

# Measuring $M_{NS}$ , $R_{NS}$ , $M_{NS}/R_{NS}$ or $R_{\infty}$

Sebastien Guillot

Advisor: Robert Rutledge

*Galileo Galilei Institute, Firenze*

*March 2014*

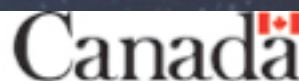


Government  
of Canada

Vanier Canada  
Graduate Scholarships

Gouvernement  
du Canada

Bourses d'études  
supérieures du Canada Vanier



# Some Reviews

Lattimer and Prakash, 2007

Miller C., 2013

Heinke et al., 2013

## Reminder Labels



# Measuring $M_{NS}$



# Double neutron stars binary systems lead to precise $M_{NS}$ measurements

## Double NS system PSR B1913+16

Measured Orbital Parameters for PSR B1913+16

Fitted Parameter	Value
$a_p \sin i$ (s) .....	2.3417725 (8)
$e$ .....	0.6171338 (4)
$T_0$ (MJD) .....	52144.90097844 (5)
$P_b$ (d) .....	0.322997448930 (4)
$\omega_0$ (deg) .....	292.54487 (8)
$\langle \dot{\omega} \rangle$ (deg/yr) .....	4.226595 (5)
$\gamma$ (s) .....	0.0042919 (8)
$\dot{P}_b$ ( $10^{-12}$ s/s)...	-2.4184 (9)

Depends on  $M_{PSR}$  and  $M_{comp}$

### Double-NS system PSR B1913+16

Best  $M_{NS}$  measurement

$$M_{PSR} = 1.4414 \pm 0.0002 M_{\odot}$$

(Weisberg et al. 2005)

### Double-Pulsar system PSRJ0737-3039

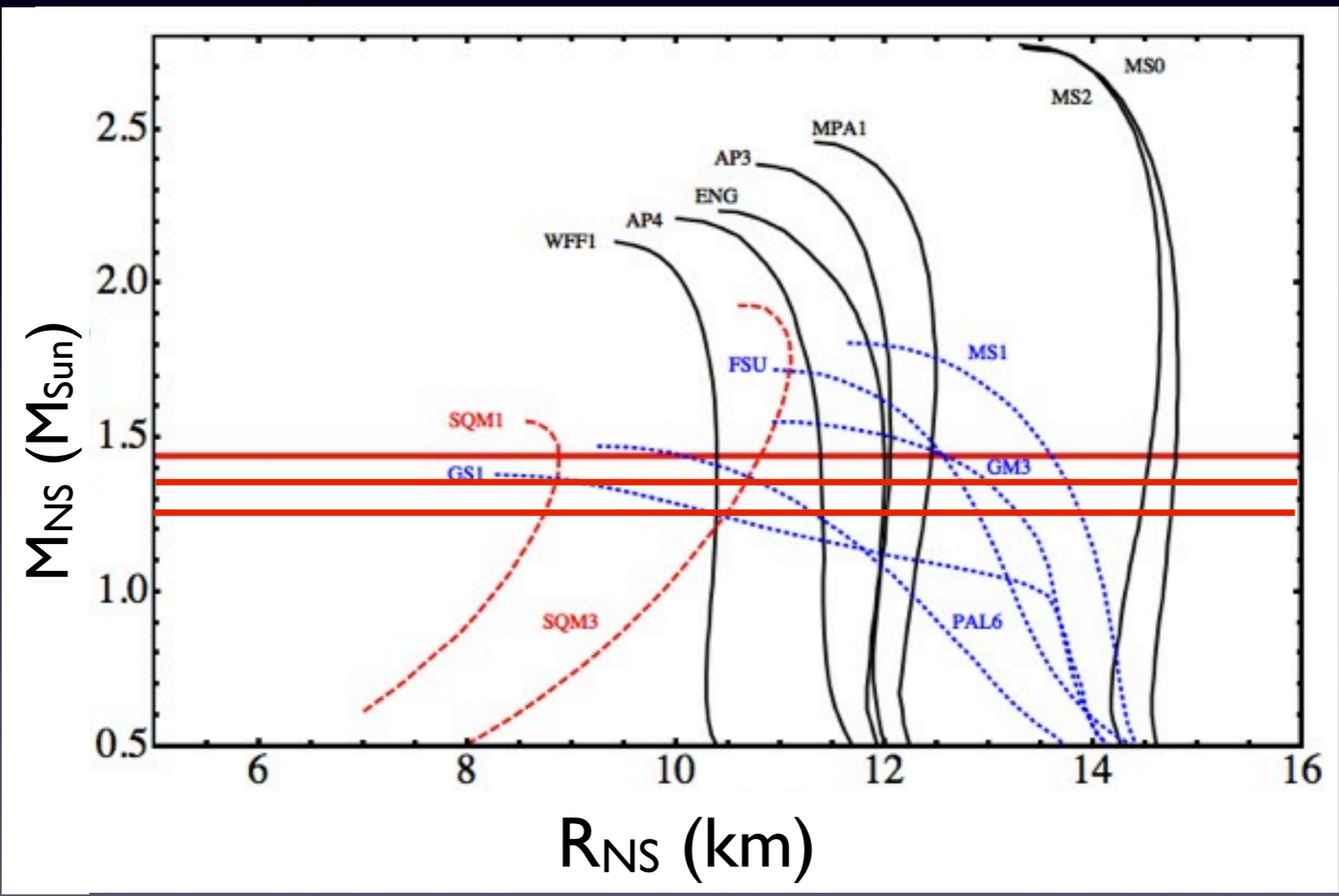
$$M_{PSRA} = 1.3381 \pm 0.0007 M_{\odot}$$

$$M_{PSRB} = 1.2489 \pm 0.0007 M_{\odot}$$

(Kramer et al. 2006)



# Double neutron stars binary systems lead to precise $M_{NS}$ measurements



Not  
constraining  
enough!!

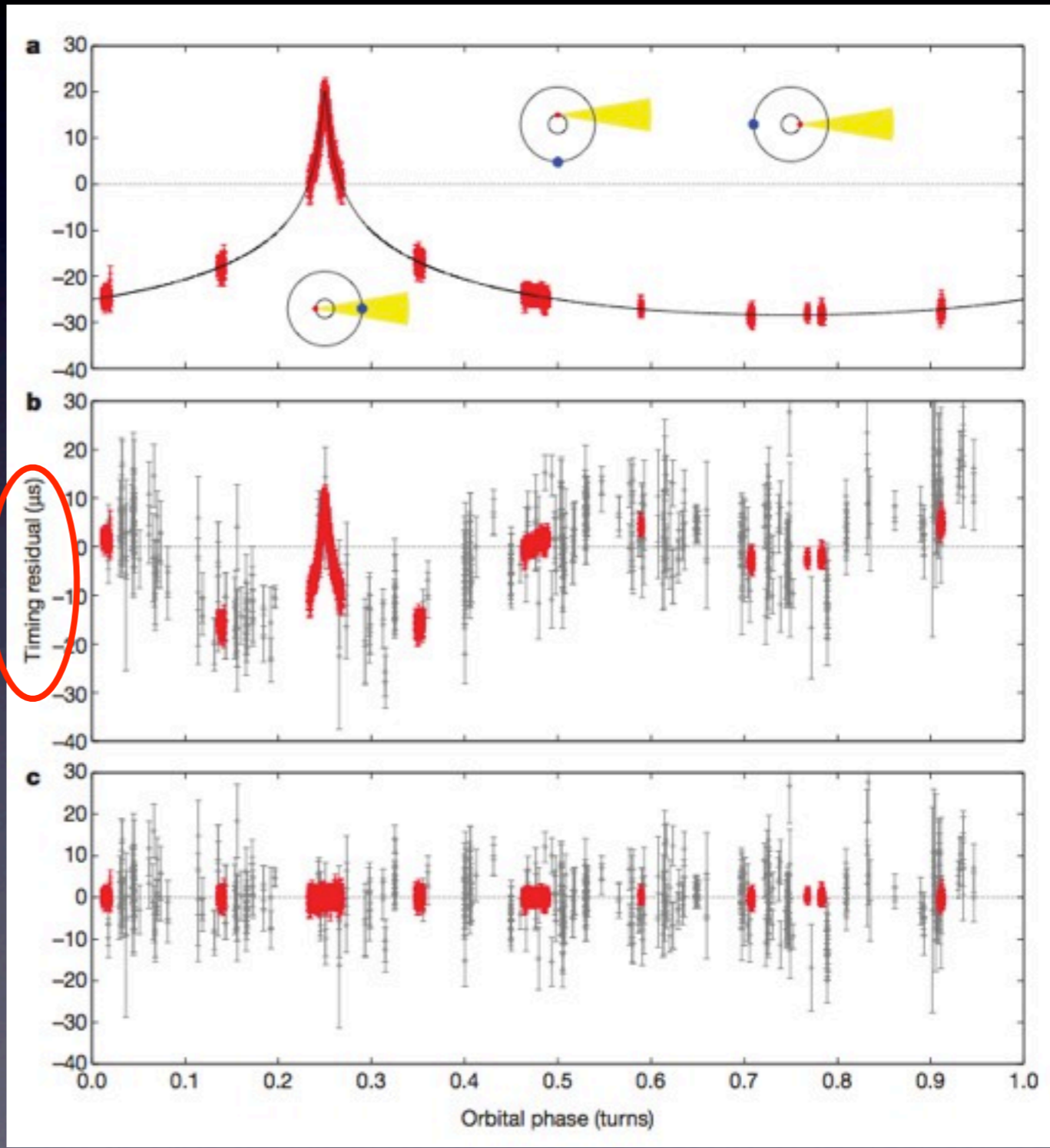


# Neutron stars in binary systems need additional input to get $M_{NS}$

Additional post-Keplerian parameters, e.g. Shapiro delay, to break the degeneracies between mass ratio and inclination

$$M_{PSR} = 1.97 \pm 0.04 M_{\odot}$$

(Demorest et al. 2010)



# Neutron stars in binary systems need additional input to get $M_{\text{NS}}$

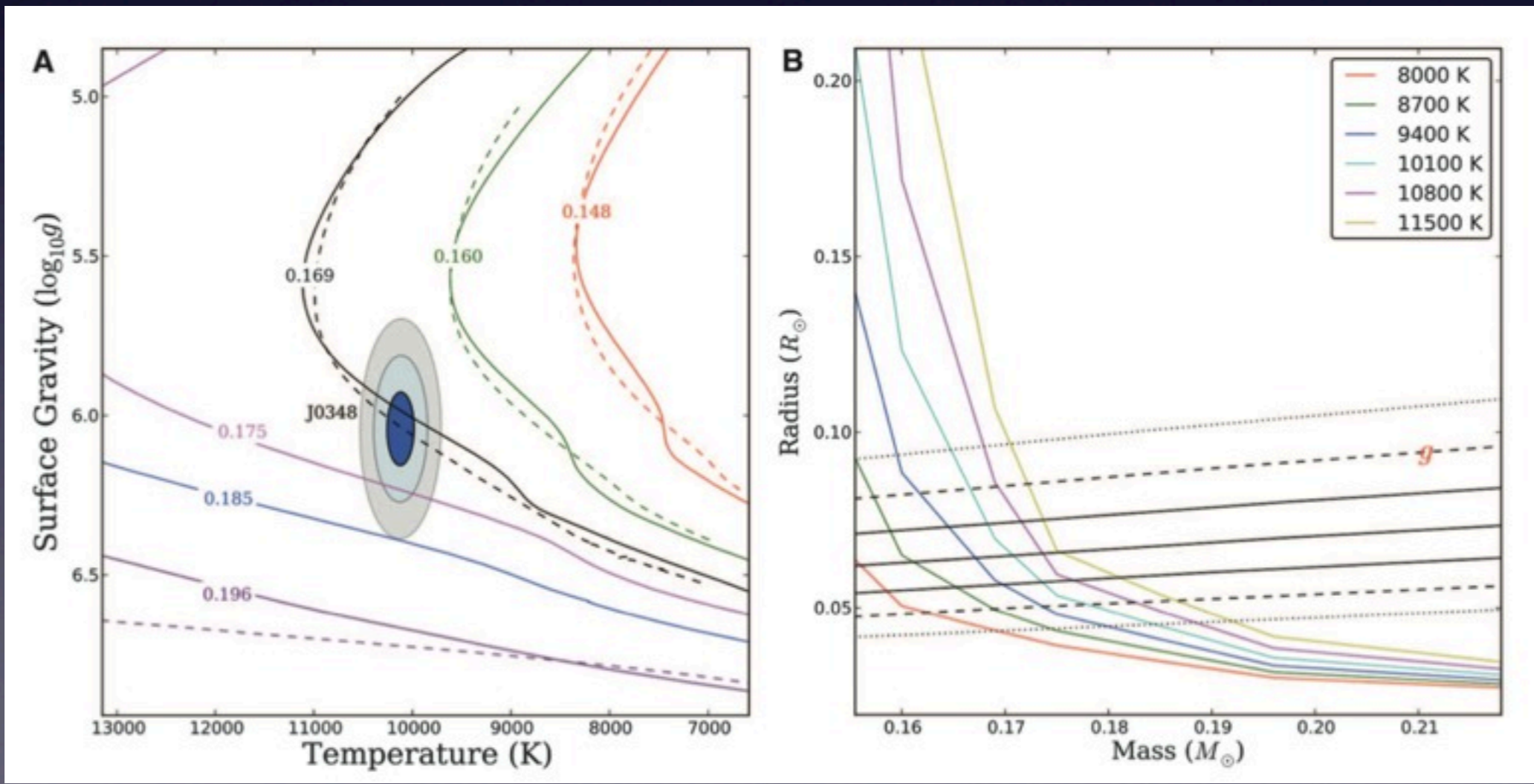


Radio  
Optical

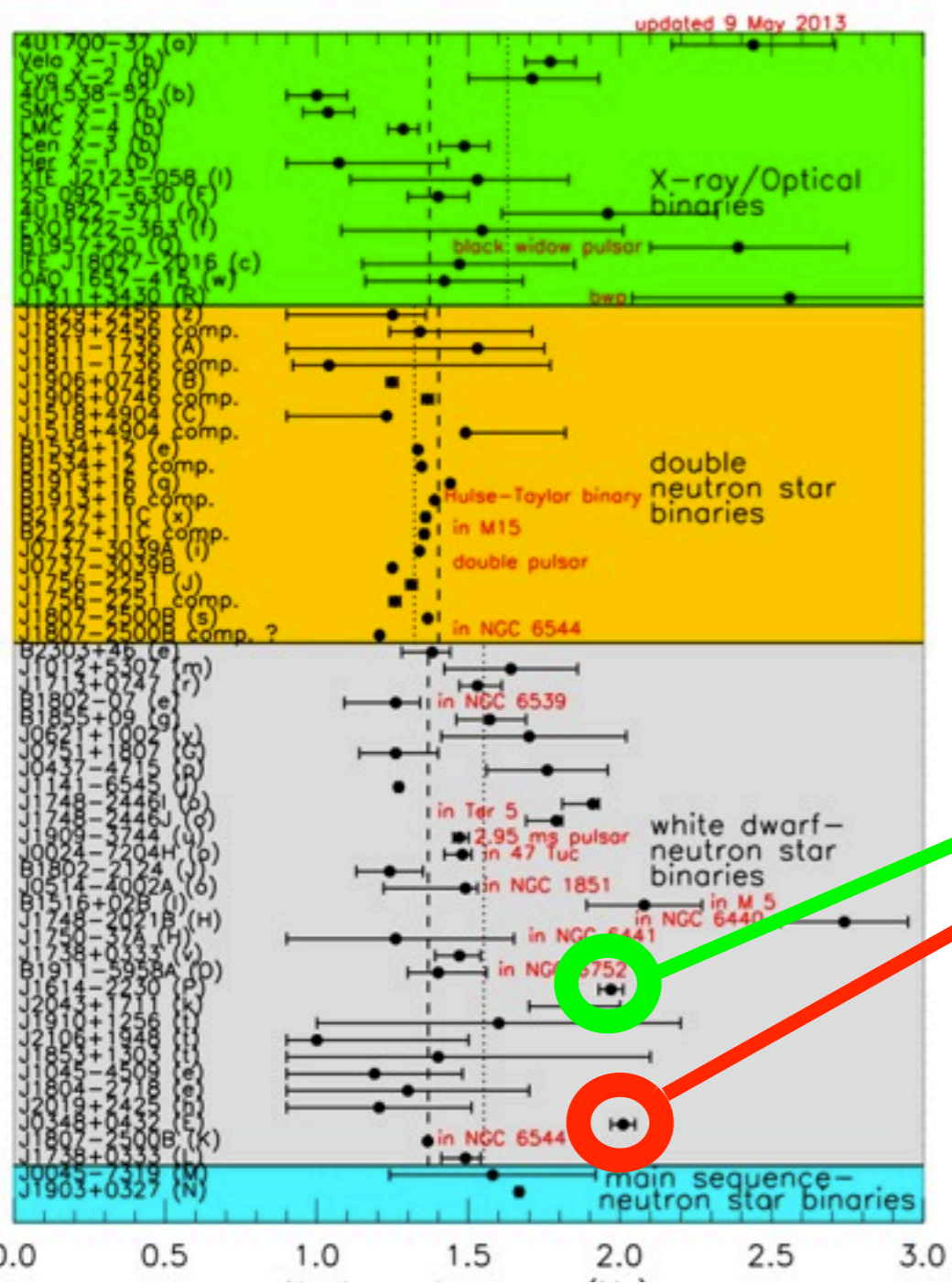
Independent measure of  $M_{\text{comp}}$

e.g., for WD companion to PSR J0348+0432

$$M_{\text{PSR}} = 2.01 \pm 0.04 M_{\odot} \text{ (Antoniadis et al. 2013)}$$



# Only new $M_{NS}$ measurements larger than previous ones improve constraints on the dense matter EoS



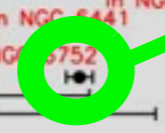
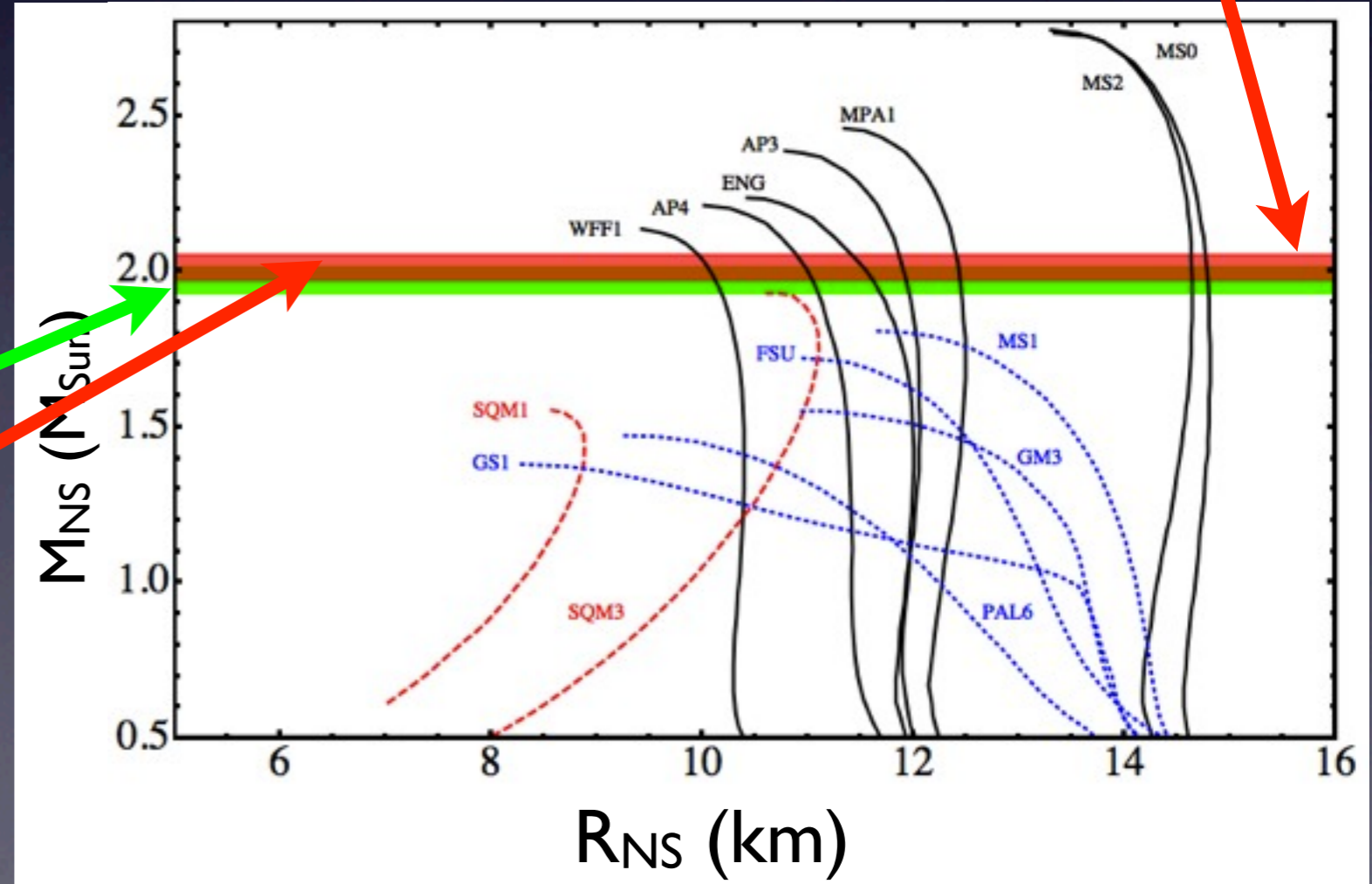
Mass ( $M_{\odot}$ ) Lattimer 2011

PSR J1614-2230

$M_{PSR} = 1.97 \pm 0.04 M_{\odot}$   
(Demorest et al. 2010)

PSR J0348+0432

$M_{PSR} = 2.01 \pm 0.04 M_{\odot}$   
(Antoniadis et al. 2013)





Enough with  $M_{NS}$ ,  
Let's measure  $M_{NS}$  and  $R_{NS}$

Two observables are necessary to measure both  $M_{\text{NS}}$  and  $R_{\text{NS}}$  from Type-I X-ray bursts with PRE



