Physical ageing in non-equilibrium statistical systems without detailed balance

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MH, J.D. NOH and M. PLEIMLING, Phys. Rev. **E85**, 030102(R) (2012)
N. ALLEGRA, J.-Y. FORTIN and MH, J. Stat. Mech. P02018 (2014)

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Overview :

1. Ageing phenomena
2. Interface growth (KPZ universality class)
3. Interface growth on semi-infinite substrates
4. Interface growth and Arcetri model
5. Conclusions

1. Ageing phenomena

known & practically used since prehistoric times (metals, glasses)
systematically studied in physics since the 1970s STRUIK '78

discovery : ageing effects **reproducible** & **universal** !

occur in widely different systems
(structural glasses, spin glasses, polymers, simple magnets, ...)

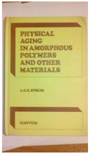
Three defining properties of **ageing** :

1. slow relaxation (non-exponential !)
2. **no** time-translation-invariance (TTI)
3. dynamical scaling

without fine-tuning of parameters

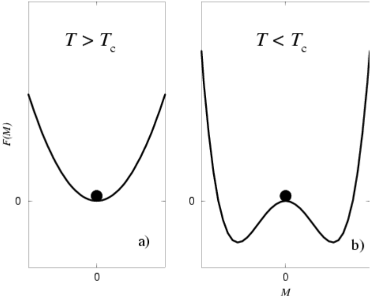
Most existing studies on 'magnets' : relaxation towards equilibrium

Question : what can be learned about intrisically **irreversible** systems by studying their ageing behaviour ?



consider a simple magnet (ferromagnet, i.e. Ising model)

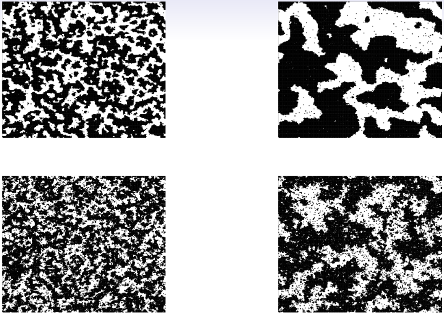
1. prepare system initially at high temperature $T \gg T_c > 0$
2. quench to temperature $T < T_c$ (or $T = T_c$)
   $\rightarrow$ non-equilibrium state
3. fix T and observe dynamics



competition :
at least 2 equivalent ground states
local fields lead to rapid local ordering
no global order, relaxation time ∞

formation of ordered domains, of linear size $L = L(t) \sim t^{1/z}$
dynamical exponent z

$t = t_1$ $\quad$ $t = t_2 > t_1$

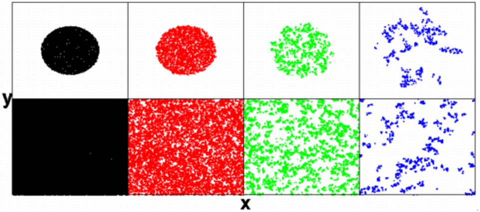


magnet $T < T_c$

$\longrightarrow$ ordered cluster

magnet $T = T_c$

$\longrightarrow$ correlated cluster



critical contact process

$\Longrightarrow$ **cluster dilution**

voter model, contact process,. . .

$$\boxed{L(t) \sim t^{1/z}}$$

common feature : growing length scale

z : dynamical exponent

Two-time observables : analogy with 'magnets'

time-dependent order-parameter $\phi(t, \mathbf{r})$

two-time **correlator** $\quad C(t,s) := \langle \phi(t,\mathbf{r})\phi(s,\mathbf{r})\rangle - \langle \phi(t,\mathbf{r})\rangle \langle \phi(s,\mathbf{r})\rangle$

two-time **response** $\quad R(t,s) := \left. \frac{\delta \langle \phi(t,\mathbf{r})\rangle}{\delta h(s,\mathbf{r})}\right|_{h=0} = \left\langle \phi(t,\mathbf{r})\widetilde{\phi}(s,\mathbf{r})\right\rangle$

t : observation time, s : waiting time

a) system **at** equilibrium : **fluctuation-dissipation theorem**

$$R(t-s) = \frac{1}{T}\frac{\partial C(t-s)}{\partial s} \quad , \qquad T \text{ : temperature}$$

b) **far from equilibrium** : C and R **independent** !

