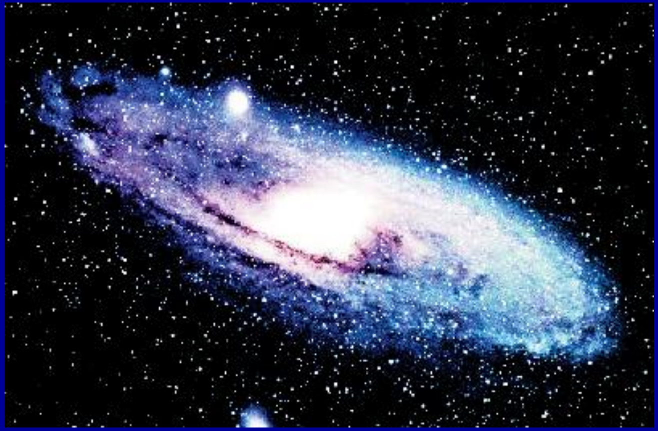


THEORY OF COOLING NEUTRON STARS

Self-similarity and model-independent data analysis

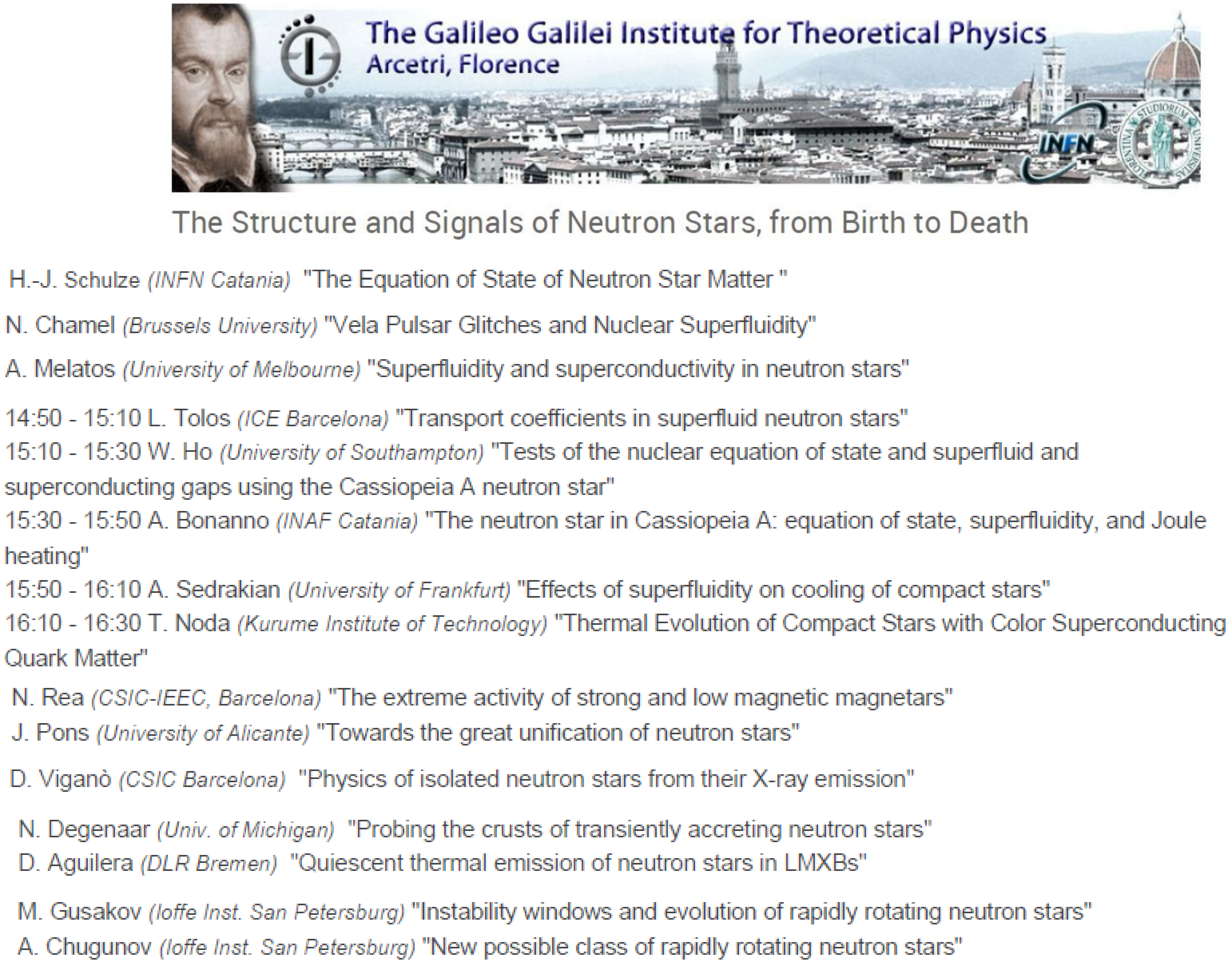
D.G. Yakovlev and P.S. Shternin

Ioffe Physical Technical Institute, St.-Petersburg, Russia

- **Introduction**
- **Neutrino emission**
- **Superfluidity**
- **Standard neutrino candle**
- **Selfsimilarity**
- **Types of cooling neutron stars**
- **Conclusions**

Florence – GGI – March 25, 2014

This conference and cooling neutron stars



The Galileo Galilei Institute for Theoretical Physics
Arcetri, Florence

The Structure and Signals of Neutron Stars, from Birth to Death

H.-J. Schulze *(INFN Catania)* "The Equation of State of Neutron Star Matter "

N. Chamel *(Brussels University)* "Vela Pulsar Glitches and Nuclear Superfluidity"

A. Melatos *(University of Melbourne)* "Superfluidity and superconductivity in neutron stars"

14:50 - 15:10 L. Tolos *(ICE Barcelona)* "Transport coefficients in superfluid neutron stars"
15:10 - 15:30 W. Ho *(University of Southampton)* "Tests of the nuclear equation of state and superfluid and superconducting gaps using the Cassiopeia A neutron star"
15:30 - 15:50 A. Bonanno *(INAF Catania)* "The neutron star in Cassiopeia A: equation of state, superfluidity, and Joule heating"
15:50 - 16:10 A. Sedrakian *(University of Frankfurt)* "Effects of superfluidity on cooling of compact stars"
16:10 - 16:30 T. Noda *(Kurume Institute of Technology)* "Thermal Evolution of Compact Stars with Color Superconducting Quark Matter"

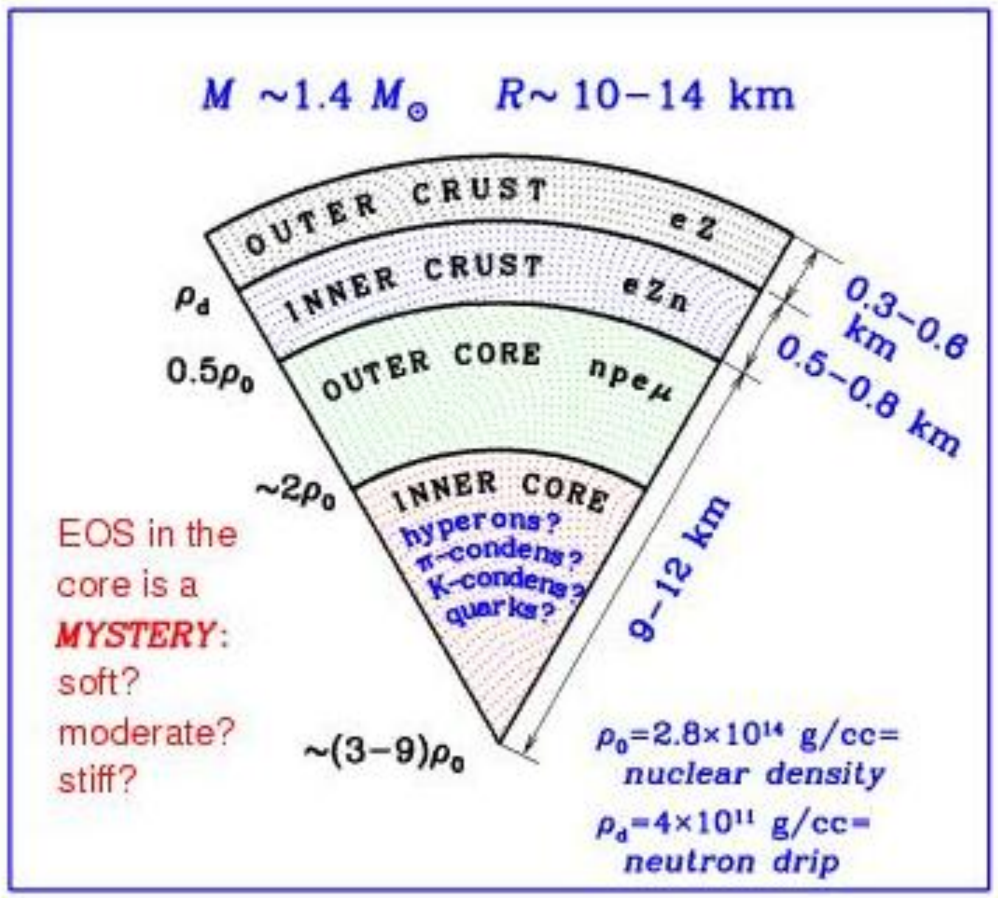
N. Rea *(CSIC-IEEC, Barcelona)* "The extreme activity of strong and low magnetic magnetars"
J. Pons *(University of Alicante)* "Towards the great unification of neutron stars"

D. Viganò *(CSIC Barcelona)* "Physics of isolated neutron stars from their X-ray emission"

N. Degenaar *(Univ. of Michigan)* "Probing the crusts of transiently accreting neutron stars"
D. Aguilera *(DLR Bremen)* "Quiescent thermal emission of neutron stars in LMXBs"

M. Gusakov *(Ioffe Inst. San Petersburg)* "Instability windows and evolution of rapidly rotating neutron stars"
A. Chugunov *(Ioffe Inst. San Petersburg)* "New possible class of rapidly rotating neutron stars"

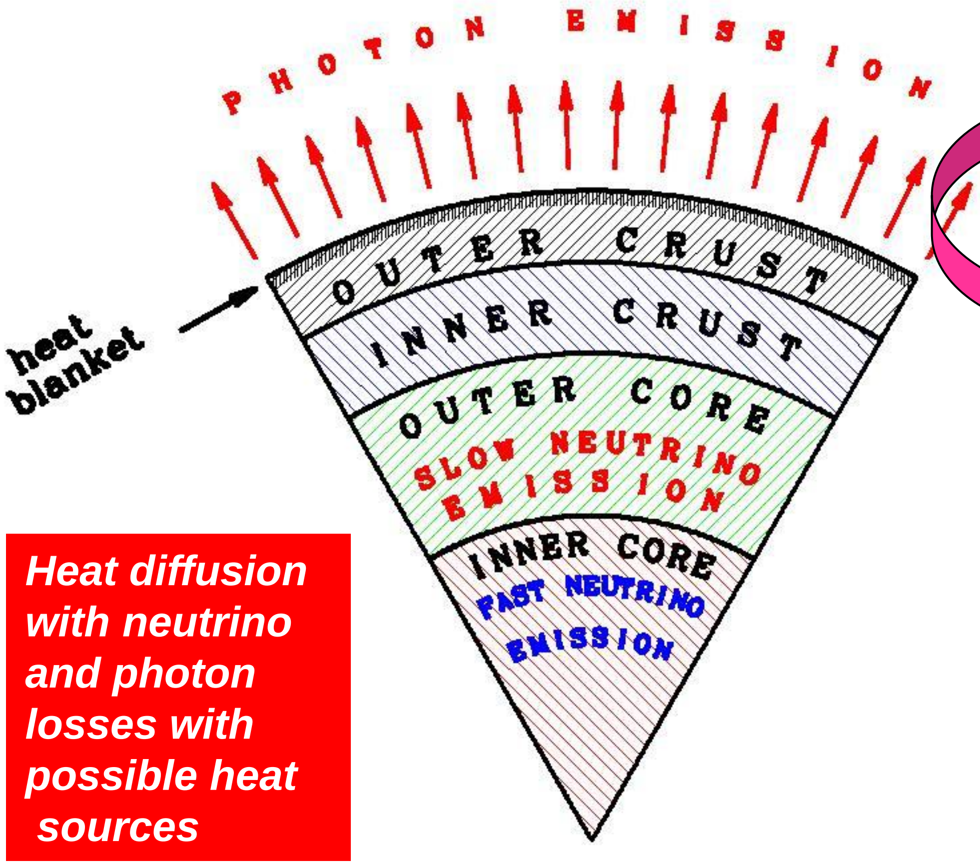
Neutron star structure



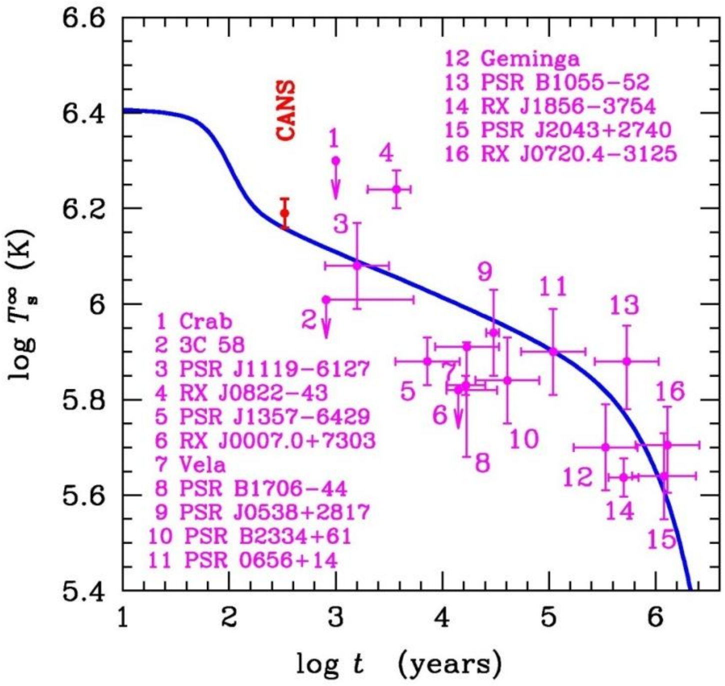
Mystery:
EOS of superdense matter in the core

For simplicity, consider
nucleon core:
neutrons
protons
electrons
muons
EOS=?
Superlfuidity=?

