# Dilatons and Fine Tuning

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with
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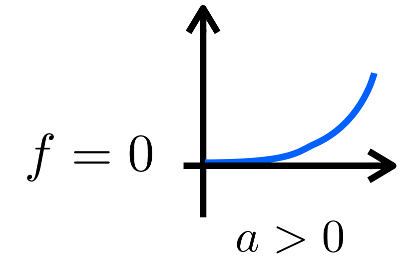
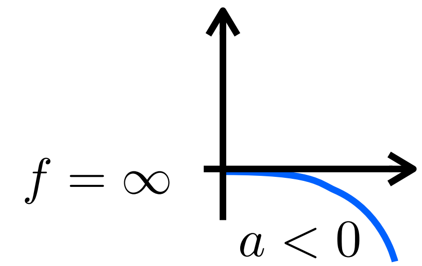
Jay Hubisz, Javi Serra

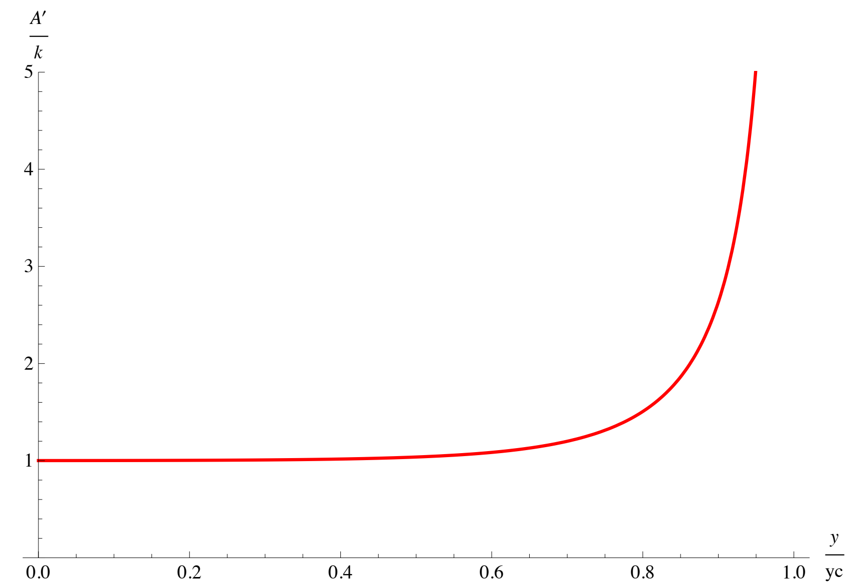
hep-ph/1209.3299 and 1305.3919

# Review of broken CFT's

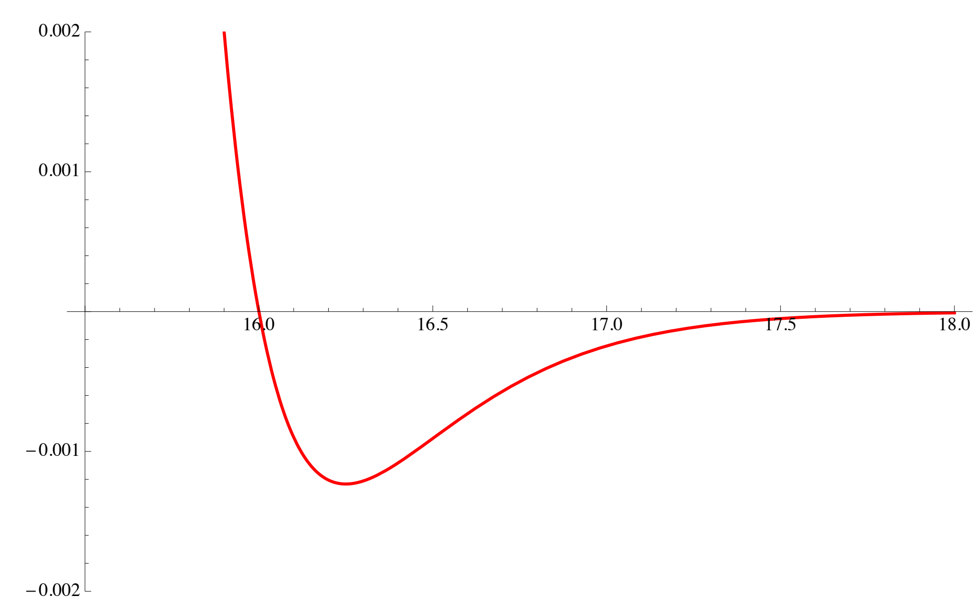



# New 5D Dual of a Spontaneously Broken CFT




# Reducing the cosmological constant



# Relevant operators cause problems

$$\mathcal{L} = \Lambda + m^2 H^\dagger H + \mathcal{L}_{d=4} + \mathcal{L}_{d=4+n}$$

With a consistent effective theory up to scale $\mu$,
without tuning coefficients go like:

$$\frac{g^2}{16\pi^2}\left( \quad \mu^4, \qquad \mu^2, \qquad \ln\mu\,, \qquad \frac{1}{\mu^n} \right)$$

observed values:

$$(10^{-12}\,\mathrm{GeV})^4 \qquad (126\,\mathrm{GeV})^2 \qquad \mathcal{O}(1) \qquad < \frac{1}{(10\,\mathrm{TeV})^n}$$

# The Plan

string theory

$\downarrow$

unique vacuum

$\downarrow$

?

$\uparrow$

understand
the weak scale

$\uparrow$

LHC

# The Plan

string theory

↓

unique vacuum

↓

?

↑

understand
the weak scale

↑

find a bunch of
new particles

LHC

# The Plan

string theory

$\downarrow$ a miracle occurs

unique vacuum

$\downarrow$

?

$\uparrow$

understand
the weak scale

$\uparrow$ find a bunch of new particles

LHC

# Plan B

string theory

$\downarrow$

unusual vacuum

one out of $e^{500}$

?

