Lattice QCD with light sea quarks?

Martin Lüscher

CERN — Theory Division

- ⋆ Chiral symmetry on the lattice, and why QCD simulations are so difficult
- ⋆ Domain decomposition: a new technology in lattice QCD
- ⋆ Making contact with chiral perturbation theory
- ⋆ Conclusions & perspectives

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Chiral symmetry on the lattice

The Wilson–Dirac operator

$$D_{\rm w} = \tfrac{1}{2}\{\gamma_\mu(\nabla_\mu^* + \nabla_\mu) - a\nabla_\mu^*\nabla_\mu\} + m_0$$

violates the isovector chiral symmetry

$$\left\langle \left\{ \partial_\mu A_\mu^k(x) - 2mP^k(x) \right\} \Phi_1(y_1)\ldots \right\rangle = \text{contact terms} + \mathrm{O}(a)$$

Wilson '74

Bochicchio, Maiani, Martinelli & Testa '85

In QCD this is not a fundamental problem, but the effects are large at the accessible lattice spacings

Can do better by including O(a) counterterms

$$D_{\rm w} \rightarrow D_{\rm w} + a c_{\rm sw} \tfrac{i}{4} \sigma_{\mu\nu} F_{\mu\nu}$$

$$A_\mu^k \rightarrow A_\mu^k + a c_{\rm A} \partial_\mu P^k$$

With properly tuned $c_{\rm sw}$ and $c_{\rm A}$

$$\left\langle \left\{ \partial_\mu A_\mu^k(x) - 2mP^k(x) \right\} \Phi_1(y_1) \ldots \right\rangle = \text{contact terms} + {\rm O}(a^2)$$

Symanzik '80

Sheikholeslami & Wohlert '85; ML, Sint, Sommer & Weisz '96; ...

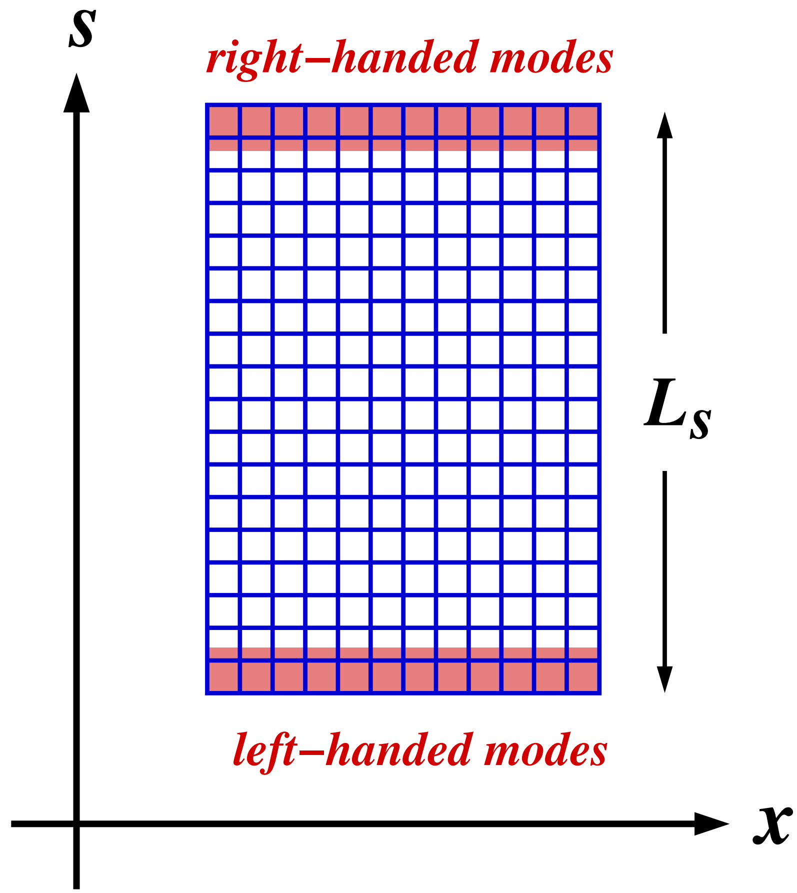
The residual symmetry violations are small at $a \leq 0.1\,{\rm fm}$

. . . can actually do much better

Domain-wall fermions

4d lattice Dirac operator D
satisfying $\{\gamma_5, D\} = aD\gamma_5 D$

Exact chiral symmetry



Ginsparg & Wilson '82; Kaplan '92; Shamir '93

Hasenfratz '98; Hasenfratz, Niedermayer & Laliena '98; Neuberger '98; ML '98; . . .

