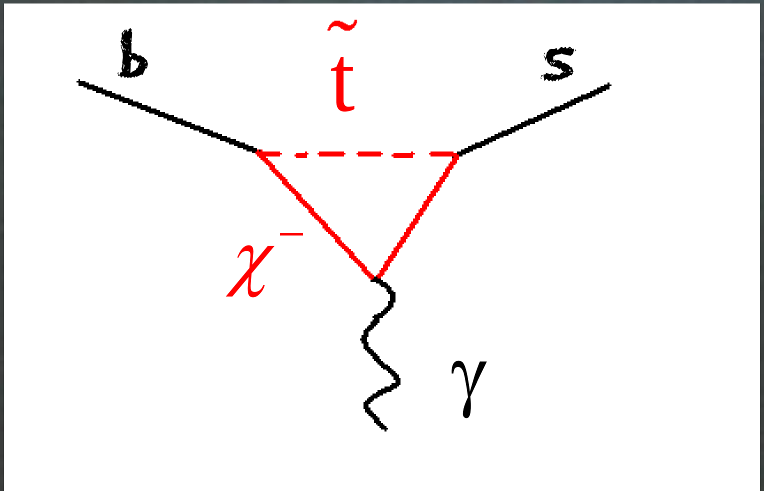
$$\tan\beta \approx 50$$

Heavy scalars

Argue need Heavy scalars !

$$BR(B \to X_s \gamma) = \left(3.55 \pm 0.26\right) \times 10^{-4} \quad \text{Exp.}$$

$$BR(B \to X_s \gamma)_{SM} = \left(3.15 \pm 0.23\right) \times 10^{-4} \quad \text{NNLO Th.}$$



$$C_7^{eff} = C_7^{SM} + C_7^{SUSY}$$

$$C_7^{eff} \approx \mp C_7^{SM}$$

$$C_7^{\chi^+} \propto \frac{\mu A_t}{m^2} \tan\beta \times \mathrm{sign}\left(C_7^{SM}\right) \approx \begin{Bmatrix} -2C_7^{SM} \\ 0 \end{Bmatrix}$$

$$\mu M_g > 0 \Rightarrow \mu A_t < 0$$

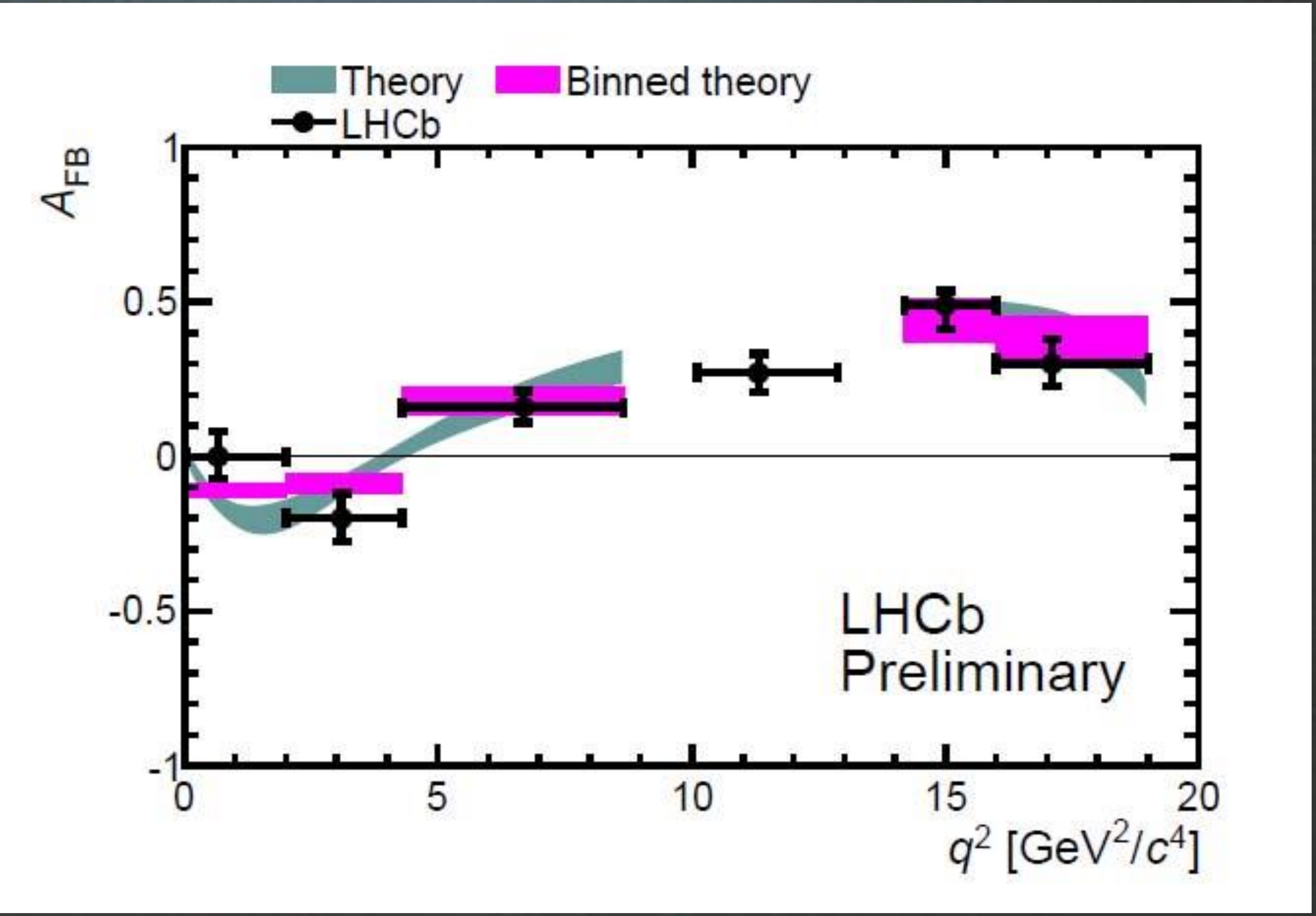
$$m_{16} \sim 4-5 \text{ TeV}$$

light squarks and sleptons !!