Cooling of isolated neutron stars



Heat diffusion with neutrino and photon losses with possible heat sources



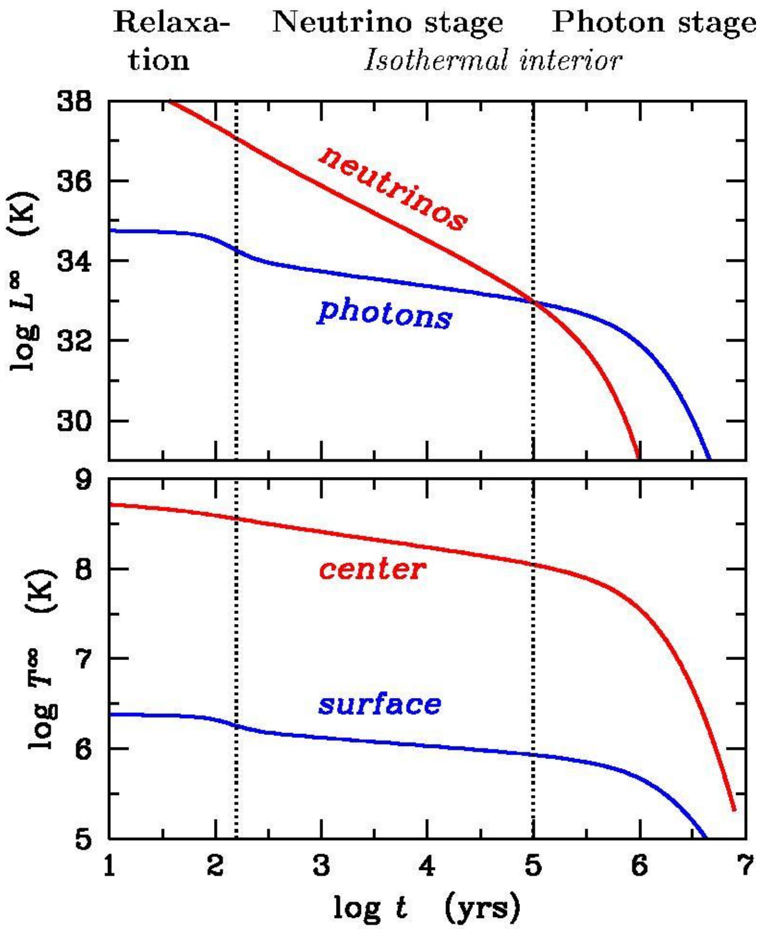
Cooling regulators:

- ***EOS***
- ***Neutrino emission***
- ***Heat capacity***
- ***Thermal conductivity***
- ***Superfluidity***
- ***Internal heat sources?***

What information can be extracted from observations?

THREE COOLING STAGES

Example: ***non-superfluid NS***
Modified Urca cooling



Stage	Duration	Physics
Relaxation	10—100 yr	Crust
Neutrino	10-100 kyr	Core, surface
Photon	infinite	Surface, core, reheating

Isothermal Interior

Isothermal interior:

$\tilde{T}(t) = T(r,t)\ \exp\Phi(r)$ ***=redshifted internal temperature, independent of r***

Equations of thermal evolution reduce to global thermal balance:

$$\boxed{C(\tilde{T})\ \frac{d\tilde{T}}{dt} = -L_\nu^\infty(\tilde{T}) + L_h^\infty - L_\gamma^\infty(T_s)}$$

$$L_\nu^\infty(\tilde{T}) = \int dV\ \ Q(T)\ \mathrm{e}^{2\Phi},\ \ L_h^\infty = \int dV\, Q_h\ \mathrm{e}^{2\Phi},\ \ \mathrm{C}(\tilde{T}) = \int dV\, c_T(T)$$

= redshifted total neutrino luminosity, heating power and heat capacity of NS

$L_\gamma^\infty(T_s) = 4\pi\sigma R_\infty^2 (T_s^\infty)^4$ ***= redshifted thermal photon luminosity of NS***

$T_s = T_s(\tilde{T}) \sim \sqrt{\tilde{T}}$ ***= solution for heat blanket***

$$R_\infty = R\ \sqrt{1-2GM/Rc^2}$$

$dV = \dfrac{4\pi r^2\ dr}{\sqrt{1-2Gm/c^2 r}}$ ***= proper volume element***

$$T_s^\infty = T_s\sqrt{1-2GM/Rc^2}$$

Cooling of isolated middle-aged neutron star

Global thermal balance:

$$C(\tilde{T})\ \frac{d\tilde{T}}{dt} = -L_{\nu}^{\infty}(\tilde{T}) \boxed{\xcancel{+ L_{h}^{\infty} - L_{\gamma}^{\infty}(T_s)}}$$

Star is cooling from inside via neutrino emission from its core

The equation is immediately integrated ⇒

$$\frac{d\tilde{T}}{dt} = -l(\tilde{T}), \quad l(\tilde{T}) \equiv \frac{L_{\nu}^{\infty}}{C} = \frac{\int dV\ Q\ \exp(2\Phi)}{\int dV\ c_T}$$

$\boxed{l(\tilde{T}, M)}$ ***= neutrino cooling rate [K/yr]= the only one function that drives cooling***

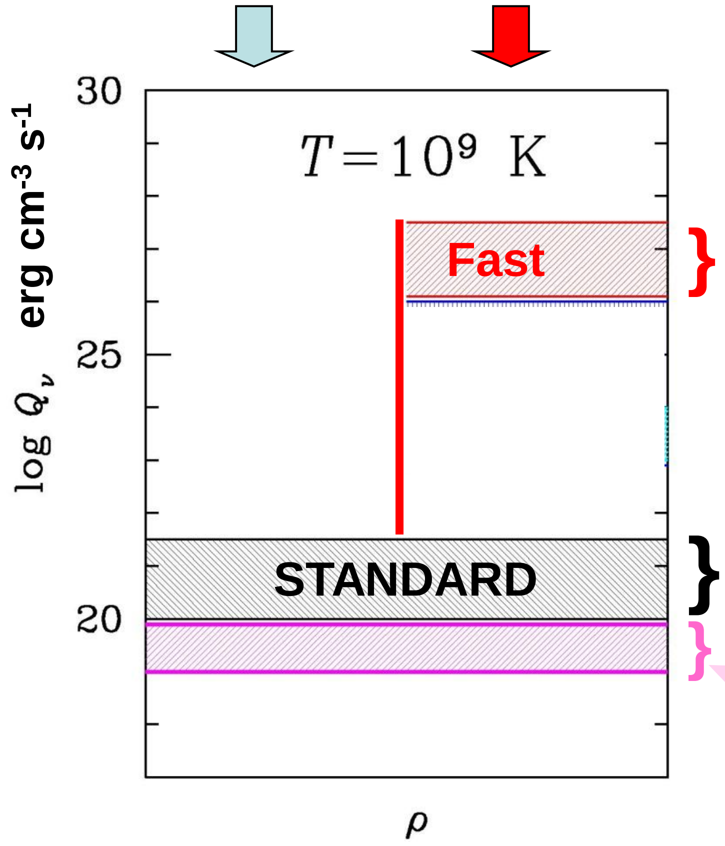
Using cooling theory one can only determine this one function, and nothing else!

The function is insensitive to details of NS structure;
but sensitive to extraordinary things: EOS, neutrino emission, superfluidity

Neutrino emission from cores of non-superfluid NSs

Outer core
Slow emission

Inner core
Fast emission



NS with nucleon core:
N=n, p

Direct Urca

$$n \rightarrow p + e + \bar{\nu}_e \qquad p + e \rightarrow n + \nu_e$$

Modified Urca

$$nN \rightarrow pNe\bar{\nu} \qquad pNe \rightarrow nN\nu$$

NN bremsstrahlung

$$N + N \rightarrow N + N + \nu + \bar{\nu}$$

$$\left\{ \begin{bmatrix} n+n \\ n+p \\ p+p \end{bmatrix} \right.$$

Enhanced emission in inner cores of massive neutron stars:

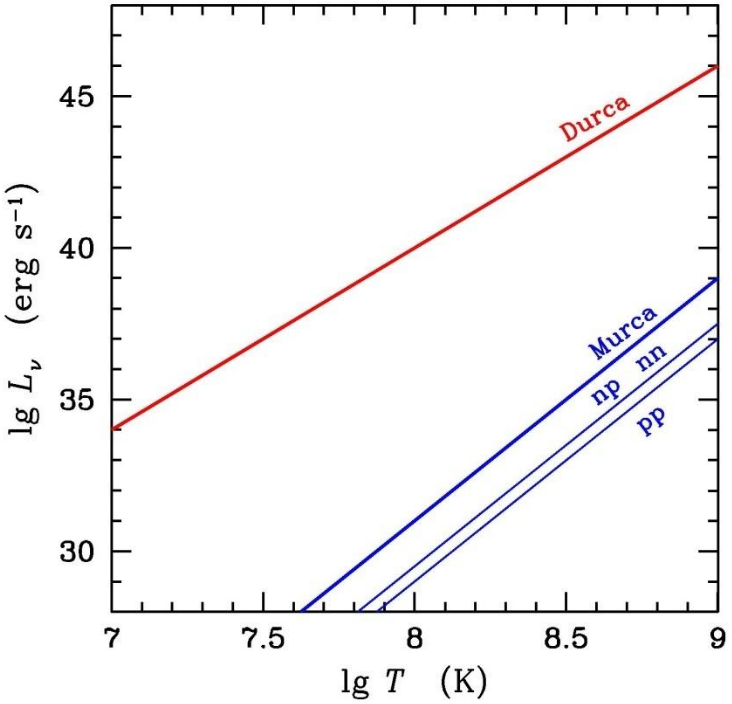
$$Q_{\mathrm{FAST}} = Q_{0\mathrm{F}} T^6 \qquad L_{\mathrm{FAST}} = L_{0\mathrm{F}} T^6$$

Everywhere in neutron star cores:

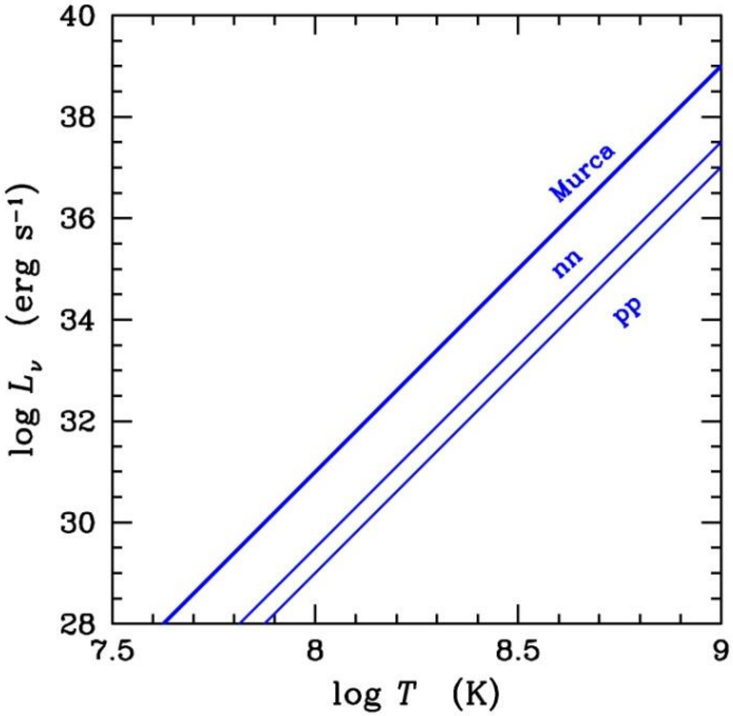
$$Q_{\mathrm{SLOW}} = Q_{0\mathrm{S}} T^8 \qquad L_{\mathrm{SLOW}} = L_{0\mathrm{S}} T^8$$

Neutrino emission of non-superfluid neutron star

Direct Urca is allowed (maximal cooling)



Direct Urca is forbidden (minimal cooling)



1. $nN \to pNe\bar{\nu}$ $pNe \to nN\nu$

2. $nn \to nn\nu\bar{\nu}$ 3. $np \to np\nu\bar{\nu}$ 4. $pp \to pp\nu\bar{\nu}$

Superfluid neutron stars

***Mechanism of superfluidity:* Cooper pairing of degenerate neutrons and/or protons due to nuclear attraction**

***Any superfluidity* is defined by critical temperature T_C, that depends on density**

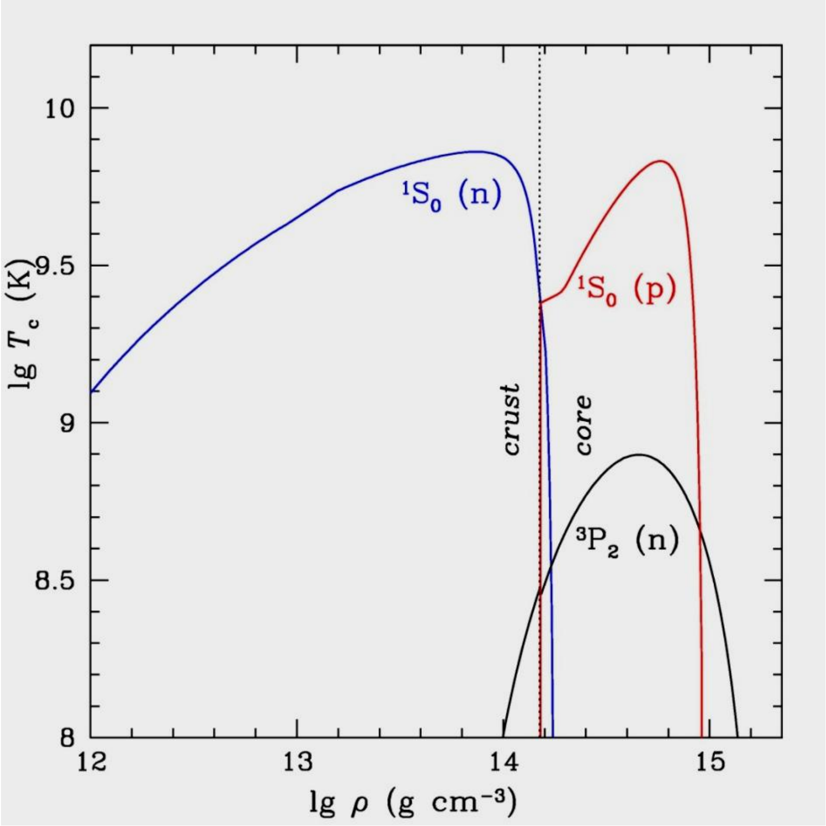
Main effects:

