Free fields, Quivers and Riemann surfaces

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"Quivers as Calculators : Counting, correlators and Riemann surfaces," arxiv:1301.1980,
J. Pasukonis, S. Ramgoolam

Introduction and Summary

4D gauge theory ($U(N)$ and $\prod_a U(N_a)$ groups) problems – counting and correlators of local operators in the free field limit – theories associated with Quivers (directed graphs) -

2D gauge theory (with S_n gauge groups) - topological lattice gauge theory, with defect observables associated with subgroups $\prod_i S_{n_i}$ - on Riemann surface obtained by thickening the quiver. n is related to the dimension of the local operators. For a given 4D theory, we need all n.

1D Quiver diagrammatics - quiver decorated with S_n data - is by itself a powerful tool. 2D structure specially useful for large N questions.

Mathematical models of gauge-string duality

OUTLINE

Part 1 : 4D theories - examples and motivations

Introduce some examples of the 4D gauge theories and motivate the study of these local operators.

- AdS/CFT and branes in dual AdS background.

- SUSY gauge theories, chiral ring

Motivations for studying the free fixed point :

- non-renormalization theorems

- a stringy regime of AdS/CFT - supergravity is not valid. Dual geometry should be constructed from the combinatoric data of the gauge theory.

- A point of enhanced symmetry and enhanced chiral ring.

OUTLINE

Part 2 : 2d lattice TFT - and defects - generating functions for 4D QFT counting

- Introduce the 2d lattice gauge theories and defect observables.
- 2d TFTs : counting and correlators of the 4d CFTs at large N.
- Generating functions for the counting at large N.

OUTLINE

Part 3 : Quiver - as calculator

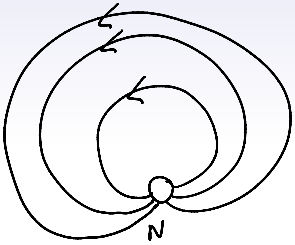
- Finite N counting with decorated Quiver.
- Orthogonal basis of operators and Quiver characters.

Part 4 : 2d TFT and models of gauge-string duality

- 2d TFTs with permutation groups - related to covering spaces of the 2d space.
- Covering spaces can be interpreted as string worldsheets.
- Quiver gauge theory combinatorics provides mathematical models of AdS/CFT.

Part 1 : Examples

Simplest theory of interest is $U(N)$ gauge theory, with $\mathcal{N} = 4$ supersymmetry. As an $\mathcal{N} = 1$ theory, it has 3 chiral multiplets in the adjoint representation.



Dual to string theory on $AdS_5 \times S^5$ by AdS/CFT. Half-BPS (maximally super-symmetric sector) reduces to a single arrow – Contains dynamics of gravitons and super-symmetric branes (giant gravitons).

$ADS_5 \times S^5 \leftrightarrow$ CFT : $N = 4$ SYM $U(N)$ gauge group on $R^{3,1}$

Radial quantization in (euclidean) CFT side :

$ADS_5 \times S^5 \leftrightarrow$ CFT : $N = 4$ SYM $U(N)$ gauge group on $R^{3,1}$

Radial quantization in (euclidean) CFT side :

Time is radius
Energy is scaling dimension Δ.

Local operators e.g. $tr(F^2)$, TrX_a^n correspond to quantum states.

Part 1 : 4D theories

Half-BPS states are built from matrix $Z = X_1 + iX_2$. Has $\Delta = 1$. Generate short representations of supersymmetry, which respect powerful non-renormalization theorems.

Holomorphic gauge invariant states :

$$\Delta = 1 \quad : \quad \mathrm{tr}\, Z$$

$$\Delta = 2 \quad : \quad \mathrm{tr}\, Z^2, \mathrm{tr}\, Z \mathrm{tr}\, Z$$

$$\Delta = 3 \quad : \quad \mathrm{tr}\, Z^3, \mathrm{tr}\, Z^2 \mathrm{tr}\, Z, (\mathrm{tr}\, Z)^3$$

For $\Delta = n$, number of states is

$p(n)$ = number of partitions of n

The number $p(n)$ is also the number of irreps of S_n and the number of conjugacy lasses.

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To see S_n – Any observable built from n copies of Z can be constructed by using a permutation.

$$\mathcal{O}_\sigma = Z^{i_1}_{i_{\sigma(1)}} Z^{i_2}_{i_{\sigma(2)}} \cdots Z^{i_n}_{i_{\sigma(n)}}$$

All indices contracted, but lower can be a permutation of upper indices.