$$C_7^{\chi^+} \approx -2C_7^{SM} \quad \text{or}$$

$$C_7 = C_7^{SM} + C_7^{\chi^+} \approx -C_7^{SM}$$

LHCb BR($B \to K^* \mu^+ \mu^-$) favors $C_7 \approx +C_7^{SM}$



tension between $b \to s\gamma$ & $b \to s\,l^+ l^-$

Albrecht, Altmannshofer, Buras, Guadagnoli, & Straub

JHEP 0710:055 (2007)

$C_7^{\chi^+} \approx 0$ or

$$C_7 = C_7^{\mathrm{SM}} + C_7^{\chi^+} \approx +C_7^{\mathrm{SM}}$$

$$\Rightarrow \quad m_{16} \geq 8\ \mathrm{TeV}$$

Observable	Exp. value	Fit value	Pull (σ)
M_W	80.403	80.6	0.5
M_Z	91.1876	90.7	1.1
$G_F \times 10^5$	1.16637	1.16	0.3
$1/\alpha_{em}$	137.036	136.8	0.4
$\alpha_s(M_Z)$	0.1176	0.117	0.2
M_t	170.9	170.6	0.2
$m_b(m_b)$	4.2	4.22	0.3
$m_c(m_b)$	1.25	1.14	1.2
$m_s(2\,\mathrm{GeV})$	0.095	0.107	0.5
$m_d(2\,\mathrm{GeV})$	0.005	0.00741	1.2
$m_u(2\,\mathrm{GeV})$	0.00225	0.00461	**3.1**
M_τ	1.777	1.78	0.1
M_μ	0.10566	0.106	0.1
M_e	0.000511	0.000511	0.0
$\lvert V_{us}\rvert$	0.2258	0.225	0.6
$\lvert V_{ub}\rvert \times 10^3$	4.1	3.26	**2.1**
$\lvert V_{cb}\rvert$	0.0416	0.0416	0.1
$\sin 2\beta$	0.675	0.639	1.4
$\Delta m^2_{31} \times 10^{21}$	2.6	2.6	0.0
$\Delta m^2_{21} \times 10^{23}$	7.9	7.9	0.0
$\sin^2 2\theta_{12}$	0.852	0.852	0.0
$\sin^2 2\theta_{23}$	0.996	1.0	0.2
$\epsilon_K \times 10^3$	2.229	2.33	0.4
$\mathrm{BR}(B \to X_s\gamma) \times 10^4$	3.55	2.86	1.3
$\mathrm{BR}(B \to X_s\ell^+\ell^-) \times 10^6$	1.6	1.62	0.0
$\Delta M_s/\Delta M_d$	35.05	31.1	1.1
$\mathrm{BR}(B^+ \to \tau^+\nu) \times 10^4$	1.31	0.517	1.7
		total χ^2:	**27.4**

m_{16}	10000
μ	1200
$\mathrm{BR}(B_s \to \mu^+\mu^-) \times 10^8$	2.1
$\hat{s}_0$	0.14
$\mathrm{BR}(\mu \to e\gamma) \times 10^{13}$	0.0026
$\delta a_\mu^{\mathrm{SUSY}} \times 10^{10}$	+0.52
M_{h_0}	129
M_A	842
$m_{\tilde{t}_1}$	1903
$m_{\tilde{b}_1}$	2366
$m_{\tilde{\tau}_1}$	3933
$m_{\tilde{\chi}^0_1}$	60
$m_{\tilde{\chi}^+_1}$	120
$m_{\tilde{g}}$	506

Light Higgs

SM-like

Light Higgs mass

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_{SUSY}^2}{m_t^2}\right) + \frac{X_t^2}{M_{SUSY}^2}\left(1 - \frac{X_t^2}{12 M_{SUSY}^2}\right)\right]$$

$$X_t = A_t - \mu / \tan\beta \qquad X_t / M_{SUSY} \sim -\sqrt{6} \quad \text{Max mixing}$$

Large A_t & M_{SUSY} $\Rightarrow$ $m_h \simeq 125$ GeV   Easy

Br(B_s -> $\mu^+ \mu^-$):
Light Higgs SM-like

SM : 3×10^{-9} MSSM : $\sim (\tan\beta)^6 / m_A^4$

$$\text{CDF}\ \ 1.8\,{}^{+1.8}_{-0.9} \times 10^{-8}\ (95\%\,CL) \quad \text{w/}\ \ 7\ \text{fb}^{-1}$$

$$LHCb \quad \left(3.2\,{}^{+1.5}_{-1.2} \pm 0.2\right) \times 10^{-9}$$

$$\text{w/ } 1\ \text{fb}^{-1}\,(7\text{TeV}) + 1.1\text{fb}^{-1}\,(8\text{TeV})$$

