Jarzynski Equality and the Landauer's bound: an experimental approach

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About Stefano

Workshop on Chaos and Complexity
1988, Villa Gualino

R. Livi; S. Ciliberto; S. Ruffo

A cellular automaton model of a fluid experiment,
F. Bagnoli, Francescato, S. Ciliberto, R.Livi, S.Ruffo

Phase transitions in convection experiments,
F. Bagnoli, S. Ciliberto, R. Livi and S. Ruffo,
Outline

- Landauer’s principle
- How to realise it?
- Experimental set-up
- Data analysis
- Comparison with numerical results
- Landauer’s limit and the Jarzynski equality
- Conclusions
Landauer’s Principle and The Maxwell’s Demon

- slow molecules
- fast molecules
The Landauer’s principle (I)

Any logically irreversible transformation of classical information is necessarily accompanied by the dissipation of at least $k_B T \cdot \ln 2$ of heat per lost bit (about $3 \cdot 10^{-21}$ Joules at room temperature)

Typical examples of logically irreversible transformations are Boolean functions such as AND, NAND, OR and NOR.

They map several input states onto the same output state.

The erasure of information, the RESET TO ONE operation, is logically irreversible and leads to an entropy production of $k_B \cdot \ln 2$ per erased bit.
Landauer’s principle II

Landauer’s principle is a central result which exorcises the Maxwell’s demon

It has been criticised and never tested in a real experiment

Questions

• Can the Landauer’s limit be reached in any experiment?
• Does any experimentally feasible procedure allow us to reach the limit?

Following Bennett we use in our experiment the RESET to ONE operation

The Bennett’s erasure procedure

Initial state is 0 or 1 with equal probability $1/2$

$$S_i = k_B \cdot \ln 2$$

Final state is 1 with probability 1

$$S_f = 0$$

Thus $$\Delta S_{\text{min}} = -k_B \cdot \ln 2$$
Procedure

Quasi Static: \(-T\Delta S = Q\)

Energy variation: \(\Delta U = 0\)

First principle: \(\Delta U = -Q + W\)

\[ \downarrow \]

In average: \(<W> = <Q> = -T \Delta S \geq k_B T \ln(2)\)

Numerical result:
*Memory Erasure in Small Systems*,
Camera rapide

\[ U(x) = \frac{k}{2} x^2 \]
Brownian particle trapped by two laser beams

The Kramers time

$$\tau_K = \tau_o \exp\left[\frac{\delta U}{k_B T}\right]$$

with $\tau_o = 1 \text{ s}$

Potential measured using detailed balance

$$\Delta U_{i,j} = U(x_i) - U(x_j)$$

$$U_o(x) = a \ x^4 - b \ x^2 + d \ x$$

$$\delta U \sim 2 \ K_B T$$

$$\frac{\omega_{i \rightarrow j}}{\omega_{j \rightarrow i}} = e^{-\frac{\Delta U_{ij}}{k_B T}}$$
The cell for the bead
The Erasure Procedure

Initial state

$U(x) \ (k_B T)$

- $0$
- $1$
- $a$

$x \ (\mu m)$

$0 \ 0.5$
The Erasure Procedure

Initial state

U(x) (k_BT)

0               1

0  1

reduction of the barrier
The Erasure Procedure

Initial state

Progressive tilt of the potential

U(x) (k_BT)

x (µm)
The Erasure Procedure

Initial state

Increasing of the barrier

Final state
Potential external control as a function of time

The laser intensity controls the barrier height

The potential tilt is produced by a linearly increasing external force $f$, applied on the time $\tau$. 

$$\tau_{\text{cycle}} = \tau + 2 \text{ s}$$

The force $f$ is created by displacing the cell with respect to the laser, thus

$$f = -\nu \nu \quad \text{with} \quad \nu = 6\pi R \mu$$

Two control parameters: 

- $\tau$ the time of application of $f$
- $F_{\text{max}}$ the maximum applied force
Bead trajectories

0 to 1 transition

position (μm)

-0.5 0 0.5

0 10 20
time (s)