Type-I X-ray Burst with Photospheric Radius Expansion

$$L_{\text{Edd}} = \frac{4\pi GcM_{\text{NS}}}{\kappa}$$

HARDY

# Two observables are necessary to measure both $M_{\text{NS}}$ and $R_{\text{NS}}$ from Type-I X-ray bursts



## Two observables

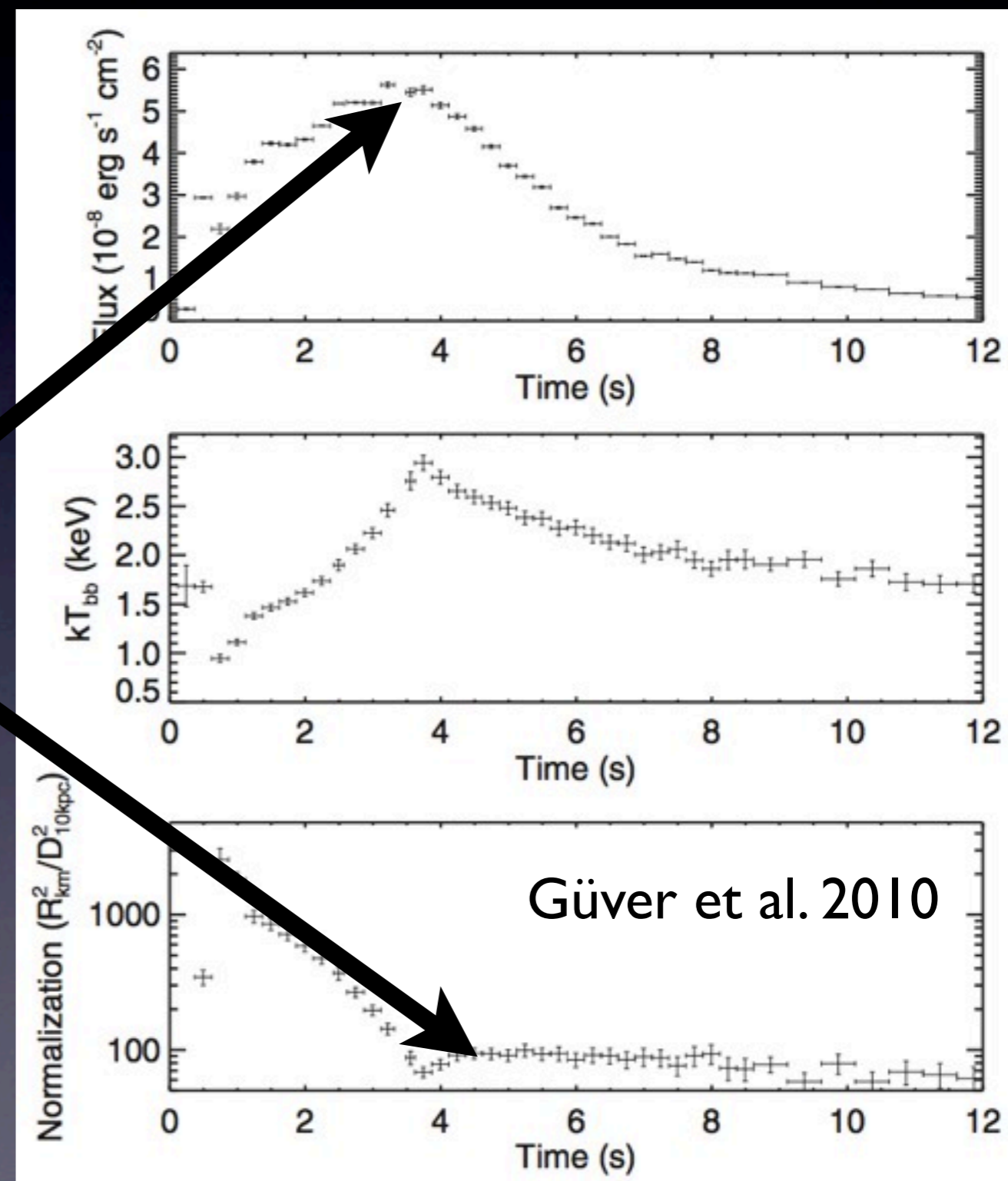
$$F_{\text{Edd},\infty} = \frac{GcM_{\text{NS}}}{\kappa D^2} \frac{1}{(1+z)}$$

$$A_{\infty} = \frac{R^2}{f_c^4 D^2} (1+z)^2$$

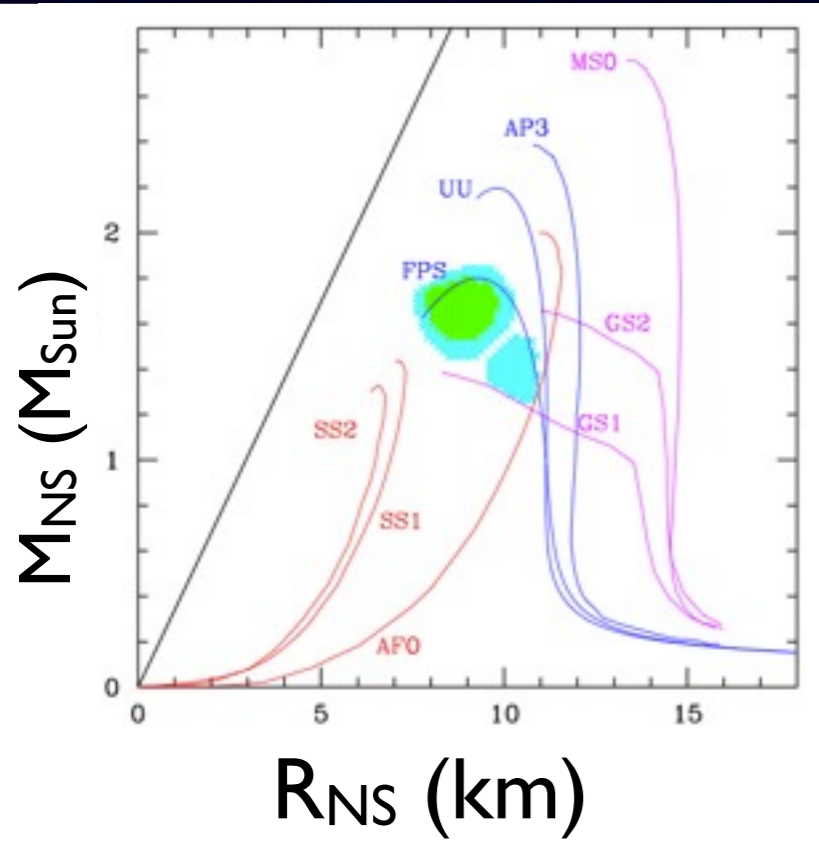
model dependent

with

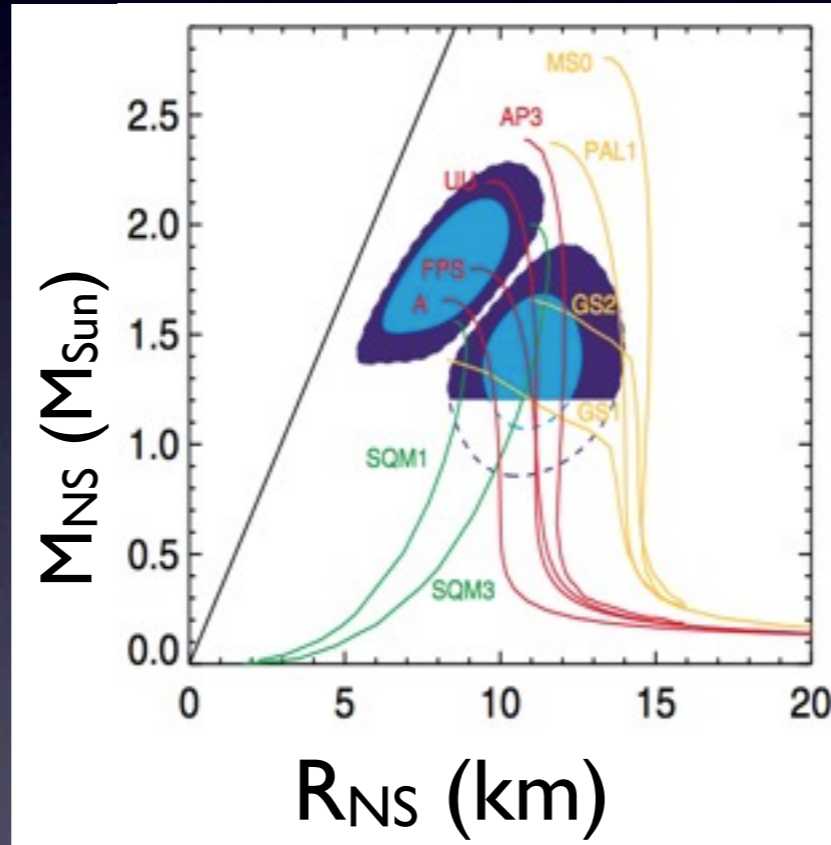
$$(1+z) = \left(1 + \frac{2GM_{\text{NS}}}{c^2 R_{\text{NS}}}\right)^{-1/2}$$



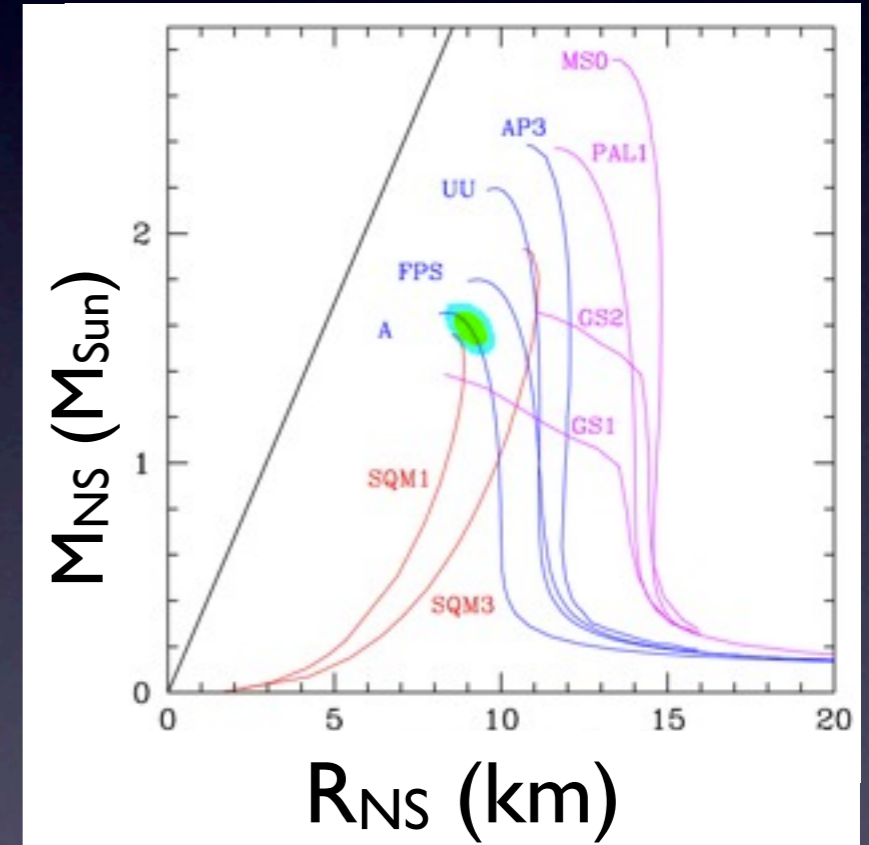
# $M_{\text{NS}}$ and $R_{\text{NS}}$ measurements with “known distances” are very constraining... Or are they?



EXO 1745-348  
in globular cluster  
Terzan 5  
(Özel et al. 2009)

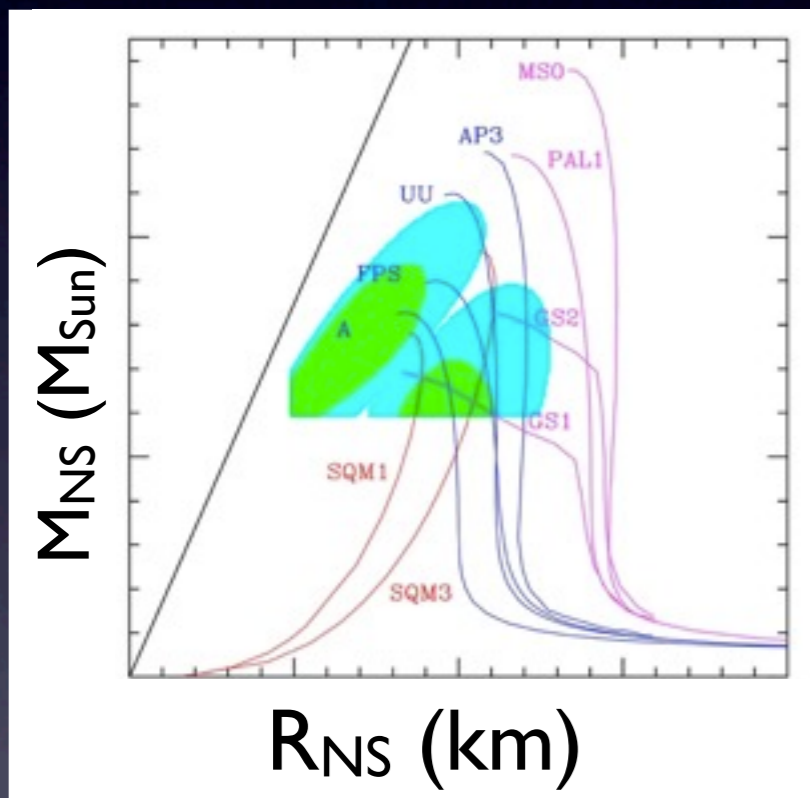


SAX J1748.9-2021  
in globular cluster  
NGC 6440  
(Güver & Özel, 2013)

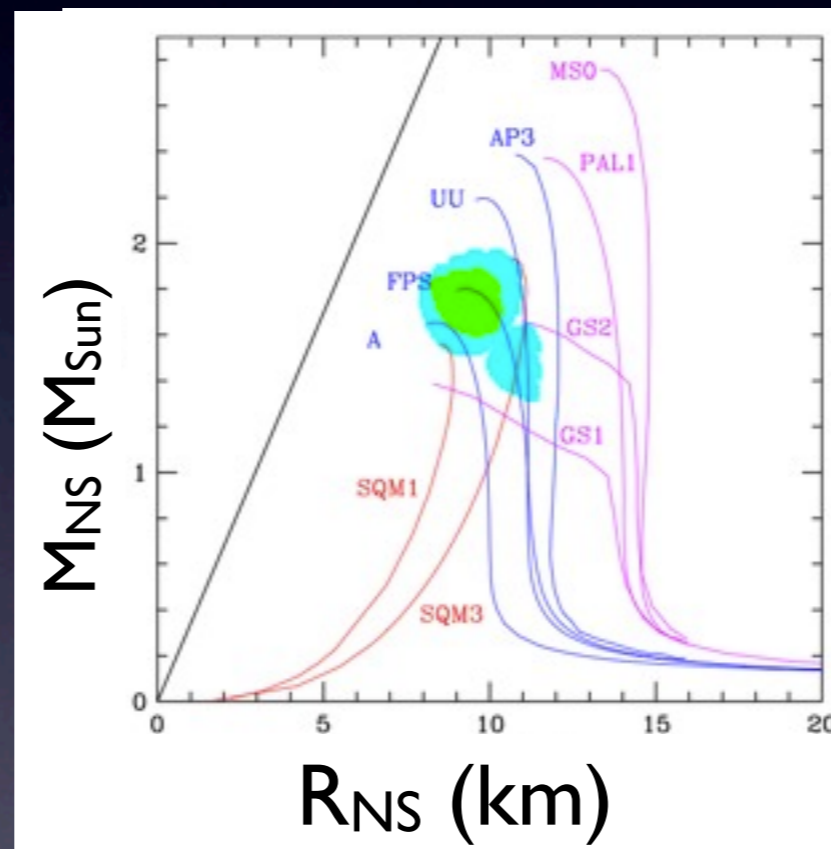


4U 1820-30  
in globular cluster  
NGC 6624  
(Güver et al. 2010a)

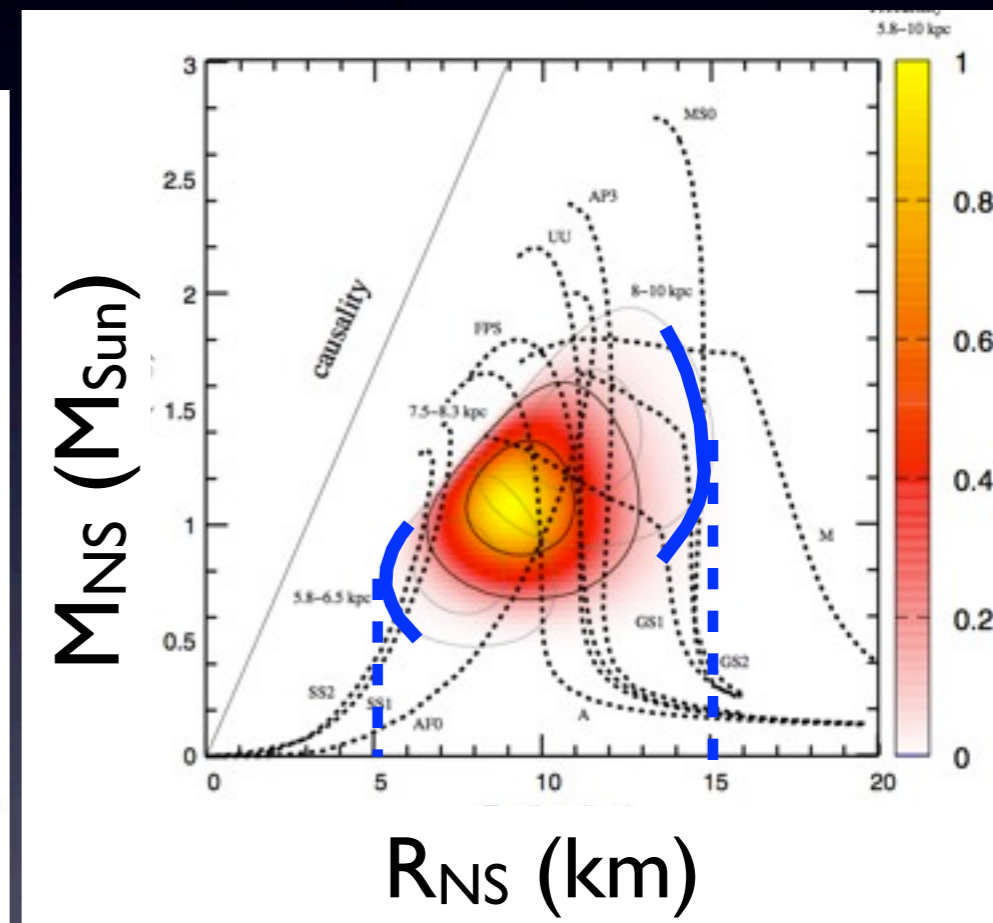
# $M_{\text{NS}}$ and $R_{\text{NS}}$ measurements with “known distances” are very constraining... Or are they?



KS 1731-260  
Distance estimated from distribution of surrounding stars (Özel et al. 2012)



4U 1608-53  
Using surrounding red clump stars (Güver et al. 2010a)



Rapid Buster  
Assuming a wide range of distances (Sala et al. 2013)

# When the distance is unknown, an additional observable is necessary!



$$M = \frac{c^5}{4G\kappa} \left(\frac{R_\infty}{d}\right)^2 \frac{1}{F_{\text{Edd},\infty}} \frac{1}{(1+z)^3} [1 - (1+z)^{-2}]^2$$

$$R = \frac{c^3}{2\kappa} \left(\frac{R_\infty}{d}\right)^2 \frac{1}{F_{\text{Edd},\infty}} \frac{1}{(1+z)^3} [1 - (1+z)^{-2}]^2$$

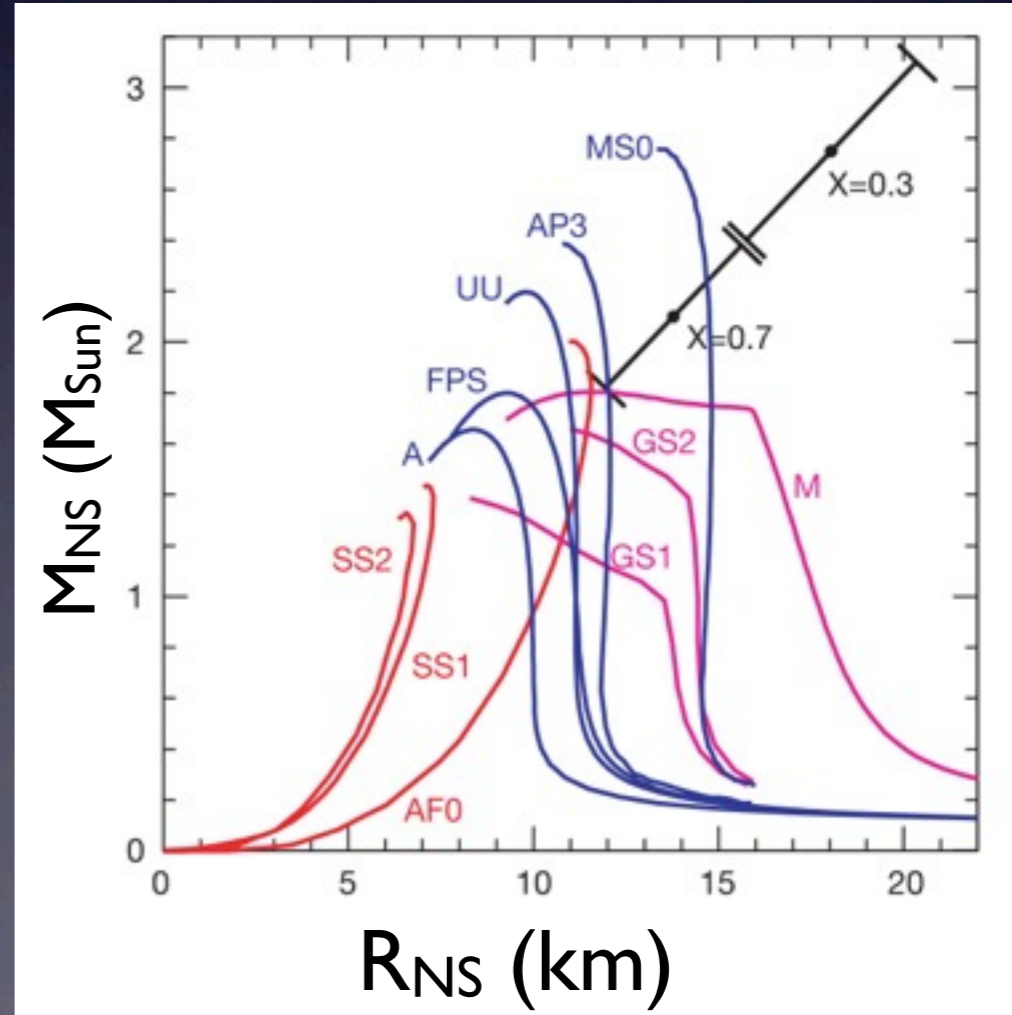
$$d = \frac{c^3}{2G\kappa} \left(\frac{R_\infty}{d}\right) \frac{1}{F_{\text{Edd},\infty}} \frac{1}{(1+z)^2} [1 - (1+z)^{-2}]$$

opacities  $\kappa$  still depends on unknown composition

Gravitational redshift measured from spectral lines in EXO 0748-676  
 $z = 0.35$  (Cottam et al. 2002)

used to measure  $M_{\text{NS}}-R_{\text{NS}}$   
(Ozel et al. 2006)

However, those lines were not confirmed later on (Cottam et al. 2008)

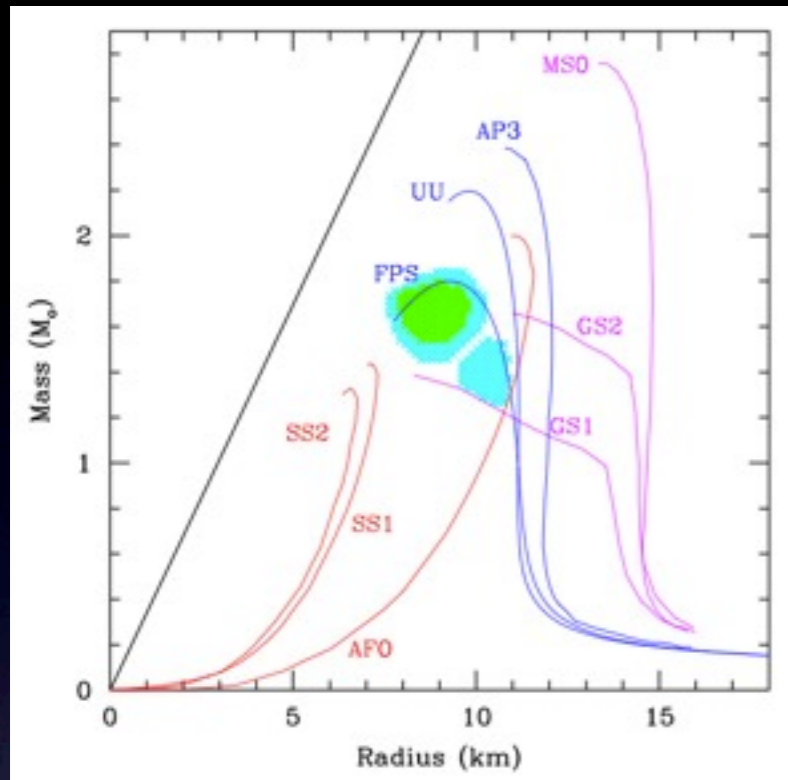




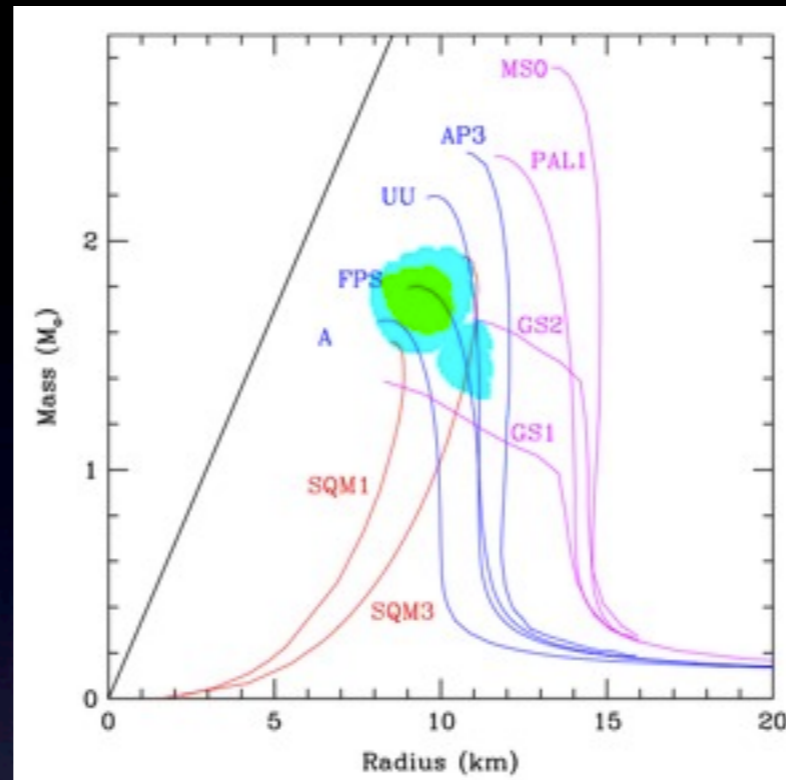
# Type-I X-ray bursts are controversial for $M_{NS}$ and $R_{NS}$ measurements!