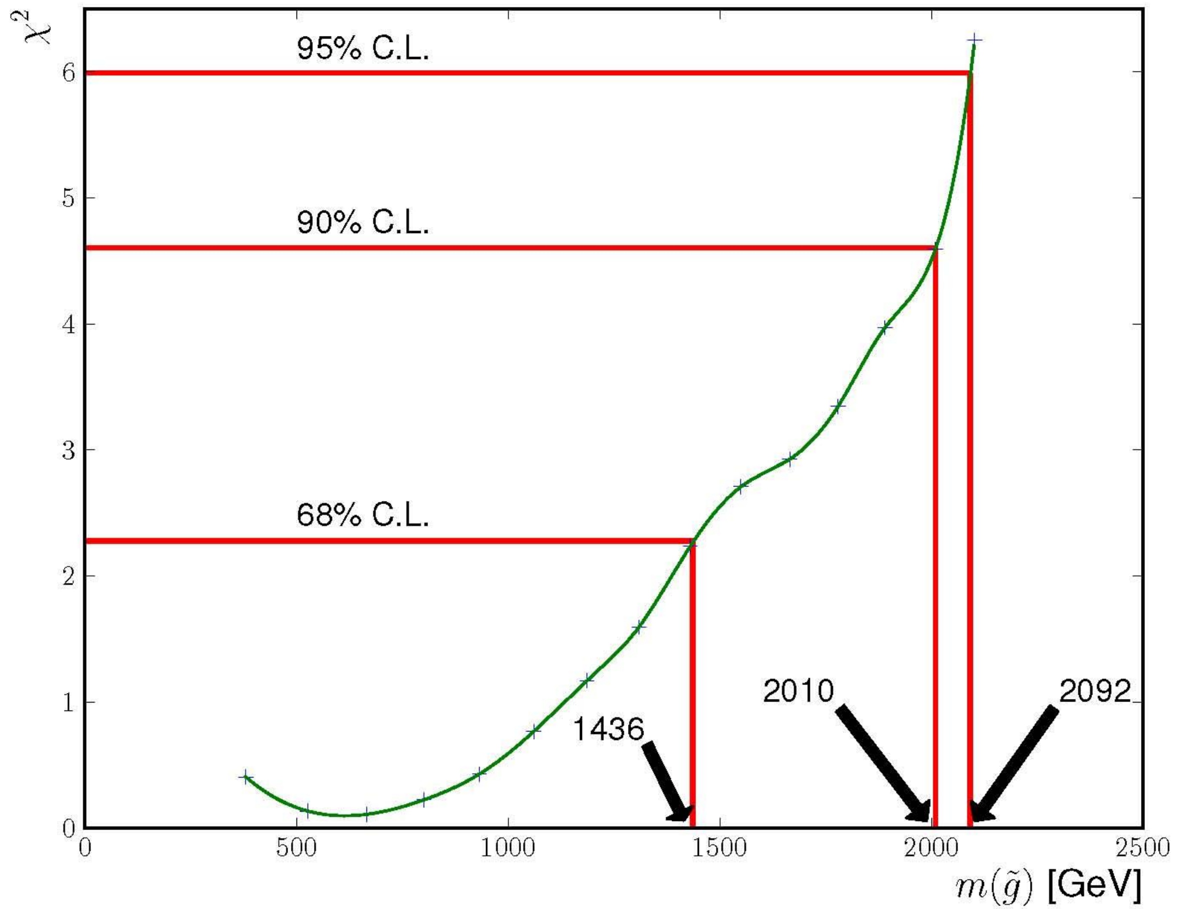
$$m_A \geq 1\ \text{TeV}$$

$$m_A \sim m_H \sim m_{H^\pm} \ \Rightarrow\ h \quad \text{SM-like}$$

Gluino mass $\leq$ 2 TeV

Gluino mass bound

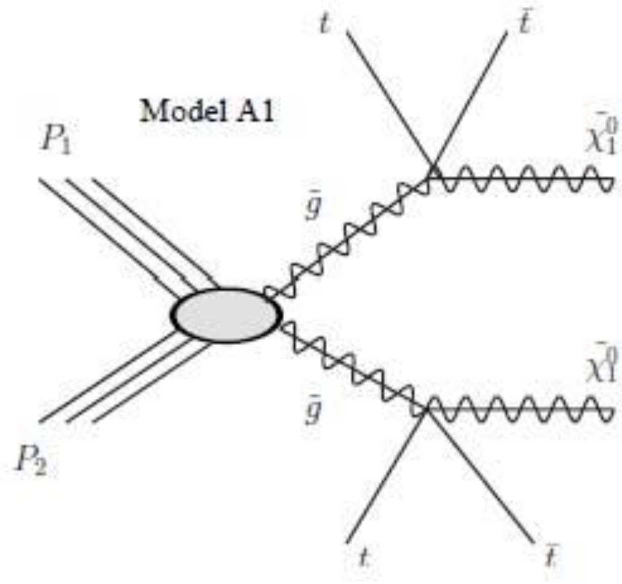
$M_{16} = 20$ TeV



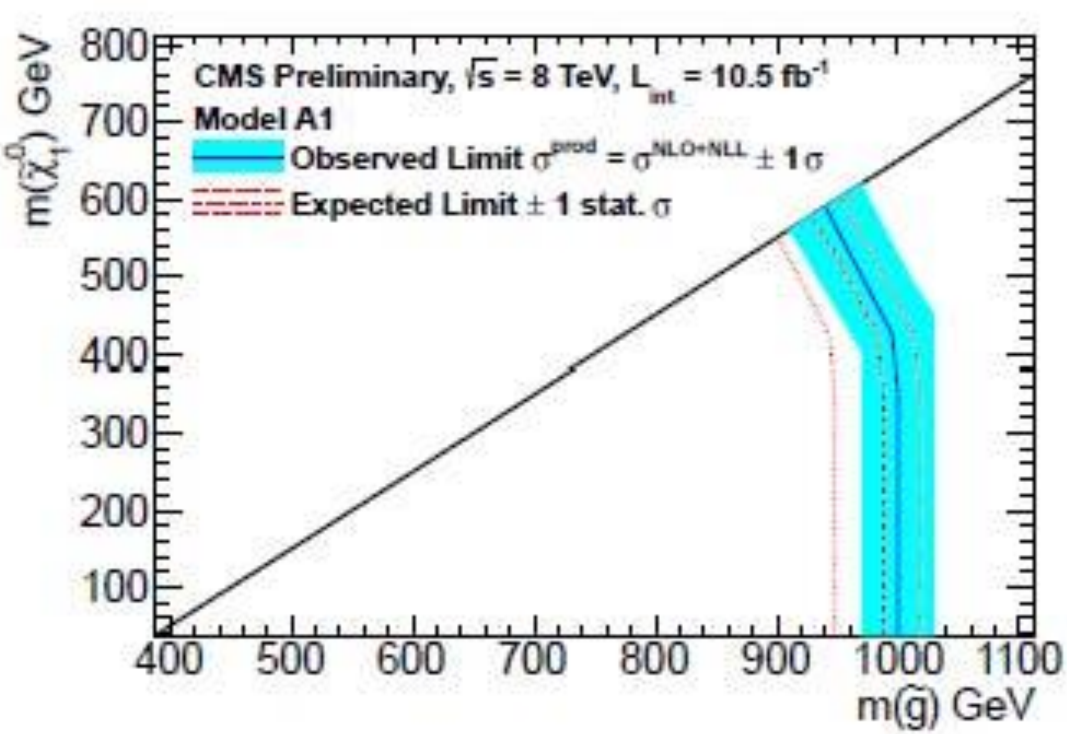
Simplified model bound

CMS PAS SUS-12-029

same sign di-leptons



$$m_{\tilde{g}} \geq 1000 \text{ GeV}$$



Benchmark point

$\alpha_G^{-1}, M_G, \varepsilon_3 :\ 25.9,\ 2.9\times10^{16}\,\mathrm{GeV},\ -0.01$

$\lambda,\ \tan\beta,\ \mathrm{M}_{1/2},\ \mathrm{m}_{16} :\ 0.6,\ 49.7,\ 150\ \mathrm{GeV},\ 20\,\mathrm{TeV}$

$\mathrm{m}_{\mathrm{H_d}},\ \mathrm{m}_{\mathrm{H_u}},\ \mathrm{A}_0 :\ 1.9\,\mathrm{m}_{16},\ 1.6\,\mathrm{m}_{16},\ -41\ \mathrm{TeV}$

g	801			
$\chi^{\pm}$	264	877		
χ^0	129 (bino)	264	876	873
U	2×10^4	4.7×10^3	3775	
D	2×10^4	4.6×10^3	4962	
E	2×10^4	1.2×10^4	7.8×10^3	
N	2×10^4	1.2×10^4		