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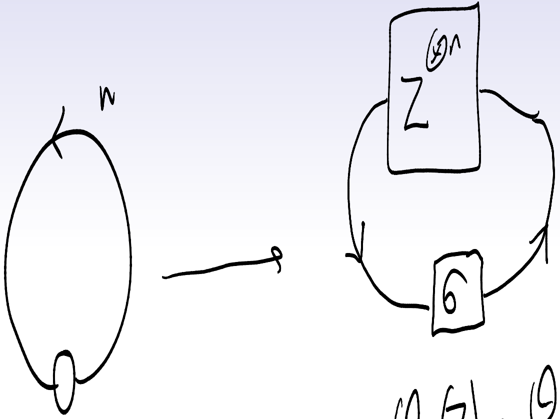
e.g

$$\begin{aligned} (\mathrm{tr}\, Z)^2 &= Z^{i_1}_{i_1} Z^{i_2}_{i_2} &= Z^{i_1}_{i_{\sigma(1)}} Z^{i_2}_{i_{\sigma(2)}} \quad \text{for } \sigma = (1)(2) \\ \mathrm{tr}\, Z^2 &= Z^{i_1}_{i_2} Z^{i_2}_{i_1} &= Z^{i_1}_{i_{\sigma(1)}} Z^{i_2}_{i_{\sigma(2)}} \quad \text{for } \sigma = (12) \end{aligned}$$

Part 1 : 4D theories



$$\mathcal{O}_\sigma(Z) = \mathcal{O}_{\gamma\sigma\gamma^{-1}}(Z)$$

Conjugacy classes are Cycle structures

For $n = 3$, permutations have 3 possible cycle structures.

$$(123), (132)$$

$$(12)(3), (13)(2), (23)(1)$$

$$(1)(2)(3)$$

Hence 3 operators we saw.

More generally - in the eighth-BPS sector - we are interested in classification/correlators of the local operators made from X, Y, Z.

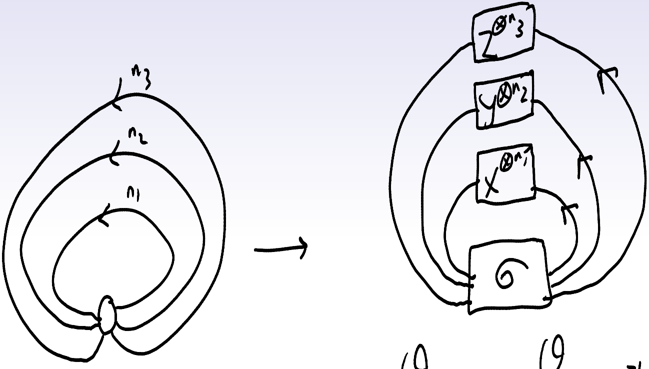
Viewed as an $\mathcal{N} = 1$ theory, this sector forms the chiral ring.

Away from the free limit, we can treat the X, Y, Z as commuting matrices, and get a spectrum of local operators in correspondence with functions on $S^N(\mathbb{C}^3)$ - the symmetric product.

This is expected since $\mathcal{N} = 4$ SYM arises from coincident 3-branes with a transverse $\mathbb{C}^3$.

At zero coupling, we cannot treat the X, Y, Z as commuting, and the chiral ring - or spectrum of eight-BPS operators - is enhanced compared to nonzero coupling.

Part 1 : 4D theories



$$\mathcal{O}_\sigma = \mathcal{O}_{\gamma\sigma\gamma^{-1}}$$

$$\gamma \in S_{n_1} \times S_{n_2} \times S_{n_3}$$

$$\sigma \in S_{n_1+n_2+n_3}$$

Part 1 : 4D theories

Conifold Theory :

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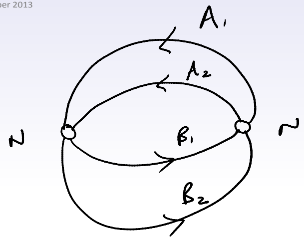

$$\underline{U(N) \times U(N)} :$$

$$A_i \longrightarrow U_1 A_i U_2^{\dagger}$$

$$B_i \longrightarrow U_2 B_i U_1^{\dagger}$$

Specify n_1, n_2, m_1, m_2, numbers of A_1, A_2, B_1, B_2, and want to count holomorphic gauge invariants.