we don't understand

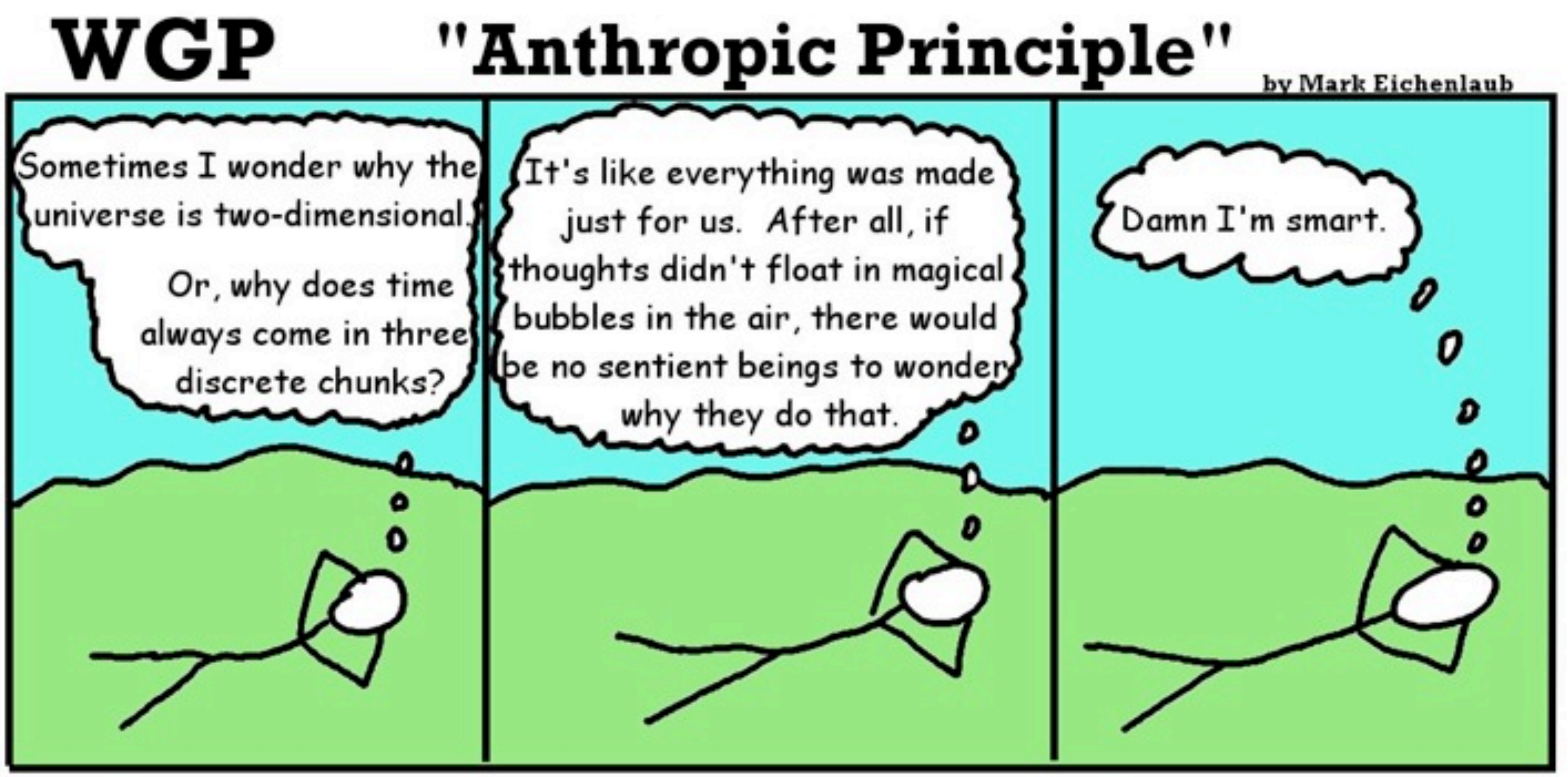
the weak scale

$\uparrow$

LHC

# Anthropic Speculations:

The Universe must be such that life can be advanced enough to contemplate the Universe and primitive enough to contemplate the anthropic principle.



# Approximate Symmetries can lead to large suppressions

terms can be forbidden to leading order

Pseudo-Nambu-Goldstone bosons can have suppressed masses

# Symmetry checklist

| | $\Lambda$ | $m^2 H^\dagger H$ |
|---|---|---|
| supersymmetry | $\checkmark$ | $\checkmark$ |
| little Higgs | | $\checkmark$ |
| extra dim. gauge | | $\checkmark$ |
| conformal | $\checkmark$ | $\checkmark$ |

we'll explore spontaneously broken CFTs

# Effective theory for broken scale invariance

$$\langle \mathcal{O} \rangle = f^n$$

Goldstone boson $$\sigma(x) \to \sigma(e^\alpha x) + \alpha f$$

non-linear realization $$f \to f\,\chi \equiv f\, e^{\sigma/f}$$

$$\mathcal{L}_{eff} = \sum_{n,m \geqslant 0} \frac{a_{n,m}}{(4\pi)^{2(n-1)}\, f^{2(n-2)}} \frac{\partial^{2n}\chi^m}{\chi^{2n+m-4}}$$

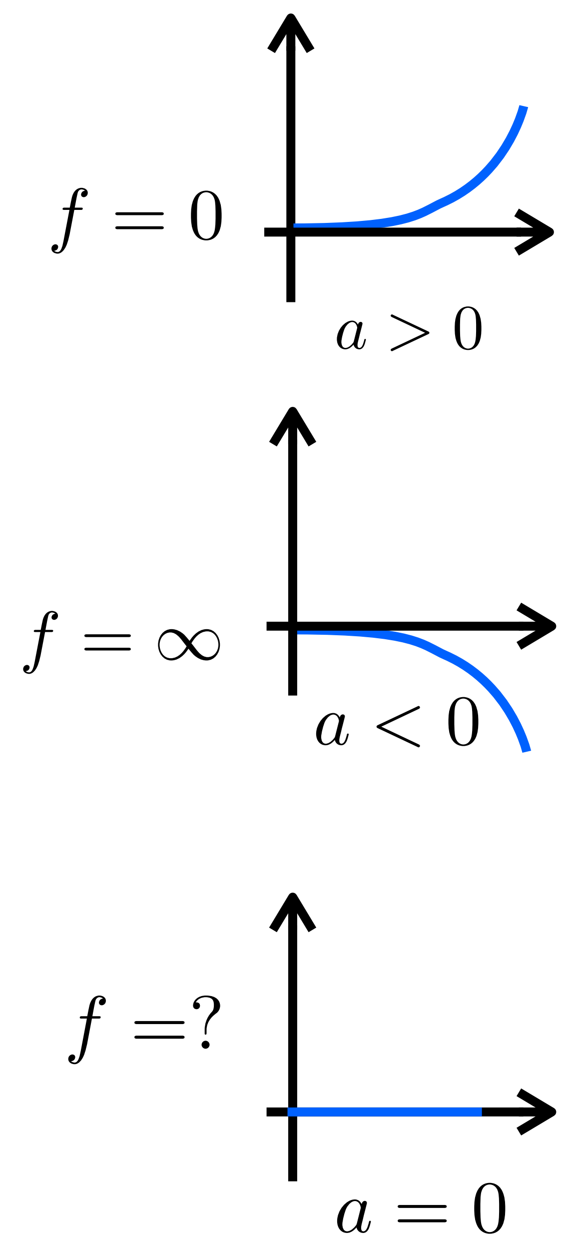
$$= -a_{0,0}\,(4\pi)^2 f^4 \chi^4 + \frac{f^2}{2}(\partial_\mu \chi)^2 + \frac{a_{2,4}}{(4\pi)^2}\frac{(\partial\chi)^4}{\chi^4} + \ldots$$

a la Callan, Coleman, Wess, and Zumino

# Runaway Goldstone Boson

$$\mathcal{L}_{\rm eff} = -a_{0,0}\,(4\pi)^2 f^4 \chi^4 + \frac{f^2}{2}(\partial_\mu \chi)^2 + \ldots$$