LHC - Gluino decay modes

using SDecay

$$\tilde{g} \to t\bar{t}\ \tilde{\chi}^0_{1,2} \quad 26\%$$

$$t\bar{b}\ \tilde{\chi}^-_{1,2} \quad 26\%$$

$$b\bar{t}\ \tilde{\chi}^+_{1,2} \quad 26\%$$

$$g\ \tilde{\chi}^0_{1,2} \quad 20\%$$

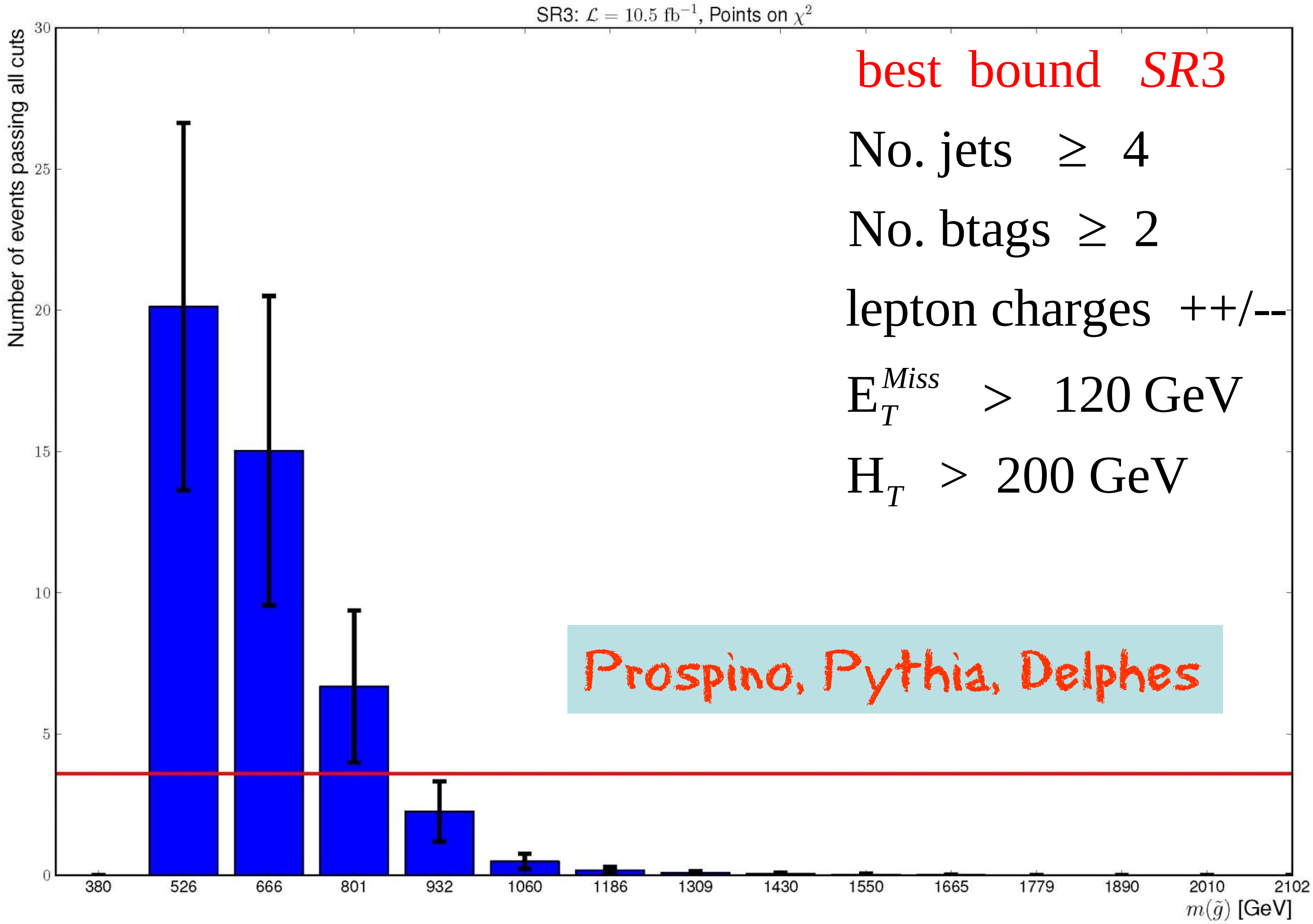
NOT Simplified Model

CMS PAS SUS-12-029 – same sign di-leptons

Table 2: A summary of the combination of results for this search. For each signal region (SR), we show its most distinguishing kinematic requirements, the prediction for the three background (BG) components as well as the total, and the observed number of events. Note that the count of the number of jets on the first line of the table includes both tagged and untagged jets.

SR3

				SR3					
No. of jets	≥ 2	≥ 2	≥ 2	≥ 4	≥ 4	≥ 4	≥ 4	≥ 3	≥ 4
No. of btags	≥ 2	≥ 2	≥ 2	≥ 2	≥ 2	≥ 2	≥ 2	≥ 3	≥ 2
Lepton charges	$++/--$	$++/--$	$++$	$++/--$	$++/--$	$++/--$	$++/--$	$++/--$	$++/--$
E_T^{miss}	> 0 GeV	> 30 GeV	> 30 GeV	> 120 GeV	> 50 GeV	> 50 GeV	> 120 GeV	> 50 GeV	> 0 GeV
H_T	> 80 GeV	> 80 GeV	> 80 GeV	> 200 GeV	> 200 GeV	> 320 GeV	> 320 GeV	> 200 GeV	> 320 GeV
Charge-flip BG	3.35 ± 0.67	2.70 ± 0.54	1.35 ± 0.27	0.04 ± 0.01	0.21 ± 0.05	0.14 ± 0.03	0.04 ± 0.01	0.03 ± 0.01	0.21 ± 0.05
Fake BG	24.77 ± 12.62	19.18 ± 9.83	9.59 ± 5.02	0.99 ± 0.69	4.51 ± 2.85	2.88 ± 1.69	0.67 ± 0.48	0.71 ± 0.47	4.39 ± 2.64
Rare SM BG	11.75 ± 5.89	10.46 ± 5.25	6.73 ± 3.39	1.18 ± 0.67	3.35 ± 1.84	2.66 ± 1.47	1.02 ± 0.60	0.44 ± 0.39	3.50 ± 1.92