Part 1 : 4D theories

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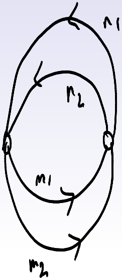

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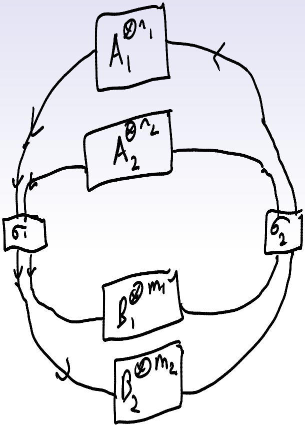


$\gamma_1 \in S_{n_1} \times S_{n_2}$

$\gamma_2 \in S_{m_1} \times S_{m_2}$

$$= \mathcal{O}_{(\sigma_1, \sigma_2)} \sim \mathcal{O}_{(\gamma_1 \sigma_1 \gamma_2^{-1}, \gamma_2 \sigma_2 \gamma_1^{-1})}$$

Part 1 : 4D theories

Having specified (m_1, m_2, n_1, n_2) we want to know the number of invariants under the $U(N) \times U(N)$ action $N(m_1, m_2, n_1, n_2)$

Counting is simpler when $m_1 + m_2 = n_1 + n_2 \leq N$. In that case, we can get a nice generating function - via 2d TFT.

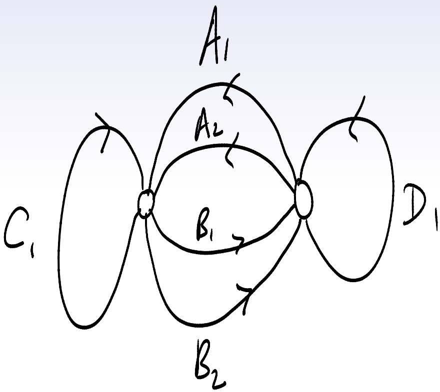
Also want to know about the matrix of 2-point functions :

$$< \mathcal{O}_\alpha(A_1, A_2, B_1, B_2)\mathcal{O}^\dagger_\beta(A_1, A_2, B_1, B_2) >$$
$$\sim \frac{M_{\alpha\beta}}{|x_1 - x_2|^{2(n_1+n_2+m_1+m_2)}}$$

The quiver diagrammatic methods produce a diagonal basis for this matrix.

Part 1 : 4D theories

$\mathbb{C}^3/Z_2$



Part 2 : 2D TFT from lattice gauge theory, 4D large N, generating functions

Edges $\rightarrow$ group elements $\sigma_{ij} \in G = S_n$

σ_P : product of group elements around plaquette.

Partition function Z :

$$Z = \sum_{\{\sigma_{ij}\}} \prod_P Z(\sigma_P)$$

Plaquette weight invariant under conjugation e.g trace in some representation.

Take the group $G = S_n$ for some integer n.

Symmetric Group of $n!$ rearrangements of $\{1, 2, \cdots, n\}$.

Plaquette action :

$$\begin{aligned} Z_P(\sigma_P) &= \delta(\sigma_P) \\ \delta(\sigma) &= 1 \ \text{ if } \sigma = 1 \\ &= 0 \quad \text{otherwise} \end{aligned}$$

Partition function :

$$Z = \frac{1}{n!^V} \sum_{\{\sigma_{ij}\}} \prod_P Z_P(\sigma_P)$$

This simple action is topological. Partition function is invariant under refinement of the lattice.

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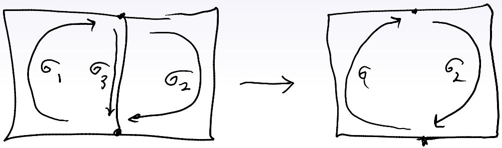


$$\sum_{\sigma_3} \delta(\sigma_1 \sigma_3)\, \delta(\sigma_3^{-1} \sigma_2) \longrightarrow \delta(\sigma_1 \sigma_2)$$

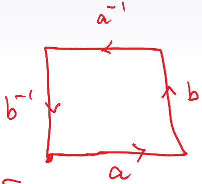
integrating out an edge $\longrightarrow$ Plaquette weight of new Plaquette

Part 2 : 2d TFTs ... gen. functions

The partition function – for a genus G surface– is

$$Z_G = \frac{1}{n!} \sum_{s_1, t_2, \cdots, s_G, t_G \in S_n} \delta(s_1 t_1 s_1^{-1} t_1^{-1} s_2 t_2 s_2^{-1} t_2^{-1} \cdots s_G t_G s_G^{-1} t_G^{-1})$$