ENDELYON

CNRS
Bead trajectories

0 to 1 transition

1 to 1 transition
The work on the erasure cycle

\[ \nu \dot{x} = -\frac{\partial U_o(x, t)}{\partial x} + f(t) + \eta \]

multiplying by \( \dot{x} \) and integrating for a time \( \tau \) we get:

\[ \Delta U_\tau = W_\tau - Q_\tau \quad \text{Stochastic thermodynamics} \]

\[ \Delta U_\tau = -\int_0^\tau \frac{\partial U_o}{\partial x} \dot{x} \, dt \quad W_\tau = \int_0^\tau f \, \dot{x} \, dt \]

\[ Q_\tau = \int_0^\tau \nu \dot{x}^2 \, dt - \int_0^\tau \eta \dot{x} \, dt \]

The work on the erasure cycle

The two erasure cycles have been considered

\[ \Delta U_\tau = - \int_0^{\tau_{\text{cycle}}} \frac{\partial U_0}{\partial x} \dot{x} \, dt \]

and

\[ W_F = - \int_0^{\tau_{\text{cycle}}} \nu \, v(t) \dot{x} \, dt = \int_0^{\tau_{\text{cycle}}} F_{\text{max}} \frac{t}{\tau} \dot{x} \, dt \]

Landauer’s limit
Results of the erasure procedure

Success rate \( r = \frac{\text{number of successful cycles}}{\text{Total number of cycle}} \)

Qualitative observations:
• At constant \( \tau \): \( W \) and \( r \) increase with \( F_{\text{max}} \)
• At constant \( F_{\text{max}} \): \( W \) decreases and \( r \) increases for increasing \( \tau \)

Landauer’s limit

\( + \ r > 0.9 \)
\( x \ r > 0.85 \)
\( o \ r > 0.75 \)
Landauer’s limit as a function of $r$

$$< Q >^r_{\text{Landauer}} = kT[\ln 2 + r \ln r + (1 - r) \ln(1 - r)]$$

At $r=0.5$ 

$$< Q >^r_{\text{Landauer}} = 0$$

Indeed the Erasure Procedure left the initial state unchanged.
As $\Delta U = 0$ then $\Delta F = -T \Delta S$ and $< Q > = < W > \simeq kT \ln 2 + B/\tau$

Landauer’s limit

$\tau \rightarrow \infty$


$< W > \simeq \Delta F + B/\tau$

$< Q > = < Q >_{\text{Landauer}} + B/\tau$

Asymptotic behaviour

$r > 0.9$
$x \ r > 0.85$
$o \ r > 0.75$
The success rate $r$

Why in the experiment $r < 1$?

Is this result produced by 3D effects of the trap?

Is the finite height of the initial barrier responsible of $r < 1$?
The Erasure Procedure

Initial state

Final state
The success rate $r$

Why in the experiment $r < 1$?

Is this result produced by 3D effects of the trap?

Is the finite height of the initial barrier responsible of $r < 1$?

Numerical test

$$\nu \dot{x} = -\frac{\partial U_o(x, t)}{\partial x} + \eta$$

We use all the experimental parameters and procedure with two different initial barriers $8k_B T$ and $15k_B T$
The success rate $r$

Why in the experiment $r < 1$?

Is this result produced by 3D effects of the trap?

Is the finite height of the initial barrier responsible of $r < 1$?

\[ \nu \dot{x} = -\frac{\partial U_o(x, t)}{\partial x} + \eta \]

Numerical test

initial barrier 15$k_B T$

initial barrier 8$k_B T
Conclusions (partials)

- Our experimental results indicate that the thermodynamic limit to information erasure, the Landauer bound, can be approached in the quasistatic regime, but not exceeded.

- The asymptotic limit is reached in $1/\tau$

- The fact that $r<1$ is due to the finite height of the initial barrier

- Thermal fluctuations play an important role to reach the limit

Question: Does Jarzynski equality compute the right $\Delta F$?
Landauer’s limit and the Jarzynski equality

\[
< \exp(-W_s) >= \exp(-\Delta F)
\]

with

\[
W_s = -\int_0^{\tau_{cycle}} \dot{\lambda} \frac{\partial H(x, \lambda)}{\partial \lambda} dt
\]

In our case this equality transforms

\[
W_s = \int_0^{\tau} \dot{x} \, dt = [f \ x]_0^{\tau} - \int_0^{\tau} f \ \dot{x} \, dt = -W_f
\]

Since the height of the barrier is always finite there is no change in the equilibrium of the system between the beginning and the end of the procedure.

\[
< \exp(-W_s) >= \frac{\rho_{eq}(\tau)}{\rho(\tau)} \exp(-\Delta F)
\]