Total BG	39.87 ± 13.94	32.34 ± 11.16	17.67 ± 6.06	2.22 ± 0.96	8.07 ± 3.39	5.67 ± 2.24	1.73 ± 0.77	1.18 ± 0.61	8.11 ± 3.26
Event yield	43	38	14	1	10	7	1	1	9
N_{UL} (13% unc.)	27.2	26.0	9.9	3.6	10.8	8.6	3.6	3.7	9.6
N_{UL} (20% unc.)	28.2	27.2	10.2	3.6	11.2	8.9	3.7	3.8	9.9
N_{UL} (30% unc.)	30.4	29.6	10.7	3.8	12.0	9.6	3.9	4.0	10.5



CMS PAS SUS-12-029 - same sign di-leptons

Dark Matter ?

LSP bino

Over-closes the universe

Axion, axino DM ???

Or non-thermal DM !

Anandakrishnan & Raby

arXiv:1303.5125

Anandakrishnan, Carpenter & Raby

in preparation

New boundary conditions at M_{GUT}

Non-Universal Gaugino Masses
"Mirage" mediation

$$M_i = \left(1 + \frac{g_G^2\, b_i\, \alpha}{16\pi^2} \log\left(\frac{M_{Pl}}{m_{16}}\right)\right) M_{1/2}, \quad b_i = \left(33/5,\, 1,\, -3\right)$$

Note

$$\alpha \geq 4,\ M_{1/2} < 0 \;\Rightarrow\; M_3 > 0,\ M_{1,2} < 0$$

$$\mu < 0 \;\Rightarrow\; \mu M_3 < 0,\ \mu M_{1,2} > 0$$

Bottom mass corrections

$$\delta m_b / m_b \propto \frac{\alpha_3 \mu M_{\mathrm{g}} \tan\beta}{m_{\tilde{b}}^2} + \frac{\lambda_t^2 \mu A_t \tan\beta}{m_{\tilde{t}}^2} + \log corr.$$

$$\delta m_b / m_b \leq -2\%$$

Needed to fit data

$$\mu M_g < 0 \quad \Rightarrow \quad \mu A_t \geq 0$$

Non-Universal Higgs Mass

$$m^2_{H_{u(d)}} = m^2_{10} - (+)\, 2D$$

Squark & Slepton masses

$$m^2_a = m^2_{16}$$

"Just-so" Higgs splitting

$$m^2_a = m^2_{16} + Q_a\, D$$

D term splitting

$$\left\{ Q_a = +1, \left\{ Q, \bar{u}, \bar{e} \right\}; -3, \left\{ L, \bar{d} \right\} \right\}$$