$G = 1$

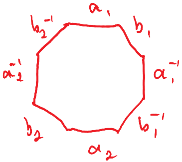


$a \to s_1$

$b \to t_1$

$\sigma_p = s_1 t_1 s_1^{-1} t_1^{-1}$

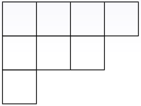
$G = 2$



$\longrightarrow \sigma_p = (s_1 t_1 s_1^{-1} t_1^{-1} s_2 t_2 s_2^{-1} t_2^{-1})$

Part 2 : 2d TFTs ... gen. functions

The delta-function can also be expanded in terms of characters of S_n in irreps. There is one irreducible rep for each Young diagram with n boxes. e.g for S_8 we can have



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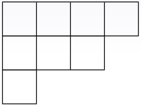

Label these R. For each partition of n

$$n = p_1 + 2p_2 + \cdots + np_n$$

there is a Young diagram.

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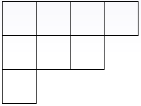

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there is a Young diagram.

The delta function is a class function :

$$\delta(\sigma) = \sum_{R \vdash n} \frac{d_R \chi_R(\sigma)}{n!}$$

The partition function

$$Z_G = \sum_{R \vdash n} (\frac{d_R}{n!})^{2-2G}$$

Fix a circle on the surface, and constrain the permutation associated with it to live in a subgroup.

$$Z(T^2, S_{n_1} \times S_{n_2}; S_{n_1+n_2}) = \frac{1}{n_1!n_2!} \sum_{\gamma \in S_{n_1} \times S_{n_2}} \sum_{\sigma \in S_n} \delta(\gamma\sigma\gamma^{-1}\sigma^{-1})$$

Part 2 : 2d TFTs .... gen functions

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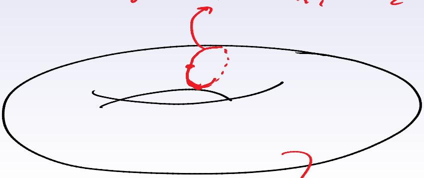
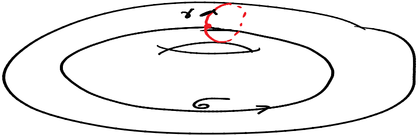
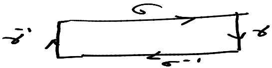
$\gamma \in H = S_{n_1} \times S_{n_2}$



$G = S_{n_1+n_2}$





Back to 4D

Start with simplest quiver. One-node, One edge. Gauge invariant operators $\mathcal{O}_\sigma$ with equivalence

$$\mathcal{O}_\sigma = \mathcal{O}_{\gamma\sigma\gamma^{-1}}$$

The set of $\mathcal{O}_\sigma$'s is acted on by γ. Burnside Lemma gives number of orbits as the average of the number of fixed points of the action.

number of orbits $= \frac{1}{n!}$ number of fixed points of the γ action on the set of σ

Hence number of distinct operators

$$
\begin{aligned}
p(n) &= \frac{1}{n!} \sum_{\sigma,\gamma \in S_n} \delta(\gamma\sigma\gamma^{-1}\gamma^{-1}) \\
&= Z_{TFT2}(T^2, S_n)
\end{aligned}
$$

In the case of $\mathbb{C}^3$, we specify n_1, n_2, n_3, the numbers of X, Y, Z and we can construct any observable $\mathcal{O}_\sigma(X, Y, Z)$ by using a permutation $\sigma \in S_n$, where $n = n_1 + n_2 + n_3$.

There are equivalences

$$\sigma \sim \gamma\sigma\gamma^{-1}$$

where $\gamma \in H \equiv S_{n_1} \times S_{n_2} \times S_{n_3} \subset S_n$.

Again using Burnside Lemma

$$\begin{aligned} N(n_1, n_2, n_3) &= \frac{1}{n_1!n_2!n_3!} \sum_{\gamma \in H} \sum_{\sigma \in S_n} \delta(\gamma\sigma\gamma^{-1}\sigma^{-1}) \\ &= Z_{TFT2}(T^2, H, S_n) \end{aligned}$$