The **fluctuation-dissipation ratio** (FDR) — Cugliandolo, Kurchan, Parisi '94

$$X(t,s) := \frac{TR(t,s)}{\partial C(t,s)/\partial s}$$

measures the distance with respect to equilibrium : $\boxed{X_{\rm eq} = X(t-s) = 1}$

Scaling regime : $\boxed{t, s \gg \tau_{\rm micro} \text{ and } t - s \gg \tau_{\rm micro}}$

$$C(t,s) = s^{-b} f_C\left(\frac{t}{s}\right) \quad , \quad R(t,s) = s^{-1-a} f_R\left(\frac{t}{s}\right)$$

asymptotics : $\boxed{f_C(y) \sim y^{-\lambda_C/z} \;,\;\; f_R(y) \sim y^{-\lambda_R/z}}$ for $y \gg 1$

λ_C : autocorrelation exponent, λ_R : autoresponse exponent,
z : dynamical exponent, a, b : ageing exponents

Question : in critical **magnets**, typically find $a = b$ and $\lambda_C = \lambda_R$
* ? what can happen when relaxation towards **non**-equilibrium state ?
* ? are λ_C, λ_R independent of stationary exponents ?

Ex. critical contact process, initial particle density > 0 Baumann & Gambassi 07

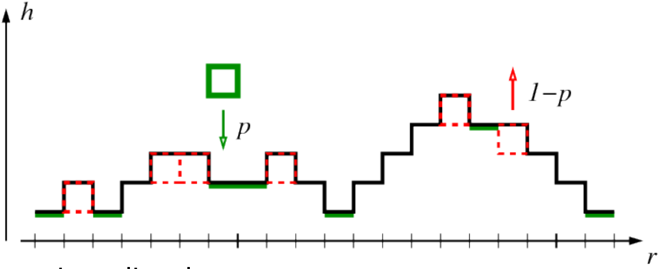
$$\lambda_C = \lambda_R = d + z + \beta/\nu_\perp \quad , \quad b = 2\beta'/\nu_\|$$

$\longrightarrow$ stationary-state critical exponents $\beta, \beta', \nu_\perp, \nu_\| = z\nu_\perp$

2. Interface growth

deposition (evaporation) of particles on a substrate $\to$ height profile $h(t, \mathbf{r})$

generic situation : RSOS (**r**estricted **s**olid-**o**n-**s**olid) model — Kim & Kosterlitz 89



p = deposition prob.

$1 - p$ = evap. prob.

here $p = 0.98$

some universality classes :

(a)	**KPZ**	$\partial_t h = \nu \boldsymbol{\nabla}^2 h + \frac{\mu}{2} (\boldsymbol{\nabla} h)^2 + \eta$	Kardar, Parisi, Zhang 86
(b)	**EW**	$\partial_t h = \nu \boldsymbol{\nabla}^2 h + \eta$	Edwards, Wilkinson 82
(c)	**MH**	$\partial_t h = -\nu \boldsymbol{\nabla}^4 h + \eta$	Mullins, Herring 63 ; Wolf, Villain 80

η is a gaussian white noise with $\langle \eta(t, \mathbf{r}) \eta(t', \mathbf{r}') \rangle = 2\nu T \delta(t - t') \delta(\mathbf{r} - \mathbf{r}')$

Family-Viscek scaling on a spatial lattice of extent L^d : $\overline{h}(t) = L^{-d} \sum_j h_j(t)$

FAMILY & VISCEK 85

$$w^2(t;L) = \frac{1}{L^d} \sum_{j=1}^{L^d} \left\langle \left(h_j(t) - \overline{h}(t) \right)^2 \right\rangle = L^{2\zeta} f\left(tL^{-z} \right) \sim \begin{cases} L^{2\zeta} & ; \text{if } tL^{-z} \gg 1 \\ t^{2\beta} & ; \text{if } tL^{-z} \ll 1 \end{cases}$$