quartic coupling

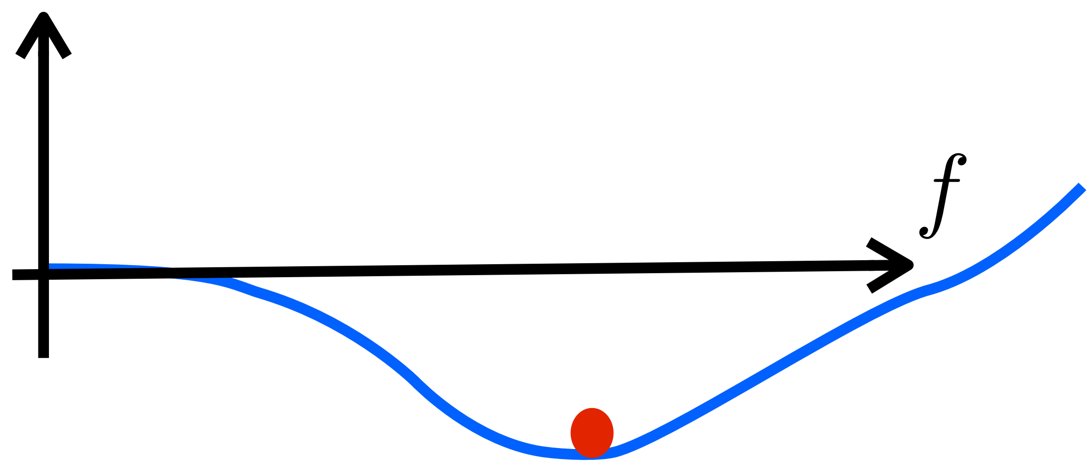



for exact conformal symmetry
need a flat direction to have
spontaneous breaking

# Perturbation by an almost marginal operator

$$\delta\mathcal{L} = \lambda(\mu)\mathcal{O}$$

$$V = a\, f^4 \to V = a(\lambda(f))\, f^4$$




$$V' = f^3\,[4a(\lambda(f)) + \beta a'(\lambda(f))] = 0$$

$$m^2_{\mathrm{dil}} = f^2\beta\,[\beta a'' + 4a' + \beta' a']$$

$$\simeq 4f^2\beta a'(\lambda(f)) = -16 f^2 a(\lambda(f)) \; = \mathcal{O}(16\pi^2) f^2$$

still need approximate flat direction

QCD-like theories don't have light dilatons, Phys.Lett. B200 (1988) 338

# Assuming an approximate flat direction

$$\beta(\lambda) = \frac{d\lambda}{d\ln\mu} = \epsilon\lambda + \frac{b_1}{4\pi}\lambda^2 + \mathcal{O}\lambda^3$$

$$a(\lambda) = (4\pi)^2 \left[c_0 + \sum_n c_n \left(\frac{\lambda}{4\pi}\right)^n\right]$$

$$V' = f^3\left[4a + \beta a'\right] \approx (4\pi)^2 f^3 \left[4c_0 + \frac{c_1}{4\pi}\lambda_0 \left(\frac{f}{\mu_0}\right)^\epsilon\right]$$

$$f \approx \mu_0 \left(\frac{-16\pi c_0}{\lambda(\mu_0)c_1}\right)^{1/\epsilon}$$

need a marginal operator, small $\beta$ function,
and a tuned flat direction

# Are there non-SUSY theories with approximate flat directions?

Turn to the AdS/CFT correspondence

$$\langle e^{\int d^4x\, \phi_0(x)\mathcal{O}(x)} \rangle_{\mathrm{CFT}} \approx e^{-S_{\mathrm{5Dgrav}}[\phi(x,z)|_{z=0}=\phi_0(x)]}$$

$$ds^2 = \frac{R^2}{z^2}\left(dx^2 - dz^2\right)$$

$\mathcal{O} \subset \mathrm{CFT} \leftrightarrow \phi$ $\mathrm{AdS}_5$ field, $\phi_0(x)$ is boundary value

# With bulk mass $m^2\,\phi^2$

$$\phi = \phi_0\, z^{4-\Delta} + c\, z^{\Delta}$$

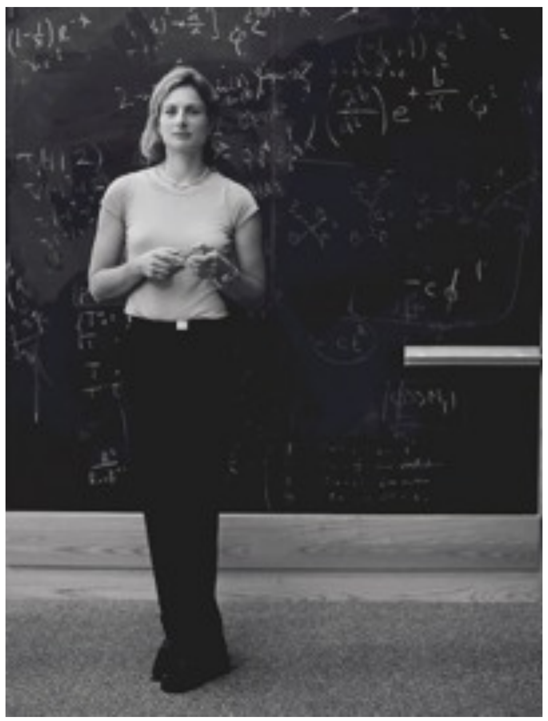
source condensate

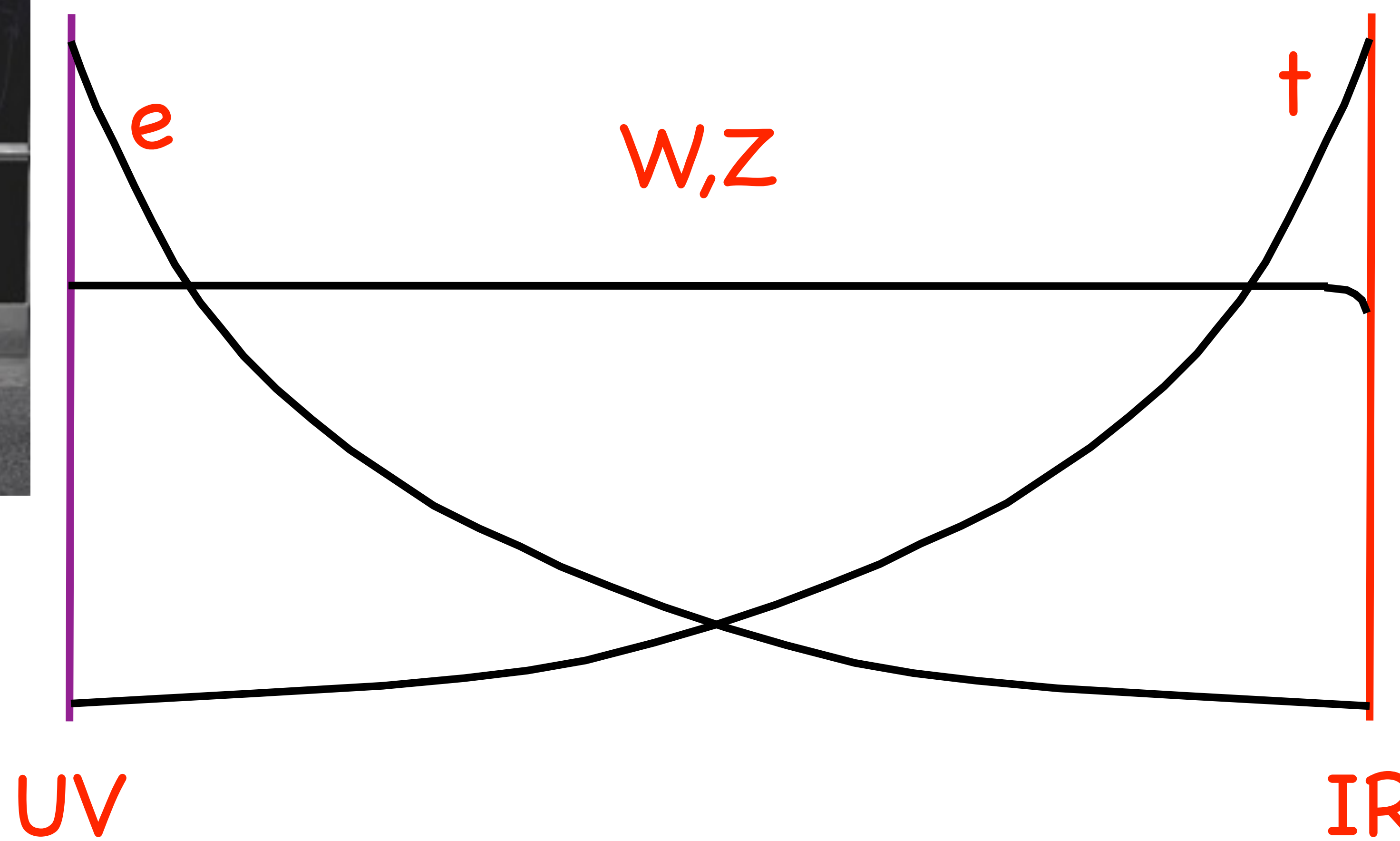
$$\Delta = 2 + \sqrt{4 + \frac{m^2}{k^2}}$$