Part 2 : 2d TFTs ....4D ... gen functions

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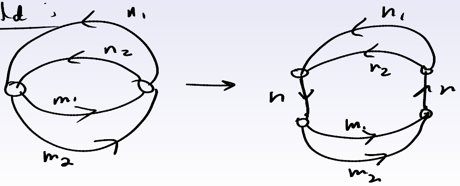
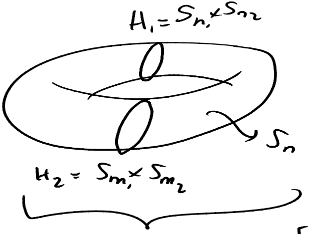
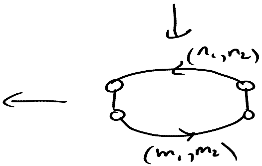
Conifold :



$$n = n_1 + n_2 = m_1 + m_2$$





$$Z_{TFT2} = N(n_1, n_2, m_1, m_2)$$

$$\boxed{n < N}$$

In terms of delta functions

$$N_{conifold}(n_1, n_2, m_1, m_2) = \sum_{\sigma_1 \in S_n} \sum_{\sigma_2 \in S_n} \sum_{\gamma_1 \in S_{n_1} \times S_{n_2}} \sum_{\gamma_2 \in S_{m_1} \times m_2} \delta(\gamma_1 \sigma_1 \gamma_2^{-1} \sigma_1^{-1}) \delta(\gamma_2 \sigma_2 \gamma_1^{-1} \sigma_2^{-1})$$

One delta function for each gauge group.
One permutation σ_a contracting the upper with lower indices for each $U(N_a)$. Equivalences

$$(\prod_b \gamma_{ba}) \sigma_a \prod_b \gamma_{ab}^{-1} \sim \sigma_a$$

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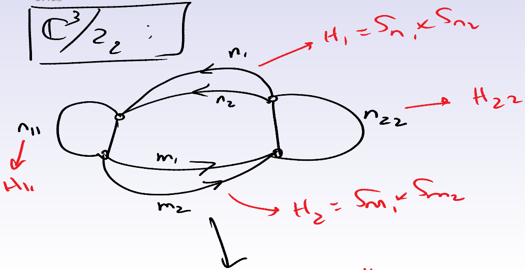
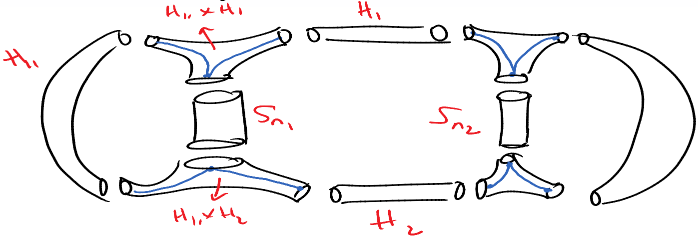
$\boxed{\mathbb{C}^3/\mathbb{Z}_2 :}$





$n_1 = n_1 + n_2 + n_{11}$
$\quad = m_1 + m_2 + n_{11}$

$n_2 = n_1 + n_2 + n_{22}$
$\quad = m_1 + m_2 + n_{22}$

These large N formulae in terms of delta functions can be used to derive simple generating functions - in the form of infinite products. The form of the denominators are simply related to the structure of the quiver - will illustrate by examples (general formula in 1301.1980).

1-node, 1-edge (Half-BPS)

$$\prod_{i=1}^{\infty} \frac{1}{(1-t^i)}$$

1-node, 3-edges (eighth-BPS)

$$\prod_{i=1}^{\infty} \frac{1}{(1-t_1^i-t_2^i-t_3^i)}$$

This formula was first written in F. Dolan 2005

Conifold case

$$\sum_{n_1,n_2,m_1,m_2} N(n_1,n_2,m_1,m_2) a_1^{n_1} a_2^{n_2} b_1^{m_1} b_2^{m_2}$$
$$= \prod_{i=1}^{\infty} \frac{1}{(1 - a_1^i b_1^i - a_1^i b_2^i - a_2^i b_1^i - a_2^i b_2^i)}$$

$\mathbb{C}^3/Z_2$ case

$$\mathcal{N}_{\mathbb{C}^3/Z_2}(a_1, a_2, b_1, b_2, c, d) = \prod_{i=1}^{\infty} \frac{1}{1 - a_1^i b_1^i - a_1^i b_2^i - a_2^i b_1^i - a_2^i b_2^i - c^i - d^i + c^i d^i}$$

The terms in the denominator are related to simple loops in the quiver. (which do not visit any node more than once). Sum over subsets of the set of nodes. For each subset, sum over permutations of that subset - for each permutation there is a term in the denominator. (arxiv-1301.1980)