β : growth exponent, ζ : roughness exponent, $\boxed{\zeta = \beta z}$

two-time correlator :

limit $L \to \infty$

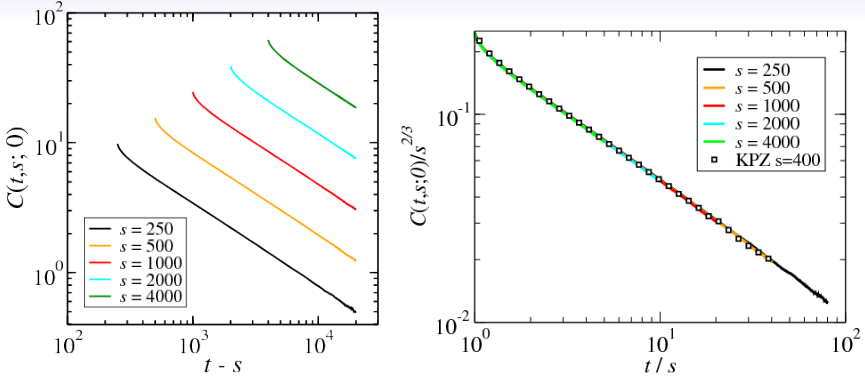
$$C(t,s;\mathbf{r}) = \left\langle \left(h(t,\mathbf{r}) - \langle \overline{h}(t) \rangle \right) \left(h(s,\mathbf{0}) - \langle \overline{h}(s) \rangle \right) \right\rangle = s^{-b} F_C\left(\frac{t}{s}, \frac{\mathbf{r}}{s^{1/z}} \right)$$

with ageing exponent : $\boxed{b = -2\beta}$

KALLABIS & KRUG 96

expect for $y = t/s \gg 1$: $F_C(y,\mathbf{0}) \sim y^{-\lambda_C/z}$ autocorrelation exponent

$1D$ relaxation dynamics, starting from an initially flat interface



observe all **3** properties of **ageing** : $\left\{ \begin{array}{l} \text{slow dynamics} \\ \text{no TTI} \\ \text{dynamical scaling} \end{array} \right.$

confirm **simple ageing** for the $1D$ KPZ universality class

pars pro toto

Kallabis & Krug 96 ; Krech 97 ; Bustingorry *et al.* 07-10 ; Chou & Pleimling 10 ; D'Aquila & Täuber 11/12 ; h.n.p. 12

extend Family-Viscek scaling to two-time responses :

analogue : TRM integrated response in magnetic systems

two-time integrated response :

* sample **A** with deposition rates $p_i = p \pm \epsilon_i$, up to time s,
* sample **B** with $p_i = p$ up to time s ;

then switch to common dynamics $p_i = p$ for all times $t > s$

$$\chi(t,s;\mathbf{r}) = \int_0^s \mathrm{d}u\, R(t,u;\mathbf{r}) = \frac{1}{L}\sum_{j=1}^{L}\left\langle \frac{h_{j+r}^{(\mathbf{A})}(t;s) - h_{j+r}^{(\mathbf{B})}(t)}{\epsilon_j}\right\rangle = s^{-a} F_\chi\left(\frac{t}{s}, \frac{|\mathbf{r}|^z}{s}\right)$$

with a : ageing exponent

expect for $y = t/s \gg 1$: $F_R(y,\mathbf{0}) \sim y^{-\lambda_R/z}$ autoresponse exponent

? Values of these exponents **?**

Effective action of the KPZ equation :

$$\mathcal{J}[\phi,\widetilde{\phi}] = \int \mathrm{d}t\mathrm{d}\mathbf{r}\ \left[\widetilde{\phi}\left(\partial_t\phi - \nu\boldsymbol{\nabla}^2\phi - \frac{\mu}{2}(\boldsymbol{\nabla}\phi)^2\right) - \nu T\widetilde{\phi}^{\,2}\right]$$

$\Longrightarrow$ $\boxed{\text{Very special properties of KPZ in } d=1 \text{ spatial dimension !}}$

Exact critical exponents $\boxed{\beta = 1/3,\ \zeta = 1/2,\ z = 3/2,\ \lambda_C = 1}$ KPZ 86 ; Krech 97

related to precise symmetry properties :

A) tilt-invariance (Galilei-invariance) Forster, Nelson, Stephen 77

kept under renormalisation **!** Medina, Hwa, Kardar, Zhang 89

$\Rightarrow$ exponent relation $\boxed{\zeta + z = 2}$ (holds for any dimension d)