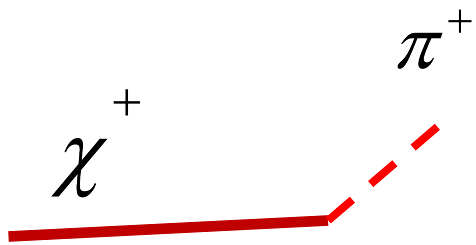
Free parameters - Non-Universal gaugino masses

Sector	Third Family Analysis
gauge	α_G, M_G, ϵ_3
SUSY (GUT scale)	m_{16}, $M_{1/2}$, α, A_0, m_{10}, D
textures	λ
SUSY (EW scale)	$\tan\beta$, μ
Total #	12

Benchmark points with $\chi^2 << 1$

NUHM	"Just-so"	D-term
m_{16}	5.00 TeV	5.00 TeV
D	3.53 TeV^2	1.85 TeV^2
m_{10}	6.10 TeV	5.58 TeV
A_0	8.07 TeV	5.69 TeV
μ	-615 GeV	-2.42 TeV
$M_{1/2}$	-105 GeV	-157 GeV
α	11.59	9.29
M_A	1558	1285
$m_{\tilde{t}_1}$	1975	3852
$m_{\tilde{b}_1}$	2049	3200
$m_{\tilde{\tau}_1}$	2473	4552
$m_{\tilde{u}}$	4905	5106
$m_{\tilde{d}}$	4975	4361
$m_{\tilde{e}}$	5075	5228
$m_{\tilde{\chi}_1^0}$	231.98	292.39
$m_{\tilde{\chi}_1^+}$	232.05	292.39
$\Delta m \equiv m_{\tilde{\chi}^+} - m_{\tilde{\chi}^0}$	519 MeV	451 MeV
$M_{\tilde{g}}$	882	954

$$\tilde{\chi}^{+} \to \tilde{\chi}^{0} + \pi^{+}$$



Disappearing track or Kink

$$\tilde{g} \to \{63\% \to \tilde{\chi}^{0} g,$$
$$\text{and the rest to} \to \tilde{\chi}^{+} b\bar{t}, \; \to \tilde{\chi}^{-} t\bar{b}\}$$

Just-so splitting

$$\tilde{g} \to \{56\% \to \tilde{\chi}^{+} b\bar{t}, \to \tilde{\chi}^{-} t\bar{b};\; 17\% \to \tilde{\chi}^{0} t\bar{t};\; 10\% \to \tilde{\chi}^{0} b\bar{b},$$
$$\text{and the rest to light quarks}\}$$

D term splitting

Dark Matter ?

LSP wino

Abundance ~ 10^{-5}

Non-thermal DM ???

Conclusions

- SO(10) Yukawa unification
- Boundary conditions at M_{GUT}
    1) Universal gaugino masses
    2) effective "Mirage" mediation
- Light Higgs - SM-like
- simplified models NOT applicable
    a) gluino w/ multi-leptons
       LSP - bino
    b) chargino-neutralino degenerate
       LSP - wino

Three family model gives good fits to Low energy data

3 Family SO(10) + family symmetry

Dermisek & Raby PLB 622:327 (2005)

Dermisek, Harada & Raby PRD74, 035011 (2006)

Albrecht, Altmannshofer, Buras, Guadagnoli & Straub JHEP 0710:055 (2007)

Anandakrishnan, Raby & Wingerter arXiv:1212.0542

3 family SO_{10} SUSY Model

- $D_3 \times U(1)$ Family Symmetry
- Superpotential
- Yukawa couplings
- χ^2 analysis
- Charged fermion masses & mixing
- Neutrino masses & mixing