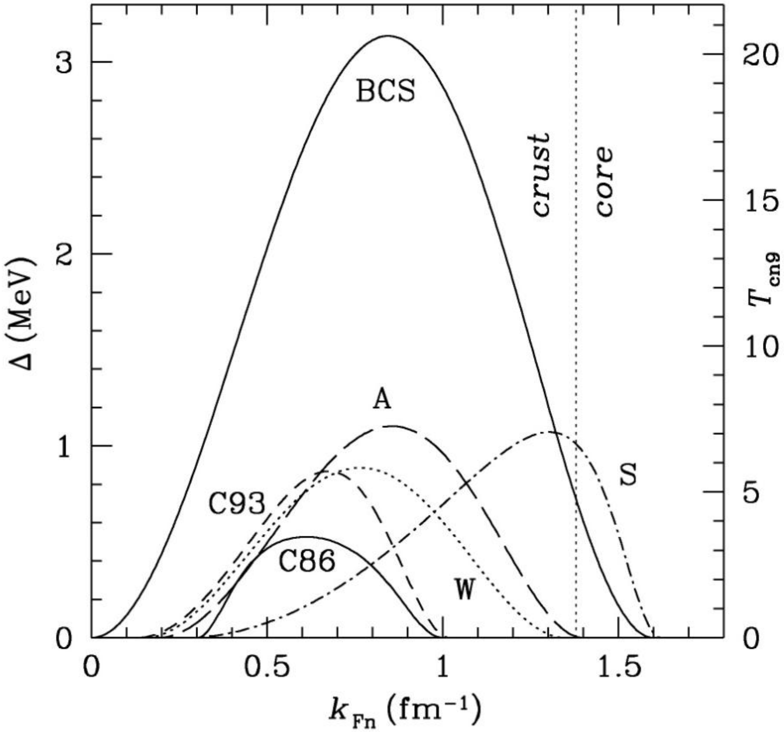
- **has almost no effect of EOS and hydrostatic structure of neutron stars**
- **suppresses ordinary neutrino processes (especially at $T \ll T_c$)**
- **creates a new mechanism of neutrino emission due to Cooper pairing of nucleons**
- **affects heat capacity**

Superfluidity – Critical temperatures

Dependence of T_c on density

$\Delta_0 \sim 1\ \text{MeV} \qquad T_c \sim 10^{10}\ \text{K} \quad \text{high}\ \ T_c\,!!!$





At high densities superfluidity disappears

After Lombardo & Schulze (2001)
A=Ainsworth, Wambach, Pines (1989)
S=Schulze et al. (1996)
W=Wambach, Ainsworth, Pines (1993)
C86=Chen et al. (1986)
C93=Chen et al. (1993)

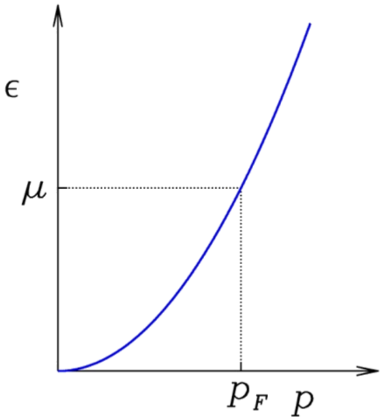
Our task is to study $T_{cn}(\rho),\ T_{cp}(\rho)$ ***in neutron star core***

Superfluidity – microscopic manifestations

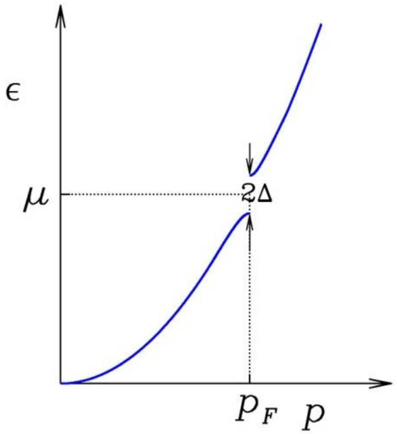
Creates gap $\Delta(\rho,T)$
in energy spectrum
near Fermi level

$$\varepsilon = \frac{p^2}{2m}$$

T=0



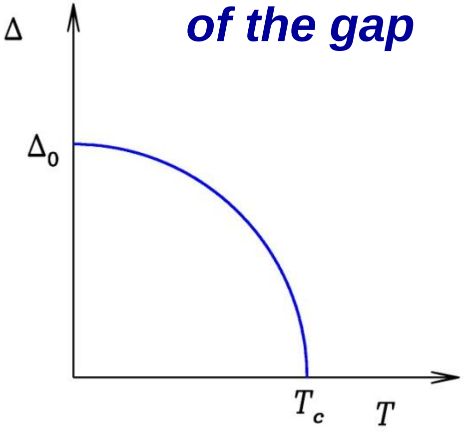
Free Fermi gas



Superfluid Fermi gas

Microscopic calculations of the gap ***are very model dependent*** *(nuclear interaction; many-body effects)*

Temperature dependence of the gap



Neutrino emission due to Cooper pairing

Flowers, Ruderman and Sutherland (1976)

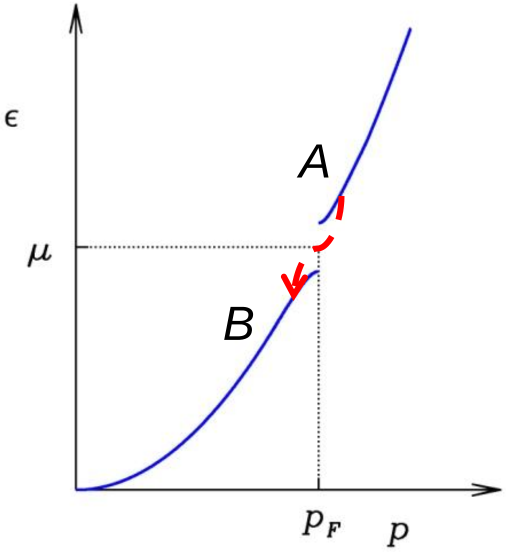
Voskresensky and Senatorov (1987)

Schaab et al. (1997)

$$\tilde{n}+\tilde{n}\rightarrow\nu+\bar{\nu}$$

Physics:

Jumping over cliff from branch A to B



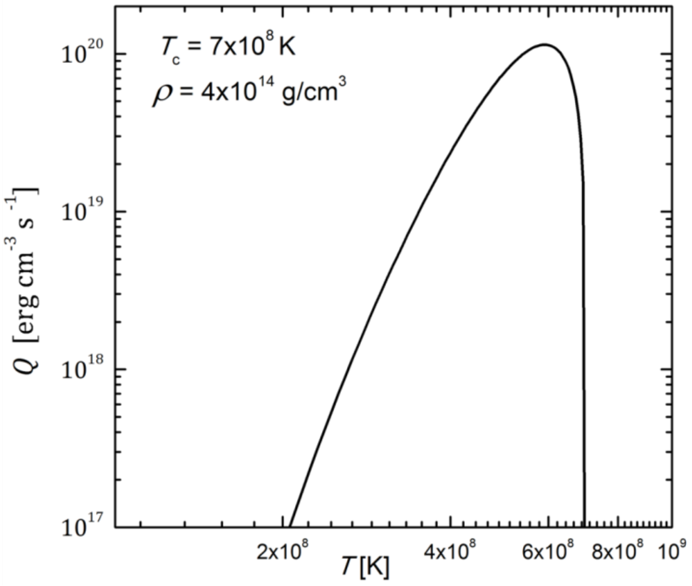
Features:

- ***Efficient only for triplet-state pairing of neutrons***
- ***Non-monotonic Q(T)***
- ***Strong many-body effects***

Single state proton superfluidity suppresses neutrino emission

Triple state neutron superfluidity can enhance

Leinson (2001)
Leinson and Perez (2006)
Sedrakian, Muether, Schuck (2007)
Kolomeitsev, Voskresensky (2008)
Steiner, Reddy (2009)
Leinson (2010) S



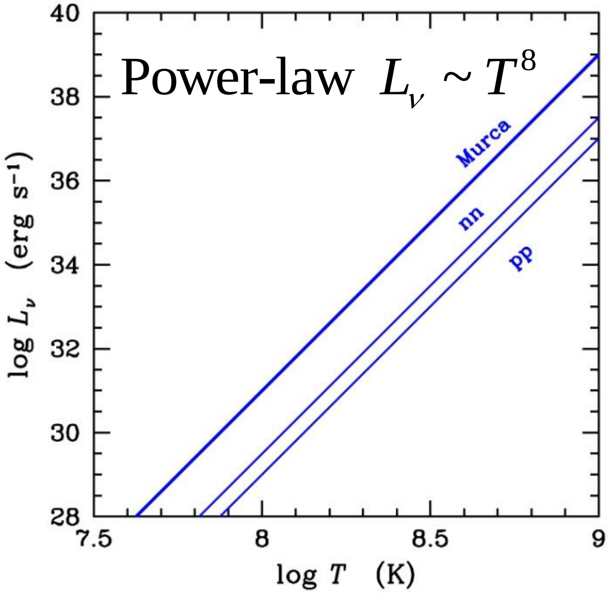
Temperature dependence of neutrino emissivity due to Cooper pairing

Neutrino luminosity of superfluid neutron star

Non-superfluid star with nucleon core

Standard Murca cooling

$$L_\nu = L_\nu^{Murca}$$



$\boxed{nN \to pNe\bar{\nu} \quad pNe \to nN\nu}$

$nn \to nn\nu\bar{\nu}$

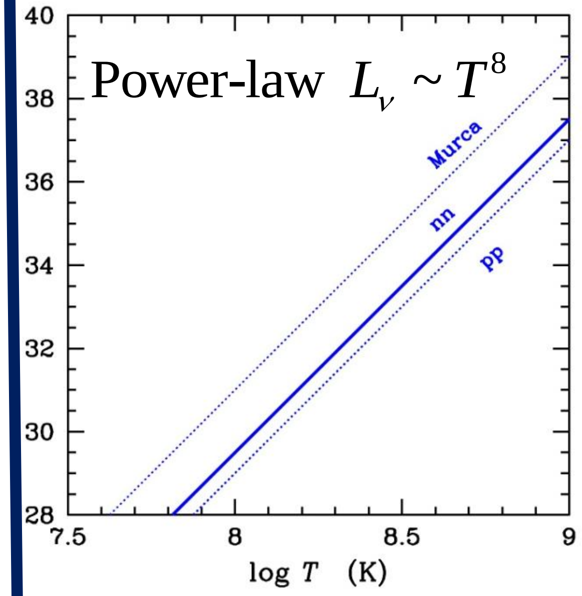
$np \to np\nu\bar{\nu}$

$pp \to pp\nu\bar{\nu}$

Add strong proton superfluidity

Very slow cooling

$$L_\nu \sim 0.01\ L_\nu^{Murca}$$



~~$nN \to pNe\bar{\nu} \quad pNe \to nN\nu$~~

$\boxed{nn \to nn\nu\bar{\nu}}$

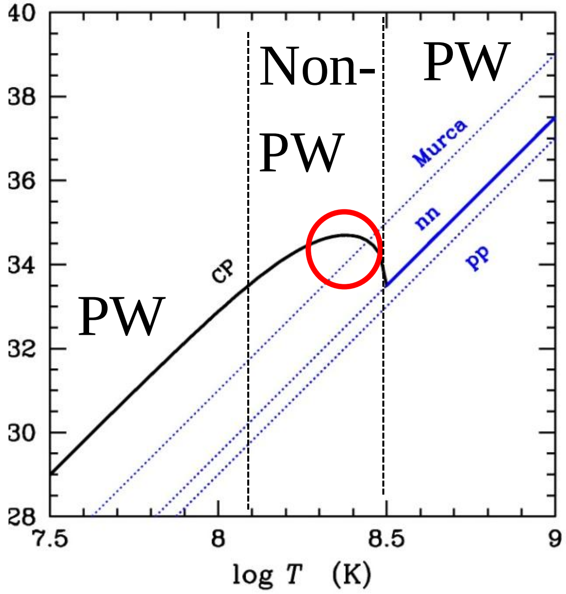
~~$np \to np\nu\bar{\nu}$~~

~~$pp \to pp\nu\bar{\nu}$~~

Add moderate neutron superfluidity:

CP neutrino outburst

$$L_\nu^{Cooper} \sim (10-100)\ L_\nu^{Murca}$$



~~$nN \to pNe\bar{\nu} \quad pNe \to nN\nu$~~

~~$nn \to nn\nu\bar{\nu}$~~

~~$np \to np\nu\bar{\nu}$~~

~~$pp \to pp\nu\bar{\nu}$~~

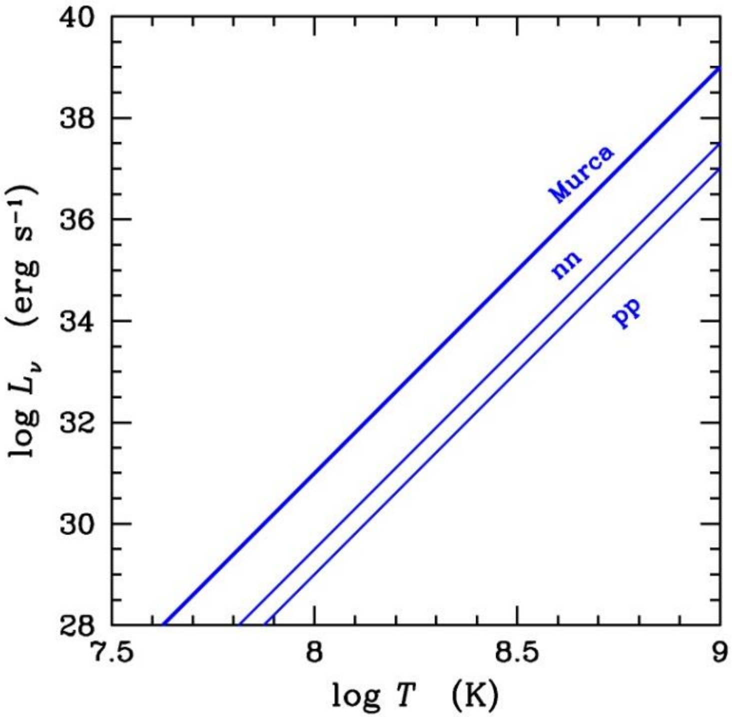
$\boxed{\tilde{n}\tilde{n} \to \nu\bar{\nu}}$

Pow-law $L_\nu \sim T^8$ is violated only when superfluidity appears

STANDARD NEUTRINO CANDLE

Def: ***Standard neutrino candle = a neutron star which cools as a nonsuperfluid star through modified Urca process at given M and R*** *= convenient cooling model to compare with observations*

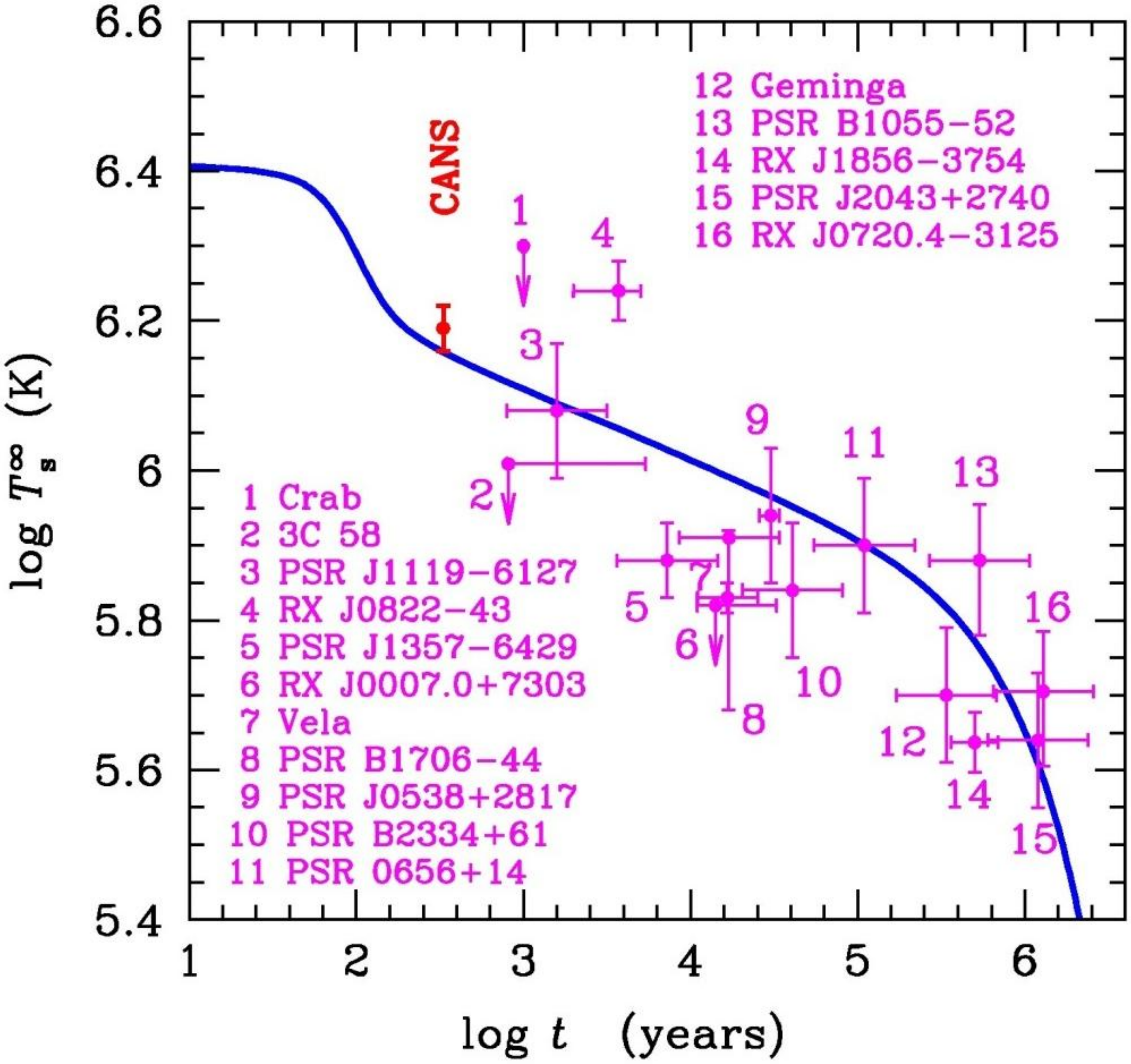
Nonsuperfluid star
with nucleon core
Murca cooling



BASIC COOLING CURVE AND STANDARD NEUTRINO CANDLE
Universal at neutrino cooling stage with isothermal interior

Nonsuperfluid star
Nucleon core
EOS PAL (1988)

Standard neutrino candle

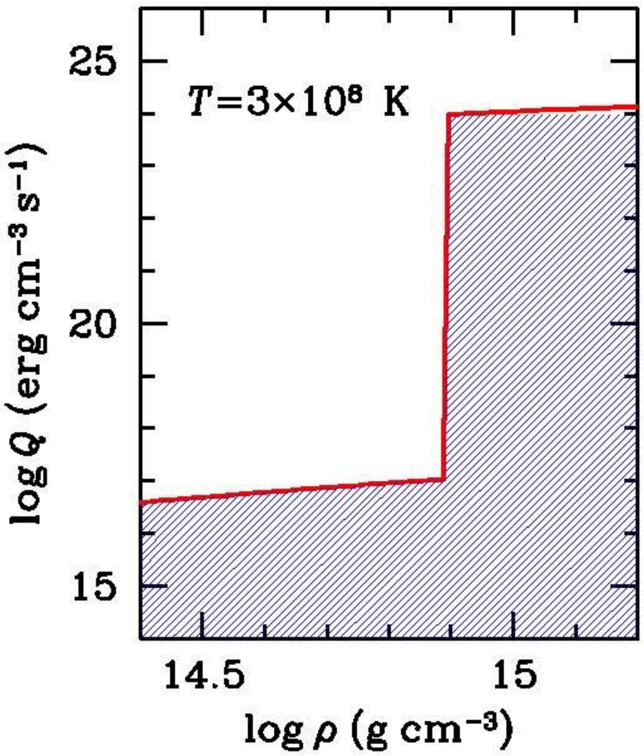


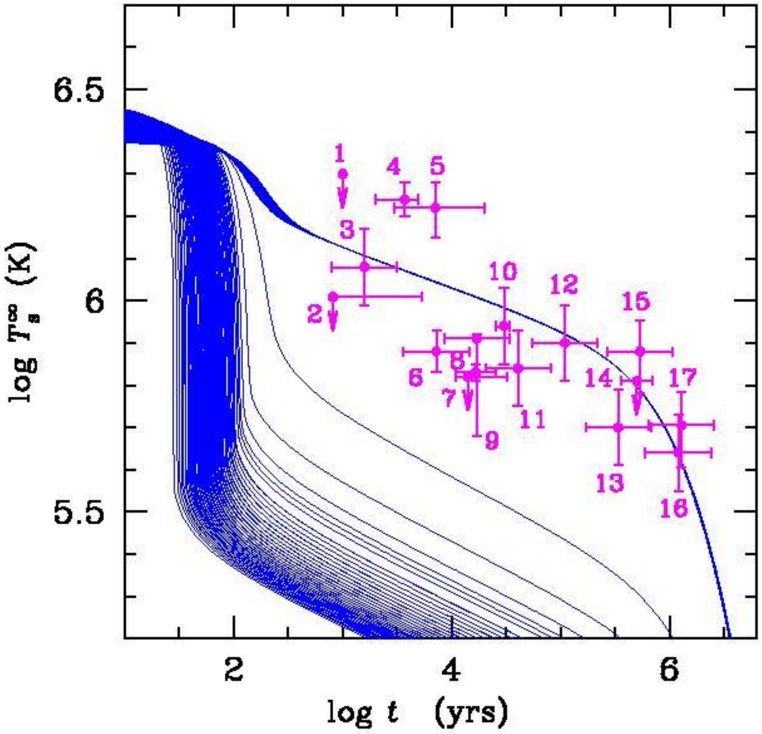
Oleg Gnedin

Cooling code

Unsuccessful explanation:
Mucra and Durca, no superfluidity

1=Crab
2=PSR J0205+6449
3=PSR J1119-6127
4=RX J0822-43
5=1E 1207-52
6=PSR J1357-6429
7=RX J0007.0+7303
8=Vela
9=PSR B1706-44
10=PSR J0538+2817
11=PSR B2234+61
12=PSR 0656+14
13=Geminga
14=RX J1856.4-3754
15=PSR 1055-52
16=PSR J2043+2740
17=PSR J0720.4-3125





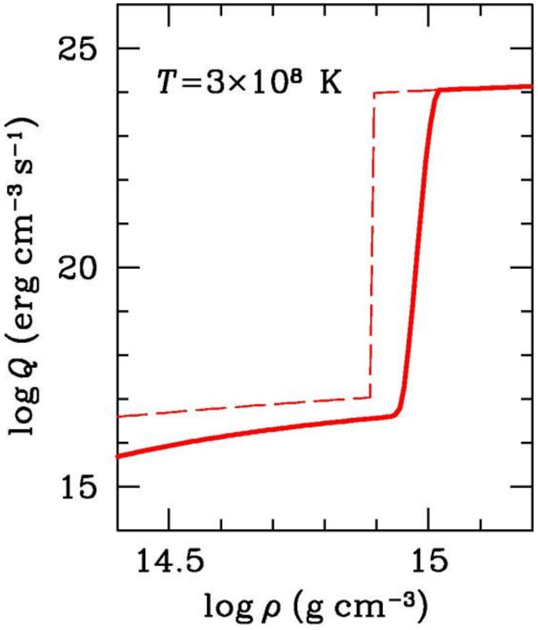
Does not explain the data

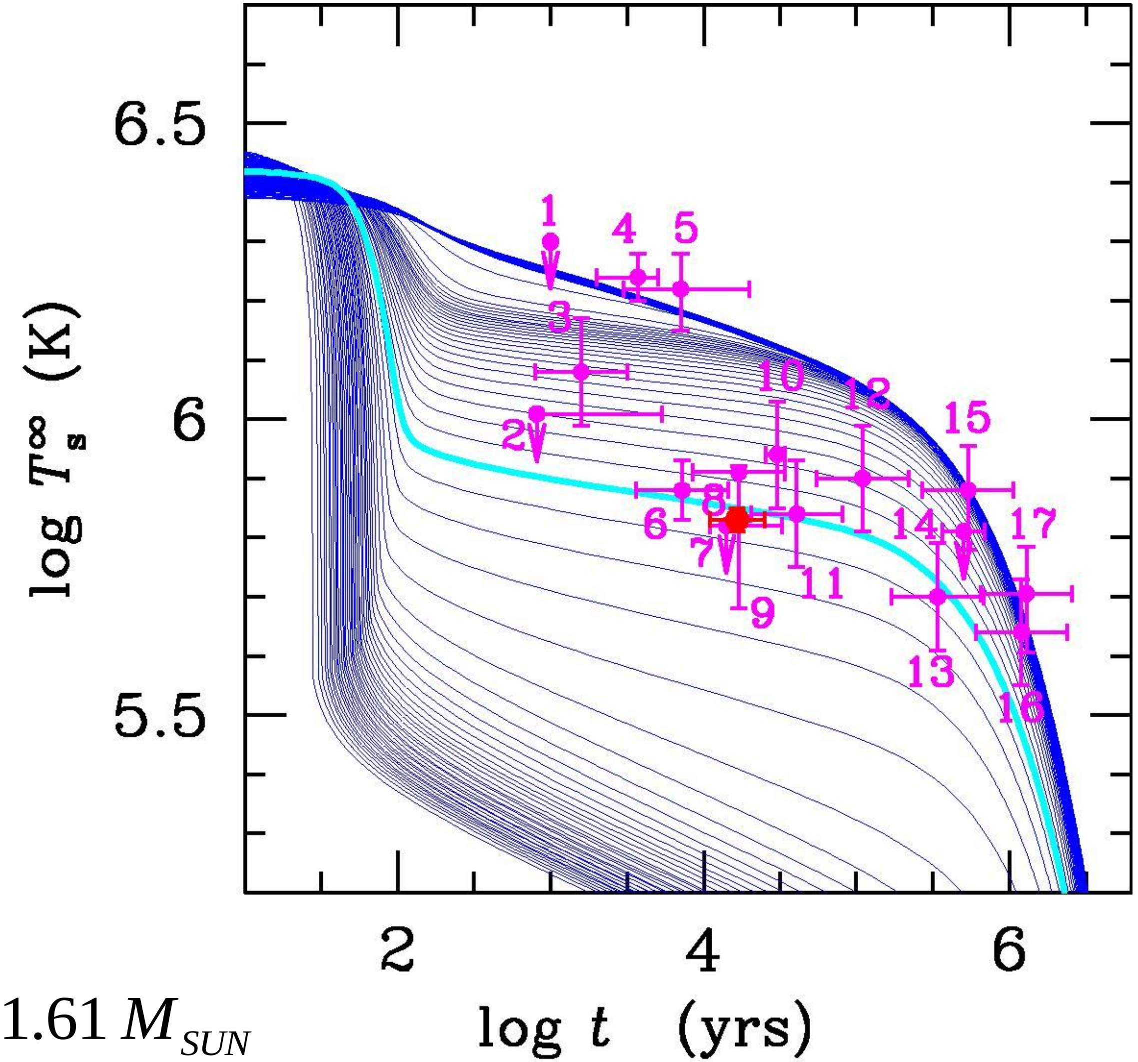
$$M_{\mathrm{MAX}} = 1.977\, M_{SUN} \qquad \rho_{\mathrm{c}} = 2.578 \times 10^{15}\ \mathrm{g/cc}$$

$$M_{\mathrm{D}} = 1.358\, M_{SUN} \qquad \rho_{\mathrm{c}} = 8.17 \times 10^{14}\ \mathrm{g/cc}$$