B) time-reversal invariance Lvov, Lebedev, Paton, Procaccia 93; Frey, Täuber, Hwa 96

special property in $1D$, where also $\zeta = \frac{1}{2}$

Special KPZ symmetry in $1D$: let $v = \frac{\partial \phi}{\partial r}$, $\widetilde{\phi} = \frac{\partial}{\partial r}\left(\widetilde{p} + \frac{v}{2T}\right)$

$$\mathcal{J} = \int \mathrm{d}t\mathrm{d}r \left[\widetilde{p}\partial_t v - \frac{\nu}{4T}\left(\partial_r v\right)^2 - \frac{\mu}{2}v^2\partial_r\widetilde{p} + \nu T\left(\partial_r\widetilde{p}\right)^2\right]$$

is invariant under **time-reversal**

$$t \mapsto -t \ \ , \ \ v(t,r) \mapsto -v(-t,r) \ \ , \ \ \widetilde{p} \mapsto +\widetilde{p}(-t,r)$$

$\Rightarrow$ **fluctuation-dissipation relation** for $t \gg s$

$$\boxed{TR(t,s;r) = -\partial_r^2 C(t,s;r)}$$

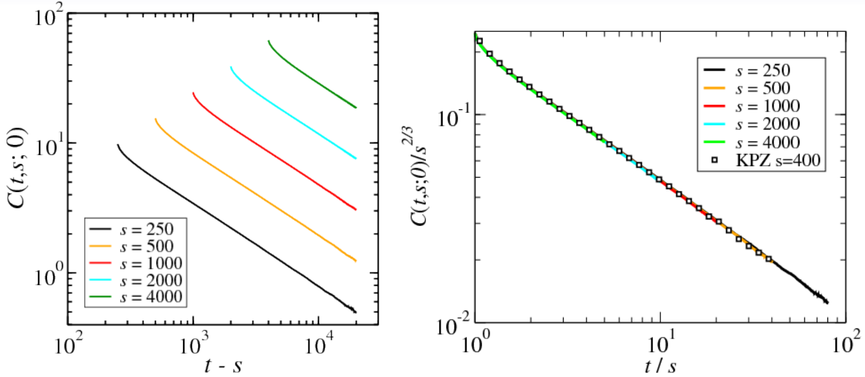
distinct from the equilibrium FDT $TR(t-s) = \partial_s C(t-s)$

Combination with ageing scaling, gives the ageing exponents :

$\boxed{\lambda_R = \lambda_C = 1}$ and $\boxed{1 + a = b + \frac{2}{z}}$

Kallabis, Krug 96 mh, Noh, Pleimling '12

$1D$ relaxation dynamics, starting from an initially flat interface



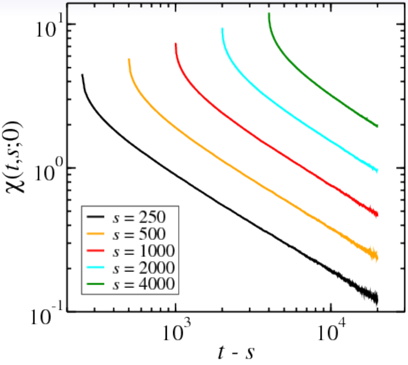
confirm simple ageing in the autocorrelator
confirm expected exponents $b = -2/3$, $\lambda_C/z = 2/3$

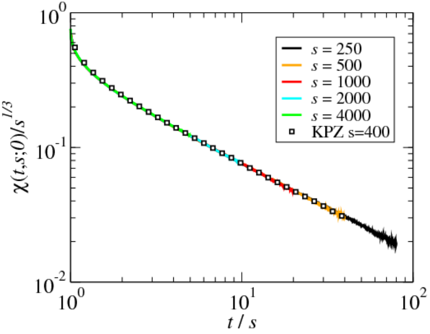
N.B. : this confirmation is out of the stationary state

Kallabis & Krug 96 ; Krech 97 ; Bustingorry *et al.* 07-10 ; Chou & Pleimling 10 ; D'Aquila & Täuber 11/12 ; h.n.p. 12