Superpotential for charged fermion Yukawa couplings

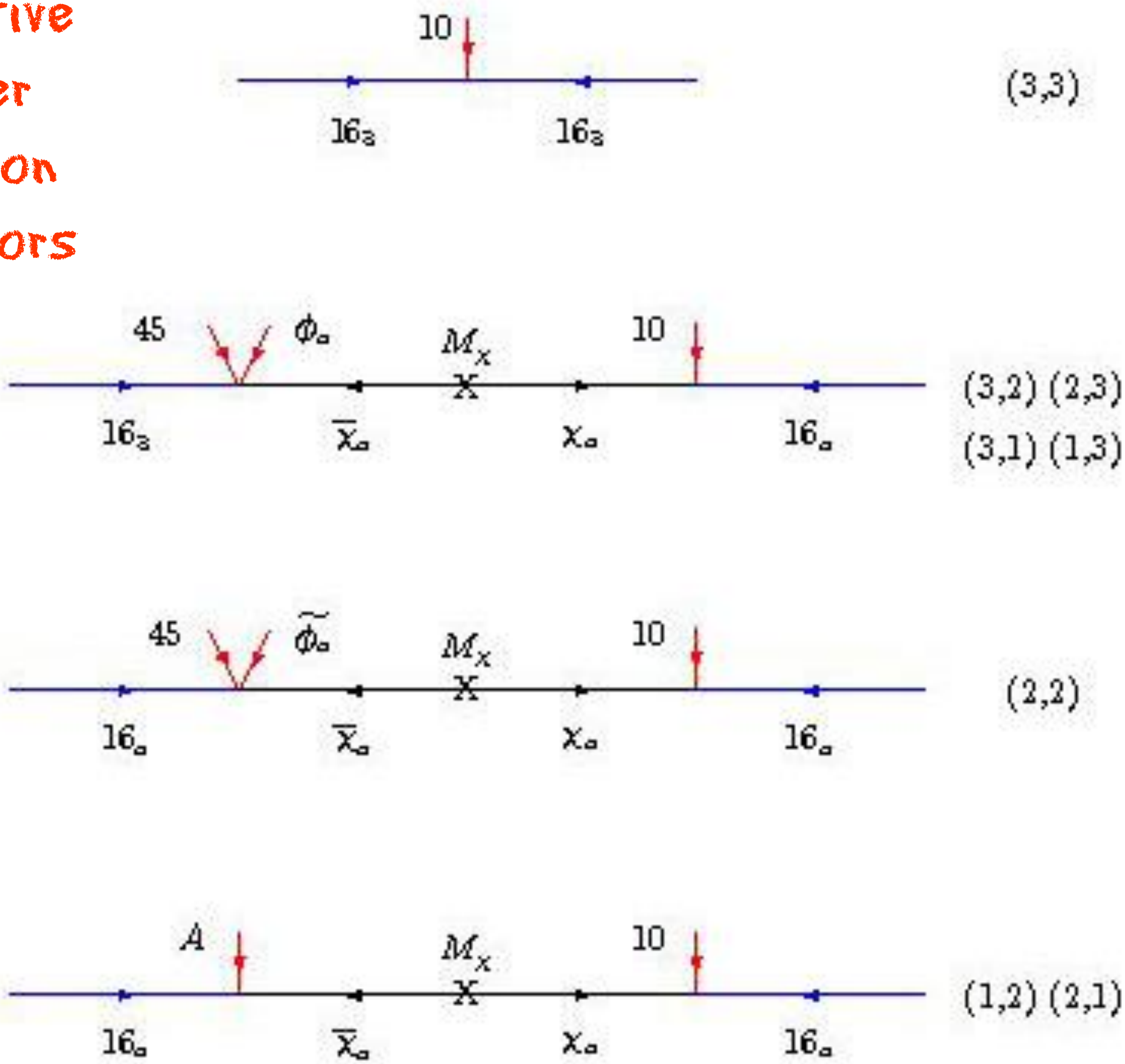
$$W_{ch.\,fermions} = 16_3 10 16_3 + 16_a 10 \chi_a$$

$$+ \overline{\chi_a}\left(M_\chi \chi_a + 45 \frac{\phi_a}{M} 16_3 + 45 \frac{\phi_a}{M} 16_a + A 16_a \right)$$

$$\langle \phi \rangle = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \qquad \langle \phi \rangle = \begin{pmatrix} 0 \\ \phi_2 \end{pmatrix} \qquad \langle 45 \rangle = (B-L) M_G$$

Familon VEVs assumed

Effective higher dimension operators



SO(10) x (D_3 x U(1) family sym.)
Yukawa Unification for 3rd Family

7 real para's

\+ 4 phases

\+ 3 real Majorana Neutrino masses

Dermisek & Raby
PLB 622:327 (2005).

$$Y_u = \begin{pmatrix} 0 & \epsilon' \rho & -\epsilon \xi \\ -\epsilon' \rho & \tilde{\epsilon} \rho & -\epsilon \\ \epsilon \xi & \epsilon & 1 \end{pmatrix} \lambda$$

$$Y_d = \begin{pmatrix} 0 & \epsilon' & -\epsilon \xi \sigma \\ -\epsilon' & \tilde{\epsilon} & -\epsilon \sigma \\ \epsilon \xi & \epsilon & 1 \end{pmatrix} \lambda$$

$$Y_e = \begin{pmatrix} 0 & -\epsilon' & 3 \epsilon \xi \\ \epsilon' & 3 \tilde{\epsilon} & 3 \epsilon \\ -3 \epsilon \xi \sigma & -3 \epsilon \sigma & 1 \end{pmatrix} \lambda$$

$$Y_\nu = \begin{pmatrix} 0 & -\epsilon' \omega & \frac{3}{2} \epsilon \xi \omega \\ \epsilon' \omega & 3 \tilde{\epsilon} \omega & \frac{3}{2} \epsilon \omega \\ -3 \epsilon \xi \sigma & -3 \epsilon \sigma & 1 \end{pmatrix} \lambda$$

Extend to neutrino sector

$$W_{neutrino} = \overline{16}\left(\lambda_2 N_a 16_a + \lambda_3 N_3 16_3\right) + \tfrac{1}{2}\left(S_a N_a N_a + S_3 N_3 N_3\right)$$

$$\langle S_a \rangle = M_a \qquad \langle S_3 \rangle = M_3 \qquad \langle \overline{16} \rangle = v_{16}$$