- Composition of the atmosphere
- Color correction factor  $f_c$  (constant or not)?
- Distance measurement used (and uncertainties)
- Analysis not self-consistent (Steiner et al. 2010)
- Short bursts do not match passive cooling theory (Suleimanov et al. 2011)

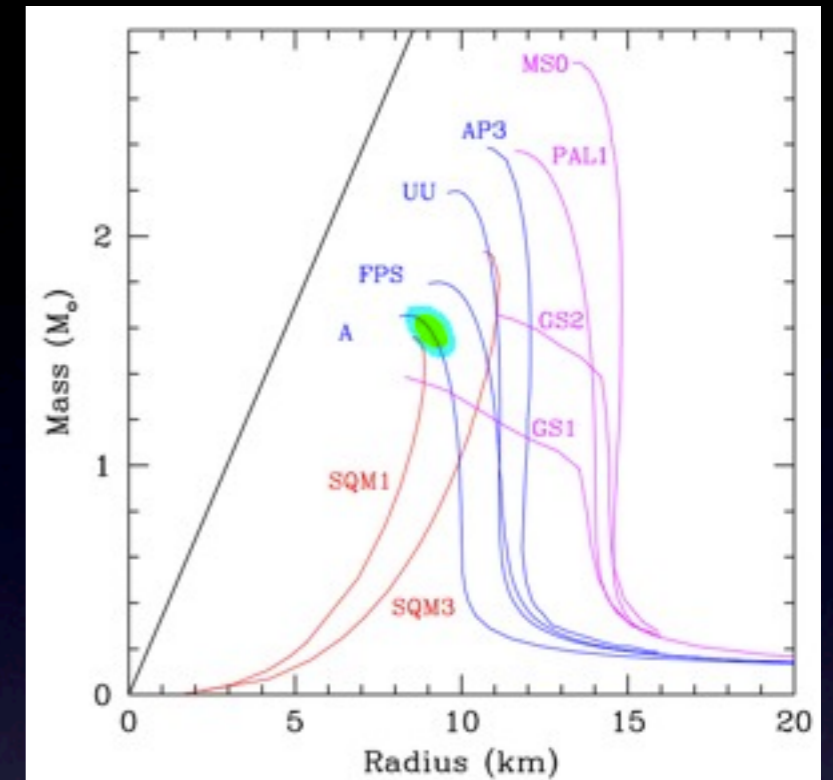
# $M_{\text{NS}}-R_{\text{NS}}$ contours are fixed by relaxing $R_{\text{touchdown}} = R_{\text{NS}}$



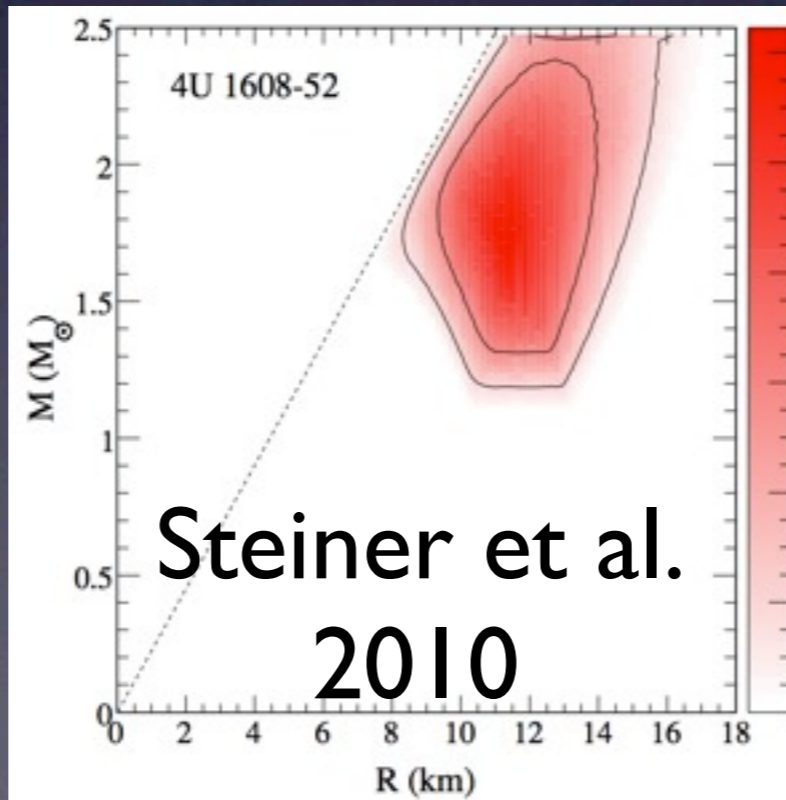
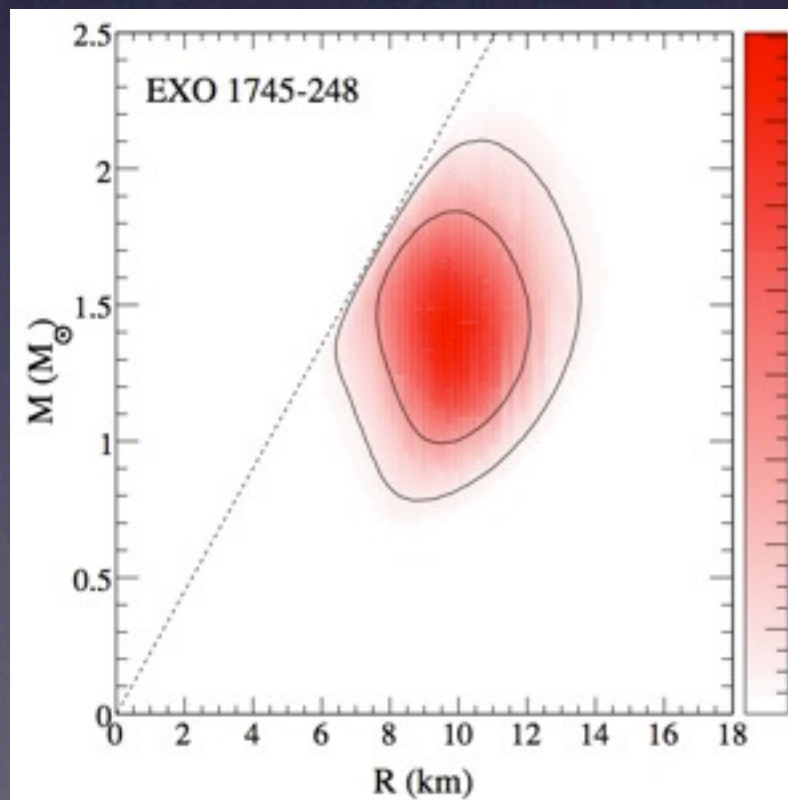
EXO 1745-348  
Özel et al. 2009



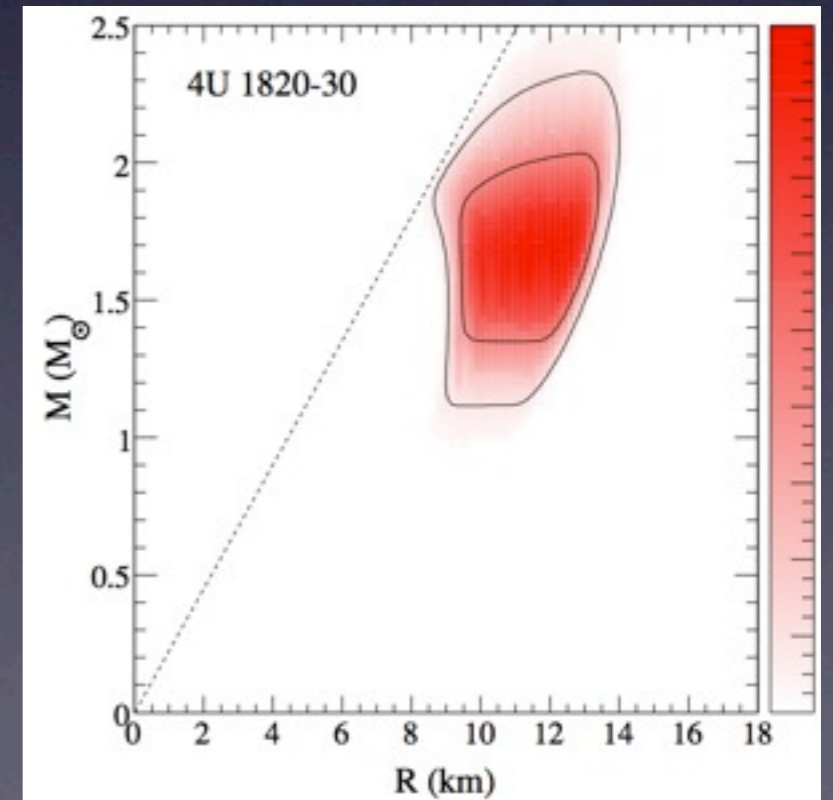
4U 1608-53  
Güver et al. 2010a



4U 1820-30  
Güver et al. 2010a

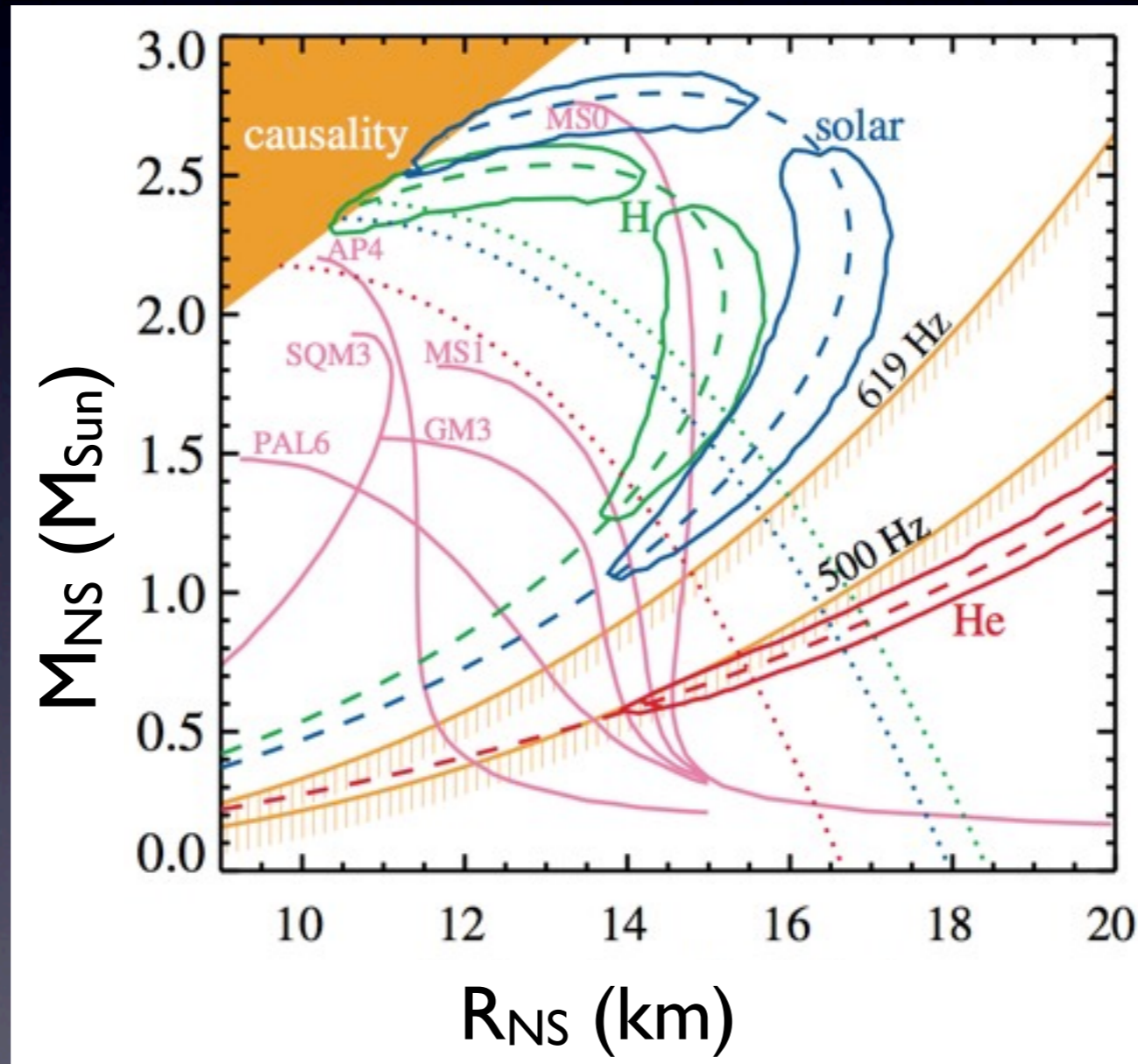


Steiner et al.  
2010





Different constraints are obtained when using long X-ray bursts instead of short X-ray bursts, and by fitting the entire cooling tail



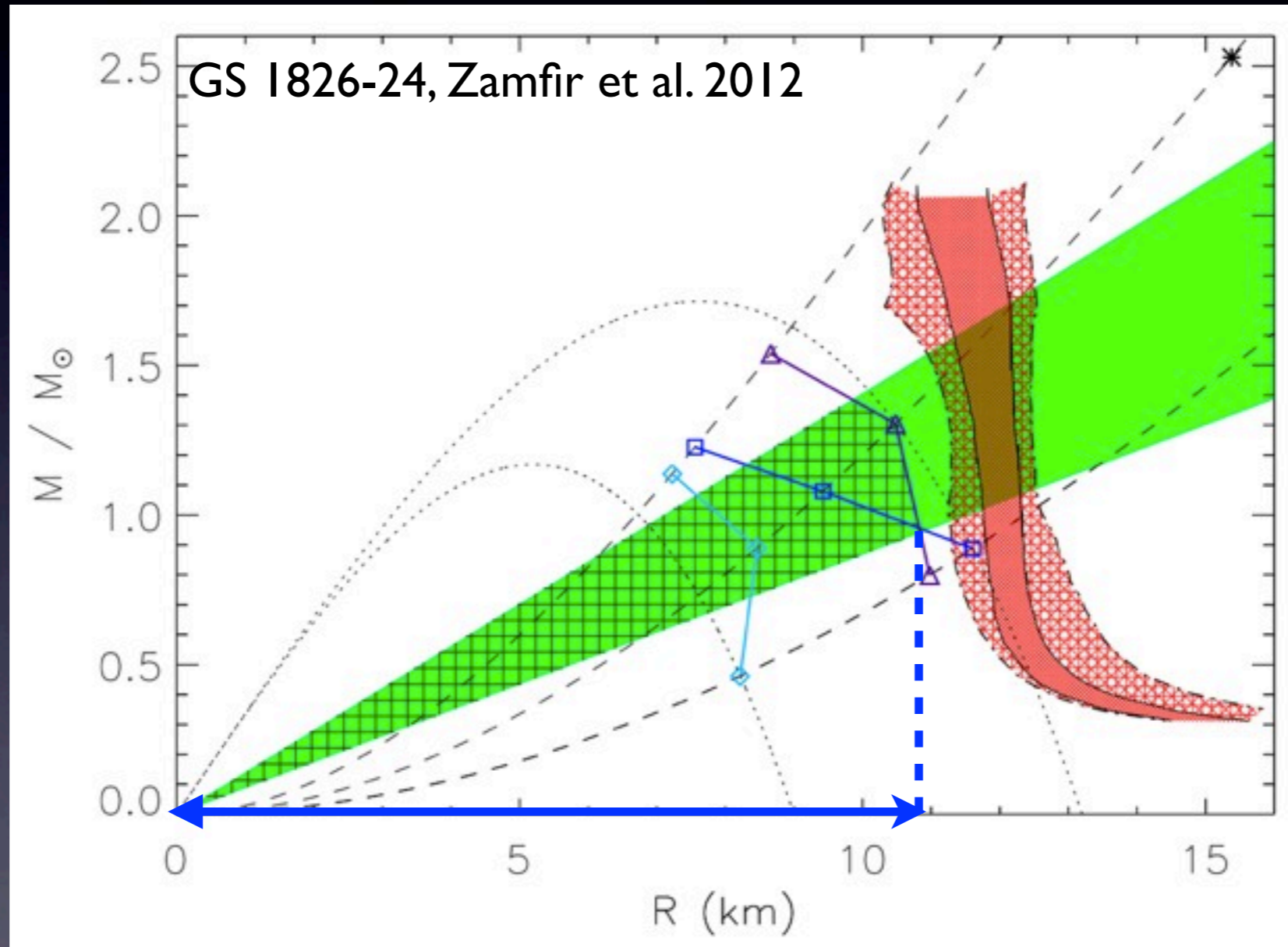
4U 1724-307

(Suleimanov et al. 2012)

Sub-Eddington bursts can also be used to provide distance independent measurements

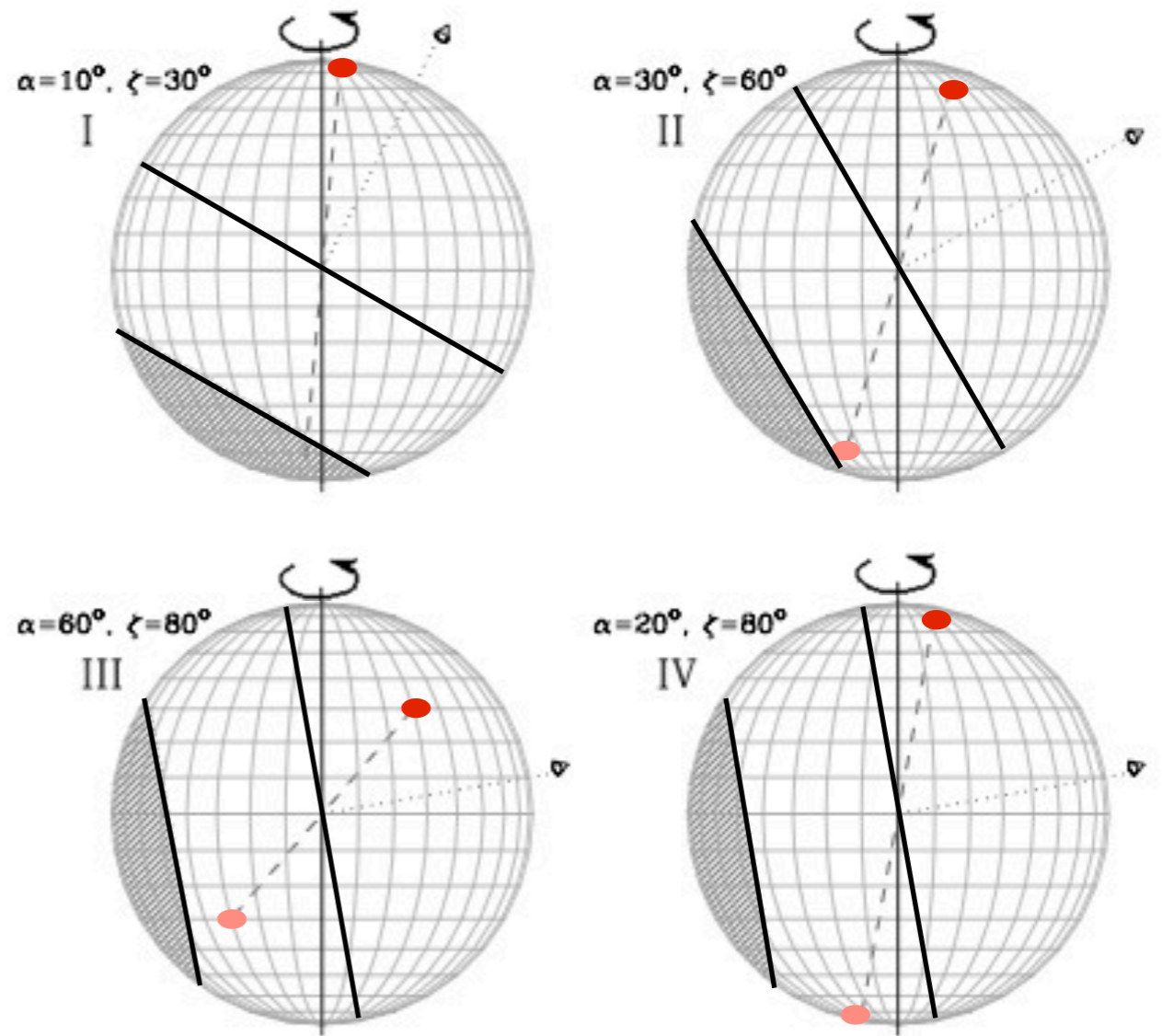
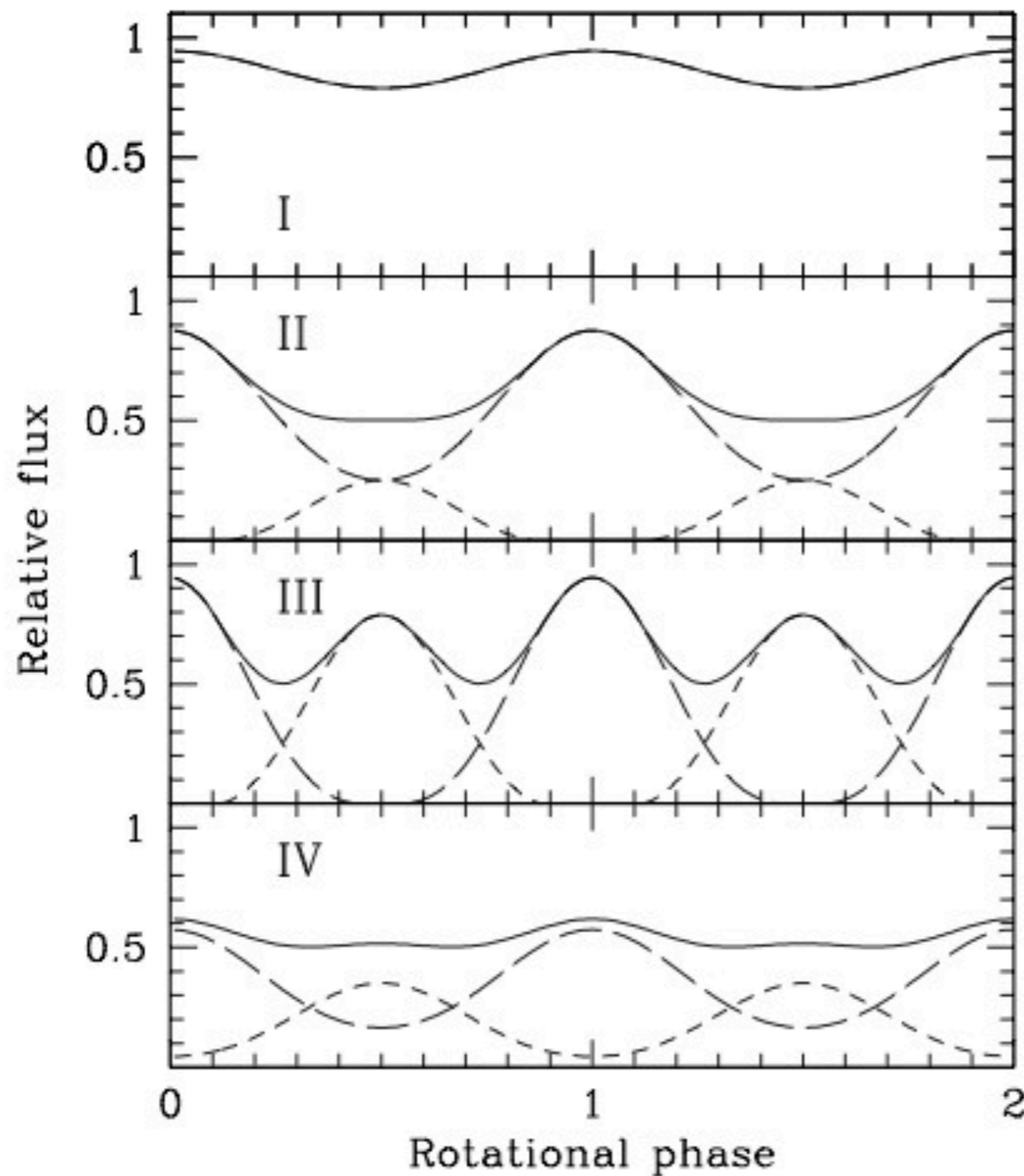