Part 3 : Quiver as Calculators - Finite N counting and orthogonal bases

The above formulae are valid when N is sufficiently large. The finite N counting formulae can be written in terms of Littlewood Richardson coefficients - the form of the expression can be read off from the quiver diagram.
for the 1-node, 1-edge quiver

$$N(n, N) = p_N(n) = \sum_{\substack{R \vdash n \\ l(R) \leq N}} 1$$

giant graviton physics in AdS/CFT - stringy exclusion principle
For the 1-node, 3-edge quiver

$$N(n_1, n_2, n_3, N) = \sum_{r_1 \vdash n_1} \sum_{r_2 \vdash n_2} \sum_{r_3 \vdash n_3} \sum_{\substack{R \vdash n \\ l(R) \leq N}} g(r_1, r_2, r_3; R)^2$$

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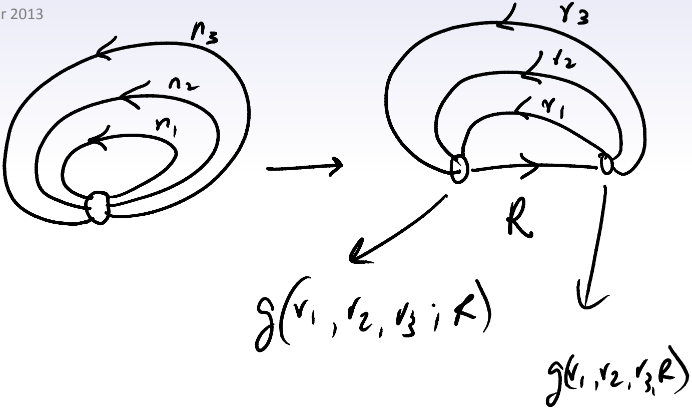


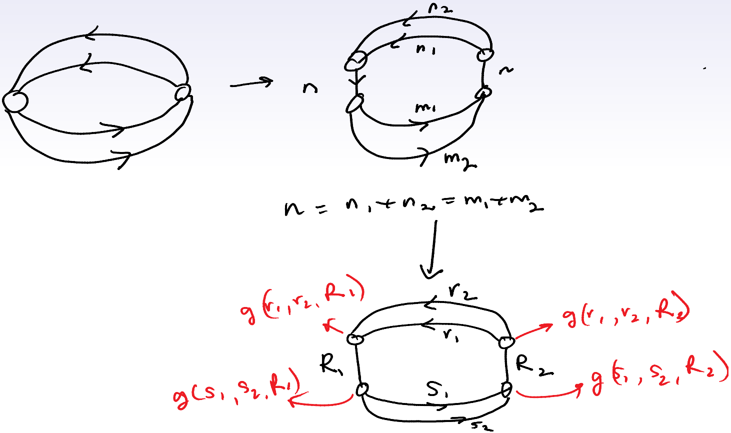
For conifold :

$$N(n_1, n_2, m_1, m_2) = \sum_{\substack{R_1 \vdash n \\ l(R_1) \leq N}} \sum_{\substack{R_2 \vdash n \\ l(R_2) \leq N}} \sum_{r_1 \vdash n_1} \sum_{r_2 \vdash n_2} \sum_{s_1 \vdash m_1} \sum_{s_2 \vdash m_2}$$

$$g(r_1, r_2, R_1) g(r_1, r_2, R_2) g(s_1, s_2, R_1) g(s_1, s_2, R_2)$$

$n = n_1 + n_2 = m_1 + m_2$.

For the conifold



For the $\mathbb{C}^3/Z_2$ case

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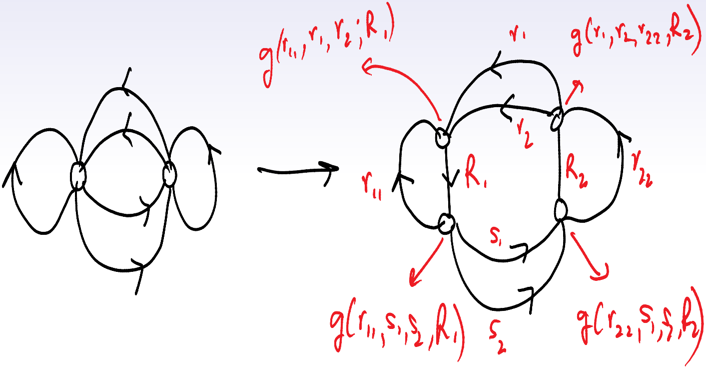


Part 3 b : Orthogonal bases

Back to 1-node, 1-edge quiver :
Using Wick's theorem and the basic 2-point function

$$< Z^i_j (Z^\dagger)^k_l >= \delta^k_j \delta^i_l$$

we can calculate the correlators

$$< \mathcal{O}_{\sigma_1} \mathcal{O}^\dagger_{\sigma_2} >$$

which give an inner product on the space of local operators.