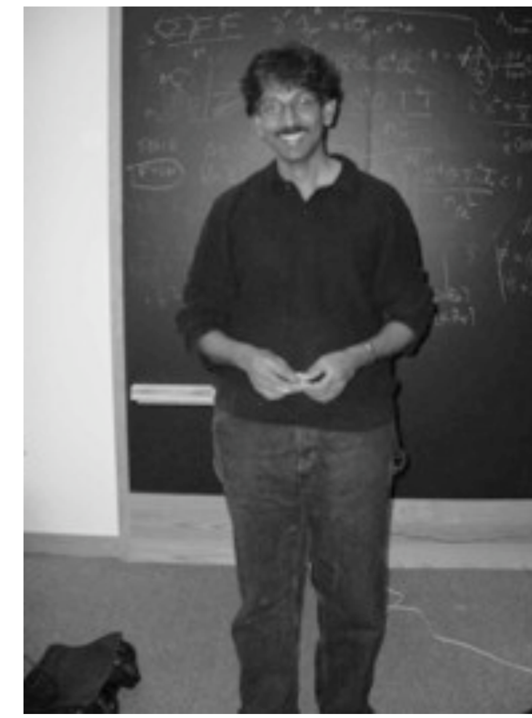
CFT operator $\mathcal{O}$ has dimension $\Delta$

how do we spontaneously break the CFT?

# Randall-Sundrum is dual to a spontaneously broken CFT





why is there a flat direction without SUSY?

# Tuning a flat direction in RS

$$V_{eff} = V_0 + V_1 \left(\frac{R}{R'}\right)^4 + \Lambda_{(5)} R \left(1 - \left(\frac{R}{R'}\right)^4\right)$$

brane tensions 5D cosmo. constant

$$V_{eff} = V_0 + \Lambda_{(5)} R + f^4 \chi^4 \left(V_1 R^4 - \Lambda_{(5)} R^5\right)$$

UV cosmo. constant quartic coupling

this solution is not stable to perturbations

Csaki, Graesser, Kolda, JT hep-ph/9906513, hep-ph/9911406

# Goldberger-Wise stabilized RS

$$m^2 = -4\,\epsilon\,k^2$$

$$\Delta \approx 4 - \epsilon$$

$$ds^2 = e^{-2A(y)}dx^2 - dy^2 \qquad\qquad \phi = \phi_0\,e^{\epsilon k y}$$

brane potentials

$$\begin{aligned} V_0 &= \lambda_0(\phi - v_0)^2 \\ V_1 &= \lambda_1(\phi - v_1)^2 \end{aligned} \qquad \lambda_0, \lambda_1 \to \infty$$

two fine tunings

$$V_{eff} = V_0 + \Lambda_{(5)}R + f^4\chi^4\left(V_1 R^4 - \Lambda_{(5)}R^5\right)$$

hep-ph/9907447

# Goldberger-Wise stabilized RS

brane potentials

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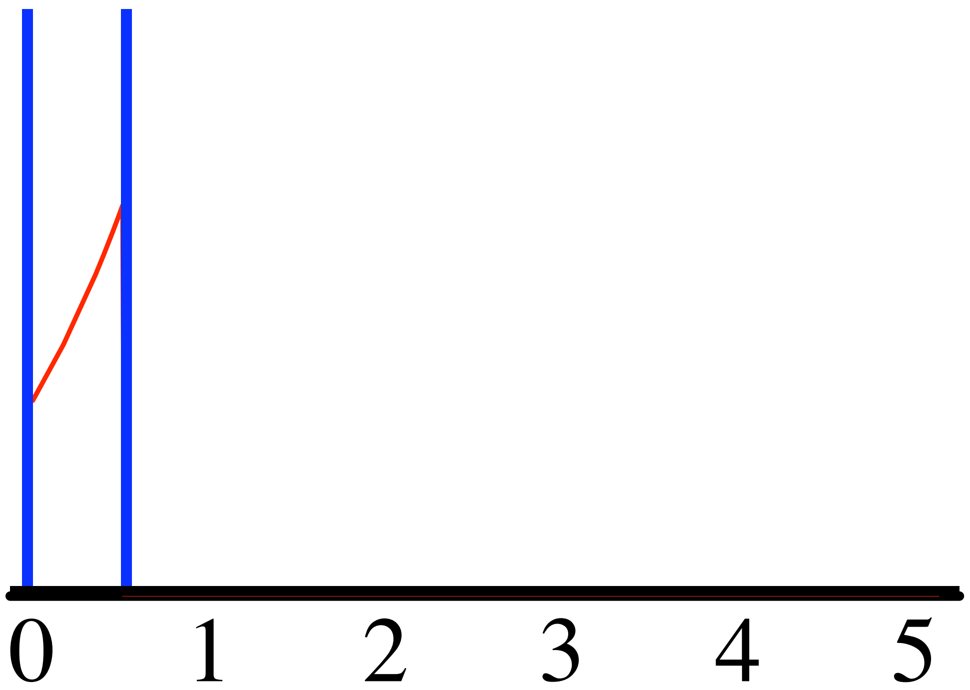