From 1.1 M_{SUN} to 1.98 M_{SUN} with step $\Delta M = 0.01\, M_{SUN}$

Direct Urca and strong ptoton SF in outer core



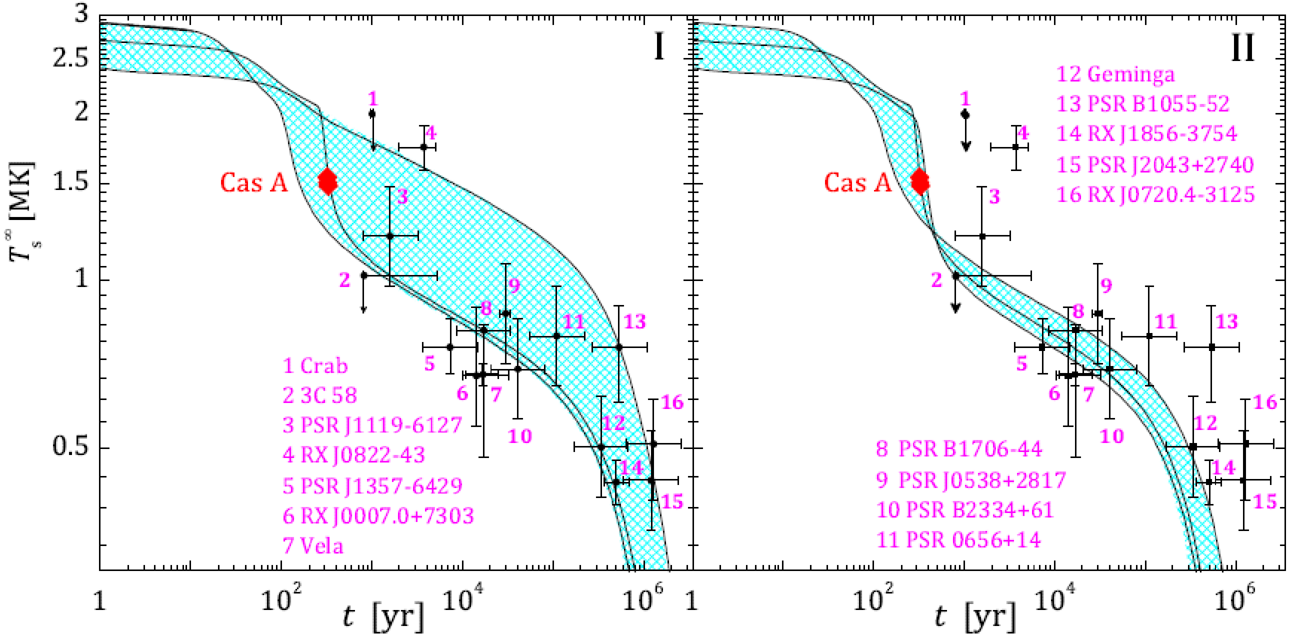


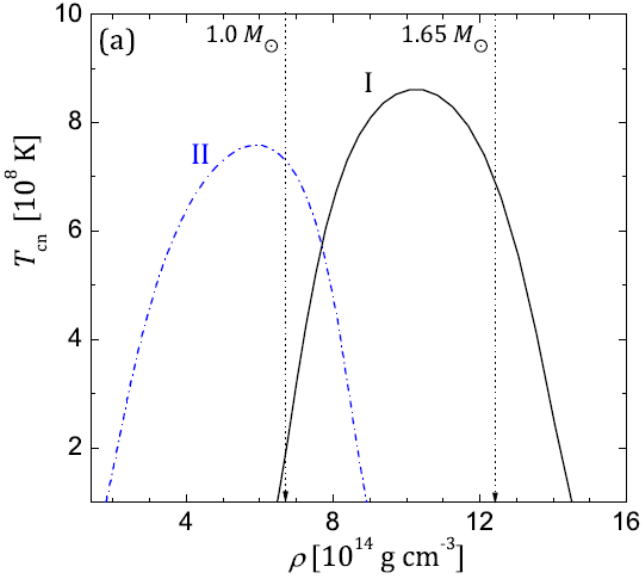
$$M_{\mathrm{VELA}} = 1.61\, M_{SUN}$$

Strong proton SF, moderate neutron SF, no Durca

$$M = 1.0\,M_\odot - M_{\max}$$

One model of superfluidity for all neutron stars





Only T_{cn} superfluidity I explains all the sources

Gusakov et al. (2004)

Alternatively: wider $T_{cn}(\rho)$ profile, but the efficiency of CP neutrino emission at low densities is weak

MODEL-INDEPENDENT STANDARD NEUTRINO CANDLE

Isothermal interior, neutrino cooling stage, lower-law cooling

$$\frac{d\tilde{T}}{dt} = -l(\tilde{T}), \quad l(\tilde{T}) \equiv \frac{L_\nu^\infty}{C}$$

= cooling equation for INSs

Assume: $L_\nu^\infty(\tilde{T}) \sim \tilde{T}^n, \quad C(\tilde{T}) \sim \tilde{T} \quad \Rightarrow \quad l(\tilde{T}) = q\tilde{T}^{n-1}$

Solution: $\tilde{T}(t) = \dfrac{1}{[(n-2)qt]^{1/(n-2)}}, \quad l(t) = \dfrac{\tilde{T}(t)}{(n-2)t}$

Case n=8: Slow cooling $\tilde{T}(t) \sim t^{-1/6}, \quad T_s^\infty(t) \sim t^{-1/12}$ $\quad T_s^\infty \sim \sqrt{\tilde{T}}$

Model-independent solution for standard candles (many EOSs):

$$\tilde{T}_{SC}(t) = 3.45 \times 10^8 \ \mathrm{K} \ (1-x) \left[1 + 0.12 \left(\frac{R}{10 \ \mathrm{km}}\right)^2\right] \left(\frac{t_c}{t}\right)^{1/6}$$

$$x = \frac{2GM}{Rc^2}$$

$$t_c = 330 \ \mathrm{yrs}$$

Yakovlev, Ho, Shternin, Heinke, Potekhin (2011)

Extracting neutrino luminosity function (power-law cooling)

***Step 1.* Observe thermal emission of NS. Assume some M, R + NS atmosphere model and infer T_s (or L_s).**

***Step 2.* Assume some composition of the heat blanketing envelope, use the theoretical $T_s – T_b$ relation and find the internal current temperature of the star *T(t)*.**

***Step 3.* Assume standard cooling ($T(t) \sim t^{-1/6}$) and determine the internal current neutrino luminosity function *l(M,R,t)*.**

***Step 4.* Use the theory and find the internal current temperature $T_{SC}(t)$ of the standard candle for given M, R, t.**

***Step 5.* Compare *T(t)* with $T_{SC}(t)$ and determine the neutrino luminosity function in units of standard candles,**

$$f_l = l(\tilde{T}) / l_{SC}(\tilde{T})$$

***Congrats! You can now reconstruct the cooling history of the star in a model-independent way.* The problem you solved is selfsilimar, with one selfsimilarity parameter f_l.**

You can obtain f_l and analyze it later

Analyzing model-independent results (PL cooling)

You have some f_l =1, >1 or <1 . How to analyze?

If $f_l \approx 1 \;\Rightarrow$ consistent with standard non-SF star cooling via modified Urca

If $0.01 \le f_l \le 1 \;\Rightarrow$ warmer star, can cool via NN bremsstrahlung, modified Urca suppressed by SF

If $f_l \le 0.01 \;\Rightarrow$ very warm star, needs internal reheating

If $1 \le f_l \le 100 \;\Rightarrow$ colder star, can cool via CP neutrino emission

If $f_l > 100 \;\Rightarrow$ very cold star, needs enhanced cooling inside (direct Urca or something like)

EXAMPLES

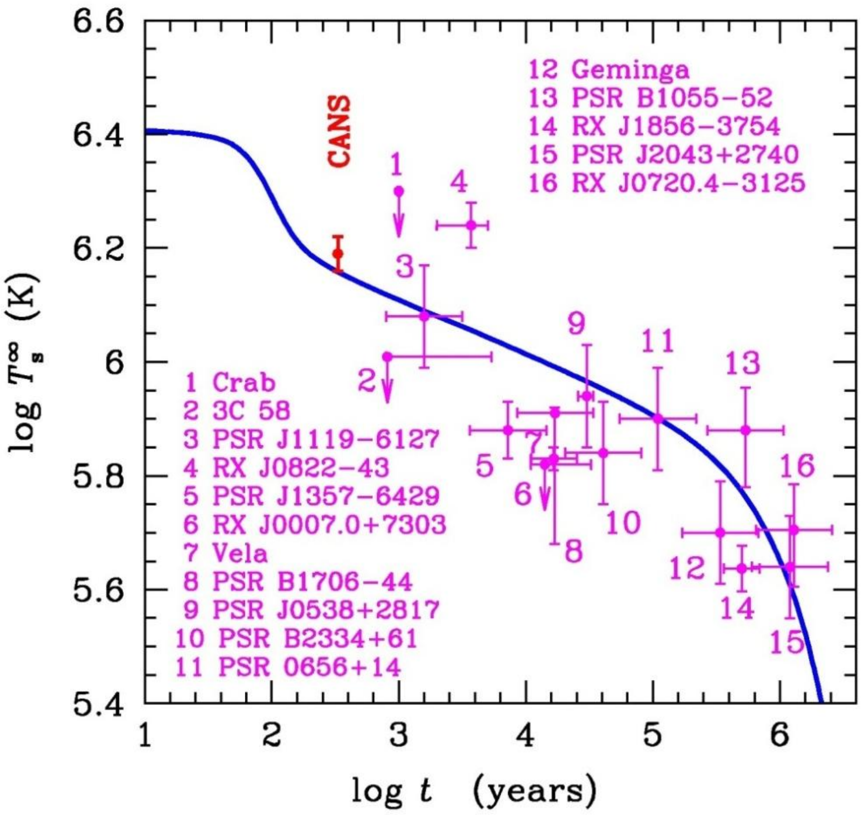
of model-independent analysis

Fe heat blanketing envelope

Vela: ***$f_l=100$ = enhanced cooling***

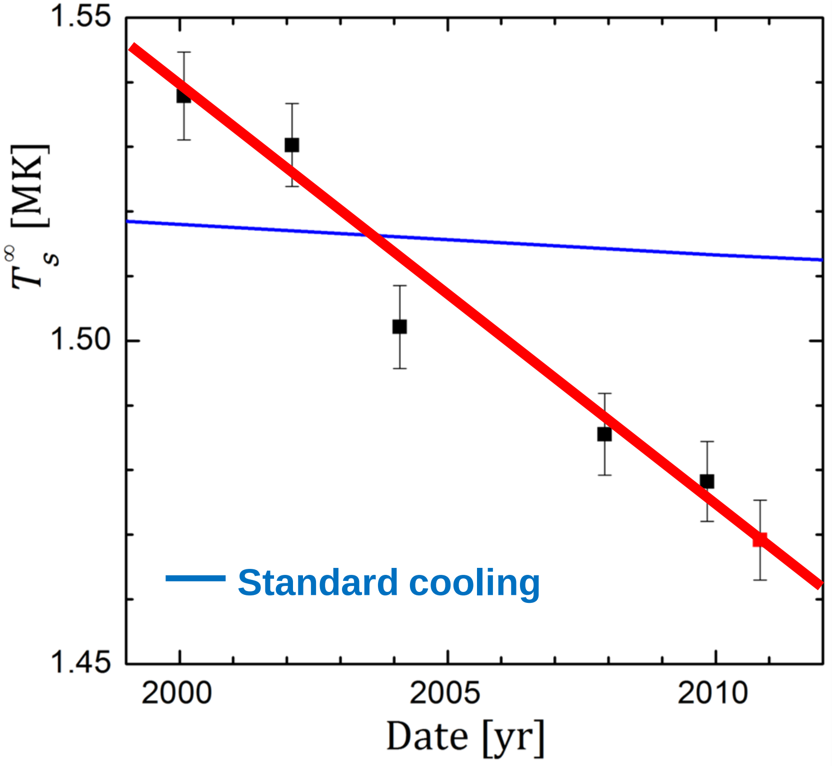
Cas A: $f_l \sim 1$ = good standard candle?

INSs hotter for their age:
$f_l \sim 0.01$ = strong proton SF inside?