relaxation of the integrated response, $1D$

MH, NOH, PLEIMLING 12



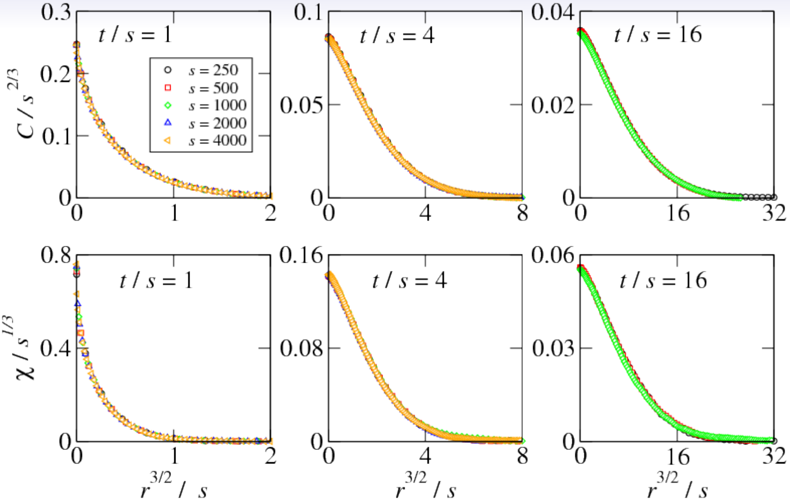


observe all **3** properties of **ageing** : $\left\{ \begin{array}{l} \text{slow dynamics} \\ \text{no TTI} \\ \text{dynamical scaling} \end{array} \right.$

exponents $a = -1/3$, $\lambda_R/z = 2/3$, as expected from FDR

N.B. : numerical tests for 2 models in KPZ class

Simple ageing is also seen in space-time observables



$$\left.\begin{array}{l}\text{correlator } C(t,s;r) = s^{2/3} F_C\left(\frac{t}{s}, \frac{r^{3/2}}{s}\right) \\ \text{integrated response } \chi(t,s;r) = s^{1/3} F_\chi\left(\frac{t}{s}, \frac{r^{3/2}}{s}\right)\end{array}\right\} \text{ confirm } z = 3/2$$

Values of some growth and ageing exponents in $1D$

model	z	a	b	$\lambda_R = \lambda_C$	β	ζ
KPZ	$3/2$	$-1/3$	$-2/3$	1	$1/3$	$1/2$
exp 1			$\approx -2/3^{\dagger}$	$\approx 1^{\dagger}$	0.336(11)	0.50(5)
exp 2	1.5(2)				0.32(4)	0.50(5)
EW	2	$-1/2$	$-1/2$	1	$1/4$	$1/2$
MH	4	$-3/4$	$-3/4$	1	$3/8$	$3/2$

liquid crystals — **Takeuchi, Sano, Sasamoto, Spohn 10/11/12**

cancer cells — **Huergo, Pasquale, Gonzalez, Bolzan, Arvia 12**

† scaling holds only for flat interface

Two-time space-time responses and correlators consistent with simple ageing for $1D$ KPZ

Similar results known for EW and MH universality classes

Roethlein, Baumann, Pleimling 06

3. Interface growth on semi-infinite substrates

properties of growing interfaces near to a boundary ?

$\rightarrow$ crystal dislocations, face boundaries ...

Experiments : $\boxed{\text{Family-Vicsek scaling not always sufficient}}$

FERREIRA *et. al.* 11
RAMASCO *et al.* 00, 06
YIM & JONES 09, ...

$\rightarrow$ **distinct** global and **local** interface fluctuations

$\left\{ \begin{array}{l} \textbf{anomalous scaling}, \text{ growth exponent } \beta \text{ larger than expected} \\ \textbf{grainy interface morphology}, \text{ facetting} \end{array} \right.$

! analyse simple models on a **semi**-infinite substrate **!**

frame co-moving with average interface deep in the bulk

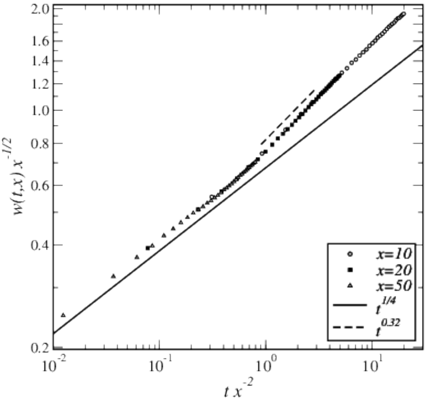
characterise interface by

$$\left\{ \begin{array}{ll} \text{height profile} & \langle h(t,\mathbf{r}) \rangle \\ \text{width profile} & w(t,\mathbf{r}) = \left\langle [h(t,\mathbf{r}) - \langle h(t,\mathbf{r})\rangle]^2 \right\rangle^{1/2} \end{array} \right.$$