However, this adds an extra dimension $\Rightarrow$ "expensive"

Why are QCD simulations so difficult?

MC methods require $\mathbb{C}$-number fields & non-negative measures

Light-quark determinant

$$(\det D_{\rm w})^2 = \int {\rm D}[\phi]\, {\rm e}^{-S_{\rm pf}[\phi]} \qquad \text{(if } m_u = m_d = m\text{)}$$

$$S_{\rm pf}[\phi] = a^4 \sum_x \phi(x)^\dagger (D_{\rm w}^\dagger D_{\rm w})^{-1} \phi(x)$$

There are pseudo-fermion representations for the heavier quarks too, and also for $m_u \neq m_d$

The total action is now real and bounded from below but non-local

There is a hierarchy of scales

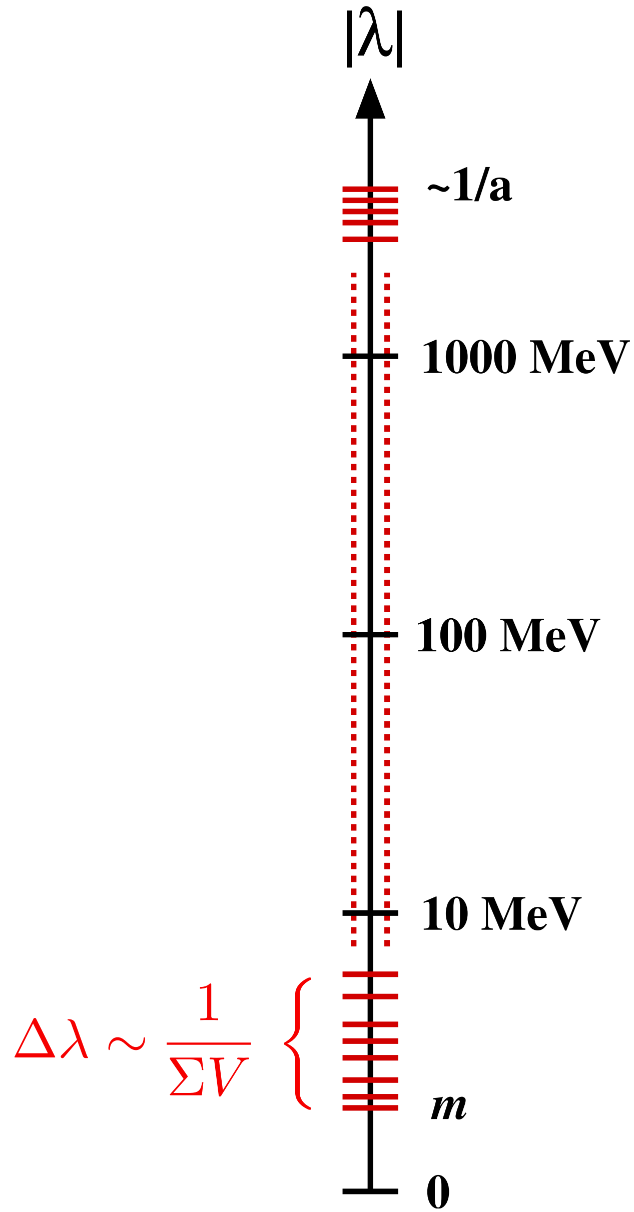
$$m_u, m_d \ll m_\pi \ll 4\pi F_\pi$$

linked to the spontaneous breaking of chiral symmetry

Leutwyler '74; Leutwyler & Smilga '92

⇒ condition number $\lambda_{\max}/\lambda_{\min}$ is large

⇒ computation of $D_{\rm w}^{-1}\phi$ is expensive



> *Simulation cost scales like* $a^{-7}m^{-3}\,V^{1.25}$
>
> *Need* $100\,\mathrm{Tflops}$ *computers for realistic simulations*
>
> *Lattice conference Berlin 2001*