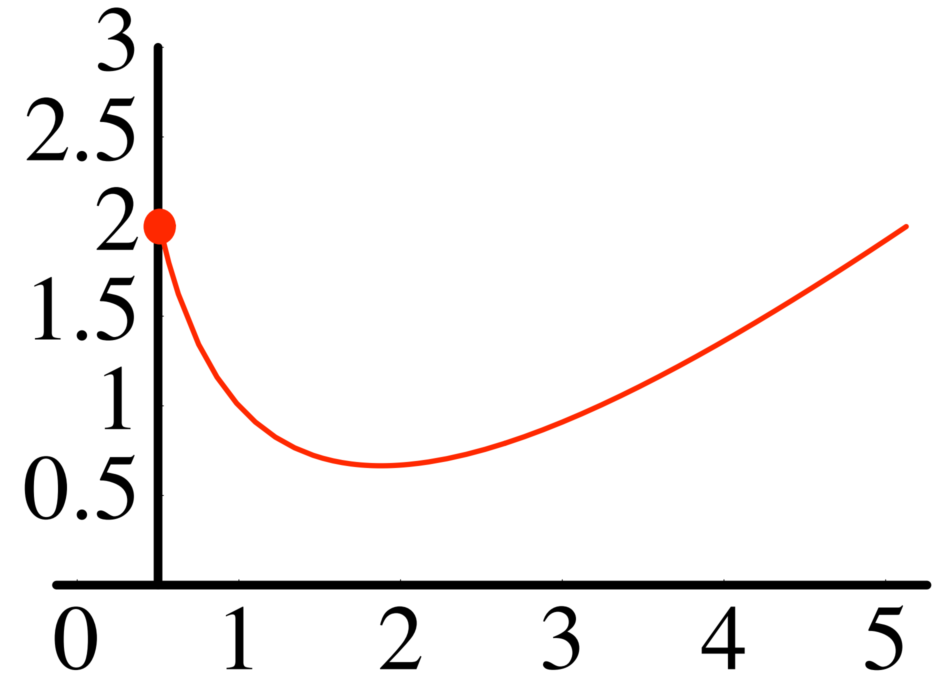

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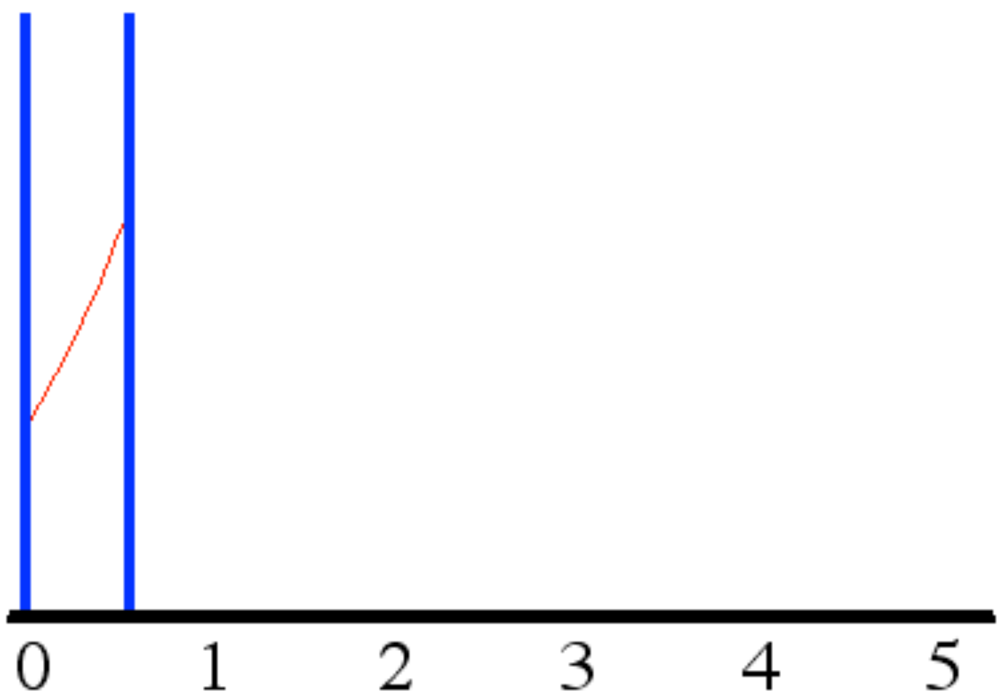


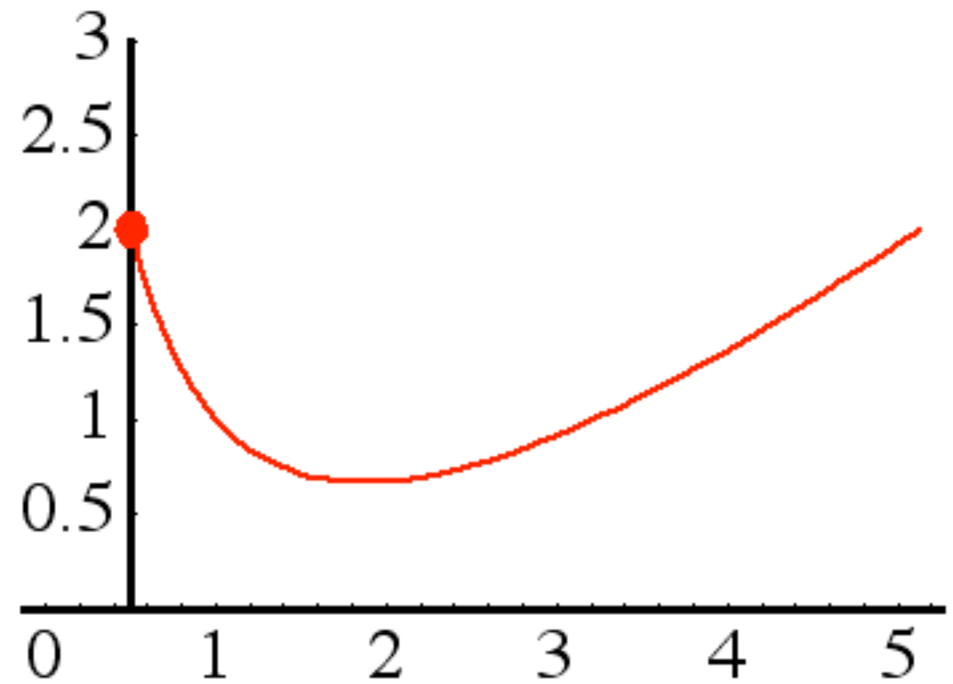

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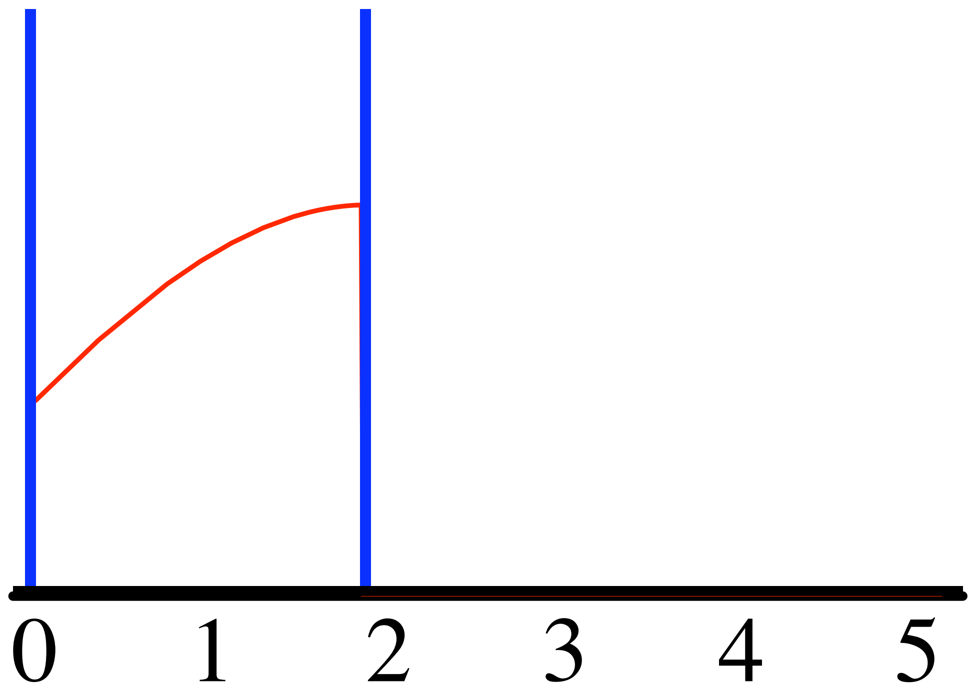
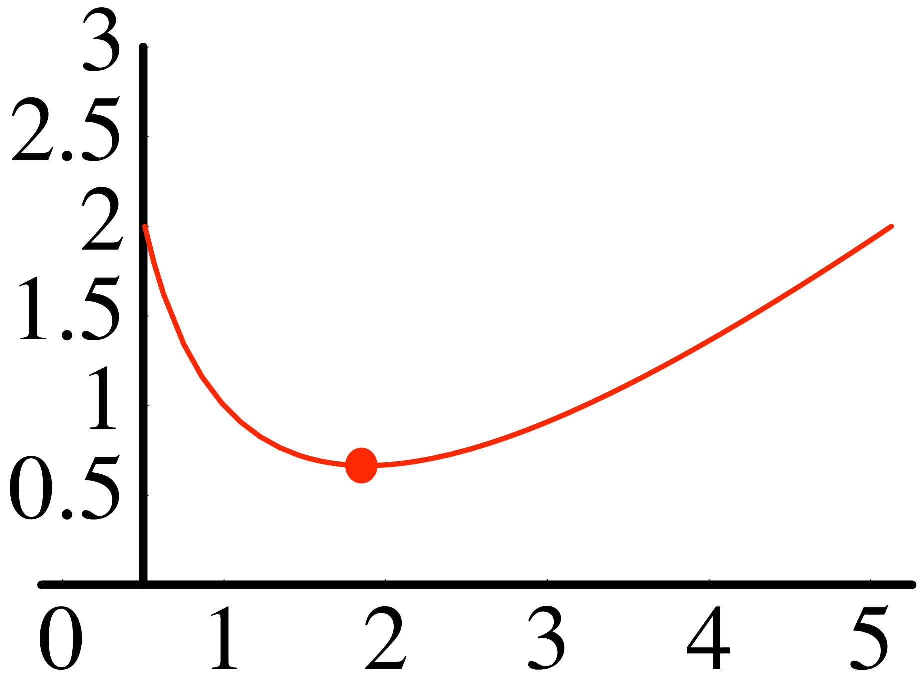