$h \rightarrow 0$ as $|\mathbf{r}| \rightarrow \infty$

specialise to $d = 1$ space dimensions ; boundary at $x = 0$, bulk $x \to \infty$

cross-over for the phenomenological growth exponent β near to boundary



bulk behaviour $w \sim t^{\beta}$

'surface behaviour' $w_1 \sim t^{\beta_1}$?

cross-over, if causal interaction with boundary

experimentally observed, e.g. for semiconductor films

Nascimento, Ferreira, Ferreira 11

EW-class Allegra, Fortin, mh 13

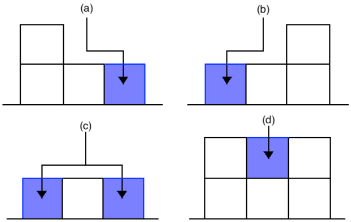
values of growth exponents (bulk & surface) :

$\beta = 0.25 \quad \beta_{1,\text{eff}} \simeq 0.32$ Edwards-Wilkinson class

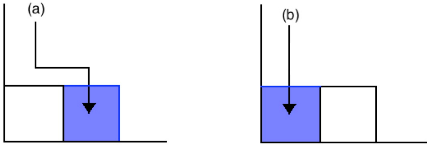
$\beta \simeq 0.32 \quad \beta_{1,\text{eff}} \simeq 0.35$ Kardar-Parisi-Zhang class

simulations of RSOS models :

well-known bulk adsorption processes (& immediate relaxation)



description of immediate relaxation if particle is adsorbed at the boundary

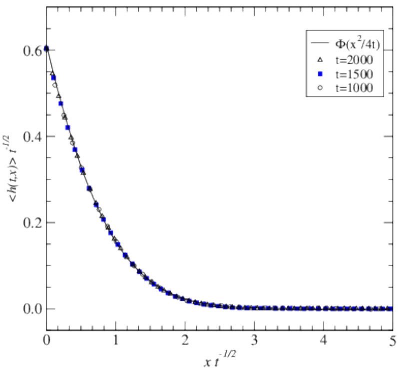


explicit boundary interactions in Langevin equation $h_1(t) = \partial_x h(t,x)|_{x=0}$

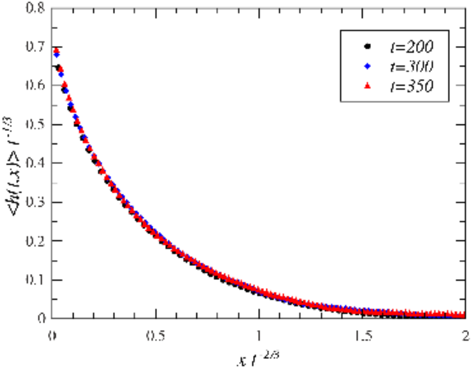
$$\left(\partial_t - \nu\partial_x^2\right) h(t,x) - \frac{\mu}{2}\left(\partial_x h(t,x)\right)^2 - \eta(t,x) = \nu\left(\kappa_1 + \kappa_2 h_1(t)\right)\delta(x)$$

height profile $\langle h(t,x)\rangle = t^{1/\gamma}\Phi\left(xt^{-1/z}\right)$, $\gamma = \frac{z}{z-1} = \frac{\zeta}{\zeta-\beta}$

EW & exact solution, $h(t,0) \sim \sqrt{t}$ self-consistently



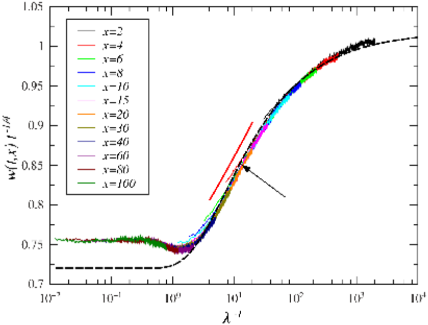
KPZ

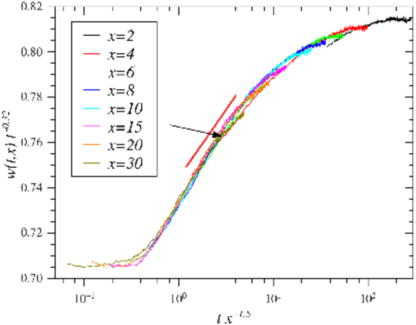


Scaling of the width profile :