Current strategies in lattice QCD

Modify the lattice theory

so as to avoid $a \ll 0.1\mathrm{fm}$

Block spin RG, perfect action approach

Wilson '79

Hasenfratz & Niedermayer '94

May be too complicated

Staggered quarks + fat links + 4^{th}-root

HPQCD, MILC, UKQCD & Fermilab collaborations '04

Violates basic principles

Build your own computer

The latest machines

- apeNEXT INFN '05
- QCDOC Columbia '05
- PACS-CS Tsukuba '06

deliver $\sim 10\,\mathrm{Tflops}$

Increasingly hard to beat the computer industry

Develop better methods

but keep theory simple

Preconditioning, error reduction techniques

Hasenbusch '01; . . .

Finite-size scaling

ALPHA collaboration '92

Try to teach physics to the algorithms

Looking for better techniques . . .

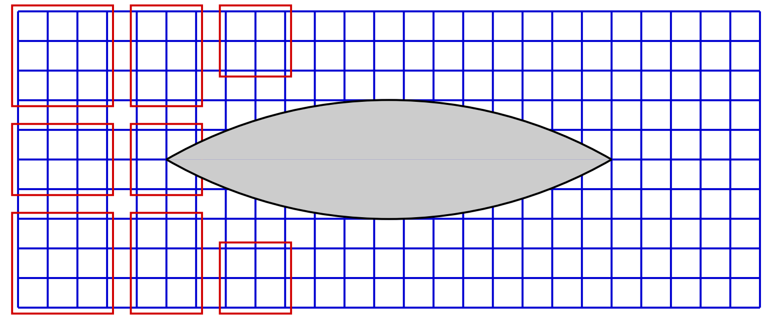