now we can parameterize a broken CFT

# Reducing the Cosmological Constant

Riccardo Rattazzi

perturbing the CFT and allowing sufficient running allows the IR brane to sit at a location with a vanishing c.c.

Planck conference 2010

# Weinberg's No-Go Theorem

You can't have your cake and eat it too!

Exact conformal symmetry can remove the c.c.

but to have a unique vacuum it must be broken.

S. Weinberg Rev. Mod. Phys. 61, 1 (1989)

# Back reaction in $AdS_5$

$$ds^2 = e^{-2A(y)} dx^2 - dy^2$$

$$\begin{aligned} A'^2 &= \frac{\kappa^2 \phi'^2}{12} - \frac{\kappa^2}{6} V(\phi) \\ \phi'' &= 4A'\phi' + \frac{\partial V}{\partial \phi} \end{aligned}$$

$$V_{eff}(y_1) = e^{-4A(y_1)} \left[ V_1\left(\phi(y_1)\right) + \frac{6}{\kappa^2} A'(y_1) \right]$$

assuming we have fine-tuned the UV contributions

# UV behavior with a bulk mass

$$\phi'' \underbrace{- 4A'\phi' + 4\epsilon k^2\phi}_{\text{dominant balance}} = 0$$

$$\begin{aligned}\phi &= \phi_0\, e^{\epsilon k y} \\ A &= ky - \frac{\kappa^2\phi_0^2}{12}\left(e^{2\epsilon k y} - 1\right)\end{aligned}$$

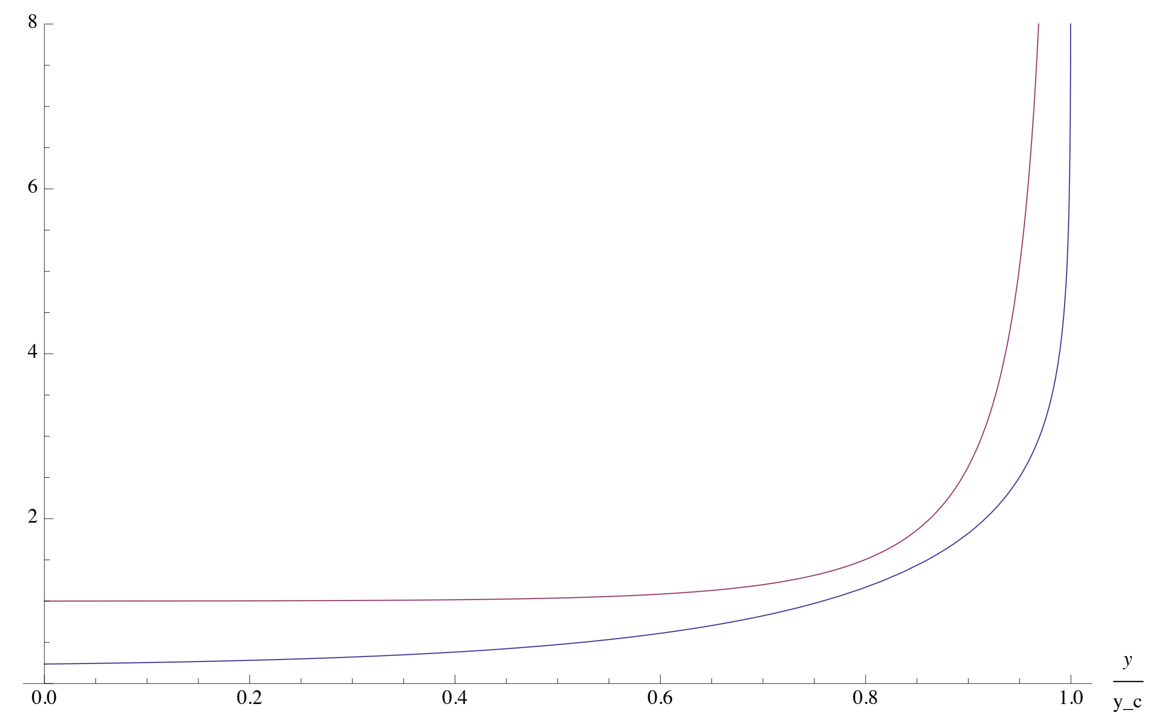
perturbative RG evolution

# IR behavior with a bulk mass

$$\underbrace{\phi'' - 4A'\phi'}_{\text{dominant balance}} + 4\epsilon k^2\phi = 0$$

$$\begin{aligned} A'(y) &= -k\coth\left(4k(y-y_c)\right) \\ \phi(y) &= \frac{c}{\kappa} - \frac{\sqrt{3}}{2\kappa}\log\left(\tanh\left(2k(yc-y)\right)\right) \end{aligned}$$