## X-ray burster GS 1826-24



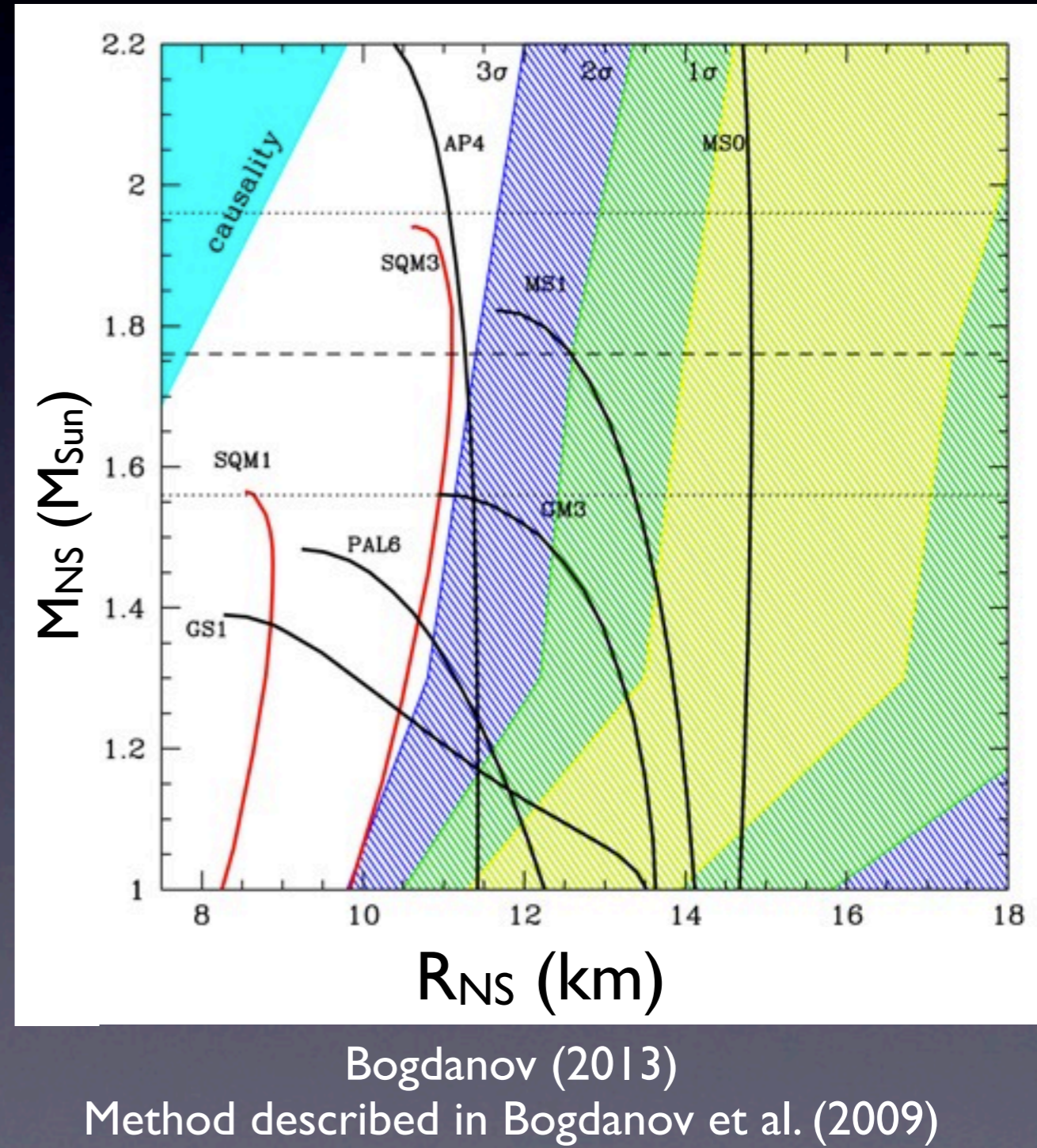
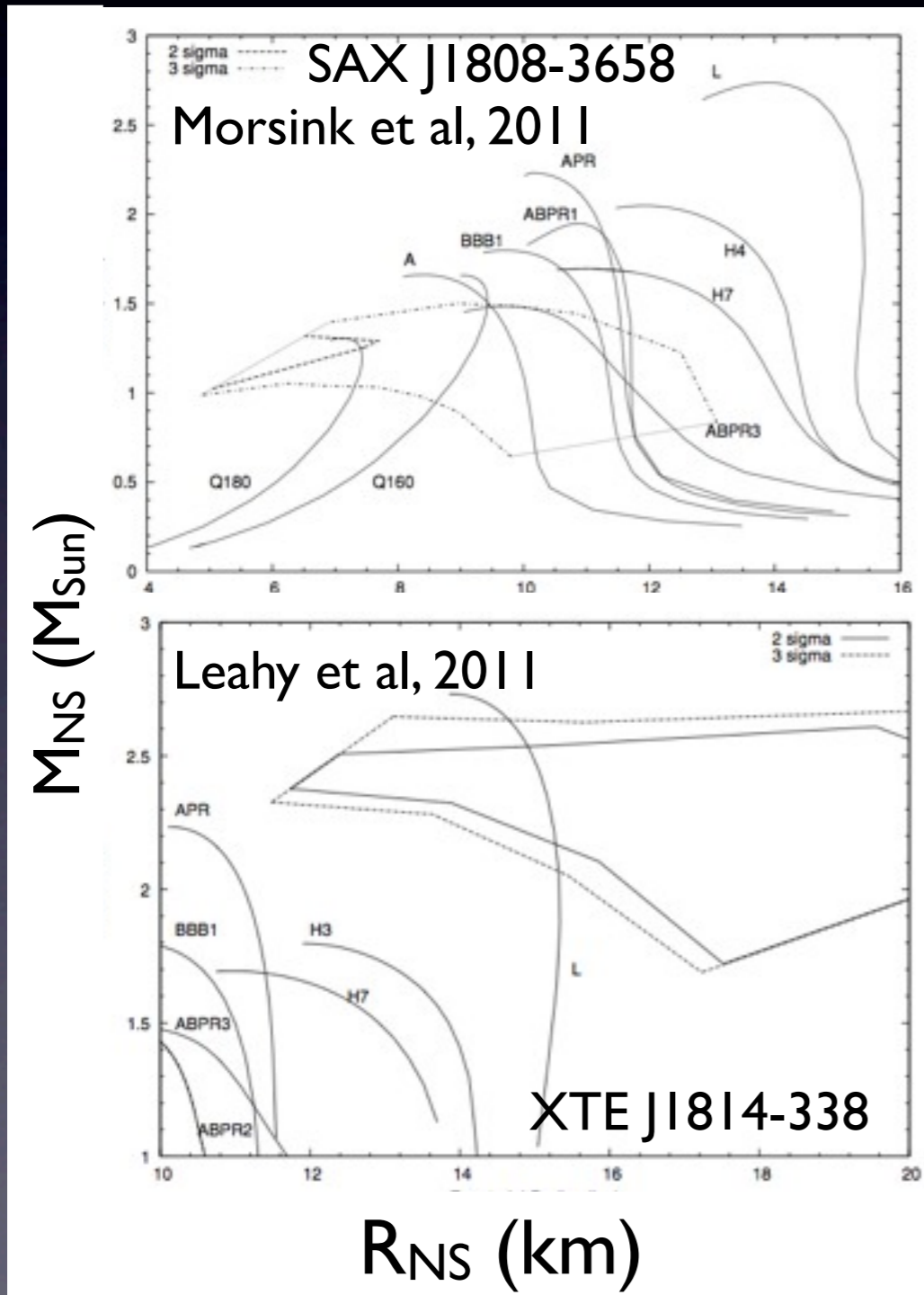
Now, measuring  
 $M_{NS}/R_{NS}$

Analyzing the pulse profiles caused by hot spots on a rotating neutron star can be used to measure the compactness.



$M_{NS}=1.4M_{\odot}, R_{NS}=10\text{km}$   
(Bodganov et al. 2008)

# Analyzing the pulse profiles caused by hot spots on a rotating neutron star can be used to measure the compactness.



Or placing limits on  
 $M_{NS}$  and  $R_{NS}$

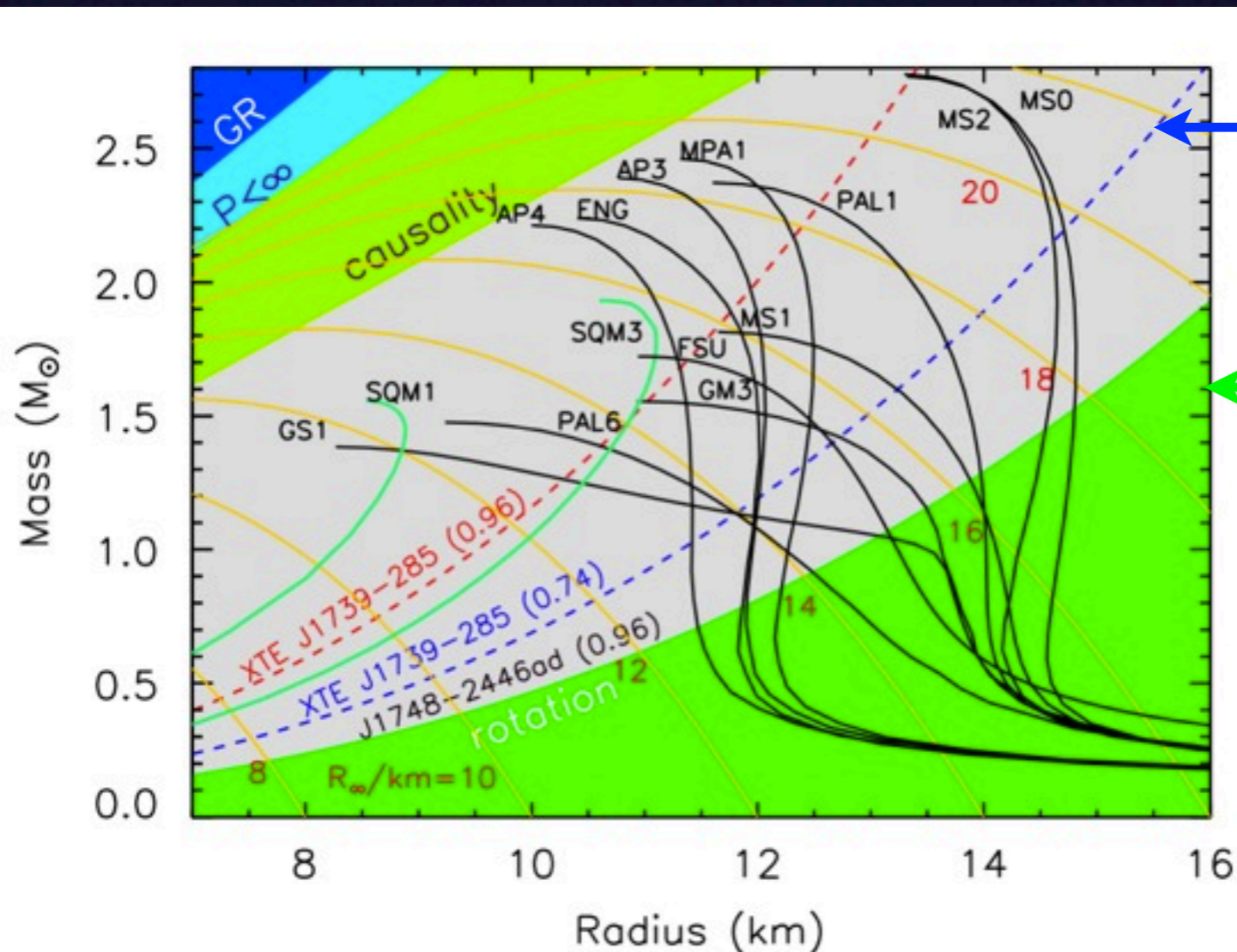
# Extremely fast rotating neutron star can place limits on $M_{NS}$ and $R_{NS}$



$$R_{\text{sph}} < 10.4 \left( \frac{1000 \text{ Hz}}{\nu} \right)^{2/3} \left( \frac{M_{\text{sph}}}{M_{\odot}} \right)^{1/3} \text{ km},$$

Because

$$v_K = (2\pi)^{-1} \sqrt{GM/R^3}$$



1122 Hz  
not confirmed!

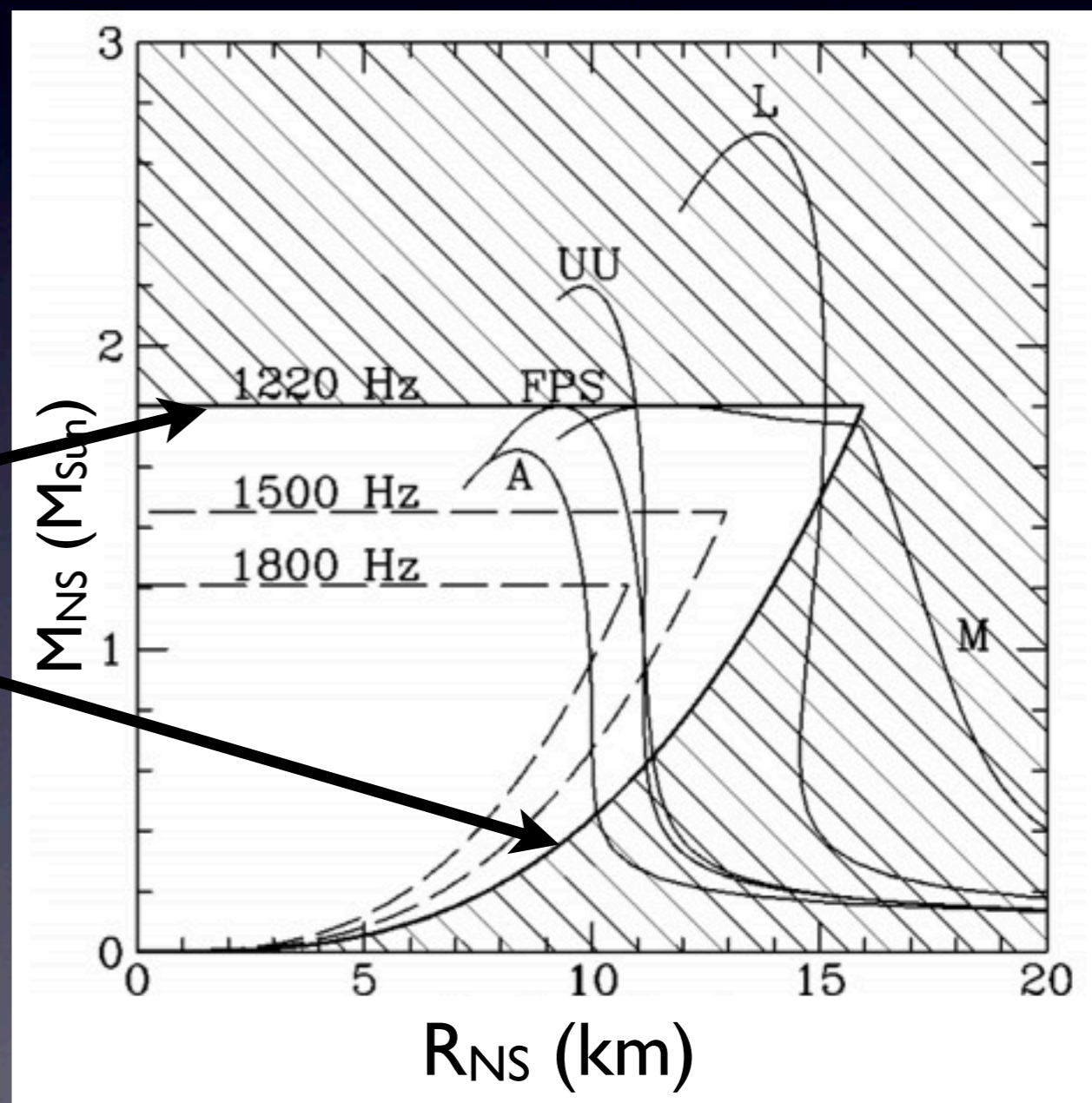
716 Hz



# kHz quasi periodic oscillations could also constrain $M_{NS}$ and $R_{NS}$

$$M \leq \left( \frac{2200 \text{ Hz}}{\nu_{\text{ISCO}}} \right) \left( 1 + 0.75 \frac{2\pi c f_{\text{spin}} I}{GM^2} \right) M_{\odot}$$

$$R \leq \left( \frac{1950 \text{ Hz}}{\nu_{\text{ISCO}}} \right) \left( 1 + 0.20 \frac{2\pi c f_{\text{spin}} I}{GM^2} \right) \text{ km}$$



Highest frequency QPO is 1310 Hz, 4U 1728-34 (Barret et al. 2006)

van der Klis 2000



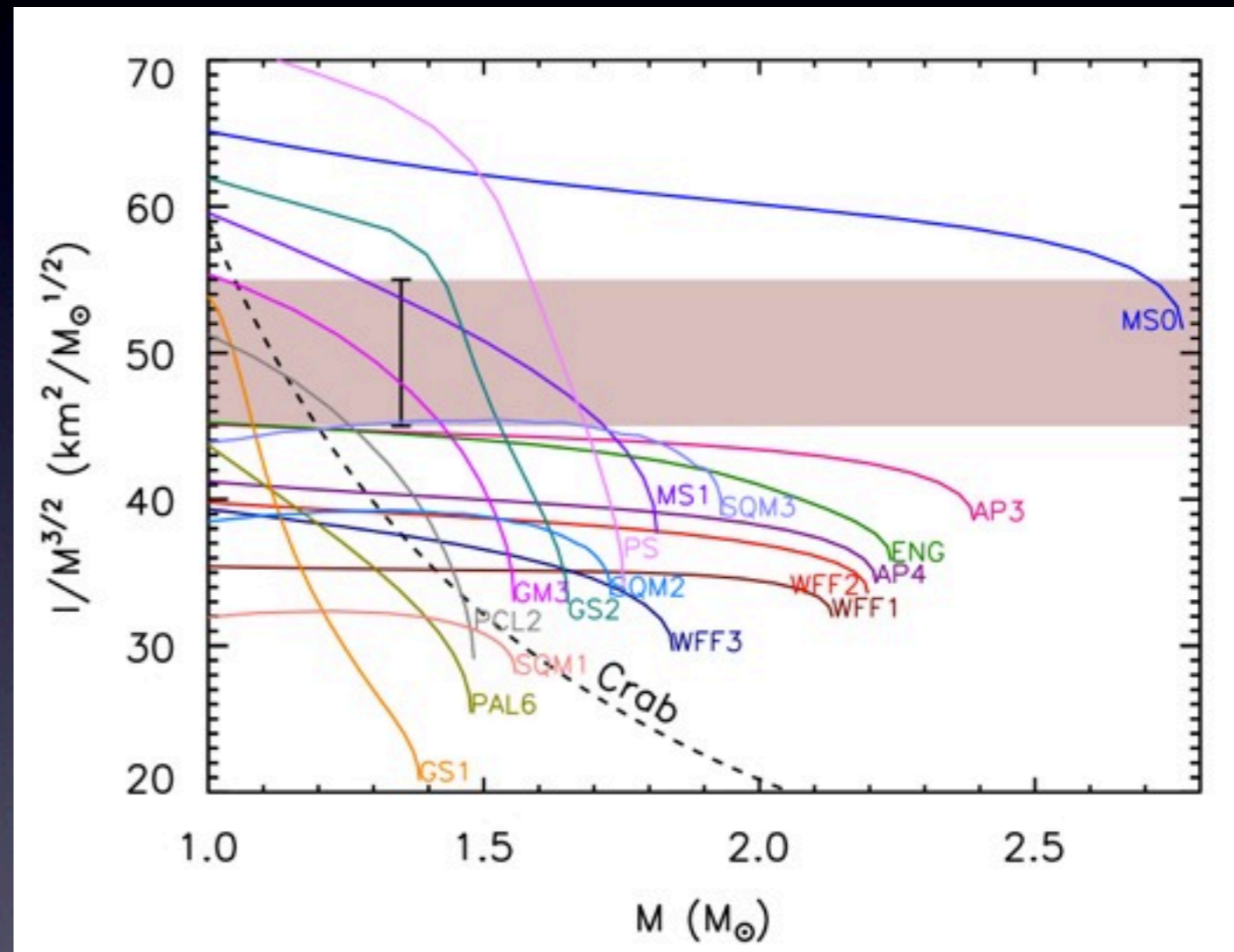
Measuring  $I_{NS}$  directly

# Spin-orbit coupling measurements can be used to determine the moment of inertia



Combining  $I_{NS}$  to known  $M_{NS}$  can be very constraining!

But, the acceleration of the centre of mass of the binary system in the gravitational potential of the Galaxy is unknown!