© Alinghi team

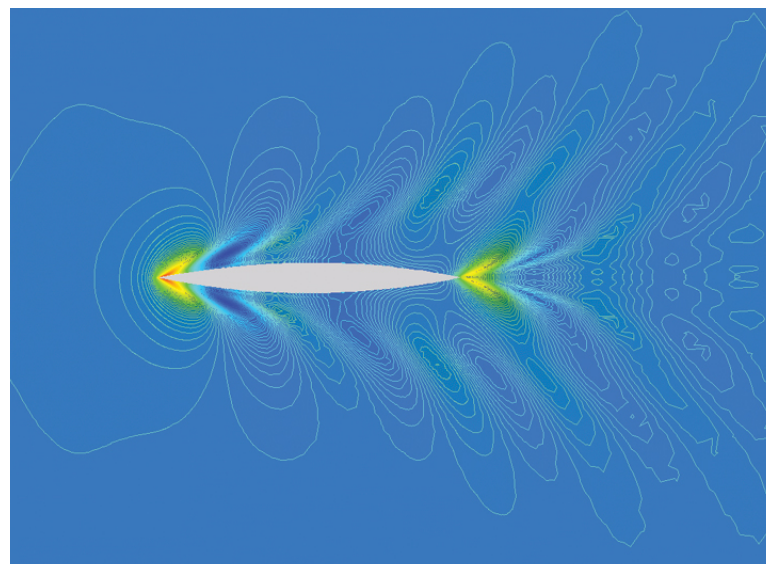
Water flow and wave calculation

* Solve Reynolds-Averaged Navier-Stokes equations

* Mesh discretization

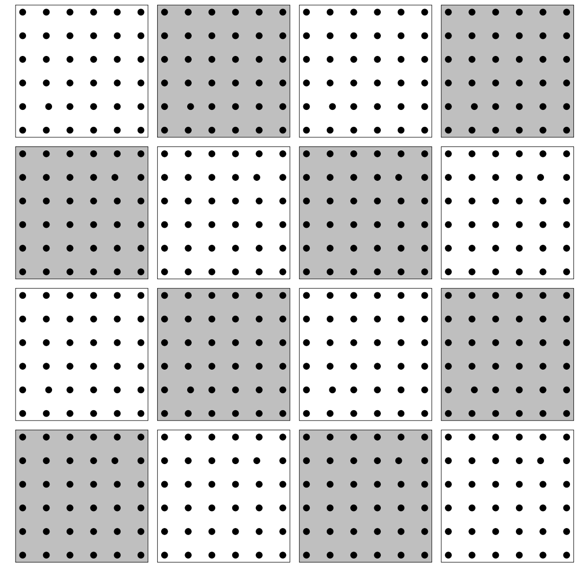
* Domain decomposition and multigrid methods





EPFL, J. Wynne '03

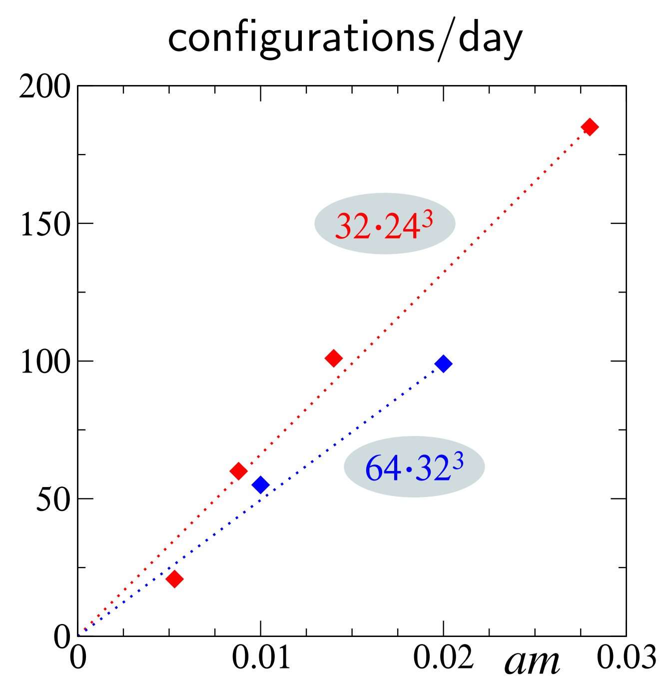
Using domain decomposition methods in lattice QCD



- Computation of $D_{\mathrm{w}}^{-1}\phi$
- Simulation algorithm $(m_u = m_d)$
  - ∗ Effort grows like $\sim m^{-1}$ only
  - ∗ High parallel efficiency

ML CPC 156 (2004) 209; CPC 165 (2005) 199

del Debbio, Giusti, ML, Petronzio & Tantalo '05



Let's go into some details . . .

The quark determinant factorizes

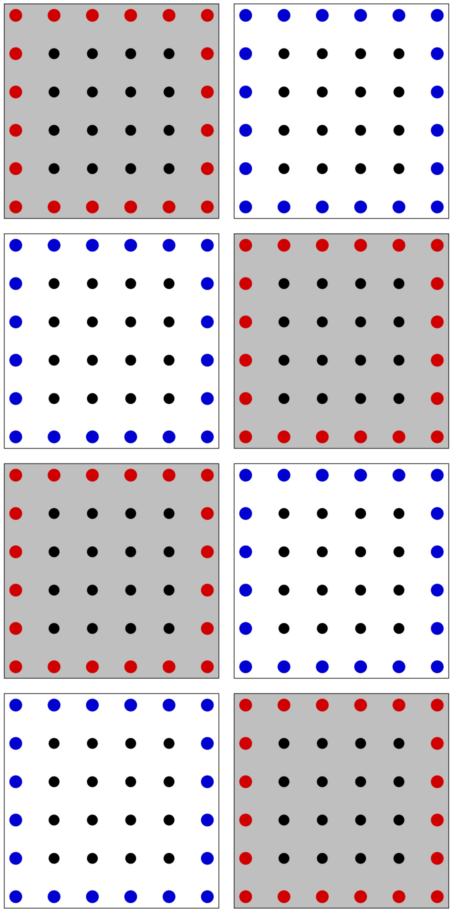
$$\det D_\mathrm{w} = \prod_{\text{blocks } \Lambda} \det D_\Lambda \times \det R$$