Generalized Jarzynski

Landauer’s limit and the Jarzynski equality

We consider the erasure procedure $0 \rightarrow 0$ from $1 \rightarrow 0$

If the final state is $0$ then $\rho = r \approx 1$, $\rho_{eq} = 1/2$, $\Delta F = 0$

and the Generalized Jarzynski is: $\langle \exp(-W_s) \rangle_{\rightarrow 0} = \frac{1/2}{r}$

from Jensen inequality $\langle W_s \rangle_{\rightarrow 0} \geq (\ln 2 + \ln r)$

$$\frac{1}{2} < \exp(-W_{1,0}) > + \frac{1}{2} < \exp(-W_{0,0}) > = \frac{1}{2}$$

Work done if the particle makes the jump from $1$ to $0$

Work done when the particle starts in the final state
Landauer’s limit and the Jarzynski equality

\[-\ln \left( \frac{< \exp(-W_{0,0}) > + < \exp(-W_{1,0}) >}{2} \right) = \Delta F_{eff}\]
Landauer’s limit and the Jarzynski equality
Landauer’s limit and the Jarzynski equality

We consider the erasure procedure $0 \rightarrow 0$

If the final state is 0 then $\rho = r \approx 1$, $\rho_{eq} = 1/2$, $\Delta F = 0$

$\langle \exp(-W_s) \rangle \rightarrow_0 = \frac{1/2}{r}$

and

$\langle W_s \rangle \rightarrow_0 \geq (\ln 2 + \ln r)$

If the final state is 1 then $\rho = (1 - r) \approx 0$, $\rho_{eq} = 1/2$, $\Delta F = 0$

$\langle \exp(-W_s) \rangle \rightarrow_1 = \frac{1/2}{1 - r}$

and

$\langle W_s \rangle \rightarrow_1 \geq \ln 2 + \ln(1 - r)$

Total work $\langle W_s \rangle = r \langle W_s \rangle \rightarrow_0 + (1 - r) \langle W_s \rangle \rightarrow_1$

using the inequalities $\langle W_s \rangle \geq \ln 2 + r \ln r + (1 - r) \ln(1 - r)$

The generalized Landauer’s bound
Conclusions

• Our experimental results indicate that the thermodynamic limit to information erasure, the Landauer bound, can be approached in the quasistatic regime, but not exceeded.

• The asymptotic limit is reached in $1/\tau$

• The fact that $r<1$ is due to the finite height of the initial barrier

• Thermal fluctuations play an important role to reach the limit

• Jarzinsky equality computes the Landauer limit independently of the rapidity of the procedure

Question: Does any procedure allows us to reach the Landauer’s limit?
Answer: NO. The barrier reduction and tilt must be two separate process

See recent paper on optimisation:
Other procedure (I)

Relative change of the laser intensity

The ramping time of the laser intensity has been changed from 1s to 50s
Other procedure (II)

The work is mainly due to the jump of the particle
The Landauer limit can never be reached
Memory Erasure in Small Systems,

Potential:

$$V(x, t) = -\frac{1}{2}g(t)x^2 + \frac{1}{4}x^4$$

External force:

$$A f(t)$$
Non-dimensional numbers and the success rate

\[ \tau = \frac{\tau}{\tau_k} \] Possibility of jumping the barrier without force

\[ \bar{F} = \frac{\delta x \ F_{\text{max}}}{\Delta U} \] The maximum external work overcomes the barrier

\[ \tau_K = \tau_o \exp\left[\frac{\Delta U}{k_B T}\right] \] is the Kramers time with \( \tau_o \simeq 1 \text{s} \)

\[ \delta x \] is the distance of the potential minima

One can think that the success rate is:

\[ r = \frac{1}{2} \left[ 1 + \exp\left( -\frac{1}{\bar{\tau} \bar{F}} \right) \right] \]
Non-dimensional numbers and the success rate

Experimentally

\[ r = \frac{1}{2} \left[ 1 + \exp\left( -\frac{a}{\bar{T}F^2} \right) \right] \]
Conclusions

• Our experimental results indicate that the thermodynamic limit to information erasure, the Landauer bound, can be approached in the quasistatic regime, but not exceeded.

• The asymptotic limit is reached in $1/\tau$ for $\tau > 3\tau_k$

• The fact that $r<1$ is due to the finite height of the initial barrier

• Thermal fluctuations play an important role to reach the limit

Question: Does any procedure allows us to reach the Landauer’s limit?

Answer: NO. The barrier reduction and tilt must be two separate process