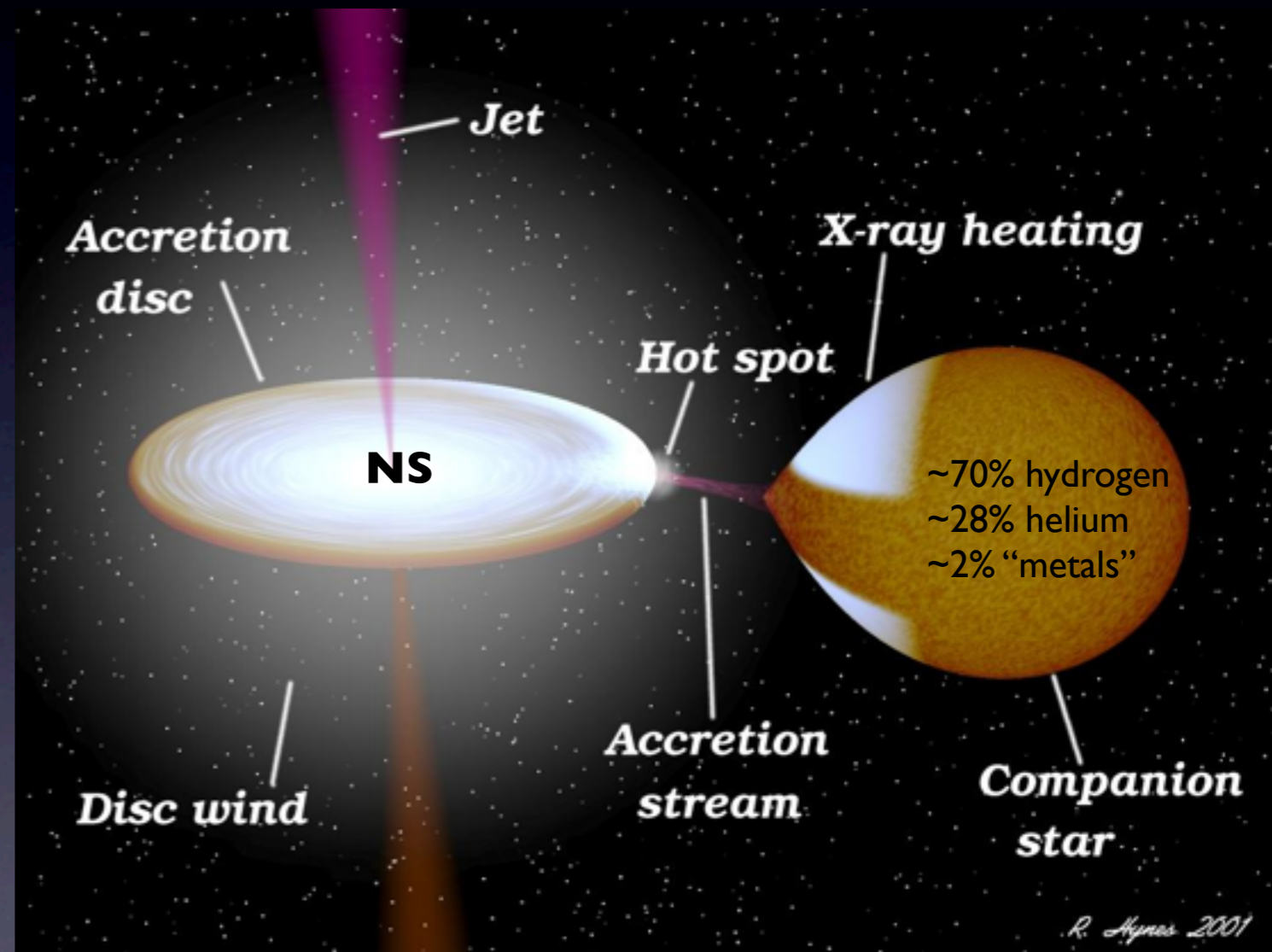
Hypothetical  $I_{NS}$  measurement with 10% precision for double pulsar system (Lattimer & Schutz, 2005)



# Measuring $R_\infty$

$$R_\infty = R_{\text{NS}} (1 + z) = R_{\text{NS}} \left( 1 - \frac{2GM_{\text{NS}}}{R_{\text{NS}} c^2} \right)^{-1/2}$$

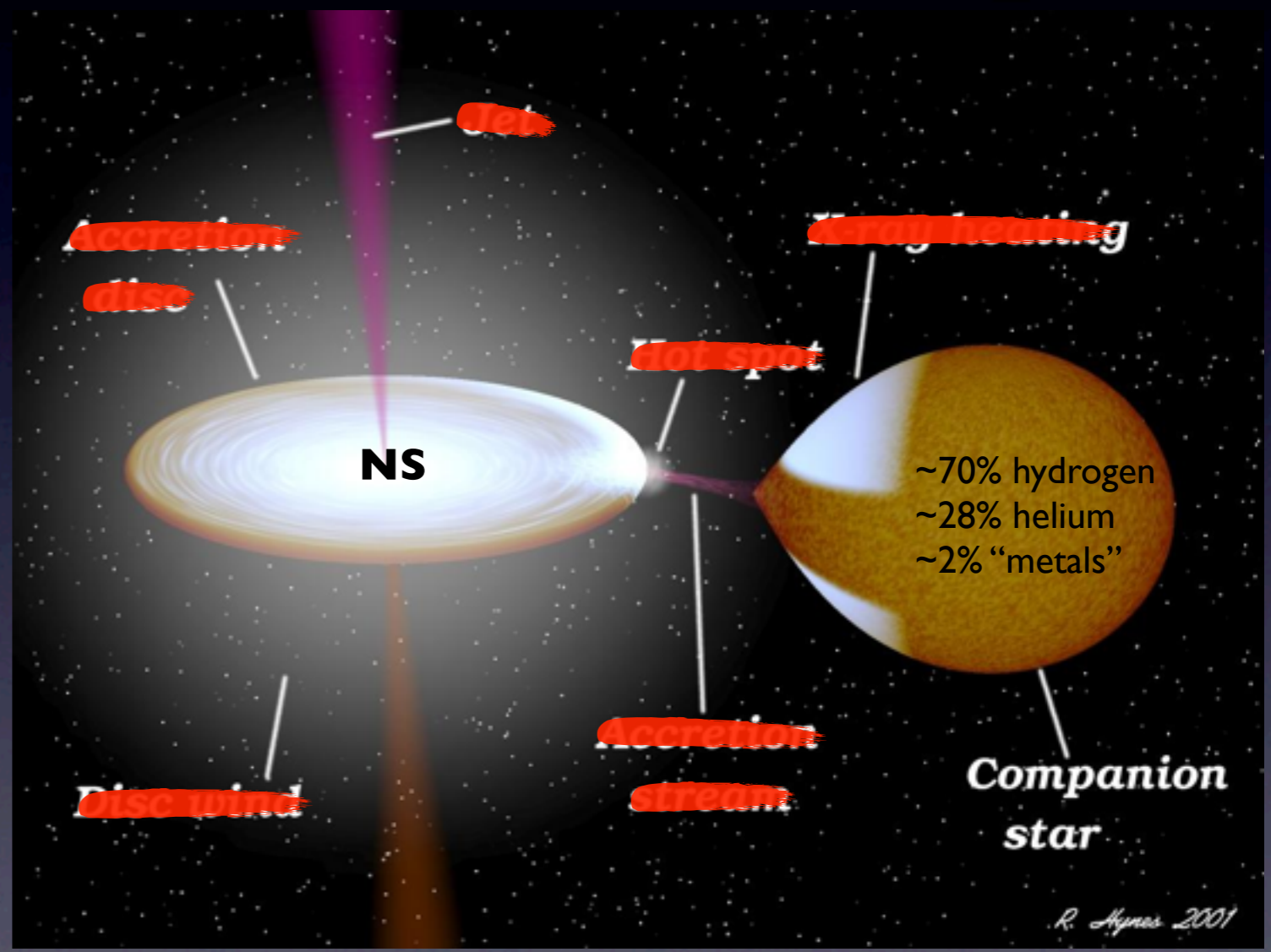
Quiescent low-mass X-ray binaries are ideal systems for  $R_\infty$  measurements.





# Quiescent low-mass X-ray binaries are ideal systems for $R_\infty$ measurements.

- In quiescence, LMXBs have low mass accretion rate
- Thermal emission powered by deep crustal heating
- Surface thermal emission comes from a pure hydrogen atmosphere with  $L_x = 10^{32-33}$  erg/sec
- Neutron star has a weak magnetic field





# The thermal emission from qLMXB is powered by Deep Crustal Heating.

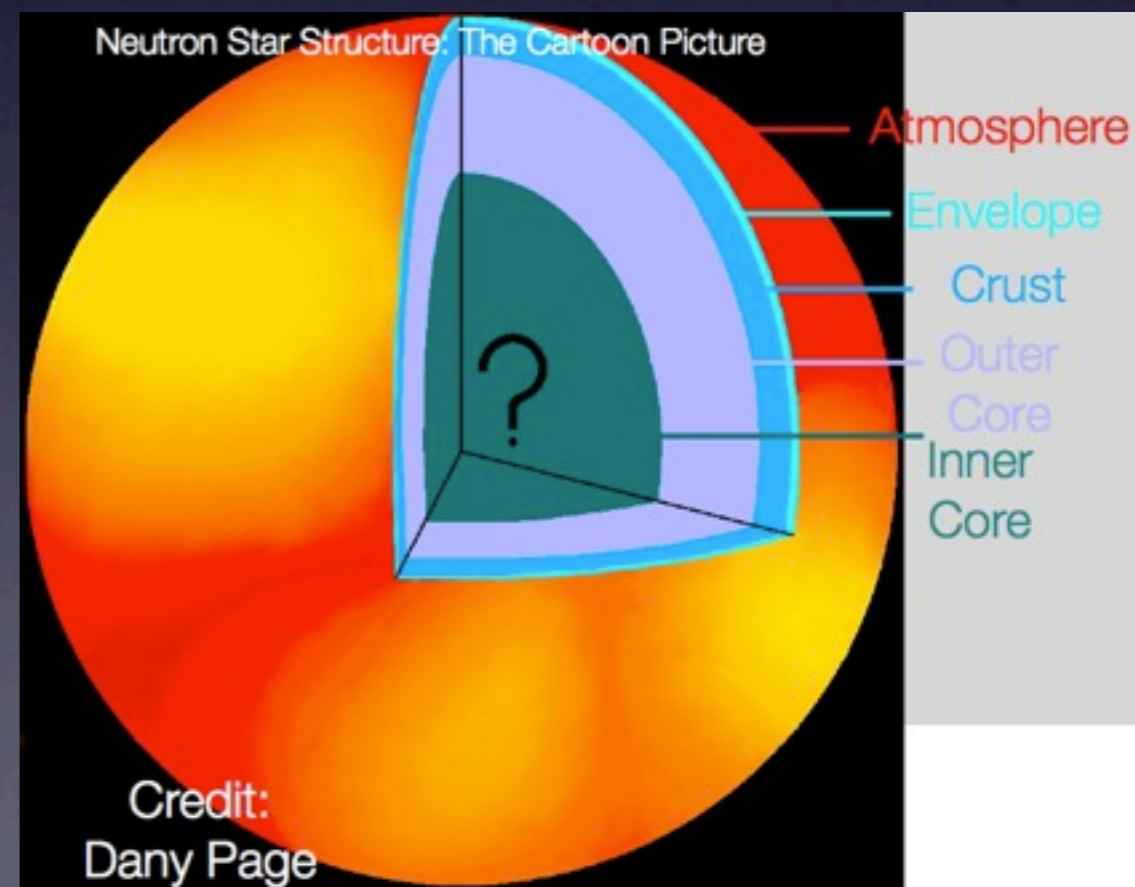
Brown et al. 1998

## Non-Equilibrium Processes in the Outer Crust Beginning with $^{56}\text{Fe}$ (Haensel & Zdunik 1990, 2003)

$\rho$ (g cm $^{-3}$ )	Reaction	$\Delta\rho/\rho$	Q (Mev/np)
$1.5 \cdot 10^9$	$^{56}\text{Fe} \Rightarrow ^{56}\text{Cr} - 2e^- + 2\nu_e$	0.08	0.01
$1.1 \cdot 10^{10}$	$^{56}\text{Cr} \Rightarrow ^{56}\text{Ti} - 2e^- + 2\nu_e$	0.09	0.01
$7.8 \cdot 10^{10}$	$^{56}\text{Ti} \Rightarrow ^{56}\text{Ca} - 2e^- + 2\nu_e$	0.10	0.01
$2.5 \cdot 10^{10}$	$^{56}\text{Ca} \Rightarrow ^{56}\text{Ar} - 2e^- + 2\nu_e$	0.11	0.01
$6.1 \cdot 10^{10}$	$^{56}\text{Ar} \Rightarrow ^{52}\text{S} + 4n - 2e^- + 2\nu_e$	0.12	0.01

## Non-Equilibrium Processes in the Inner Crust

$\rho$ (g cm $^{-3}$ )	Reaction	$X_n$	Q (Mev/np)
$9.1 \cdot 10^{11}$	$^{52}\text{S} \Rightarrow ^{46}\text{Si} + 6n - 2e^- + 2\nu_e$	0.07	0.09
$1.1 \cdot 10^{12}$	$^{46}\text{Si} \Rightarrow ^{40}\text{Mg} + 6n - 2e^- + 2\nu_e$	0.07	0.09
$1.5 \cdot 10^{12}$	$^{40}\text{Mg} \Rightarrow ^{34}\text{Ne} + 6n - 2e^- + 2\nu_e$		
	$^{34}\text{Ne} + ^{34}\text{Ne} \Rightarrow ^{68}\text{Ca}$	0.29	0.47
$1.8 \cdot 10^{12}$	$^{68}\text{Ca} \Rightarrow ^{62}\text{Ar} + 6n - 2e^- + 2\nu_e$	0.39	0.05
$2.1 \cdot 10^{12}$	$^{62}\text{Ar} \Rightarrow ^{56}\text{S} + 6n - 2e^- + 2\nu_e$	0.45	0.05
$2.6 \cdot 10^{12}$	$^{56}\text{S} \Rightarrow ^{50}\text{Si} + 6n - 2e^- + 2\nu_e$	0.50	0.06
$3.3 \cdot 10^{12}$	$^{50}\text{Si} \Rightarrow ^{44}\text{Mg} + 6n - 2e^- + 2\nu_e$	0.55	0.07
$4.4 \cdot 10^{12}$	$^{44}\text{Mg} \Rightarrow ^{38}\text{Ne} + 6n - 2e^- + 2\nu_e$		
	$^{38}\text{Ne} + ^{38}\text{Ne} \Rightarrow ^{72}\text{Ca}$		
	$^{68}\text{Ca} \Rightarrow ^{62}\text{Ar} + 6n - 2e^- + 2\nu_e$	0.61	0.28
$5.8 \cdot 10^{12}$	$^{62}\text{Ar} \Rightarrow ^{60}\text{S} + 6n - 2e^- + 2\nu_e$	0.70	0.02
$7.0 \cdot 10^{12}$	$^{60}\text{S} \Rightarrow ^{54}\text{Si} + 6n - 2e^- + 2\nu_e$	0.73	0.02
$9.0 \cdot 10^{12}$	$^{54}\text{Si} \Rightarrow ^{48}\text{Mg} + 6n - 2e^- + 2\nu_e$	0.76	0.03
$1.1 \cdot 10^{13}$	$^{48}\text{Mg} + ^{48}\text{Mg} \Rightarrow ^{96}\text{Cr}$	0.79	0.11
$1.1 \cdot 10^{13}$	$^{96}\text{Cr} \Rightarrow ^{88}\text{Ti} + 8n - 2e^- + 2\nu_e$	0.80	0.01

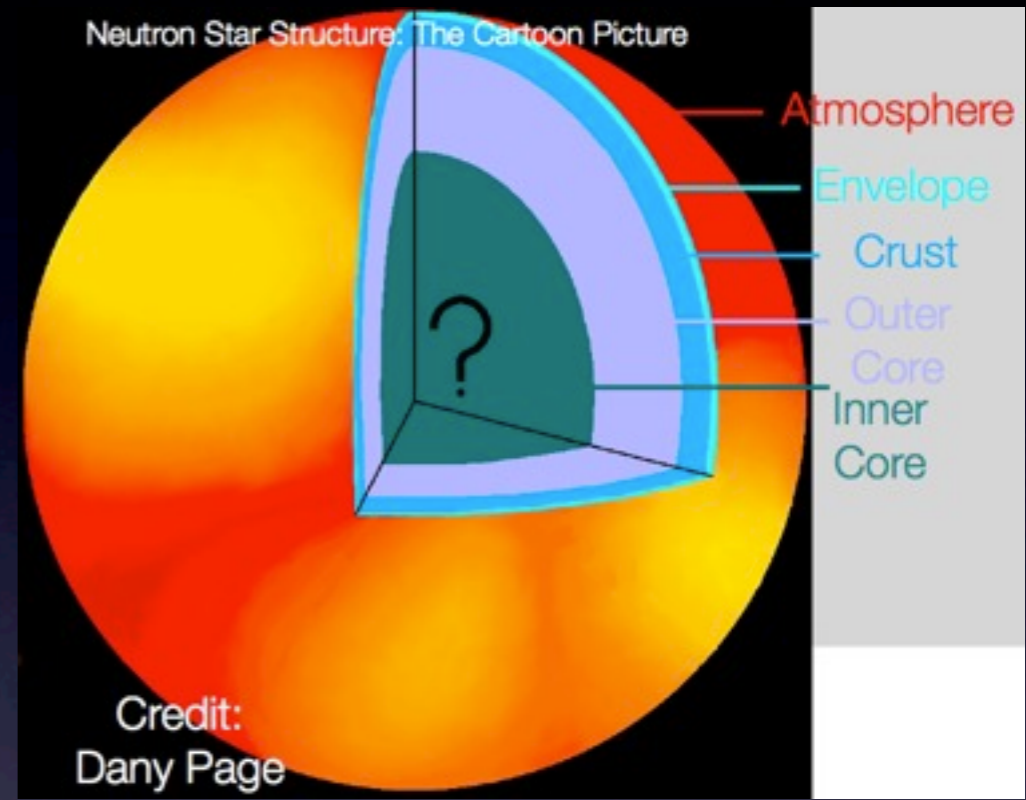




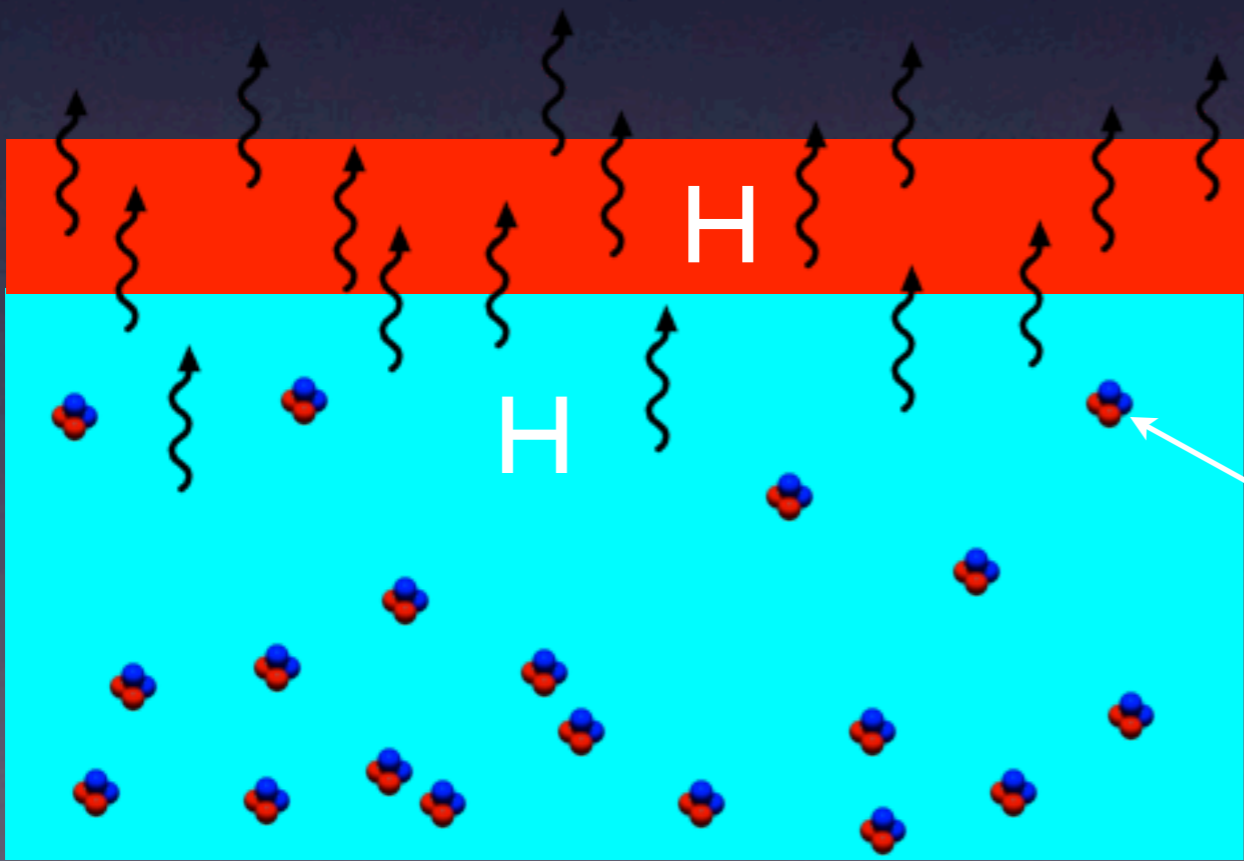
# The atmosphere of the neutron star in a qLMXB is composed of pure hydrogen.