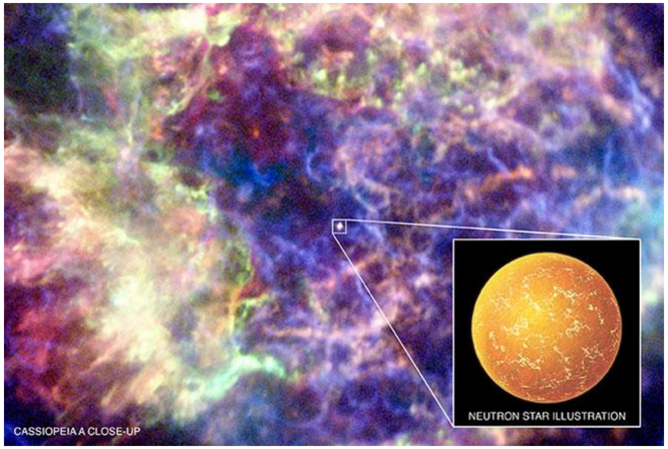


Cas A NS (2010)

Heinke & Ho, ApJL (2010): **Surface temperature decline by 4% over 10 years**



M, R, d, N_H are fixed



"Standard cooling" cannot explain these observations

Observed cooling curve slope

$$s = -\frac{d\ln T_s}{d\ln t} \approx 1.35 \pm 0.15\ (2\sigma)$$

Standard cooling curve slope

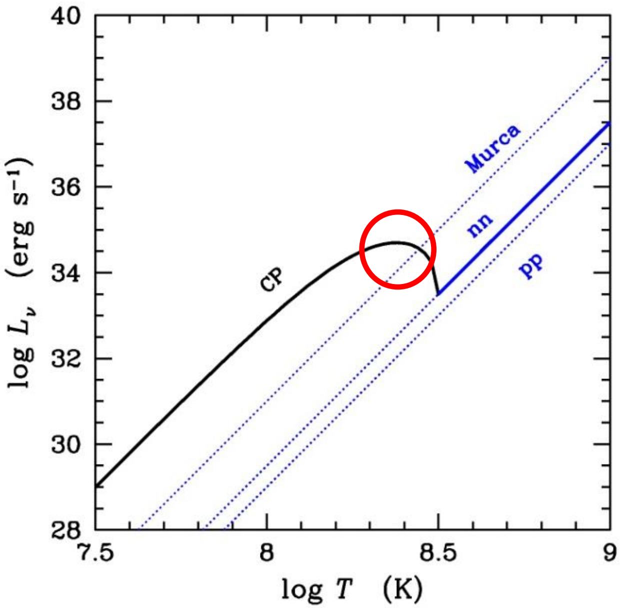
$$s = -\frac{d\ln T_s}{d\ln t} \approx 0.1$$

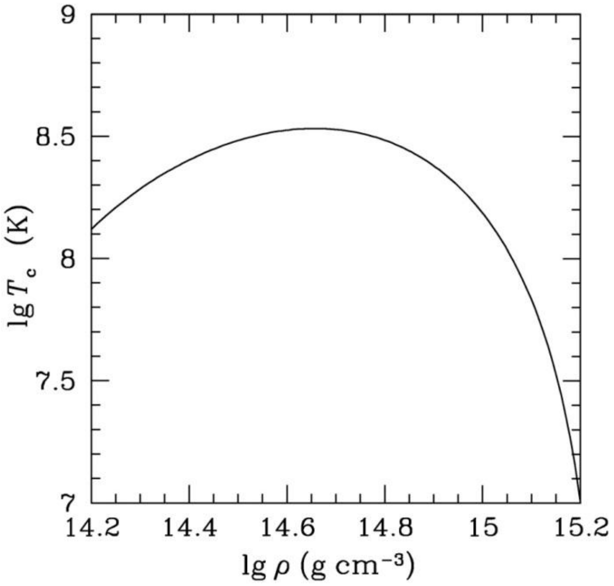
Cas A neutron star:

1. ***Is warm as for standard cooling***
2. ***Cools much faster than for standard cooling***

Slow cooling accelerated by neutron superfluidity

Cooling is accelerated by neutron superfluidity (Page et al. 2011, Shternin et al. 2011):

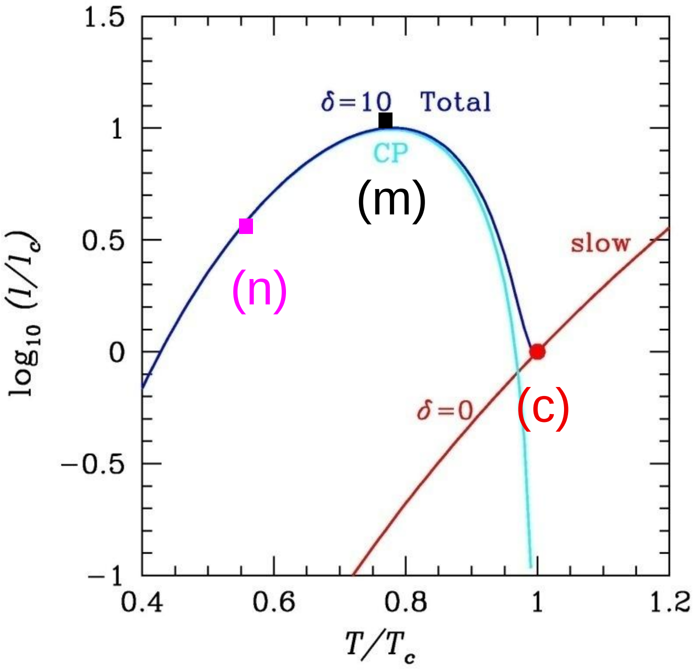




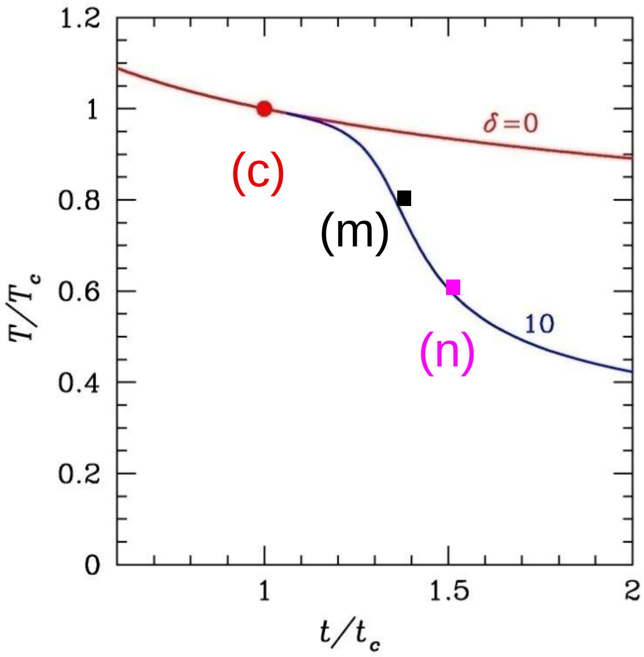
At $0.6T_C<T<T_C$ the problem is selfsimilar again

Self-similarity after superfluidity onset regulated by delta

Neutrino luminosity function



Cooling curve



Three points:

(c) Superfluidity onset (t=t$_C$, T=T$_C^{max}$ =T$_C$)

(m) Maximum of neutrino luminosity function (T=0.765 T$_C$)

(n) Observation (=now)

T= convenient independent variable

$\delta = l_{CP}(T_m)/l_0(T_c)$ ***= convenient parameter of power of CP neutrinos***

with $l_{CP}(T_m)/l_0(T_m) = 5.6\delta$ and $l_{CP}(0.6\ T_c)/l_0(0.6\ T_c) = 18.56\delta$

Self-similarity after superfluidity onset

Analytic neutrino cooling function after superfluidity onset: Gusakov et al. (2004)

$$l(T) = l_c \left(\frac{T}{T_c}\right)^7 \left(1+116\,\delta\left(1-\frac{T}{T_c}\right)^2\right)$$

Convenient universal fit at 0.6 T_c<T<T_c

Slow cooling

Cooper pairing

$$T_C = T_C^{\max}$$

Then cooling equation is integrated and analytic self-similar solution emerges:

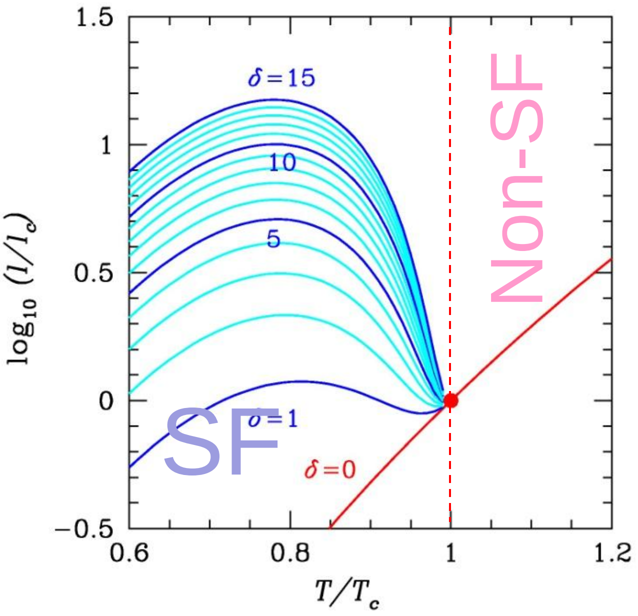
$$\frac{t}{t_c} = 1 + 6 I_7\left(\frac{T}{T_c}\right) \quad \text{at} \;\; 0.6T_c \le T \le T_c$$

$$I_m(\tau) = \int_\tau^1 \frac{dx}{x^m\,[1+116\,\delta\,(1-x)^2]}$$

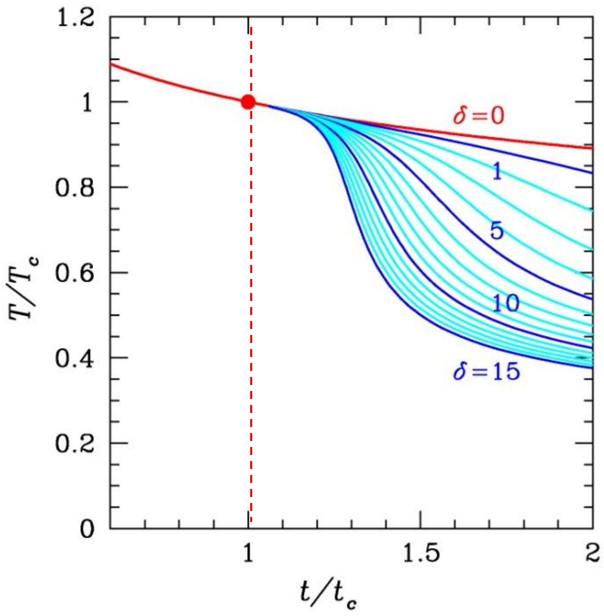
Analytic function

Self-similar solutions after SF onset

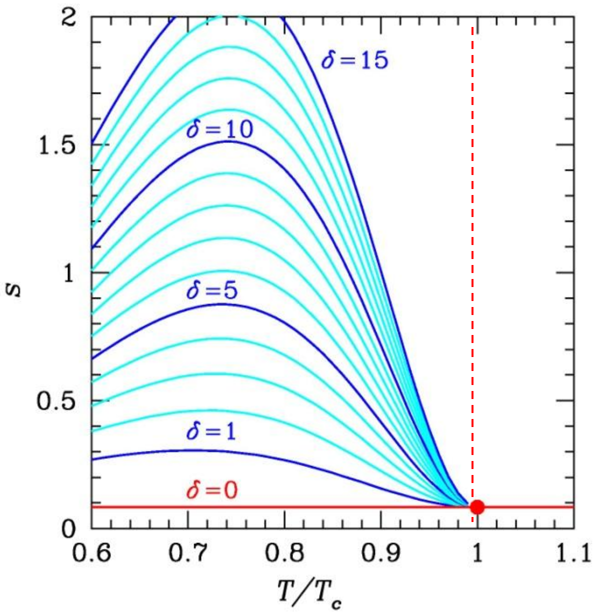
Neutrino luminosity function



Cooling curves



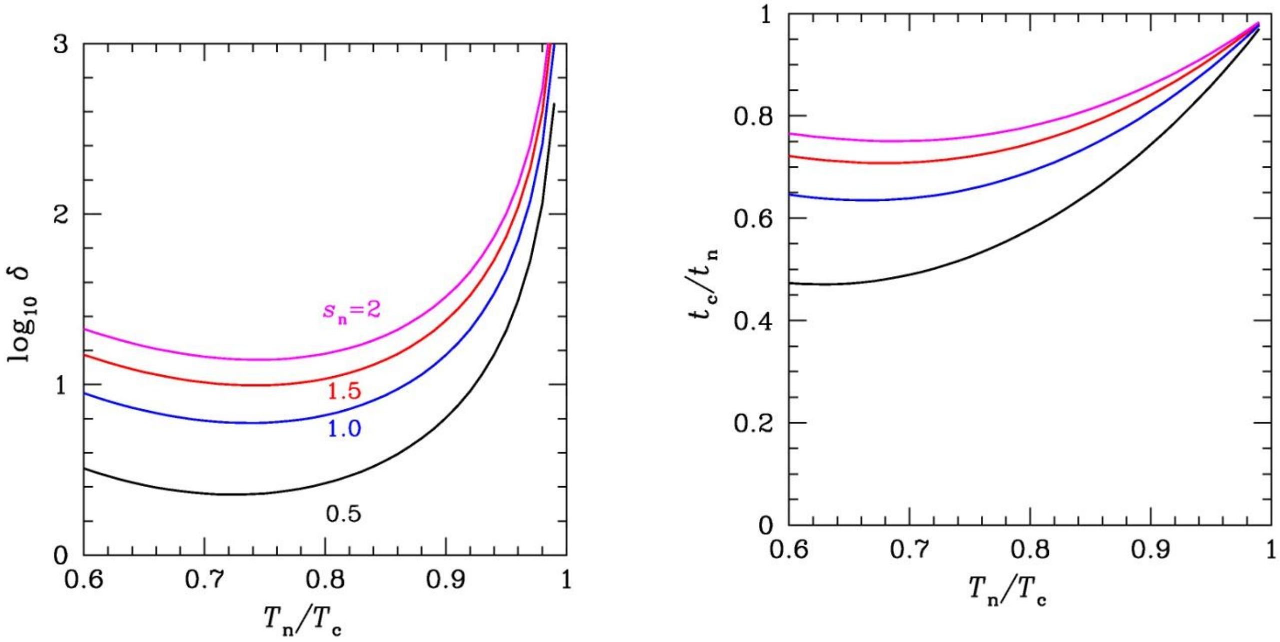
Slopes of cooling curve



$$s = -\frac{d \ln T_s}{d \ln t}$$

= slope of cooling curve,
observable?