EW & exact solution $\lambda^{-1} = 4tx^{-2}$ KPZ

bulk boundary





same growth scaling exponents in the bulk and near to the boundary
large **intermediate scaling regime** with effective exponent (**slopes**)

agreement with RG for non-disordered, local interactions — Lopéz, Castro, Gallego 05

$\boxed{\text{? ageing behaviour near to a boundary ?}}$

4. Interface growth & Arcetri model

? KPZ $\longrightarrow$ $\boxed{\text{intermediate model}}$ $\longrightarrow$ EW **?**

preferentially exactly solvable, and this in $d \geq 1$ dimensions

inspiration : **spherical model** of a ferromagnet

Berlin & Kac 52
Lewis & Wannier 52

Ising spins $s_i = \pm 1$ obey $\sum_i s_i^2 = \mathcal{N} = \#$ sites

spherical spins $s_i \in \mathbb{R}$ spherical constraint $\left\langle \sum_i s_i^2 \right\rangle = \mathcal{N}$

hamiltonian $\mathcal{H} = -J \sum_{(i,j)} s_i s_j - \lambda \sum_i s_i^2$ Lagrange multiplier λ

$$\begin{cases} \text{gives critical point } T_c > 0 \text{ for } d > 2 \\ \text{exponents } \textbf{non}\text{-mean-field for } 2 < d < 4 \end{cases}$$

kinetic spherical model : write Langevin equation

$$\partial_t \phi = -D \frac{\delta \mathcal{H}[\phi]}{\delta \phi} + \mathfrak{z}(t)\phi + \eta$$

η is the standard white noise : $\langle \eta(t, \mathbf{r}) \rangle = 0$,

$\langle \eta(t, \mathbf{r}) \eta(t, \mathbf{r}) \rangle = 2DT \delta(t - t') \delta(\mathbf{r} - \mathbf{r}')$

with Lagrange multiplier $\mathfrak{z}(t)$, fixed by spherical constraint

auxiliary function $g(t) = \exp\left(-2 \int_0^t \mathrm{d}\tau\, \mathfrak{z}(\tau)\right)$, satisfies Volterra equation

$$g(t) = f(t) + 2T \int_0^t \mathrm{d}\tau\, g(\tau) f(t - \tau) \;\; , \;\; f(t) := \left(e^{-4t} I_0(4t)\right)^d$$

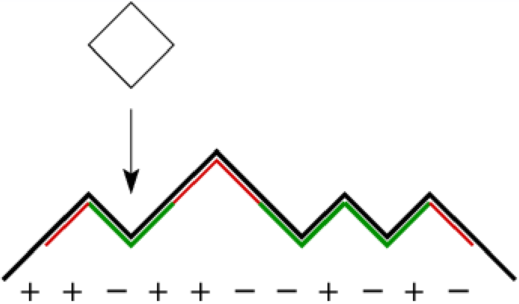
* all equilibrium and ageing exponents exactly known,

for both $T < T_c$ and $T = T_c$ Godrèche & Luck '00

* for $d = 3$: same universality class as 'spherical spin glass'

Cugliandolo & Dean '95

consider RSOS-adsorption process :



use not the heights $h_n(t) \in \mathbb{N}$ on a discrete lattice,

but rather the slopes $u_n(t) = \frac{1}{2}\left(h_{n+1}(t) - h_{n-1}(t)\right)$

? can one let $u_n(t) \in \mathbb{R}$, but impose a spherical constraint ?

? consequences of the 'hardening' of a soft EW-interface by a 'spherical constraint' on the u_n ?

since $u(t,x) = \partial_x h(t,x)$: go from KPZ to Burgers' equation, and

replace its non-linearity by a mean spherical condition

$$
\begin{aligned}
\partial_t u_n(t) &= \nu\left(u_{n+1}(t) + u_{n-1}(t) - 2u_n(t)\right) + \mathfrak{z}(t) u_n(t) \\
&\quad + \frac{1}{2}\left(\eta_{n+1}(t) - \eta_{n-1}(t)\right) \\
\sum_n \left\langle u_n(t)^2 \right\rangle &= N
\end{aligned}
$$