H-atmosphere  
thermal spectrum  
seen by observer



Gravity  
↓



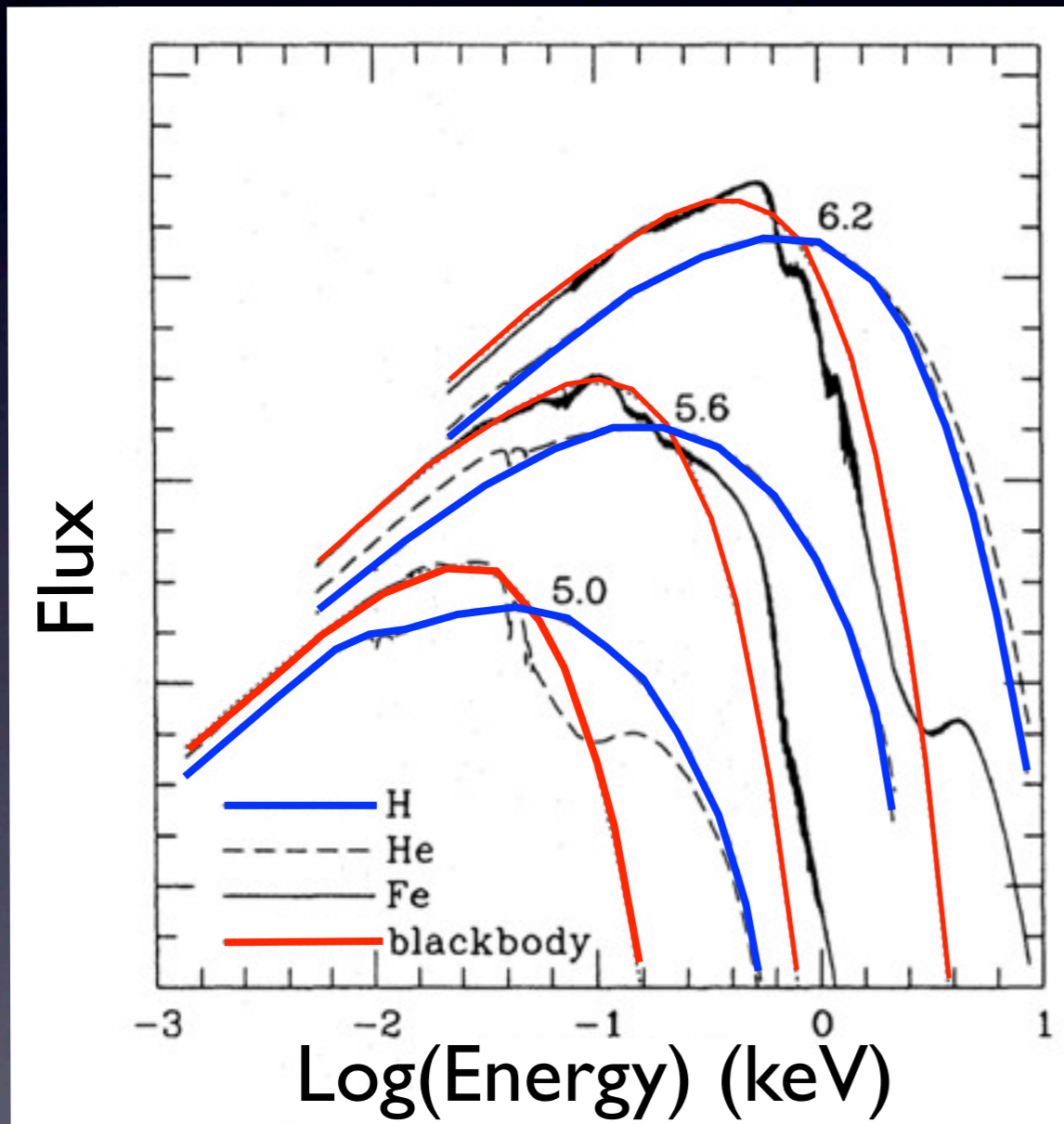
Photosphere ~ 10 cm

Helium

# The thermal emission from a NS surface is modelled with atmosphere models.



Models by Zavlin et al. (1996), Heinke et al. (2006), Haakonsen et al. (2012)



Spectral fitting of the thermal emission gives us  $T_{\text{eff}}$  and  $(R_{\infty}/D)^2$

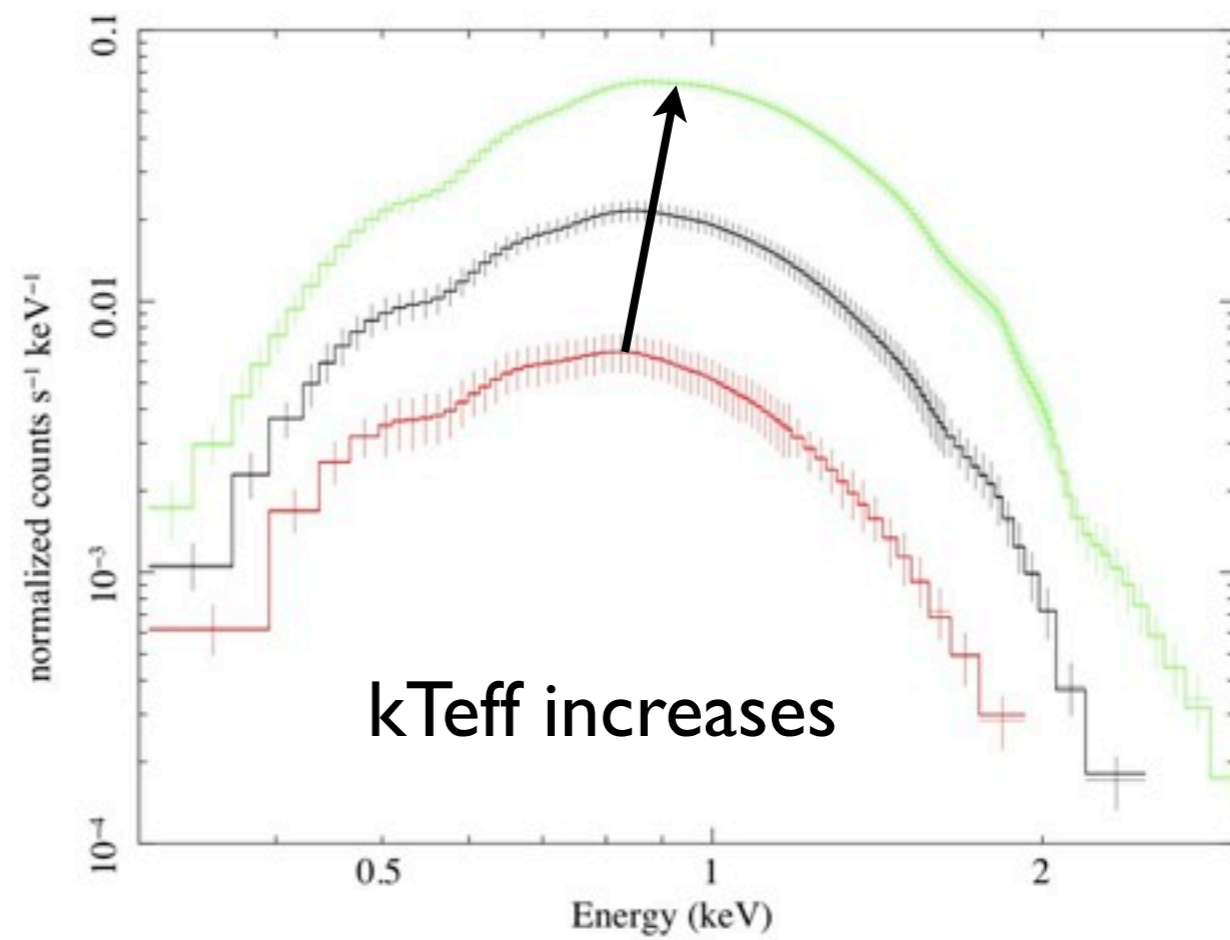
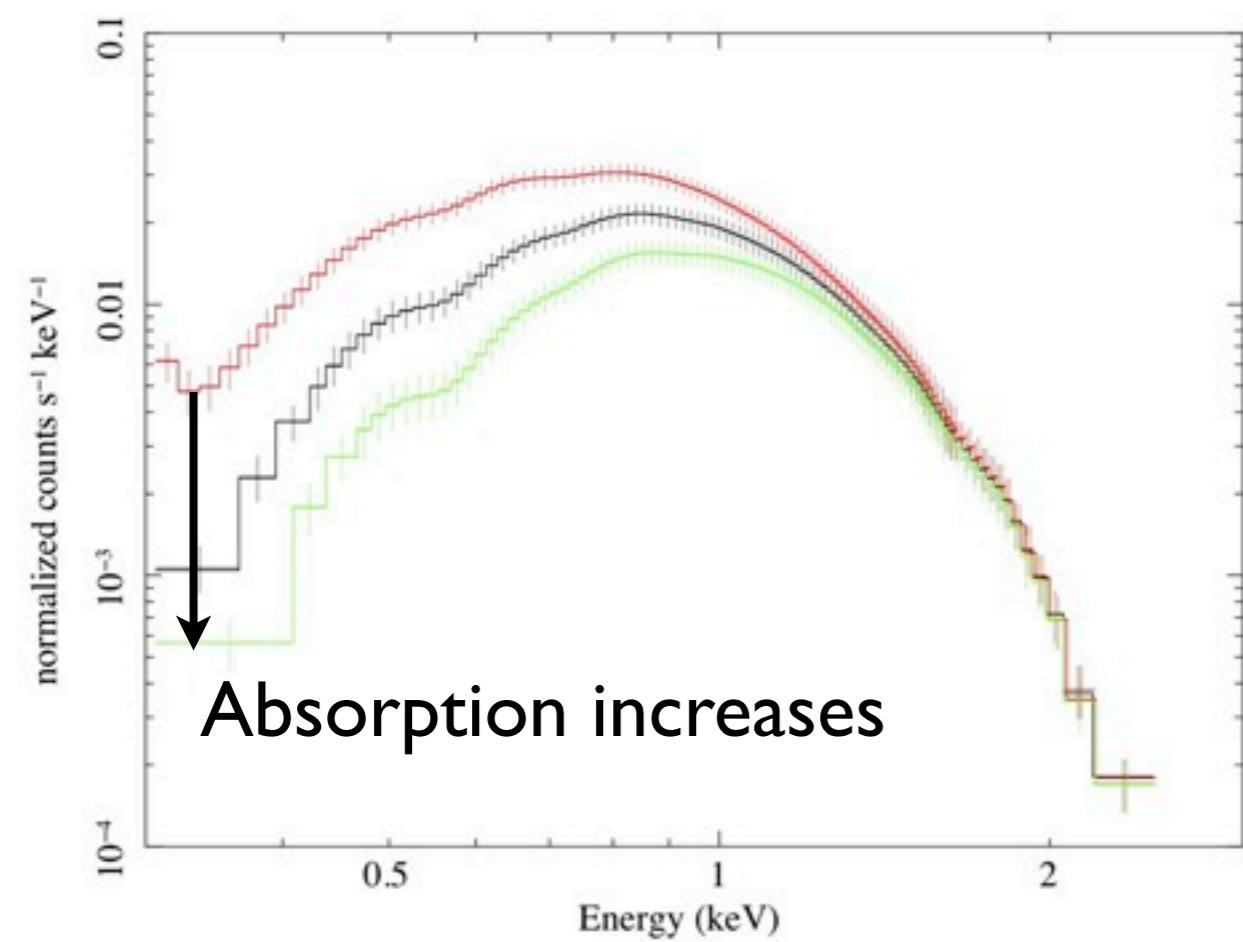
$$R_{\infty} = R_{\text{NS}} \left( 1 - \frac{2GM_{\text{NS}}}{R_{\text{NS}} c^2} \right)^{-1/2}$$

NS H-atmosphere model parameters are:

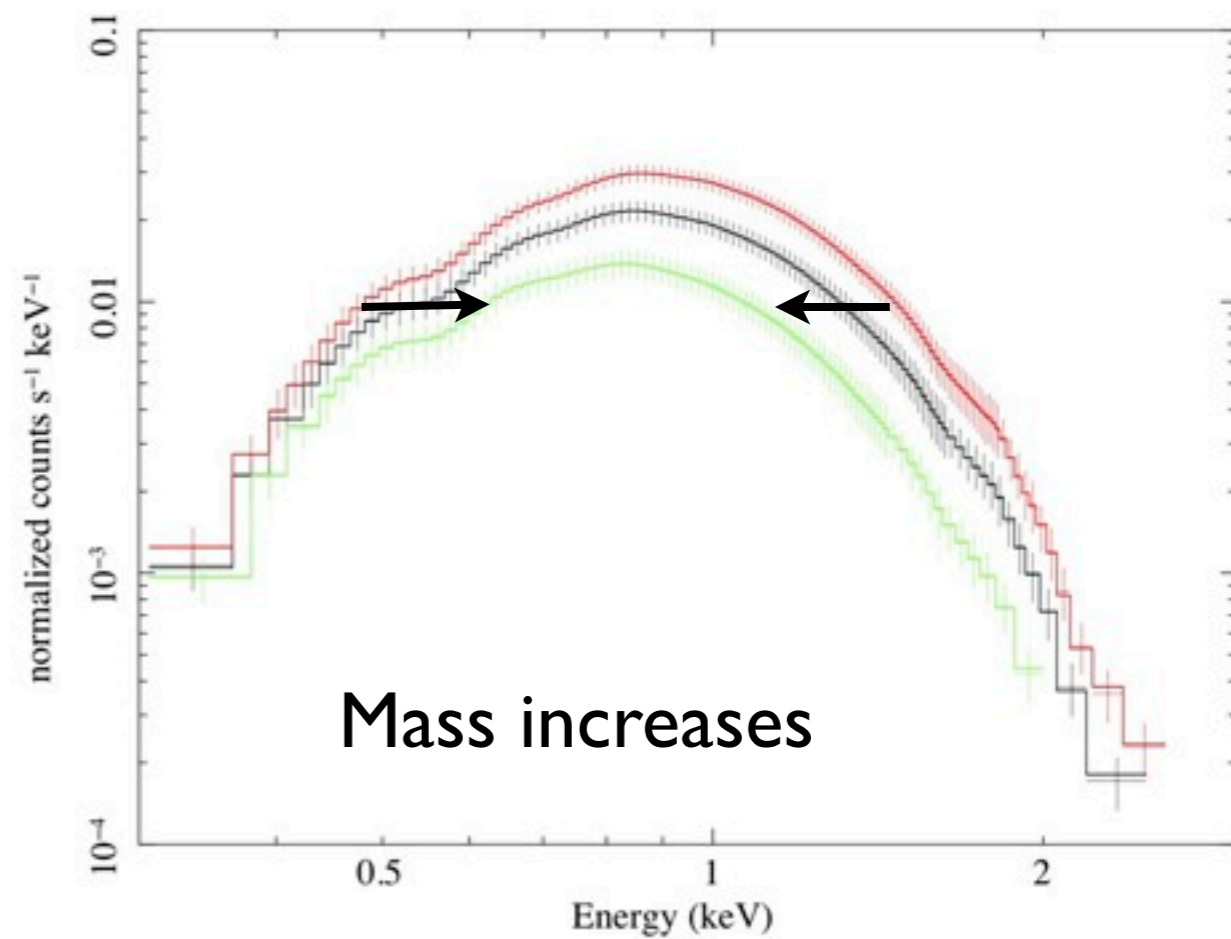
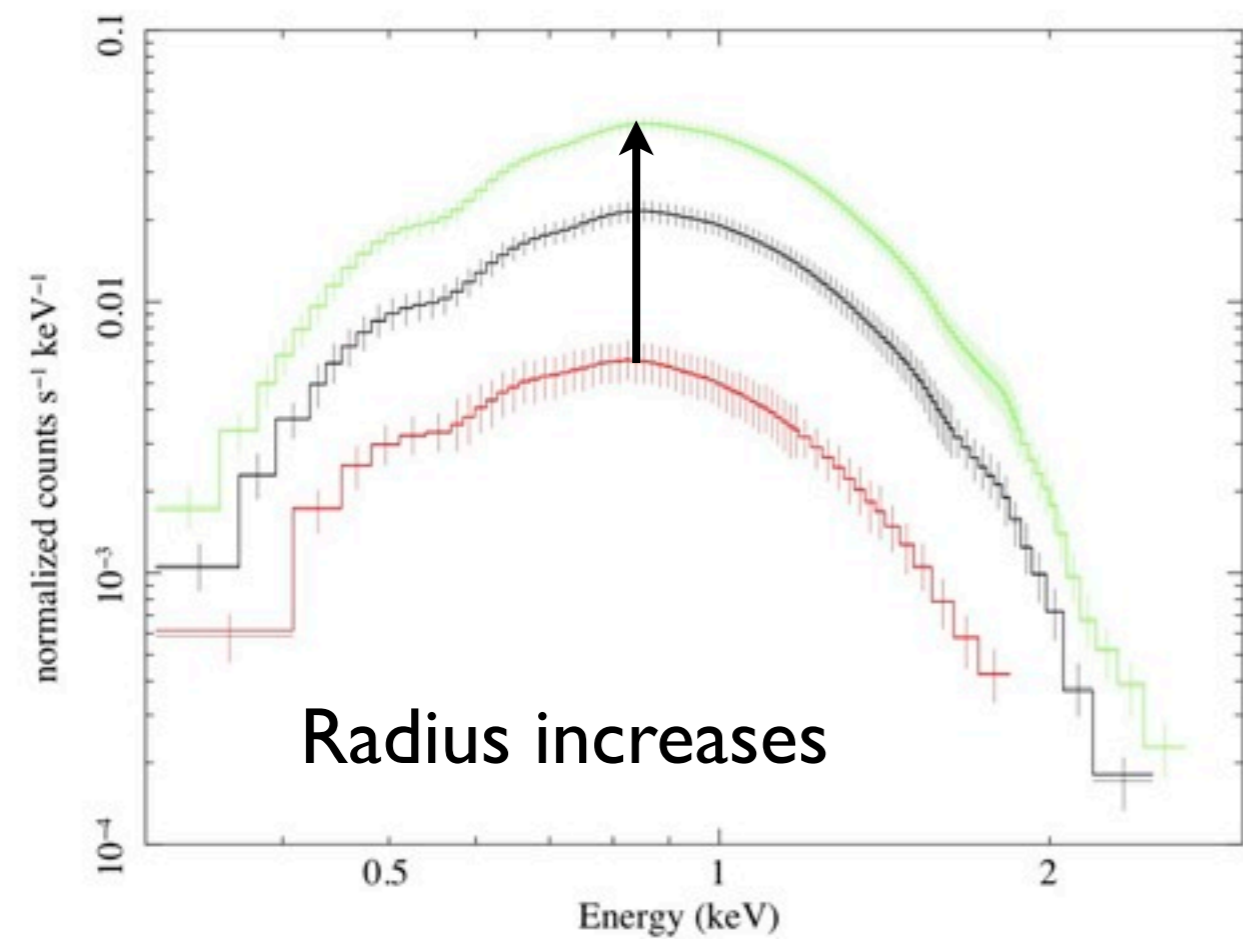
- Effective temperature  $kT_{\text{eff}}$
- Mass  $M_{\text{NS}}$  ( $M_{\odot}$ )
- Radius  $R_{\text{NS}}$  (km)
- Distance  $D$  (kpc)

NSA, NSAGRAV models  
Zavlin et al 1996, A&A 315

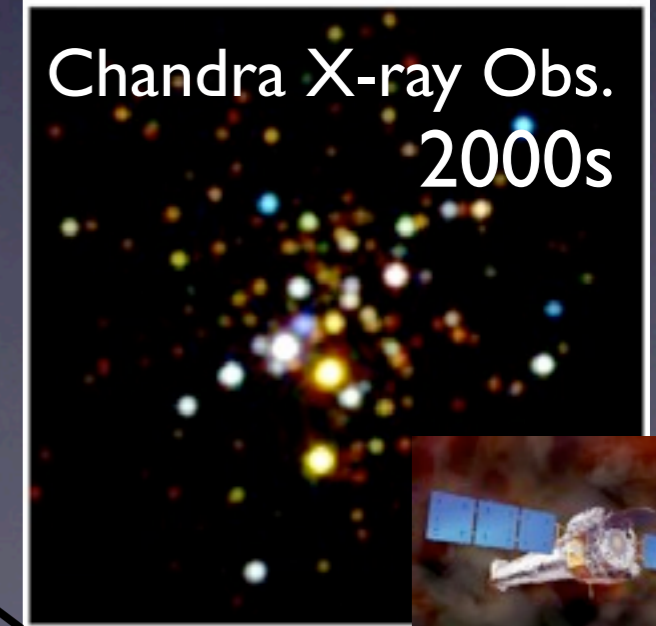
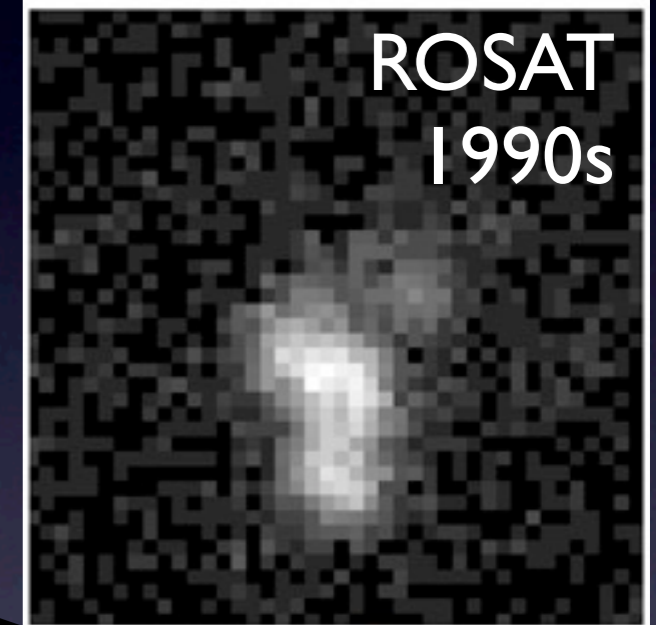
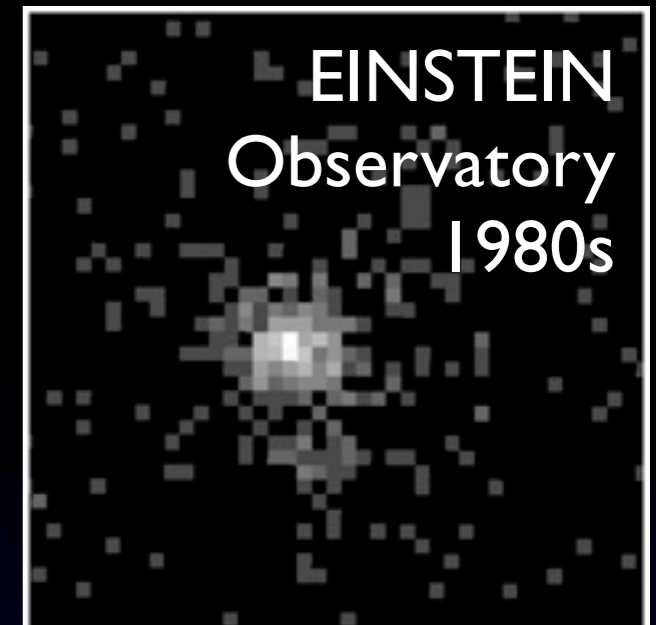
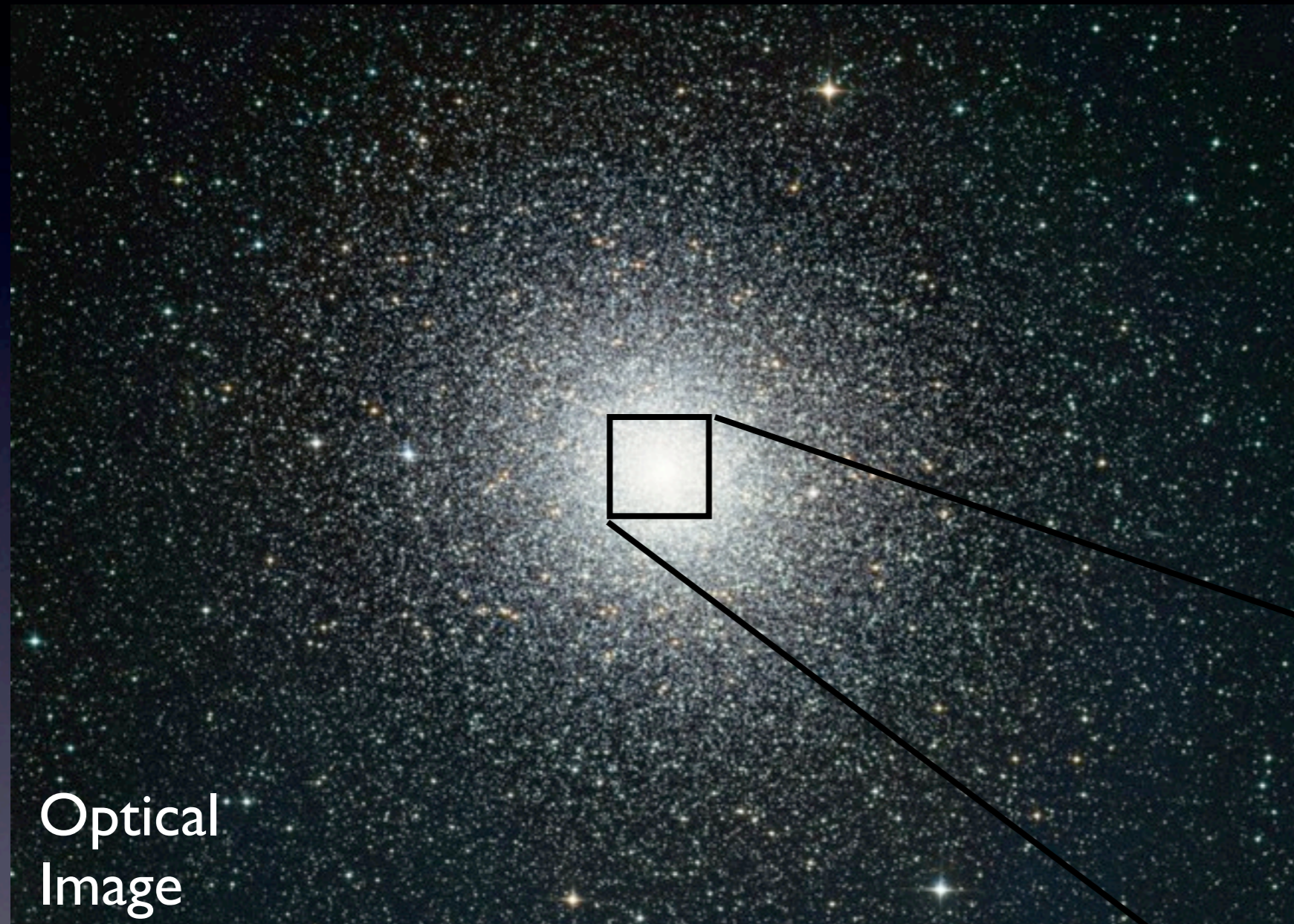




**Neutron stars properties are extracted from the spectra.**



# Globular clusters host an overabundance of LMXB systems...

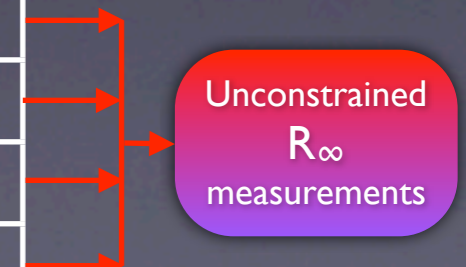


...and they have well-measured distances.