This inner product is diagonalized by

$$\mathcal{O}_R = \sum_{\sigma} \chi_R(\sigma) \mathcal{O}_\sigma$$

$$< \mathcal{O}_R \mathcal{O}_S^\dagger >= f_R \delta_{RS}$$

Proof uses orthogonality properties of characters e.g.

$$\frac{1}{n!} \sum_{\sigma} \chi_R(\sigma) \chi_S(\sigma) = \delta_{RS}$$

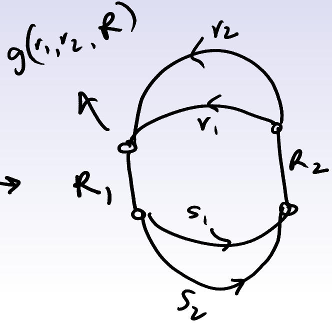
This diagonalization was done and used to propose a map between Young diagram operators and giant gravitons in AdS/CFT
Corley, Jevicki, Ramgoolam 2001
extended to half-BPS sugra backgrounds Lin, Lunin, Maldacena 2004
Recent tests (2011-2012) using DBI in $AdS \times S$ - Bissi ,Kristkjanssen, Young, Zoubos ; Caputa, de Mello Koch, Zoubos ; Hai Lin

Part 3 : Quivers as calculators, finite N, orthogonality

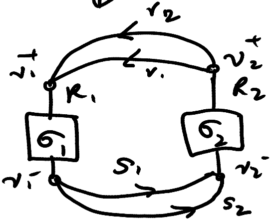
For general quivers, the $\chi_R(\sigma)$ are replaced by what we called Quiver characters, which are obtained by inserting permutations in the quiver diagram, interpreting the resulting in terms of $D^R_{ij}(\sigma)$ and branching coefficients $B^{R \to r_1, r_2 \cdots ; \nu}_{i, i_1, i_2 \cdots}$

The quiver characters have analogous orthogonality properties to ordinary S_n characters. And lead to orthogonal multi-matrix operators for quiver theories.

For the multi-edge single node quiver, this was understood in 2007/2008,
Kimura, Ramgoolam
Brown,Heslop,Ramgoolam
Collins, De Mello Koch, Bhattacharyya, Stephanou



$$\chi^{\nu_1^{\pm},\nu_2^{\pm}}_{R_1,R_2,\vec{r},\vec{s}} =$$



Part 4 : Models of gauge-string duality

$$\langle \mathcal{O}_{\sigma_1}(x_1)\mathcal{O}_{\sigma_2}(x_2)\rangle = \frac{1}{|x_1 - x_2|^{2n}} \times \frac{n!}{|T_1||T_2|} \sum_{\sigma_1' \in T_1, \sigma_2' \in T_2, \sigma_3 \in S_n} \delta(\sigma_1'\sigma_2'\sigma_3) N^{C_{\sigma_3}}$$

• Space-time dependence determined by conformal invariance : Combinatoric factor non-trivial.

• T_i : set of all permutations in the conjugacy class of σ_1, σ_2.

• Third permutation summed over entire group. C_{σ_3} is the number of cycles in the permutation. $\sum_\sigma N^{C_\sigma}\sigma$ an observable in S_n TFT

• Leading term comes from σ_3 having maximum number of cycles, i.e identity permutation. Then $T_1 = T_2$.

$$\langle (trZ)^{p_1} (trZ^2)^{p_2} \cdots (trZ^n)^{p_n} \cdot (trZ^\dagger)^{q_1} (trZ^{\dagger\ 2})^{q_2} \cdots (trZ^{\dagger\ n})^{q_n} \rangle$$

$$= \delta_{p_1,q_1} \delta_{p_2,q_2} \cdots \delta_{p_n,q_n}\ N^n \prod_i i^{p_i} p_i! (1 + \mathcal{O}(1/N^2))$$

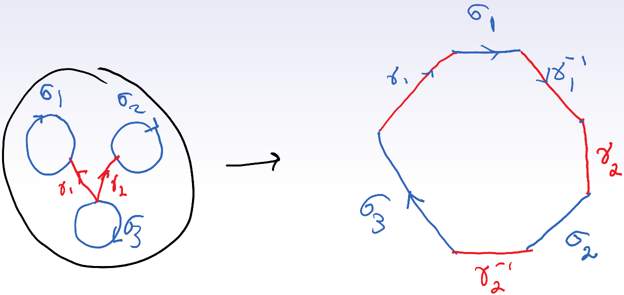
• This is large N factorization. Different trace structures do not mix in the 2-point function.