Extension to $d \geq 1$ dimensions :

define gradient fields $u_a(t,\mathbf{r}) := \nabla_a h(t,\mathbf{r})$, $a = 1,\ldots,d$:

$$
\begin{aligned}
\partial_t u_a(t,\mathbf{r}) &= \nu \nabla_{\mathbf{r}} \cdot \nabla_{\mathbf{r}} u_a(t,\mathbf{r}) + \mathfrak{z}(t) u_a(t,\mathbf{r}) + \nabla_a \eta(t,\mathbf{r}) \\
\sum_{a=1}^{d} \left\langle u_a(t,\mathbf{r})^2 \right\rangle &= N^d
\end{aligned}
$$

interface height : $\boxed{\widehat{u}_a(t,\mathbf{p}) = \mathrm{i} \sin p_a \, \widehat{h}(t,\mathbf{p})}$ in Fourier space exact solution :

$$\widehat{h}(t,\mathbf{p}) = \widehat{h}(0,\mathbf{p})e^{-2t\omega(\mathbf{p})}g(t)^{-1/2} + \int_0^t \mathrm{d}\tau\; \widehat{\eta}(\tau,\mathbf{p})\sqrt{\frac{g(\tau)}{g(t)}}\, e^{-2(t-\tau)\omega(\mathbf{p})}$$

in terms of the auxiliary function $g(t) = \exp\left(-2\int_0^t \mathrm{d}\tau\, \mathfrak{z}(\tau)\right)$,
satisfies Volterra equation

$$g(t) = f(t) + 2T\int_0^t \mathrm{d}\tau g(\tau) f(t-\tau) \;\;,\;\; f(t) := d\,\frac{e^{-4t} I_1(4t)}{4t}\left(e^{-4t} I_0(4t)\right)^{d-1}$$

* for $d=1$, identical to 'spherical spin glass', with $T = 2T_{\mathrm{SG}}$:
  hamiltonian $\mathcal{H} = -\frac{1}{2}\sum_{i,j} J_{ij} s_i s_j$; J_{ij} random matrix, its eigenvalues distributed
  according to Wigner's semi-circle law — Cugliandolo & Dean 95
* correspondence spherical spins $s_i \leftrightarrow$ slopes u_n.
* kinetics of heights $h_n(t)$ is driven by phase-ordering of the spherical spin glass = $3D$ kinetic spherical model

phase transition : long-range correlated surface growth for $T \leq T_c$

$$\frac{1}{T_c(d)} = \frac{d}{2}\int_0^\infty \mathrm{d}t\; e^{-dt} t^{-1} I_1(t) I_0(t)^{d-1} \quad ; \quad T_c(1) = 2,\; T_c(2) = \frac{\pi}{\pi - 2}$$

Some results :

upper critical dimension $d^* = 2$

1. $T = T_c$, $d < 2$: sub-diffusive interface motion $\langle h(t)\rangle \sim t^{(2-d)/4}$

   interface width $w(t) = t^{(2-d)/4} \Longrightarrow \beta = \frac{2-d}{4}$

   ageing exponents $a = b = \frac{d}{2} - 1$, $\lambda_R = \lambda_C = \frac{3d}{2} - 1$, $z = 2$

2. $T = T_c$, $d > 2$:

   interface width $w(t) = \mathrm{cste.} \Longrightarrow \beta = 0$

   ageing exponents $a = b = \frac{d}{2} - 1$, $\lambda_R = \lambda_C = d$, $z = 2$

3. $T < T_c$, $d < 2$:

   sub-diffusive interface motion $\langle h(t)\rangle \sim (1 - T/T_c) t^{(d+2)/4}$

   interface width $w(t) = (1 - T/T_c) t \Longrightarrow \beta = \frac{1}{2}$

   ageing exponents $a = b = \frac{d}{2} - 1$, $\lambda_R = \lambda_C = \frac{d-2}{2}$, $z = 2$

5. Conclusions

- physical ageing occurs naturally in many **irreversible** systems relaxing towards **non**-equilibrium stationary states
  considered here : absorbing phase transitions & surface growth
- scaling phenomenology analogous to simple magnets
- **but** finer differences in relationships between non-equilibrium exponents
- **surprises** in scaling near a boundary : height/width profiles
- the **Arcetri model** captures at least some qualitative properites of KPZ :
  - sub-diffusive motion of the interface
  - interface becomes more smooth as $d \to d^* = 2$
  - at $T = T_c$, the stationary exponents (β, z) are those of EW, but the ageing exponents are different
  - new kind of behaviour at $T < T_c$

studies of the ageing properties, via two-time observables, might become a **new tool**, also for the analysis of complex systems !