singularity at $y_c$

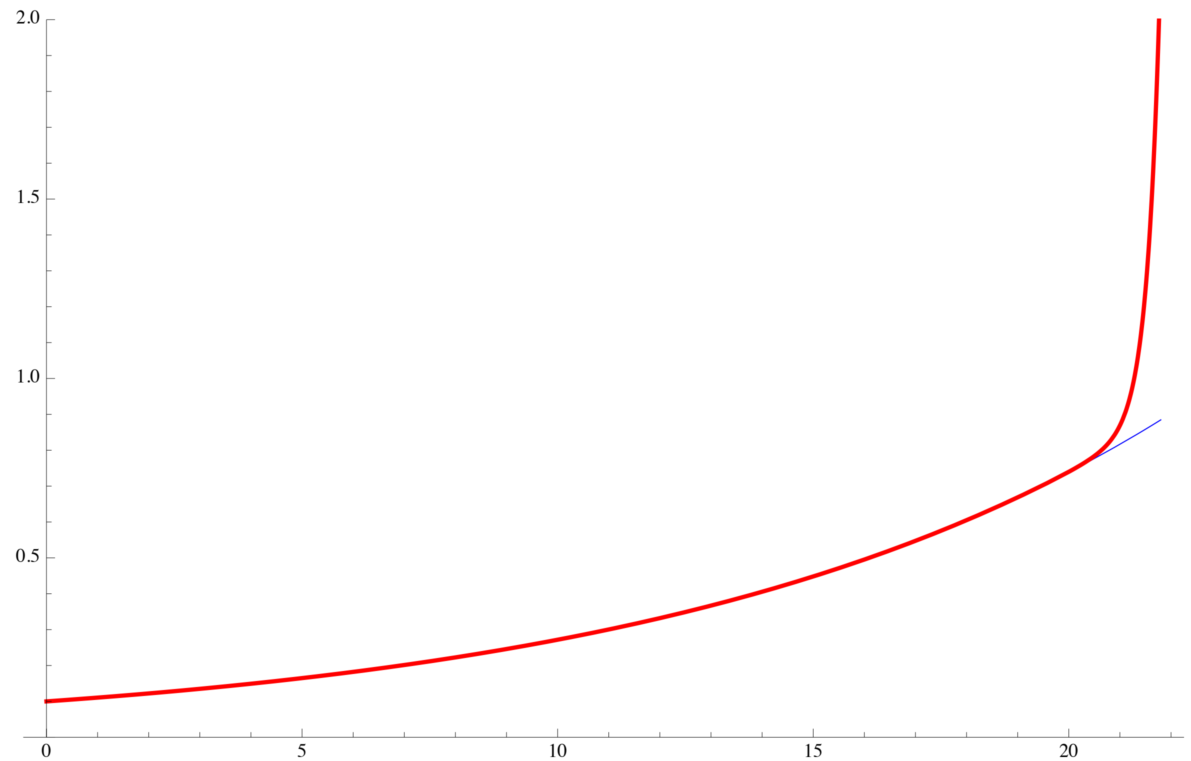



Csaki, Erlich, Grojean, Hollowood hep-th/0004133

# Boundary Layer Matching

$$y_c = \frac{1}{\epsilon k} \log \left( \frac{c}{\kappa\phi_0} - \frac{\sqrt{3}}{2\kappa\phi_0} \log \left[ \frac{1-(\kappa\phi_0)^4}{1+(\kappa\phi_0)^4} \right] \right)$$

$$\phi(y) = \phi_0 e^{\epsilon k y} - \frac{\sqrt{3}}{2\kappa} \log \left[ \tanh \left( 2k(y_c - y) \right) \right] - \phi_0 e^{\epsilon k y_c} + \frac{c}{\kappa}$$




cf. Chacko, Mishra, Stolarski hep-ph/1304.1795

# 5D Dual of Spontaneously Broken Scale Invariance

$$
\begin{aligned}
A'(y) &= k \coth\left(4k(y_c - y)\right) \\
\phi(y) &= \phi_0\, e^{\epsilon k y} - \frac{\sqrt{3}}{2\kappa} \log\left(\tanh\left(2k(y_c - y)\right)\right)
\end{aligned}
$$

$$
V_{\rm eff} = e^{-4A(y_1)} \left[\Lambda_1 + \frac{6}{\kappa^2} A'(y_1, y_c)\right] = e^{-4A(y_1)} V_b
$$

$$
V'_{\rm eff} = e^{-4A(y_1)} \left[-4A' V_b + \frac{d}{dy_1} V_b + \underbrace{\frac{dy_c}{dy_1}\frac{d}{dy_c} V_b}_{\substack{\| \\ 0}}\right]
$$

by $\phi$ boundary condition

# 5D Dual of Spontaneously Broken Scale Invariance

$$\begin{aligned} A'(y) &= k \coth\left(4k(y_c - y)\right) \\ \phi(y) &= \phi_0\, e^{\epsilon k y} - \frac{\sqrt{3}}{2\kappa} \log\left(\tanh\left(2k(y_c - y)\right)\right) \end{aligned}$$

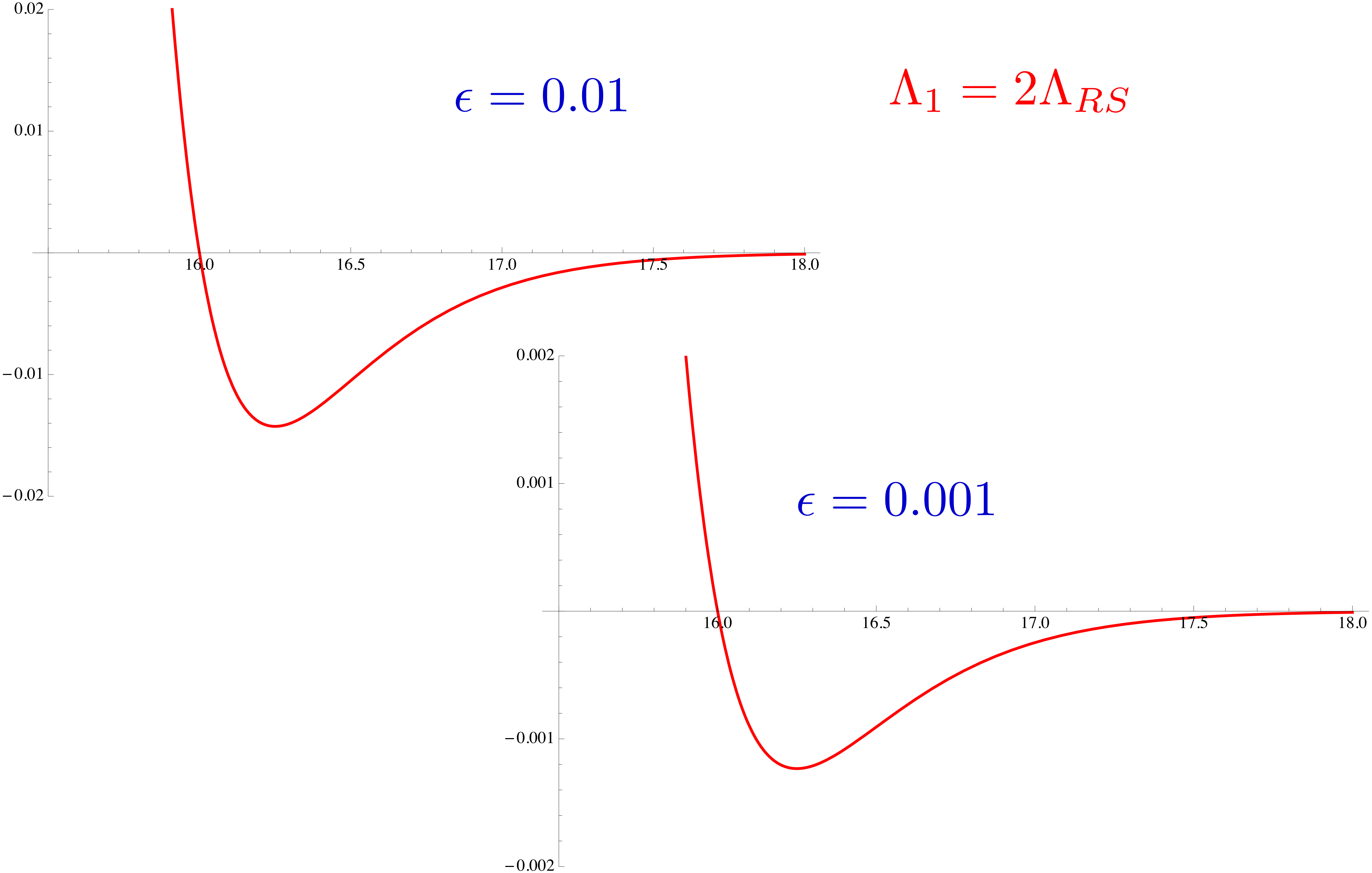
$$V \sim \epsilon \, (\mathrm{TeV})^4$$

$$m_{\rm dil}^2 \sim \epsilon \, (\mathrm{TeV})^2$$

potentially 24 orders of magnitude better than SUSY

# Cosmological Constant in TeV$^4$

$$\Lambda_1 = 2\Lambda_{RS}$$



# Weinberg's No-Go Theorem

You can't have your cake and eat it too!