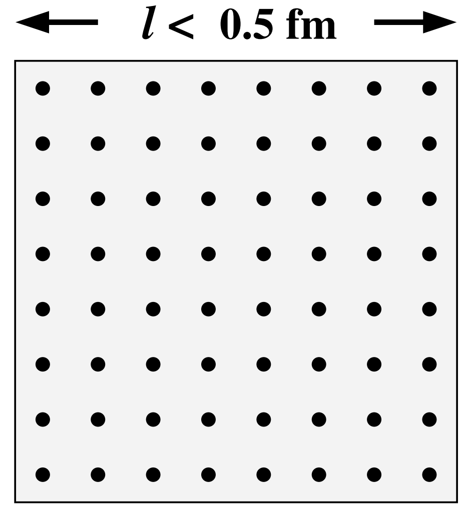
$\uparrow$

D_w with Dirichlet b.c.

where the block interaction is given by

$$R = 1 - \sum_{\text{pairs } \Lambda,\Lambda^*} D_\Lambda^{-1} D_{\partial\Lambda} D_{\Lambda^*}^{-1} D_{\partial\Lambda^*}$$





On the blocks an infrared cutoff

$$q \geq \pi/l > 1\,\mathrm{GeV}$$

is implied by the boundary conditions

⇒ *theory is weakly coupled*

⇒ *easy to simulate at all quark masses*

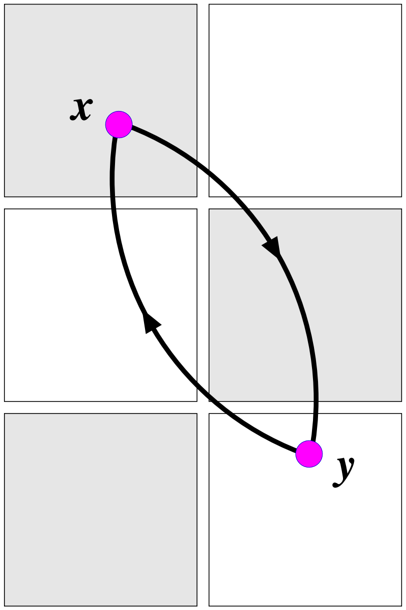
In other words

$$\det D_{\mathrm{w}} = \underbrace{\prod_{\text{blocks }\Lambda} \det D_{\Lambda}}_{\text{easy}} \times \underbrace{\det R}_{\text{long range}}$$

The block-interactions are actually weak

$$\frac{\delta^2 (\ln \det D_{\rm w})}{\delta A_\mu^a(x) \delta A_\nu^b(y)} =$$

$$\mathrm{tr}\{T^a \gamma_\mu S(x,y) T^b \gamma_\nu S(y,x)\} \sim |x-y|^{-6}$$



$\Rightarrow$ $\det R$ is a small correction

•
•
•

$\Rightarrow$ exact simulation algorithm that exploits these facts

First studies using the new algorithm

Del Debbio, Giusti, M.L., Petronzio, Tantalo [CERN – Tor Vergata]

Two-flavour QCD, $m_u = m_d$, without O(a) counterterms

lattice	a [fm]	$\sim m/m_{\mathrm{s}}$	m_π [MeV]	N_{cnfg}
$32 \cdot 24^3$	0.080	0.93	676	64
		0.48	484	95
		0.30	381	94
		0.17	294	100
$64 \cdot 32^3$	0.064	0.75	606	100
		0.38	429	101
		0.25	350	running