# 29 quiescent LMXBs are known within globular clusters of the Milky Way.

Host Globular Cluster	Distance (kpc)	Proxy for Absorption $N_H$ ( $10^{22} \text{ cm}^{-2}$ )	Number of qLMXBs	"Useful"	Observational Difficulties	Need Chandra
$\omega$ Cen	5.3	0.09	1	Green		NO
M13	7.7	0.01	1	Green		NO
M28	5.5	0.26	1	Green	Moderate pile-up	YES
NGC 6304	6.0	0.27	1	Green		YES
NGC 6397	2.5	0.14	1	Green		YES
NGC 6553	6.0	0.35	1	Green	<i>NEEDS TO BE CONFIRMED</i>	YES
47 Tuc	4.5	0.03	2 (+3?)	Orange	Important pile-up	YES
M30	9.0	0.03	1	Orange	Large distance	YES
M80	10.3	0.09	2	Orange	Large distance	YES
NGC 362	8.6	0.03	1	Orange	Large distance	YES
NGC 2808	9.6	0.82	1	Red	Large distance and $N_H$	YES
NGC 3201	5.0	1.17	1	Red	Very Large $N_H$	NO
NGC 6440	8.5	0.70	8	Red	Large distance and $N_H$	YES
Terzan 5	8.7	1.20	4	Red	Large distance and $N_H$	YES



qLMXBs inside globular clusters are observed with Chandra, and sometimes with XMM-Newton.



Chandra X-Ray Observatory



1" angular resolution

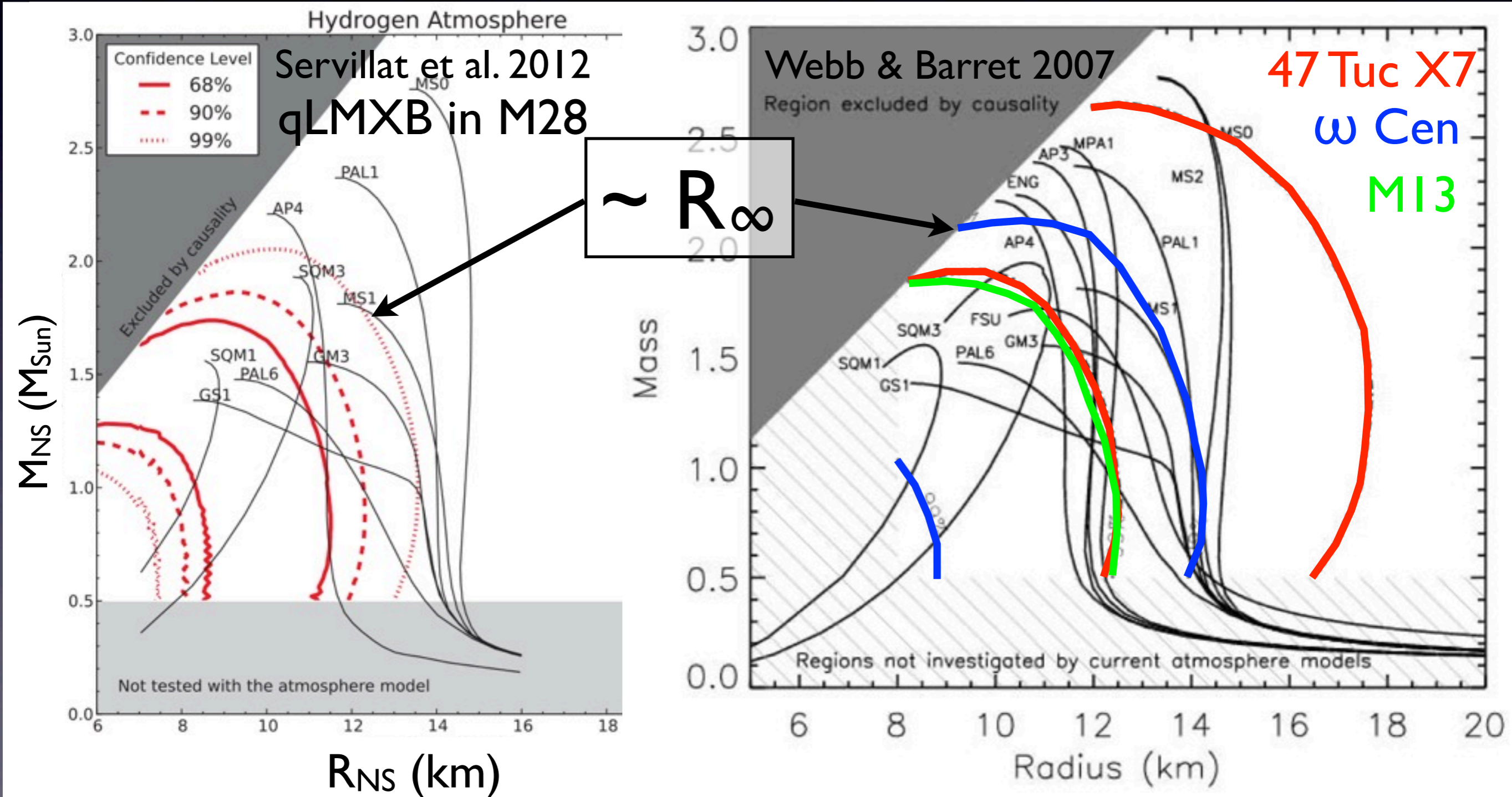
XMM-Newton



6" angular resolution  
4x effective area of Chandra

In spectral imaging mode, photons are time-tagged with  $\sim 0.1$ –3sec resolution, and energy resolution of about 150eV at 1keV

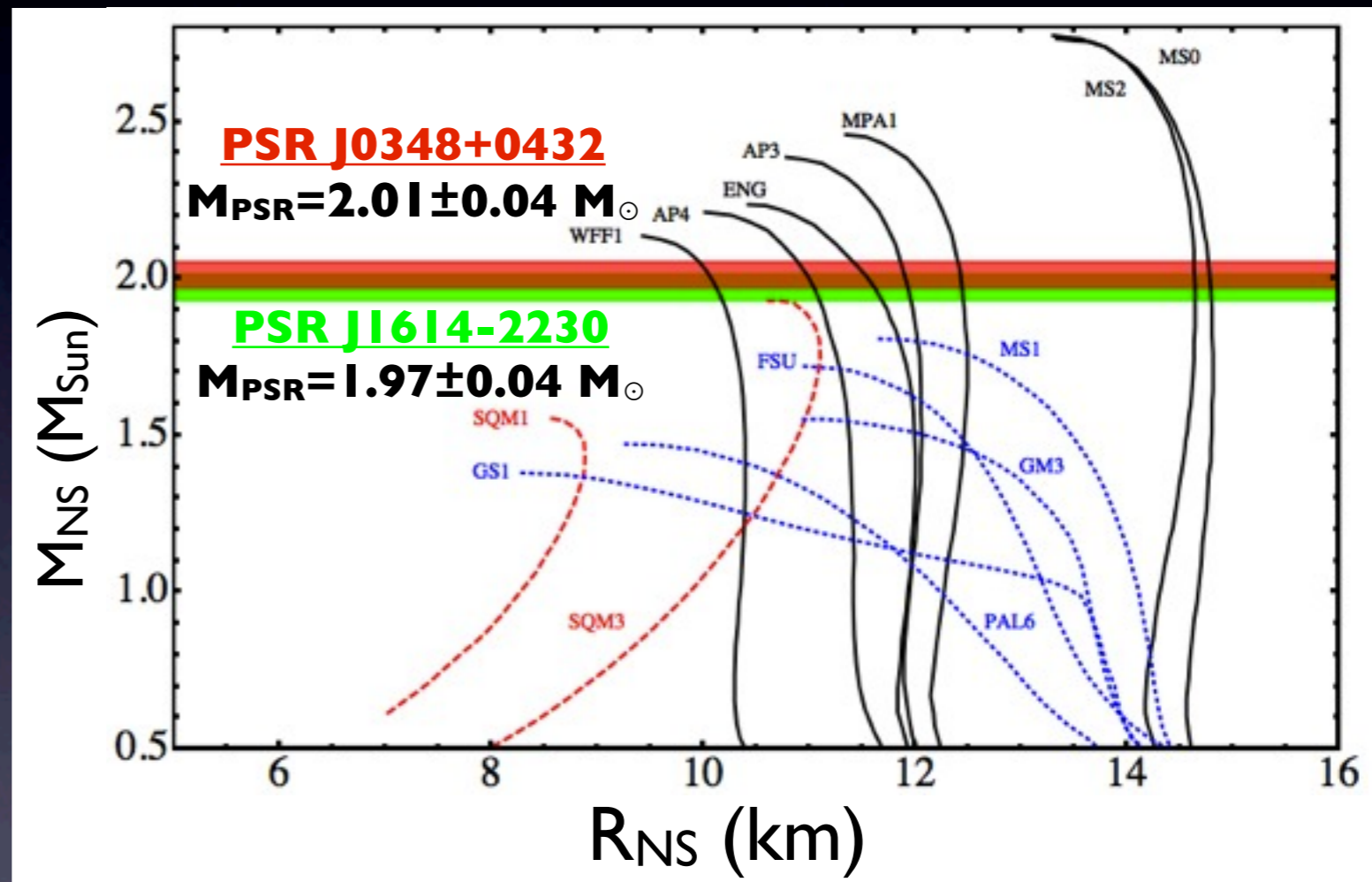
Quiescent LMXBs are routinely used for  $M_{\text{NS}}-R_{\text{NS}}$  measurements, but only place weak constraints on the dense matter EoS.



In Guillot et al (2013), we follow a simplified parametrization for the EoS.

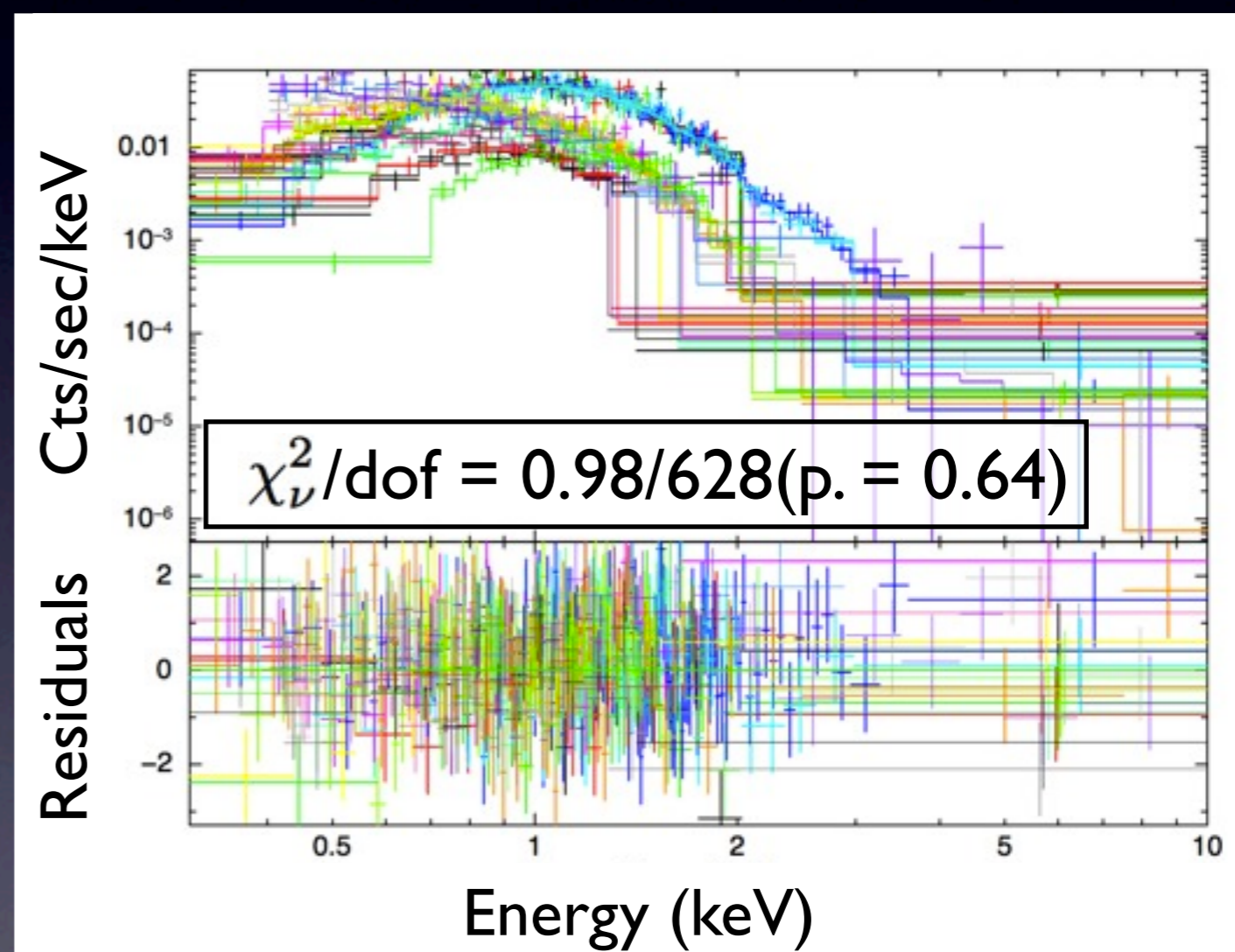
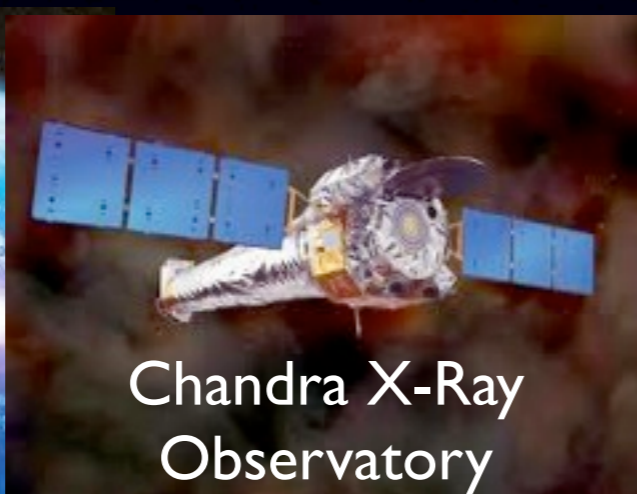


Equations of state consistent with  $\sim 2M_{\text{sun}}$  are those described by a constant radius for a wide range of masses.



We assume that  
all neutron stars have the same radius

# We simultaneously fit the spectra of 5 qLMXBs with H-atmosphere model

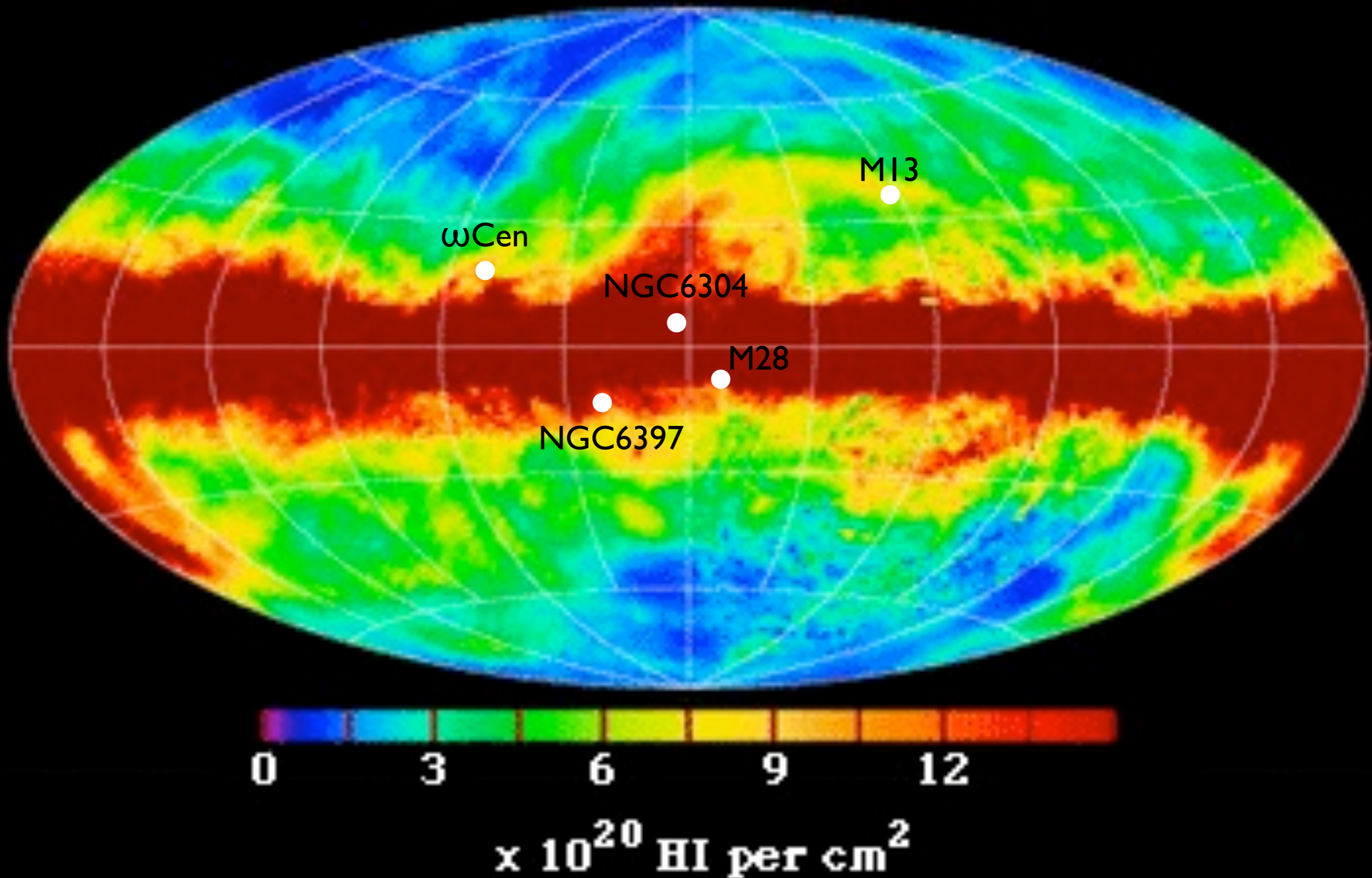


**One radius to fit them all!**

Five parameters per target:

$T_{\text{eff}}$ ,  $M_{\text{NS}}$ ,  $N_{\text{H}}$ , distance,  
power-law component

# Targeted Globular Clusters





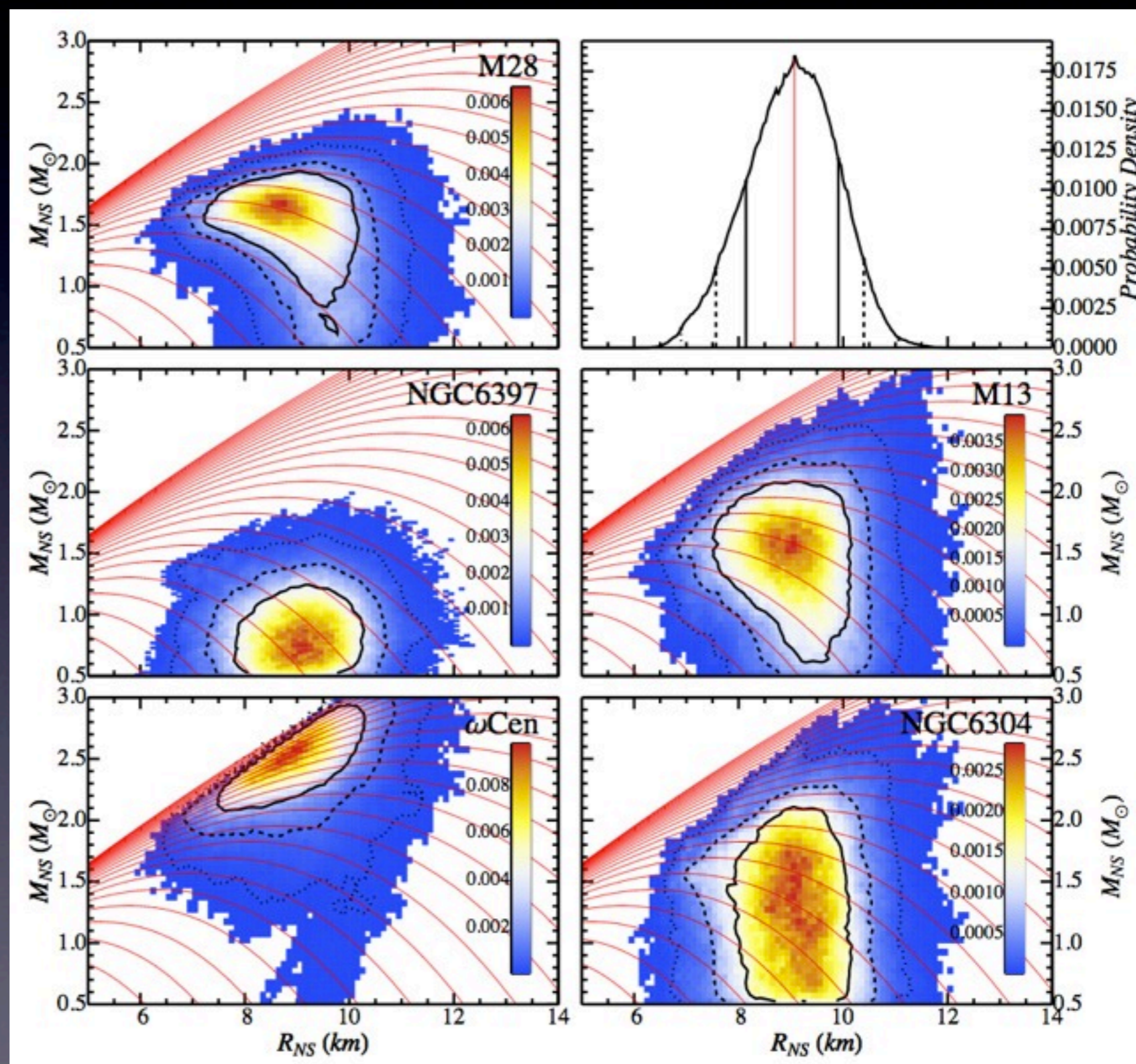
Our most conservative radius measurement relies on the least number of assumptions.