$\epsilon = 0$ can remove the c.c.

but to have a unique vacuum $\epsilon \neq 0$

$$\epsilon = 10^{-12} \quad ?$$

# Conclusions

we have a new 5D dual of a
spontaneously broken CFT

conformal symmetry is better than SUSY
for reducing the cosmological constant
but not nearly a solution by itself

# Backup

# Soluble Conformal Field Theory

$$\mathcal{L} = \frac{1}{2}\,\partial^\nu\phi\,\partial_\nu\phi$$

$$\langle\phi\rangle = v$$

$\phi$ is a massless Nambu-Goldstone boson

# SUSY 3-2 Model is a Broken CFT

| | $SU(3)$ | $SU(2)$ | $U(1)$ | $U(1)_R$ |
|---|---|---|---|---|
| $Q$ | $\square$ | $\square$ | $1/3$ | $1$ |
| $L$ | $\mathbf{1}$ | $\square$ | $-1$ | $-3$ |
| $\overline{U}$ | $\overline{\square}$ | $\mathbf{1}$ | $-4/3$ | $-8$ |
| $\overline{D}$ | $\overline{\square}$ | $\mathbf{1}$ | $2/3$ | $4$ |

$$W = \frac{\Lambda_3^7}{\det(\overline{Q}Q)} + \lambda\, Q\bar{D}L$$

$$\begin{aligned} V &= \left|\frac{\partial W}{\partial Q}\right|^2 + \left|\frac{\partial W}{\partial \overline{U}}\right|^2 + \left|\frac{\partial W}{\partial \overline{D}}\right|^2 + \left|\frac{\partial W}{\partial L}\right|^2 \\ &\approx \frac{\Lambda_3^{14}}{\phi^{10}} + \lambda\frac{\Lambda_3^7}{\phi^3} + \lambda^2\phi^4 \end{aligned}$$

$$\langle\phi\rangle = \frac{\Lambda_3}{\lambda^{1/7}}$$

# SUSY 3-2 Model is a Broken CFT

| | $SU(3)$ | $SU(2)$ | $U(1)$ | $U(1)_R$ |
|---|---|---|---|---|
| $Q$ | $\square$ | $\square$ | $1/3$ | $1$ |
| $L$ | $\mathbf{1}$ | $\square$ | $-1$ | $-3$ |
| $\overline{U}$ | $\overline{\square}$ | $\mathbf{1}$ | $-4/3$ | $-8$ |
| $\overline{D}$ | $\overline{\square}$ | $\mathbf{1}$ | $2/3$ | $4$ |

$$\langle\phi\rangle = \frac{\Lambda_3}{\lambda^{1/7}}$$

$$m_{\text{dil}} = \lambda\langle\phi\rangle = \lambda^{6/7}\Lambda_3$$

$$V_{min} \approx \lambda^2 < \phi >^4$$

# Scale invariant action

$$x \to x' = e^{-\alpha} x$$

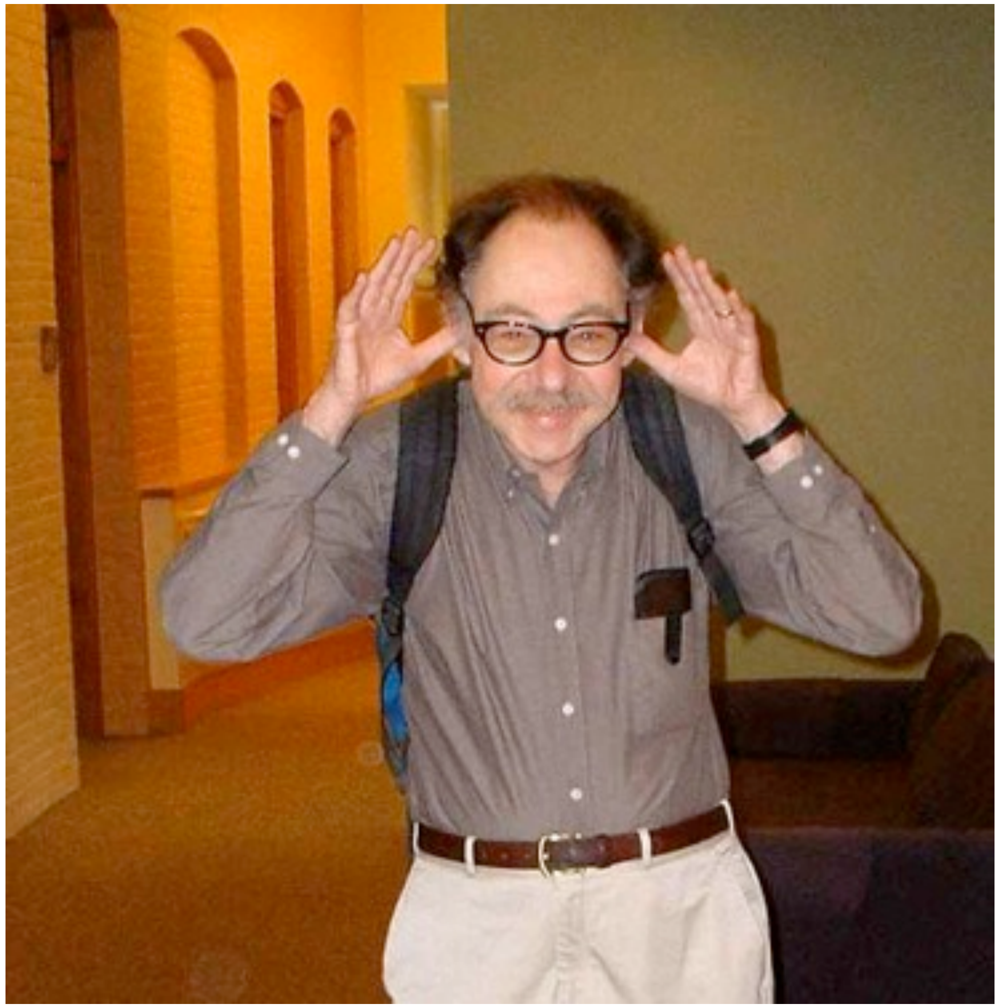
$$\mathcal{O}(x) \to \mathcal{O}'(x) = e^{\alpha\Delta} \mathcal{O}(e^{\alpha} x)$$

$$S = \sum_i \int d^4x \, g_i \mathcal{O}_i(x) \longrightarrow S' = \sum_i \int d^4x e^{\alpha(\Delta_i - 4)} g_i \mathcal{O}_i(x)$$

invariance requires $\Delta_i = 4$

# 5 spontaneously broken conformal generators give one Goldstone Boson

$$
\begin{aligned}
M_{\mu\nu} &= -i(x_\mu \partial_\nu - x_\nu \partial_\mu) \\
P_\mu &= -i\partial_\mu \\
K_\mu &= -i(x^2 \partial_\mu - 2x_\mu x_\alpha \partial^\alpha) \\
D &= i x_\alpha \partial^\alpha
\end{aligned}
$$



Sydney Coleman, "Aspects of Symmetry"
see also Ian Low, hep-th/0110285