Simulations performed on 8 nodes of a PC cluster at the ITP Bern and on 64 nodes at the Fermi Institute

Chiral behaviour of m_π and F_π

SU(2) ChPT predicts

$$m_\pi^2 = M^2 R_\pi, \quad M^2 = 2Bm$$

$$R_\pi = 1 + \frac{M^2}{32\pi^2 F^2} \ln(M^2/\Lambda_\pi^2) + \ldots$$

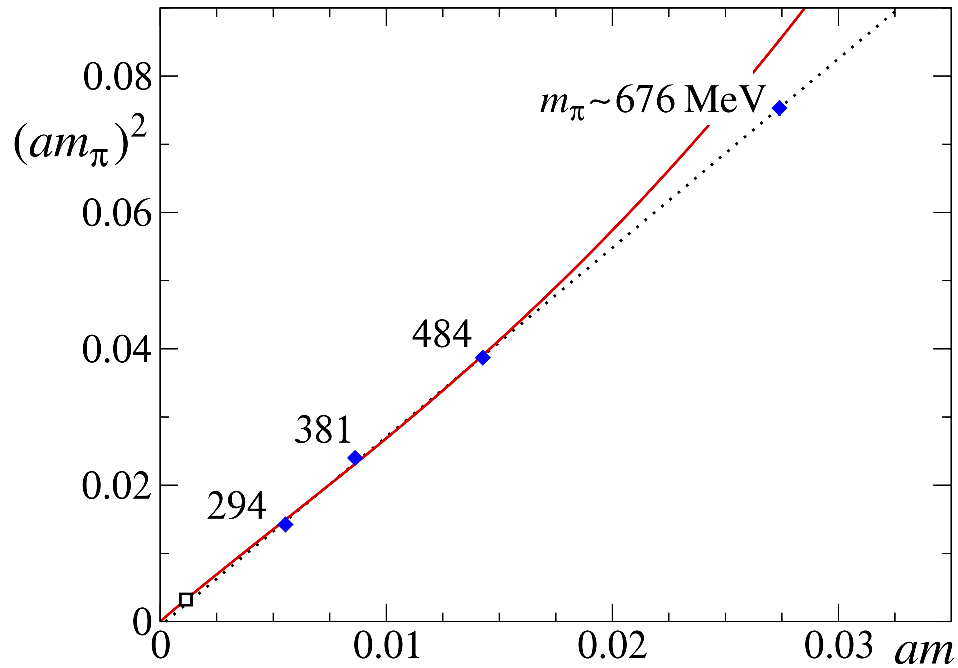
where, in real-world QCD,

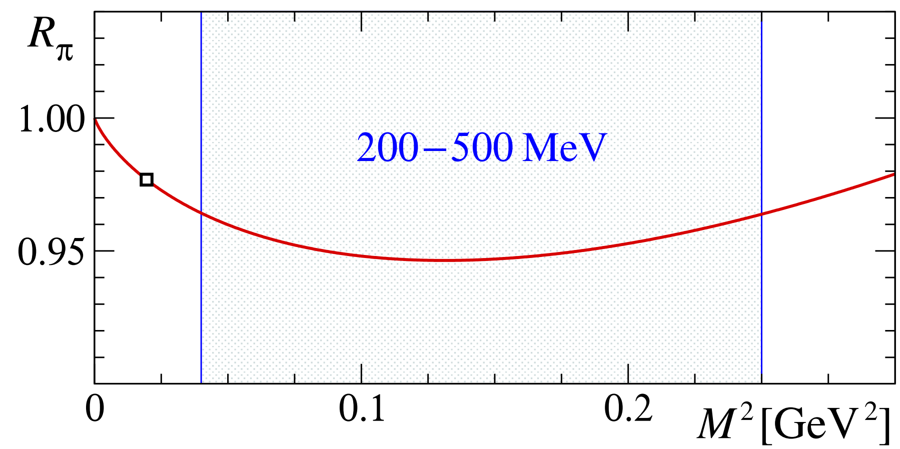
$$\ln(\Lambda_\pi^2/M^2)\big|_{M=140\,\mathrm{MeV}} \simeq 2.9 \pm 2.4$$