Most conservative  
NS radius  
measurement is

$$R_{\text{NS}} = 9.1^{+1.3}_{-1.5} \text{ km}$$

90% conf. level





Our most conservative  $R_{NS}$  measurement includes most sources of uncertainty

We included the uncertainties linked to:

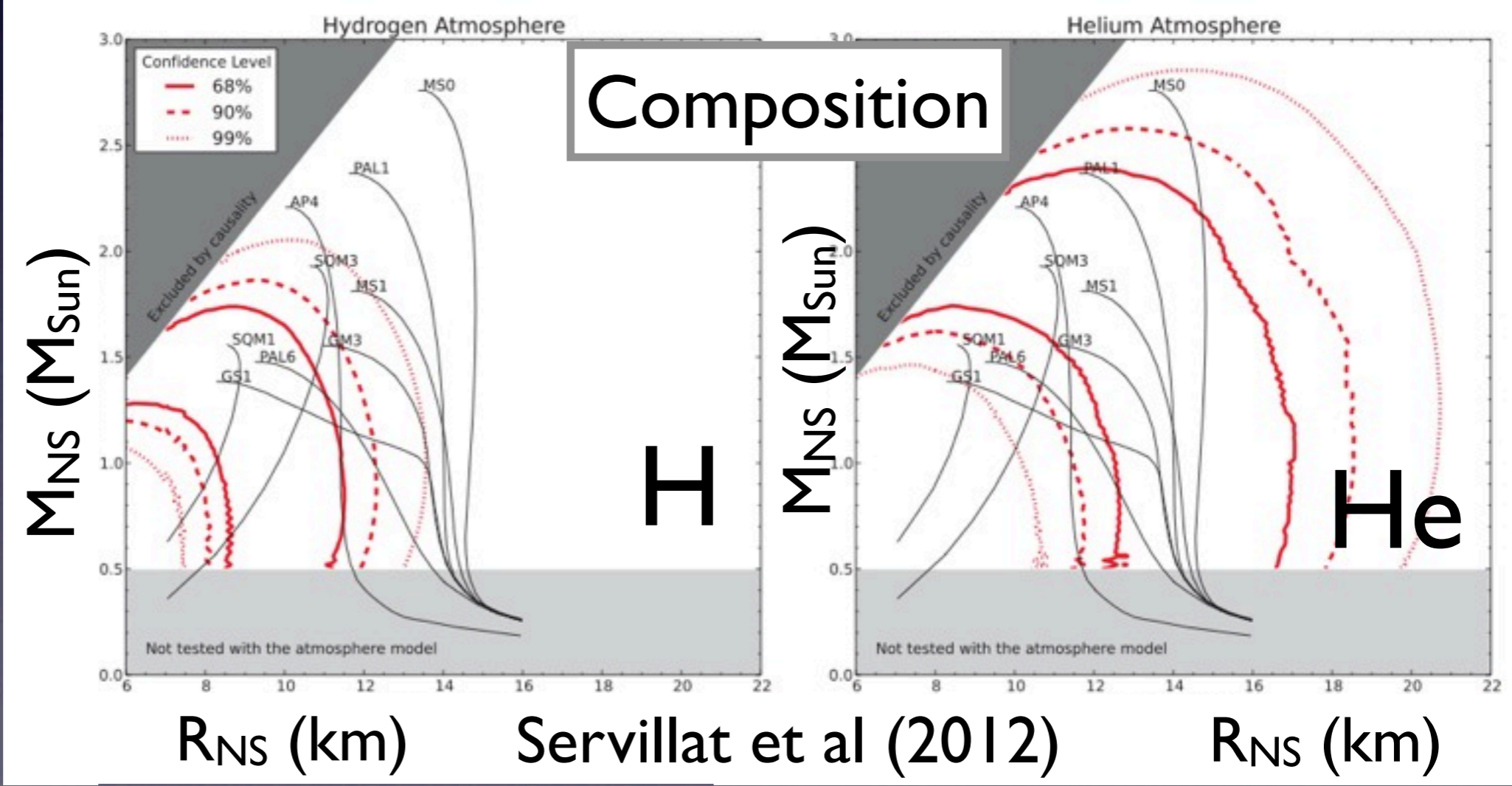
- Galactic absorption
- Distances of the host clusters
- Possible power-law component
- Calibration of X-ray detectors



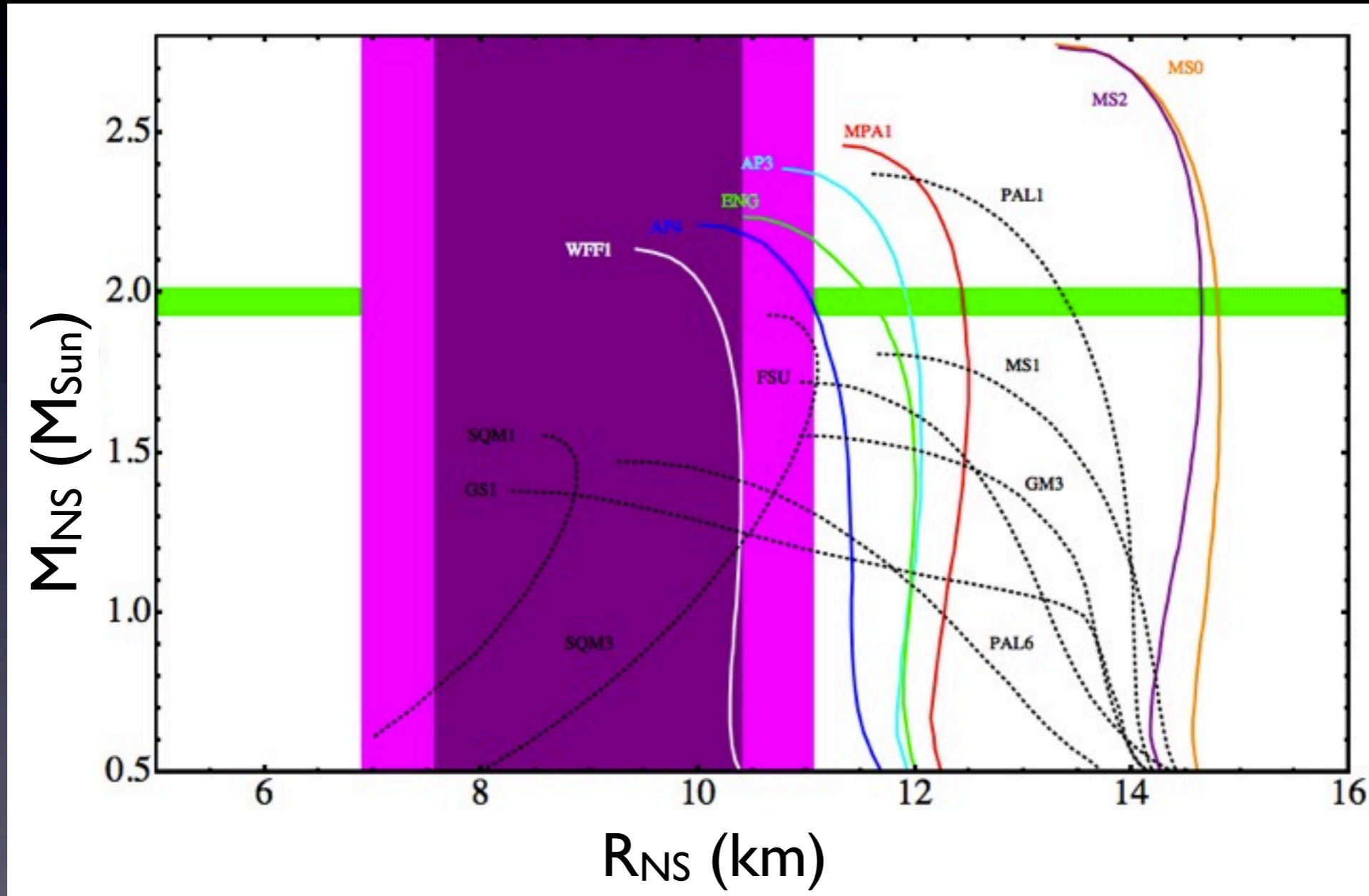
# There are analysis assumptions

NS surface emits isotropically

Negligible magnetic field



Our most conservative radius measurement places important constraints on the dense matter equation of state.



$R_{NS}$  in the 7-11 km range at the 99%-confidence level

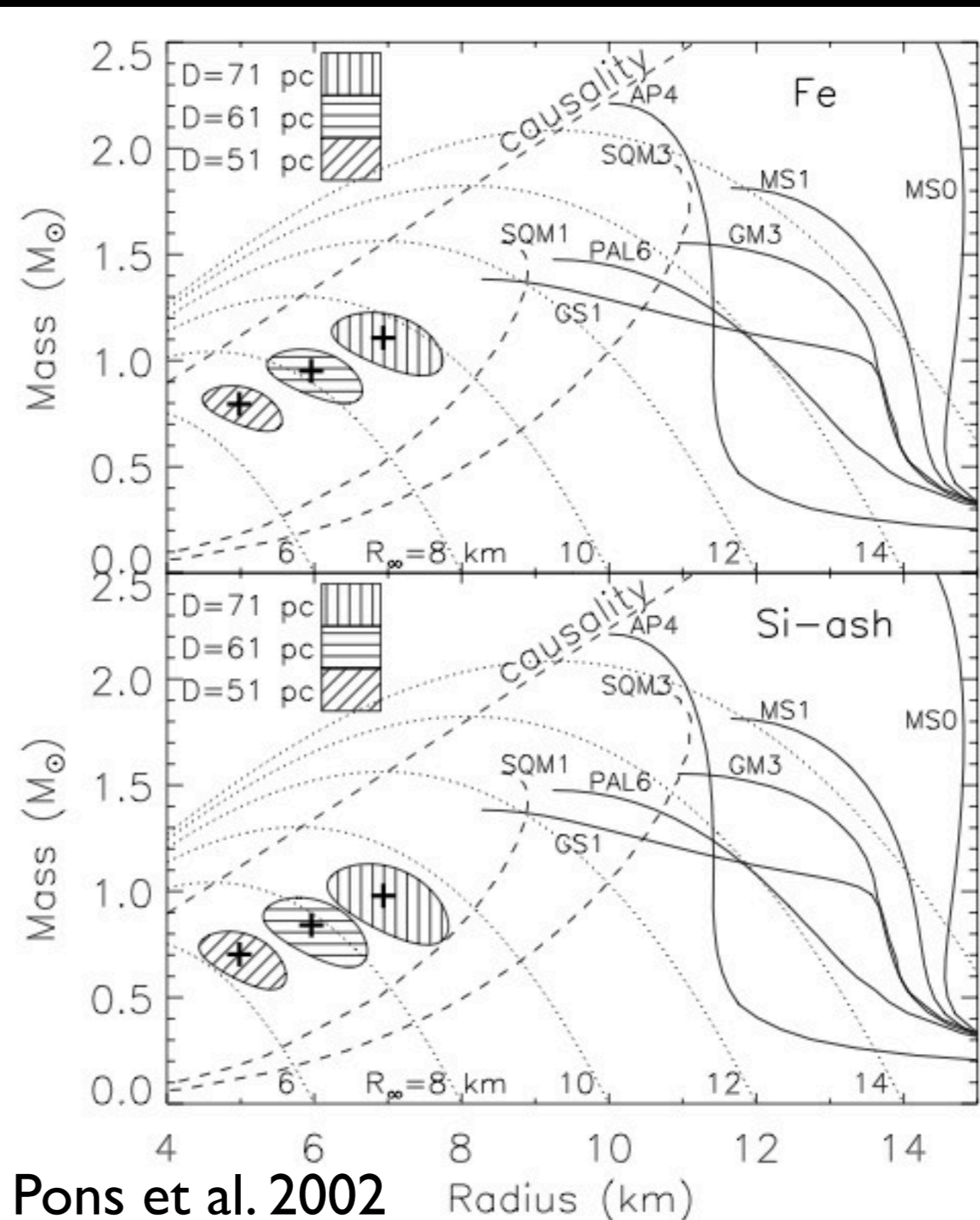
# What about thermally-cooling isolated neutron stars?



Only RX J 185635-3754 is remotely useful for this

## Problems:

- What is the distance?  
 $d = 51-177$  pc
- What is the composition?
- What are the effects of the magnetic field?



Combining previous  
results...

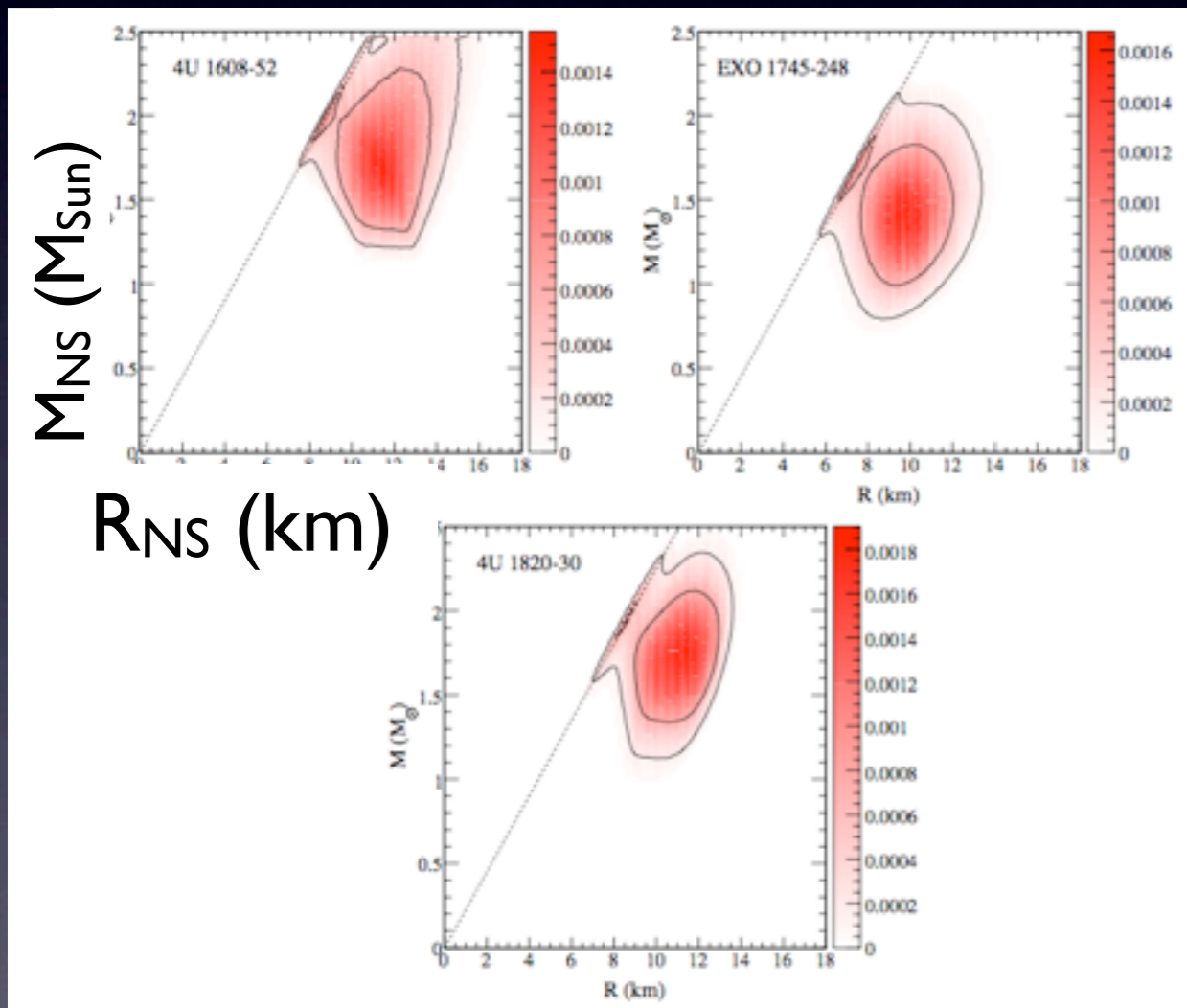
# $M_{\text{NS}}-R_{\text{NS}}$ contours can be combined to parametrize the EoS.



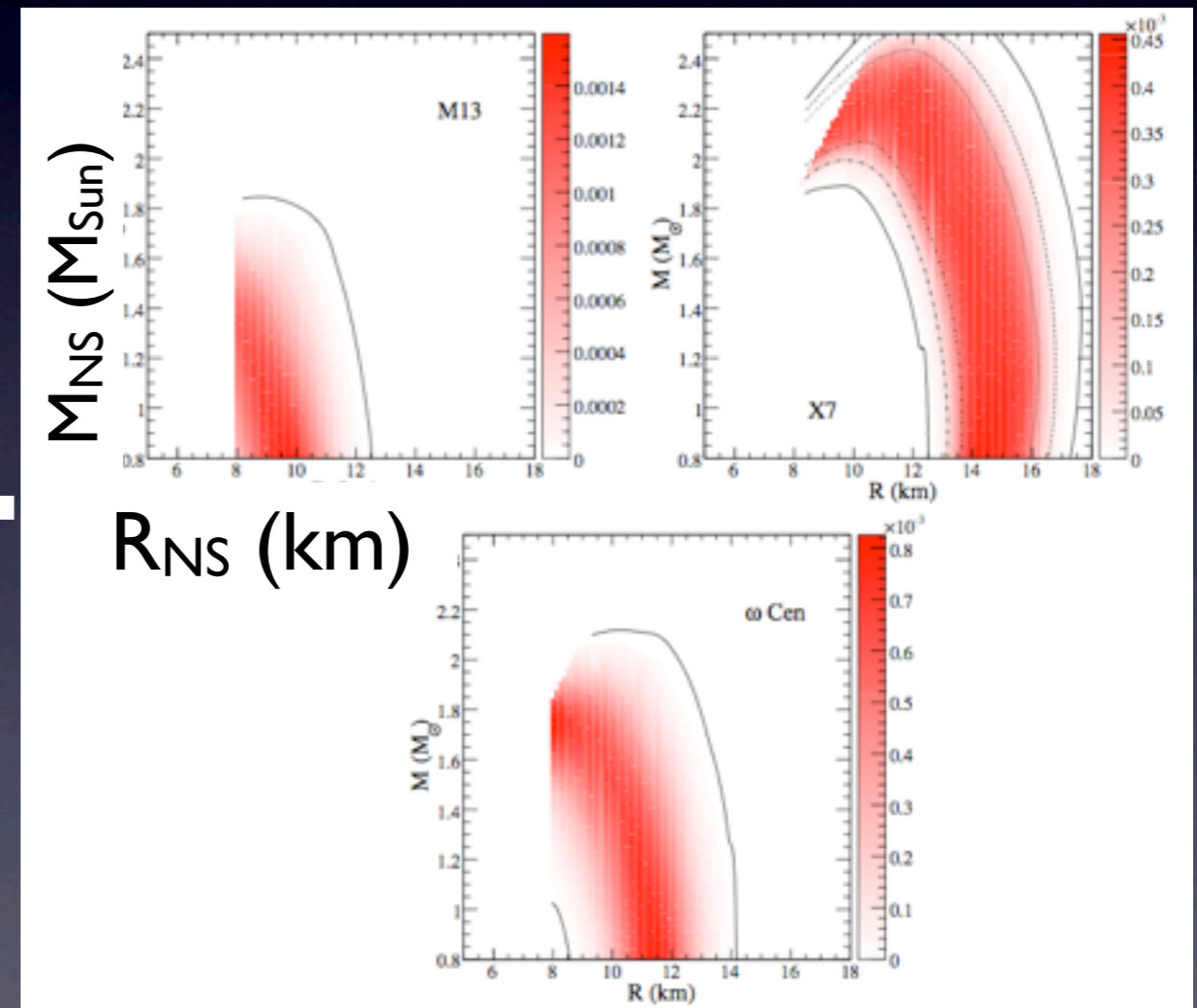
Steiner et al 2010, 2012

Type I X-ray bursts

Quiescent LMXBs



+



+ M-R contour of X-ray burst KSI731-260  
Özel et al. 2012

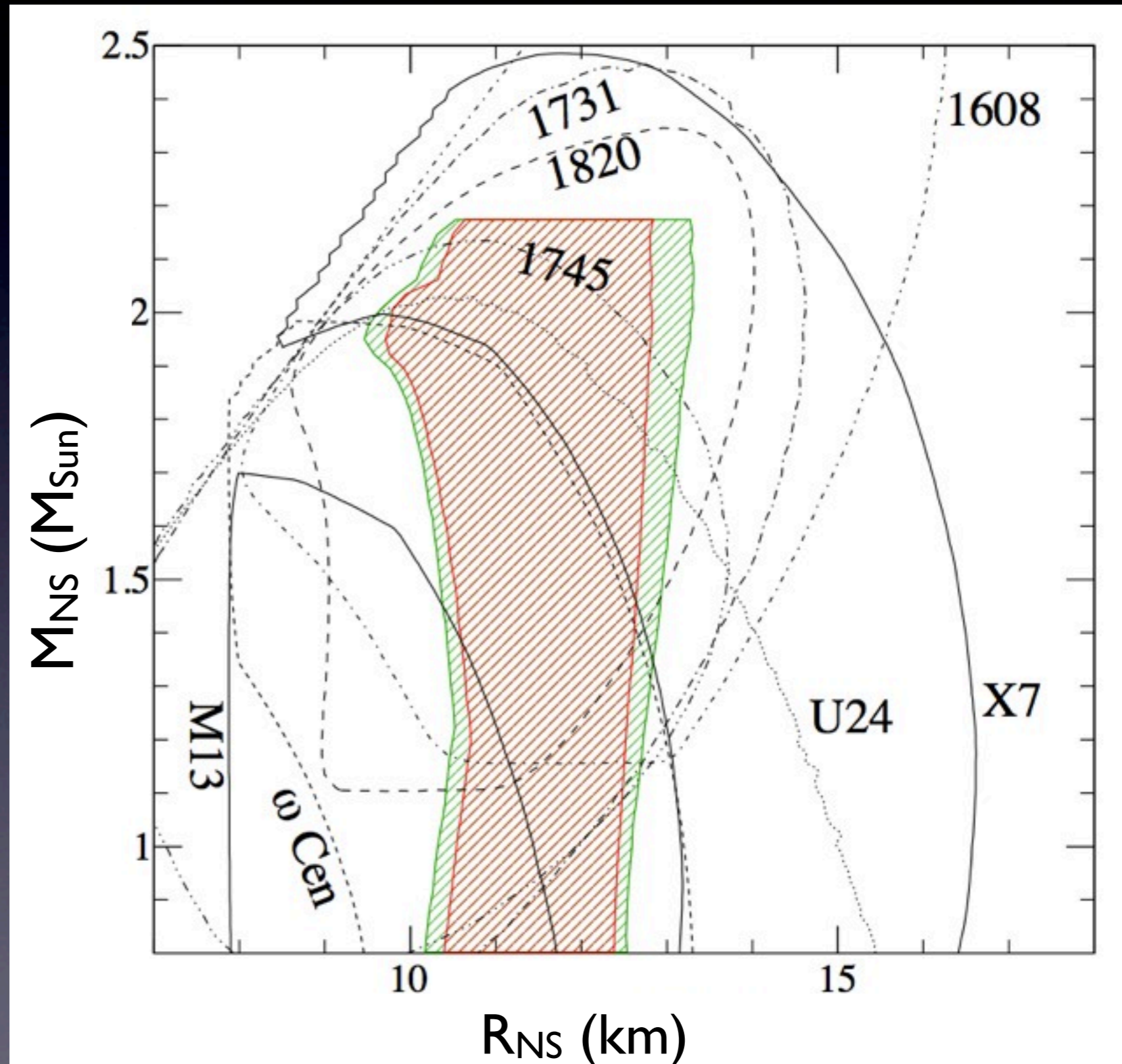
+ M-R contour of qLMXB in NGC6397  
from Guillot et al. 2011

# $M_{\text{NS}}-R_{\text{NS}}$ contours can be combined to parametrize the EoS.



$R_{\text{NS}}$  is roughly constrained between 10 and 13 km for a wide range of masses,

$R_{\text{NS}}$  mostly insensitive to the exclusion of extremum contours (like M13, or 47Tuc), or to the exclusion of type-I X-ray burst sources



Steiner et al 2012



# Summary of methods

Measurement	Type of neutron star	Limitations
$M_{\text{NS}}$	Radio timing of pulsars	Only "higher than before" $M_{\text{NS}}$ are useful
$M_{\text{NS}}$ and $R_{\text{NS}}$	Type-I X-ray bursts	Modelling issues
Limits on $M_{\text{NS}}$ and $R_{\text{NS}}$	Millisecond radio pulsars	Only "higher than before" $f_{\text{spin}}$ are useful
	kHz QPOs (X-ray)	Model assumptions
$I_{\text{NS}}$	Radio timing of double NS systems	Difficult measurement
$R_{\text{NS}}$ and $M_{\text{NS}}/R_{\text{NS}}$	Pulse-profile analysis (X-ray)	Need high S/N pulse profiles and several assumptions
$R_{\infty}$	Quiescent low-mass X-ray binaries	Need high S/N spectra Not useful individually
	Isolated neutron stars (X-ray)	Distance? Magnetic fields?

See talks during conference March 24-28, by:

C. Heinke: "Probing the neutron star equation of state through X-ray spectroscopy"

D. Chakrabarty: "Probing the neutron star equation of state through X-ray timing"

Z. Arzoumanian: "The Neutron Star Interior Composition Explorer: An X-ray Astrophysics Facility Dedicated to Neutron Star Science"

M. Chakraborty: "X-ray Bursts from Accreting Neutron star LMXBs"

H. Stiele: "Millihertz Quasi-periodic Oscillations in 4U 1636-536: Pulse Profile and Energy Spectrum"