• The delta-formula contains all the $1/N$ corrections and is a 2d-TFT partition :

$$Z_{TFT}(S^2 \setminus \ \text{3 points}\ : T_1, T_2, T_3) = \sum_{\sigma_1 \in T_1, \sigma_2 \in T_2, \sigma_3 \in T_3} \delta(\sigma_1 \sigma_2 \sigma_3)$$

Part 4 : Mathematical Models of gauge-string duality

09 April 2013
08:30



$$\delta(\gamma_1 \sigma_1 \gamma_1^{-1} \gamma_2 \sigma_2 \gamma_2^{-1} \sigma_3)$$

$$\downarrow$$

$$\delta(\sigma_1 \sigma_2 \sigma_3)$$

• Similar symmetric group delta functions arise in the large N 2d YM and were used to argue for a string interpretation of the large N expansion (Gross-Taylor 1992-1994).

• 2dYM with $U(N)$ gauge group is solvable. Partition function of on a surface of genus G with area A is

$$Z(G, A) \sim \sum_{R} (DimR)^{2-2G} e^{-g_{YM}^2 A C_2(R)}$$

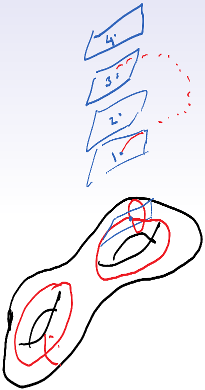
• Sum over all irreps of $U(N)$.

• In leading large N limit, and $A \to 0$, one gets again S_n-TFT (al n summed).

$$Z(G, A = 0) = \sum_n \frac{N^{n(2-2G)}}{n!} \sum_{s_i, t_i \in S_n} \delta(\prod_{i=1}^{G} s_i t_i s_i^{-1} t_i^{-1})$$

• This is interpreted in terms of n fold covers of Σ_G. The covering space is string worldsheet.

04 April 2013
18:30



- Covers of Σ_G can be described by permutations – s_i, t_i – one for each non-trivial cycle –

$s_1 t_1 s_1^{-1} t_1^{-1} s_2 t_2 s_2^{-1} t_2^{-1}$ is a trivial path

→ hence δ γ^{-1}

Similar logic here :
Half-BPS operators in $N = 4$ SYM.

• Correlators $\leftrightarrow$ TFT on 3-holed sphere.

• Holomorphic maps from worldsheet to to 2-sphere with 3 branch points. (Belyi maps)

de Mello Koch, Ramgoolam, 2010
Brown, 2010

These relations should be understood better e.g.

- or in terms of LLM coordinates for $AdS_5 \times S^5$, where the space transverse to $S^3 \times S^3$ could conceivably contain the above combinatoric T^2 or S^2 ?
- A topological string sector of the $AdS_5 \times S^5$ string ? vskip.2cm
- in terms of 6D - 4D relation of $(0, 2)$ theory compactified on T^2 or S^2 ?

For any free quiver theory, we have S_n data on a TFT2 on thickened quiver – for counting and also correlators.

So some sort of covering spaces with n-sheets – need to interpret the H-defects in terms of covering spaces.

What is the precise mathematical formulation of the TFT2 with defects (as a functor between geometrical and algebraic categories) ? S_n – all n ; subgroups on 1-dimensional subspaces; N-dependent sums over conjugacy classes.

What is the TFT2 living on the worldsheets ?

Some neat “permutation TFT2” formulations of 4D QFT combinatorics away from zero coupling also known – integrability in giant graviton dynamics.

Giant graviton oscillators - Giatanagas, de Mello Koch , Dessein, Mathwin (2011)
A double coset ansatz for integrability in AdS/CFT - de Mello Koch, Ramgoolam (2012)

Also shows up Feynman graph counting problems ..

There is a lot of stringy geometry in 4D QFT combinatorics – permutation TFT2 is a constructive tool to expose some of it ....

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How far does this story go ?

How much does it know about AdS/CFT ? Does it link up with M5-branes ? and dimensional reductions ?