Self-similar solutions for given s



If $s=s_n$ is known from observations: (e.g., 2, 1.5, 1, 0.5)

There is a family of solutions parameterized by T_n/T_c and s
For each solution we know all dimensionless quantities,
e.g., δ, t_c/t_n , …

Self-similar solutions after SF onset

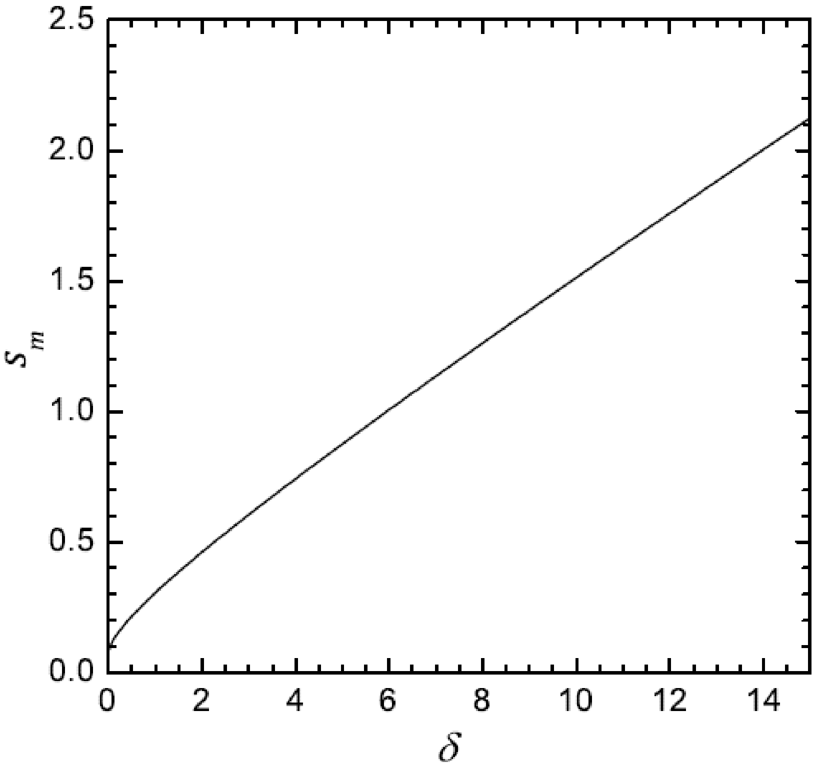
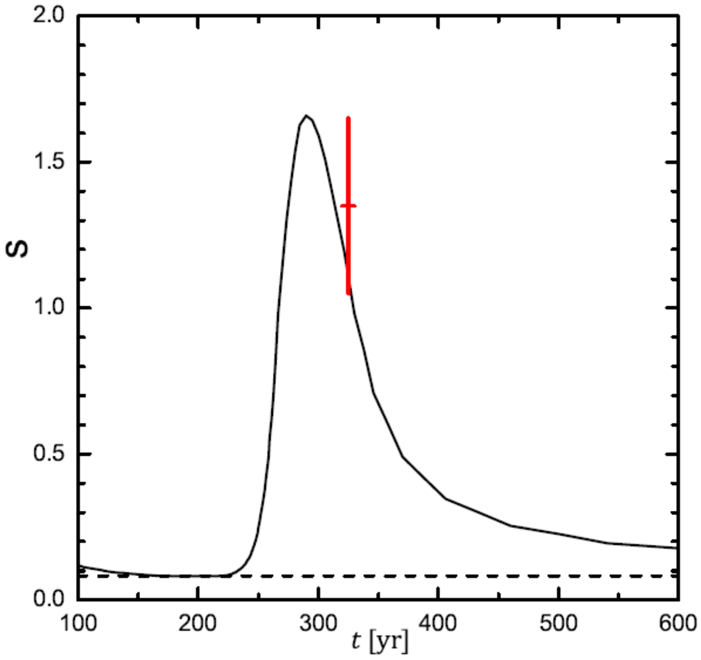


Figure 4. The maximum value s_m of the surface temperature decline versus δ.



Theoretical model for Cas A NS
Shternin et al. (2011)
Now: s = 1.35 = very big number
=> unique phenomenon!

Self-similar solutions for given s

Model-independent determination of neutrino emission rate

Step 1. Assume M, R, composition of heat blanket, find T_{sn} , s_n, t_n

Step2. Use s_n , assume T_n/T_c , and perform dimensionless analysis

Step3. Use T_{sn} , properties of heat blanket, and find T_n and T_c

Step 4. Use t_n and t_c/t_n to find t_c

Step 5. Use t_c and T_c to find neutrino emission level at slow cooling (before superfluidity onset) [Yakovlev et al. 2011]

Step 6. Use δ and find neutrino emission level due to Cooper pairing

Now neutrino cooling function is reconstructed (at any given s_n and T_n/T_c) in terms of parameters which are independent of specific EOS and $T_c(\rho)$ model

Use physical constraints to reject unphysical solutions!

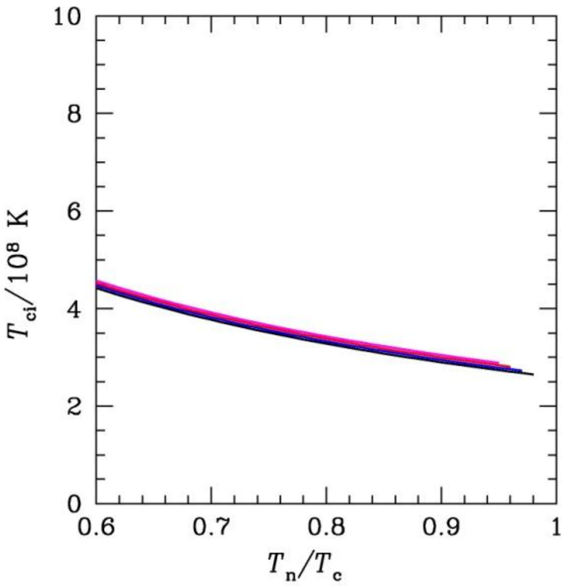
Example: Self-similar solutions for Cas A NS

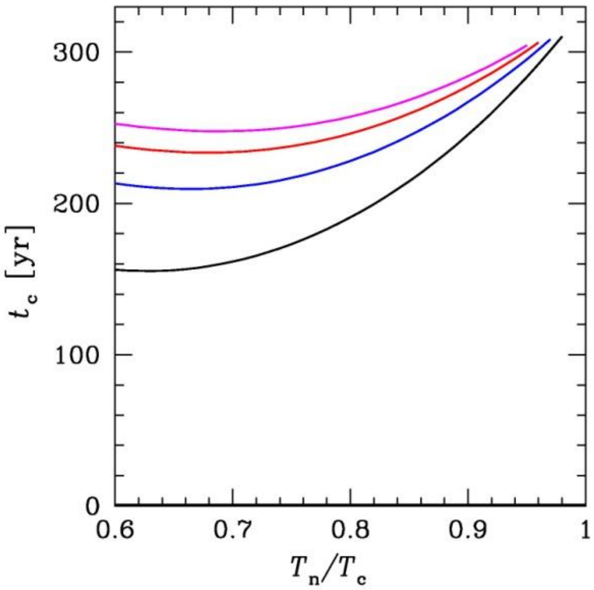
APR I: $M = 1.65\ M_{SUN};\quad R = 11.8\ \text{km}$

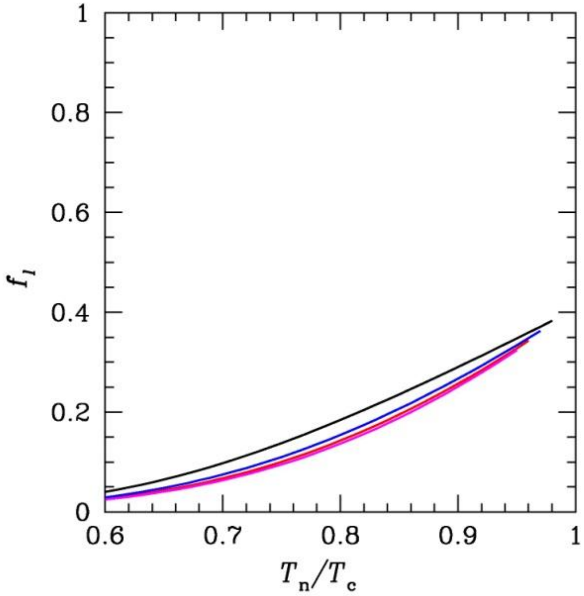
Fe heat blanketing envelope

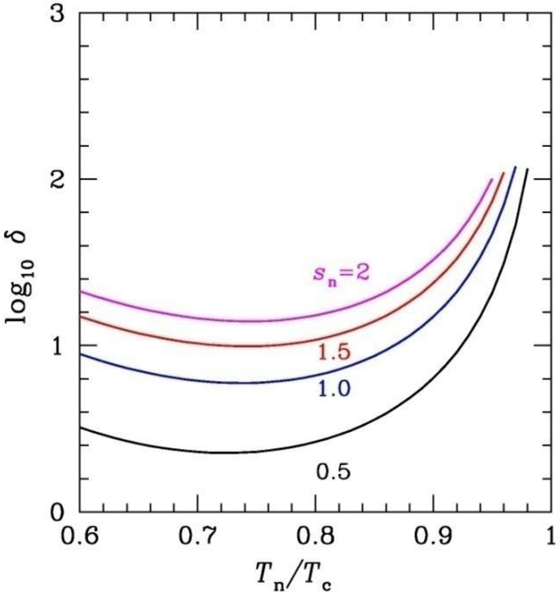
$$T_s = 2\times10^6\ \text{K} \;\Rightarrow\; T_b = 3.6\times10^8\ \text{K}$$

Four cases s=s$_n$ =2, 1.5, 1, 0.5









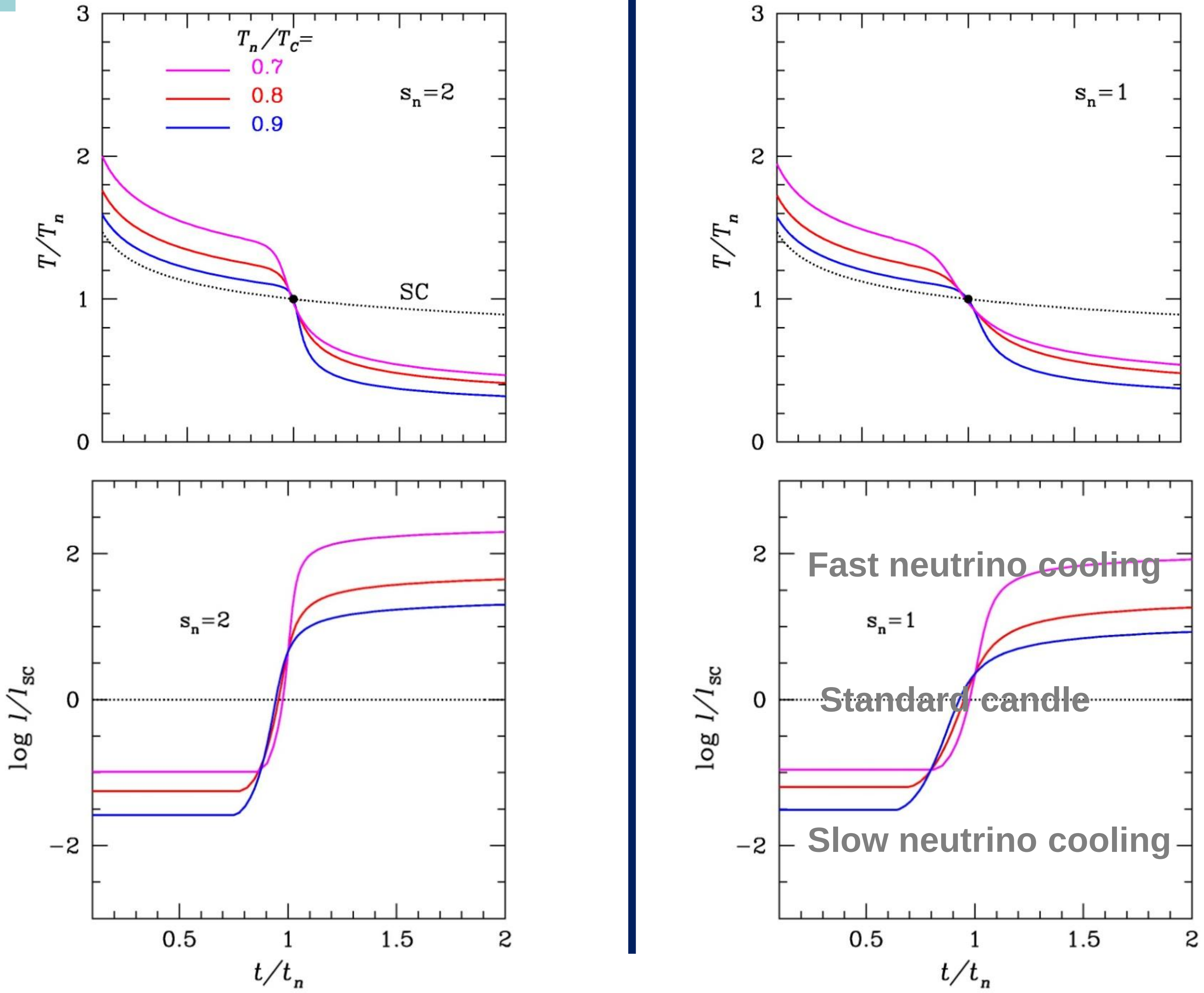
Example: Self-similar solutions for Cas A NS

S_n=2

S_n=1

Internal temperature drop

Neutrino emission level evolution



Fast neutrino cooling

Standard candle

Slow neutrino cooling

Is there rapid cooling of Cas A NS real (2013)?