Gasser & Leutwyler '84

$\Rightarrow$ $R_\pi \simeq$ constant $= 0.956(8)$ in the range $M = 200 - 500\,\mathrm{MeV}$

$32 \cdot 24^3$ lattice





For F_π we expect

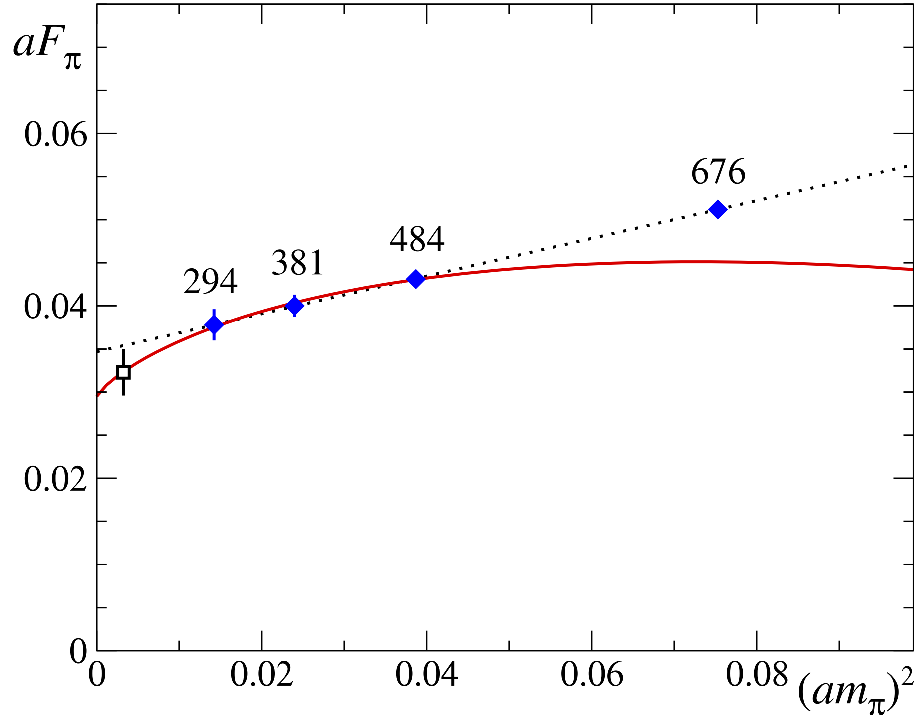
$$F_\pi = F - \frac{M^2}{16\pi^2 F} \ln(M^2/\Lambda_F^2) + \dots$$

$$\ln(\Lambda_F^2/M^2)\big|_{M=140\,\mathrm{MeV}} \simeq 4.6 \pm 0.9$$

This fits the last three points

$$\Rightarrow F_\pi\big|_{M=140\,\mathrm{MeV}} = 80(7)\,\mathrm{MeV}$$

$32 \cdot 24^3$ lattice



Up to $m_\pi \sim 500\,\mathrm{MeV}$, the data are compatible with 1-loop ChPT

Needs to be confirmed at smaller masses and several lattice spacings

Conclusions & perspectives

Numerical simulations of lattice QCD with light sea quarks are much less "expensive" than previously estimated**!**

$\Rightarrow$ it is now possible to reach the chiral regime on large lattices

Example

$96 \cdot 48^3$ lattice, $a = 0.06\,\mathrm{fm}$, $m_\pi = 200 - 300\,\mathrm{MeV}$

To simulate this lattice, a (current) PC cluster with 288 nodes should be sufficient

What next?

A wide range of physics questions may now be addressed

- $\pi\pi$ scattering & the ρ resonance
- Properties of the nucleons
- Charm physics

More technical directions to explore are

- Including O(a) counterterms
- Adding the strange sea quark
- Ginsparg–Wilson valence fermions (B_K, $K \to \pi\pi$, ...)