Assume 3 new real para's

$$W_{neutrino} = \nu m_\nu \bar{\nu} + \bar{\nu} V N + \tfrac{1}{2} N M_N N$$

$$Y_\upsilon = \begin{pmatrix} 0 & -\varepsilon'\omega & \tfrac{3}{2}\varepsilon\xi\omega \\ \varepsilon'\omega & 3\varepsilon\omega & \tfrac{3}{2}\varepsilon\omega \\ -3\varepsilon\xi\sigma & -3\varepsilon\sigma & 1 \end{pmatrix} \lambda$$

$$\omega = {2\sigma}/{(2\sigma - 1)}$$

$$m_\nu = Y_\nu \frac{v}{\sqrt{2}} \sin\beta$$

$$V = v_{16} \begin{pmatrix} 0 & \lambda_2 & 0 \\ \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

$$M_N = diag\begin{pmatrix} M_1 & M_2 & M_3 \end{pmatrix}$$

$$M_\nu = U_e^T \left(m_\nu \left(V^T \right)^{-1} M_N V^{-1} m_\nu^T \right) U_e$$

Using χ^2 analysis, fit

15 charged fermion & 5 neutrino low energy observables with

11 arbitrary Yukawa & 3 Majorana mass parameters

⇒ 4 & 2 d. o. f. or 6 predictions

Global χ^2 analysis

Sector	#	Parameters
gauge	3	α_G, M_G, ϵ_3,
SUSY (GUT scale)	5	m_{16}, $M_{1/2}$, A_0, m_{H_u}, m_{H_d},
textures	11	ϵ, ϵ', λ, ρ, σ, $\tilde{\epsilon}$, ξ,
neutrino	3	M_{R_1}, M_{R_2}, M_{R_3},
SUSY (EW scale)	2	$\tan\beta$, μ

24 parameters at GUT scale

compared to SM - 27 parameters

CMSSM - 32 parameters

$\chi^2/\text{dof} = 2$

$m_{16} = 20$ TeV

Observable	Fit value	Exp value	Pull	Sigma
M_Z	91.1876	91.1876	0.0000	0.4559
M_W	80.5452	80.3850	0.3982	0.4022
$1/\alpha_{em}$	137.0725	137.0360	0.0533	0.6852
$G_\mu \times 10^5$	1.1713	1.1664	0.4250	0.0117
α_3	0.1184	0.1184	0.0467	0.0009
M_t	174.0184	173.5000	0.3916	1.3238
$m_b(m_b)$	4.1849	4.1800	0.1334	0.0366
M_τ	1.7755	1.7768	0.1462	0.0089
$m_c(m_c)$	1.2547	1.2750	0.7876	0.0258
m_s	0.0964	0.0950	0.2807	0.0050
m_d/m_s	0.0692	0.0526	2.9891	0.0055
$1/Q^2$	0.0018	0.0019	0.4749	0.0001
M_μ	0.1056	0.1057	0.1049	0.0005
$M_e \times 10^4$	5.1122	5.1100	0.0862	0.0255
$\lvert V_{us}\rvert$	0.2243	0.2252	0.5964	0.0014
$\lvert V_{cb}\rvert$	0.0415	0.0406	0.4511	0.0020
$\lvert V_{ub}\rvert \times 10^3$	3.2023	3.7700	0.6678	0.8502
$\lvert V_{td}\rvert \times 10^3$	8.9819	8.4000	0.9675	0.6015
$\lvert V_{ts}\rvert$	0.0407	0.0429	0.8518	0.0026
$\sin 2\beta$	0.6304	0.6790	2.3959	0.0203
ϵ_K	0.0023	0.0022	0.3823	0.0002
$\Delta M_{Bs}/\Delta M_{Bd}$	39.4933	35.0600	0.6311	7.0246
$\Delta M_{Bd} \times 10^{13}$	3.9432	3.3370	0.9072	0.6682
$m_{21}^2 \times 10^5$	7.5126	7.5450	0.0593	0.5463
$m_{31}^2 \times 10^3$	2.4828	2.4800	0.0135	0.2104
$\sin^2\theta_{12}$	0.2949	0.3050	0.2880	0.0350
$\sin^2\theta_{23}$	0.5156	0.5050	0.0640	0.1650
$\sin^2\theta_{13}$	0.0131	0.0230	1.4134	0.0070
M_h	124.07	125.30	0.4010	3.0676
$BR(B \to X_s\gamma) \times 10^4$	3.4444	3.4300	0.0088	1.6374
$BR(B_s \to \mu^+\mu^-) \times 10^9$	1.6210	3.2000	0.9682	1.6308
$BR(B_d \to \mu^+\mu^-) \times 10^{10}$	1.0231	8.1000	0.0000	5.2559
$BR(B \to \tau\nu) \times 10^5$	6.3855	16.6000	1.1436	8.9320
$BR(B \to K^*\mu^+\mu^-)$(low) $\times 10^8$	5.1468	19.7000	1.2123	12.0051
$BR(B \to K^*\mu^+\mu^-)$(high) $\times 10^8$	7.7469	12.0000	0.5839	7.2835
$q_0^2(B \to K^*\mu^+\mu^-)$	4.5168	4.9000	0.2945	1.3009
Total χ^2			26.5812	

m_{16}	10 TeV	15 TeV	20 TeV	25 TeV	30 TeV
χ^2	49.65	31.02	26.58	27.93	29.48
M_A	2333	3662	1651	2029	2036
$m_{\tilde{t}_1}$	1681	2529	3975	4892	5914
$m_{\tilde{b}_1}$	2046	2972	5194	6353	7660
$m_{\tilde{\tau}_1}$	3851	5576	7994	9769	11620
$m_{\tilde{\chi}_1^0}$	133	134	137	149	167
$m_{\tilde{\chi}_1^+}$	260	263	279	309	351
$M_{\tilde{g}}$	853	850	851	910	1004

SO(10) Yukawa unification

- Still alive after LHC 7, 8
  - Light Higgs is SM-like !
- Gluino mass < 2 TeV
  - Great fun at LHC 13 !!
- 3rd family scalars lighter than first two
- NOT simplified model !!!