K. G. Elshamouty,
C. Heinke et al.

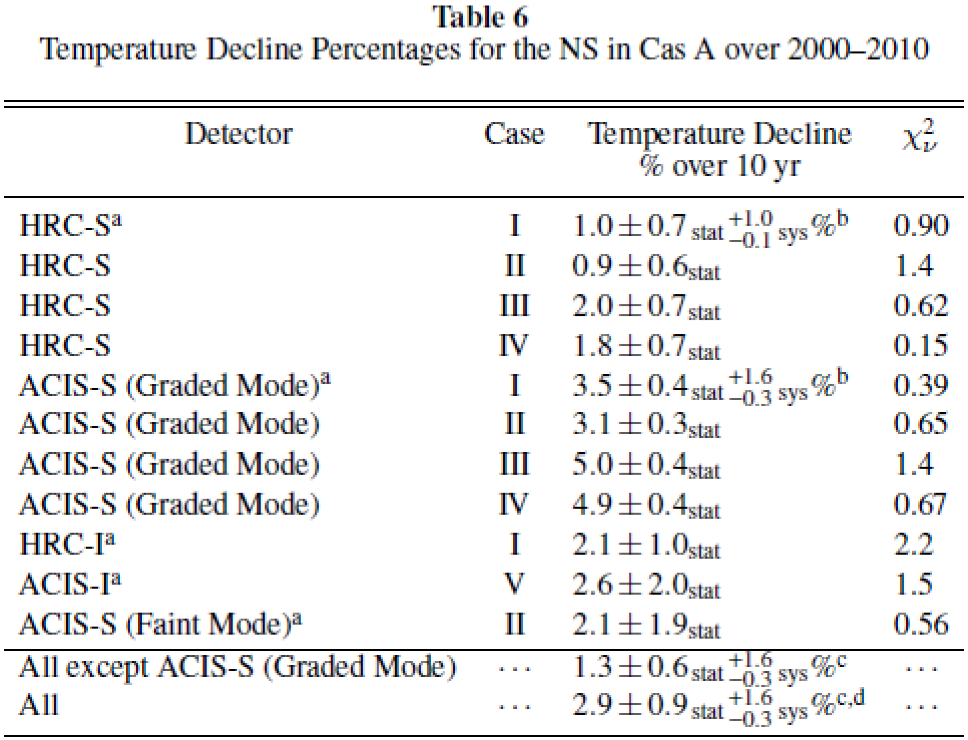
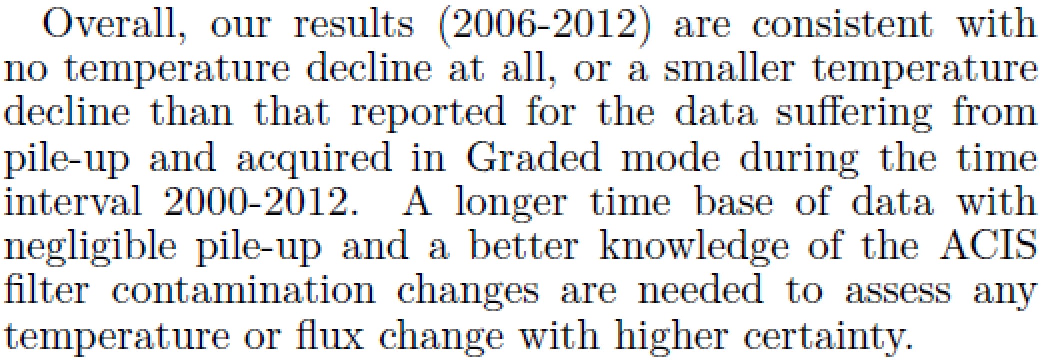


Table 6
Temperature Decline Percentages for the NS in Cas A over 2000–2010

Detector	Case	Temperature Decline % over 10 yr	χ^2_ν
HRC-S[a]	I	$1.0 \pm 0.7_{\rm stat}\,{}^{+1.0}_{-0.1}{}_{\rm sys}$%[b]	0.90
HRC-S	II	$0.9 \pm 0.6_{\rm stat}$	1.4
HRC-S	III	$2.0 \pm 0.7_{\rm stat}$	0.62
HRC-S	IV	$1.8 \pm 0.7_{\rm stat}$	0.15
ACIS-S (Graded Mode)[a]	I	$3.5 \pm 0.4_{\rm stat}\,{}^{+1.6}_{-0.3}{}_{\rm sys}$%[b]	0.39
ACIS-S (Graded Mode)	II	$3.1 \pm 0.3_{\rm stat}$	0.65
ACIS-S (Graded Mode)	III	$5.0 \pm 0.4_{\rm stat}$	1.4
ACIS-S (Graded Mode)	IV	$4.9 \pm 0.4_{\rm stat}$	0.67
HRC-I[a]	I	$2.1 \pm 1.0_{\rm stat}$	2.2
ACIS-I[a]	V	$2.6 \pm 2.0_{\rm stat}$	1.5
ACIS-S (Faint Mode)[a]	II	$2.1 \pm 1.9_{\rm stat}$	0.56
All except ACIS-S (Graded Mode)	...	$1.3 \pm 0.6_{\rm stat}\,{}^{+1.6}_{-0.3}{}_{\rm sys}$%[c]	...
All	...	$2.9 \pm 0.9_{\rm stat}\,{}^{+1.6}_{-0.3}{}_{\rm sys}$%[c,d]	...

B. Posselt,
G. Pavlov,
V. Suleimanov,
O. Korgaltsev

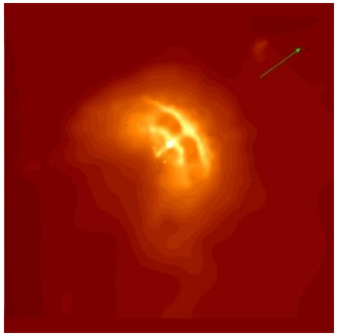


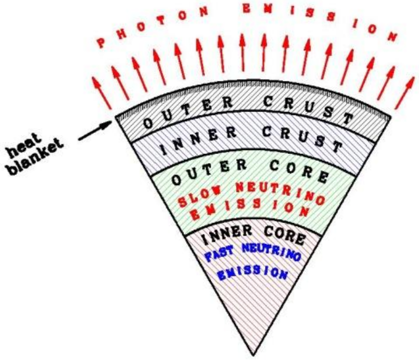
Overall, our results (2006-2012) are consistent with no temperature decline at all, or a smaller temperature decline than that reported for the data suffering from pile-up and acquired in Graded mode during the time interval 2000-2012. A longer time base of data with negligible pile-up and a better knowledge of the ACIS filter contamination changes are needed to assess any temperature or flux change with higher certainty.

Main Cooling Objects

- *Isolated (cooling) neutron stars*
- *Accreting neutron stars in X-ray transients (heated inside)*
- *Sources of superbursts*
- *Magnetars (heated inside)*

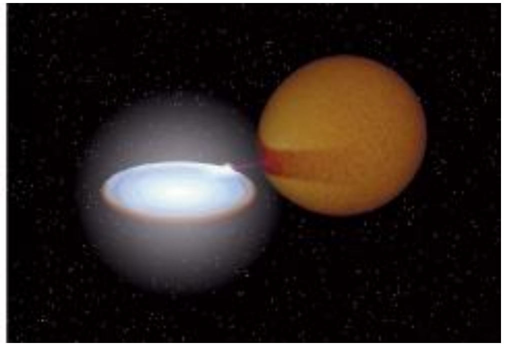
INSs

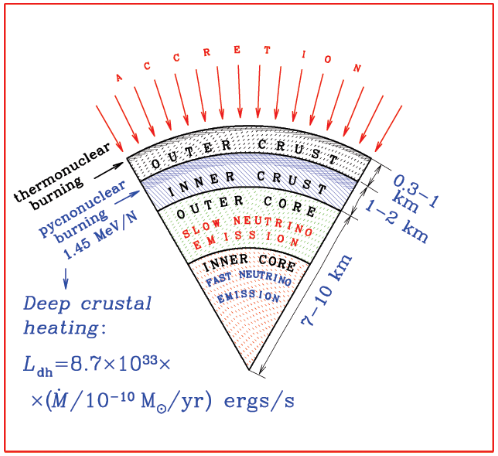




Cooling of initially hot star

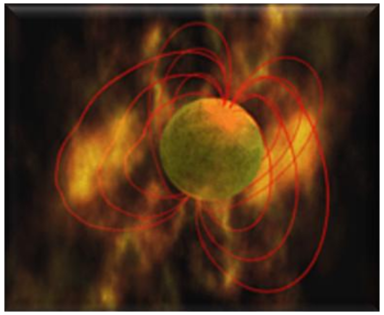
XRTs

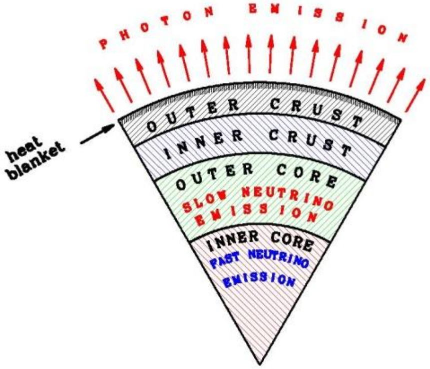




Deep crustal heating:
Theory: Haensel & Zdunik (1990)
Applications to XRTs:
Brown, Bildsten & Rutledge (1998)

Magnetars





Magnetic heating

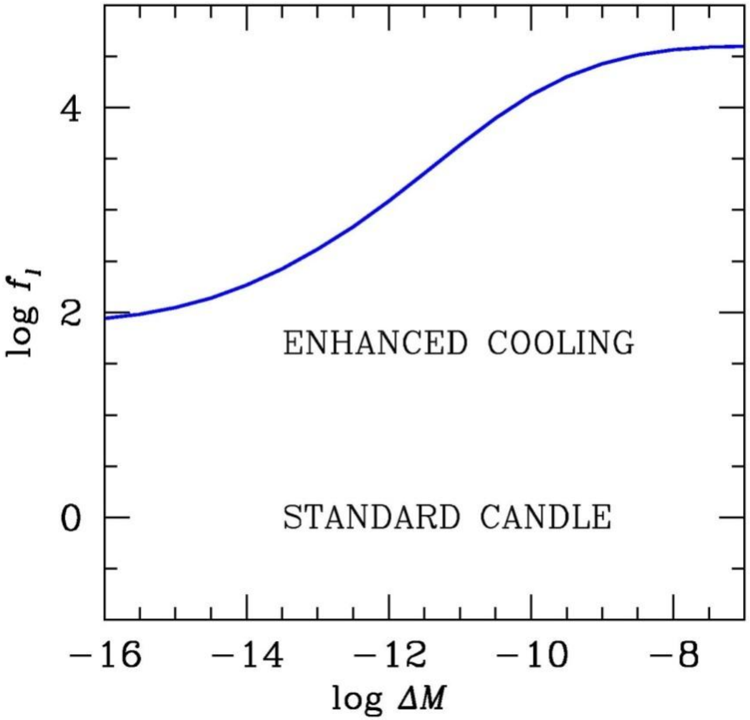
Main Cooling Objects

Objects	Physics which is tested
Middle-aged isolated NSa	Neutrino luminosity function Composition and B-field in heat-blanketing envelopes
Young isolated NSs	Crust
Quasistationary XRTs	Neutrino luminosity function Composition and B-field in heat-blanketing envelopes Deep crustal heating
Quasipersistent XRTs KS 1731—260; MXB 1659—29	Crust Deep crustal heating
Superbursts	Crust
Magnetar outbursts	Crust
Magnetars in quasistationary states	Crustal heating

Important warning about heat blankets

Chemical composition of heat blanketing envelope is basically unknown =>
extracting neutrino cooling rate from observations is basically uncertain =>
internal neutrino cooling is disguised by by unknown thermal insulation

Important issue: to study composition of envelopes (Bildsten et al.)



Vela pulsar, M=1.4 M_{SUN}, APR I EOS

$$T_s^{\infty} \approx 6.8 \times 10^5 \text{ K}$$

CONCLUSIONS

OBSERVATIONS

- *Include sources of different types*

- *Observations seem more important that theory*

- *Are still insufficient to solve NS problem*

THEORY

- *Main cooling regulator: neutrino luminosity function*

- *Warmest observed stars INSs are low-massive; their neutrino luminosity <= 0.01 of modified Urca*

- *Coldest observed stars INSs are massive; their neutrino luminosity >= 100 of modified Urca*

Evidence for SF from NS cooling

- *Warmest observed isolated INSs and NSs in quasi-stationary XRTs require very slow cooling => consistent with strong proton superfluidity which suppresses many neutrino processes*

- *Unusual cooling behavior of Cas A NS*

CONCLUSIONS

Warning

- *Do not expect from cooling theory more than it can give*
- *Cooling theory by itself will hardy allow accurate mass and radius measurements – it has to be calibrated by independent measurements and/or reliable theory of nuclear matter*
- *Info on internal neutrino emission is disguised by unknown composition of heat blanket*

Future

- *New observations => good practical theories of dense matter*
- *Proper inclusion of B-field, rotation, superfluidity*
- *Independent measurements of masses, radii, etc.*
- *Good luck*